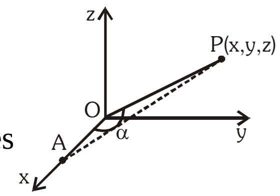
Important point :

Direction cosines of a line have two sets but direction ratios of a line have infinite possible sets.

7. PROJECTIONS :

(a) Projection of line segment OP on co-ordinate axes :



Let line segment make angle α with x-axis

Thus, the projections of line segment OP on axes are the absolute values of the co-ordinates of P. i.e.

Projection of OP on x-axis = $|x|$

Projection of OP on y-axis = $|y|$

Projection of OP on z-axis = $|z|$

Now, in ΔOAP, angle A is a right angle and OA = x

$OP = \sqrt{x^2 + y^2 + z^2}$

$$\therefore \quad \cos\alpha = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{|OP|}$$

if $|OP| = r$, then $x = |OP|\cos\alpha = \ell r$

Similarly $y = |OP|\cos\beta = mr$, $z = nr$, where ℓ, m, n are DC's of line

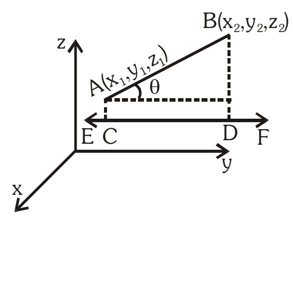
(b) Projection of a line segment AB on coordinate axes :

Projection of the point $A(x_1, y_1, z_1)$ on x-axis is $E(x_1, 0, 0)$. Projection of point $B(x_2, y_2, z_2)$ on x-axis is $F(x_2, 0, 0)$.

Hence projection of AB on x-axis is $EF = |x_2 - x_1|$.

Similarly, projection of AB on y and z-axis are $|y_2 - y_1|$, $|z_2 - z_1|$ respectively.

(c) Projection of line segment AB on a line having direction cosines ℓ, m, n :



Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$.

Now projection of AB on EF = CD = AB $\cos\theta$

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \times \frac{|(x_2-x_1)\ell+(y_2-y_1)m+(z_2-z_1)n|}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

$$= |(x_2-x_1)\ell+(y_2-y_1)m+(z_2-z_1)n|$$

Illustration 5 : A line OP makes with the x-axis an angle of measure 120° and with y-axis an angle of measure 60°. Find the angle made by the line with the z-axis.

Solution : $\alpha = 120°$ and $\beta = 60°$

$\therefore \cos\alpha = \cos 120° = -\frac{1}{2}$ and $\cos\beta = \cos 60° = \frac{1}{2}$ but $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2} \Rightarrow \cos\gamma = \pm\frac{1}{\sqrt{2}}$

$\therefore \gamma = 45°$ or $135°$ **Ans.**

Illustration 6 : Find the length of projection of the line segment joining the points (–1, 0, 3) and (2, 5, 1) on the line whose direction ratios are 6, 2, 3.

Solution : The direction cosines ℓ, m, n of the line are given by $\frac{\ell}{6} = \frac{m}{2} = \frac{n}{3} = \frac{\sqrt{\ell^2+m^2+n^2}}{\sqrt{6^2+2^2+3^2}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$

$\therefore \ell = \frac{6}{7}, m = \frac{2}{7}, n = \frac{3}{7}$

The required length of projection is given by

$= |\ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)| = \left|\frac{6}{7}[2-(-1)] + \frac{2}{7}(5-0) + \frac{3}{7}(1-3)\right|$

$= \left|\frac{6}{7}\times 3 + \frac{2}{7}\times 5 + \frac{3}{7}\times -2\right| = \left|\frac{18}{7} + \frac{10}{7} - \frac{6}{7}\right| = \left|\frac{18+10-6}{7}\right| = \frac{22}{7}$ **Ans.**

Do yourself - 2 :

(i) Find the length of projections of the line segment joining the origin O to the point P(3, 2, –5) on the axes.

(ii) Find the length of projections of the line joining the points P(3, 2, 5) and Q(0, –2, 8) on the axes.

(iii) Find the direction ratios & direction cosines of the line joining the points O(0, 0, 0) and P(2, 3, 4).

11. ANGLE BETWEEN TWO LINES :

Let θ be the angle between the lines with d.c.'s ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 then $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$.

If a_1, b_1, c_1 and a_2, b_2, c_2 be D.R.'s of two lines then angle θ between them is given by

$$\cos\theta = \frac{(a_1a_2 + b_1b_2 + c_1c_2)}{\sqrt{(a_1^2+b_1^2+c_1^2)}\sqrt{(a_2^2+b_2^2+c_2^2)}}$$

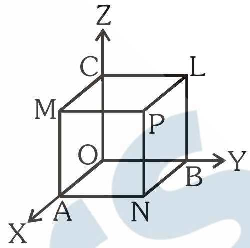
Illustration 7 : If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ equals -

(A) 3 (B) 4 (C) 4/3 (D) 3/4

Solution : Let OA, OB, OC be coterminous edges of a cube and OA = OB = OC = a, then co-ordinates of its vertices are O(0, 0, 0), A(a, 0, 0), B(0, a, 0), C(0, 0, a), L(0, a, a), M(a, 0, a), N(a, a, 0) and P (a, a, a)

Direction ratio of diagonal AL, BM, CN and OP are

$$\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right),\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$$



Let ℓ, m, n be the direction cosines of the given line, then

$$\cos\alpha = \ell\left(-\frac{1}{\sqrt{3}}\right)+m\left(\frac{1}{\sqrt{3}}\right)+n\left(\frac{1}{\sqrt{3}}\right)=\frac{-\ell+m+n}{\sqrt{3}}$$

Similarly $\cos\beta = \frac{\ell-m+n}{\sqrt{3}}$, $\cos\gamma=\frac{\ell+m-n}{\sqrt{3}}$ and $\cos\delta=\frac{\ell+m+n}{\sqrt{3}}$

$\therefore$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$ **Ans. (C)**

Illustration 8 : (a) Find the acute angle between two lines whose direction ratios are 2, 3, 6 and 1, 2, 2 respectively.

(b) Find the measure of the angle between the lines whose direction ratios are 1, –2, 7 and 3, –2, –1.

Solution : (a) $a_1 = 2, b_1 = 3, c_1 = 6; a_2 = 1, b_2 = 2, c_2 = 2.$

If θ be the angle between two lines whose d.r's are given, then

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}=\frac{2\times1+3\times2+6\times2}{\sqrt{2^2+3^2+6^2}\sqrt{1^2+2^2+2^2}}=\frac{2+6+12}{7\times3}=\frac{20}{21}$$

$$\therefore\quad \theta=\cos^{-1}\left(\frac{20}{21}\right)$$

(b) $\sqrt{1^2+(-2)^2+7^2}=\sqrt{54}$

$\sqrt{3^2+(-2)^2+(-1)^2}=\sqrt{14}$

$\therefore$ The actual direction cosines of the lines are

$$\frac{1}{\sqrt{54}},\frac{-2}{\sqrt{54}},\frac{7}{\sqrt{54}} \quad \text{and} \quad \frac{3}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-1}{\sqrt{14}}$$

If θ is the angle between the lines, then

$$\cos\theta=\left(\frac{1}{\sqrt{54}}\right)\left(\frac{3}{\sqrt{14}}\right)+\left(\frac{-2}{\sqrt{54}}\right)\left(\frac{-2}{\sqrt{14}}\right)+\left(\frac{7}{\sqrt{54}}\right)\left(\frac{-1}{\sqrt{14}}\right)$$

$$=\frac{3+4-7}{\sqrt{54}.\sqrt{14}}=0 \Rightarrow \theta=90^\circ$$ **Ans.**

12. PERPENDICULAR AND PARALLEL LINES :

Let the two lines have their d.c.'s given by ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 respectively then they are perpendicular if $\theta = 90°$ i.e. $\cos\theta = 0$, i.e. $\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$.

Also the two lines are parallel if $\theta = 0$ i.e. $\sin\theta = 0$, i.e. $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

Note: If instead of d.c.'s, d.r.'s a_1, b_1, c_1 and a_2, b_2, c_2 are given, then the lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ and parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Illustration 9 : If the lines whose direction cosines satisfies the equations $a\ell + bm + cn = 0$ and $fmn + gn\ell + h\ell m = 0$ are perpendicular, then $\frac{f}{a} + \frac{g}{b} + \frac{h}{c}$ equals -

(A) 0 (B) –1 (C) 1 (D) none of these

Solution : Eliminating n between the given relations, we find that $(fm + g\ell)\left(\frac{-a\ell - bm}{c}\right) + h\ell m = 0$

or $ag\left(\frac{\ell}{m}\right)^2 + (af + bg - ch)\left(\frac{\ell}{m}\right) + bf = 0$ ....(i)

Let $\frac{\ell_1}{m_1}$ and $\frac{\ell_2}{m_2}$, are roots of (i), then $\frac{\ell_1}{m_1} \cdot \frac{\ell_2}{m_2} = \frac{bf}{ag}$

$\Rightarrow \frac{\ell_1\ell_2}{f/a} = \frac{m_1m_2}{g/b}$ ....(ii)

Similarly $\frac{m_1m_2}{g/b} = \frac{n_1n_2}{h/c}$ ....(iii)

From (ii) and (iii), we get $\frac{\ell_1\ell_2}{f/a} = \frac{m_1m_2}{g/b} = \frac{n_1n_2}{h/c} = \lambda$

$\Rightarrow \ell_1\ell_2 = \lambda.f/a$; $m_1m_2 = \lambda.g/b$; $n_1n_2 = \lambda.h/c$

$\Rightarrow \ell_1\ell_2 + m_1m_2 + n_1n_2 = \lambda\left(\frac{f}{a} + \frac{g}{b} + \frac{h}{c}\right)$

$\Rightarrow \frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ $\{\because \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0\}$ **Ans. (A)**

Do yourself - 3 :

(i) Find the angle between the lines whose direction ratios are 1, –2, 1 and 4, 3, 2.

(ii) If a line makes α, β and γ angle with axes, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

(iii) Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to (1, –2, –2) & (0, 2, 1).

PLANE

13. DEFINITION :

A plane is a surface such that a line segment joining any two points on the surface lies wholly on it.

14. EQUATIONS OF A PLANE :

The equation of every plane is of the first degree i.e. of the form ax + by + cz + d = 0, in which a, b, c are constants, not all zero simultaneously.

(a) Equation of plane passing through a fixed point :

Vector form : If $\vec{a}$ is the position vector of a point on the plane and $\vec{n}$ is a vector normal to the plane then its vectorial equation is given by $(\vec{r}-\vec{a}).\vec{n}=0 \Rightarrow \mathbf{\vec{r}.\vec{n}=d}$, where $d=\vec{a}.\vec{n}=$ constant.

Cartesian form : If $\vec{a}(x_1,y_1,z_1)$ and $\vec{n}=a\hat{i}+b\hat{j}+c\hat{k}$, then cartesian equation of plane will be

$\mathbf{a(x-x_1)+b(y-y_1)+c(z-z_1)=0}$

(b) Plane Parallel to the Coordinate Planes :

(i) Equation of yz plane is x = 0.

(ii) Equation of zx plane is y = 0.

(iii) Equation of xy plane is z = 0.

(iv) Equation of the plane parallel to xy plane at a distance c is z = c or z = –c.

(v) Equation of the plane parallel to yz plane at a distance c is x = c or x = –c

(vi) Equation of the plane parallel to zx plane at a distance c is y = c or y = –c.

(c) Equations of Planes Parallel to the Axes :

If a = 0, the plane is parallel to x-axis i.e. equation of the plane parallel to x-axis is **by + cz + d = 0**.

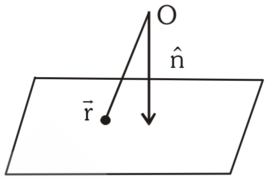
Similarly, equations of planes parallel to y-axis and parallel to z-axis are **ax + cz + d = 0** and **ax + by + d = 0**, respectively.

(d) Equation of a Plane in Intercept Form :

Equation of the plane which cuts off intercepts a, b, c from the axes x, y, z respectively is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$$

(e) Equation of a Plane in Normal Form :



Vector form : If $\hat{n}$ is a unit vector normal to the plane from the origin and d be the perpendicular distance of plane from origin then its vector equation is $\vec{r}\cdot\hat{n} = d$.

Cartesian form : If the length of the perpendicular distance of the plane from the origin is p and direction cosines of this perpendicular are (ℓ, m, n), then the equation of the plane is $\ell x + my + nz = p$.

(f) Equation of a Plane through three points :

Vector form : If A, B, C are three points having P.V.'s $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively, then vector equation of the plane is $[\vec{r}\ \vec{a}\ \vec{b}] + [\vec{r}\ \vec{b}\ \vec{c}] + [\vec{r}\ \vec{c}\ \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]$.

Cartesian form : The equation of the plane through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

Illustration 10 : Find the equation of the plane through the points A(2, 2, –1), B(3, 4, 2) and C(7, 0, 6).

Solution : The general equation of a plane passing through (2, 2, –1) is

$a(x-2) + b(y-2) + c(z+1) = 0$ ....(i)

It will pass through B (3, 4, 2) and C (7, 0, 6) if

$a(3-2) + b(4-2) + c(2+1) = 0$ or $a + 2b + 3c = 0$ ....(ii)

and $a(7-2) + b(0-2) + c(6+1) = 0$ or $5a - 2b + 7c = 0$ ....(iii)

Solving (ii) and (iii) by cross-multiplication, we have

$$\frac{a}{14+6} = \frac{b}{15-7} = \frac{c}{-2-10} \text{ or } \frac{a}{5} = \frac{b}{2} = \frac{c}{-3} = \lambda \quad \text{(say)}$$

$\Rightarrow$ $a = 5\lambda$, $b = 2\lambda$ and $c = -3\lambda$

Substituting the values of a, b and c in (i), we get

$5\lambda(x-2) + 2\lambda(y-2) - 3\lambda(z+1) = 0$

or $5(x-2) + 2(y-2) - 3(z+1) = 0$

$\Rightarrow$ $5x + 2y - 3z = 17$, which is the required equation of the plane **Ans.**

Illustration 11 : A plane meets the co-ordinates axes in A,B,C such that the centroid of the Δ ABC is the point (p,q,r) show that the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r} = 3$

Solution : Let the required equation of plane be :

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1 \qquad \text{......(i)}$$

Then, the co-ordinates of A, B and C are A(a, 0, 0), B(0, b, 0), C(0, 0, c) respectively

So the centroid of the triangle ABC is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

But the co-ordinate of the centroid are (p,q,r)

$$\frac{a}{3} = p,\ \frac{b}{3} = q,\ \frac{c}{3} = r$$

Putting the values of a, b and c in (i), we get the required plane as $\frac{x}{3p}+\frac{y}{3q}+\frac{z}{3r} = 1$

$$\Rightarrow \quad \frac{x}{p}+\frac{y}{q}+\frac{z}{r} = 3$$

Ans.

Do yourself - 4 :

(i) Equation of a plane is 3x + 4y + 5z = 7.

(a) Find the direction cosines of its normal

(b) Find the points where it intersects the axes.

(c) Find its intercept form.

(d) Find its equation in normal form (in cartesian as well as in vector form)

(ii) Find the equation of the plane passing through the points (2, 3, 1), (3, 0, 2) and (–1, 2, 3).

15. ANGLE BETWEEN TWO PLANES :

Vector form : If $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$ be two planes, then angle between these planes is the angle between their normals

$$\cos\theta = \frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$\therefore$ Planes are perpendicular if $\vec{n}_1.\vec{n}_2 = 0$ and they are parallel if $\vec{n}_1 = \lambda\vec{n}_2$.

Cartesian form : Consider two planes ax + by + cz + d = 0 and a' x + b' y + c' z + d' = 0. Angle between these planes is the angle between their normals.

$$\cos\theta = \frac{aa'+bb'+cc'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}$$

$\therefore$ Planes are perpendicular if aa' + bb' + cc' = 0 and they are parallel if $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$.

Planes parallel to a given Plane :

Equation of a plane parallel to the plane ax + by + cz + d = 0 is ax + by + cz + d' = 0. d' is to be found by other given condition.

Illustration 12 : Find the angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$

Solution : We know that the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

Therefore, angle between $x + y + 2z = 9$ and $2x - y + z = 15$ is given by

$$\cos\theta = \frac{(1)(2) + (1)(-1) + (2)(1)}{\sqrt{1^2 + 1^2 + 2^2}\sqrt{2^2 + (-1)^2 + 1^2}} = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$$ **Ans.**

Illustration 13: Find the equation of the plane through the point (1, 4, –2) and parallel to the plane $-2x + y - 3z = 7$.

Solution : Let the equation of a plane parallel to the plane $-2x + y - 3z = 7$ be $-2x + y - 3z + k = 0$

This passes through (1, 4, –2), therefore $(-2)(1) + 4 - 3(-2) + k = 0$

$\Rightarrow \quad -2 + 4 + 6 + k = 0 \quad \Rightarrow \quad k = -8$

Putting $k = -8$ in (i), we obtain $\quad -2x + y - 3z - 8 = 0 \quad$ or $\quad -2x + y - 3z = 8$

Ans.

This is the equation of the required plane.

Do yourself - 5 :

(i) Prove that the planes $3x - 2y + z + 17 = 0$ and $4x + 3y - 6z - 25 = 0$ are perpendicular.

(ii) Find the angle between the planes $3x + 4y + z + 7 = 0$ and $-x + y - 2z = 5$

16. A PLANE THROUGH THE LINE OF INTERSECTION OF TWO GIVEN PLANES :

Consider two planes $u \equiv ax + by + cz + d = 0$ and $v \equiv a'x + b'y + c'z + d' = 0$.

The equation $u + \lambda v = 0$, λ a real parameter, represents the plane passing through the line of intersection of given planes and if planes are parallel, this represents a plane parallel to them.

Illustration 14 : Find the equation of plane containing the line of intersection of the plane $x + y + z - 6 = 0$ and $2x + 3y + 4z + 5 = 0$ and passing through (1,1,1).

Solution : The equation of the plane through the line of intersection of the given planes is,

$$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0 \qquad ......(i)$$

If it is passes through (1,1,1)

$\Rightarrow \quad (1 + 1 + 1 - 6) + \lambda(2 + 3 + 4 + 5) = 0 \quad \Rightarrow \quad \lambda = \dfrac{3}{14}$

Putting $\lambda = 3/14$ in (i); we get $(x + y + z - 6) + \dfrac{3}{14}(2x + 3y + 4z + 5) = 0$

$\Rightarrow \quad 20x + 23y + 26z - 69 = 0$ **Ans.**

17. PERPENDICULAR DISTANCE OF A POINT FROM THE PLANE :

Vector form : If $\vec{r}.\vec{n} = d$ be the plane, then perpendicular distance p, of the point A($\vec{a}$)

$$p = \frac{|\vec{a}.\vec{n} - d|}{|\vec{n}|}$$

Distance between two parallel planes $\vec{r}.\vec{n} = d_1$ & $\vec{r}.\vec{n} = d_2$ is $\left|\frac{d_1 - d_2}{|\vec{n}|}\right|$.

Cartesian form : Perpendicular distance p, of the point $A(x_1, y_1, z_1)$ from the plane $ax + by + cz + d = 0$

is given by $p = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{(a^2 + b^2 + c^2)}}$

Distance between two parallel planes $ax + by + cz + d_1 = 0$ & $ax + by + cz + d_2 = 0$ is $\left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right|$

Illustration 15 : Find the perpendicular distance of the point (2, 1, 0) from the plane $2x + y + 2z + 5 = 0$

Solution : We know that the perpendicular distance of the point (x_1, y_1, z_1) from the plane

$ax + by + cz + d = 0$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

so required distance = $\frac{|2 \times 2 + 1 \times 1 + 2 \times 0 + 5|}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{10}{3}$ **Ans.**

Illustration 16: Find the distance between the parallel planes $2x - y + 2z + 3 = 0$ and $4x - 2y + 4z + 5 = 0$.

Solution : Let $P(x_1, y_1, z_1)$ be any point on $2x - y + 2z + 3 = 0$, then $2x_1 - y_1 + 2z_1 + 3 = 0$

The length of the perpendicular from $P(x_1, y_1, z_1)$ to $4x - 2y + 4z + 5 = 0$ is

$$\left|\frac{4x_1 - 2y_1 + 4z_1 + 5}{\sqrt{4^2 + (-2)^2 + 4^2}}\right| = \left|\frac{2(2x_1 - y_1 + 2z_1) + 5}{\sqrt{36}}\right| = \frac{|2(-3) + 5|}{6} = \frac{1}{6} \text{ [using (i)]}$$

Therefore, the distance between the two given parallel planes is $\frac{1}{6}$ **Ans.**

Do yourself - 6 :

(i) Find the perpendicular distance of the point P(1, 2, 3) from the plane $2x + y + z + 1 = 0$.

(ii) Find the equation of the plane passing through the line of intersection of the planes $x + y + z = 5$ and $2x + 3y + z + 5 = 0$ and passing through the point (0, 0, 0).

18. BISECTORS OF ANGLES BETWEEN TWO PLANES :

Let the equations of the two planes be $ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$.

Then equations of bisectors of angles between them are given by

$$\frac{ax + by + cz + d}{\sqrt{(a^2 + b^2 + c^2)}} = \pm \frac{a_1x + b_1y + c_1z + d_1}{\sqrt{(a_1^2 + b_1^2 + c_1^2)}}$$

(a) **Equation of bisector of the angle containing origin :** First make both constant terms positive. Then positive sign give the bisector of the angle which contains the origin.

(b) **Bisector of acute/obtuse angle :** First making both constant terms positive,

$aa_1 + bb_1 + cc_1 > 0 \quad \Rightarrow$ origin lies in obtuse angle

$aa_1 + bb_1 + cc_1 < 0 \quad \Rightarrow$ origin lies in acute angle

Illustration 17: Find the equation of the bisector planes of the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$ and specify the plane which bisects the acute angle and the plane which bisects the obtuse angle.

Solution : The two given planes are $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$

where $d_1, d_2 > 0$

and $a_1a_2 + b_1b_2 + c_1c_2 = 6 + 2 + 12 > 0$

$$\therefore \quad \frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = -\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}} \quad \text{(acute angle bisector)}$$

$$\text{and} \quad \frac{a_1x+b_1y+c_1z+d}{\sqrt{a_1^2+b_1^2+c_1^2}} = \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}} \quad \text{(obtuse angle bisector)}$$

$$\text{i.e.,} \quad \frac{2x-y+2z+3}{\sqrt{4+1+4}} = \pm\frac{3x-2y+6z+8}{\sqrt{9+4+36}}$$

$\Rightarrow \quad (14x - 7y + 14z + 21) = \pm (9x - 6y + 18z + 24)$

Taking positive sign on the right hand side,

we get $\quad 5x - y - 4z - 3 = 0 \quad$ (obtuse angle bisector)

and taking negative sign on the right hand side,

we get $\quad 23x - 13y + 32z + 45 = 0 \quad$ (acute angle bisector) **Ans.**

19. POSITION OF TWO POINTS W.R.T. A PLANE :

Two points $P(x_1, y_1, z_1)$ & $Q(x_2, y_2, z_2)$ are on the same or opposite sides of a plane $ax + by + cz + d = 0$ according to $ax_1 + by_1 + cz_1 + d$ & $ax_2 + by_2 + cz_2 + d$ are of same or opposite signs. The plane divides the line joining the points P & Q externally or internally according to P and Q lying on same or opposite sides of the plane.

Do yourself - 7 :

(i) Find the position of the point P(2, –2, 1), Q(3, 0, 1) and R(–12, 1, 8) w.r.t. the plane $2x - 3y + 4z - 7 = 0$.

(ii) Two given planes are $-2x + y - 2z + 5 = 0$ and $6x - 2y + 3z - 7 = 0$. Find

(a) equation of plane bisecting the angle between the planes.

(b) equation of a plane parallel to the plane bisecting the angle between both the two planes and passing through the point (3, 2, 0).

(c) specify which plane is acute angle bisector and which one is obtuse angle bisector.

STRAIGHT LINE

20. DEFINITION :

A straight line in space is characterised by the intersection of two planes which are not parallel and, therefore, the equation of a straight line is present as a solution of the system constituted by the equations of the two planes : $a_1 x + b_1 y + c_1 z + d_1 = 0$; $a_2 x + b_2 y + c_2 z + d_2 = 0$

This form is also known as **unsymmetrical form**.

Some particular straight lines :

	Straight lines	Equation
(i)	Through the origin	$y = mx,\ z = nx$
(ii)	x-axis	$y = 0, z = 0$ or $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$
(iii)	y-axis	$x = 0, z = 0$ or $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$
(iv)	z-axis	$x = 0, y = 0$ or $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$
(v)	parallel to x-axis	$y = p, z = q$
(vi)	parallel to y-axis	$x = h, z = q$
(vii)	parallel to z-axis	$x = h, y = p$

21. EQUATION OF A STRAIGHT LINE IN SYMMETRICAL FORM :

(a) **One point form :** Let $A(x_1, y_1, z_1)$ be a given point on the straight line and ℓ, m, n be the d.c's of the line, then its equation is

$$\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r \quad \text{(say)}$$

It should be noted that $P(x_1 + \ell r, y_1 + mr, z_1 + nr)$ is a general point on this line at a distance r from the point $A(x_1, y_1, z_1)$ i.e. AP = r. One should note that for AP = r; ℓ, m, n must be d.c.'s not d.r.'s. If a, b, c are direction ratios of the line, then equation of the line is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r \quad \text{but here } AP \neq r$$

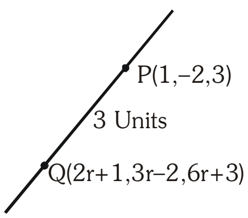
(b) Equation of the line through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Illustration 18 : Find the co-ordinates of those points on the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6}$ which is at a distance of 3 units from point (1,–2, 3).

Solution : Here, $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6}$ ....(i)

is the given straight line



Let, P = (1,–2,3) on the straight line

Here direction ratios of line (i) are (2,3,6)

$\therefore$ Direction cosines of line (i) are : $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

$\Rightarrow$ Equations of line (i) can be written as

$$\frac{x-1}{2/7} = \frac{y+2}{3/7} = \frac{z-3}{6/7} \qquad ......(ii)$$

Co-ordinates of any point on the line (ii) can be taken as $\left(\frac{2}{7}r+1, \frac{3}{7}r-2, \frac{6}{7}r+3\right)$

Let, $Q\left(\frac{2}{7}r+1, \frac{3}{7}r-2, \frac{6}{7}r+3\right)$

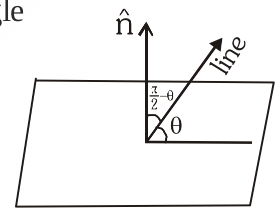
Given $|\vec{r}| = 3, \therefore r = \pm 3$

Putting the value of r, we have $Q\left(\frac{13}{7}, -\frac{5}{7}, \frac{39}{7}\right)$ or $Q = \left(\frac{1}{7}, -\frac{23}{7}, \frac{3}{7}\right)$ **Ans.**

---

22. ANGLE BETWEEN A LINE AND A PLANE :

Let equations of the line and plane be $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and ax + by + cz + d = 0 respectively and θ be the angle which line makes with the plane. Then $(\pi/2 - \theta)$ is the angle between the line and the normal to the plane.



So, $\sin\theta = \frac{a\ell + bm + cn}{\sqrt{(a^2+b^2+c^2)}\sqrt{(\ell^2+m^2+n^2)}}$

Line is parallel to plane if $\theta = 0$ i.e. if $a\ell + bm + cn = 0$.

Line is perpendicular to the plane if line is parallel to the normal of the plane i.e. if $\frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$.

Illustration 19 : Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$ and the plane 3x + 4y + z + 5 = 0.

Solution : The given line is $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$ .... (i)

and the given plane is 3x + 4y + z + 5 = 0 .... (ii)

If the line (i) makes angle θ with the plane (ii), then the line (i) will make angle $(90° - \theta)$ with the normal to the plane (i). Now direction-ratios of line (i) are 3, –1, –2 and direction-ratios of normal to plane (ii) are 3, 4, 1

$$\therefore \quad \cos(90° - \theta) = \frac{(3)(3)+(-1)(4)+(-2)(1)}{\sqrt{9+1+4}\sqrt{9+16+1}} \quad \Rightarrow \quad \sin\theta = \frac{9-4-2}{\sqrt{14}\sqrt{26}} = \frac{3}{\sqrt{14}\sqrt{26}}$$

Hence $\theta = \sin^{-1}\left(\frac{3}{\sqrt{14}\sqrt{26}}\right)$ **Ans.**

Do yourself - 8 :

(i) Find the equation of the line passing through the point (4, 2, 3) and having direction ratios 1, –1, 2

(ii) Find the symmetrical form of the line $x - y + 2z = 5$, $3x + y + z = 6$.

(iii) Find the angle between the plane $3x + 4y + 5 = 0$ and the line $\frac{x-1}{2} = \frac{y-2}{0} = \frac{z-1}{1}$.

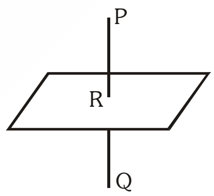
(iv) Prove that the line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ is parallel to the plane $4x + 4y - 5z + 2 = 0$.

23. CONDITION THAT A LINE LIES ON THE GIVEN PLANE :

The line $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ will lie on the plane $Ax + By + Cz + D = 0$ if

(a) $A\ell + Bm + Cn = 0$ and **(b)** $Ax_1 + By_1 + Cz_1 + D = 0$

24. IMAGE OF A POINT IN THE PLANE :



In order to find the image of a point $P(x_1, y_1, z_1)$ in a plane $ax + by + cz + d = 0$, assume it as a mirror. Let $Q(x_2, y_2, z_2)$ be the image of the point $P(x_1, y_1, z_1)$ in the plane, then

(a) Line PQ is perpendicular to the plane. Hence equation of PQ is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = r$

(b) Hence, Q satisfies the equation of line then $\frac{x_2-x_1}{a} = \frac{y_2-y_1}{b} = \frac{z_2-z_1}{c} = r$. The plane passes through the middle point of line PQ, therefore the middle point satisfies the equation of the plane i.e. $a\left(\frac{x_2+x_1}{2}\right) + b\left(\frac{y_2+y_1}{2}\right) + c\left(\frac{z_2+z_1}{2}\right) + d = 0$. The co-ordinates of Q can be obtained by solving these equations.

25. FOOT, LENGTH AND EQUATION OF PERPENDICULAR FROM A POINT TO A LINE:

Let equation of the line be $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r$ ....(i)

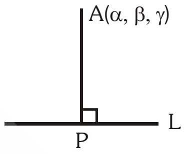
and A (α, β, γ) be the point. Any point on the line (i) is $P(\ell r + x_1, mr + y_1, nr + z_1)$ ....(ii)

If it is the foot of the perpendicular, from A on the line, then AP is ⊥ to the line, so

$$\ell(\ell r + x_1 - \alpha) + m(mr + y_1 - \beta) + n(nr + z_1 - \gamma) = 0$$

i.e. $r = (\alpha - x_1)\ell + (\beta - y_1)m + (\gamma - z_1)n$

since $\ell^2 + m^2 + n^2 = 1$



Putting this value of r in (ii), we get the foot of perpendicular from point A to the line.

Length : Since foot of perpendicular P is known, length of perpendicular,

$$AP = \sqrt{[(\ell r + x_1 - \alpha)^2 + (mr + y_1 - \beta)^2 + (nr + z_1 - \gamma)^2]}$$

Equation of perpendicular is given by

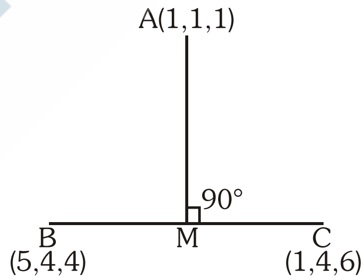
$$\frac{x-\alpha}{\ell r + x_1 - \alpha} = \frac{y-\beta}{mr + y_1 - \beta} = \frac{z-\gamma}{nr + z_1 - \gamma}$$

Illustration 20 : Find the co-ordinates of the foot of the perpendicular from (1, 1, 1) on the line joining (5, 4, 4) and (1, 4, 6).

Solution : Let A (1, 1, 1), B (5, 4, 4) and C (1, 4, 6) be the given points. Let M be the foot of the perpendicular from A on BC.

If M divides BC in the ratio λ : 1, then

co-ordinates of M are $\left(\frac{\lambda+5}{\lambda+1}, \frac{4\lambda+4}{\lambda+1}, \frac{6\lambda+4}{\lambda+1}\right)$



Direction ratios of BC are 1 – 5, 4 – 4, 6 – 4

i.e. –4, 0, 2

D.R.'s of AM are $\frac{\lambda+5}{\lambda+1} - 1, \frac{4\lambda+4}{\lambda+1} - 1, \frac{6\lambda+4}{\lambda+1} - 1$

$\Rightarrow \quad \frac{4}{\lambda+1}, \frac{3\lambda+3}{\lambda+1}, \frac{5\lambda+3}{\lambda+1} \quad \Rightarrow \quad 4, 3\lambda + 3, 5\lambda + 3$

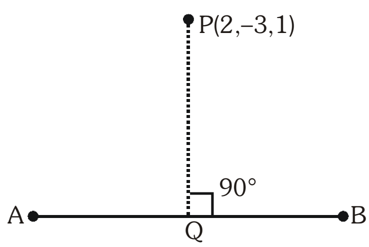
Since AM ⊥ BC

$\therefore \quad 2(4) + 0(3\lambda + 3) - 1(5\lambda + 3) = 0 \quad \Rightarrow 8 - 5\lambda - 3 = 0 \quad \Rightarrow \quad \lambda = 1$

Hence the co-ordinates of M are (3, 4, 5) **Ans.**

Illustration 21 : Find the length of perpendicular from P(2, – 3,1) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+2}{-1}$

Solution : Given line is $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+2}{-1}$ ....(i)



and P(2, – 3,1)

Co-ordinates of any point on (i) may be taken as

(2r–1,3r+3,–r–2)

Let Q = (2r –1, 3r + 3,–r – 2)

Direction ratio's of PQ are : (2r –3, 3r +6, –r –3)

Direction ratio's of AB are : (2, 3, –1)

Since, $PQ \perp AB$

$$2(2r-3)+3(3r+6)-1(-r-3)=0$$

$$\Rightarrow \quad r=-\frac{15}{14}$$

$$\therefore \quad Q=\left(-\frac{22}{7},-\frac{3}{14},-\frac{13}{14}\right)$$

$$PQ^2=\left(2+\frac{22}{7}\right)^2+\left(-3+\frac{3}{14}\right)^2+\left(1+\frac{13}{14}\right)^2=\frac{531}{14}$$

$$PQ=\sqrt{\frac{531}{14}} \text{ units}$$

Ans.

Do yourself - 9 :

(i) Find the image of point P(1, 3, 2) in the plane $2x-y+z+3=0$ as well as the foot of the perpendicular drawn from the point (1, 3, 2).

(ii) Find the distance of the point (1, –2, 3) from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

(iii) Prove that $\frac{x+1}{-2}=\frac{y+2}{3}=\frac{z+5}{4}$ lies in the plane $x+2y-z=0$.

26. EQUATION OF PLANE CONTAINING TWO INTERSECTING LINES :

Let the two lines be

$$\frac{x-\alpha_1}{\ell_1}=\frac{y-\beta_1}{m_1}=\frac{z-\gamma_1}{n_1} \qquad \text{.......... (i)}$$

and $$\frac{x-\alpha_2}{\ell_2}=\frac{y-\beta_2}{m_2}=\frac{z-\gamma_2}{n_2} \qquad \text{.......... (ii)}$$

These lines will coplanar if $\begin{vmatrix}\alpha_2-\alpha_1 & \beta_2-\beta_1 & \gamma_2-\gamma_1\\ \ell_1 & m_1 & n_1\\ \ell_2 & m_2 & n_2\end{vmatrix}=0$ (It is condition for intersection of two lines)

the plane containing the two lines is $\begin{vmatrix}x-\alpha_1 & y-\beta_1 & z-\gamma_1\\ \ell_1 & m_1 & n_1\\ \ell_2 & m_2 & n_2\end{vmatrix}=0$

Illustration 22: Find the equation of the plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+1}{2}$ and parallel to the line $\frac{x-4}{2}=\frac{y-1}{-3}=\frac{z+3}{5}$.

Solution : Any plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+1}{2}$ is

$$a(x-1) + b(y+6) + c(z+1) = 0 \quad \text{....... (i)}$$

where, $3a + 4b + 2c = 0$ .... (ii)

Also, it is parallel to the second line and hence, its normal is perpendicular to this line

$\therefore$ $2a - 3b + 5c = 0$ .... (iii)

Solving (ii) & (iii) by cross multiplication, we get $\frac{a}{26}=\frac{b}{-11}=\frac{c}{-17}=k$

$\Rightarrow$ $a = 26k$, $b = -11k$ & $c = -17k$

Putting these values in (i), we get $26k(x-1) - 11k(y+6) - 17k(z+1) = 0$

$\Rightarrow$ $26x - 11y - 17z = 109$, which is the required equation of the plane.

27. LINE OF GREATEST SLOPE :

Consider two planes G-plane and H-plane. H-plane is treated as a horizontal plane or reference plane. G-plane is a given plane. Let AB be the line of intersection of G-plane & H-plane. Line of greatest slope is a line which is contained by G-plane & perpendicular to line of intersection of G-plane & H-plane. Obviously, infinitely many such lines of greatest slopes are contained by G-plane. Generally an additional information is given in problem so that a unique line of greatest slope can be found out.

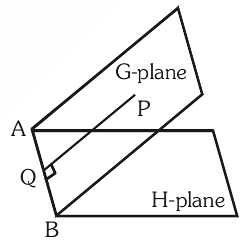


Illustration 23: Assuming the plane $4x-3y+7z=0$ to be horizontal, find the equation of the line of greatest slope on the plane $2x+y-5z=0$, passing through the point (2,1,1).

Solution : The required line passing through the point P (2,1,1) in the plane $2x + y - 5z = 0$ and is having greatest slope, so it must be perpendicular to the line of intersection of the planes

$2x + y - 5z = 0$ ......(i)

and $4x - 3y + 7z = 0$ ......(ii)

Let the d.r'.s of the line of intersection of (i) and (ii) be a, b, c

$\Rightarrow$ $2a + b - 5c = 0$ and $4a - 3b + 7c = 0$

{as dr'.s of straight line (a, b,c) is perpendicular to d.r'.s of normal to both the planes}

$\Rightarrow$ $\frac{a}{4}=\frac{b}{17}=\frac{c}{5}$

Now let the direction ratio of required line be proportional to ℓ, m, n then its

equation be $\frac{x-2}{\ell}=\frac{y-1}{m}=\frac{z-1}{n}$

where $2\ell + m - 5n = 0$ and $4\ell + 17m + 5n = 0$

so, $\frac{\ell}{3} = \frac{m}{-1} = \frac{n}{1}$

Thus the required line is $\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z-1}{1}$ **Ans.**

28. AREA OF TRIANGLE :

To find the area of a triangle in terms of its projections on the co-ordinates planes.

Let Δ_x, Δ_y, Δ_z be the projections of the plane area of the triangle on the planes yOz, zOx, xOy respectively.Let ℓ, m, n be the direction cosines of the normal to the plane of the triangle.

Then the angle between the plane of the triangle and yOz plane is the angle between the normal to the plane of the triangle and the x-axis.

$\therefore \quad \Delta_x = \Delta\ell$

Similarly $\Delta_y = \Delta m$; $\Delta_z = \Delta n \Rightarrow \Delta = \sqrt{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}$

If A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3) be the three vertices of the triangle then

$$\Delta_x = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}, \Delta_y = \frac{1}{2}\begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix}, \Delta_z = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Do yourself - 10 :

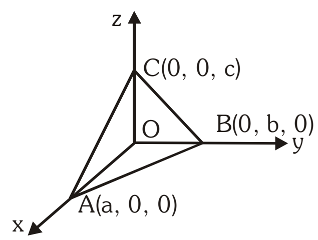
(i) Prove that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ are coplanar. Find their point of intersection.

(ii) Find the area of the triangle whose vertices are the points (1, 2, 3), (–2, 1, –4), (3, 4, –2).

Miscellaneous Illustrations :

Illustration 24 : If a variable plane cuts the coordinate axes in A, B and C and is at a constant distance p from the origin, find the locus of the centre of the tetrahedron OABC.

Solution : Let A ≡ (a, 0, 0), B ≡ (0, b, 0) and C ≡ (0, 0, c)



$\therefore$ Equation of plane ABC is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Now p = length of perpendicular from O to this plane

$$= \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \quad \text{or} \quad p^2 = \frac{1}{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} \quad ...(i)$$

Let G(α, β, γ) be the centre of the tetrahedron OABC, then

$$\alpha = \frac{a}{4},\ \beta = \frac{b}{4},\ \gamma = \frac{c}{4} \qquad \left[\because\ \alpha = \frac{a+0+0+0}{4} = \frac{a}{4}\right]$$

or, $a = 4\alpha,\ b = 4\beta,\ c = 4\gamma$

Putting these values of a, b, c in equation (i), we get

$$p^2 = \frac{16}{\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}\right)} \quad \text{or} \quad \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} = \frac{16}{p^2}$$

∴ locus of (α, β, γ) is $x^{-2} + y^{-2} + z^{-2} = 16\,p^{-2}$ **Ans.**

Illustration 25 : Through a point P(h, k, ℓ) a plane is drawn at right angles to OP to meet the coordinate axes in A, B and C. If OP = p, show that the area of ΔABC is $\frac{p^5}{2|hk\ell|}$.

Solution : $OP = \sqrt{h^2+k^2+\ell^2} = p$

Direction cosines of OP are $\frac{h}{\sqrt{h^2+k^2+\ell^2}}, \frac{k}{\sqrt{h^2+k^2+\ell^2}}, \frac{\ell}{\sqrt{h^2+k^2+\ell^2}}$

Since OP is normal to the plane, therefore, equation of the plane will be,

$$\frac{h}{\sqrt{h^2+k^2+\ell^2}}x + \frac{k}{\sqrt{h^2+k^2+\ell^2}}y + \frac{\ell}{\sqrt{h^2+k^2+\ell^2}}z = \sqrt{h^2+k^2+\ell^2}$$

or, $hx + ky + \ell z = h^2 + k^2 + \ell^2 = p^2$

$$\therefore\ A \equiv \left(\frac{p^2}{h}, 0, 0\right), B \equiv \left(0, \frac{p^2}{k}, 0\right), C \equiv \left(0, 0, \frac{p^2}{\ell}\right)$$

Now area of ΔABC, $\Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2$

Now A_{xy} = area of projection of ΔABC on xy-plane = area of ΔAOB

$$= \text{Mod of } \frac{1}{2}\begin{vmatrix} \frac{p^2}{h} & 0 & 1 \\ 0 & \frac{p^2}{k} & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\frac{p^4}{|hk|}$$

Similarly, $A_{yz} = \frac{1}{2}\frac{p^4}{|k\ell|}$ and $A_{zx} = \frac{1}{2}\frac{p^4}{|\ell h|}$

$$\therefore\ \Delta^2 = \frac{1}{4}\frac{p^8}{h^2k^2} + \frac{1}{4}\frac{p^8}{k^2\ell^2} + \frac{1}{4}\frac{p^8}{h^2\ell^2} = \frac{p^{10}}{4h^2k^2\ell^2}$$

or $\Delta = \frac{p^5}{2|hk\ell|}$ **Ans.**

Illustration 26: Find the locus of a point, the sum of squares of whose distances from the planes : $x - z = 0$, $x - 2y + z = 0$ and $x + y + z = 0$ is 36

Solution : Given planes are $x - z = 0$, $x - 2y + z = 0$ and, $x + y + z = 0$

Let the point whose locus is required be $P(\alpha,\beta,\gamma)$. According to question

$$\frac{|\alpha-\gamma|^2}{2}+\frac{|\alpha-2\beta+\gamma|^2}{6}+\frac{|\alpha+\beta+\gamma|^2}{3} = 36$$

or $3(\alpha^2+\gamma^2-2\alpha\gamma)+\alpha^2+4\beta^2+\gamma^2-4\alpha\beta-4\beta\gamma+2\alpha\gamma+2(\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\alpha\gamma) = 36\times 6$

or $6\alpha^2 + 6\beta^2 + 6\gamma^2 = 36 \times 6$

or $\alpha^2 + \beta^2 + \gamma^2 = 36$

Hence, the required equation of locus is $x^2 + y^2 + z^2 = 36$ **Ans.**

Illustration 27 : Direction ratios of normal to the plane which passes through the point (1, 0, 0) and (0, 1, 0) which makes angle $\pi/4$ with $x + y = 3$ are -

(A) 1, 1, 2 (B) $\sqrt{2}$, 1, 1 (C) 1, $\sqrt{2}$, 1 (D) 1, 1, $\sqrt{2}$

Solution : The plane by intercept form is $\frac{x}{1}+\frac{y}{1}+\frac{z}{c}=1$

direction ratios of normal of this plane are $1, 1, \frac{1}{c}$ and that of given plane are 1, 1, 0.

$$\therefore \quad \cos\frac{\pi}{4} = \frac{1.1+1.1+0\cdot\frac{1}{c}}{\sqrt{1+1+\frac{1}{c^2}}\sqrt{1+1+0}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2+\frac{1}{c^2}}\sqrt{2}} \Rightarrow 2+\frac{1}{c^2}=4 \Rightarrow \quad c=\pm\frac{1}{\sqrt{2}}$$

$\therefore$ d.r.'s are 1, 1, $\sqrt{2}$ **Ans. (D)**

ANSWERS FOR DO YOURSELF

1: **(i)** $2\sqrt{21}$ **(iii)** $8x + 2y + 24z \pm 2k^2 + 9 = 0$ **(iv)** $\left(-2, \frac{8}{3}, 5\right)$ & $\left(-1, \frac{10}{3}, 6\right)$

(v) **(a)** 7 : 8, externally **(b)** 2 : 3 internally

2 : **(i)** 3, 2, 5 **(ii)** 3, 4, 3 **(iii)** 2, 3, 4 & $\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$

3 : **(i)** $\theta = \frac{\pi}{2}$ **(iii)** $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

4 : **(i)** **(a)** $\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$ **(b)** $\left(\frac{7}{3}, 0, 0\right), \left(0, \frac{7}{4}, 0\right)$ & $\left(0, 0, \frac{7}{5}\right)$ **(c)** $\frac{x}{7/3} + \frac{y}{7/4} + \frac{z}{7/5} = 1$

(d) $\frac{3x}{5\sqrt{2}} + \frac{4y}{5\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{7}{5\sqrt{2}}$ & $\vec{r}.\left(\frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right) = \frac{7}{5\sqrt{2}}$

(ii) $x + y + 2z = 7$

5: **(ii)** $\theta = \cos^{-1}\left(\frac{1}{\sqrt{156}}\right)$

6 : **(i)** $\frac{8}{\sqrt{6}}$ **(ii)** $3x + 4y + 2z = 0$

7 : **(i)** P, Q same side & R opposite side

(ii) **(a)** $4x + y - 5z + 14 = 0$ & $32x - 13y + 23z - 56 = 0$

(b) $4x + y - 5z - 14 = 0$ & $32x - 13y + 23z - 70 = 0$

(c) $4x + y - 5z + 14 = 0$ (obtuse angle bisector) & $32x - 13y + 23z - 56 = 0$ (acute angle bisector)

8 : **(i)** $\frac{x-4}{1} = \frac{y-2}{-1} = \frac{z-3}{2}$ **(ii)** $\frac{x - 11/4}{-3} = \frac{y + 9/4}{5} = \frac{z-0}{4}$ **(iii)** $\theta = \sin^{-1}\left(\frac{6}{5\sqrt{5}}\right)$

9 : **(i)** $\left(\frac{-5}{3}, \frac{13}{3}, \frac{2}{3}\right)$ & $\left(\frac{-1}{3}, \frac{11}{3}, \frac{4}{3}\right)$ **(ii)** 1

10: **(i)** $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$ **(ii)** $\frac{\sqrt{1218}}{2}$

EXERCISE (O-1)

[STRAIGHT OBJECTIVE TYPE]

1. Consider three vectors $\vec{p}=\hat{i}+\hat{j}+\hat{k}$, $\vec{q}=2\hat{i}+4\hat{j}-\hat{k}$ and $\vec{r}=\hat{i}+\hat{j}+3\hat{k}$. If $\vec{p}$, $\vec{q}$ and $\vec{r}$ denotes the position vector of three non-collinear points then the equation of the plane containing these points is

(A) $2x - 3y + 1 = 0$ (B) $x - 3y + 2z = 0$ (C) $3x - y + z - 3 = 0$ (D) $3x - y - 2 = 0$

2. The intercept made by the plane $\vec{r}.\vec{n}=q$ on the x-axis is

(A) $\frac{q}{\hat{i}.\vec{n}}$ (B) $\frac{\hat{i}.\vec{n}}{q}$ (C) $(\hat{i}.\vec{n})q$ (D) $\frac{q}{|\vec{n}|}$

3. If the distance between the planes

$$8x + 12y - 14z = 2$$

and

$$4x + 6y - 7z = 2$$

can be expressed in the form $\frac{1}{\sqrt{N}}$ where N is natural then the value of $\frac{N(N+1)}{2}$ is

(A) 4950 (B) 5050 (C) 5150 (D) 5151

4. A plane passes through the point P(4, 0, 0) and Q(0, 0, 4) and is parallel to the y-axis. The distance of the plane from the origin is

(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

5. If from the point P (f, g, h) perpendiculars PL, PM be drawn to yz and zx planes then the equation to the plane OLM is

(A) $\frac{x}{f}+\frac{y}{g}-\frac{z}{h}=0$ (B) $\frac{x}{f}+\frac{y}{g}+\frac{z}{h}=0$ (C) $\frac{x}{f}-\frac{y}{g}+\frac{z}{h}=0$ (D) $-\frac{x}{f}+\frac{y}{g}+\frac{z}{h}=0$

6. If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(k)$ with x-axis, then k is equal to

(A) $\sqrt{3}/2$ (B) 2/7 (C) $\sqrt{2}/3$ (D) 1

7. The plane XOZ divides the join of (1, –1, 5) and (2, 3, 4) in the ratio $\lambda : 1$, then λ is

(A) – 3 (B) – 1/3 (C) 3 (D) 1/3

8. A variable plane forms a tetrahedron of constant volume 64 K^3 with the coordinate planes and the origin, then locus of the centroid of the tetrahedron is

(A) $x^3 + y^3 + z^3 = 6K^3$ (B) $xyz = 6k^3$

(C) $x^2 + y^2 + z^2 = 4K^2$ (D) $x^{-2} + y^{-2} + z^{-2} = 4K^{-2}$

9. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be 3, 4 and 5 sq. units respectively. Then the area of the triangle BCD, is

(A) $5\sqrt{2}$ (B) 5 (C) $5/\sqrt{2}$ (D) 5/2

10. Equation of the line which passes through the point with p. v. (2, 1, 0) and perpendicular to the plane containing the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is

(A) $\vec{r}$ = (2, 1, 0) + t (1, –1, 1) (B) $\vec{r}$ = (2, 1, 0) + t (–1, 1, 1)

(C) $\vec{r}$ = (2, 1, 0) + t (1, 1, –1) (D) $\vec{r}$ = (2, 1, 0) + t (1, 1, 1)

where t is a parameter

11. Which of the following planes are parallel but not identical?

$P_1 : 4x - 2y + 6z = 3$

$P_2 : 4x - 2y - 2z = 6$

$P_3 : -6x + 3y - 9z = 5$

$P_4 : 2x - y - z = 3$

(A) P_2 & P_3 (B) P_2 & P_4 (C) P_1 & P_3 (D) P_1 & P_4

12. A parallelopiped is formed by planes drawn through the points (1, 2, 3) and (9, 8, 5) parallel to the coordinate planes then which of the following is not the length of an edge of this rectangular parallelopiped

(A) 2 (B) 4 (C) 6 (D) 8

13. Vector equation of the plane $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$ in the scalar dot product form is

(A) $\vec{r}.(5\hat{i} - 2\hat{j} + 3\hat{k}) = 7$ (B) $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 7$

(C) $\vec{r}.(5\hat{i} - 2\hat{j} - 3\hat{k}) = 7$ (D) $\vec{r}.(5\hat{i} + 2\hat{j} + 3\hat{k}) = 7$

14. The vector equations of the two lines L_1 and L_2 are given by

$L_1 : \vec{r} = 2\hat{i} + 9\hat{j} + 13\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$; $L_2 : \vec{r} = -3\hat{i} + 7\hat{j} + p\hat{k} + \mu(-\hat{i} + 2\hat{j} - 3\hat{k})$

then the lines L_1 and L_2 are

(A) skew lines for all $p \in R$

(B) intersecting for all $p \in R$ and the point of intersection is (–1, 3, 4)

(C) intersecting lines for $p = -2$

(D) intersecting for all real $p \in R$

15. Consider the plane $(x, y, z) = (0, 1, 1) + \lambda(1, -1, 1) + \mu(2, -1, 0)$. The distance of this plane from the origin is

(A) 1/3 (B) $\sqrt{3}/2$ (C) $\sqrt{3/2}$ (D) $2/\sqrt{3}$

16. The value of 'a' for which the lines $\frac{x-2}{1} = \frac{y-9}{2} = \frac{z-13}{3}$ and $\frac{x-a}{-1} = \frac{y-7}{2} = \frac{z+2}{-3}$ intersect, is

(A) – 5 (B) – 2 (C) 5 (D) – 3

17. Given A (1, –1, 0) ; B(3, 1, 2) ; C(2, –2, 4) and D(–1, 1, –1) which of the following points neither lie on AB nor on CD?

(A) (2, 2, 4) (B) (2, –2, 4) (C) (2, 0,1) (D) (0, –2, –1)

18. For the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$, which one of the following is incorrect?

(A) it lies in the plane $x - 2y + z = 0$ (B) it is same as line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

(C) it passes through (2, 3, 5) (D) it is parallel to the plane $x - 2y + z - 6 = 0$

19. Given planes

$P_1 : cy + bz = x$

$P_2 : az + cx = y$

$P_3 : bx + ay = z$

P_1, P_2 and P_3 pass through one line, if

(A) $a^2 + b^2 + c^2 = ab + bc + ca$ (B) $a^2 + b^2 + c^2 + 2abc = 1$

(C) $a^2 + b^2 + c^2 = 1$ (D) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + 2abc = 1$

20. The line $\frac{x-x_1}{0}=\frac{y-y_1}{1}=\frac{z-z_1}{2}$ is

(A) parallel to x-axis
(B) perpendicular to x-axis
(C) perpendicular to YOZ plane
(D) parallel to y-axis

21. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(A) k = 0 or – 1 (B) k = 1 or – 1 (C) k = 0 or – 3 (D) k = 3 or – 3

22. The line which contains all points (x, y, z) which are of the form (x, y, z) = (2, –2, 5) + λ(1, –3, 2) intersects the plane 2x – 3y + 4z = 163 at P and intersects the YZ plane at Q. If the distance PQ is $a\sqrt{b}$ where $a, b \in N$ and a > 3 then (a + b) equals

(A) 23 (B) 95 (C) 27 (D) none

23. Let L_1 be the line $\vec{r}_1 = 2\hat{i}+\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{k})$ and let L_2 be the line $\vec{r}_2 = 3\hat{i}+\hat{j}+\mu(\hat{i}+\hat{j}-\hat{k})$.
Let Π be the plane which contains the line L_1 and is parallel to L_2. The distance of the plane Π from the origin is

(A) 1/7 (B) $\sqrt{2/7}$ (C) $\sqrt{6}$ (D) none

24. The value of m for which straight line 3x – 2y + z + 3 = 0 = 4x – 3y + 4z + 1 is parallel to the plane 2x – y + mz – 2 = 0 is

(A) –2 (B) 8 (C) – 18 (D) 11

25. The distance of the point (–1, –5, – 10) from the point of intersection of the line $\frac{x-2}{2}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane x – y + z = 5 is

(A) $2\sqrt{11}$ (B) $\sqrt{126}$ (C) 13 (D) 14

26. $P(\vec{p})$ and $Q(\vec{q})$ are the position vectors of two fixed points and $R(\vec{r})$ is the position vector of a variable point. If R moves such that $(\vec{r}-\vec{p})\times(\vec{r}-\vec{q})=0$ then the locus of R is

(A) a plane containing the origin 'O' and parallel to two non collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$
(B) the surface of a sphere described on PQ as its diameter.
(C) a line passing through the points P and Q
(D) a set of lines parallel to the line PQ.

[MATRIX MATCH TYPE]

27. Consider the following four pairs of lines in **column-I** and match them with one or more entries in **column-II**.

	Column-I		Column-II
(A)	$L_1 : x = 1 + t,\ y = t,\ z = 2 - 5t$ $L_2 : \vec{r} = (2,1,-3) + \lambda(2, 2, -10)$	(P)	non coplanar lines
(B)	$L_1 : \frac{x-1}{2}=\frac{y-3}{2}=\frac{z-2}{-1}$ $L_2 : \frac{x-2}{1}=\frac{y-6}{-1}=\frac{z+2}{3}$	(Q)	lines lie in a unique plane
(C)	$L_1 : x = -6t,\ y = 1 + 9t,\ z = -3t$ $L_2 : x = 1 + 2s,\ y = 4 - 3s,\ z = s$	(R)	infinite planes containing both the lines
(D)	$L_1 : \frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ $L_2 : \frac{x-3}{-4}=\frac{y-2}{-3}=\frac{z-1}{2}$	(S)	lines are not intersecting at a unique point

EXERCISE (O-2)

[MULTIPLE OBJECTIVE TYPE]

1. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are A(1, 0, 1) ; B(2,0, 0) and C(0, 1, 0) then the position vectors of the vertex A_1 can be:

(A) (2, 2, 2) (B) (0, 2, 0)

(C) (0, –2, 2) (D) (0, –2, 0)

2. Consider Lines $L_1 : \frac{x-\alpha}{1} = \frac{y}{-2} = \frac{z+\beta}{2}$, $L_2 : x = \alpha, \frac{y}{-\alpha} = \frac{z+\alpha}{2-\beta}$, plane $P : 2x + 2y + z + 7 = 0$. Let line L_2 lies in plane P, then

(A) $\alpha = -7$

(B) $\alpha = 7$

(C) minimum distance between line L_1 and plane P is 11.

(D) minimum distance between line L_1 and plane P is $\frac{23}{3}$

3. The point $\vec{A}$ (3,4,7), $\vec{B}$ (4,5,9) and $\vec{C}$ (1,2,–1) are three vertices of a parallelogram ABCD, then -

(A) vector equation of line AB is $\vec{r} = 4\hat{i} + 5\hat{j} + 9\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right)$

(B) cartesian equation of line BC is $\frac{x-4}{3} = \frac{x-5}{3} = \frac{z-9}{10}$

(C) coordinates of D are (0,1,–3)

(D) ABCD is rectangle

4. Let two planes $P_1 : 2x - y + z - 2 = 0$ and $P_2 : x + 2y - z - 3 = 0$ are given then-

(A) The equation of the plane through line of intersection of $P_1 = 0$ and $P_2 = 0$ and the point (3,2,1) is $x - 3y + 2z + 1 = 0$

(B) The equation of the plane through line of intersection of $P_1 = 0$ and $P_2 = 0$ and the point (3,2,1) is $3x - y + 2z - 9 = 0$

(C) The equation of acute angle bisector plane of $P_1 = 0$ and $P_2 = 0$ is $x - 3y + 2z + 1 = 0$

(D) The equation of acute angle bisector plane of $P_1 = 0$ and $P_2 = 0$ is $x + 3y + 2z + 2 = 0$

5. A variable point P(4 cost, 4sint, 4sint) moves in space, now which of the following holds good ?

(A) Point P moves on plane $ax + by + cz + d = 0$

(B) Point 'P' traces a circle.

(C) Area enclosed by P is $16\sqrt{2}\pi$

(D) Point P cannot lie on a fixed plane.

6. The projection of line $\frac{x}{2} = \frac{y-1}{2} = \frac{z-1}{1}$ on a plane 'P' is $\frac{x}{1} = \frac{y-1}{1} = \frac{z-1}{-1}$. If the plane P passes through (k, –2, 0), then k is greater than -

(A) 2 (B) 3 (C) 5 (D) 4

7. A line segment has length 6 and direction ratios are –3,4,6, then the component of the line vector are-

(A) $\frac{-18}{\sqrt{61}},\frac{24}{\sqrt{61}},\frac{36}{\sqrt{61}}$ (B) 27,–18,–54 (C) 27,–18,54 (D) $\frac{18}{\sqrt{61}},\frac{-24}{\sqrt{61}},\frac{-36}{\sqrt{61}}$

8. Which of the following is (are) correct -

(A) If two lines in space are not intersecting, then they must be skew lines.

(B) If two lines are parallel to a plane 'P', then their direction ratios will be proportional

(C) If two lines are perpendicular to a plane 'P', then their direction ratios will be proportional

(D) Equation $\frac{x+1}{a}=\frac{y-1}{b}=\frac{z}{c}$, where a, b, c are real parameters, represents a family of concurrent lines in space

9. Given the equations of the line $3x - y + z + 1 = 0$, $5x + y + 3z = 0$.

Then which of the following is correct ?

(A) Symmetrical form of the equations of line is $\frac{x}{2}=\frac{y-\frac{1}{8}}{-1}=\frac{z+\frac{5}{8}}{1}$

(B) symmetrical form of the equations of line is $\frac{x+\frac{1}{8}}{1}=\frac{y-\frac{5}{8}}{1}=\frac{z}{-2}$

(C) equation of the plane through (2, 1, 4) and prependicular to the given lines is $2x - y + z - 7 = 0$

(D) equation of the plane through (2, 1, 4) and prependicular to the given lines is $x + y - 2z + 5 = 0$

10. Consider the family of planes $x + y + z = c$ where is a parameter intersecting the coordinate axes at P, Q,R and α,β,γ are the angles made by each member of this family with positive x,y and z axis. Which of the following interpretations hold good for this family -

(A) each member of this family is equally inclined with the coordinate axes.

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$

(C) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

(D) for c = 3 area of the triangle PQR is $3\sqrt{3}$ sq. units.

11. Consider a plane P passing through $A(\lambda,3,\mu)$, B(–1,3,2) and C(7,5,10) and a straight line L with positive direction cosines passing through A, bisecting BC and makes equal angles with the coordinate axes. Let L_1 be a line parallel to L and passing through origin. Which of the following is(are) correct?

(A) The value of $(\lambda + \mu)$ is equal to 5.

(B) Equation of straight line L_1 is $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{1}$.

(C) Equation of the plane perpendicular to the plane P and containing line L_1 is $x - 2y + z = 0$

(D) Area of triangle ABC is equal to $3\sqrt{2}$.

12. A line L passing through the point P(1,4,3), is perpendicular to both the lines

$\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{4}$ and $\frac{x+2}{3}=\frac{y-4}{2}=\frac{z+1}{-2}$.

If the position vector of point Q on L is(a_1,a_2,a_3) such that $(PQ)^2 = 357$, then $(a_1 + a_2 + a_3)$ can be-

(A) 16 (B) 15 (C) 2 (D) 1

[MATRIX MATCH TYPE]

13. P(0, 3, – 2); Q(3, 7, – 1) and R(1, – 3, – 1) are 3 given points. Let L_1 be the line passing through P and Q and L_2 be the line through R and parallel to the vector $\vec{V} = \hat{i} + \hat{k}$.

Column-I		Column-II	
(A)	perpendicular distance of P from L_2	(P)	$7\sqrt{3}$
(B)	shortest distance between L_1 and L_2	(Q)	2
(C)	area of the triangle PQR	(R)	6
(D)	distance from (0, 0, 0) to the plane PQR	(S)	$\frac{19}{\sqrt{147}}$

EXERCISE (S-1)

1. Find the angle between the two straight lines whose direction cosines ℓ, m, n are given by $2\ell + 2m - n = 0$ and $mn + n\ell + \ell m = 0$.

2. The plane denoted by $\Pi_1 : 4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $\Pi_2 : 5x + 3y + 10z = 25$. If the plane in its new position be denoted by Π, and the distance of this plane from the origin is $\sqrt{k}$ where $k \in N$, then find k.

3. Find the equations of the straight line passing through the point (1,2,3) to intersect the straight line $x + 1 = 2(y - 2) = z + 4$ and parallel to the plane $x + 5y + 4z = 0$.

4. A variable plane is at a constant distance p from the origin and meets the coordinate axes in points A,B and C respectively. Through these points, planes are drawn parallel to the coordinates planes. Find the locus of their point of intersection.

5. Find the value of p so that the lines $\frac{x+1}{-3} = \frac{y-p}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are in the same plane. for this value of p, find the coordinates of their point of intersection and the equation of the plane containing them.

6. Find the equations to the line of greatest slope through the point (7, 2, –1) in the plane $x - 2y + 3z = 0$ assuming that the axes are so placed that the plane $2x + 3y - 4z = 0$ is horizontal.

7. Let L be the line given by $\vec{r} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and let P be the point (2,–1,1). Also suppose that E be the plane containing three non collinear points A(0,1,1) ; B(1,2,2) and C(1,0,1).

Find

(a) Distance between the point P and the line L.

(b) Equation of the plane E.

(c) Equation the plane F containing the line L and the point P.

(d) Acute angle between the plane E and F.

(e) Volume of the parallelopiped by A,B,C and the point D(–3,0,1).

8. The position vectors of the four angular points of a tetrahedron OABC are (0, 0, 0); (0, 0, 2); (0, 4, 0) and (6, 0, 0) respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the value of 'r'.

9. Let the equation of the plane containing the line $x - y - z - 4 = 0 = x + y + 2z - 4$ and is parallel to the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$ be $x + Ay + Bz + C = 0$ Compute the value of $|A + B + C|$.

10. Find the equation of the line which is reflection of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$.

11. Find the equation of the plane containing the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{2}$ and parallel to the line $\frac{x-3}{2} = \frac{y}{5} = \frac{z-2}{4}$.

Find also the S.D. between the two lines.

12. Consider the plane

$$E : \vec{r} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Let F be the plane containing the point A(–4,2,2) and parallel to E.

Suppose the point B is on the plane E such that B has a minimum distance from the point A. If C(–3,0,4) lies in the plane F. Find the area of the triangle ABC.

13. The equation of the plane which has the property that the point Q(5,4,5) is the reflection of point P(1,2,3) through that plane, is $ax + by + cz = d$ where $a,b,c,d \in N$. Find the least value of $(a + b + c + d)$.

14. Find the equation of the line passing through the point (4,–14,4) and intersecting the line of intersection of the planes : $3x + 2y - z = 5$ and $x - 2y - 2z = -1$ at right angles.

15. Find the point where the line of intersection of the planes $x - 2y + z = 1$ and $x + 2y - 2z = 5$, intersects the plane $2x + 2y + z + 6 = 0$.

16. Feet of the perpendicular drawn from the point P(2,3,–5) on the axes of coordinates are A,B and C. Find the equation of the plane passing through their feet and the area of ΔABC.

17. Find the equation of the plane containing the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ and perpendicular to the plane $x - y + z + 2 = 0$.

EXERCISE (S-2)

1. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

2. Let $\Pi : x+y-z-4=0$ be the equation of a plane and A be the point with position vector $\hat{i}+2\hat{j}-3\hat{k}$. L is a line which passes through the point (1,2,3) with direction ratios 3,–1 and 4. If the distance of the point A from the line L measured parallel to the plane Π is d_1 and the distance of the point A from the plane Π measured parallel to the line L is d_2, then find the value of $\sqrt{d_1^2-d_2^2}$.

3. The three planes $kx + y + z = 2$, $x + y - z = 3$, $x + 2z = 2$ form a triangular prism and area of the normal section (where normal section of the triangular prism is the plane parallel to the base of the triangular prism) be k_1. Then value of $2\sqrt{14}\,(k.k_1)$ is

4. The line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

5. **(a)** Consider a plane passing through three points A(a,0,0), B(0,b,0), C(0,0,c) with $a > 0$, $b > 0$, $c > 0$. Let d be the distance between the origin O and the plane and m be the distance between the origin O and the point M(a,b,c). If a,b,c vary in the range of any positive numbers, then find the minimum value of $\left(\frac{m}{d}\right)^2$.

(b) Let A_1,A_2,A_3,A_4 be the areas of the triangular faces of a tetrahedron and h_1,h_2,h_3,h_4 be corresponding altitudes of the tetrahedron. If volume of tetrahedron is 5 cubic units then find the minimum value of $(A_1 + A_2 + A_3+A_4)(h_1+ h_2 + h_3 + h_4)$ (in cubic units).

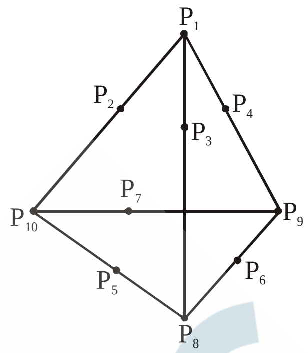
6. If the angle between the planes given by $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ is $\cos^{-1}(k)$, then the value of 'k' is equal to

7. Planes P_1', P_2', P_3' are drawn parallel to the planes $P_1 : x + y + z = 3$, $P_2 : x - y + z = 1$ & $P_3 : x + y - z = 2$ respectively from the point (2,2,3). If d_1,d_2,d_3 are distances of P_1',P_2',P_3' from (1,1,2) respectively then $\left(d_1^2+\frac{1}{d_2^2}+\frac{1}{d_3^2}\right)$ is equal to

8. Faces ABC and BCD of a tetrahedron ABCD meet at an angle of 30°. The area of face ABC is 120 and the area of face BCD is 80 and BC = 10, then the volume of tetrahedron is

9. Through a point $P(\alpha, \beta, \gamma)$ a plane is drawn at right angle to OP to meet the axes in A, B, C. If the area of ΔABC can be written as $\frac{(OP)^m}{n.\alpha.\beta.\gamma}$ (where O is origin, $m, n \in N$), then the value of $(m^2 + n^2)$ is

10. **(i)** Points P_1, P_2, P_3 ..... P_{10} are either lying along vertices or midpoints of the edges of a tetrahedron as shown in the diagram, then the number of groups of four distinct points (where each group of four points contains point P_1) which lies on the same plane is equal to



(ii) Let A, B, C, D be four non-coplanar points. Then the number of planes which are equidistant from all the four points is equal to

EXERCISE (JM)

1. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals **[AIEEE-2009]**

(1) (5, – 15) (2) (–5, 5) (3) (6, –17) (4) (–6, 7)

2. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are :- **[AIEEE-2009]**

(1) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$ (2) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (3) 6, –3, 2 (4) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$

3. **Statement–1 :** The point A(3, 1, 6) is themirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$. **[AIEEE-2010]**

Statement–2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(2) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is false, Statement–2 is true.

4. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals:- **[AIEEE-2011]**

(1) $\frac{2}{5}$ (2) $\frac{5}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

5. **Statement-1 :** The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Statement-2 : The line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B(1, 6, 3). **[AIEEE-2011]**

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

6. The distance of the point (1, –5, 9) from the plane x – y + z = 5 measured along a straight line x = y = z is : **[AIEEE-2011]**

(1) $3\sqrt{5}$ (2) $10\sqrt{3}$ (3) $5\sqrt{3}$ (4) $3\sqrt{10}$

7. An equation of a plane parallel to the plane x – 2y + 2z – 5 = 0 and at a unit distance from the origin is : **[AIEEE-2012]**

(1) x – 2y + 2z + 5 = 0 (2) x – 2y + 2z – 3 = 0

(3) x – 2y + 2z + 1 = 0 (4) x – 2y + 2z – 1 = 0

8. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to : **[AIEEE-2012]**

(1) 0 (2) – 1 (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

9. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is :- **[JEE-MAIN 2013]**

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

10. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have : **[JEE-MAIN 2013]**

(1) any value (2) exactly one value

(3) exactly two values (4) exactly three values.

11. A vector $\vec{n}$ is inclined to x-axis at 45º, to y –axis at 60º and at an acute angle to z-axis. If $\vec{n}$ is a normal to a plane passing through the point ($\sqrt{2}$, –1, 1), then the equation of the plane is : **[JEE-MAIN Online 2013]**

(1) $\sqrt{2}x-y-z=2$ (2) $\sqrt{2}x+y+z=2$

(3) $3\sqrt{2}x-4y-3z=7$ (4) $4\sqrt{2}x+7y+z=2$

12. The acute angle between two lines such that the direction cosines ℓ, m, n of each of them satisfy the equations $\ell + m + n = 0$ and $\ell^2 + m^2 - n^2 = 0$ is :- **[JEE-MAIN Online 2013]**

(1) 30° (2) 45° (3) 60° (4) 15°

13. Let Q be the foot of perpendicular from the origin to the plane 4x – 3y + z + 13 = 0 and R be a point (–1, 1, –6) on the plane. Then length QR is :- **[JEE-MAIN Online 2013]**

(1) $3\sqrt{\frac{7}{2}}$ (2) $\sqrt{14}$ (3) $\sqrt{\frac{19}{2}}$ (4) $\frac{3}{\sqrt{2}}$

14. If the projections of a line segment on thex, y and z-axes in 3-dimensional space are 2, 3 and 6 respectively, then the length ofthe line segment is : **[JEE-MAIN Online 2013]**

(1) 7 (2) 9 (3) 12 (4) 6

15. If two lines L_1 and L_2 in space, are definedby **[JEE-MAIN Online 2013]**

$L_1 = \{x = \sqrt{\lambda}\, y + (\sqrt{\lambda} - 1)$

$z = (\sqrt{\lambda} - 1)\, y + \sqrt{\lambda}\}$ and

$L_2 = \{x = \sqrt{\mu}\, y + (1 - \sqrt{\mu})$

$z = (1 - \sqrt{\mu})\, y + \sqrt{\mu}\}$, then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that :

(1) $\lambda = \mu$ (2) $\lambda \neq \mu$ (3) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (4) $\lambda + \mu = 0$

16. The equation of a plane through the line of intersection of the planes $x + 2y = 3$, $y - 2z + 1 = 0$, and perpendicular to the first plane is : **[JEE-MAIN Online 2013]**

(1) $2x - y + 7z = 11$ (2) $2x - y + 10z = 11$ (3) $2x - y - 9z = 10$ (4) $2x - y - 10z = 9$

17. Let ABC be a triangle with vertices at points A (2, 3, 5), B (-1, 3, 2) and C (λ, 5, μ) in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ.) is equal to :

[JEE-MAIN Online 2013]

(1) (10, 7) (2) (7.5) (3) (7, 10) (4) (5,7)

18. The angle between the lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $\ell^2 = m^2 + n^2$ is : **[JEE-MAIN 2014]**

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

19. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line : **[JEE-MAIN 2014]**

(1) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$ (2) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

(3) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$ (4) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

20. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is : **[JEE(Main)-2015]**

(1) $x + 3y + 6z = 7$ (2) $2x + 6y + 12z = -13$

(3) $2x + 6y + 12z = 13$ (4) $x + 3y + 6z = -7$

21. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is : **[JEE(Main)-2015]**

(1) $3\sqrt{21}$ (2) 13 (3) $2\sqrt{14}$ (4) 8

22. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : **[JEE(Main)-2016]**

(1) $\frac{20}{3}$ (2) $3\sqrt{10}$ (3) $10\sqrt{3}$ (4) $\frac{10}{\sqrt{3}}$

23. If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to :- **[JEE(Main)-2017]**

(1) $6\sqrt{5}$ (2) $3\sqrt{5}$ (3) $2\sqrt{42}$ (4) $\sqrt{42}$

24. The distantce of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is :- **[JEE(Main)-2017]**

(1) $\frac{10}{\sqrt{74}}$ (2) $\frac{20}{\sqrt{74}}$ (3) $\frac{10}{\sqrt{83}}$ (4) $\frac{5}{\sqrt{83}}$

25. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is : **[JEE(Main)-2018]**

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

26. If L_1 is the line of intersection of the planes 2x – 2y + 3z – 2 = 0, x – y + z + 1 = 0 and L_2 is the line of intersection of the planes x + 2y – z – 3 =0, 3x – y + 2z – 1 = 0, then the distance of the origin from the plane, containing the lines L_1 and L_2 is : **[JEE(Main)-2018]**

(1) $\frac{1}{3\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4\sqrt{2}}$

27. The equation of the line passing through (–4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$ is : **[JEE(Main)-Jan 19]**

(1) $\frac{x+4}{-1}=\frac{y-3}{1}=\frac{z-1}{1}$ (2) $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$ (3) $\frac{x+4}{1}=\frac{y-3}{1}=\frac{z-1}{3}$ (4) $\frac{x-4}{2}=\frac{y+3}{1}=\frac{z+1}{4}$

28. If the lines x = ay + b, z = cy + d and x=a'z + b', y = c'z + d' are perpendicular, then : **[JEE(Main)-Jan 19]**

(1) cc' + a + a' = 0 (2) aa' + c + c' = 0
(3) ab' + bc' + 1 = 0 (4) bb' + cc' + 1 = 0

29. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1,1,2) and O(0, 0, 0). The angle between the faces OPQ and PQR is : **[JEE(Main)-Jan 19]**

(1) $\cos^{-1}\left(\frac{9}{35}\right)$ (2) $\cos^{-1}\left(\frac{19}{35}\right)$ (3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

30. A plane which bisects the angle between the two given planes 2x – y + 2z – 4 = 0 and x + 2y + 2z – 2 = 0, passes through the point : **[JEE(Main)-Apr 19]**

(1) (2,4,1) (2) (2, – 4, 1) (3) (1, 4, – 1) (4) (1, – 4, 1)

EXERCISE (JA)

1. **(a)** Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0
(C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

(b) If the distance of the point P(1,–2,1) from the plane x + 2y – 2z = α, where α > 0, is 5, then the foot of the perpendicular from P to the plane is-

(A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ (C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

(c) If the distance between the plane Ax – 2y + z = d and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then |d| is

(d) Match the statements in **Column-I** with the values in **Column-II**.

Column-I	**Column-II**
(A) A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p) –4
(B) The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\left\|(\vec{b}+\vec{c})\right\|=\left\|(\vec{b}-\vec{a})\right\|$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r) 4
(D) Let f be the function on $[-\pi,\pi]$ given by	(s) 5
f(0) = 9 and $f(x)=\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$.	(t) 6
The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is	

[JEE 2010, 3+5+3+(2+2+2+2)]

2. **(a)** The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1,–1,4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2,1,4) to QR, then the length of the line segment PS is -

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

(b) The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is

(A) 5x – 11y + z = 17 (B) $\sqrt{2}x+y=3\sqrt{2}-1$

(C) $x + y + z = \sqrt{3}$ (D) $x-\sqrt{2}y=1-\sqrt{2}$

(c) If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) y + 2z = –1 (B) y + z = –1 (C) y – z = –1 (D) y – 2z = –1

[JEE 2012, 3+3+4]

3. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line **[JEE-Advanced 2013, 2]**

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

(C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

4. A line ℓ passing through the origin is perpendicular to the lines

$\ell_1:(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}, -\infty<t<\infty$

$\ell_2:(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}, -\infty<s<\infty$

Then , the coordinate(s) of the point(s) on ℓ_2 at a distance of $\sqrt{17}$ from the point of intersection of ℓ and ℓ_1 is(are) - **[JEE-Advanced 2013, 4, (–1)]**

(A) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (B) (–1,–1,0) (C) (1,1,1) (D) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

5. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

[JEE-Advanced 2013, 3, (–1)]

(A) 1 (B) 2 (C) 3 (D) 4

6. Consider the lines $L_1:\frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}, L_2:\frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}$ and the planes P_1:7x+y + 2z = 3, P_2 : 3x + 5y – 6z = 4. Let ax + by + cz = d be the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2.
Match List-I with List-II and select the correct answer using the code given below the lists.

	List-I		List-II
P.	a =	1.	13
Q.	b =	2.	–3
R.	c =	3.	1
S.	d =	4.	–2

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

[JEE-Advanced 2013, 3, (–1)]

7. From a point P(λ,λ,λ), perpendiculars PQ and PR are drawn respectively on the lines y = x, z = 1 and y = –x, z = –1. If P is such that ∠QPR is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1 (C) –1 (D) $-\sqrt{2}$

[JEE(Advanced)-2014, 3]

ANSWER KEY

EXERCISE (O-1)

1. D **2.** A **3.** D **4.** D **5.** A **6.** B **7.** D **8.** B
9. A **10.** A **11.** C **12.** B **13.** C **14.** C **15.** C **16.** D
17. A **18.** C **19.** B **20.** B **21.** C **22.** A **23.** B **24.** A
25. C **26.** C **27.** (A) R,S; (B) Q; (C) Q,S; (D) P,S

EXERCISE (O-2)

1. A,D **2.** A,D **3.** A,B,C **4.** A,C **5.** A,C **6.** A,B,D **7.** A,D **8.** C,D
9. B,D **10.** A,B,C **11.** B,C,D **12.** B,D **13.** (A) R; (B) Q; (C) P ; (D) S

EXERCISE (S-1)

1. $\theta = 90°$ **2.** 212 **3.** $\frac{x-1}{2}=\frac{y-2}{2}=\frac{z-3}{-3}$ **4.** $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

5. p = 3, (2,1,–3); x + y + z = 0 **6.** $\frac{x-7}{22}=\frac{y-2}{5}=\frac{z+1}{-4}$

7. **(a)** $\sqrt{3}$; **(b)** x + y – 2z + 1 = 0; **(c)** x – 2y + z = 5; **(d)** $\pi/3$; **(e)** 4 **8.** $\frac{2}{3}$ **9.** 11

10. $\frac{x-4}{9}=\frac{y+1}{-1}=\frac{z-7}{-3}$ **11.** x – 2y + 2z – 1 = 0; 2 units **12.** 9/2 **13.** 17

14. $\frac{x-4}{3}=\frac{y+14}{10}=\frac{z-4}{4}$ **15.** (1, –2, –4) **16.** $\frac{x}{2}+\frac{y}{3}+\frac{z}{-5}=1$, Area $=\frac{19}{2}$ sq. units

17. 2x + 3y + z + 4 = 0

EXERCISE (S-2)

1. $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ or $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ **2.** 10 **3.** 18 **4.** $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$; $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

5. **(a)** 9; **(b)** 240 **6.** 0 **7.** 9 **8.** 320 **9.** 29 **10.** **(i)** 33 **(ii)** 7

EXERCISE (JM)

1. 4 **2.** 1 **3.** 2 **4.** 3 **5.** 4 **6.** 2 **7.** 2 **8.** 4
9. 3 **10.** 3 **11.** 2 **12.** 3 **13.** 1 **14.** 1 **15.** 1,4 **16.** 2
17. 3 **18.** 1 **19.** 1 **20.** 1 **21.** 2 **22.** 3 **23.** **3** **24.** 3
25. 3 **26.** 1 **27.** 2 **28.** 2 **29.** 2 **30.** 2

EXERCISE (JA)

1. **(a)** C; **(b)** A; **(c)** 6; **(d)** (A) t (B) p,r (C) q (D) r **2.** **(a)** A; **(b)** A; **(c)** B,C **3.** D
4. B,D **5.** A,D **6.** A **7.** C **8.** B,D **9.** A,B **10.** B,C,D **11.** C
12. **A** **13.** C,D **14.** 8 **15.** 0.5 **16.** 1,2,4 **17.** 0.75