$$\frac{1}{3}\ell n\left|\frac{y-3}{y}\right|=\frac{1}{4}\ell n\left|\frac{x-2}{x+2}\right|+\frac{1}{4}\ell n 3$$

$\Rightarrow x = 10$

$$\frac{1}{3}\ell n\left|\frac{y-3}{y}\right|=\frac{1}{4}\ell n\left|\frac{2}{3}\right|+\frac{1}{4}\ell n 3$$

$$\frac{1}{3}\ell n\left|\frac{y-3}{y}\right|=\frac{1}{4}\ell n 2$$

$$\ell n\left|\frac{y-3}{y}\right|=\ell n 2^{\frac{3}{4}}$$

$$\left|\frac{y-3}{y}\right|=2^{\frac{3}{4}}$$

$$-y+3=8^{\frac{1}{4}}y$$

$$y=\frac{3}{1+8^{\frac{1}{4}}}$$

**22.** Three lines $2x - y - 3 = 0$, $6x + 3y + 4 = 0$, $\alpha x + 2y + 4 = 0$ does not form triangle then find $[\Sigma\alpha^2]$ (where [.] denotes the greatest integer function)

**Ans.** **(32)**

**Sol.** If two lines are parallel

$$\frac{2}{\alpha}=\frac{-1}{2}\Rightarrow \alpha=-4$$

$$\frac{6}{\alpha}=\frac{3}{2}\Rightarrow \alpha=4$$

If lines are concurrent

$$\begin{vmatrix}2 & -1 & -3\\ 6 & 3 & 4\\ \alpha & 2 & 4\end{vmatrix}=0$$

$2(12-8)+1(24-4\alpha)-3(12-3\alpha)=0$

$8+24-4\alpha-36+9\alpha=0$

$5\alpha=4\Rightarrow \alpha=\frac{4}{5}$

$\Sigma\alpha^2=16+16+\frac{16}{25}$

$[\Sigma\alpha^2]=32$

**23.** Let $f(x) = \int_0^x g(t)\ \log\left(\frac{1-t}{1+t}\right)$ dt, (where g(x) is cont. odd function).

If $\int_{-\frac{\pi}{2}}^{\pi/2}\left(f(x)+\frac{x^2\cos x}{1+e^x}\right)dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then find $\alpha$

**Ans.** $\alpha = 2$

**Sol.** $I = \int_0^{\pi/2}\left(f(x)+f(-x)+\frac{x^2\cos x}{1+e^x}+\frac{x^2\cos x}{1+e^{-x}}\right)dx = \int_0^{\pi/2}(f(x) + f(-x) + x^2\cos x)\,dx$ ....(i)

Now $f(-x) = \int_0^{-x} g(t)\ \log\left(\frac{1-t}{1+t}\right)$ dt

$t = -p$

$= \int_0^x -g(-p)\log\left(\frac{1+p}{1-p}\right)dp = -f(x)$

$\therefore$ (i) becomes $I = \int_0^{\pi/2} x^2\cos x\,dx = x^2\sin x - 2\int x\ \sin x\,dx = (x^2\sin x - 2)\,(-x\cos x + \sin x)\Big|_0^{\pi/2}$

$= \frac{\pi^2}{4} - 2 \Rightarrow \frac{\pi^2}{4} - 2$