Function

Excerise-1: Single Choice Problems

1. Range of the function $f(x) = \log_2(2 - \log_{\sqrt{2}}(16\sin^2 x + 1))$is:
(a) [0, 1] (b) (-∞, 1] (c) [-1, 1] (d) (-∞, ∞)

2. The value of a and b for which $\left|e^{|x-b|} - a\right| = 2$, has four distinct solutions, are:
(a) $a \in (-3, \infty), b = 0$ (b) $a \in (2, \infty), b = 0$
(c) $a \in (3, \infty), b \in R$ (d) $a \in (2, \infty), b = a$

3. The range of the function:
$$f(x) = \tan^{-1}x + \frac{1}{2}\sin^{-1}x$$
(a) $(-\pi/2, \pi/2)$ (b) $[-\pi/2, \pi/2] - \{0\}$
(c) $[-\pi/2, \pi/2]$ (d) $(-3\pi/4, 3\pi/4)$

4. Find the number of real ordered pair(s) (x, y) for which:
$16^{x^2+y} + 16^{x+y^2} = 1$
(a) 0 (b) 1 (c) 2 (d) 3

5. The complete range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|} = x^2 - a$ is satisfied for maximum number of values of x is:
(a) $(-\infty, -1)$ (b) $(-\infty, \infty)$ (c) $(-1, 1)$ (d) $(-1, \infty)$

6. For a real number x, let [x] denotes the greatest integer less than or equal to x. Let $f: R \longrightarrow R$ be defined by $f(x) = 2x + [x] + \sin x \cos x$. then f is:
(a) One-one but not onto (b) Onto but not one-one
(c) Both one-one and onto (d) Neither one-one nor onto

7. The maximum value of $\sec^{-1}\left(\frac{7-5(x^2+3)}{2(x^2+2)}\right)$ is:
(a) $\frac{5\pi}{6}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{2\pi}{3}$

8. Number of ordered pair (a, b) from the set A = {1, 2, 3, 4, 5} so that function $f(x) = \frac{x^3}{3} + \frac{a}{2}x^2 + bx + 10$ is an injective mapping $\forall x \in R$:

(a) 13 (b) 14 (c) 15 (d) 16

9. Let A be the greatest value of the function $f(x) = \log_x[x]$, (where [.] denotes greatest integer function) and B be the least value of the function $g(x) = |\sin x| + |\cos x|$, then:
(a) A > B (b) A < B (c) A = B (d) 2A + B = 4

10. Let $A = [a, \infty)$ denotes domain, then $f: [a, \infty) \to B, f(x) = 2x^3 - 3x^2 + 6$ will have an inverse for the smallest real value of a, if:
(a) $a = 1, B = [5, \infty)$ (b) $a = 2, B = [10, \infty)$
(c) $a = 0, B = [6, \infty)$ (d) $a = -1, B = [1, \infty)$

11. Solution of the in equation $\{x\}\ (\{x\} - 1)(\{x\} + 2) \geq 0$
(where $\{.\}$ denotes fractional part function) is:
(a) $x \in (2,1)$ (b) $x \in I$(I denote set of integers)
(c) $x \in [0,1)$ (d) $x \in [-2, 0)$

12. Let f(x), g(x) be two real valued functions then the function $h(x) = 2\max\{f(x) - g(x), 0\}$ is equal to:
(a) $f(x) - g(x) - |g(x) - f(x)|$ (b) $f(x) + g(x) - |g(x) - f(x)|$
(c) $f(x) - g(x) + |g(x) - f(x)|$ (d) $f(x) + g(x) + |g(x) - f(x)|$

13. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set $A = \{1, 2, 3, 4\}$.
The relation R is:
(a) a function (b) reflexive (c) not symmetric (d) transitive

14. The true set values of 'K' for which $\sin^{-1}\left(\frac{1}{1+\sin^2 x}\right) = \frac{K\pi}{6}$ may have a solution is:
(a) $\left[\frac{1}{4}, \frac{1}{2}\right]$ (b) [1, 3] (c) $\left[\frac{1}{6}, \frac{1}{2}\right]$ (d) [2, 4]

15. A real valued function f(x) satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where 'a' is a given constant and $f(0) = 1, f(2a - x)$ is equal to:
(a) $-f(x)$ (b) $f(x)$ (c) $f(a) + f(a - x)$ (d) $f(-x)$

16. Let g:R→R be given by $g(x) = 3 + 4x$ if $g^n(x) = gogogo \ldots \ldots \ldots og(x)$n times.
Then inverse of $g^n(x)$ is equal to:
(a) $(x + 1 - 4^n).\ 4^{-n}$ (b) $(x - 1 + 4^n)\ 4^{-n}$

(c) $(x+1+4^n)\,4^{-n}$ (d) None of these

17. Let f:D → R be defined as : $f(x)=\frac{x^2+2x+a}{x^2+4x+3a}$ where D and R denote the domain of f and the set of all real numbers respectively. If f is surjective mapping, then the complete range of a is:

(a) $0\le a\le 1$ (b) $0<a\le 1$ (c) $0\le a<1$ (d) $0<a<1$

18. If $f:(-\infty,2]\longrightarrow(-\infty,4]$, where $f(x)=x(4-x)$, then $f^{-1}(x)$ is given by:

(a) $2-\sqrt{4-x}$ (b) $2+\sqrt{4-x}$ (c) $2-\sqrt{4-x}$ (d) $-2-\sqrt{4-x}$

19. If [5 sin x] + [cos x] + 6 = 0, then range of $f(x)=\sqrt{3}\cos x+\sin x$ corresponding to solution set of the given equation is: (where [.] denotes greatest integer function)

(a) $[-2,-1]$ (b) $\left(-\frac{3\sqrt{3}+2}{5},-1\right)$ (c) $[-2,-\sqrt{3}]$ (d) $\left(-\frac{3\sqrt{3}+4}{5},-1\right)$

20. If $f: R\to R, f(x)=ax+\cos x$ is an invertible function, then complete set of values of a is:

(a) $(-2,-1]\cup[1,2)$ (b) $[-1,1]$

(c) $(-\infty,-1]\cup[1,\infty)$ (d) $(-\infty,-2]\cup[2,\infty)$

21. The range of function $f(x)=[1+\sin x]+\left[2+\sin\frac{x}{2}\right]+\left[3+\sin\frac{x}{2}\right]+\cdots+\left[n+\sin\frac{x}{2}\right]\ \forall\, x\in[0,\pi], n\in N$([.] denotes greatest integer function) is:

(a) $\left\{\frac{n^2+n-2}{2},\frac{n(n+1)}{2}\right\}$ (b) $\left\{\frac{n(n+1)}{2}\right\}$

(c) $\left\{\frac{n(n+1)}{2},\frac{n^2+n+2}{2},\frac{n^2+n+4}{2}\right\}$ (d) $\left\{\frac{n(n+1)}{2},\frac{n^2+n+2}{2}\right\}$

22. If $f: R\to R, f(x)=\frac{x^2+ax+1}{x^2+x+1}$, then the complete set of values of ‘a’ such that f(x) is onto is :

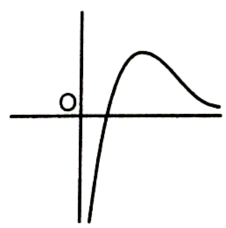
(a) $(-\infty,\infty)$ (b) $(-\infty,0)$

(c) $(0,\infty)$ (d) not possible

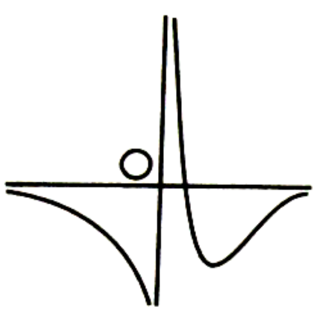
23. If $f(x)$ and $g(x)$ are two functions such that $f(x) = [x] + [-x]$ and $g(x) = \{x\}\ \forall\ x \in R$ and $h(x) = f(g(x))$; then which of the following is incorrect?
([.] Denotes greatest integer function and {.} denotes fractional part function)
(a) $f(x)$ and $h(x)$ are identical function
(b) $f(x) = g(x)$ has no solution
(c) $f(x) + h(x) > 0$ has no solution
(d) $f(x) - h(x) > 0$ is a periodic function

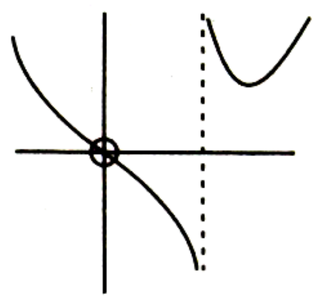
24. Number of elements in the range set of $f(x) = \left[\frac{x}{15}\right]\left[-\frac{15}{x}\right]\ \forall\ x \in (0, 90)$; (where [.] denotes greatest integer function):
(a) 5
(b) 6
(c) 7
(d) Infinite

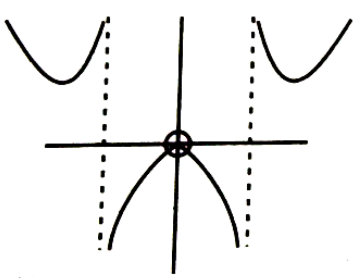
25. The graph of function $f(x)$ is shown below:

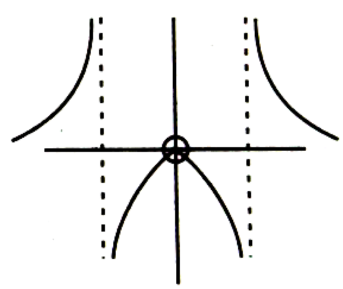


Then the graph of $g(x) = \frac{1}{f(|x|)}$ is :

(a)


(b)


(c)


(d)


26. Which of the following function is homogeneous?

(a) $f(x) = x\sin y + y\sin x$ (b) $g(x) = xe^{\frac{y}{x}} + ye^{\frac{x}{y}}$

(c) $h(x) = \frac{xy}{x+y^2}$ (d) $\emptyset(x) = \frac{x-y\cos x}{y\sin x+y}$

27. Let $f(x) = \begin{bmatrix} 2x+3 & ; & x \le 1 \\ a^2x+1 & ; & x > 1 \end{bmatrix}$. If the range of $f(x) = R$ (set of real numbers) then number of integral value (s), which a may take:;

(a) 2 (b) 3

(c) 4 (d) 5

28. The maximum integral value of x in the domain of $f(x) = \log_{10}(\log_{\frac{1}{3}}(\log_4(x-5))$ is :

(a) 5 (b) 7

(c) 8 (d) 9

29. Range of the function $f(x) = \log_2\left(\frac{4}{\sqrt{x+2}+\sqrt{2-x}}\right)$ is :

30. Number of integers statisfying the equation $|x^2+5x| + |x-x^2| = 6x$ is :

(a) 3 (b) 5

(c) 7 (d) 9

31. Which of the following is not an odd function ?

(a) In $\left(\frac{x^4+x^2+1}{(x^2+x+1)^2}\right)$

(b) sgn (sgn (x))

(c) $\sin(\tan x)$

(d) $f(x)$, where $f(x) + f\left(\frac{1}{x}\right) = f(x).f\left(\frac{1}{x}\right)$ $\forall\ x \in R - \{0\}$ and $f(2) = 33$

32. Which of the following function is periodic with fundamental period π?

(a) $f(x) = \cos x + \left[\!\left[\frac{\sin x}{2}\right]\!\right]$; where [.] denotes greatest integer function

(b) $g(x) = \frac{\sin x+\sin 7x}{\cos x+\cos 7x} + |\sin x|$

(c) $h(x) = \{x\} + |\cos x|$; where {.} denotes fractional part function

(d) $\emptyset(x) = |\cos x| + \text{In}(\sin x)$

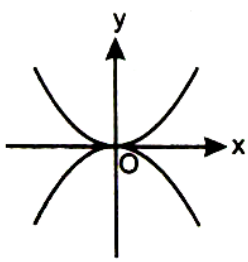
33. Let $f: N \longrightarrow Z$ and $f(x) = \begin{cases} \frac{x-1}{2} & ; \quad \text{when x is odd} \\ -\frac{x}{2} & ; \quad \text{when x is even} \end{cases}$, then :

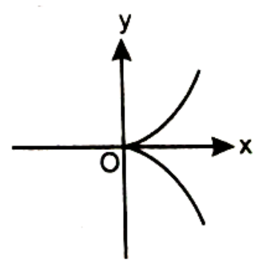
(a) f(x) is bijective
(b) f(x) is injective but not surjective
(c) f(x) is not injective but surjective
(d) f(x) is neither injective nor surjective

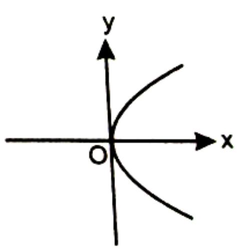
34. Let g(x) be the inverse of $f(x) = \frac{2^{x+1} - 2^{1-x}}{2^x + 2^{-x}}$ then g(x) be :

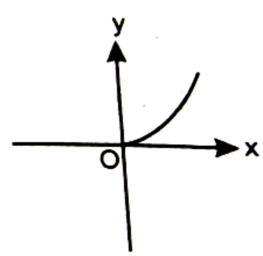
(a) $\frac{1}{2}\log_2\left(\frac{2+x}{2-x}\right)$ (b) $-\frac{1}{2}\log_2\left(\frac{2+x}{2-x}\right)$
(c) $\log_2\left(\frac{2+x}{2-x}\right)$ (d) $\log_2\left(\frac{2-x}{2+x}\right)$

35. Which of the following is the graph of the curve $\sqrt{|y|} = x$ is?

(a)


(b)


(c)


(d)


36. Range of $f(x) = \log_{[x]}(9 - x^2)$; where [.] denotes G.I.F. is :

(a) $\{1, 2\}$ (b) $\{-\infty, 2\}$
(c) $(-\infty, \log_2 5]$ (d) $[\log_2 5, 3]$

37. If $e^x + e^{f(x)} = e$, ten for f(x):

(a) Domain is $(-\infty, 1)$ (b) Range is $(-\infty, 1]$
(c) Domain is $(-\infty, 0]$ (d) Range is $(-\infty, 0]$

38. If high voltage current is applied on the field given by the graph $y + |y| - x - |x| = 0$. On which of the following curve a person can move so that he remains safe?

(a) $y = x^2$ (b) $y = \text{sgn}(-e^2)$

(c) $y = \log_{1/3} x$ (d) $y = m + |x|; m > 3$

39. If $|f(x) + 6 - x^2] = |f(x)| + |4 - x^2| + 2$, then f(x) is necessarily non-negative for :

(a) $x \in [-2, 2]$ (b) $x \in (-\infty, -2) \cup (2, \infty)$

(c) $x \in [-\sqrt{6}, \sqrt{6}]$ (d) $x \in [-5, -2] \cup [2, 5]$

40. Let $f(x) = \cos(px) + \sin x$ be periodic, then p must be :

(a) Positive real number (b) Negative real number

(c) Rational (d) Prime

41. The domain of f(x) is (0, 1), therefore, the domain of $y = f(e^x) + f(\text{In}|x|)$ is :

(a) $\left(\frac{1}{e}, 1\right)$ (b) $(-e, -1)$

(c) $\left(-1, -\frac{1}{e}\right)$ (d) $(-e, -1) \cup (1, e)$

42. Let $A = \{1, 2, 3, 4\}$ and $f: A \longrightarrow A$ satisfy $f(1) = 2, f(2) = 3, f(3) = 4 = 1$.
Suppose $g: A \longrightarrow A$ satisfies $g(1) = 3$ and $fog = gof$, then $g =$

(a) $\{(1,3), (2,1), (3,2), (4,4)\}$ (b) $\{(1,3), (2,4), (3,1), (4,2)\}$

(c) $\{(1,3), (2,2), (3,4), (4,3)\}$ (d) $\{(1,3), (2,4), (3,2), (4,1)\}$

43. The number of solutions of the equation $[y + [y]] = 2\cos x$ is :
(where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ and $[.]$ = greatest integer function)

(a) 0 (b) 1

(c) 2 (d) Infinite

44. The function, $f(x)=\begin{cases}\frac{(x^{2n})}{(x^{2n}\operatorname{sgn} x)^{2n+1}}\left(\frac{e^{\frac{1}{x}}-e^{-\frac{1}{x}}}{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}\right) & x\neq 0\\ 1 & x=0\end{cases}$ $n\in N$:

(a) Odd function
(b) Even function
(c) Neither odd nor even function
(d) Constant function

45. Let $f(1)=1$, and $f(n)=2\sum_{r=1}^{n-1}f(r)$. Then $\sum_{r=1}^{m}f(r)$ is equal to :

(a) $\frac{3^m-1}{2}$
(b) 3^m
(c) 3^{m-1}
(d) $\frac{3^{m-1}-1}{2}$

46. Let $f(x)=\frac{x}{\sqrt{1+x^2}}$, then $\underbrace{fofofo\ldots.of\,(x)}$ is :

(a) $\frac{x}{\sqrt{1+(\sum_{r=1}^{n}r)x^2}}$
(b) $\frac{x}{\sqrt{1+(\sum_{r=1}^{n}1)x^2}}$
(c) $\left(\frac{x}{\sqrt{1+x^2}}\right)^n$
(d) $\frac{nx}{\sqrt{1+nx^2}}$

47. Let $f: R\to R, f(x)=2x+|\cos x|$, then f is:

(a) One-one and into
(b) One-one and onto
(c) Many-one and into
(d) Many –one and onto

48. Let $f: R\to R, f(x)=x^3+x^2+3x+\sin x$, then f is :

(a) One-one and into
(b) One-one and onto
(c) Many-one and into
(d) Many-one and onto

49. $f(x)=\{x\}+\{x+1\}+\{x+2\}+\cdots+\{x+99\}$, then $\left[f(\sqrt{2})\right]$, (where $\{.\}$ denotes fractional part function and [.] denotes the greatest integer function) is equal to :

(a) 5050
(b) 4950
(c) 41
(d) 14

50. If $|\cot x+\operatorname{cosec} x|=|\cot x|+|\operatorname{cosec} x|$; $x\in[0,2\pi]$, then complete set of values of x is :

(a) $[0,\pi]$
(b) $\left(0,\frac{\pi}{2}\right]$
(c) $\left(0,\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2},2\pi\right)$
(d) $\left(0,\frac{3\pi}{2}\right]\cup\left[\frac{7\pi}{4},2\pi\right]$

51. The function $f(x) = 0$ has eight distinct real solution and f also satisfy $f(4 + x) = f(4 - x)$. The sum of all the eight solution of $f(x) = 0$ is :

(a) 12 (b) 32

(c) 16 (d) 15

52. Let $f(x)$ be a polynomial of degree 5 with leading coefficient unity such that $f(1) = 5, f(2) = 4, f(3) = 3, f(4) = 2, f(5) = 1$. Then $f(6)$ is equal to :

(a) 0 (b) 24

(c) 120 (d) 720

53. Let $f: A \to B$ be a function such that $f(x) = \sqrt{x-2} + \sqrt{4-x}$, is invertible, then which of the following is not possible ?

(a) $A = [3, 4]$ (b) $A = [2, 3]$

(c) $A = [2, 2\sqrt{3}]$ (d) $\left[2, 2\sqrt{2}\right]$

54. The number of positive integrals values of x satisfying $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$ is :

(where [.] denotes greatest integer function)

(a) 21 (b) 22

(c) 23 (d) 24

55. The domain of function $f(x) = \log_{\left[x+\frac{1}{2}\right]}(2x^2 + x - 1)$, where [.] denotes the greatest integer function is :

(a) $\left[\frac{3}{2}, \infty\right)$ (b) $(2, \infty)$

(c) $\left(-\frac{1}{2}, \infty\right) - \left\{\frac{1}{2}\right\}$ (d) $\left(\frac{1}{2}, 1\right) \cup (1, \infty)$

56. The solution set of the equation $[x]^2 + [x + 1] - 3 = 0$, where [.] represents greatest integer function is :

(a) $[-1,0) \cup [1,2)$ (b) $[-2, -1) \cup [1,2)$

(c) $[1,2)$ (d) $[-3, -2) \cup [2,3)$

57. Which among the following relations is a function ?

(a) $x^2 + y^2 = r^2$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r^2$

(c) $y^2 + 4ax$ (d) $x^2 = 4ay$

58. A function $f: R \longrightarrow R$ is defined as $f(x) = 3x^2 + 1$. Then $f^{-1}(x)$ is :

(a) $\frac{\sqrt{x-1}}{3}$ (b) $\frac{1}{3}\sqrt{x} - 1$

(c) f^{-1} does not exist (d) $\sqrt{\frac{x-1}{3}}$

59. If $f(x) = \begin{cases} 2+x, & x \geq 0 \\ 4-x, & x < 0 \end{cases}$, then $f(f(x))$ is given by :

(a) $f(f(x)) = \begin{cases} 4+x & , & x \geq 0 \\ 6-x & , & x < 0 \end{cases}$ (b) $f(f(x)) = \begin{cases} 4+x & , & x \geq 0 \\ x & , & x < 0 \end{cases}$

(c) $f(f(x)) = \begin{cases} 4-x & , & x \geq 0 \\ x & , & x < 0 \end{cases}$ (d) $f(f(x)) = \begin{cases} 4-2x & , & x \geq 0 \\ 4+2x & , & x < 0 \end{cases}$

60. The function $f: R \rightarrow R$ defined as $f(x) = \frac{3x^2+3x-4}{3+3x-4x^2}$ is :

(a) One to one but not onto (b) Onto but not one to one

(c) Both one to one and onto (d) Neither one to one nor onto

61. The number of solutions of the equation $e^2 - \log|x| = 0$ is :

(a) 0 (b) 1

(c) 2 (d) 3

62. If complete solution set of $e^{-x} \leq 4 - x$ is $[\alpha, \beta]$ then $[\alpha] + [\beta]$ is equal to :
(where [.] denotes greatest integer function)

(a) 0 (b) 2

(c) 1 (d) 4

63. Range of $f(x) = \sqrt{\sin\left(\log_7\left(\cos(\sin x)\right)\right)}$ is :

(a) $[0, 1)$ (b) $\{0, 1\}$

(c) $\{0\}$ (d) $[1, 7]$

64. If domain of $y = f(x)$ is $x \in [-3,2]$, then domain of $y = f(|[x]|)$:
(where [.] denotes greatest integer function)

(a) $[-3, 2]$ (b) $[-2, 3)$

(c) $[-3,3]$ (d) $[-2,3]$

65. Range of the function $f(x) = \cot^{-1}\{-x\} + \sin^{-1}\{x\} + \cos^{-1}\{x\}$, where $\{.\}$ denotes fractional part function :

(a) $\left(\frac{3\pi}{4}, \pi\right)$ (b) $\left[\frac{3\pi}{4}, \pi\right)$

(c) $\left[\frac{3\pi}{4}, \pi\right]$ (d) $\left(\frac{3\pi}{4}, \pi\right]$

66. Let $f: R - \left\{\frac{3}{2}\right\} \to R, f(x) = \frac{3x+5}{2x-3}$. Let $f_1(x) = f(x), f_n(x) = f(f_{n-1}(x))$ for $n \geq 2, n \in N$, then $f_{2008}(x) + f_{2009}(x) =$

(a) $\frac{2x^2+5}{2x-3}$ (b) $\frac{x^2+5}{2x-3}$

(c) $\frac{2x^2-5}{2x-3}$ (d) $\frac{x^2-5}{2x-3}$

67. Range of the function, $f(x) = \frac{(1+x+x^2)(1+x^4)}{x^3}$, for $x > 0$ is :

(a) $[0, \infty)$ (b) $[2, \infty)$

(c) $[4, \infty)$ (d) $[6, \infty)$

68. The functionf: $(-\infty, 3] \to (0, e^7]$ defined by $f(x) = e^{x^3-3x^2-9x+2}$ is :

(a) Many-one and onto (b) Many-one and into

(c) One to one and onto (d) One to one and into

69. If $f(x) = \sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}$; $x \in R$, then range of f(x) is given by :

(a) $[-1,1]$ (b) $[0,1]$

(c) $(-1,1)$ (d) None of these

70. Set of values of 'a' for which the function $f: R \to R$, given by $f(x) = x^3 + (a+2)x^2 + 3ax + 10$ is one –one is given by :

(a) $(-\infty, 1] \cup [4, \infty)$ (b) $[1, 4]$

(c) $[1, \infty)$ (d) $[-\infty, 4]$

71. If the range of the function $f(x) = \tan^{-1}(3x^2 + bx + c)$ is $\left[0, \frac{\pi}{2}\right)$; (domain is R), then :

(a) $b^2 = 3c$ (b) $b^2 = 4c$

(c) $b^2 = 12c$ (d) $b^2 = 8c$

72. Let $f(x) = \sin^{-1} x - \cos^{-1} x$, then the set of values of k for which of $|f(x)| = k$ has exactly two distinct solutions is :

(a) $\left(0, \frac{\pi}{2}\right]$ (b) $\left(0, \frac{\pi}{2}\right)$

(c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (d) $\left[\pi, \frac{3\pi}{2}\right]$

73. Let $f: R \to R$ is defined by $(x) = \begin{cases} (x+1)^3 & ; \quad x \leq 1 \\ \text{In } x + (b^2 - 3b + 10) & ; \quad x > 1 \end{cases}$.

If f(x) is invertible, then the set of all values of $'b'$ is :

(a) $\{1, 2\}$ (b) $\emptyset$

(c) $\{2, 5\}$ (d) None of these

74. Let f(x) is continuous function with range $[-1, 1]$ and f(x) is defined $\forall\, x \in R$. If $(x) = \frac{e^{f(x)} - e^{|f(x)|}}{e^{f(x)} + e^{|f(x)|}}$, then range of g(x) is :

(a) $[0, 1]$ (b) $\left[0, \frac{e^2+1}{e^2-1}\right]$

(c) $\left[0, \frac{e^2-1}{e^2+1}\right]$ (d) $\left[\frac{-e^2+1}{e^2-1}, 0\right]$

75. Consider all functions $f: \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

if f(k) is odd then f(k + 1) is even, k = 1, 2, 3.

The number of such functions is :

(a) 4 (b) 8

(c) 12 (d) 16

76. Consider the function $f: R - \{1\} \longrightarrow R - \{2\}$ given by $(x) = \frac{2x}{x-1}$. Then :

(a) f is one-one but not onto (b) f is onto but not one-one

(c) f is neither one-one nor onto (d) f is both one-one and onto

77. If range of function f(x) whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \frac{1}{2} f(2x + 1)$ is equal to :

(a) $[-2, 4]$ (b) $[-1, 2]$

(c) $[-3, 9]$ (d) $[-2, 2]$

78. Let $f: R \longrightarrow R$ and $f(x) = \frac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, f(x) is :

(a) One-one, into
(b) Many-one, onto
(c) One-one, onto
(d) Many one, into

79. Let f(x) be defined as :

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1| + |x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

Then range of function $g(x) = \sin(7(f(x))$ is :

(a) $[0, 1]$
(b) $[-1, 0]$
(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(d) $[-1, 1]$

80. If $[x]^2 - 7[x] + 10 < 0$ and $4[y]^2 - 16[y] + 7 < 0$, then $[x + y]$ cannot be ([.] denotes greatest integer function) :

(a) 7
(b) 8
(c) 9
(d) both (b) and (c)

81. Let $f: R \rightarrow R$ be a function defined by $f(x) = \frac{e^{|x|}-e^{-x}}{e^x+e^{-x}}$, then

(a) f(x) is many one, onto function
(b) f(x) is many one, into function
(c) f(x) is decreasing function $\forall\, x \in R$
(d) f(x) is bijective function

82. The function f(x) satisfy the equation $f(1-x) + 2f(x) = 3x \;\forall\, x \in R$, then $f(0) =$

(a) -2
(b) -1
(c) 0
(d) 1

83. Let $f: [0, 5] \rightarrow [0, 5]$ be an invertible function defined by $f(x) = ax^2 + bx + c$, where $a, b, c \in R$, $abc \neq 0$, then one of the root of the equation $cx^2 + bx + a = 0$ is :

(a) a
(b) b
(c) c
(d) $a + b + c$

84. Let $f(x) = x^2 + \lambda x + \mu \cos x, \lambda$ being an integer and μ is a real number. The number of ordered pairs (λ, μ) for which the equation $f(x) = 0$ and $f(f(x)) = 0$ have the same (non empty) set of real roots is :

(a) 2 (b) 3
(c) 4 (d) 6

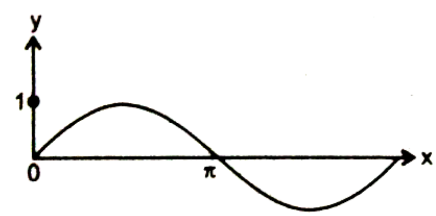
85. Consider all function $f: \{1, 2, 3, 4\} \longrightarrow \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property :

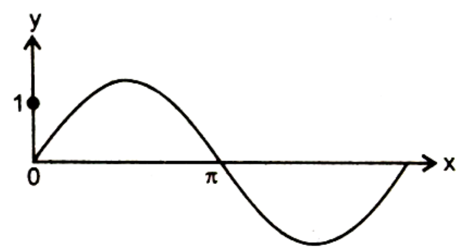
if $f(k)$ is odd then $f(k + 1)$ is even, $k = 1, 2, 3.$

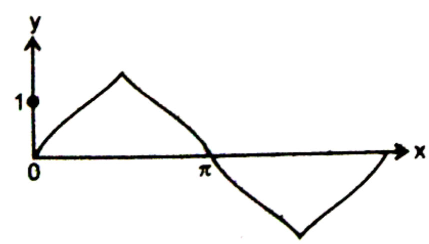
The number of such function is :

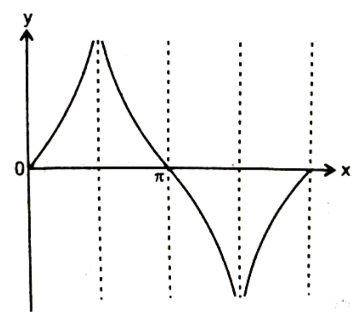
(a) 4 (b) 8
(c) 12 (d) 16

86. Which of the following is closest to the graph of $y = \tan(\sin x), x > 0$?

(a)


(b)


(c)


(d)


87. Consider the function $f: R - \{1\} \longrightarrow R - \{2\}$ given by $(x) = \frac{2x}{x-1}$. Then

(a) f is one-one but not onto (b) f is onto but one-one
(c) f is neither one-one nor onto (d) f is both one-one and onto

88. If range of function $f(x)$ whose domain is set of all real numbers is $[-2, 4]$, then range of function $g(x) = \frac{1}{2}f(2x+1)$ is equal to :

(a) $[-2, 4]$ (b) $[-1, 2]$
(c) $[-3, 9]$ (d) $[-2, 2]$

89. Let $f: R \to R$ and $f(x) = \frac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then $f(x)$ is :

(a) One-one, into (b) Many one, onto
(c) One-one, onto (d) Many one, into

90. Let $f(x)$ be defined as

$$f(x) = \begin{cases} |x| & 0 \le x < 1 \\ |x-1| + |x-2| & 1 \le x < 2 \\ |x-3| & 2 \le x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x))$ is :

(a) $[0, 1]$ (b) $[-1, 0]$
(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-1, 1]$

91. The number of integral values of x in the domain of function f defined as $f(x) = \sqrt{\text{In}\left|\text{In}\,|x|\right|} + \sqrt{7|x| - |x|^2 - 10}$ is :

(a) 5 (b) 6
(c) 7 (d) 8

92. The complete set of values of x in the domain of function $f(x) = \sqrt{\log_{x+2\{x\}}([x]^2 - 5[x+7])}$ (where [.] denote greatest integer function and {.} denote fraction part function) is :

(a) $\left(-\frac{1}{3}, 0\right) \cup \left(\frac{1}{3}, 1\right) \cup (2, \infty)$ (b) $(0, 1) \cup (1, \infty)$
(c) $\left(-\frac{2}{3}, 0\right) \cup \left(\frac{1}{3}, 1\right) \cup (1, \infty)$ (d) $\left(-\frac{1}{3}, 0\right) \cup \left(\frac{1}{3}, 1\right) \cup (1, \infty)$

93. The number of integral ordered pair (x, y) that satisfy the system of equation $|x + y - 4| = 5$ and $|x - 3| + |y - 1| = 5$ is/are :

(a) 2 (b) 4
(c) 6 (d) 12

94. Let f: R → R, where $(x) = \frac{x^2+ax+1}{x^2+x+1}$. Then the complete set of values of 'a' such that f(x) is onto is :

(a) $(-\infty, \infty)$ (b) $(-\infty, 0)$
(c) $(0, \infty)$ (d) Empty set

95. If $A = \{1, 2, 3, 4\}$ and $f: A \to A$, then total number invertible function 'f' such that $f(2) \neq 2, f(4) \neq 4, f(1) = 1$ is equal to :

(a) 1 (b) 2
(c) 3 (d) 4

96. The domain of definition of $f(x) = \log_{(x^2-x+1)}(2x^2 - 7x + 9)$ is :

(a) R (b) $R - \{0\}$
(c) $R - \{0, 1\}$ (d) $R - \{1\}$

97. If $A = \{1, 2, 3, 4\}, B = \{1, 2, 3, 4, 5,6\}$ and $f: A \to B$ is an injective mapping satisfying $f(i) \neq i$, then number of such mappings are :

(a) 182 (b) 181
(c) 183 (d) None of these

98. Let $f(x) = x^2 - 2x - 3; x \geq 1$ and $g(x) = 1 + \sqrt{x + 4}; x \geq -4$ then the number of real solutions of equation $f(x) = g(x)$ is are

(a) 0 (b) 1
(c) 2 (d) 4

Answer

1.	(b)	2.	(c)	3.	(c)	4.	(b)	5.	(d)	6.	(a)	7.	(d)	8.	(c)	9.	(c)	10.	(a)
11.	(b)	12.	(c)	13.	(c)	14.	(b)	15.	(a)	16.	(a)	17.	(d)	18.	(a)	19.	(d)	20.	(c)
21.	(d)	22.	(d)	23.	(b)	24.	(b)	25.	(c)	26.	(b)	27.	(c)	28.	(c)	29.	(b)	30.	(c)
31.	(d)	32.	(b)	33.	(a)	34.	(c)	35.	(b)	36.	(c)	37.	(a)	38.	(d)	39.	(a)	40.	(c)
41.	(b)	42.	(b)	43.	(a)	44.	(b)	45.	(c)	46.	(b)	47.	(b)	48.	(b)	49.	(c)	50.	(c)
51.	(b)	52.	(c)	53.	(c)	54.	(d)	55.	(a)	56.	(b)	57.	(d)	58.	(c)	59.	(a)	60.	(b)
61.	(b)	62.	(c)	63.	(c)	64.	(b)	65.	(d)	66.	(a)	67.	(d)	68.	(a)	69.	(a)	70.	(b)
71.	(c)	72.	(a)	73.	(a)	74.	(d)	75.	(c)	76.	(d)	77.	(b)	78.	(d)	79.	(d)	80.	(c)
81.	(b)	82.	(b)	83.	(a)	84.	(c)	85.	(c)	86.	(b)	87.	(d)	88.	(b)	89.	(d)	90.	(d)
91.	(b)	92.	(d)	93.	(d)	94.	(d)	95.	(c)	96.	(c)	97	(b)	98.	(b)				

Excerise-2: One or More than One Answer is/are Correct

1. f(x) is an even periodic function with period 10.

   In $[0,5], f(x) = \begin{cases} 2x & 0 \le x < 2 \\ 3x^2 & 2 \le x < 4 \\ 10x & 4 \le x \le 5 \end{cases}$. Then :

   (a) $f(-4) = 40$ (b) $\frac{f(-13)-f(11)}{f(13)+f(-11)} = \frac{17}{21}$

   (c) $f(5)$ is not defined (d) Range of $f(x)$is $[0,50]$

2. Let $f(x) = \left||x^2 - 4x + 3| - 2\right|$. Which of the following is /are correct ?

   (a) $f(x) = m$ has exactly two real solutions is/are correct ?

   (b) $f(x) = m$ has exactly two real solutions $\forall\ m \in (2,\infty) \cup \{0\}$

   (c) $f(x) = m$ has no solutions $\forall\ m < 0$

   (d) $f(x) = m$ has four distinct real solution $\forall\ m \in (0,1)$

3. Let $f(x) = \cos^{-1}\left(\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}\right)$

   Which of the following statement (s) is/are correct about f(x)?

   (a) Domain is R (b) Range is $[0,\pi]$

   (c) f(x) is even (d) f(x) is derivable in $(\pi, 2\pi)$

4. $|\log_e|x|| = |k-1| - 3$ has four distinct roots then k satisfies :

   (where $|x| < e^2, x \ne 0$)

   (a) $(-4,-2)$ (b) $(4,6)$

   (c) (e^{-1}, e) (d) (e^{-2}, e^{-1})

5. Which of the following functions are defined for all $x \in R$?

   (Where [.] = denotes greatest integer function)

   (a) $f(x) = \sin[x] + \cos[x]$ (b) $f(x) = \sec^{-1}(1+\sin^2 x)$

   (c) $f(x) = \sqrt{\frac{9}{8} + \cos x + \cos 2x}$ (d) $f(x) = \tan(\text{In }(1+|x|))$

6. Let $f(x) = \begin{cases} x^2 & 0 < x < 2 \\ 2x-3 & 2 \le x < 3 \\ x+2 & x \ge 3 \end{cases}$, then the true equations :

(a) $f\left(f\left(f\left(\frac{3}{2}\right)\right)\right) = f\left(\frac{3}{2}\right)$ (b) $1 + f\left(f\left(f\left(\frac{3}{2}\right)\right)\right) = f\left(\frac{5}{2}\right)$

(c) $f\left(f\left(f(2)\right)\right) = f(1)$ (d) $\underbrace{f(f(f(\ldots f(f(4))\ldots) = 2012}$

7. Let $f: \left[\frac{2\pi}{3}, \frac{5\pi}{3}\right] \longrightarrow [0, 4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$, then :

(a) $f^{-1}(1) = \frac{4\pi}{3}$ (b) $f^{-1}(1) = \pi$

(c) $f^{-1}(2) = \frac{5\pi}{6}$ (d) $f^{-1}(2) = \frac{7\pi}{6}$

8. Let f(x) be invertible function and let $f^{-1}(x)$ be its inverse. Let equation $f\left(f^{-1}(x)\right) = f^{-1}(x)$ has two real roots α and β (with in domain of f(x)), then:

(a) $f(x) = x$ also have same two real roots

(b) $f^{-1}(x) = x$ also have same two real roots

(c) $f(x) = f^{-1}(x)$ also have same two real roots

(d) Area of triangle formed by $(0, 0)$, $\left(\alpha, f(\alpha)\right)$, and $(\beta, f(\beta))$ is 1 unit.

9. The function $f(x) = \cos^{-1}x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right)$, then :

(a) Range of f(x) is $\left[\frac{\pi}{3}, \frac{10\pi}{3}\right]$ (b) Range of f(x) is $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

(c) f(x) is one-one for $x \in \left[-1, \frac{1}{2}\right]$ (d) f(x) is one-one for $x \in \left[\frac{1}{2}, 1\right]$

10. Let $f: R \rightarrow R$ defined by $f(x) = \cos^{-1}(-\{x\})$, where $\{x\}$ is fractional part function. Then which of the following is/are correct?

(a) f is many –one but not even function

(b) Range of f contains two prime numbers

(c) f is a periodic

(d) Graph of f does not lie below x-axis

11. Which option (s) is/are true ?

(a) $f: R \to R, f(x) = e^{|x|} - e^{-x}$ is many-one into function

(b) $f: R \to R, f(x) = 2x + |\sin x|$ is one-one onto

(c) $f: R \to R, f(x) = \frac{x^2+4x+30}{x^2-8x+18}$ is many-one onto

(d) $f: R \to R, f(x) = \frac{2x^2-x+5}{7x^2+2x+10}$ is many-one onto

12. If $h(x) = \left[\ln \frac{x}{e}\right] + \left[\ln \frac{e}{x}\right]$, where [.] denotes greatest integer function, then which of the following are true ?

(a) range of $h(x)$ is $\{-1, 0\}$

(b) If $h(x) = 0$, then x must be irrational

(c) If $h(x) = -1$, then x can be rational as well as irrational

(d) $h(x)$ is periodic function

13. If $f(x) = \begin{cases} x^3 & ; \quad x \in Q \\ -x^3 & ; \quad x \notin Q \end{cases}$, then :

(a) $f(x)$ is periodic

(b) $f(x)$ is many-one

(c) $f(x)$ is one-one

(d) range of the function is R

14. Let $f(x)$ be a real valued continuous function such that $f(0) = \frac{1}{2}$ and $f(x + y) = f(x)f(a - y) + f(y)f(a - x)\ \forall\ x, y \in R$, then for some real a :

(a) $f(x)$ is a periodic function

(b) $f(x)$ is a constant function

(c) $f(x) = \frac{1}{2}$

(d) $f(x) = \frac{\cos x}{2}$

15. $f(x)$ is an even periodic function with period 10. In $[0, 5]$,

$$f(x) = \begin{cases} 2x & 0 \le x < 2 \\ 3x^2 - 8 & 2 \le x < 4 \\ 10x & 4 \le x \le 5 \end{cases}$$. Then :

(a) $f(-4) = 4 =$

(b) $\frac{f(-13)-f(11)}{f(13)+f(-11)} = \frac{17}{21}$

(c) $f(5)$ is not defined

(d) Range of $f(x)$ is $[0, 50]$

16. For the equation $\frac{e^{-x}}{1+x} = \lambda$ which of the following statements(s) is/are correct?
(a) when $\lambda \in (0, \infty)$ equation has 2 real and distinct roots
(b) when $\lambda \in (-\infty, -e^2)$ equation has 2 real and distinct roots
(c) when $\lambda \in (0, \infty)$ equation has 1 real root
(d) when $\lambda \in (-e, 0)$ equation has no real root

17. For $x \in R^+$, if $x, [x], \{x\}$ are in harmonic progression then the value of x can not be equal to :
(where [.] denotes greatest integer function, {.} denotes fractional part function)
(a) $\frac{1}{\sqrt{2}} \tan \frac{\pi}{8}$
(b) $\frac{1}{\sqrt{2}} \cot \frac{\pi}{8}$
(c) $\frac{1}{\sqrt{2}} \tan \frac{\pi}{12}$
(d) $\frac{1}{\sqrt{2}} \cot \frac{\pi}{12}$

18. The equation $\left||x-1|+a\right| = 4, a \in R$, has :
(a) 3 distinct real roots for unique value of a.
(b) 4 distinct real roots for $a \in (-\infty, -4)$
(c) 2 distinct real roots for $|a| < 4$
(d) no real roots for $a > 4$

19. Let $f_n(x) = (\sin x)^{1/n} + (\cos x)^{1/n}, x \in R$, then :
(a) $f_2(x) > 1$ for all $x \in (2k\pi, (4k+1)\frac{\pi}{2}), k \in I$
(b) $f_2(x) = 1$ for $x = 2k\pi, k \in I$
(c) $f_2(x) > f_3(x)$ for all $x \in (2k\pi, (4k+1)\frac{\pi}{2}), k \in I$
(d) $f_3(x) \geq f_5(x)$ for all $x \in (2k\pi, (4k+1)\frac{\pi}{2}), k \in I$
(where I denotes set of integers)

20. If the domain of $f(x) = \frac{1}{\pi} \cos^{-1}\left[\log_3\left(\frac{x^2}{3}\right)\right]$ where, $x > 0$ is $[a, b]$ and the range of $f(x)$ is $[c, d]$, then :
(a) a, b are the roots of the equation $x^4 - 3x^3 - x + 3 = 0$
(b) a, b are the roots of the equation $x^4 - x^3 - x^2 - 2x + 1 = 0$
(c) $a^3 + d^3 = 1$
(d) $a^2 + b^2 + c^2 + d^2 = 11$

21. The number of real values of x satisfying the equation ;
$\left[\frac{2x+1}{3}\right] + \left[\frac{4x+5}{6}\right] = \frac{3x-1}{2}$ are greater than or equal to ([.] denotes greatest integer function) :

(a) 7 (b) 8

(c) 9 (d) 10

22. Let $f(x) = \sin^6\left(\frac{x}{4}\right) + \cos^6\left(\frac{x}{4}\right)$. If $f^n(x)$ denotes n^{th} derivative of f evaluated at x. Then which of the following hold ?

(a) $f^{2014}(0) = -\frac{3}{8}$ (b) $f^{2015}(0) = \frac{3}{8}$

(c) $f^{2010}\left(\frac{\pi}{2}\right) = 0$ (d) $f^{2011}\left(\frac{\pi}{2}\right) = \frac{3}{8}$

23. Which of the following is(are) incorrect ?

(a) If $f(x) = \sin x$ and $g(x) = \text{In}\, x$ then range of $g(f(x))$ is $[-1, 1]$

(b) If $x^2 + ax + 9 > x \forall x \in R$ then $-5 < a < 7$

(c) If $f(x) = (2011 - x^{2012})^{\frac{1}{2012}}$ then $f\big(f(2)\big) = \frac{1}{2}$

(d) The function $f: R \to R$ defined as $f(x) = \frac{x^2+4x+30}{x^2-8x+18}$ is not surjective.

24. If [x] denotes the integral part of x for real x, and

$$S = \left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{200}\right] + \left[\frac{1}{4} + \frac{1}{100}\right] + \left[\frac{1}{4} + \frac{3}{200}\right] \ldots\ldots + \left[\frac{1}{4} + \frac{199}{200}\right] \text{ then}$$

(a) S is a composite number (b) Exponent of S in ⌊100 is 12⌋

(c) Number of factors of S is 10 (d) ${}^{2S}C_r$ is max when $r = 51$

Answers

1.	(a, b, d)	2.	(a, b, c)	3.	(c, d)	4.	(a, b)	5.	(a, b, c)	6.	(a, b, c, d)
7.	(a, d)	8.	(a, b, c)	9.	(b, c)	10.	(a, b, d)	11.	(a, b, d)	12.	(a, d)
13.	(c, d)	14.	(a, b, c)	15.	(a, b, d)	16.	(b, c, d)	17.	(a, c, d)	18.	(a, b, c, d)
19.	(a, b)	20.	(a, d)	21.	(a, b, c)	22.	(a, c, d)	23.	(a, b)	24.	(a, b)