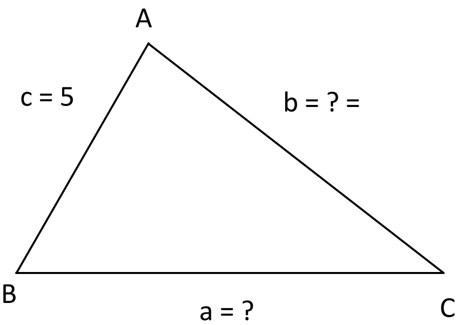
MATHEMATICS
JEE-MAIN (July-Attempt) 20 July
(Shift-1) Paper

SECTION - A

1. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circum circle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is:

(1) $6+8\sqrt{3}$ (2) $8+2\sqrt{2}$ (3) $4+2\sqrt{3}$ (4) $10+6\sqrt{2}$

Sol. (1)



$\cos B = \frac{3}{5} \Rightarrow \sin B = \frac{4}{5}$

Now, $\frac{b}{\sin B} = 2R \Rightarrow \boxed{b = 2(5)\left(\frac{4}{5}\right) = 8}$

Now, by cosine formula

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$

$\Rightarrow a^2 - 6a - 3g = 0$

$\therefore = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$

$\boxed{3+4\sqrt{3}}$ (Reject a = $3 - 4\sqrt{3}$)

Now, $\Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(2+4\sqrt{3})$

$\Rightarrow \Delta = (6+8\sqrt{3})$

2. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is:

(1) $\frac{1}{9}$ (2) $\frac{1}{66}$ (3) $\frac{2}{11}$ (4) $\frac{1}{11}$

Sol. (4)

AAEIIMNNOTX

Total words $= \frac{11:}{2:2:21} = n(s)$

$\_\ \_\ \_\ \underline{M}\ \_\ \_\ \_\ \_\ \_\ \_\ \_$

Total words with M at fourth place = $\frac{10!}{2!2!2!} = n(A)$

Probability = $\frac{10!}{11!} = \frac{1}{11}$

3. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:

(1) 10, 11 (2) 8, 13 (3) 1, 20 (4) 3, 18

Sol. **(1)**

Let other two numbers be a, (21-a)

Now,

$$10.25 = \frac{\left(4+16+25+49+a^2+(21-a)^2\right)}{6}$$

(Using formula for variance)

$\Rightarrow 6(10.25)+6(6.5)^2=94+a^2+(21-a)^2$

$\Rightarrow$ a2 + (21 – a^2) = 221

$\therefore$ a = 10 and (21-a) = 21 – 10 = 11

so, remaining two observations are 10, 11.

4. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|$, $|\vec{c}-\vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a}\times\vec{b})\times\vec{c}|$ is:

(1) $\frac{2}{3}$ (2) 4 (3) 3 (4) $\frac{3}{2}$

Sol. **(4)**

$|\vec{a}| = a; \vec{a}.\vec{c} = c$

Now $|\vec{c}-\vec{a}| = 2\sqrt{2}$

$\Rightarrow c^2 + a^2 - 2\vec{c}.\vec{a} = 8$

$\Rightarrow$ **c^2 + 9 – 2 (c) = 8**

$\Rightarrow$ **C^2 – 2C + 1 = 0 $\Rightarrow$ C = *1*** $\Rightarrow |\vec{c}| = 1$

Also, $\vec{a}\times\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\sin\frac{\pi}{6}$

= (3) (1) (1/2)

= 3/2

5. The value of the integral $\int_{-1}^{1} \log_e\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$ is equal to:

(1) $2\log_e 2 + \frac{\pi}{4} - 1$

(2) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$

(3) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$

(4) $\log_e 2 + \frac{\pi}{2} - 1$

Sol. **(4)**

Let I = $2\int_0^1 \underbrace{In\left(\sqrt{1-x}+\sqrt{1+x}\right)}_{I} \underbrace{1}_{(II)} dx$

(I.B.P.)

$\therefore I = \left| x.In\left(\sqrt{1-x}+\sqrt{1-x}\right)\right)_0^1$

$-\int_0^1 x.\left(\frac{1}{\sqrt{1-x}+\sqrt{1+x}}\right).\left(\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}\right)dx\Big]$

$= 2\left(In\sqrt{2}-0\right) - \frac{2}{2}\int_0^1 \frac{x\sqrt{1-x}-\sqrt{1+x}dx}{\sqrt{1-x}+\sqrt{1+x}\sqrt{1-x^2}}$

$= 2(\log_e 2)\int_0^1 \frac{x.\left(2-2\sqrt{1-x^2}\right)}{-2x\sqrt{1-x^2}}dx$

(After rationalisation)

$= (\log_e 2) + \int_0^1 \left(\frac{\left(1-\sqrt{1-x^2}\right)}{\sqrt{1-x^2}}\right)dx$

$= (\log_e 2) + \left(\sin^{-1} x\right)_0^1 - 1$

$= \log_e 2 + \left(\frac{\pi}{2} - 0\right) - 1$

$\therefore$ I =$(\text{lo}_e 2) + \frac{\pi}{2} - 1$

6. The probability of selecting integers a $\in$[−5, 30] such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all x $\in$ R, is :

(1) $\frac{1}{4}$

(2) $\frac{7}{36}$

(3) $\frac{2}{9}$

(4) $\frac{1}{6}$

Sol. **(3)**

D <0

$\Rightarrow 4(a+4)^2 - 4(-5a+64) < 0$

$\Rightarrow a^2 + 16 + 8a + 5a - 64 < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\Rightarrow (a+16)(a-3) < 0$

$\Rightarrow a \in (-16, 3)$

$\therefore$ Possible a : {-5, -4,….,2}

$\therefore$ Required probability = $\frac{8}{36}$

$= \frac{2}{9}$

7. Let y = y(x) be the solution of the differential equation

$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx$, $-1 \le x \le 1$, $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

Then the area of the region bounded by the curves x= 0, $x = \frac{1}{\sqrt{2}}$ and y = y(x) in the upper half plane is :

(1) $\frac{1}{12}(\pi - 3)$ (2) $\frac{1}{6}(\pi - 1)$ (3) $\frac{1}{8}(\pi - 1)$ (4) $\frac{1}{4}(\pi - 2)$

Sol. **(3)**

We have

$$\tan\left(\frac{y}{x}\right)(xdy - ydx) = -xdx$$

$$\Rightarrow \tan\left(\frac{y}{x}\right)\left(\frac{xdy - ydx}{x^2}\right) = -\frac{x}{x^2}dx$$

$$\Rightarrow \int \tan\left(\frac{y}{x}\right)\left(d\left(\frac{y}{x}\right)\right) = \int -\frac{1}{x}dx$$

$$\Rightarrow \ell n\,|\sec(y/x)| = -\ln x + C$$

$$\Rightarrow \ell n\,|x\sec(y/x)| = C$$

Now $y = \left(\frac{1}{2}\right)$ & $x = \pi/6$

As $\ell n\left|\frac{1}{2}.\sec\left(\frac{\pi}{3}\right)\right| = C \Rightarrow \boxed{C = 0}$

$$\therefore \sec\left(\frac{y}{x}\right) = \frac{1}{x}$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = x$$

$$\therefore \boxed{y = x\cos^{-1}(x)}$$

So, required bounded area

$$A = \int_0^{\frac{1}{\sqrt{2}}} \underset{(II)}{x}\,\underset{(I)}{\left(\cos^{-1}\right)}dx = \left(\frac{\pi - 1}{8}\right)$$

(I.B.P.)

8. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to:

(1) 56×3^{25} (2) 52×3^{24} (3) 56×3^{24} (4) 28×3^{25}

Sol. **(2)**

As, $(a^2 + \sqrt{3}) = -(3)^{1/4}.\alpha$

$\Rightarrow (\alpha^2 + 2\sqrt{3}\,\alpha^2 + 3) = \sqrt{3}\,\alpha^2$(On squaring)

$\therefore (a^4 + 3) = (-)\sqrt{3}\,\alpha^2)$

$\Rightarrow \alpha^8 + 6\alpha^4 + 9 = 3\alpha^2$(Again squaring)

$\therefore \alpha^8+3\alpha^4 + 9 = 0$

$\Rightarrow \alpha^8 = -9 - 3\alpha^4$

(Multiply by α^4)

So, $\alpha^{12} = -9\alpha^4 - 3\alpha^8$

$\therefore \alpha^{12} = -9\alpha^4 -3(-9-3\alpha^4)$

$\Rightarrow \alpha^{12} = \cancel{-9\alpha^4} + 27 + \cancel{9\alpha^4}$

Hence, $\alpha^{12} = (27)$

$\Rightarrow (\alpha^{12})^{18} = (27)^8$

$\Rightarrow \alpha^{96} = (3)^{24}$

Similarly$\beta^{96} = (3)^{24}$

$\therefore \alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)=(3)^{24}\times 52$

9. Let a function f: R $\to$ R be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a+[-x] & \text{if } 0<x<1 \\ 2x-b & \text{if } x \ge 1 \end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous on R, then (a+b) is equal to:

(1) 5 (2) 3 (3) 2 (4) 4

Sol. **(2)**

Continuous at x = 0

$f(0^+) = f(0^-) \Rightarrow a - 1 = 0 - e^0$

$\Rightarrow a = 0$

Continuous at x = 1

$f(1^+) = f(1^-)$

$\Rightarrow 2(1) - b = a + (-1)$

$\Rightarrow b = 2 - a + 1 \Rightarrow b = 3$

$\therefore a + b = 3$

10. Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right) dy = 0$, y(1) = –1. Then the value of $(y(3))^2$ is equal to:

(1) $1 + 4e^3$ (2) $1 + 4e^6$ (3) $1 - 4e^6$ (4) $1 - 4e^3$

Sol. **(3)**

$$e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0$$

$$\Rightarrow e^x\sqrt{1-y^2}dx + \frac{-y}{x}dy = 0$$

$$\Rightarrow \int \frac{ydy}{\sqrt{1-y^2}} = \int x.e^x dx$$

$$\Rightarrow \int \frac{-y}{\sqrt{1-y^2}}dy = \int xe^x dx$$

$\Rightarrow \sqrt{1-y^2} = e^x(x-1)+c$

Given: At x = 1, y = - 1

$\Rightarrow$ 0 = 0 + c $\Rightarrow$ c = 0

$\therefore \sqrt{1-y^2} = e^x(x-1)$

At x = 3, 1 – y^2 = $(e^3 2)^2 \Rightarrow y^2 = 1 - 4e^6$

11. If z and ω are two complex numbers such that $|z\omega|$ = 1 and arg(z) – arg(ω) = $\frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is:

(Here arg(z) denotes the principal argument of complex number z)

(1) $\frac{3\pi}{4}$ (2) $-\frac{\pi}{4}$ (3) $-\frac{3\pi}{4}$ (4) $\frac{\pi}{4}$

Sol. **(3)**

As $|z\omega| = 1$

$\Rightarrow$ |z|= r, then |ω| = $\frac{1}{r}$

Let arg(z) = q

$\therefore$ arg(ω) = $\left(\theta - \frac{3\pi}{2}\right)$

So, z = $re^{i\theta}$

$\Rightarrow \bar{z} = re^{i(-\theta)}$

$\omega = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)}$

Now, consider

$$\frac{1-w\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1-3e^{i\left(-\frac{3\pi}{2}\right)}} = \left(\frac{1-2i}{1+3i}\right)$$

$\therefore$ prin arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

= prin arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$

= $\left(-\frac{1}{2}(1+i)\right)$

= $-\left(\pi - \frac{\pi}{4}\right) = \frac{-3\pi}{4}$

12. Let [x] denote the greatest integer ≤ x, where x ∈ R. If the domain of the real valued function $f(x) = \sqrt{\frac{[[x]]-2}{[[x]]-3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, a < b < c, then the value of a + b + c is:

(1) −3 (2) 1 (3) −2 (4) 8

Sol. **(3)**

For domain,

$$\frac{|[x]|-2}{|[x]|-3} \geq 0$$

Case I: When $|[x]| - 2 \geq 0$

and $|[x]| - 3 > 0$

$\therefore x \in (-\infty, -3) \cup [4, \infty]$ .....(1)

Case II: When $|[x]| - 2 \leq 0$

and $|[x]| - 3 < 0$

$\therefore x \in [-2, 3)$ .... (2)

So, from (1) and (2)

We get

Domain of function

$= (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$\therefore (a+b+c) = -3 + (-2) + 3 = -2$ $(a<b<c)$

13. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is:

(1) 0 (2) 4 (3) 1 (4) 2

Sol. **(1)**

$$\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$$

For equation to be defined,

$x^3 + x \geq 0$

$\Rightarrow x^2 + x + 1 \geq 1$

$\therefore$ Only possibility that the equation is defined

$x^2 + x = 0 \Rightarrow x = 0;\ x = -1$

None of these values satisfy

$\therefore$ No of roots = 0

14. The coefficient of x^{256} in the expansion of $(1 - x)^{101}(x^2+x+1)^{100}$ is:

(1) $-{}^{100}C_{16}$ (2) ${}^{100}C_{16}$ (3) ${}^{100}C_{15}$ (4) $-{}^{100}C_{15}$

Sol. **(3)**

$$y = (1-x)(1-x)^{100}(x^2+x+1)^{100}$$

$$y = (1-x)(x^3-1)^{100}$$

$$y = (x^3-1)^{100} - x(x^3-1)^{100}$$

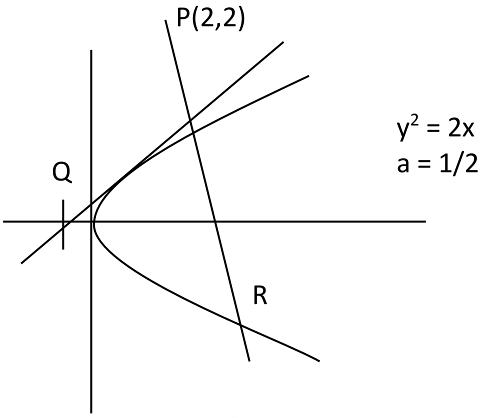
Coff. Of x^{256} in y = - coff of x^{255} in $(x^3-1)^{100}$

$$= {}^{-100}C_{85}(-1)^{15} = {}^{100}C_{15}$$

15. Let the tangent to the parabola S : $y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to:

(1) 25 (2) $\frac{25}{2}$ (3) $\frac{15}{2}$ (4) $\frac{35}{2}$

Sol. **(2)**



Tangent at P: y(2) = 2 (1/2) (x+2)

$\Rightarrow$ 2y = x + 2

$\therefore$ Q = (-2, 0)

Normal at P:$y - 2 = -\frac{(2)}{2.1/2}(x-2)$

$\Rightarrow$ y – 2 = - 2(x - 2)

$\Rightarrow$ y = 6 – 2x

$\therefore$ Solving with $y^2 = 2x \Rightarrow R\left(\frac{9}{2} - 3\right)$

$$\therefore \text{Ar}(\Delta PQR) = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 1 & 1 \\ \frac{9}{2} & 3 - & 1 \end{vmatrix}$$

$= \frac{25}{2}$ sq. units

16. Let a be a positive real number such that

$\int_0^a e^{x-[x]} dx = 10e - 9$

where [x] is the greatest integer less than or equal to x. Then a is equal to:

(1) $10 + \log_e 3$ (2) $10 - \log_e(1 + e)$ (3) $10 + \log_e 2$ (4) $10 + \log_e(1 + e)$

Sol. **(3)**

a>0

Let ≥ a < n+ 1, n ∈ W

$\therefore a = \underset{\text{G.I.F}}{\underset{\Downarrow}{[a]}} + \underset{\text{Fractional part}}{\underset{\Downarrow}{\{a\}}}$

Here [a] = n

Now, $\int_0^a e^{x-[x]}dx = 10e - 9$

$\Rightarrow \int_0^a e^x dx + \int_n^a e^{x-[x]}dx = 10e - 9$

$\therefore n\int_0^1 e^x dx + \int_n^a e^{x-n}dx = 10e - 9$

$\Rightarrow n(e - 1) + (e^{a-n} - 1) = 10e - 9$

$\therefore$ n = 10 and $\{a\} = \log_e - 9$

So, $a = [a] + \{a\} = (10 + \log_e 2)$

17. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15$, $x \in R$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15$, $x \in R$ has a:

(1) local minimum at $x = -\frac{3}{4}$

(2) local maximum at $x = \frac{3}{4}$

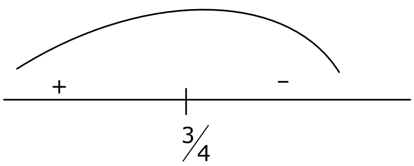
(3) local minimum at $x = \frac{3}{4}$

(4) local maximum at $x = -\frac{3}{4}$

Sol. **(4)**

$f(x) = ax^2 + 6x - 15$

$f' = 2ax + 6 = 2a(x + \frac{3}{a})$

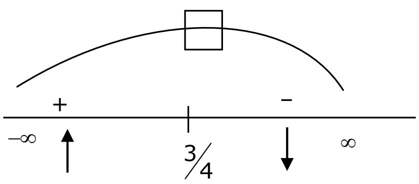


$\Rightarrow -\frac{3}{a} = \frac{3}{4} \Rightarrow a = -4$

Now $g(x) = -4x^2 - 6x + 15$

$g'(x) = -8x - 6$

$= -2\{4x + 3\}$



18. Let A = $[a_{ij}]$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \text{ if } i = j \\ -x & , \text{ if } |i-j| = 1 \\ 2x+1, & \text{otherwise} \end{cases}$$

Let a function f: R → R be defined as f(x) = det(A). Then the sum of maximum and minimum values of f on R is equal to:

(1) $\frac{20}{27}$ (2) $-\frac{88}{27}$ (3) $-\frac{20}{27}$ (4) $\frac{88}{27}$

Sol. **(2)**

$$\begin{bmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{bmatrix}$$

$|A| = 4x^3 - 4x^2 - 4x = f(x)$

$f(x) = 4(3x^2-2x-1) = 0$

$\Rightarrow x = 1;\ x = \frac{-1}{3}$

$\therefore \underbrace{f(1) = -4}_{\text{min}};\ f\ ;\underbrace{f\left(-\frac{1}{3}\right) = \frac{20}{27}}_{\text{max}}$

$$\text{Sum} = -4 + \frac{20}{27} = -\frac{88}{27}$$

19. Let A = $\begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, a ∈ R be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to:

(1) 24 (2) 18 (3) 45 (4) 36

Sol. **(4)**

A = $\begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, a ∈ R

and $and\, P \frac{A + A^T}{2} = \begin{bmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix}$

and $and\, Q \frac{A - A^T}{2} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$

As, det (Q) = 9

$\Rightarrow (a-3)^2 = 36$

$\Rightarrow a = 3 \pm 6$

$\therefore a = 9, -3$

$$\det(P) = \begin{vmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{vmatrix}$$

$= 0 - \frac{(a+3)^2}{4} = 0$, for a = -3 $\Rightarrow$ det (P) = 0

$= \;\; 0 - \frac{(a+3)^2}{4} = \frac{1}{4}(12)^2, \textit{for } a = 9 \;\Rightarrow$ det (P) = 36

$\therefore$ Modulus of the sum of all possible values of det. (P) = |36|+|0|= 36 Ans.

20. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to:

(1) $\sim q \Rightarrow p$ (2) $p \Rightarrow q$ (3) $p \Rightarrow \sim q$ (4) $q \Rightarrow p$

Sol. **(2)**

p	q	~p	~q	p∧~p	(p∨~q)	(p∧~p)⇒(qv~p)	p⇒q
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
T	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T

(P∧~q) (q∨~p)

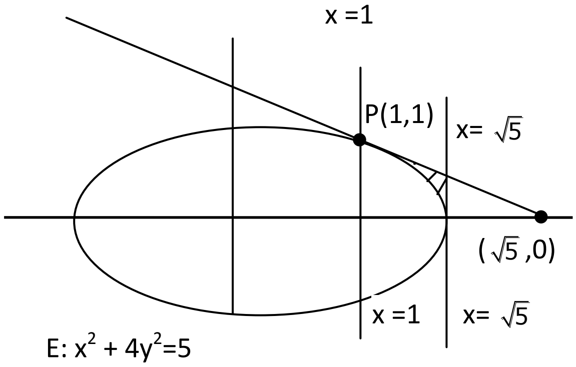
$\equiv p \Rightarrow q$

SECTION - B

1. Let T be the tangent to the ellipse E : $x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and x = $\sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to ____________.

Sol. **(1) NTA**

(1.25) Motion or Bonus



Tangent at P: x + 4y = 5

Required Area

$$= \int_1^{\sqrt{5}} \left(\frac{5-x}{4} - \frac{\sqrt{5-x^2}}{2} \right) dx$$

$$= \left[\frac{5x}{4} - \frac{x^2}{8} - \frac{x}{4}\sqrt{5-x^2} - \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}} \right]_1^{\sqrt{5}}$$

$= \frac{5}{4}\sqrt{5} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

It we assume $\alpha, \beta, \gamma, \in Q$ (Not given in question)

then $\alpha = \frac{5}{4}, \beta = -\frac{5}{4}$ & $\gamma = -\frac{5}{4}$

$|\alpha+\beta+\gamma| = 1.25$

2. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is ________.

Sol. **(21)**

$\left(4^{1/4} + 5^{1/6}\right)^{120}$

$T_{r+1} = {}^{120}C_r \, (2^{1/2})^{120-r} \, (5)^{r/6}$

for rational terms $r = 6\lambda$ $0 \le r \le 120$

so total no of forms are 21.

3. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is __________.

Sol. **(777)**

15: Players

6: Bowlers

7: Bastman

2: Wicket keepers

Total number of ways for:

at least 4 bowlers, 5 bastsman & 1 wicket keeper

${}^6C_4 . {}^7C_5 . {}^2C_2 + {}^6C_4 . {}^7C_6 . {}^2C_1$

$+ {}^6C_5 . {}^7C_5 . {}^2C_1 + {}^6C_5 . {}^7C_4 . {}^2C_2$

$+ {}^6C_6 . {}^7C_4 . {}^2C_1 + {}^6C_6 . {}^7C_3 . {}^2C_2$

$= \boxed{777}$

4. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to ___________

Sol. **(4)**

$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\left(\vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{c}\right) = 3$

$\Rightarrow \left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{3}$

$\vec{a}\left(\vec{a}+\vec{b}+\vec{c}\right) = \left|\vec{a}\right| + \left|\vec{a}+\vec{b}+\vec{c}\right|\cos\theta$

$\Rightarrow 1 = \sqrt{3}\cos\theta$

$\Rightarrow \cos 2\theta = -\frac{1}{3}$

$\Rightarrow 36\cos^2 2\theta = \boxed{4}$

5. Let P be a plane passing through the points (1, 0, 1), (1, −2, 1) and (0, 1, −2). Let a vector $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\vec{a}.\left(\hat{i}+\hat{j}+2\hat{k}\right)$ = 2, then $(\alpha - \beta + \gamma)^2$ equals __________.

Sol. **(81)**

$\bar{a} = \bar{n}_P \times (\hat{i}+2\hat{j}+3\hat{k})$

$\bar{a} = (\overline{AB} \times \overline{AC}) \times (\hat{i}+2\hat{j}+3\hat{k})$

$\bar{a} = \left((-2\hat{j}) \times (-\hat{i}+\hat{j}-3\hat{k})\right) \times (\hat{i}+2\hat{j}+3\hat{k})$

$$\bar{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 0 \\ -1 & 1 & -3 \end{vmatrix} \times (\hat{i}+2\hat{j}+3\hat{k})$$

$\bar{a} = (3\hat{i}-\hat{k}) \times (\hat{i}+2\hat{j}+3\hat{k})$

$$\Rightarrow \bar{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 2 & 3 \end{vmatrix}$$

$\bar{a} = (2\hat{i}-10\hat{j}+6\hat{k})$

$\boxed{\bar{a} = (1,-5,3)}$ in S.F.

6. Let a, b, c, d be in arithmetic progression with common difference λ. If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2,$$

then value of λ^2 is equal to __________.

Sol. **(1)**

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+C \end{vmatrix} = 2$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} x-2\lambda & 1 & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix} = 2$$

$\Rightarrow 1(4\lambda - 4\lambda^2 + 2\lambda) = 2 \Rightarrow \boxed{\lambda^2 = 1}$

7. If the value of $\lim_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to_______.

Sol. **(3)**

$\lim_{x\to 0}\left(2-\cos x\sqrt{\cos x}\right)^{\frac{x+2}{x^2}}$

form 1^∞

$e^{\lim_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)\times(x+2)}$

Now limt $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$\underset{x\to 0}{\text{lim}t}\frac{\sin x\sqrt{\cos 2}x-\cos x\times\frac{1}{2\sqrt{\cos 2x}}\times(-2\sin 2x)}{x^2}$$

(by L' Hospital Rule)

$$\underset{x\to 0}{\text{lim}t}\frac{\sin x\cos 2x+\sin 2x.\cos x}{2x}$$

$= \frac{1}{2}+1=\frac{3}{2}$

So, $e^{\underset{x\to 0}{\text{lim}it}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)(x+2)}$

$= e^{\frac{3}{2}\times 2} = e^3$

$\Rightarrow \boxed{a=3}$

8. If the shortest distance between the lines $\vec{r}_1=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$, $\lambda\in$ R, α > 0 and $\vec{r}_2=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$, $\mu\in$ R is 9, then α is equal to ______.

Sol. **(6)**

If $\vec{r}=\vec{a}+\lambda\vec{b}$ and $\vec{r}=\vec{c}+\lambda\vec{d}$

then shortest distance between two lines is

$L=\frac{(\vec{a}-\vec{c}).(\vec{b}\times\vec{d})}{|b\times d|}$

$\therefore \vec{a}-\vec{c}=\left((\alpha+4)\hat{i}+2\hat{j}+3\hat{k}\right)$

$\frac{\vec{b}\times\vec{d}}{|b\times d|}=\frac{(2\hat{i}+2\hat{j}+\hat{k})}{3}$

$\therefore \left((\alpha+4)\hat{i}+2\hat{j}+3\hat{j}\right).\frac{(2\hat{i}+2\hat{j}+\hat{k})}{3}=9$

or α = 6

9. Let y = mx + c, m > 0 be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m+c)$ is equal to _______.

Sol. **(34)**

y2 = -64

focus : (-16, 0)

y = mx + c is focal chord

$\Rightarrow$ c = 16m ……(1)

y = mx + c is tangent to $(x + 10)^2 + y^2 = 4$

$\Rightarrow y - m(x+10) \pm 2\sqrt{1+m^2}$

$\Rightarrow c = 10m \pm 2\sqrt{1+m^2}$

$\Rightarrow 16m = 10 \pm 2\sqrt{1+m^2}$

$\Rightarrow 6m = 2\sqrt{1+m^2}$ (m>0)

$\Rightarrow 9m^2 = 1 + m^2$

$\Rightarrow m = \frac{1}{2\sqrt{2}} \; \& \; c = \frac{8}{\sqrt{2}}$

$4\sqrt{2}(m+c) = 4\sqrt{2}\left(\frac{17}{2\sqrt{2}}\right) = \boxed{34}$

10. Let A = $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and B = $7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3 × 3.

If B = $[b_{ij}]$, then b_{13} is equal to ________.

Sol. **(910)**

Let A = $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = 1 + C$

Where I = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$C^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$,

$C^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, C^4 = C^5 = \ldots..$

$B = 7A^{20} - 20A^7 + 2I$

$= 7(1+c)^{20} - 20(1+C)^7 + 2I$

So

$B13 = 7 \times {}^{20}C_2 - 20 \times {}^{7}C_2 = \boxed{910}$