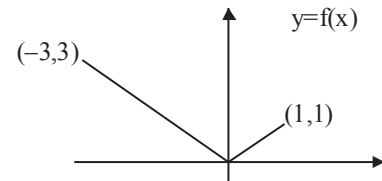
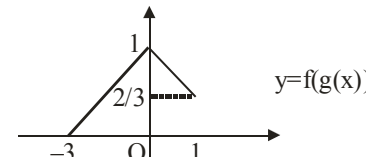


**JEE Main 2024**  
**(Shift - 01 Mathematics Paper)**

**29.01.2024**

MATHEMATICS	TEST PAPER WITH SOLUTION
<p align="center"><b>SECTION-A</b></p> <p><b>1.</b> If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to</p> <p>(1) 7                                  (2) 4 (3) 5                                  (4) 6</p> <p><b>Ans. (4)</b></p> <p><b>Sol.</b> <math>a + ar + ar^2 + ar^3 + \dots + ar^{63}</math>  <math>= 7(a + ar^2 + ar^4 + \dots + ar^{62})</math>  <math>\Rightarrow \frac{a(1-r^{64})}{1-r} = \frac{7a(1-r^{64})}{1-r^2}</math>  <math>r = 6</math></p> <p><b>2.</b> In an A.P., the sixth terms <math>a_6 = 2</math>. If the <math>a_1, a_4, a_5</math> is the greatest, then the common difference of the A.P., is equal to</p> <p>(1) <math>\frac{3}{2}</math>                  (2) <math>\frac{8}{5}</math>                  (3) <math>\frac{2}{3}</math>                  (4) <math>\frac{5}{8}</math></p> <p><b>Ans. (2)</b></p> <p><b>Sol.</b> <math>a_6 = 2 \Rightarrow a + 5d = 2</math>  <math>a_1 a_4 a_5 = a(a + 3d)(a + 4d)</math>  <math>= (2 - 5d)(2 - 2d)(2 - d)</math>  <math>f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3</math>  <math>f'(d) = -2(5d - 8)(3d - 2)</math></p> <p align="center"> <math>\overline{\begin{matrix} - &amp; + &amp; - \\ &amp; 2/3 &amp; 8/5 \end{matrix}}</math> </p> <p><math>d = \frac{8}{5}</math></p>	<p><b>3.</b> If <math>f(x) = \begin{cases} 2+2x, &amp; -1 \leq x &lt; 0 \\ 1-\frac{x}{3}, &amp; 0 \leq x \leq 3 \end{cases}</math> ; <math>g(x) = \begin{cases} -x, &amp; -3 \leq x \leq 0 \\ x, &amp; 0 &lt; x \leq 1 \end{cases}</math>, then range of (f○g(x)) is</p> <p>(1) (0, 1]                                  (2) [0, 3) (3) [0, 1]                                  (4) [0, 1]</p> <p><b>Ans. (3)</b></p> <p><b>Sol.</b> <math>f(g(x)) = \begin{cases} 2+2g(x), &amp; -1 \leq g(x) &lt; 0 \dots\dots(1) \\ 1-\frac{g(x)}{3}, &amp; 0 \leq g(x) \leq 3 \dots\dots(2) \end{cases}</math></p> <p>By (1) <math>x \in \phi</math>      And by (2) <math>x \in [-3, 0]</math> and <math>x \in [0, 1]</math></p> <p align="center">  </p> <p align="center">  </p> <p>Range of <math>f(g(x))</math> is <math>[0, 1]</math></p> <p><b>4.</b> A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is</p> <p>(1) <math>\frac{5}{6}</math>                                  (2) <math>\frac{1}{6}</math>                                  (3) <math>\frac{5}{11}</math>                                  (4) <math>\frac{6}{11}</math></p> <p><b>Ans. (3)</b></p> <p><b>Sol.</b> Required probability =</p> <p align="center"> <math>\frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \dots</math> </p> <p align="center"> <math>= \frac{1}{6} \times \frac{5}{1 - \frac{25}{36}} = \frac{5}{11}</math> </p>

5. If  $z = \frac{1}{2} - 2i$ , is such that  $|z+1| = \alpha z + \beta(1+i)$ ,  $i = \sqrt{-1}$  and  $\alpha, \beta \in \mathbb{R}$ , then  $\alpha + \beta$  is equal to

- (1) -4 (2) 3  
(3) 2 (4) -1

Ans. (2)

Sol.  $z = \frac{1}{2} - 2i$

$$|z+1| = \alpha z + \beta(1+i)$$

$$\left| \frac{3}{2} - 2i \right| = \frac{\alpha}{2} - 2\alpha i + \beta + \beta i$$

$$\left| \frac{3}{2} - 2i \right| = \left( \frac{\alpha}{2} + \beta \right) + (\beta - 2\alpha) i$$

$$\beta = 2\alpha \text{ and } \frac{\alpha}{2} + \beta = \sqrt{\frac{9}{4} + 4}$$

$$\alpha + \beta = 3$$

6.  $\lim_{x \rightarrow \frac{\pi}{2}^-} \left( \frac{1}{\left(x - \frac{\pi}{2}\right)^2} \int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos\left(\frac{1}{t^3}\right) dt \right)$  is equal to

- (1)  $\frac{3\pi}{8}$  (2)  $\frac{3\pi^2}{4}$   
(3)  $\frac{3\pi^2}{8}$  (4)  $\frac{3\pi}{4}$

Ans. (3)

Sol. Using L'hospital rule

$$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{0 - \cos x \times 3x^2}{2\left(x - \frac{\pi}{2}\right)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sin\left(x - \frac{\pi}{2}\right)}{2\left(x - \frac{\pi}{2}\right)} \times \frac{3\pi^2}{4}$$

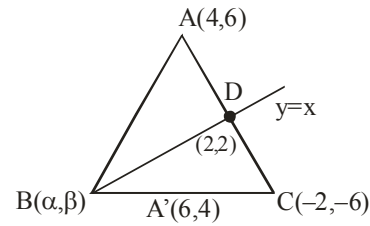
$$= \frac{3\pi^2}{8}$$

7. In a  $\Delta ABC$ , suppose  $y = x$  is the equation of the bisector of the angle B and the equation of the side AC is  $2x - y = 2$ . If  $2AB = BC$  and the point A and B are respectively  $(4, 6)$  and  $(\alpha, \beta)$ , then  $\alpha + 2\beta$  is equal to

- (1) 42 (2) 39  
(3) 48 (4) 45

Ans. (1)

Sol.



$$AD : DC = 1 : 2$$

$$\frac{4 - \alpha}{6 - \alpha} = \frac{10}{8}$$

$$\alpha = \beta$$

$$\alpha = 14 \text{ and } \beta = 14$$

8. Let  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  be three non-zero vectors such that  $\vec{b}$  and  $\vec{c}$  are non-collinear. If  $\vec{a} + 5\vec{b}$  is collinear with  $\vec{c}$ ,  $\vec{b} + 6\vec{c}$  is collinear with  $\vec{a}$  and  $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$ , then  $\alpha + \beta$  is equal to

- (1) 35 (2) 30  
(3) -30 (4) -25

Ans. (1)

Sol.  $\vec{a} + 5\vec{b} = \lambda\vec{c}$

$$\vec{b} + 6\vec{c} = \mu\vec{a}$$

Eliminating  $\vec{a}$

$$\lambda\vec{c} - 5\vec{b} = \frac{6}{\mu}\vec{c} + \frac{1}{\mu}\vec{b}$$

$$\therefore \mu = \frac{-1}{5}, \lambda = -30$$

$$\alpha = 5, \beta = 30$$

9. Let  $\left(5, \frac{a}{4}\right)$ , be the circumcenter of a triangle with vertices  $A(a, -2)$ ,  $B(a, 6)$  and  $C\left(\frac{a}{4}, -2\right)$ . Let  $\alpha$  denote the circumradius,  $\beta$  denote the area and  $\gamma$  denote the perimeter of the triangle. Then  $\alpha + \beta + \gamma$  is
- (1) 60 (2) 53  
(3) 62 (4) 30

Ans. (2)

Sol.  $A(a, -2)$ ,  $B(a, 6)$ ,  $C\left(\frac{a}{4}, -2\right)$ ,  $O\left(5, \frac{a}{4}\right)$

$AO = BO$

$$(a-5)^2 + \left(\frac{a}{4} + 2\right)^2 = (a-5)^2 + \left(\frac{a}{4} - 6\right)^2$$

$a = 8$

$AB = 8$ ,  $AC = 6$ ,  $BC = 10$

$\alpha = 5$ ,  $\beta = 24$ ,  $\gamma = 24$

10. For  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , if

$$y(x) = \int \frac{\operatorname{cosec} x + \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx \text{ and}$$

$\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^-} y(x) = 0$  then  $y\left(\frac{\pi}{4}\right)$  is equal to

- (1)  $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$  (2)  $\frac{1}{2} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$   
(3)  $-\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$  (4)  $\frac{1}{\sqrt{2}} \tan^{-1}\left(-\frac{1}{2}\right)$

Ans. (4)

Sol.  $y(x) = \int \frac{(1 + \sin^2 x) \cos x}{1 + \sin^4 x} dx$

Put  $\sin x = t$

$$= \int \frac{1+t^2}{t^4+1} dt = \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{t-\frac{1}{t}}{\sqrt{2}} \right) + C$$

$x = \frac{\pi}{2}$ ,  $t = 1$   $\therefore C = 0$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \tan^{-1} \left( -\frac{1}{2} \right)$$

11. If  $\alpha$ ,  $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$  is the solution of  $4\cos\theta + 5\sin\theta = 1$ , then the value of  $\tan\alpha$  is

- (1)  $\frac{10-\sqrt{10}}{6}$  (2)  $\frac{10-\sqrt{10}}{12}$   
(3)  $\frac{\sqrt{10}-10}{12}$  (4)  $\frac{\sqrt{10}-10}{6}$

Ans. (3)

Sol.  $4 + 5 \tan \theta = \sec \theta$

Squaring :  $24 \tan^2 \theta + 40 \tan \theta + 15 = 0$

$$\tan \theta = \frac{-10 \pm \sqrt{10}}{12}$$

and  $\tan \theta = -\left(\frac{10 + \sqrt{10}}{12}\right)$  is Rejected.

(3) is correct.

12. A function  $y = f(x)$  satisfies

$f(x) \sin 2x + \sin x - (1 + \cos^2 x) f'(x) = 0$  with condition

$f(0) = 0$ . Then  $f\left(\frac{\pi}{2}\right)$  is equal to

- (1) 1 (2) 0 (3) -1 (4) 2

Ans. (1)

Sol.  $\frac{dy}{dx} - \left(\frac{\sin 2x}{1 + \cos^2 x}\right) y = \sin x$

I.F. =  $1 + \cos^2 x$

$$y \cdot (1 + \cos^2 x) = \int (\sin x) dx$$

=  $-\cos x + C$

$x = 0$ ,  $C = 1$

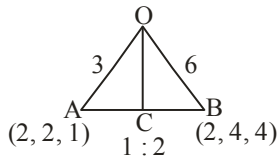
$$y\left(\frac{\pi}{2}\right) = 1$$

13. Let O be the origin and the position vector of A and B be  $2\hat{i} + 2\hat{j} + \hat{k}$  and  $2\hat{i} + 4\hat{j} + 4\hat{k}$  respectively. If the internal bisector of  $\angle AOB$  meets the line AB at C, then the length of OC is

- (1)  $\frac{2}{3} \sqrt{31}$  (2)  $\frac{2}{3} \sqrt{34}$   
(3)  $\frac{3}{4} \sqrt{34}$  (4)  $\frac{3}{2} \sqrt{31}$

Ans. (2)

Sol.



$$\text{length of } OC = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$$

14. Consider the function  $f: \left[\frac{1}{2}, 1\right] \rightarrow \mathbb{R}$  defined by

$$f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1. \text{ Consider the statements}$$

(I) The curve  $y = f(x)$  intersects the x-axis exactly at one point

(II) The curve  $y = f(x)$  intersects the x-axis at

$$x = \cos \frac{\pi}{12}$$

Then

- (1) Only (II) is correct
- (2) Both (I) and (II) are incorrect
- (3) Only (I) is correct
- (4) Both (I) and (II) are correct

Ans. (4)

Sol.  $f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \geq 0$  for  $\left[\frac{1}{2}, 1\right]$

$$f\left(\frac{1}{2}\right) < 0$$

$$f(1) > 0 \Rightarrow \text{(A) is correct.}$$

$$f(x) = \sqrt{2}(4x^3 - 3x) - 1 = 0$$

$$\text{Let } \cos \alpha = x,$$

$$\cos 3\alpha = \cos \frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$$

$$x = \cos \frac{\pi}{12}$$

(4) is correct.

15. Let  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$  and  $|2A|^3 = 2^{21}$  where  $\alpha, \beta \in \mathbb{Z}$ ,

Then a value of  $\alpha$  is

- (1) 3
- (2) 5
- (3) 17
- (4) 9

Ans. (2)

Sol.  $|A| = \alpha^2 - \beta^2$

$$|2A|^3 = 2^{21} \Rightarrow |A| = 2^4$$

$$\alpha^2 - \beta^2 = 16$$

$$(\alpha + \beta)(\alpha - \beta) = 16 \Rightarrow \alpha = 4 \text{ or } 5$$

16. Let PQR be a triangle with  $R(-1, 4, 2)$ . Suppose  $M(2, 1, 2)$  is the mid point of PQ. The distance of the centroid of  $\Delta PQR$  from the point of intersection of the line

$$\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1} \text{ and } \frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1} \text{ is}$$

- (1) 69
- (2) 9
- (3)  $\sqrt{69}$
- (4)  $\sqrt{99}$

Ans. (3)

Sol. Centroid G divides MR in 1 : 2

$$G(1, 2, 2)$$

Point of intersection A of given lines is  $(2, -6, 0)$

$$AG = \sqrt{69}$$

17. Let R be a relation on  $Z \times Z$  defined by

$(a, b)R(c, d)$  if and only if  $ad - bc$  is divisible by 5.

Then R is

- (1) Reflexive and symmetric but not transitive
- (2) Reflexive but neither symmetric nor transitive
- (3) Reflexive, symmetric and transitive
- (4) Reflexive and transitive but not symmetric

Ans. (1)

**Sol.** (a, b)R(a, b) as  $ab - ab = 0$

Therefore reflexive

Let (a,b)R(c,d)  $\Rightarrow ad - bc$  is divisible by 5

$\Rightarrow bc - ad$  is divisible by 5  $\Rightarrow (c,d)R(a,b)$

Therefore symmetric

Relation not transitive as (3,1)R(10,5) and (10,5)R(1,1) but (3,1) is not related to (1,1)

**18.** If the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left( \frac{x^2 \cos x}{1 + \pi^x} + \frac{1 + \sin^2 x}{1 + e^{\sin x^{2023}}} \right) dx = \frac{\pi}{4}(\pi + a) - 2,$$

then the value of a is

- (1) 3      (2)  $-\frac{3}{2}$       (3) 2      (4)  $\frac{3}{2}$

**Ans. (1)**

**Sol.**  $I = \int_{-\pi/2}^{\pi/2} \left( \frac{x^2 \cos x}{1 + \pi^x} + \frac{1 + \sin^2 x}{1 + e^{\sin x^{2023}}} \right) dx$

$$I = \int_{-\pi/2}^{\pi/2} \left( \frac{x^2 \cos x}{1 + \pi^{-x}} + \frac{1 + \sin^2 x}{1 + e^{\sin(-x)^{2023}}} \right) dx$$

On Adding, we get

$$2I = \int_{-\pi/2}^{\pi/2} (x^2 \cos x + 1 + \sin^2 x) dx$$

On solving

$$I = \frac{\pi^2}{4} + \frac{3\pi}{4} - 2$$

a = 3

**19.** Suppose

$$f(x) = \frac{(2^x + 2^{-x}) \tan x \sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3},$$

Then the value of  $f'(0)$  is equal to

- (1)  $\pi$       (2) 0  
 (3)  $\sqrt{\pi}$       (4)  $\frac{\pi}{2}$

**Ans. (3)**

**Sol.**  $f'(0) = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$

$$= \lim_{h \rightarrow 0} \frac{(2^h + 2^{-h}) \tan h \sqrt{\tan^{-1}(h^2 - h + 1)} - 0}{(7h^2 + 3h + 1)^3 h}$$

$$= \sqrt{\pi}$$

**20.** Let A be a square matrix such that  $AA^T = I$ . Then

$\frac{1}{2}A[(A+A^T)^2 + (A-A^T)^2]$  is equal to

- (1)  $A^2 + I$       (2)  $A^3 + I$   
 (3)  $A^2 + A^T$       (4)  $A^3 + A^T$

**Ans. (4)**

**Sol.**  $AA^T = I = A^T A$

On solving given expression, we get

$$\frac{1}{2}A[A^2 + (A^T)^2 + 2AA^T + A^2 + (A^T)^2 - 2AA^T]$$

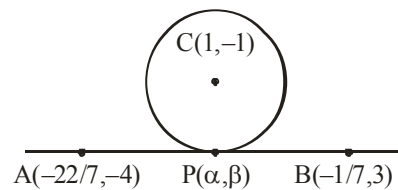
$$= A[A^2 + (A^T)^2] = A^3 + A^T$$

**SECTION-B**

**21.** Equation of two diameters of a circle are  $2x - 3y = 5$  and  $3x - 4y = 7$ . The line joining the points  $(-\frac{22}{7}, -4)$  and  $(-\frac{1}{7}, 3)$  intersects the circle at only one point P( $\alpha, \beta$ ). Then  $17\beta - \alpha$  is equal to

**Ans. (2)**

**Sol.** Centre of circle is (1, -1)



Equation of AB is  $7x - 3y + 10 = 0$  ... (i)

Equation of CP is  $3x + 7y + 4 = 0$  ... (ii)

Solving (i) and (ii)

$$\alpha = \frac{-41}{29}, \beta = \frac{1}{29} \quad \therefore 17\beta - \alpha = 2$$

22. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

**Ans. (553)**

**Sol.** Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

GTWENTY = 1

Total = 553

23. Let  $\alpha, \beta$  be the roots of the equation  $x^2 - x + 2 = 0$  with  $\text{Im}(\alpha) > \text{Im}(\beta)$ . Then  $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$  is equal to

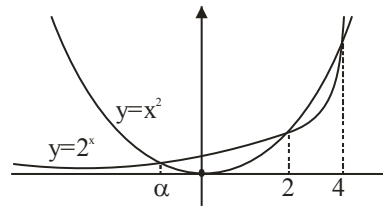
**Ans. (13)**

**Sol.**  $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$   
 $= \alpha^4(\alpha - 2) + \alpha^4 - 5\alpha^2 + (\beta - 2)^2$   
 $= \alpha^5 - \alpha^4 - 5\alpha^2 + \beta^2 - 4\beta + 4$   
 $= \alpha^3(\alpha - 2) - \alpha^4 - 5\alpha^2 + \beta - 2 - 4\beta + 4$   
 $= -2\alpha^3 - 5\alpha^2 - 3\beta + 2$   
 $= -2\alpha(\alpha - 2) - 5\alpha^2 - 3\beta + 2$   
 $= -7\alpha^2 + 4\alpha - 3\beta + 2$   
 $= -7(\alpha - 2) + 4\alpha - 3\beta + 2$   
 $= -3\alpha - 3\beta + 16 = -3(1) + 16 = 13$

24. Let  $f(x) = 2^x - x^2, x \in \mathbb{R}$ . If  $m$  and  $n$  are respectively the number of points at which the curves  $y = f(x)$  and  $y = f'(x)$  intersects the x-axis, then the value of  $m + n$  is

**Ans. (5)**

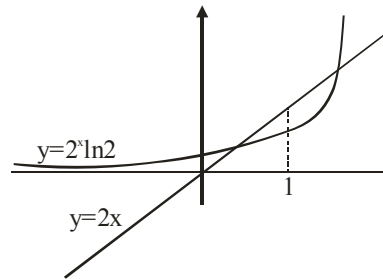
**Sol.**



$\therefore m = 3$

$f'(x) = 2^x \ln 2 - 2x = 0$

$2^x \ln 2 = 2x$



$\therefore n = 2$

$\Rightarrow m + n = 5$

25. If the points of intersection of two distinct conics

$x^2 + y^2 = 4b$  and  $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$  lie on the curve

$y^2 = 3x^2$ , then  $3\sqrt{3}$  times the area of the rectangle formed by the intersection points is \_\_\_

**Ans. (432)**

**Sol.** Putting  $y^2 = 3x^2$  in both the conics

We get  $x^2 = b$  and  $\frac{b}{16} + \frac{3}{b} = 1$

$\Rightarrow b = 4, 12$  ( $b = 4$  is rejected because curves coincide)

$\therefore b = 12$

Hence points of intersection are

$(\pm\sqrt{12}, \pm 6) \Rightarrow \text{area of rectangle} = 432$

26. If the solution curve  $y=y(x)$  of the differential equation  $(1+y^2)(1+\log_e x)dx + x dy = 0$ ,  $x > 0$  passes through the point  $(1, 1)$  and

$$y(e) = \frac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}, \text{ then } \alpha + 2\beta \text{ is}$$

Ans. (3)

Sol.  $\int \left( \frac{1}{x} + \frac{\ln x}{x} \right) dx + \int \frac{dy}{1+y^2} = 0$

$$\ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = C$$

Put  $x = y = 1$

$$\therefore C = \frac{\pi}{4}$$

$$\Rightarrow \ln x + \frac{(\ln x)^2}{2} + \tan^{-1} y = \frac{\pi}{4}$$

Put  $x = e$

$$\Rightarrow y = \tan\left(\frac{\pi}{4} - \frac{3}{2}\right) = \frac{1 - \tan\frac{3}{2}}{1 + \tan\frac{3}{2}}$$

$$\therefore \alpha = 1, \beta = 1$$

$$\Rightarrow \alpha + 2\beta = 3$$

27. If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60,  $\alpha, \beta, 60$  where  $\alpha > \beta$  are 56 and 66.2 respectively, then  $\alpha^2 + \beta^2$  is equal to

Ans. (6344)

Sol.  $\bar{x} = 56$

$$\sigma^2 = 66.2$$

$$\Rightarrow \frac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$$

$$\therefore \alpha^2 + \beta^2 = 6344$$

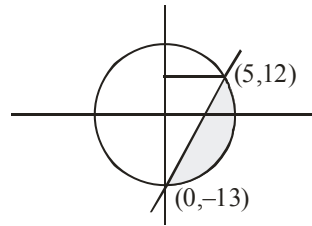
28. The area (in sq. units) of the part of circle  $x^2 + y^2 = 169$  which is below the line  $5x - y = 13$  is

$$\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta} \sin^{-1}\left(\frac{12}{13}\right) \text{ where } \alpha, \beta \text{ are coprime}$$

numbers. Then  $\alpha + \beta$  is equal to

Ans. (171)

Sol.



$$\text{Area} = \int_{-13}^{12} \sqrt{169 - y^2} dy - \frac{1}{2} \times 25 \times 5$$

$$= \frac{\pi}{2} \times \frac{169}{2} - \frac{65}{2} + \frac{169}{2} \sin^{-1} \frac{12}{13}$$

$$\therefore \alpha + \beta = 171$$

29. If  $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \dots + \frac{{}^{11}C_9}{10} = \frac{n}{m}$  with  $\gcd(n, m) = 1$ ,

then  $n + m$  is equal to

Ans. (2041)

Sol.  $\sum_{r=1}^9 \frac{{}^{11}C_r}{r+1}$

$$= \frac{1}{12} \sum_{r=1}^9 {}^{12}C_{r+1}$$

$$= \frac{1}{12} [2^{12} - 26] = \frac{2035}{6}$$

$$\therefore m + n = 2041$$

30. A line with direction ratios 2, 1, 2 meets the lines  $x = y + 2 = z$  and  $x + 2 = 2y = 2z$  respectively at the point P and Q. If the length of the perpendicular from the point (1, 2, 12) to the line PQ is  $l$ , then  $l^2$  is

Ans. (65)

Sol. Let  $P(t, t - 2, t)$  and  $Q(2s - 2, s, s)$

D.R's of PQ are 2, 1, 2

$$\frac{2s - 2 - t}{2} = \frac{s - t + 2}{1} = \frac{s - t}{2}$$

$$\Rightarrow t = 6 \text{ and } s = 2$$

$$\Rightarrow P(6, 4, 6) \text{ and } Q(2, 2, 2)$$

$$PQ: \frac{x - 2}{2} = \frac{y - 2}{1} = \frac{z - 2}{2} = \lambda$$

Let  $F(2\lambda + 2, \lambda + 2, 2\lambda + 2)$

$A(1, 2, 12)$

$$\vec{AF} \cdot \vec{PQ} = 0$$

$$\therefore \lambda = 2$$

So  $F(6, 4, 6)$  and  $AF = \sqrt{65}$

