DIFFERENTIAL EQUATIONS

Excerise-1: Single Choice Problems

1. $\frac{dy}{dx}\left(\frac{1+\cos x}{y}\right) = -\sin x$ and $f\left(\frac{\pi}{2}\right) = -1$, then $f(0)$ is :
   (a) 2 (b) 1
   (c) 3 (d) 4

2. The differential equation satisfied by family of curves $y = Ae^{x} + Be^{3x} + Ce^{5x}$ where A, B, C are arbitrary constants is :
   (a) $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 23\frac{dy}{dx} + 15y = 0$ (b) $\frac{d^3y}{dx^3} + 9\frac{d^2y}{dx^2} - 23\frac{dy}{dx} - 15y = 0$
   (c) $\frac{d^3y}{dx^3} + 9\frac{d^2y}{dx^2} - 23\frac{dy}{dx} + 15y = 0$ (d) $\frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} + 23\frac{dy}{dx} - 15y = 0$

3. If $y = (x)$ and it follows the relation $e^{xy^2} + y\cos(x^2) = 5$ then $y'(0)$ is equal to :
   (a) 4 (b) -16
   (c) -4 (d) 16

4. $(x^2 + y^2)dy = xydx$. If $y(x_0) = e, y(1) = 1$, then the value of x_0 is equal to :
   (a) $\sqrt{3}e$ (b) $\sqrt{e^2 - \frac{1}{2}}$
   (c) $\sqrt{\frac{e^2-1}{2}}$ (d) $\sqrt{e^2 + \frac{1}{2}}$

5. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with :
   (a) Variable radii and fixed centre at (0, 1)
   (b) Variable radii and fixed centre at (0, -1)
   (c) Fixed radius 1 and variable centres along x-axis
   (d) Fixed radius 1 and variable centres along y-axis

6. Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 y' + y = 0$ is :

(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

(c) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$ (d) $(-\pi, \pi)$

7. A function $y = f(x)$ satisfies the differential equation $(x+1)f'(x) - 2(x^2 + x)f(x) = \frac{e^{x^2}}{(x+1)}$; $\forall\, x > -1$. If $f(0) = 5$, then $f(x)$ is :

(a) $\left(\frac{3x+5}{x+1}\right).e^{x^2}$ (b) $\left(\frac{6x+5}{x+1}\right).e^{x^2}$

(c) $\left(\frac{6x+5}{(x+1)^2}\right).e^{x^2}$ (d) $\left(\frac{5-6x}{x+1}\right).e^{x^2}$

8. The solution of the differential equation $2x^2 y \frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$ given $y(1) = \sqrt{\frac{\pi}{2}}$ is :

(a) $\sin(x^2y^2) - 1 = 0$ (b) $\cos\left(\frac{\pi}{2} + x^2y^2\right) + x = 0$

(c) $\sin(x^2y^2) = e^{x-1}$ (d) $\sin(x^2y^2) = e^{2(x-1)}$

9. The differential equation whose general solution is given by $y = C_1 \cos(x + C_2) - C_3 e^{-x+C_4} + C_5 \sin x$, where $C_1, C_2, \dots, C_5$ are constants is :

(a) $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

(c) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ (d) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

10. If $y = e^{(\alpha+1)x}$ be solution of differential equation $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} + 4y = 0$; then α is :

(a) 0 (b) 1

(c) -1 (d) 2

11. The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^{1/3} - 4\frac{d^2y}{dx^2} - 7x = 0$ are α and β, then the value of $(\alpha + \beta)$ is :

(a) 3 (b) 4

(c) 2 (d) 5

12. General solution of differential equation of $f(x)\frac{dy}{dx} = f^2(x) + f(x)y + f'(x)y$ is:

(c being arbitrary constant.)

(a) $y = f(x) + ce^x$ (b) $y = -f(x) + ce^x$

(c) $y = -f(x) + ce^x f(x)$ (d) $y = c\,f(x) + e^x$

13. The order and degree respectively of the differential equation of all tangent lines to parabola $x^2 = 2y$ is :

(a) 1, 2 (b) 2, 1

(c) 1, 1 (d) 1, 3

14. The general solution of the differential equation $\frac{dy}{dx} + x(x+y) = x(x+y)^3 - 1$ is :

(a) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^4}\right| = x^2 + C$ (b) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x^2 + C$

(c) $2\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^2}\right| = x^2 + C$ (d) $\ln\left|\frac{(x+y+1)(x+y-1)}{(x+y)^4}\right| = x + C$

15. The general solution of $\frac{dy}{dx} = 2y\tan x + \tan^2 x$ is :

(a) $y\cos^2 x = \frac{x}{2} - \frac{\sin 2x}{4} + C$ (d) $y\sec^2 x = \frac{x}{2} - \frac{\sin 2x}{4} + C$

(c) $y\cos^2 x = \frac{x}{2} - \frac{\cos 2x}{4} + C$ (d) $y\cos^2 x = \frac{x}{2} - \frac{\sin 2x}{2} + C$

(where C is arbitrary constant.)

16. The solution of differential equation $\frac{d^2y}{dx^2} = \frac{dy}{dx}, y(0) = 3$ and $y'(0) = 2$:

(a) is a periodic function

(b) approaches to zero as $x \to -\infty$

(c) has an asymptote parallel to $x -$ axis

(d) has an asymptote parallel to $y -$ axis

17. The solution of the differential equation $(x^2+1)\frac{d^2y}{dx^2} = 2x\left(\frac{dy}{dx}\right)$ under the conditions $y(0) = 1$ and $y'(0) = 3$, is :

(a) $y = x^2 + 3x + 1$ (b) $y = x^3 + 3x + 1$

(c) $y = x^4 + 3x + 1$ (d) $y = 3\tan^{-1}x + x^2 + 1$

18. The differential of the family of curves $cy^2 = 2x + c$ (where c is an arbitrary constant.) is :

(a) $\frac{xdy}{dx} = 1$ (b) $\left(\frac{dy}{dx}\right)^2 = \frac{2xdy}{dx} + 1$

(c) $y^2 = 2xy\frac{dy}{dx} + 1$ (d) $y^2 = \frac{2ydy}{dx} + 1$

19. The solution of the equation $\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\tan y \sin y$ is :

(a) $2y = \sin y(1 - 2cx^2)$ (b) $2x = \cot y(1 + 2cx^2)$

(c) $2x = \sin y(1 - 2cx^2)$ (d) $2x \sin y = 1 - 2cx^2$

20. Solution of the differential equation $xdy - ydx - \sqrt{x^2 + y^2} = cx$

(a) $y - \sqrt{x^2 + y^2} = cx^2$ (b) $y + \sqrt{x^2 + y^2} = cx$

(c) $x - \sqrt{x^2 + y^2} = cx^2$ (d) $y + \sqrt{x^2 + y^2} = cx^2$

21. Let f(x) be differentiable function on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t\to x}\left(\frac{t^3 f(x) - x^3 f(t)}{t^2 - x^2}\right) = \frac{1}{2}\ \forall\ x > 0$, then f(x) is :

(a) $\frac{1}{4x} + \frac{3x^2}{4}$ (b) $\frac{3}{4x} + \frac{x^3}{4}$

(c) $\frac{1}{4x} + \frac{3x^3}{4}$ (d) $\frac{1}{4x^3} + \frac{3x}{4}$

22. The population p(t) at time 't' of a certain mouse species satisfies the differential equation $\frac{d}{dt}p(t) = 0.5p(t) - 450.$ If $p(0) = 850$, then the time at which the population becomes zero is :

(a) $\frac{1}{2}$ In 18 (b) In 18

(c) 2 In 18 (d) In 9

23. The solution of the differential equation $\sin 2y\frac{dy}{dx} + 2\tan x \cos^2 y = 2\sec x \cos^3 y$ is :

(where C is arbitrary constant)

(a) $\cos y\ \sec x = \tan x + C$ (b) $\sec y \cos x = \tan x + C$

(c) $\sec y \sec x = \tan x + C$ (d) $\tan y \sec x = \sec x + C$

24. The solution of the differential equation $\frac{dy}{dx} = (4x + y + 1)^2$ is :
(where C is arbitrary constant)
(a) $4x + y + 1 = 2\tan(2x + y + C)$ (b) $4x + y + 1 = 2\tan(x + 2y + C)$
(c) $4x + y + 1 = 2\tan(2y + C)$ (d) $4x + y + 1 = 2\tan(2x + C)$

25. If a curve is such that line joining origin to any point $P(x, y)$ on the curve and the line parallel to $y -$ axis through P are equally inclined to tangent to curve at P, then the differential equation of the curve is :
(a) $x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = x$ (b) $x\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} = x$
(c) $y\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} = x$ (d) $y\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = x$

26. If $y = f(x)$ satisfy the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2; f(1) = 1;$ then value of $f(3)$ equals :
(a) 7 (b) 5
(c) 9 (d) 27

27. Let $y = f(x)$ and $\frac{x}{y}\frac{dy}{dx} = \frac{3x^2 - y}{2y - x^2}; f(1) = 1$ then the possible value of $\frac{1}{3}f(3)$ equals:
(a) 9 (b) 4
(c) 3 (d) 2

Answer

1.	(a)	2.	(d)	3.	(b)	4.	(a)	5.	(c)	6.	(a)	7.	(b)	8.	(c)	9.	(c)	10.	(b)
11.	(d)	12.	(c)	13.	(a)	14.	(b)	15.	(a)	16.	(c)	17.	(b)	18.	(c)	19.	(c)	20.	(d)
21.	(c)	22.	(c)	23.	(c)	24.	(d)	25.	(a)	26.	(a)	27.	(c)						

Excerise-2: One or More than One Answer is/are Correct

1. Let $y = f(x)$ be a real valued function satisfying $x\frac{dy}{dx} = x^2 + y - 2, , f(1) = 1,$ then :

   (a) $f(x)$ is minimum at $x = 1$ (b) $f(x)$ is maximum at $x = 1$

   (c) $f(3) = 5$ (d) $f(2) = 3$

2. Solution of differential equation $x\cos x\left(\frac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$:

   (a) $xy = \sin x + c\cos x$ (b) $xy\sec x = \tan x + c$

   (c) $xy + \sin x + c\cos x = 0$ (d) None of these

3. If a differential function satisfies $(x - y)f(x + y) - (x + y)f(x - y) = 2(x^2y - y^3)\forall\, x, y \in R$ and $f(1) = 2$, then :

   (a) $f(x)$ must be polynomial function

   (b) $f(3) = 12$

   (c) $f(0) = 0$

   (d) $f(3) = 13$

4. A function $y = f(x)$ satisfies the differential equation

   $$f(x)\sin 2x - \cos x + (1 + \sin^2 x)f'(x) = 0$$

   with $f(0) = 0$. The value of $f\left(\frac{\pi}{6}\right)$ equals to :

   (a) $\frac{2}{5}$ (b) $\frac{3}{5}$

   (c) $\frac{1}{5}$ (d) $\frac{4}{5}$

5. Solution of the differential equation $\left(2 + 2x^2\sqrt{y}\right)ydx + \left(x^2\sqrt{y} + 2\right)x\,dy = 0$ is are :

   (a) $xy\left(x^2\sqrt{y} + 5\right) = c$ (b) $xy\left(x^2\sqrt{y} + 3\right) = c$

   (c) $xy\left(y^2\sqrt{x} + 3\right) = c$ (d) $xy\left(y^2\sqrt{x} + 5\right) = c$

6. If y(x) satisfies the differential equation $\frac{dy}{dx} -= \sin 2x + 3y \cot x$ and $y\left(\frac{\pi}{2}\right) = 2$ then which of the following statement(s) is/are correct ?

(a) $y\left(\frac{\pi}{6}\right) = 0$

(b) $y'\left(\frac{\pi}{3}\right) = \frac{9-3\sqrt{2}}{2}$

(c) y(x) increases in the interval

(d) $\int_{-\pi/2}^{\pi/2} y(x)dx = x$

Answers

1.	(a, c)	2.	(a, b)	3.	(a, b, c)	4	(a)	5.	(b)	6.	(a, c)