

## Limit

### Exerise-1: Single Choice Problems

1.  $\lim_{x \rightarrow 0} \frac{\cos(\tan x) - \cos x}{x^4} =$

(a)  $\frac{1}{6}$

(b)  $-\frac{1}{3}$

(c)  $-\frac{1}{6}$

(d)  $\frac{1}{3}$

2. The value of  $\lim_{x \rightarrow 0} \frac{(\sin x - \tan x)^2 - (1 - \cos 2x)^4 + x^5}{(\tan^{-1} x)^7 + (\sin^{-1} x)^6 + 3 \sin^5 x}$  equal to:

(a) 0

(b) 1

(c) 2

(d)  $\frac{1}{3}$

3. Let  $a = \lim_{x \rightarrow 0} \frac{\ln(\cos 2x)}{3x^2}$ ,  $b = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 - e^x)}$ ,  $c = \lim_{x \rightarrow 1} \frac{\sqrt{x} - x}{\ln x}$ .

Then a, b, c satisfy :

(a)  $a < b < c$

(b)  $b < c < a$

(c)  $a < c < b$

(d)  $b < a < c$

4. If  $f(x) = \cot^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right)$  and  $g(x) = \cos^{-1} \left( \frac{1 - x^2}{1 + x^2} \right)$ ,

then  $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{g(x) - g(a)}$ ,  $0 < a < \frac{1}{2}$  is :

(a)  $\frac{3}{1(2+a^2)}$

(b)  $\frac{3}{2}$

(c)  $\frac{-3}{2(1+a^2)}$

(d)  $-\frac{3}{2}$

5.  $\lim_{x \rightarrow 0} \left( \frac{(1+x)^{\frac{2}{x}}}{e^2} \right)^{\frac{4}{\sin x}}$  is:

(a)  $e^4$

(b)  $e^{-4}$

(c)  $e^8$

(d)  $e^{-8}$

6.  $\lim_{x \rightarrow \infty} \frac{3 \left[ \frac{x}{4} \right]}{x} = \frac{p}{q}$  (where  $[.]$  denotes greatest integer function), then  $p + q$

(where  $p, q$  are relative prime) is :

- (a) 2 (b) 7  
(c) 5 (d) 6

7.  $f(x) = \lim_{n \rightarrow \infty} \frac{x^n + \left(\frac{\pi}{3}\right)^n}{x^{n-1} + \left(\frac{\pi}{3}\right)^{n-1}}$ , ( $n$  is an even integer), then which of the following is

incorrect ?

- (a) If  $f: \left[\frac{\pi}{3}, \infty\right) \rightarrow \left[\frac{\pi}{3}, \infty\right)$ , then function is invertible  
(b)  $f(x) = f(-x)$  has infinite number of solutions  
(c)  $f(x) = |f(-x)|$  has infinite number of solutions  
(d)  $f(x)$  is one-one function for all  $x \in \mathbb{R}$

8.  $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2} =$

- (a)  $\pi$  (b)  $\frac{\pi}{4}$   
(c)  $\frac{\pi}{2}$  (d) None of these

9. If  $f(x) = \begin{cases} \frac{(e^{(x+3)\ln 27})^{\frac{x}{27}} - 9}{3^x - 27} & ; x < 3 \\ \frac{1 - \cos(x-3)}{(x-3)\tan(x-3)} & ; x > 3 \end{cases}$

If  $\lim_{x \rightarrow 3} f(x)$  exist, then  $\lambda =$

- (a)  $\frac{9}{2}$  (b)  $\frac{2}{9}$   
(c)  $\frac{2}{3}$  (d) None of these

10.  $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin\left(\frac{\pi}{3} - x\right)}{2 \cos x - 1}$  is equal to:

- (a)  $\frac{2}{\sqrt{3}}$  (b)  $\frac{1}{\sqrt{3}}$   
(c)  $\sqrt{3}$  (d)  $\frac{1}{2}$

11.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(\frac{\pi}{3}-x)}{2 \cos x - 1}$  is equal to :

(a)  $\frac{2}{\pi}$

(b) 1

(c)  $\frac{4}{\pi}$

(d) does not exist

12. Let  $f$  be a continuous function on  $\mathbb{R}$  such that  $f\left(\frac{1}{4^n}\right) = (\sin e^n) e^{-n^2} + \frac{n^2}{n^2+1}$ ,  
then  $f(0) =$

(a) 1

(b) 0

(c) -1

(d)  $\frac{1}{4}$

13.  $\lim_{x \rightarrow I^-} \frac{e^{\{x\}} - \{x\} - 1}{\{x\}^2}$  equals, where  $\{ \}$  is fractional part function and  $I$  is an integer, to:

(a)  $\frac{1}{2}$

(b)  $e - 2$

(c)  $I$

(d) does not exist

14.  $\lim_{x \rightarrow \infty} (e^{11x} - 7x)^{\frac{1}{3x}}$  is equal to :

(a)  $\frac{11}{3}$

(b)  $\frac{3}{11}$

(c)  $e^{\frac{3}{11}}$

(d)  $e^{\frac{11}{3}}$

15. The value of  $\lim_{x \rightarrow 0} \left[ (1 - 2x)^n \sum_{r=0}^n n C_r \left( \frac{x+x^2}{1-2x} \right)^r \right]^{1/x}$  is :

(a)  $e^n$

(b)  $e^{-n}$

(c)  $e^{3n}$

(d)  $e^{-3n}$

16. For a certain value of 'c',  $\lim_{x \rightarrow \infty} [x^5 + 7x^4 + 2]^c - x$  is finite and non-zero.

Then the value of limit is :

(a)  $\frac{7}{5}$

(b) 1

(c)  $\frac{2}{5}$

(d) None of these

17. The number of non-negative integral values of  $n$  for which

$$\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n} = 0 \text{ is :}$$

- (a) 1 (b) 2  
(c) 3 (d) 4

18. The value of  $\lim_{x \rightarrow \infty} \left( \frac{\sin x}{x} \right)^{\frac{1}{1 - \cos x}}$  :

- (a)  $e^{-1/3}$  (b)  $e^{1/3}$   
(c)  $e^{-1/6}$  (d)  $e^{1/6}$

19. If  $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x + 1} - ax - b) = 0$ , then for  $k \geq 2, (k \in \mathbb{N}) \lim_{n \rightarrow \infty} \sec^{2n}(k! \pi b) =$

- (a)  $a$  (b)  $-a$   
(c)  $2a$  (d)  $b$

20. If  $f$  is a positive function such that  $f(x + T) = f(x)(T > 0), \forall x \in \mathbb{R}$ , then

$$\lim_{n \rightarrow \infty} n \left( \frac{f(x+T) + 2f(x+2T) + \dots + nf(x+nT)}{f(x+T) + 4f(x+4T) + \dots + n^2f(x+n^2T)} \right) =$$

- (a) 2 (b)  $\frac{2}{3}$   
(c)  $\frac{3}{2}$  (d) None of these

21. Let  $f(x) = 3x^{10} - 7x^8 + 5x^6 - 21x^3 + 3x^2 - 7$

$$265 \left( \lim_{h \rightarrow 0} \frac{h^4 + 3h^2}{(f(1-h) - f(1)) \sin 5h} \right) =$$

- (a) 1 (b) 2  
(c) 3 (d)  $-3$

22.  $\lim_{x \rightarrow 0} \left( \frac{\cos x - \sec x}{x^2(x+1)} \right) =$

- (a) 0 (b)  $-\frac{1}{2}$   
(c)  $-1$  (d)  $-2$

23. Let  $f(x)$  be a continuous and differentiable function satisfying  $f(x+y) = f(x)f(y) \forall x, y \in \mathbb{R}$  if  $f(x)$  can be expressed as  $f(x) = 1 + xP(x) + x^2Q(x)$  where  $\lim_{x \rightarrow 0} P(x) = a$  and  $\lim_{x \rightarrow 0} Q(x) = b$ , then

$f'(x)$  is equal to :

- (a)  $a f(x)$  (b)  $b f(x)$   
 (c)  $(a + b)f(x)$  (d)  $(a + 2b)f(x)$

24.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{(1 - \tan \frac{x}{2})(1 - \sin x)}{(1 + \tan \frac{x}{2})(\pi - 2x)^3} =$

- (a) not exist (b)  $\frac{1}{8}$   
 (c)  $\frac{1}{16}$  (d)  $\frac{1}{32}$

25.  $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2}\right)^x$  is equal to:

- (a)  $e$  (b)  $e^{-1}$   
 (c)  $e^{-5}$  (d)  $e^5$

26.  $\lim_{x \rightarrow \frac{\pi}{2}} (\cos x)^{\cos x}$  is :

- (a) 1 (b) 0  
 (c)  $\frac{1}{e}$  (d)  $\frac{2}{e}$

27. If  $\lim_{x \rightarrow c^-} \{ \ln x \}$  and  $\lim_{x \rightarrow c^+} \{ \ln x \}$  exists finitely but they are not equal (where  $\{.\}$

denotes fractional part function), then :

- (a) 'c' can take only rational values  
 (b) 'c' can take only irrational values  
 (c) 'c' can take infinite values in which only one is irrational  
 (d) 'c' can take infinite values in which only one is rational

28.  $\lim_{x \rightarrow 0} \left(1 + \frac{a \sin bx}{\cos x}\right)^{\frac{1}{x}}$ , where a, b are non-zero constants is equal to :

- (a)  $e^{a/b}$  (b) ab  
(c)  $e^{ab}$  (d)  $e^{b/a}$

29. The value of  $\lim_{x \rightarrow 0} \left( (\cos x)^{\frac{1}{\sin^2 x}} + \frac{\sin 2x + 2 \tan^{-1} 3x + 3x^2}{\ln(1 + 3x + \sin^2 x) + xe^x} \right)$  is :

- (a)  $\sqrt{e} + \frac{3}{2}$  (b)  $\frac{1}{\sqrt{e}} + \frac{3}{2}$   
(c)  $\sqrt{e} + 2$  (d)  $\frac{1}{\sqrt{e}} + 2$

30. Let  $a = \lim_{x \rightarrow 1} \left( \frac{x}{\ln x} - \frac{1}{x \ln x} \right)$ ;  $b = \lim_{x \rightarrow 0} \frac{x^3 - 16x}{4x + x^2}$ ;  $c = \lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{x}$  and

$d = \lim_{x \rightarrow 1} \frac{(x+1)^3}{3[\sin(x+1) - (x+1)]}$  then the matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is :

- (a) Idempotent (b) Involutory  
(c) Non-singular (d) Nilpotent

31. The integral value of n so that  $\lim_{x \rightarrow 0} f(x)$  where  $f(x) = \frac{(\sin x - x)(2 \sin x - \ln \frac{1+x}{1-x})}{x^n}$  is a finite non-zero number, is :

- (a) 2 (b) 4  
(c) 6 (d) 8

32. Consider the function  $f(x) = \begin{cases} \frac{\max(x, \frac{1}{x})}{\min(x, \frac{1}{x})} & , \text{ if } x \neq 0 \\ 1 & , \text{ if } x = 0 \end{cases}$ ,

then  $\lim_{x \rightarrow 0^-} \{f(x)\} + \lim_{x \rightarrow 1^-} \{f(x)\} + \lim_{x \rightarrow 1^-} [f(x)] =$

(where  $\{.\}$  denotes fraction part function and  $[.]$  denotes greatest integer function )

- (a) 0 (b) 1  
(c) 2 (d) 3

$$33. \lim_{x \rightarrow \left(\frac{1}{\sqrt{2}}\right)^+} \frac{\cos^{-1}(2x\sqrt{1-x^2})}{\left(x - \frac{1}{\sqrt{2}}\right)} - \lim_{x \rightarrow \left(\frac{1}{\sqrt{2}}\right)^+} \frac{\cos^{-1}(2x\sqrt{1-x^2})}{\left(x - \frac{1}{\sqrt{2}}\right)} =$$

(a)  $\sqrt{2}$

(b)  $2\sqrt{2}$

(c)  $4\sqrt{2}$

(d) 0

$$34. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \sin \frac{\pi}{2k} - \cos \frac{\pi}{2k} - \sin \left( \frac{\pi}{2(k+2)} \right) + \cos \frac{\pi}{2(k+2)} \right) =$$

(a) 0

(b) 1

(c) 2

(d) 3

$$35. \lim_{x \rightarrow 0^+} [1 + [x]]^{2/x}, \text{ where } [.] \text{ is greatest integer function, is equal to :}$$

(a) 0

(b) 1

(c)  $e^2$

(d) Does not exist

$$36. \text{ If } m \text{ and } n \text{ are positive integers, then } \lim_{x \rightarrow 0} \frac{(\cos x)^{1/m} - (\cos x)^{1/n}}{x^2} \text{ equals to :}$$

(a)  $m - n$

(b)  $\frac{1}{n} - \frac{1}{m}$

(c)  $\frac{m-n}{2mn}$

(d) None of these

$$37. \text{ The value of ordered pair } (a, b) \text{ such that } \lim_{x \rightarrow 0} \frac{x(1+a \cos x) - b \sin x}{x^3} = 1 \text{ is:}$$

(a)  $\left(-\frac{5}{2}, -\frac{3}{2}\right)$

(b)  $\left(\frac{5}{2}, \frac{3}{2}\right)$

(c)  $\left(-\frac{5}{2}, \frac{3}{2}\right)$

(d)  $\left(\frac{5}{2}, -\frac{3}{2}\right)$

$$38. \text{ What is the value of } a + b, \text{ if } \lim_{x \rightarrow 0} \frac{\sin(ax) - \ln(e^x \cos x)}{x \sin(bx)} = \frac{1}{2} ?$$

(a) 1

(b) 2

(c) 3

(d)  $-\frac{1}{2}$

$$39. \text{ Let } \alpha = \lim_{n \rightarrow \infty} \frac{(1^3 - 1^2) + (2^3 - 2^2) + \dots + (n^3 - n^2)}{n^4}, \text{ then } \alpha \text{ is equal to :}$$

(a)  $\frac{1}{3}$

(b)  $\frac{1}{4}$

(c)  $\frac{1}{2}$

(d) non existent

40. The value of  $\lim_{x \rightarrow 0} \frac{\cos(\sin x) - \cos x}{x^4}$  is equal to:

(a)  $\frac{1}{5}$

(b)  $\frac{1}{6}$

(c)  $\frac{1}{4}$

(d)  $\frac{1}{12}$

41. The value of ordered pair (a, b) such that  $\lim_{x \rightarrow 0} \frac{x(1+a\cos x) - b \sin x}{x^3} = 1$ , is:

(a)  $\left(-\frac{5}{2}, -\frac{3}{2}\right)$

(b)  $\left(\frac{5}{2}, \frac{3}{2}\right)$

(c)  $\left(-\frac{5}{2}, \frac{3}{2}\right)$

(d)  $\left(\frac{5}{2}, -\frac{3}{2}\right)$

42. Consider the sequence :

$$u_n = \sum_{r=1}^n \frac{r}{2^r}, n \geq 1$$

Then the limit of  $u_n$  as  $n \rightarrow \infty$  is :

(a) 1

(b) e

(c)  $\frac{1}{2}$

(d) 2

43. The value of  $\lim_{x \rightarrow 0} \left( (\cos x)^{\frac{1}{\sin^2 x}} + \frac{\sin 2x + 2 \tan^{-1} 3x + 3x^2}{\ln(1+3x+\sin^2 x) + xe^x} \right)$  is :

(a)  $\sqrt{e} + \frac{3}{2}$

(b)  $\frac{1}{\sqrt{e}} + \frac{3}{2}$

(c)  $\sqrt{e} + 2$

(d)  $\frac{1}{\sqrt{e}} + 2$

44. For  $n \in \mathbb{N}$ , let  $f_n(x) = \tan \frac{x}{2} (1 + \sec x)(1 + \sec 2x)(1 + \sec 4x) \dots (1 + \sec^{2^n} x)$ ,

the  $\lim_{x \rightarrow 0} \frac{f_n(x)}{2x}$  is equal to :

(a) 0

(b)  $2^n$

(c)  $2^{n-1}$

(d)  $2^{n+1}$



45. The value of  $\lim_{x \rightarrow \frac{\pi}{4}} (1 + [x])^{\frac{1}{\ln(\tan x)}}$  is

(where  $[.]$  denotes greatest integer function).

- (a) 0 (b) 1  
(c) e (d)  $\frac{1}{e}$

46. If  $\lim_{x \rightarrow 0} \frac{\{(a-n)x - \tan x\} \sin nx}{x^2} = 0$ ,  $n \neq 0$  then a is equal to :

- (a) 0 (b)  $1 + \frac{1}{n}$   
(c) n (d)  $n + \frac{1}{n}$

47. The value of  $\lim_{n \rightarrow \infty} \left(\frac{n!}{n^n}\right)^{\frac{3n^3+4}{4n^4-1}}$ ,  $n \in \mathbb{N}$  is equal to :

- (a)  $\left(\frac{1}{e}\right)^{3/4}$  (b)  $e^{3/4}$   
(c)  $e^{-1}$  (d) 0

48. The value of  $\lim_{x \rightarrow \infty} \frac{ax^2+bx+c}{dx+e}$  ( $a, b, c, d, e \in \mathbb{R} - \{0\}$ ) depends on the sign of :

- (a) a only (b) d only  
(c) a and d only (d) a, b and d only

49. Let  $f(x) = \lim_{n \rightarrow \infty} \tan^{-1} \left(4n^2 \left(1 - \cos \frac{x}{n}\right)\right)$  and  $g(x) = \lim_{n \rightarrow \infty} \frac{n^2}{2} \ln \cos \left(\frac{2x}{n}\right)$

Then  $\lim_{x \rightarrow 0} \frac{e^{-2g(x)} - e^{f(x)}}{x^6}$  equals.

- (a)  $\frac{8}{3}$  (b)  $\frac{7}{3}$   
(c)  $\frac{5}{3}$  (d)  $\frac{2}{3}$

50. If  $f(x)$  be a cubic polynomial and  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{f(x)} = \frac{1}{3}$  then  $f(1)$  can not be equal to:

- (a) 5 (b) -5  
(c) 3 (d) -2

51.  $\lim_{x \rightarrow 0} \frac{2e^{\sin x} - e^{-\sin x} - 1}{x^2 + 2x}$  equals to:

- (a)  $\frac{3}{2}$  (b)  $e^{3/2}$   
(c) 2 (d)  $e^2$

52. If  $x_1, x_2, x_3, \dots, x_n$  are the roots of  $x^n + ax + b = 0$ , then the value of

$(x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \dots (x_1 - x_n)$  is equal to :

- (a)  $nx_1 + b$  (b)  $nx_1^{n-1} + a$   
(c)  $nx_1^{n-1}$  (d)  $nx_1^{n-1}$

53.  $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1 + \sin^2 x} - \sqrt[4]{1 - 2 \tan x}}{\sin x + \tan^2 x}$  is equal to :

- (a) -1 (b) 1  
(c)  $\frac{1}{2}$  (d)  $-\frac{1}{2}$

54. If  $f(x) = \begin{vmatrix} x \cos x & 2x \sin x & \tan x \\ 1 & x & 1 \\ 1 & 2x & 1 \end{vmatrix}$ , find  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2}$ .

- (a) 0 (b) 1  
(c) -1 (d) Does not exist

## Answer

1.	(b)	2.	(d)	3.	(d)	4.	(d)	5.	(b)	6.	(b)	7.	(d)	8.	(a)	9.	(c)	10.	(b)
11.	(a)	12.	(a)	13.	(b)	14.	(d)	15.	(b)	16.	(a)	17.	(c)	18.	(a)	19.	(a)	20.	(c)
21.	(c)	22.	(c)	23.	(a)	24.	(d)	25.	(c)	26.	(a)	27.	(d)	28.	(c)	29.	(d)	30.	(d)
31.	(c)	32.	(a)	33.	(c)	34.	(d)	35.	(b)	36.	(c)	37.	(a)	38.	(b)	39.	(b)	40.	(b)
41.	(a)	42.	(d)	43.	(d)	44.	(c)	45.	(b)	46.	(d)	47.	(a)	48.	(c)	49.	(a)	50.	(c)
51.	(a)	52.	(b)	53.	(c)	54.	(c)												

## Exercise-2: One or More than One Answer is/are Correct

1. If  $\lim_{x \rightarrow 0} (p \tan qx^2 - 3 \cos^2 x + 4)^{1/(3x^2)} = e^{5/3}$ ;  $p, q \in \mathbb{R}$  then :

(a)  $p = \sqrt{2}, q = \frac{1}{2\sqrt{2}}$

(b)  $p = \frac{1}{\sqrt{2}}, q = 2\sqrt{2}$

(c)  $p = 1, q = 2$

(d)  $p = 2, q = 4$

2.  $\lim_{x \rightarrow \infty} 2(\sqrt{25x^2 + x} - 5x)$  is equal to :

(a)  $\lim_{x \rightarrow 0} \frac{2x - \log_e(1+x)^2}{5x^2}$

(b)  $\lim_{x \rightarrow 0} \frac{e^{-x} - 1 + x}{x^2}$

(c)  $\lim_{x \rightarrow 0} \frac{2(1 - \cos x^2)}{5x^2}$

(d)  $\lim_{x \rightarrow 0} \frac{\sin^x \frac{x}{5}}{x}$

3. Let  $\lim_{x \rightarrow \infty} (2^x + a^x + e^x)^{1/x} = L$

Which of the following statement(s) is (are) correct ?

(a) if  $L = a$  ( $a > 0$ ), then the range of  $a$  is  $[e, \infty)$

(b) if  $L = 2e$  ( $a > 0$ ), then the range of  $a$  is  $\{2e\}$

(c) if  $L = e$  ( $a > 0$ ), then the range of  $a$  is  $(0, e]$

(d) if  $L = 2a$  ( $a > 1$ ), then the range of  $a$  is  $(\frac{e}{2}, \infty)$

4. Let  $\tan \alpha \cdot x + \sin \alpha \cdot y = \alpha$  and  $\alpha \operatorname{cosec} \alpha \cdot x + \cos \alpha \cdot y = 1$  be two variable straight lines,  $\alpha$  being the parameter. Let  $P$  be the point of intersection of the lines. In the limiting position when  $\alpha \rightarrow 0$ , the point  $P$  lies on the line :

(a)  $x = 2$

(b)  $x = -1$

(c)  $y + 1 = 0$

(d)  $y = 2$

5. Let  $f: \mathbb{R} \rightarrow [-1, 1]$  be defined as  $f(x) = \cos(\sin x)$ , then which of the following is (are) correct ?

(a)  $f$  is periodic with fundamental period  $2\pi$

(b) Range of  $f = [\cos 1, 1]$

(c)  $\lim_{x \rightarrow \frac{\pi}{2}} \left( f\left(\frac{\pi}{2} - x\right) + f\left(\frac{\pi}{2} + x\right) \right) = 2$

(d)  $f$  is neither even nor odd function

6. Let  $f(x) = x + \sqrt{x^2 + 2x}$  and  $g(x) = \sqrt{x^2 + 2x} - x$ , then :

- (a)  $\lim_{x \rightarrow \infty} g(x) = 1$  (b)  $\lim_{x \rightarrow \infty} f(x) = 1$   
 (c)  $\lim_{x \rightarrow \infty} f(x) = -1$  (d)  $\lim_{x \rightarrow \infty} g(x) = -1$

7. Which of the following limits does not exist ?

- (a)  $\lim_{x \rightarrow \infty} \operatorname{cosec}^{-1}\left(\frac{x}{x+7}\right)$  (b)  $\lim_{x \rightarrow 1} \sec^{-1}(\sin^{-1}x)$   
 (c)  $\lim_{x \rightarrow 0^+} x^{\frac{1}{x}}$  (d)  $\lim_{x \rightarrow 0} \left(\tan\left(\frac{\pi}{8} + x\right)\right)^{\cot x}$

8. If  $f(x) = \lim_{n \rightarrow \infty} x \left(\frac{3}{2} + [\cos x](\sqrt{n^2 + 1} - \sqrt{n^2 - 3n + 1})\right)$  where  $[y]$  denotes largest integer  $x \leq y$ , then identify the correct statement(s).

- (a)  $\lim_{x \rightarrow 0} f(x) = 0$  (b)  $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \frac{3\pi}{4}$   
 (c)  $f(x) = \frac{3x}{2} \forall x \in \left[0, \frac{\pi}{2}\right]$  (d)  $f(x) = 0 \forall x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

9. Let  $f: \mathbb{R} \rightarrow \mathbb{R}; f(x) = \begin{cases} (-1)^n & \text{if } x = \frac{1}{2^{2^n}}, n = 1, 2, 3, \dots \\ 0 & \text{otherwise} \end{cases}$

then identify the correct statement (s).

- (a)  $\lim_{x \rightarrow 0} f(x) = 0$  (b)  $\lim_{x \rightarrow 0} f(x)$  does not exist  
 (c)  $\lim_{x \rightarrow 0} f(x)f(2x) = 0$  (d)  $\lim_{x \rightarrow 0} f(x)f(2x)$  does not exist

10. If  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} [f(x)]$  ( $[.]$  denotes the greatest integer function) and  $f(x)$  is non-constant continuous function, then :

- (a)  $\lim_{x \rightarrow a} f(x)$  is an integer  
 (b)  $\lim_{x \rightarrow a} f(x)$  is non-integer  
 (c)  $f(x)$  has local maximum at  $x = a$   
 (d)  $f(x)$  has local minimum at  $x = a$

11. Let  $f(x) = \frac{\cos^{-1}(1-\{x\})\sin^{-1}(1-\{x\})}{\sqrt{2\{x\}(1-\{x\})}}$  where  $\{x\}$  denotes the fractional part of  $x$ , then

:

- (a)  $\lim_{x \rightarrow 0^+} f(x) = \frac{\pi}{4}$                       (b)  $\lim_{x \rightarrow 0^+} f(x) = \sqrt{2} \lim_{x \rightarrow 0^-} f(x)$   
 (c)  $\lim_{x \rightarrow 0^-} f(x) = \frac{\pi}{4\sqrt{2}}$                       (d)  $\lim_{x \rightarrow 0^-} f(x) = \frac{\pi}{2\sqrt{2}}$

12. If  $\lim_{x \rightarrow 0} \frac{(\sin(\sin x) - \sin x)}{ax^3 + bx^5 + c} = -\frac{1}{12}$ , then :

- (a)  $a = 2$                                       (b)  $a = -2$   
 (c)  $c = 0$                                       (d)  $b \in \mathbb{R}$

13. If  $f(x) = \lim_{n \rightarrow \infty} \left( n \left( x^{\frac{1}{n}} - 1 \right) \right)$  for  $x > 0$ , then which of the following is/are true ?

- (a)  $f\left(\frac{1}{x}\right) = 0$                               (b)  $f\left(\frac{1}{x}\right) = \frac{1}{f(x)}$   
 (c)  $f\left(\frac{1}{x}\right) = -f(x)$                       (d)  $f(xy) = f(x) + f(y)$

14. The value of  $\lim_{n \rightarrow \infty} \cos^2(\pi(\sqrt[3]{n^3 + n^2 + 2n}))$  (where  $n \in \mathbb{N}$ ):

- (a)  $\frac{1}{3}$     (b)  $\frac{1}{2}$   
 (c)  $\frac{1}{4}$     (d)  $\frac{1}{9}$

15. If  $\alpha, \beta, \in \left(-\frac{\pi}{2}, 0\right)$  such that  $(\sin \alpha + \sin \beta) + \frac{\sin \alpha}{\sin \beta} = 0$  and

$(\sin \alpha + \sin \beta) + \frac{\sin \alpha}{\sin \beta} = -1$  and  $\lambda = \lim_{n \rightarrow \infty} \frac{1+(2 \sin \alpha)^{2n}}{(2 \sin \beta)^{2n}}$  then :

- (a)  $\alpha = -\frac{\pi}{6}$                                       (b)  $\pi = 2$   
 (c)  $\alpha = -\frac{\pi}{3}$                                       (d)  $\pi = 1$

