SOLVED PAPER – 2010 (VITEE)

PART - I (PHYSICS)

1. A straight wire carrying current i is turned into a circular loop. If the magnitude of magnetic moment associated with it in MKS unit is M, the length of wire will be

(a) $\frac{4\pi}{M}$ (b) $\sqrt{\frac{4\pi M}{i}}$

(c) $\sqrt{\frac{4\pi i}{M}}$ (d) $\frac{M\pi}{i}$

2. The ratio of the amounts of heat developed in the four arms of a balance Wheatstone bridge, when the arms have resistances $P=100\,\Omega$, $Q=10\,\Omega$, $R=300\,\Omega$ and $S=30\,\Omega$ respectively is

(a) 3 : 30 : 1 : 10 (b) 30 : 3 : 10 : 1

(c) 30 : 10 : 1 : 3 (d) 30 : 1 : 3 : 10

3. An electric kettle takes 4 A current at 220V. How much time will it take to boil 1 kg of water from temperature 20 °C ? The temperature of boiling water is 100°C.

(a) 12.6 min (b) 4.2 min

(c) 6.3 min (d) 8.4 min

4. Magnetic field at the centre of a circular loop of area A is B. The magnetic moment of the loop will be

(a) $\frac{BA^2}{\mu_0\pi}$ (b) $\frac{BA^{3/2}}{\mu_0\pi}$

(c) $\frac{BA^{3/2}}{\mu_0\pi^{1/2}}$ (d) $\frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$

5. In Young's double slit experiment, the spacing between the slits is d and wavelength of light used is 6000 Å. If the angular width of a fringe formed on a distance screen is 1°, then value of d is

(a) 1 mm (b) 0.05 mm

(c) 0.03 mm (d) 0.01 mm

6. An electric dipole consists of two opposite charges of magnitude $q=1\times10^{-6}$ C separated by 2.0 cm. The dipole is placed in an external field of $1\times10^5\ NC^{-1}$. What maximum torque does the field exert on the dipole? How much work must an external agent do to turn the dipole end to end, starting from position of alignment $(\theta=0°)$?

(a) 4.4×10^6N-m, 3.2×10^{-4}J

(b) -2×10^{-3}N-m, -4×10^3J

(c) 4×10^3N-m, 2×10^{-3}J

(d) 2×10^{-3}N-m, 4×10^{-3}J

7. The electron of hydrogen atom is considered to be revolving round a proton in circular orbit of radius h^2/me^2 with velocity e^2/h, where $h=\hbar/2\pi$. The current i is

(a) $\frac{4\pi^2me^5}{\hbar^2}$ (b) $\frac{4\pi^2me^5}{\hbar^3}$

(c) $\frac{4\pi^2m^2e^2}{\hbar^3}$ (d) $\frac{4\pi^2m^2e^5}{\hbar^3}$

8. In a double slit experiment, 5^{th} dark fringe is formed opposite to one of the slits, the wavelength of light is

(a) $\frac{d^2}{6D}$ (b) $\frac{d^2}{5D}$

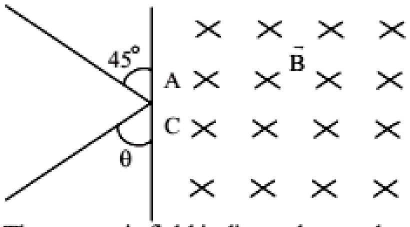
(c) $\frac{d^2}{15D}$ (d) $\frac{d^2}{9D}$

9. Which of the following rays is emitted by a human body?

(a) X-rays (b) UV rays

(c) Visible rays (d) IR rays

10. A proton of mass 1.67×10^{-27}kg enters a uniform magnetic field 1T of at point A shown in figure with a speed of 10^7 ms^{-1}.



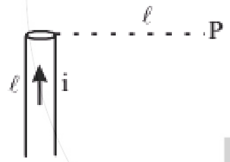
The magnetic field is directed normal to the plane of paper downwards. The proton emerges out of the magnetic field at point C, then the distance AC and the value of angle θ will respectively be

(a) 0.7 m, 45° (b) 0.7 m, 90°
(c) 0.14 m, 90° (d) 0.14 m, 45°

11. A neutral water molecule (H_2O) in its vapour state has an electric dipole moment of magnitude 6.4×10^{-30}C-m. How far apart are the molecules centres of positive and negative charges?

(a) 4m (b) 4mm
(c) 4μm (d) 4 pm

12. Figure shows a straight wire length l carrying current i. The magnitude of magnetic field produced by the current at point P is



(a) $\frac{\sqrt{2}\mu_0 i}{\pi l}$ (b) $\frac{\mu_0 i}{4\pi l}$
(c) $\frac{\sqrt{2}\mu_0 i}{8\pi l}$ (d) $\frac{\mu_0 i}{2\sqrt{2}\pi l}$

13. Zener diode is used for

(a) producing oscillations in an oscillator
(b) amplification
(c) stabilisation
(d) rectification

14. Two light sources are said to be coherent if they are obtained from

(a) two independent point sources emitting light of the same wavelength
(b) a single point source
(c) a wide source
(d) two ordinary bulbs emitting light of different wavelengths

15. A small coil is introduced between the poles of an electromagnet so that its axis coincides with the magnetic field direction. The number of turns is n and the cross-sectional area of the coil is A. When the coil turns through 180° about its diameter, the charge flowing through the coil is Q. The total resistance of the circuit is R. What is the magnitude of the magnetic induction?

(a) $\frac{QR}{nA}$ (b) $\frac{2QR}{nA}$
(c) $\frac{Qn}{2RA}$ (d) $\frac{QR}{2nA}$

16. The attenuation in optical fibre is mainly due to

(a) absorption (b) scattering
(c) neither absorption nor scattering
(d) Both (a) and (b)

17. An arc of radius r carries charge. The linear density of charge is λ and the arc subtends an angle $\frac{\pi}{3}$ at the centre. What is electric potential at the centre?

(a) $\frac{\lambda}{4\varepsilon_0}$ (b) $\frac{\lambda}{8\varepsilon_0}$
(c) $\frac{\lambda}{12\varepsilon_0}$ (d) $\frac{\lambda}{16\varepsilon_0}$

18. Sinusoidal carrier voltage of frequency 1.5 MHz and amplitude 50 V is amplitude modulated by sinusoidal voltage of frequency 10 kHz producing 50% modulation. The lower and upper side-band frequencies in kHz are

(a) 1490, 1510 (b) 1510, 1490
(c) $\frac{1}{1490}, \frac{1}{1510}$ (d) $\frac{1}{1510}, \frac{1}{1490}$

19. 50 Ω and 100 Ω resistors are connected in series. This connection is connected with a battery of 2.4 V. When a voltmeter of 100 Ω resistance is connected across 100 Ω resistor, then the reading of the voltmeter will be

(a) 1.6V (b) 1.0V
(c) 1.2V (d) 2.0V

20. In space charge limited region, the plate current in a diode is 10 mA for plate voltage 150V. If the plate voltagte is increased to 600V, then the plate current will be

(a) 10 mA (b) 40 mA
(c) 80 mA (d) 160 mA

21. Light of wavelength λ strikes a photo-sensitive surface and electrons are ejected with kinetic energy E. If the kinetic energy is to be increased to 2E, the wavelength must be changed to λ′ wher

(a) $\lambda' = \frac{\lambda}{2}$ (b) $\lambda' = 2\lambda$

(c) $\frac{\lambda}{2} < \lambda' < \lambda$ (d) $\lambda' > \lambda$

22. The maximum velocity of electrons emitted from a metal surface is v, when frequency of light falling on it is f. The maximum velocity when frequency becomes 4f is

(a) 2v (b) > 2v

(c) < 2v (d) between 2v and 4v

23. The collector plate in an experiment on photoelectric effect is kept vertically above the emitter plate. Light source is put on and a saturation photo-current is recorded. An electric field is switched on which has a vertically downward direction, then

(a) the photo-current will increase

(b) the kinetic energy of the electrons will increase

(c) the stopping potential will decrease

(d) the threshold wavelength will increase

24. A cylindrical conductor of radius R carries a current i. The value of magnetic field at a point which is $\frac{R}{4}$ distance inside from the surface is 10 T. The value of magnetic field at point which is 4R distance outside from the surface

(a) $\frac{4}{3}$T (b) $\frac{8}{3}$T

(c) $\frac{40}{3}$T (d) $\frac{80}{3}$T

25. The power of a thin convex lens ($_a n_g$ = 1.5) is 5.0 D. When it is placed in a liquid of refractive index $_a n_\ell$, then it behaves as a concave lens of focal length 100cm. The refractive index of the liquid $_a n_l$ will be

(a) 5/3 (b) 4/3

(c) $\sqrt{3}$ (d) 5/4

26. Find the value of magnetic field between plates of capacitor at a distance 1m from centre, where electric field varies by 10^{10} V/m per second.

(a) 5.56×10^{-8}T (b) 5.56×10^{-3}T

(c) 5.56 μT (d) 5.55T

27. Using an AC voltmeter the potential difference in the electrical line in a house is read to be 234V. If line frequency is known to be 50 cycles/s, the equation for the line voltage is

(a) V=165sin (100 π t)

(b) V=331sin (100 π t)

(c) V=220sin (100 π t)

(d) V=440sin (100 π t)

28. There are a 25W – 220 V bulb and a 100W–220V line. Which eletric bulb will glow more brightly?

(a) 25W bulb

(b) 100W bulb

(c) Both will have equal incadescene

(d) Neither 25 W nor 100 W bulb will give light

29. Silver has a work function of 4.7 eV. When ultraviolet light of wavelength 100 nm is incident upon it , potential of 7.7 V is required to stop photoelectrons from reaching the collector plate. The potential required to stop electrons when light of wavelength 200 nm is incident upon silver is

(a) 1.5V (b) 1.85V

(c) 1.95V (d) 2.37V

30. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, Respectively. The ratio of masses of X and Y is

(a) $(R_1/R_2)^{-2}$ (b) (R_2/R_1)

(c) $(R_1/R_2)^2$ (d) (R_1/R_2)

31. According to the Bohr's theory of hydrogen atom, the speed of the electron, energy and the radius of its orbit vary with the principal quantum number n, respectively, as

(a) $\frac{1}{n}, \frac{1}{n^2}, n^2$ (b) $\frac{1}{n}, n^2, \frac{1}{n^2}$

(c) $n^2, \frac{1}{n^2}, n^2$ (d) $n, \frac{1}{n^2}, \frac{1}{n^2}$

32. In the hydrogen atom, the electron is making 6.6×10^{15} rps. If the radius of orbit is 0.53×10^{-10} m, then magnetic field produced at the centre of the orbit is

(a) 140T (b) 12.5T

(c) 1.4T (d) 0.14T

33. Two identical light sources S_1 and S_2 emit light of same wavelength λ. These light rays will exhibit interference if

(a) their phase differences remain constant
(b) their phases are distributed randomly
(c) their light intensities remain constant
(d) their light intensities change randomly

34. In Meter bridge or Wheatstone bridge for measurement of resistance, the known and the unknown resistances are interchanged. The error so removed is

(a) end correction
(b) index error
(c) due to temperature effect
(d) random error

35. A fish, looking up through the water, sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12cm below the surface of water, the radius of the circle in centimetre is

(a) $\frac{12\times3}{\sqrt{5}}$ (b) $12\times3\times\sqrt{5}$
(c) $\frac{12\times3}{\sqrt{7}}$ (d) $12\times3\times\sqrt{7}$

36. Radio waves diffract around building althrough light waves do not. The reason is that radio waves

(a) travel with speed larger than c
(b) have much larger wavelength then light
(c) carry news
(d) are not electromagnetic waves

37. In the Bohr model of a hydrogen atom, the centripetal force is furnished by the coulomb attraction between the proton and the electron. If a_0 is the radius of the ground state orbit, m is the mass and e is charge on the electron and ε_0 is the vacuum permittivity, the speed of the electron is

(a) 0 (b) $\frac{e}{\sqrt{\varepsilon_0 a_0 m}}$
(c) $\frac{e}{\sqrt{4\pi\varepsilon_0 a_0 m}}$ (d) $\frac{\sqrt{4\pi\varepsilon_0 a_0 m}}{e}$

38. A potential difference of 2V is applied between the opposite faces of a Ge crystal plate of area 1 cm^2 and thickness 0.5 mm. If the concentration of electrons in Ge is $2\times10^{19}/m^2$ and mobilities of electrons and holes are 0.36 $m^2V^{-1}s^{-1}$ and 0.14 $m^2V^{-1}s^{-1}$ respectively, then the current flowing through the plate will be

(a) 0.25 A (b) 0.45 A
(c) 0.56 A (d) 0.64 A

39. An AM wave has 1800 W of total power content. For 100% modulation the carrier should have power content equal to

(a) 1000 W (b) 1200 W
(c) 1500 W (d) 1600 W

40. Two light rays having the same wavelength λ in vacuum are in phase initially. Then the first ray travels a path l_1 through a medium of refractive index n_1 while the second ray travels a path of length l_2 through a medium of refractive index n_2. The two waves are then combined to observe interference. The phase difference between the two waves is

(a) $\frac{2\pi}{\lambda}(l_2-l_1)$ (b) $\frac{2\pi}{\lambda}(n_1l_2-n_2l_1)$
(c) $\frac{2\pi}{\lambda}(n_2l_2-n_1l_1)$ (d) $\frac{2\pi}{\lambda}\left(\frac{l_1}{n_1}-\frac{l_2}{n_2}\right)$

PART - II (CHEMISTRY)

41. The correct formula of the complex tetraammineaquachlorocobalt (III) chloride is

(a) $[Cl(H_2O)(NH_3)_4Co]Cl$
(b) $[CoCl(H_2O)(NH_3)_4]Cl$
(c) $[Co(NH_3)_4(H_2O)Cl]Cl$
(d) $[CoCl(H_2O)(NH_3)_4]Cl_2$

42. The equivalent conductance at infinite dilution of a weak acid such as HF

(a) can be determined by extrapolation of measurements on dilute solutions of HCl, HBr and HI
(b) can be determined by measurement on very dilute HF solutions
(c) can best be determined from measurements on dilute solutions of NaF, NaCl and HCl
(d) is an undefined quantity

43. $C_2H_5I \xrightarrow[\text{KOH}]{\text{Alcoholic}} X \xrightarrow[CCl_4]{Br_2} Y \xrightarrow{\text{KCN}} Z \xrightarrow{H_3O^+} A$

The product 'A' is

(a) succinic acid (b) melonic acid
(c) oxalic acid (d) maleic acid

44. For a reaction of type A + B → products, it is observed that doubling concentration of A causes the reaction rate to be four times as great, but doubling amount of B does not affect the rate. The unit of rate constant is

(a) s^{-1} (b) s^{-1} mol L^{-1}
(c) s^{-1} mol^{-1} L (d) s s^{-1} mol^{-2} L^2

45. A chemical reaction was carried out at 320 K and 300 K. The rate constants were found to be k_1 and k_2 respectively. Then
(a) $k_2 = 4k_1$ (b) $k_2 = 2k_1$
(c) $k_2 = 0.25\,k_1$ (d) $k_2 = 0.5\,k_1$

46. The formula of ethyl carbinol is
(a) CH_3OH (b) CH_3CH_2OH
(c) $CH_3CH_2CH_2OH$ (d) $(CH_3)_3COH$

47. Which of the following gives red colour in Victor Meyer's test?
(a) n-propyl alcohol (b) Isopropyl alcohol
(c) tert-butyl alcohol (d) sec-butyl alcohol

48. Enthalpy of a compound is equal to its
(a) heat of combustion (b) heat of formation
(c) heat of reaction (d) heat of solution

49. For which one of the following reactions will there be a positive ΔS?
(a) $H_2O\,(g) \longrightarrow H_2O(l)$
(b) $H_2 + I_2 \longrightarrow 2HI$
(c) $CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$
(d) $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

50. Across the lanthanide series, the basicity of the lanthanide hydroxides
(a) increases
(b) decreases
(c) first increases and then decreases
(d) first decreases and then increases

51. When p-nitrobromobenzene reacts with sodium ethoxide, the product obtained is
(a) p-nitroanisole (b) ethyl phenyl ether
(c) p-nitrophenetole (d) no reaction occurs

52. A radioactive element X emits 3α, 1β and 1γ-particles and forms ${}_{76}Y^{235}$. Element X is
(a) ${}_{81}X^{247}$ (b) ${}_{80}X^{247}$
(c) ${}_{81}X^{246}$ (d) ${}_{80}X^{246}$

53. For the reaction,
$$2A(g) + B_2(g) \rightleftharpoons 2AB_2(g)$$
the equilibrium constant, K_p at 300 K is 16.0. The value of K_p for $AB_2\,(g) \rightleftharpoons A(g) + 1/2\,B_2\,(g)$ is
(a) 8 (b) 0.25
(c) 0.125 (d) 32

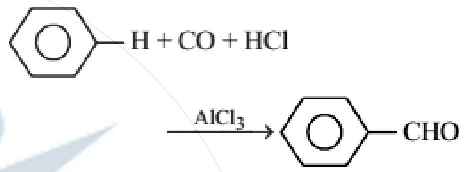
54. Frenkel defect is generally observed in
(a) AgBr (b) AgI
(c) ZnS (d) All of the above

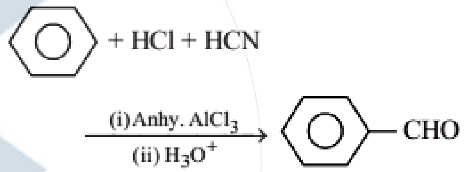
55. Most crystals show good cleavage because their atoms, ions or molecules are
(a) weakly bonded together
(b) strongly bonded together
(c) spherically symmetrical
(d) arranged in planes

56. $[Co\,(NH_3)_4Cl_2]NO_2$ and $[Co\,(NH_3)_4ClNO_2]Cl$ exhibit which type of isomerism?
(a) Geometrical (b) Optical
(c) Linkage (d) Ionisation

57. Which of the following compounds is not coloured?
(a) $Na_2[Cu(Cl_4]$ (b) $Na[Cd(Cl)_4]$
(c) $K_4[Fe(CN)_6]$ (d) $K_3[Fe(CN)_6]$

58. Which of the following is a Gattermann aldehyde synthesis?
(a) 

(b) 

(c) 

(d) 

59. Aldol is
(a) β-hydroxybutyraldehyde
(b) α-hydroxybutanal
(c) β-hydroxypropanal
(d) None of the above

60. Nitrobenzene can be converted into azobenzene by reduction with
(a) Zn, NH_4Cl, Δ
(b) $Zn/NaOH, CH_3OH$
(c) Zn/NaOH
(d) $LiAlH_4$, ether

61. The one which is least basic is
(a) NH_3 (b) $C_6H_5NH_2$
(c) $(C_6H_5)_3N$ (d) $(C_6H_5)_2NH$

62. Coordination number of Ni in $[Ni(C_2O_4)_3]^{4-}$ is
(a) 3 (b) 6
(c) 4 (d) 5

63. Mg is an important component of which biomolecule occurring extensively in living world?
(a) Haemoglobin (b) Chlorophyll
(c) Florigen (d) ATP

64. Sterling silver is
(a) $AgNO_3$
(b) Ag_2S
(c) Alloy of 80% Ag + 20% Cu
(d) AgCl

65. Identify the statement which is not correct regarding $CuSO_4$
(a) It reacts with KI to give iodine
(b) It reacts with KCl to give Cu_2Cl_2
(c) It reacts with NaOH and glucose to give Cu_2O
(d) It gives CuO on strong heating in air

66. Transition metals usually exhibit highest oxidation states in their
(a) chlorides (b) fluorides
(c) bromides (d) iodides

67. The number of Faradays needed to reduce 4 g equivalents of Cu^{2+} to Cu metal will be
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) 4

68. Which one of the following cells can convert chemical energy of H_2 and O_2 directly into electrical energy?
(a) Mercury cell (b) Daniel cell
(c) Fuel cell (d) Lead storage cell

69. On treatment of propanone with dilute $Ba(OH)_2$, the product formed is
(a) aldol
(b) phorone
(c) propionaldehyde
(d) 4-hydroxy-4-methyl-2-pentanone

70. Which of the following converts CH_3CONH_2 to CH_3NH_2?
(a) NaBr (b) NaOBr
(c) Br_2 (d) None of the above

71. Which metal aprons are worn by radiographer to protect him from radiation?
(a) Mercury coated apron
(b) Lead apron
(c) Copper apron
(d) Aluminimised apron

72. The standard Gibb's free energy change, ΔG^o is related to equilibrium constant, K_p as
(a) $K_p = -RT \ln \Delta G^o$ (b) $K_p = \left[\frac{e}{RT}\right]^{\Delta G^o}$
(c) $K_p = -\frac{\Delta G}{RT}$ (d) $K_p = e^{-\Delta G^o/RT}$

73. The yield of the product in the reaction
$$A_2(g) + 2B(g) \rightleftharpoons C(g) + Q \text{ kJ}$$
would be higher at
(a) high temperature and high pressure
(b) high temperature and low pressure
(c) low temperature and high pressure
(d) low temperature and low pressure

74. In which of the following case, does the reaction go farthest to completion?
(a) $K = 10^2$ (b) $K = 10$
(c) $K = 10^{-2}$ (d) $K = 1$

75. Formation of cyanohydrin from a ketone is an example of
(a) electrophilic addition
(b) nucleophilic addition
(c) nucleophilic substitution
(d) electrophilic substitution

76. Glycerol on treatment with oxalic acid at 110°C forms
(a) formic acid (b) allyl alcohol
(c) CO_2 and CO (d) acrolein

77. The activity of an old piece of wood is just 25% of the fresh piece of wood. If $t_{1/2}$ of C-14 is 6000 yr, the age of piece of wood is
(a) 6000 yr (b) 3000 yr
(c) 9000 yr (d) 12000 yr

78. The radius of Na^+ is 95 pm and that of Cl^- ion is 181 pm. Hence, the coordination number of Na^+ will be
(a) 4 (b) 6
(c) 8 (d) unpredictable

79. The reaction, ROH + H_2CN_2 in the presence of HBF_4, gives the following product
(a) $ROCH_3$ (b) RCH_2OH
(c) $ROHCN_2N_2$ (b) RCH_2CH_3

80. The fatty acid which shows reducing property is
(a) acetic acid (b) ethanoic acid
(c) oxalic acid (d) formic acid

PART - III (MATHEMATICS)

81. If F is function such that F (0) = 2, F (1) = 3, F (x+2)= 2F (x) - F (x+1) for $x \geq 0$, then F (5) is equal to
(a) –7 (b) –3
(c) 17 (d) 13

82. Let *S* be a set containing *n* elements. Then, number of binary operations on S is
(a) n^n (b) 2^{n^2}
(c) n^{n^2} (d) n^2

83. The numerically greatest term in the expansion of $(3-5x)^{11}$ when $x = \frac{1}{5}$, is

(a) 55×3^9 (b) 55×3^6
(c) 45×3^9 (d) 45×3^6

84. The number of solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$, is

(a) 0 (b) 1
(c) 2 (d) infinitely many

85. If $a^x = b^y = c^z = d^u$ and a, b, c, d are in GP, then x, y, z, u are in

(a) AP (b) GP
(c) HP (d) None of these

86. If z satisfies the equation $|z| - z = 1 + 2i$, then z is equal to

(a) $\frac{3}{2} + 2i$ (b) $\frac{3}{2} - 2i$
(c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$

87. If $z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}$, then arg (z) is

(a) 60° (b) 120°
(c) 240° (d) 300°

88. If $f(x) = \sqrt{\log_{10} x^2}$. The set of all values of x for which $f(x)$ is real, is

(a) $[-1, 1]$ (b) $[1, \infty]$
(c) $(-\infty, -1]$ (d) $(-\infty, -1] \cup [1, \infty)$

89. For what values of m can the expression $2x^2 + mxy + 3y^2 - 5y - 2$ be expressed as the product of two linear factors?

(a) 0 (b) ± 1
(c) ± 7 (d) 49

90. If B is a non-singular matrix and A is a square matrix, then det $(B^{-1}AB)$ is equal to

(a) det (A^{-1}) (b) det (B^{-1})
(c) det (A) (d) det (B)

91. If $f(x)$, g(x) and h (x) are three polynomials of degree 2 and

$$\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$$

then $\Delta(x)$ is a polynomial of degree

(a) 2 (b) 3
(c) 0 (d) atmost 3

92. The chances of defective screws in three boxes A, B, C are $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ respectively. A box is selected at random and a screw drawn from it at random is found to be defective. Then, the probability that it came from box A, is

(a) $\frac{16}{29}$ (b) $\frac{1}{15}$
(c) $\frac{27}{59}$ (d) $\frac{42}{107}$

93. The value of $\frac{\cos\theta}{1+\sin\theta}$ is equal to

(a) $\tan\left(\frac{\theta}{2} - \frac{\pi}{4}\right)$ (b) $\tan\left(-\frac{\pi}{4} - \frac{\theta}{2}\right)$
(c) $\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ (d) $\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

94. If $3\sin\theta + 5\cos\theta = 5$, then the value of $5\sin\theta - 3\cos\theta$ is equal to

(a) 5 (b) 3
(c) 4 (d) None of these

95. The principal value of $\sin^{-1}\left\{\sin\frac{5\pi}{6}\right\}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$
(c) $\frac{7\pi}{6}$ (d) None of these

96. A rod of length l slides with its ends on two perpendicular lines. Then, the locus of its mid point is

(a) $x^2 + y^2 = \frac{l^2}{4}$ (b) $x^2 + y^2 = \frac{l^2}{2}$
(c) $x^2 - y^2 = \frac{l^2}{4}$ (d) None of these

97. The equation of straight line through the intersection of line $2x + y = 1$ and $3x + 2y = 5$ and passing through the origin is

(a) $7x + 3y = 0$ (b) $7x - y = 0$
(c) $3x + 2y = 0$ (d) $x + y = 0$

98. The line joining (5,0) to $(10\cos\theta, 10\sin\theta)$ is divided internally in the ratio 2:3 at P. If θ varies, then the locus of P is

(a) a straight line
(b) a pair of straight lines
(c) a circle
(d) None of the above

99. If $2x + y + k = 0$ is a normal to the parabola $y^2 = -8x$, then the value of k, is
(a) 8 (b) 16
(c) 24 (d) 32

100. $\lim_{n\to\infty}\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.....+\frac{1}{n(n+1)}\right]$ is equal to
(a) 1 (b) –1
(c) 0 (d) None of these

101. The condition that the line $lx + my = 1$ may be normal to the curve $y^2 = 4ax$, is
(a) $al^3 - 2alm^2 = m^2$ (b) $al^2 + 2alm^3 = m^2$
(c) $al^3 + 2alm^2 = m^3$ (d) $al^3 + 2alm^2 = m^2$

102. If $\int f(x)dx = f(x)$, then $\int\{f(x)\}^2\,dx$ is equal to
(a) $\frac{1}{2}\{f(x)\}^2$ (b) $\{f(x)\}^3$
(c) $\frac{\{f(x)\}^3}{3}$ (d) $\{f(x)\}^2$

103. $\int \sin^{-1}\left\{\frac{(2x+2)}{\sqrt{4x^2+8x+13}}\right\}dx$ is equal to
(a) $(x+1)\tan^{-1}\left(\frac{2x+2}{3}\right)-\frac{3}{4}\log\left(\frac{4x^2+8x+13}{9}\right)+c$
(b) $\frac{3}{2}\tan^{-1}\left(\frac{2x+2}{3}\right)-\frac{3}{4}\log\left(\frac{4x^2+8x+13}{9}\right)+c$
(c) $(x+1)\tan^{-1}\left(\frac{2x+2}{3}\right)-\frac{3}{2}\log(4x^2+8x+13)+c$
(d) $\frac{3}{2}(x+1)\tan^{-1}\left(\frac{2x+2}{3}\right)-\frac{3}{4}\log(4x^2+8x+13)+c$

104. If the equation of an elipse is $3x^2+2y^2+6x-8y+5=0$, then which of the following are true ?
(a) $e = \frac{1}{\sqrt{3}}$
(b) centre is (–1, 2)
(c) foci are (–1, 1) and (–1, 3)
(d) All of the above

105. The equation of the common tangents to the two hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, are
(a) $y = \pm x \pm \sqrt{b^2-a^2}$
(b) $y = \pm x \pm \sqrt{a^2-b^2}$
(c) $y = \pm x \pm \sqrt{a^2+b^2}$
(d) $y = \pm x \pm \sqrt{a^2-b^2}$

106. Domain of the function $f(x) = \log_x \cos x$, is
(a) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)-\{1\}$ (b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{1\}$
(c) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (d) None of these

107. Range of the function $y = \sin^{-1}\left(\frac{x^2}{1+x^2}\right)$, is
(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left[0,\frac{\pi}{2}\right)$
(c) $\left(0,\frac{\pi}{2}\right]$ (d) $\left[0,\frac{\pi}{2}\right]$

108. If $x = \sec\theta - \cos\theta$, $y = \sec^n\theta - \cos^n\theta$, then $(x^2+4)\left(\frac{dy}{dx}\right)^2$ is equal to
(a) $n^2(y^2-4)$ (b) $n^2(4-y^2)$
(c) $n^2(y^2+4)$ (d) None of these

109. If $y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\ldots\infty}}}}$, then $\frac{dy}{dx}$ is equal to
(a) $\frac{y+x}{y^2-2x}$ (b) $\frac{y^3-x}{2y^2-2xy-1}$
(c) $\frac{y^3+x}{2y^2-x}$ (d) None of these

110. If $\int_1^x \frac{dt}{|t|\sqrt{t^2-1}}=\frac{\pi}{6}$, then x can be equal to
(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$
(c) 2 (d) None of these

111. The area bounded by the curve $y=|\sin x|$, x-axis and the lines $|x|=\pi$, is

(a) 2 sq unit (b) 1 sq unit
(c) 4 sq unit (d) None of these

112. The degree of the differential equation of all curves having normal of constant length c is

(a) 1 (b) 3
(c) 4 (d) None of these

113. If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$, then $\vec{a}+t\vec{b}$ is perpendicular to $\vec{c}$, if t is equal to

(a) 2 (b) 4
(c) 6 (d) 8

114. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+5\hat{j}+\hat{k})=5$, is

(a) $\frac{10}{3}$ (b) $\frac{10}{\sqrt{3}}$
(c) $\frac{10}{3\sqrt{3}}$ (d) $\frac{10}{9}$

115. The equation of sphere concentric with the sphere $x^2+y^2+z^2-4x-6y-8z-5=0$ and which passes through the origin, is

(a) $x^2+y^2+z^2-4x-6y-8z=0$
(b) $x^2+y^2+z^2-6y-8z=0$
(c) $x^2+y^2+z^2=0$
(d) $x^2+y^2+z^2-4x-6y-8z-6=0$

116. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k, is

(a) $\frac{3}{2}$ (b) $\frac{9}{2}$
(c) $-\frac{2}{9}$ (d) $-\frac{3}{2}$

117. The two curves $y=3^x$ and $y=5^x$ intersect at an angle

(a) $\tan^{-1}\left(\frac{\log 3-\log 5}{1+\log 3\log 5}\right)$
(b) $\tan^{-1}\left(\frac{\log 3+\log 5}{1-\log 3\log 5}\right)$
(c) $\tan^{-1}\left(\frac{\log 3+\log 5}{1+\log 3\log 5}\right)$
(d) $\tan^{-1}\left(\frac{\log 3-\log 5}{1-\log 3\log 5}\right)$

118. The equation $\lambda x^2+4xy+y^2+\lambda x+3y+2=0$ represents a parabola, if λ is

(a) 0 (b) 1
(c) 2 (d) 4

119. If two circles $2x^2+2y^2-3x+6y+k=0$ and $x^2+y^2-4x+10y+16=0$ cut orthogonally, then the value of k is

(a) 41 (b) 14
(c) 4 (d) 1

120. If A (–2, 1), B (2, 3) and C (–2, –4) are three points. Then, the angle between BA and BC is

(a) $\tan^{-1}\left(\frac{2}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{2}\right)$
(c) $\tan^{-1}\left(\frac{1}{3}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

SOLUTIONS

PART - I (PHYSICS)

1. (b) Here, length $l = 2\pi r$ or $r = \frac{1}{2\pi}$

Area of circular loop $A = \pi r^2$
Magnetic moment $M = iA = i\pi r^2$

$$M = i\pi \times \frac{l^2}{4\pi^2}$$

$$\therefore l = \sqrt{\frac{4\pi M}{i}}$$

2. (b) Current through arms of resistances P and Q in series

$$i_1 = \frac{i \times 330}{330 + 110} = \frac{3}{4}i$$

Here i = total current
Similarly, current through arms of resistances R and S in series

$$i_2 = \frac{i \times 110}{330 + 110} = \frac{1}{4}i$$

Heat developed per second = i^2R
∴ Ratio of heat developed per sec

$H_P : H_Q : H_R : H_S$

$$= \left(\frac{3}{4}i\right)^2 \times 100 : \left(\frac{3}{4}i\right)^2 \times 10 : \left(\frac{1}{4}i\right)^2 \times 300 : \left(\frac{1}{4}i\right)^2 \times 30$$

$= 30 : 3 : 10 : 1$

3. (c) Heat taken by water when its temperature changes from 20°C to 100°C.

$$H_1 = mc(\theta_2 - \theta_1) = 1000 \times 1 \times (100 - 20) \text{ cal}$$
$$= 1000 \times 80 \times 4.2 \text{ J}$$

Heat produced in time t due to current in resistor

$H_2 = Vit = 220 \times 4 \times t$ J

According to question,

$220 \times 4 \times t = 1000 \times 80 \times 4.2$

$$\Rightarrow t = \frac{1000 \times 80 \times 4.2}{220 \times 4} = 381.8 \text{ s} = 6.3 \text{ min}$$

4. (d) Magnetic field,

$$B = \frac{\mu_0}{4\pi}\frac{2\pi i}{r} = \frac{\mu_0 i}{2r} \text{ or } i = \frac{2Br}{\mu_0}$$

Also, $A = \pi r^2$ or $r = \left(\frac{A}{\pi}\right)^{1/2}$

Magnetic moment,

$$M = iA = \frac{2Br}{\mu_0}A$$

$$= \frac{2BA}{\mu_0} \times \left(\frac{A}{\pi}\right)^{1/2} = \frac{2BA^{3/2}}{\mu_0 \pi^{1/2}}$$

5. (c) Here, $\sin\theta = \left(\frac{Y}{D}\right)$ $\quad \therefore \Delta\theta = \frac{\Delta Y}{D}$

Angular fringe width $\theta_0 = \Delta\theta$ (width $\Delta Y = \beta$)

$$\theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

$$\theta_0 = 1° = \frac{\pi}{180} \text{ rad and } \lambda = 6 \times 10^{-7} \text{ m}$$

$$d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times 6 \times 10^{-7} = 0.03 \text{ mm}$$

6. (d) Here, charge $q = \pm 1 \times 10^{-6}$ C
$2a = 2.0$ cm $= 2.0 \times 10^{-2}$ m
$E = 1 \times 10^5$ NC^{-1}, $\tau_{max} = ?$
$W = ?$, $\theta_1 = 0°$, $\theta_2 = 180°$
$\tau_{max} = pE = q(2a)E$
$= 1 \times 10^{-6} \times 2.0 \times 10^{-2} \times 1 \times 10^5$
$= 2 \times 10^{-3}$ Nm

$W = pE(\cos\theta_1 - \cos\theta_2)$
$= (10^{-6} \times 2 \times 10^{-2})(10^5)(\cos 0° - \cos 180°)$
$= 4 \times 10^{-3}$ J

7. (b) Current, $i = \frac{e}{t} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$

Here, $v = \frac{e^2}{\hbar}$ and $r = \frac{\hbar^2}{me^2}$

$$\therefore i = \frac{e(e^2/\hbar)}{2\pi(\hbar^2/me^2)} = \frac{e^3 \times me^2}{2\pi\hbar^3} = \frac{me^5}{2\pi\hbar^3}$$

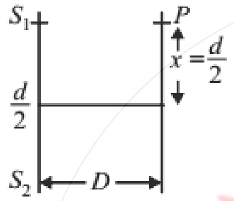
$$\because \hbar = \frac{h}{2\pi} \text{ (given)}$$

$$\therefore i = \frac{me^5}{2\pi\left(\frac{h}{2\pi}\right)^3} = \frac{4\pi^2 me^5}{h^3}$$

8. (d) For dark fringe,

$$\frac{xd}{D} = (2m-1)\frac{\lambda}{2}$$

Here, $m = 5$, $x = \frac{d}{2}$



$$\therefore \frac{d}{2} \cdot \frac{d}{D} = (2 \times 5 - 1)\frac{\lambda}{2}$$

or $\frac{d^2}{D} = 9\lambda$

$\therefore$ Wavelength, $\lambda = \frac{d^2}{9D}$

9. (d) Generally, temperature of human body is 37°C (= 98.4°F) corresponding to which IR and microwave radiations are emitted from the human body.

10. (d) The path of moving proton in a normal magnetic field is circular. If r is the radius of the circular path, then from the figure, From the symmetry of figure, the angle $\theta = 45°$.

$$AC = 2r\cos 45° = 2r \times \frac{1}{\sqrt{2}} = \sqrt{2}r \quad ...(1)$$

As $Bqv = \frac{mv^2}{r}$ or $r = \frac{mv}{Bq}$

$$AC = \frac{\sqrt{2}mv}{Bq} = \frac{\sqrt{2} \times 1.67 \times 10^{-27} \times 10^7}{1 \times 1.6 \times 10^{-19}}$$

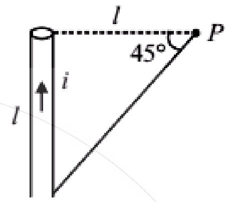
$= 0.14$ m

11. (d) In a neutral water molecule, there are 10 electrons and 10 protons.
So, its dipole moment $p = q(2l) = 10e(2l)$
Hence length of the dipole = distance between centres of positive and negative charges

$$2l = \frac{p}{10e} = \frac{6.4 \times 10^{-30}}{10 \times 1.6 \times 10^{-19}} = 4 \times 10^{-12} \text{ m}$$

12. (c) Magnetic field due to finite length of a wire,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$



Here, $\phi_1 = 0°, \phi = 45°$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\sin 0° + \sin 45°) = \frac{\mu_0}{4\pi} \cdot \frac{i}{\sqrt{2}l}$$

$$\Rightarrow B = \frac{\sqrt{2}\mu_0 i}{8\pi l}$$

13. (c) Zener diode is suitable for voltage regulating purpose. It is used as voltage stabilizer in many applications in electronics.

14. (a) If two independent sources emitting light of the same wavelength are said to be coherent.

15. (d) Induced charge

$$Q = -\frac{nBA}{R}(\cos\theta_2 - \cos\theta_1)$$

$$= -\frac{nBA}{R}(\cos 180° - \cos 0°)$$

$$\Rightarrow B = \frac{QR}{2nA}$$

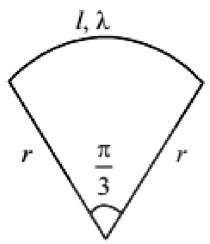
16. (d) From an optical fibre due to absorption or light leaving the fibre area resulting scattering of light sideways by impurities in the glass fibre. And due to this reason a very small part of light energy is lost.

17. (c) Length of the arc = $r\theta = \frac{r\pi}{3}$

Charge on the arc = $\frac{r\pi}{3} \times \lambda$

$\therefore$ Potential at centre v

$= \frac{kq}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{r\pi}{3}\frac{\lambda}{r} = \frac{\lambda}{12\varepsilon_0}$



18. (a) Here $f_c = 1.5$ MHz = 1500 kHz, $f_m = 10$ kHz

$\therefore$ Lower side-band frequency

$= f_c - f_m = 1500$ kHz $- 10$ kHz $= 1490$ kHz

Upper side-band frequency

$= f_c + f_m = 1500$ kHz $+ 10$ kHz $= 1510$ kHz

19. (c) Equivalent resistance of the circuit

$R_{eq} = 100\Omega$

Current through the circuit, $i = \frac{V}{R} = \frac{2.4}{100}$ A

Potential difference across combination of voltmeter and 100Ω resistance

$= \frac{2.4}{100} \times 50 = 1.2$ V

Since the voltmeter and 100Ω resistance are in parallel, the voltmeter reads the same value i.e., 1.2 V.

20. (c) In space charge limited region, the plate current is given by Child's law $i_p = KV_p^{3/2}$

Thus,

$$\frac{i_{p2}}{i_{p1}} = \left(\frac{V_{p2}}{V_{p1}}\right)^{3/2} = \left(\frac{600}{150}\right)^{3/2} = (4)^{3/2} = 8$$

or, $i_{p2} = i_{p1} \times 8 = 10 \times 8$ mA $= 80$ mA

21. (c) Here, $E = \frac{hc}{2} - W_0$ and $2E = \frac{hc}{\lambda'} - W_0$

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{E+W}{2E+W_0} \Rightarrow \lambda' = \lambda\left(\frac{1+W_0/E}{2+W_0/E}\right)$$

Since $\frac{(1+W_0/E)}{(2+W_0/E)} > \frac{1}{2}$ So $\lambda' > \frac{\lambda}{2}$

22. (b) According to Einstein's photoelectric equation,

$$E = W_0 + \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max} = \sqrt{\frac{2(hf - W_0)}{m}}$$

If frequency becomes $4f$ then

$$v' = \sqrt{\frac{2(h \times 4f - W_0)}{m}} = 2\sqrt{\frac{2\left(hf - \frac{W_0}{4}\right)}{m}}$$

$\Rightarrow v' > 2v$

23. (b) In electric field photoelectron will experience force and accelerate opposite to the field so its KE increases (i.e., stopping potential will increase), no change in photoelectric current, and threshold wavelength.

24. (b) Magnetic field inside the cyclindrical conductor $B_{in} = \frac{\mu_0}{4\pi}\frac{2ir}{R^2}$

(R = radius of cylinder and r = distance of observation point from axis of cylinder)

Magnetic field outside the cylinder at a distance r' from its axis, $B_{out} = \frac{\mu_0}{4\pi}\frac{2i}{r'}$

$$\Rightarrow \frac{B_{in}}{B_{out}} = \frac{rr'}{R^2} \Rightarrow \frac{10}{B_{out}} = \frac{\left(R - \frac{R}{4}\right)(R+4R)}{R^2}$$

$\Rightarrow B_{out} = \frac{8}{3}T$

25. (a) By using lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow 5 = (1.5-1)\left(\frac{2}{R}\right) \quad ...(i)$$

If a lens of refractive index μ_g is immersed in a liquid of refractive index μ_1, then its focal length in liquid

$$\frac{1}{f_l} = ({}_l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow -1 = \left(\frac{1.5}{n} - 1\right)\left(\frac{2}{R}\right) \quad ...(ii)$$

Dividing, (i) by (ii) $-5 = \dfrac{0.5n}{1.5 - n}$

$$\Rightarrow -7.5 + 5n = 0.5n \Rightarrow -7.5 = -4.5n$$

$$\Rightarrow n = \frac{75}{45} = \frac{5}{3}$$

26. (a) Magnetic field

$$B = \frac{\mu_0 \varepsilon_0 r}{2}\frac{dE}{dt} = \frac{1}{9\times 10^{16}\times 2}\times 10^{10}$$
$= 5.56 \times 10^{-8}$ T

27. (b) $E = E_0 \sin \omega t$

Voltmeter read rms value

$$\therefore E_0 = \sqrt{2}\times 234 \text{ V} = 331 \text{ V}$$

and $\omega t = 2\pi n t = 2\pi \times 50 \times t = 100\pi t$

Thus, the equation of the line voltage

$E = 331 \sin(100\pi t)$

28. (a) Power, $P = \dfrac{V^2}{R}$, $\Rightarrow R = \dfrac{V^2}{P}$

For the first bulb,

$$R_1 = \left(\frac{V^2}{P_1}\right) = \left(\frac{(220)^2}{25}\right) = 1936\Omega$$

For the second bulb,

$$R_2 = \left(\frac{V^2}{P_2}\right) = \left(\frac{(220)^2}{100}\right) = 484\Omega$$

Current in series combination is the same in the two bulbs,

$$i = \frac{V}{R_1 + R_2} = \frac{220}{1936 + 484} = \frac{220}{2420} = \frac{1}{11} A$$

If the actual powers in the two bulbs be P_1 and P_2 then

$$P'_1 = i^2 R_1 = \left(\frac{1}{11}\right)^2 \times 1936 = 16 \text{ W}$$

and $P'_2 = i^2 R_2 = \left(\dfrac{1}{11}\right)^2 \times 484 = 4 \text{ W}$

Since $P'_1 > P'_2$, so, 25 W bulb will glow more brightly.

29. (a) Given : $\lambda = 100$ nm $= 1000$ Å

Energy corresponding to 1000 Å

$$= \frac{12375}{1000} = 12.375 \text{ eV}$$

Now, $7.7 = 12.375 - \phi_0$

or $\phi_0 = 12.375 - 7.7 = 4.675$ eV

In the second case,

Energy corresponding to 2000 Å

$$= \frac{12375}{2000} \text{ eV} = 6.1875 \text{ eV}$$

Now, $4.7 = 6.1875 - \phi'_0$

or $\phi'_0 = 6.1875 - 4.7 = 1.4875 \simeq 1.5$ V

30. (c) As we know, $\dfrac{1}{2}mv^2 = qV$ or $v = \sqrt{\dfrac{2qV}{m}}$

Centripetal force $\dfrac{mv^2}{R} = q \times B \times v$

$$\therefore v = \frac{qBR}{m}$$

Hence, $\sqrt{\dfrac{2qV}{m}} = \dfrac{qBR}{m}$ or $R = \left(\dfrac{2mV}{q}\right)^{1/2} \times \dfrac{1}{B}$

Here, V, q and B are constants.

$\therefore R \propto m$

And, $\dfrac{m_1}{m_2} = \left(\dfrac{R_1}{R_2}\right)^2$

31. (a) According to Bohr's theory of hydrogen atom,

(i) The speed of the electron in the nth orbit

$$V_n = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0(h/2\pi)} \text{ or } v_n \propto \frac{1}{n}$$

(ii) The energy of the electron in the nth orbit

$$E_n = \frac{me^4}{8n^2\varepsilon_0^2 h^2} = \frac{-13.6}{n^2} eV \text{ or } E_n \propto \frac{1}{n^2}$$

(iii) The radius of the electron in the nth orbit

$$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2} = n^2 a_0 \text{ or } r_n \propto n^2$$

where $a_0 = \dfrac{h^2\varepsilon_0}{\pi m e} = 5.29 \times 10^{-11}$ m

32. (b) Current, $i = qv$

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 qv}{2r}$$

$$= \frac{4\pi\times10^{-7}\times1.6\times10^{-19}\times6.6\times10^{15}}{2\times0.53\times10^{-10}}$$

$$= \frac{2\pi\times1.6\times6.6}{5.3} = 12.513 \text{ T}$$

33. (a) For interference phase difference must be constant.

34. (a) To remove the error, resistance box and the unknown resistance must be interchanged and then the mean reading must be taken.

35. (c) Here, $\tan i_c = \frac{r}{h}$ or $r = h\tan i_c$

or $r = h\frac{\sin i_c}{\cos i_c}$ or $r = h\frac{\sin i_c}{\sqrt{1-\sin^2 i_c}}$

But $\sin i_c = \frac{1}{\mu}$

$$\therefore r = h\frac{\frac{1}{\mu}}{\sqrt{1-\frac{1}{\mu^2}}} = \frac{h}{\sqrt{\mu^2-1}} = \frac{12}{\sqrt{\frac{16}{9}-1}}$$

$$= \frac{12\times3}{\sqrt{7}} \text{ cm}$$

36. (b) Diffraction takes places when the wavelength of waves is comparable with the size of the obstacle in path.
$P_{radio} > P_{light}$
Hence, radio waves are diffracted around building.

37. (c) Centripetal force = force of attraction of nucleus on electron.

$$\frac{mv^2}{a_0} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{a_0^2} \Rightarrow v = \frac{e}{\sqrt{4\pi\varepsilon_0 m a_0}}$$

38. (d) As we know, conductivity

$$\sigma = ne(\mu_e + \mu_h)$$

$$= 2\times10^{19}\times1.6\times10^{-19}(0.36+0.14)$$
$$= 1.6(\Omega\text{m})^{-1}$$

$$R = \rho\frac{l}{A} = \frac{l}{\sigma A} = \frac{0.5\times10^{-3}}{1.6\times10^{-4}} = \frac{25}{8}\Omega$$

$$i = \frac{V}{R} = \frac{2}{25/8} = 0.64\text{A}$$

39. (b) Total power $P_t = P_c\left[1+\frac{ma^2}{2}\right]$ $\because ma^2 = 1$

$$\therefore 1800 = P_c\left[1+\frac{1}{2}\right] \Rightarrow P_c = 1200 \text{ W}$$

40. (b) Optical path for ray 1 $= n_1 l_1$
Optical path for ray 2 $= n_2 l_2$
Phase difference,

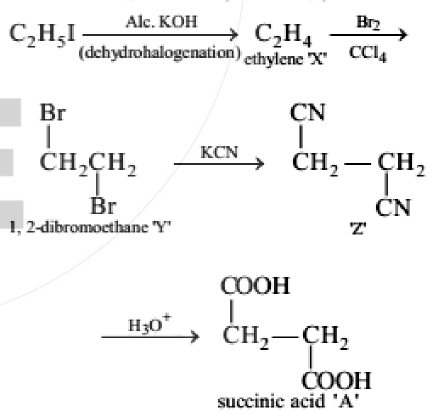
$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}(n_1 l_1 - n_2 l_2)$$

PART - II (CHEMISTRY)

41. (d) The correct formula of the given complex is tetraammine aqua chlorocobalt (III) chloride is $[CoCl(H_2O)(NH_3)_4]Cl_2$, because in it the oxidation number of Co is +3. While in rest other options O. No. of Co is +2
$[CoCl(H_2O)(NH_3)_4]Cl_2$
$\Rightarrow x + (-1) + 0 + (0\times4) + (-1)2 = 0$
$\Rightarrow x - 3 = 0 \Rightarrow x = +3$

42. (c) According to Kohlrausch's law, equivalent conductance at infinite dilution of HF,
$\Lambda^\circ_{HF} = \Lambda^\circ_{NaF} + \Lambda^\circ_{HCl} - \Lambda^\circ_{NaCl}$

43. (a)



44. (c) Let the initial rate be R
and order with respect to A be x and B be y.
Thus, rate law can be written as,
rate, $R = [A]^x[B]^y$ ...(i)
After doubling the concentration of A, rate becomes $4R$,

$4R = [2A]^x [B]^y$ ...(ii)

After doubling the concentration of B, rate remains R,

$R = [A]^x [2B]^y$ ...(iii)

From Eq. (i) and (ii), we get

$$\frac{R}{4R} = \left(\frac{1}{2}\right)^x \Rightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^x$$

So, $x = 2$

From Eq. (i) and (iii), we get

$$\frac{R}{R} = \left[\frac{1}{2}\right]^y \Rightarrow \left(\frac{1}{1}\right)^o = \left(\frac{1}{2}\right)^y$$

So, Y = 0

Hence, the rate law is, rate R = $[A]^2 [B]^0$

This clearly shows that the order of this reaction is 2 and for second order reaction units of rate constant are $mol^{-1}\ Ls^{-1}$.

45. (c) As we know that for every 10° rise in temperature, rate constant, k becomes doubled. Hence, on rising the temperature 20°, the rate constant will be four times,

i.e., $k_1 = 4k_2 \Rightarrow k_2 = \frac{1}{4}k_1 = 0.25\ k_1$

46. (c) The other name of methanol is carbinol. So, the formula of ethyl carbinol is

$\underbrace{CH_3CH_2}_{\text{ethyl}}\ \underbrace{CH_2OH}_{\text{carbinol}}$

47. (a) In Victor Meyer's test, Red colour is given by primary alcohols (1°) (alcohols having $-CH_2OH$). The structres of the given Alcohols are

(a) $CH_3CH_2CH_2OH$
n-propyl alcohol (1°)

(b) $CH_3CH(OH)-CH_3$
iso-propyl alcohol (2°)

(c) $CH_3-C(CH_3)(OH)-CH_3$
tert-butyl alcohol (3°)

(d) $CH_3CH(OH)CH_2CH_3$
sec-butyl alcohol (2°)

Hence, *n*-propyl alcohol is a 1° alcohol and gives red colour in Victor-Meyer's test.

48. (b) Heat of formation is equal to enthalpy of a compound.

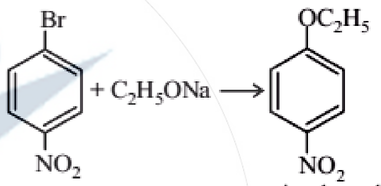
49. (c) ΔS (entropy change) is the measure of randomness and thus in solid, liquid and gas, the order of entropy is

gas > liquid > solid

Thus, ΔS is positive for the reaction given in option (c) because solid $CaCO_3$ is forming gaseous CO_2.

50. (b) We know that basicity is depend on ionic character. So, as the ionic size of lanthanide decreases, the covalent character of their hydroxide increases. Hence, their basicity decreases.

51. (c) Commonly aryl halides do not take part in Williamson's synthesis, due to their high stability but due to the presence of strong electron withdrawing group like $-NO_2$ makes the C–X bond weaker and substitution of –Br takes place by –OR.



p-nitrophenetole

52. (a) The complete nuclear reaction is

$${}_Z X^A \longrightarrow {}_{76}Y^{235} + \underset{\alpha-particle}{3\ {}_2He^4} + \underset{\beta-particle}{{}_{-1}e^0} + \underset{\gamma-particle}{\gamma}$$

On observing the reaction, mass number of '*X*' is

$A = 235 + 12 + 0 = 247$

On observing the reaction, atomic number of '*X*' is

$Z = 76 + 6 - 1 = 81$

Hence, element '*X*' is ${}_{81}X^{247}$.

53. (b) For the given reaction,

$2A(g) + B_2(g) \rightleftharpoons 2AB_2(g)$

the equilibrium constant,

$$K_p = \frac{p^2_{AB_2}}{p^2_A \cdot p_{B_2}} = 16 \qquad ...(i)$$

For the other given reaction,

$$AB_2(g) \rightleftharpoons A(g) + \frac{1}{2}B_2(g)$$

The equilibrium constant,

$$K'_p = \frac{p_A \cdot p_{B_2}^{1/2}}{p_{AB_2}} \quad ...(ii)$$

On squaring Eq. (ii), we obtain,

$$(K'_p)^2 = \frac{p_A^2 \cdot p_{B_2}}{p_{AB_2}^2} \quad ...(iii)$$

Now, from Eq. (i) and (iii), we obtain,

$$K_p.(K'_p)^2 = 1 \Rightarrow 16.(K'_p)^2 = 1$$

$$(\because K_p = 16.0)$$

$$\Rightarrow (K'_p) = \left(\frac{1}{16}\right)^{1/2} \Rightarrow K'_p = \frac{1}{4} = 0.25$$

54. (d) When the size of cation is much smaller than the anion then frenkel defect is observed. Hence, AgBr, AgI and ZnS all exhibit Frenkel defect.

55. (d) Crystals show good cleavage when, the constitutents are arranged in orderly pattern, i.e., in planes,

56. (d) On ionisation they give different ions

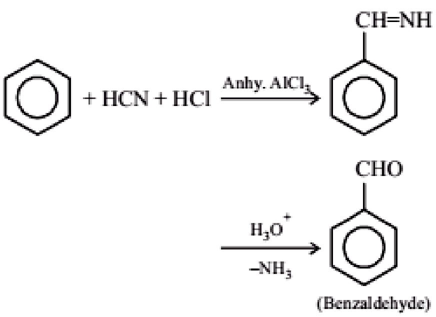
$$[Co(NH_3)_4Cl_2]NO_2 \longrightarrow [Co(NH_3)_4Cl_2]^+ + NO_2^-$$

$$[Co(NH_3)_4ClNO_2]Cl \longrightarrow [Co(NH_3)_4ClNO_2]^+ + Cl^-$$

So, they show ionisation isomerism.

57. (c) We know that complex compound having no unpaired electron is colourless. Among the given complexes, $K_4[Fe(CN)_6]$ has no unpaired electron as CN^- is a strong field ligand and causes pairing of electrons. So, it is colourless.

58. (c)

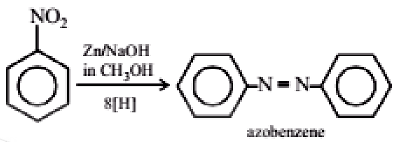


This is Gattermann aldehyde synthesis.

59. (a) Aldol is β-hydroxybutyraldehyde (or 3-hydroxybutanal) i.e.

$$CH_3-\underset{|}{\overset{OH}{C}}H-CH_2CHO$$

60. (b) Nitrobenzene can be converted into azobenzene, on reduction in the presence of Zn/NaOH in CH_3OH.



61. (c) Due to the presence of electron withdrawing group like Ph group decreases the electron density of nitrogen and hence, the lone pair of nitrogen are not available for donation.

So, $(C_6H_5)_3N$ is least basic due to the presence of three electron withdrawing $Ph(C_6H_5)$ groups.

62. (b) Coordination number of Ni in $[Ni(C_2O_4)_3]^{4-}$ is 6 because $C_2O_4^{2-}$ (oxalate) is a bidentate ligand and each have two sites to coordinated with the central atom .

63. (b) Chlorophyll is rich source of Mg, the green pigment of plants.

64. (c) An alloy of 80% Ag and 20% other metals, usually copper is sterling silver.

65. (b) As $CuSO_4$ reacts with KI to give white precipitate of Cu_2I_2 and to evolve I_2.$CuSO_4$ does not react with KCl.

66. (b) Due to highest reduction potential of fluorine. Transition metals exhibit highest oxidation states in their fluoride.

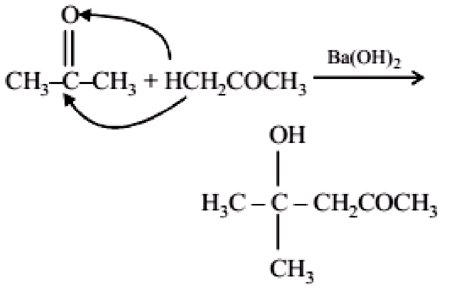
67. (d)

$$\begin{array}{ccccc} Cu^{2+} & + & 2e^- & \longrightarrow & Cu \\ 1\text{ mol} & & 2F & & 1\text{ mol} \\ \frac{1}{2}\text{ mol} & & 1F & & \frac{1}{2}\text{ mol} \\ 1\text{ gequi.} & & 1F & & 1\text{g equi.} \end{array}$$

Thus, to reduce 4 g equivalent of Cu^{2+} into Cu 4F are required.

68. (c) Fuel cell, which convert chemical energy of fuels like H_2, O_2, CH_4, etc, is converted into electric energy, e.g., H_2–O_2 fuel cell.

69. (d) In Aldol condensation propanone gives diacetone alcohol in presence of $Ba(OH)_2$



4-hydroxy-4-methyl pentanone-2 (diacetone alcohol)

70. (b) $CH_3CONH_2 \xrightarrow{NaOBr} CH_3NH_2 + Na_2CO_3$

71. (b) Radiographer to protect themself from radiation worn lead apron.

72. (d) $\Delta G°$ and K_p are related as

$$\Delta G° = -RT \ln K_p$$

$$\Rightarrow \ln K_p = \frac{\Delta G°}{RT} \Rightarrow K_p = e^{-\Delta G°/RT}$$

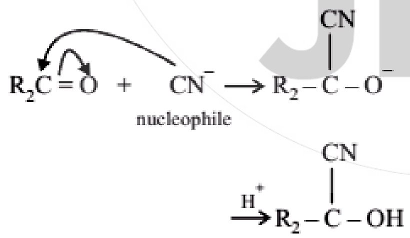
73. (c) $A_2(g) + 2B(g) \rightleftharpoons C(g) + Q\ kJ$

Since, the reaction is exothermic, So, it is favoured by low temperature.

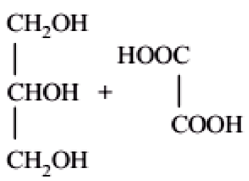
In addition, the number of moles of products is lesser than the number of moles of reactants, thus high pressure favours the forward reaction.

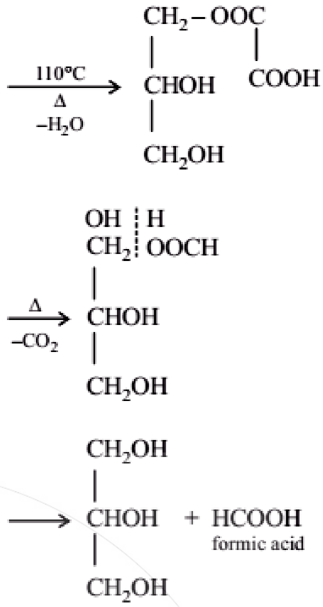
74. (a) Larger the value of K more the reaction moves towards completion.

75. (b) As CN is a nucleophile. So it is an example of nucleophilic addition



76. (a)





77. (d) As we know that,

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{6000} = 1.155 \times 10^{-4}$$

$$t = \frac{2.303}{k} \log \frac{N_0}{N} = \frac{2.303}{1.155 \times 10^{-4}} \log \frac{100}{25}$$

$= 12000$ yr (age of piece of wood)

78. (b) $\frac{\text{Radius of cation, } r^+}{\text{Radius of anion, } r^-} = \frac{95}{181} = 0.525$

As this value lies in between $0.414 - 0.732$, thus, the coordination number of Na^+ ion will be 6.

79. (a) $ROH + H_2CN_2 \xrightarrow{HBF_4} ROCH_3 + N_2$

80. (d) The compounds having –CHO group reduces Tollen's reagent, Fehling solution etc. Thus, formic acid (HCO/ OH) has reducing property.

PART - III (MATHEMATICS)

81. (d) $F(x+2) = 2F(x) - F(x+1)$ ...(i)

Putting $x = 0$, we get

$F(2) = 2F(0) - F(1)$

$\Rightarrow F(2) = 2(2) - 3$

$\{\because F(0) = 2,\ F(1) = 3\}$

$\Rightarrow F(2) = 4 - 3 \Rightarrow F(2) = 1$

Putting $x = 1$, in eq. (i), we get

$F(3) = 2F(1) - F(2)$

$= 2(3) - 1 \quad \{\because F(1) = 3,\ F(2) = 1\}$

$\Rightarrow F(3) = 5$

Putting $x = 2$, in eq. (i), we get

$F(4) = 2F(2) - F(3)$

$= 2(1) - 5 \quad \{\because F(2) = 1,\ F(3) = 5\}$

$\Rightarrow F(4) = -3$

Putting $x = 3$, in eq. (i), we get

$F(5) = 2F(3) - F(4)$

$= 2(5) + 3 \quad \{\because F(3) = 5, F(4) = -3\}$

$\Rightarrow F(5) = 13$

82. (c) The number of binary operations on a set S having n elements in n^{n^2}.

83. (a) $(3 - 5x)^{11} = 3^{11}\left(1 - \frac{5x}{3}\right)^{11}$

$= 3^{11}\left(1 - \frac{5}{3}.\frac{1}{5}\right)^{11} \quad \left\{\because x = \frac{1}{5}\right\}$

$= 3^{11}\left(1 - \frac{1}{3}\right)^{11}$

Now, $r = \frac{|x|(n+1)}{|x|+1} = \frac{\left|-\frac{1}{3}\right|(11+1)}{\left|-\frac{1}{3}\right|+1} = \frac{4}{\frac{4}{3}}$

$\Rightarrow r = 3$

Therefore, 3rd (T_3) and (3 + 1) = 4th (T_4) terms are numerically greatest in the expansion of $(3 - 5x)^{11}$.

So, greatest term = T_3

$= 3^{11}\left|{}^{11}C_2(1)^9\left(-\frac{1}{3}\right)^2\right| = 3^{11}\left|\frac{11\times 10}{1.2.9}\right|$

$= 55 \times 3^9$

and $T_4 = 3^{11}\left|{}^{11}C_3(1)^8\left(-\frac{1}{3}\right)^3\right|$

$= 3^{11}\left|\frac{11\times 10\times 9}{1.2.3}.\left(-\frac{1}{27}\right)\right| = 55\times 3^9$

$\therefore$ Greatest term (numerically)

$= T_3 = T_4 = 55 \times 3^9$

84. (a) We have, $\sin(e^x) = 5^x + 5^{-x}$ ...(i)

Let $5^x = t$, then eq. (i), reduces to

$\sin(e^x) = t + \frac{1}{t}$

$\Rightarrow \sin(e^x) = t + \frac{1}{t} - 2 + 2$

$\Rightarrow \sin(e^x) = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 2$

$\{\because 5^x > 0, \therefore \sqrt{5^x} = \sqrt{t}$ exists$\}$

$\Rightarrow \sin(e^x) \geq 2$

which is not possible as also $\sin\theta \leq 1$.

Thus, given equation has no solution.

85. (c) $a^x = b^y = c^z = d^u$

Let, $a^x = b^y = c^z = d^u = k$

$\Rightarrow a = k^{1/x},\ b = k^{1/y},\ c = k^{1/z},\ d = k^{1/u}$ ...(i)

a, b, c, d are in GP.

$\therefore \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\Rightarrow \frac{k^{1/y}}{k^{1/x}} = \frac{k^{1/z}}{k^{1/y}} = \frac{k^{1/u}}{k^{1/z}}$ {using Eq. (i)}

$\Rightarrow k^{\frac{1}{y}-\frac{1}{x}} = k^{\frac{1}{z}-\frac{1}{y}} = k^{\frac{1}{u}-\frac{1}{z}}$

$\Rightarrow \frac{1}{y} - \frac{1}{x} = \frac{1}{z} - \frac{1}{y} = \frac{1}{u} - \frac{1}{z}$

$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{u}$ are in A.P.

$\Rightarrow x, y, z, u$ are in H.P.

86. (b) **Given :** $|z| - z = 1 + 2i$

If $z = x + iy$, then this equation reduces to

$|x + iy| - (x + iy) = 1 + 2i$

$\Rightarrow (\sqrt{x^2 + y^2} - x) + (-iy) = 1 + 2i$

On comparing real and imaginary parts of both sides of this equation, we get

$\sqrt{x^2+y^2}-x=1$

$\Rightarrow \sqrt{x^2+y^2}=1+x \Rightarrow x^2+y^2=(1+x)^2$

$\Rightarrow x^2+y^2=1+x^2+2x$

$\Rightarrow y^2=1+2x$ ...(i)

and $-y=2$

$\Rightarrow y=-2$

Putting this value in eq. (i), we get

$(-2)^2=1+2x$

$\Rightarrow 2x=3 \Rightarrow x=\frac{3}{2}$

$\therefore z=x+iy=\frac{3}{2}-2i$

87. (c) $z=\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)$

$\Rightarrow z=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}$

$\Rightarrow z=\frac{1-3-2\sqrt{3}i}{1+3}=\frac{-2-2\sqrt{3}i}{4}$

$\Rightarrow z=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$\Rightarrow z=\cos 240°-i\sin 240°$

Thus, $\arg(z)=240°$

88. (d) $f(x)=\sqrt{\log_{10} x^2}$ is real, if

$\log_{10} x^2 \geq 0$

$\Rightarrow x^2 \geq 1$

$\Rightarrow x<-1$ and $x>1$

$\Rightarrow x \in (-\infty, -1] \cup [1, \infty)$

89. (c) The given expression is

$2x^2+mxy+3y^2-5y-2$

Comparing the given expression with

$ax^2+2hxy+by^2+2gx+2fy+c$,

we get

$a=2,\ h=\frac{m}{2},\ b=3,\ c=-2,\ g=0,\ f=-\frac{5}{2}$

The given expression is resolvable into linear factors, if

$abc+2fgh-af^2-bg^2-ch^2=0$

$(2)(3)(-2)+2(0)-2\left(\frac{25}{4}\right)-0-(-2)\frac{m^2}{4}=0$

$\Rightarrow -12-\frac{25}{2}+\frac{m^2}{2}=0$

$\Rightarrow \frac{m^2}{2}=\frac{49}{2} \Rightarrow m^2=49 \Rightarrow m=\pm 7$

90. (c) $\det(B^{-1}AB)=\det(B^{-1})\det A \det B$

$=\det(B^{-1})\cdot\det B\cdot\det A$

$=\det(B^{-1}B)\det A=\det I.\det A$

$=1\ .\det A=\det A$

91. (c) Since $f(x)$, $g(x)$ and $h(x)$ are the polynomials of degree 2,

therefore $f'''(x)=g'''(x)=h'''(x)=0$

Now, $\Delta(x)=\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

$\Rightarrow \Delta'(x)=\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

$+\begin{vmatrix} f(x) & g(x) & h(x) \\ f''(x) & g''(x) & h''(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$

$+\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f'''(x) & g'''(x) & h'''(x) \end{vmatrix}$

$\Rightarrow \Delta'(x)=0+0+0=0$

$\Rightarrow \Delta(x)=$ constant.

Thus, $\Delta(x)$ is the polynomial of degree zero.

92. (d) Let E_1, E_2 and E_3 denote the events of selecting boxes A, B, C respectively and A be the event that a screw selected at random is defective. Then,

$$P(E_1)=\frac{1}{3},\ P(E_2)=\frac{1}{3},\ P(E_3)=\frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right)=\frac{1}{5},\ P\left(\frac{A}{E_2}\right)=\frac{1}{6},\ P\left(\frac{A}{E_3}\right)=\frac{1}{7}$$

By Baye's rule, the required probability

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$\Rightarrow P\left(\frac{E_1}{A}\right)=\frac{\frac{1}{3}.\frac{1}{5}}{\frac{1}{3}.\frac{1}{5}+\frac{1}{3}.\frac{1}{6}+\frac{1}{3}.\frac{1}{7}}$$

$$=\frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{6}+\frac{1}{7}}=\frac{42}{107}$$

93. (c) $$\frac{\cos\theta}{1+\sin\theta}=\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{1+\cos\left(\frac{\pi}{2}-\theta\right)}$$

$$=\frac{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$$

$$=\frac{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}=\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

94. (b) $3\sin\theta+5\cos\theta=5 \Rightarrow 3\sin\theta=5(1-\cos\theta)$

$$\Rightarrow 3.2\sin\frac{\theta}{2}\cos\frac{\theta}{2}=5.2\sin^2\frac{\theta}{2}$$

$$\left(\begin{array}{l}\because \sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\\ \text{and } 1-\cos\theta=2\sin^2\frac{\theta}{2}\end{array}\right)$$

$$\Rightarrow \tan\frac{\theta}{2}=\frac{3}{5}$$

Now, $5\sin\theta-3\cos\theta$

$$=5.\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}-3.\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$=5.\frac{2.\frac{3}{5}}{\left(1+\frac{9}{25}\right)}-3.\frac{\left(1-\frac{9}{25}\right)}{\left(1+\frac{9}{25}\right)}$$

$$=\frac{6-3.\frac{16}{25}}{1+\frac{9}{25}}=\frac{150-48}{34}=\frac{102}{34}=3.$$

95. (a) $$\sin^{-1}\left\{\sin\frac{5\pi}{6}\right\}=\sin^{-1}\left\{\sin\left(\pi-\frac{\pi}{6}\right)\right\}$$

$[\because \sin(\pi-\theta)=\sin\theta]$

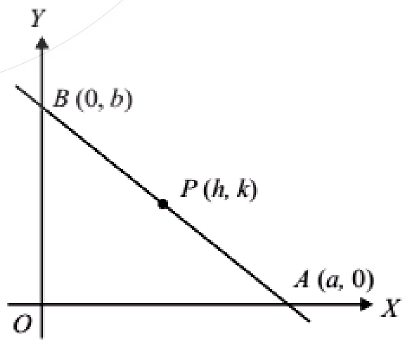
$$=\sin^{-1}\left\{\sin\frac{\pi}{6}\right\}$$

$[\because \text{Principal value} \in [0, \pi/2]]$

$$=\frac{\pi}{6}$$

which is the required principal value.

96. (a) Let both of the ends of the rod are on x-axis and y-axis. Let AB be rod of length l and coordinates of A and B be $(a, 0)$ and $(0, b)$, respectively.



Let $P(h, k)$ be the mid point of the rod AB.

Then, $\left.\begin{aligned} h &= \frac{0+a}{2} = \frac{a}{2} \\ k &= \frac{b+0}{2} = \frac{b}{2} \end{aligned}\right\}$ ...(i)

In ΔOAB,

$OA^2 + OB^2 = AB^2$

$\Rightarrow a^2 + b^2 = l^2$

$\Rightarrow (2h)^2 + (2k)^2 = l^2$ [using eq. (i)]

$\Rightarrow h^2 + k^2 = \frac{l^2}{4}$

$\therefore$ The equation of locus is

$x^2 + y^2 = \frac{l^2}{4}$

97. (a) Let $L_1 \equiv 2x + y - 1 = 0$

$L_2 \equiv 3x + 2y - 5 = 0$

The equation of straight line passing through the intersection point of the lines L_1 and L_2 is given by

$L_1 + \lambda L_2 = 0$

$\Rightarrow (2x + y - 1) + \lambda(3x + 2y - 5) = 0$

Since, this line passes through the origin also

$(0 + 0 - 1) + \lambda(0 + 0 - 5) = 0$

$\Rightarrow -5\lambda = 1 \Rightarrow \lambda = -\frac{1}{5}$

Required line is

$(2x + y - 1) - \frac{1}{5}(3x + 2y - 5) = 0$

$\Rightarrow \left(2 - \frac{3}{5}\right)x + \left(1 - \frac{2}{5}\right)y - 1 + 1 = 0$

$\Rightarrow \frac{7}{5}x + \frac{3}{5}y = 0 \Rightarrow 7x + 3y = 0$

98. (c) Let coordinates of P be (h, k), then

$h = \frac{2(10\cos\theta) + 3(5)}{2+3} = 4\cos\theta + 3$

and $k = \frac{2(10\sin\theta) + 3(0)}{2+3} = 4\sin\theta$

[Using the internal section formula]

$\Rightarrow \frac{h-3}{4} = \cos\theta$ and $\frac{k}{4} = \sin\theta$

Squaring and adding both of these equations,

$\frac{(h-3)^2}{16} + \frac{k^2}{16} = \cos^2\theta + \sin^2\theta$

$\Rightarrow (h-3)^2 + k^2 = 16$

Therefore, locus of point P is

$(x-3)^2 + y^2 = 16$ which is a circle.

99. (c) The equation of any normal to the parabola

$y^2 = -8x$ is $y = mx + 4m + 2m^3$ ...(i)

(using equation of normal of parabola in slope form $y = mx - 2am - am^3$ and $a = -2$)

The given normal is

$2x + y + k = 0 \Rightarrow y = -2x - k$ ...(ii)

Comparing eqs. (i) and (ii), we get

$m = -2$ and $-4m - 2m^3 = k$

$\Rightarrow k = 8 + 16 = 24$

100. (a) $\lim_{n\to\infty}\left[\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{n(n+1)}\right]$

$= \lim_{n\to\infty}\left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right]$

$= \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = \lim_{n\to\infty}\frac{n}{n+1}$

$= \lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)} = \lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)} = 1$

101. (d) Let $P(x_1, y_1)$ be a point on the curve

$y^2 = 4ax$ ...(i)

On differentiating $y^2 = 4ax$ w.r.t. 'x', we get

$2y\frac{dy}{dx} = 4a$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{2a}{y_1}$

Thus, the equation of normal at (x_1, y_1) is

$$y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

$$\Rightarrow y_1 x + 2ay = y_1(x_1 + 2a) \quad \text{...(ii)}$$

But $lx + my = 1$ ...(iii)

is also a normal.

Therefore, coefficients of eqs. (ii) and (iii), must be proportional.

i.e., $\frac{y_1}{l} = \frac{2a}{m} = \frac{y_1(x_1 + 2a)}{1}$

$$\Rightarrow y_1 = \frac{2al}{m} \text{ and } x_1 = \frac{1}{l} - 2a$$

Putting these values of x_1 and y_1 in eq. (i), we get

$$\left(\frac{2al}{m}\right)^2 = 4a\left(\frac{1}{l} - 2a\right)$$

$$\Rightarrow \frac{4a^2l^2}{m^2} = \frac{4a - 8a^2l}{l}$$

$$\Rightarrow al^3 = m^2 - 2am^2l \Rightarrow al^3 + 2alm^2 = m^2$$

102. (a) $\int f(x)\,dx = f(x)$

$$\Rightarrow \frac{d}{dx} f(x) = f(x)$$

$$\left[\because f(x) = \int \frac{d}{dx} f(x)dx\right]$$

Now, $\int \{f(x)\}^2 dx = \int f(x) \cdot f(x) dx$

$$= f(x)\int f(x)dx - \int\left[\frac{d}{dx} f(x)\int f(x)dx\right]dx$$

(integrating by parts)

$$= f(x)f(x) - \int f(x)f(x)dx$$

$$\Rightarrow 2\int \{f(x)\}^2 dx = \{f(x)\}^2$$

$$\Rightarrow \int \{f(x)\}^2 dx = \frac{1}{2}\{f(x)\}^2$$

103. (a) Let $I = \int \sin^{-1}\left\{\frac{2x+2}{\sqrt{4x^2+8x+13}}\right\}dx$

$$\Rightarrow I = \int \sin^{-1}\left\{\frac{2x+2}{\sqrt{4x^2+8x+4+9}}\right\}dx$$

$$\Rightarrow I = \int \sin^{-1}\left\{\frac{2x+2}{\sqrt{(2x+2)^2+3^2}}\right\}dx$$

Substituting $2x + 2 = 3\tan\theta$,

$\Rightarrow 2dx = 3\sec^2\theta d\theta$, we get

$$I = \int \sin^{-1}\left\{\frac{3\tan\theta}{3\sec\theta}\right\} \cdot \frac{3}{2}\sec^2\theta d\theta$$

$$\Rightarrow I = \frac{3}{2}\int \sin^{-1}(\sin\theta) \cdot \sec^2\theta d\theta$$

$$\Rightarrow I = \frac{3}{2}\int \theta\sec^2\theta d\theta$$

$$\Rightarrow I = \frac{3}{2}\left[\theta\tan\theta - \int \tan\theta d\theta\right]$$

(integrating by parts)

$$I = \frac{3}{2}[\theta\tan\theta - \log|\sec\theta|] + c$$

$$= \frac{3}{2}\left[\tan^{-1}\left(\frac{2x+2}{3}\right)\cdot\left(\frac{2x+2}{3}\right) - \log\sqrt{1+\left(\frac{2x+2}{3}\right)^2}\right] + c$$

$$= (x+1)\tan^{-1}\left(\frac{2x+2}{3}\right) - \frac{3}{4}\log\left(\frac{4x^2+8x+13}{9}\right) + c$$

104. (c) The equation of ellipse is

$$3x^2 + 2y^2 + 6x - 8y + 5 = 0$$

$$\Rightarrow 3(x^2 + 2x) + 2(y^2 - 4y) + 5 = 0$$

$$\Rightarrow 3(x^2 + 2x + 1) + 2(y^2 - 4y + 4) + 5 - 3 - 8 = 0$$

$\Rightarrow 3(x+1)^2+2(y-2)^2=6$

$\Rightarrow \frac{(x+1)^2}{2}+\frac{(y-2)^2}{3}=1$

Comparing with

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, we get

$h=-1, k=2, a^2=2, b^2=3$

Here, centre $(h, k)=(-1, 2)$

And using $a^2=b^2(1-e^2)$

$2=3(1-e^2)\Rightarrow e=\frac{1}{\sqrt{3}}$

And foci are $(h, k+be)$ and $(h, k-be)$

$=(-1, 2+1)$ and $(-1, 2-1)$

$=(-1, 3)$ and $(-1, 1)$

105. (b) We have the two hyperbolas as

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ...(i)

and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ ...(ii)

Any tangent to the hyperbola eq(i)

$y=mx+c$

where $c=\pm\sqrt{a^2m^2-b^2}$ ... (iii)

But this tangent touches the parabola eq. (ii) also

$\therefore \frac{(mx+c)^2}{a^2}-\frac{x^2}{b^2}=1$

$\Rightarrow b^2(m^2x^2+c^2+2mcx)-a^2x^2=a^2b^2$

$\Rightarrow (b^2m^2-a^2)x^2+2mcb^2x+b^2(c^2-a^2)=0$

For the tangency, it should have equal roots

$(2mcb^2)^2=4(b^2m^2-a^2)b^2(c^2-a^2)$

$\Rightarrow 4m^2c^2b^4=4b^2(b^2m^2c^2-b^2m^2a^2-a^2c^2+a^4)$

$\Rightarrow \quad c^2=a^2-b^2m^2$

$\Rightarrow \quad a^2m^2-b^2=a^2-b^2m^2$ [using Eq. (iii)]

$\Rightarrow \quad (a^2+b^2)m^2=a^2+b^2$

$\Rightarrow \quad m^2=1\Rightarrow m=\pm1$

Hence, the equation of common tangent are

$y=\pm x\pm\sqrt{a^2-b^2}$

106. (d) We have

$f(x)=\log_x \cos x$

$f(x)$ is defined for $\cos x>0$.

$x>0, x\neq 1$

$\cos x>0 \Rightarrow -\frac{\pi}{2}<x<\frac{\pi}{2}$

Also, $x>0, x\neq 1$

$\therefore$ Domain of f is $\left(0, \frac{\pi}{2}\right)-\{1\}$

107. (b) $y=\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$

For y to be defined $\left|\frac{x^2}{1+x^2}\right|<1$

which is true for all $x\in R$.

Now, $y=\sin^{-1}\left(\frac{x^2}{1+x^2}\right)$

$\Rightarrow \frac{x^2}{1+x^2}=\sin y\Rightarrow x=\sqrt{\frac{\sin y}{1-\sin y}}$

For the existance of x

$\sin y\geq 0$ and $1-\sin y>0$

$\Rightarrow 0\leq \sin y<1\Rightarrow 0\leq y<\frac{\pi}{2}$

Thus, range of the given function is $\left[0, \frac{\pi}{2}\right)$.

108. (c) $x=\sec\theta-\cos\theta$

$\Rightarrow \frac{dx}{d\theta}=\sec\theta\tan\theta+\sin\theta$,

$y=\sec^n\theta-\cos^n\theta$

$\Rightarrow \frac{dy}{d\theta}=n\sec^{n-1}\theta\sec\theta\tan\theta+n\cos^{n-1}\theta\sin\theta$

$\therefore \quad \frac{dy}{dx}=n\frac{(\sec^n\theta\tan\theta+\cos^{n-1}\theta\sin\theta)}{(\sec\theta\tan\theta+\sin\theta)}$

$\Rightarrow \frac{dy}{dx}=n\frac{(\sec^n\theta+\cos^n\theta)\tan\theta}{(\sec\theta+\cos\theta)\tan\theta}$

$$\Rightarrow \frac{dy}{dx} = \frac{n(\sec^n \theta + \cos^n \theta)}{(\sec\theta + \cos\theta)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{n^2\{(\sec^n \theta - \cos^n \theta)^2 + 4\}}{(\sec\theta - \cos\theta)^2 + 4}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{n^2(y^2+4)}{x^2+4}$$

$$\Rightarrow (x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

109. (d) $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\ldots\infty}}}}$

$$\Rightarrow y^2 = x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\ldots\infty}}}$$

$$\Rightarrow y^2 = x + \sqrt{y + y} \Rightarrow y^2 = x + \sqrt{2y}$$

$$\Rightarrow (y^2 - x)^2 = 2y$$

On differentiating both sides w.r.t. x, we get

$$2(y^2 - x)\left(2y\frac{dy}{dx} - 1\right) = 2\frac{dy}{dx}$$

$$\Rightarrow 2(y^3 - xy)\frac{dy}{dx} - (y^2 - x) = \frac{dy}{dx}$$

$$\Rightarrow (2y^3 - 2xy - 1)\frac{dy}{dx} = y^2 - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$$

110. (a) $\int_1^x \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{6}$

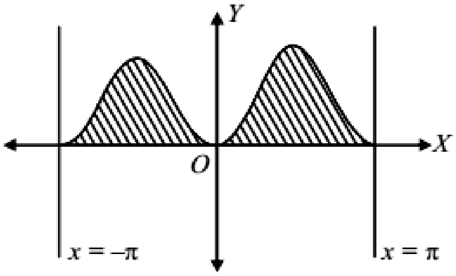
$$\Rightarrow [\sec^{-1} t]_1^x = \frac{\pi}{6}$$

$$\Rightarrow \sec^{-1} x - \sec^{-1} 1 = \frac{\pi}{6}$$

$$\Rightarrow \sec^{-1} x - 0 = \frac{\pi}{6} \Rightarrow x = \sec\frac{\pi}{6}$$

$$\Rightarrow x = \frac{2}{\sqrt{3}}$$

111. (c) Required area = Shaded area



$$= \int_{-\pi}^{\pi} |\sin x|\, dx = 2\int_0^{\pi} |\sin x|\, dx$$

$$= -2[\cos x]_0^{\pi} = -2(\cos\pi - \cos 0)$$

= 4 sq units

112. (d) Length of normal = c

$$\Rightarrow y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = c$$

$$\Rightarrow y^2\left[1 + \left(\frac{dy}{dx}\right)^2\right] = c^2$$

Clearly, this is the differential equation of degree 2.

113. (d) $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

Now,

$$\vec{a} + t\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + (-t\hat{i} + 2t\hat{j} + t\hat{k})$$

$$= (2-t)\hat{i} + (2+2t)\hat{j} + (3+t)\hat{k}$$

Since, $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$

$$(\vec{a} + t\vec{b}) \cdot \vec{c} = 0$$

$$\Rightarrow \{(2-t)\hat{i} + (2+2t)\hat{j} + (3+t)\hat{k}\} \cdot (3\hat{i} + \hat{j}) = 0$$

$$\Rightarrow 3(2-t) + 2 + 2t = 0$$

$$\Rightarrow 6 - 3t + 2 + 2t = 0 \Rightarrow t = 8$$

114. (c) The given line is

$$\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$$

On comparing it with $\vec{r} = \vec{a} + t\vec{b}$, we get

$\vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}, \quad \vec{b} = \hat{i} - \hat{j} + 4\hat{k}$

Also, the plane is

$\vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5$

On comparing it with $\vec{r} \cdot \vec{n} = d$, we get

$\vec{n} = \hat{i} + 5\hat{j} + \hat{k}$ and $d = 5$

Since, $\vec{b} \cdot \vec{n} = (\hat{i} - \hat{j} + 4\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k})$

$= 1 - 5 + 4 = 0$

$\therefore$ Given line is parallel to the given plane.

Now, distance between the line and the plane is given by required distance

$$= \left| \frac{\vec{a} \cdot \vec{n} - d}{|\vec{n}|} \right|$$

$$= \frac{|(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5|}{\sqrt{1 + 25 + 1}}$$

$$= \frac{|2 - 10 + 3 - 5|}{\sqrt{27}} = \frac{10}{3\sqrt{3}}$$

115. (a) The equation of the sphere concentric with the sphere

$x^2 + y^2 + z^2 - 4x - 6y - 8z - 5 = 0$ is

$x^2 + y^2 + z^2 - 4x - 6y - 8z + c = 0$ ...(i)

Since, this sphere eq. (i) passes through origin, therefore

$0 + 0 + 0 - 0 - 0 - 0 + c = 0$

$\Rightarrow c = 0$

Hence, the required equation of sphere is

$x^2 + y^2 + z^2 - 4x - 6y - 8z = 0$

116. (b) We have the lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} \quad \text{...(i)}$$

and $$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} \quad \text{...(ii)}$$

Let a point $(2r+1, 3r-1, 4r+1)$ be on the line Eq. (i). If this is an intersection point of both the lines, then it will lie on Eq. (ii), also

$$\therefore \frac{2r+1-3}{1} = \frac{3r-1-k}{2} = \frac{4r+1}{1} \quad \text{...(iii)}$$

Taking first and third part of eq. (iii), we get

$2r - 2 = 4r + 1$

$\Rightarrow r = -\frac{3}{2}$

Taking second and third part of eq. (iii), we get

$3r - 1 - k = 8r + 2$

$\Rightarrow 3r - 1 - k - 8r - 2 = 0 \Rightarrow k = -5r - 3$

$\Rightarrow k = -5\left(-\frac{3}{2}\right) - 3 \qquad \left(\because r = -\frac{3}{2}\right)$

$\Rightarrow k = \frac{15}{2} - 3 \Rightarrow k = \frac{9}{2}$

117. (a) Given curves are $y = 3^x$ ...(i)

and $y = 5^x$ ...(ii)

intersect at the point (0, 1).

Now, differentiating eqs. (i) and (ii) w.r.t. x, we get

$\frac{dy}{dx} = 3^x \log 3$ and $\frac{dy}{dx} = 5^x \log 5$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = \log 3$ and $\left(\frac{dy}{dx}\right)_{(0,1)} = \log 5$

$\Rightarrow m_1 = \log 3$ and $m_2 = \log 5$

Angle between these curves is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\Rightarrow \tan\theta = \frac{\log 3 - \log 5}{1 + \log 3 \cdot \log 5}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\log 3 - \log 5}{1 + \log 3 \log 5}\right)$$

118. (d) We have the given equation as

$\lambda x^2 + 4xy + y^2 + \lambda x + 3y + 2 = 0$

On comparing this equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = \lambda,\ h = 2,\ b = 1,\ g = \frac{\lambda}{2},\ f = \frac{3}{2},\ c = 2$

Since, given equation represents a parabola

$\therefore h^2 = ab \Rightarrow 4 = \lambda \cdot 1 \Rightarrow \lambda = 4$

119. (c) The given two circles are

$$2x^2+2y^2-3x+6y+k=0$$

$$\Rightarrow x^2+y^2-\frac{3}{2}x+3y+\frac{k}{2}=0 \quad \text{...(i)}$$

and $x^2+y^2-4x+10y+16=0$ ...(ii)

Since, general equation of circle is

$$x^2+y^2+2gx+2fy+c=0 \quad \text{...(iii)}$$

Therefore, comparing eqs. (i) and (ii) with eq. (iii), we get

$$g_1=-\frac{3}{4},\ f_1=\frac{3}{2},\ c_1=\frac{k}{2}$$

and $g_2=-2,\ f_2=5,\ c_2=16$

Both the circles cut orthogonally,

$$\therefore\ 2(g_1g_2+f_1f_2)=c_1+c_2$$

$$\Rightarrow 2\left(\frac{3}{2}+\frac{15}{2}\right)=\frac{k}{2}+16$$

$$\Rightarrow 18=\frac{k}{2}+16 \Rightarrow \frac{k}{2}=2 \Rightarrow k=4$$

120. (a) $A(-2, 1)$, $B(2, 3)$ and $C(-2, -4)$ are three given points.

Slope of the line BA

$$m_1=\frac{1-3}{-2-2}=\frac{1}{2}$$

$$\left(\text{Using slope formula, } m=\frac{y_2-y_1}{x_2-x_1}\right)$$

Slope of the line BC

$$m_2=\frac{-4-3}{-2-2}=\frac{7}{4}$$

Now, angle between AB and BC is given by

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{\frac{1}{2}-\frac{7}{4}}{1+\frac{1}{2}\cdot\frac{7}{4}}\right|$$

$$\Rightarrow \tan\theta=\left|-\frac{10}{15}\right| \Rightarrow \tan\theta=\left|-\frac{2}{3}\right|$$

$$\Rightarrow \theta=\tan^{-1}\left|-\frac{2}{3}\right| \Rightarrow \theta=\tan^{-1}\left(\frac{2}{3}\right)$$

$$[\because |-x|=x]$$