AREA UNDER CURVES

Excerise-1: Single Choice Problems

1. The area enclosed by the curve
$[x + 3y] = [x - 2]$ where $x \epsilon [3, 4]$ is :
(where [.] denotes greatest integer function.)
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) 1

2. The area of region enclosed by the curves $y = x^2$ and $y = \sqrt{|x|}$ is :
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{4}{3}$ (d) $\frac{16}{3}$

3. Area enclosed by the figure described by the equation $x^4 + 1 = 2x^2 + y^2$, is :
(a) 2 (b) $\frac{16}{3}$
(c) $\frac{8}{3}$ (d) $\frac{4}{3}$

4. The area defined by $|y| \leq e^{-|x|} - \frac{1}{2}$ in Cartesian co-ordinate system, is :
(a) $(4 - 2 \text{ In } 2)$ (b) $(4 - \text{ In } 2)$
(c) $(2 - \text{ In } 2)$ (d) $(2 - 2 \text{ In } 2)$

5. For each positive integer $n > 1$; A_n represents the area of the region restricted to the following to inequalities : $\frac{x^2}{n^2} + y^2 \leq 1$ and $x^2 + \frac{y^2}{n^2} \leq 1$. Find $\lim_{n\to\infty} A_n$.
(a) 4 (b) 1
(c) 2 (d) 3

6. The ratio in which the area bounded by curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$ is :
(a) 7 : 15 (b) 15 : 49
(c) 1 : 3 (d) 17 : 49

7. The value of positive real parameter ‘a’ such that area of region bounded by parabolas $y = x - ax^2, ay = x^2$ attains its maximum value is equal to :

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{1}{3}$ (d) 1

8. For $0 < r < 1$, let n_r denotes the line that is normal to the curve $y = x^r$ at the point $(1, 1)$. Let S_r denotes the region in the first quadrant bounded by the curve $y = x^r$; the x-axis and the line n_r. Then the value of r that minimizes the area of S_r is :

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2} - 1$

(c) $\frac{\sqrt{2}-1}{2}$ (d) $\sqrt{2} - \frac{1}{2}$

9. The area bounded by $|x| = 1 - y^2$ and $|x| + |y| = 1$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) 1

10. Point A lies on curve $y = e^{-x^2}$ and has the coordinate $\left(x, e^{-x^2}\right)$ where $x > 0$. Point B has coordinates $(x, 0)$. If ‘O’ is the origin, then the maximum area of ΔAOB is :

(a) $\frac{1}{\sqrt{8e}}$ (b) $\frac{1}{\sqrt{4e}}$

(c) $\frac{1}{\sqrt{2e}}$ (d) $\frac{1}{\sqrt{e}}$

11. The area enclosed between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq. unit, then the value of a is :

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) 1 (d) $\frac{1}{3}$

12. Let $f(x) = x^3 - 3x^2 + 3x + 1$ and gbe the inverse of it ; then area bounded by the curve $y = g(x)$ with x-axis between $x = 1$ to $x = 2$ is (in square units) :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) 1

13. Area bounded by $x^2y^2 + y^4 - x^2 - 5y^2 + 4 = 0$ is equal to :

(a) $\frac{4\pi}{3} + \sqrt{2}$

(b) $\frac{4\pi}{3} - \sqrt{2}$

(c) $\frac{4\pi}{3} + 2\sqrt{3}$

(d) None of these

14. Let $f: R^+ \to R^+$ is an invertible function such that $f'(x) > 0$ and $f''(x) > 0 \ \forall \ x \in [1, 5]$. If $f(1) = 1$ and $f(5) = 5$ and area bounded by $y = f(x), x - \text{axis}, x = 1$ and $x = 5$ is 8 sq. units. Then the area bounded by $y = f^{(-1)}(x), x - \text{axis}, x = 1$ and $x = 5$ is :

(a) 12

(b) 16

(c) 18

(d) 20

15. A particle centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of the upper part equals to :

(a) $\frac{\pi^2}{2}$

(b) $\frac{\pi^3}{4}$

(c) $\frac{\pi^3}{2}$

(d) $\frac{\pi^3}{8}$

16. The area of the loop formed by $y^2 = x(1 - x^3)dx$ is :

(a) $\int_0^1 \sqrt{x - x^4}\, dx$

(b) $2\int_0^1 \sqrt{x - x^4}\, dx$

(c) $\int_{-1}^1 \sqrt{x - x^4}\, dx$

(d) $4\int_0^{1/2} \sqrt{x - x^4}\, dx$

17. If $f(x) = \min\left[x^2, \sin\frac{x}{2}, (x-2\pi)^2\right]$, the area bounded by the curve $y = f(x)$, x-axis, $x = 0$ and $x = 2\pi$ is given by

(**Note :** x_1 is the point of intersection of the curves x^2 and $\sin\frac{x}{2}$; x_2 is the point of intersection of the curves $\sin\frac{x}{2}$ and $(x-2\pi)$)

(a) $\int_0^{x_1}\left(\sin\frac{x}{2}\right)dx + \int_{x_1}^{\pi} x^2 dx + \int_{\pi}^{x_2}(x-2\pi)^2 dx + \int_{x_2}^{2\pi}\left(\sin\frac{x}{2}\right)dx$

(b) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\left(\sin\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(0, \frac{\pi}{3}\right)$ and $x_2 \in \left(\frac{5\pi}{3}, 2\pi\right)$

(c) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\sin\left(\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ and $x_2 \in \left(\frac{3\pi}{2}, 2\pi\right)$

(d) $\int_0^{x_1} x^2 dx + \int_{x_1}^{x_2}\sin\left(\frac{x}{2}\right)dx + \int_{x_2}^{2\pi}(x-2\pi)^2 dx$, where $x_1 \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $x_2 \in (\pi, 2\pi)$

18. The area enclosed between the curves $|x||y| \geq 2$ and $y^2 = 4\left(1 - \frac{x^2}{9}\right)$ is :

(a) $(6\pi - 4)$sq.units
(b) $(6\pi - 8)$sq.units
(c) $(3\pi - 4)$sq.units
(d) $(3\pi - 2)$sq.units

Answer

1.	(b)	2.	(b)	3.	(c)	4.	(d)	5.	(a)	6.	(b)	7.	(d)	8.	(b)	9.	(c)	10.	(a)
11.	(d)	12.	(b)	13.	(c)	14.	(b)	15.	(c)	16.	(b)	17.	(b)	18.	(b)				

Excerise-2: One or More than One Answer is/are Correct

1. Let $f(x)$ be a polynomial function of degree 3 where $a < b < c$ and $f(a) = f(b) = f(c)$. If the graph of $f(x)$ is as shown, which of the following statements are **INCORRECT** ? (where $c > |a|$)
   (a) $\int_a^c f9x)dx = \int_b^c f(x)dx + \int_a^b f(x)dx$
   (b) $\int_a^c f(x)dx < 0$
   (c) $\int_a^b f(x)dx < \int_c^b f(x)dx$
   (d) $\frac{1}{b-a}\int_a^b f(x)dx > \frac{1}{c-b}\int_b^c f(x)dx$

2. $T_n = \sum_{r=2n}^{3n-1} \frac{r}{r^2+n^2}, S_n = \sum_{r=2n+1}^{3n} \frac{r}{r^2+n^2}$, then $\forall\ n \epsilon \{1, 2, 3, \dots\}$:
   (a) $T_n > \frac{1}{2} \text{In } 2$
   (b) $S_n < \frac{1}{2} \text{In } 2$
   (c) $T_n < \frac{1}{2} \text{In } 2$
   (d) $S_n > \frac{1}{2} \text{In } 2$

3. If a curve $y = a\sqrt{x} + bx$ passes through point $(1, 2)$ and the area bounded by curve, line $x = 4$ and $x -$ axis is 8, then :
   (a) $a = 3$
   (b) $b = 3$
   (c) $a = -3$
   (d) $b = -1$

4. Area enclosed by the curves $y = x^2 + 1$ and a normal drawn to it with gradient -1; is equal to :
   (a) $\frac{2}{3}$
   (b) $\frac{1}{3}$
   (c) $\frac{3}{4}$
   (d) $\frac{4}{3}$

Answers

1.	(b, c, d)	2.	(a, b)	3.	(a, d)	4.	(d)