MATHEMATICS
JEE-MAIN (July-Attempt) 20 July
(Shift-2) Paper

SECTION A

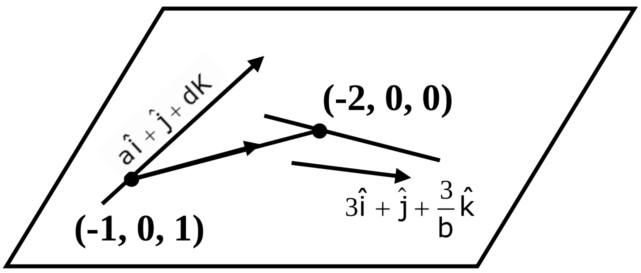
1. The lines x = ay – 1 = z – 2 and x =3y – 2 = bz – 2, $(ab \neq 0)$ are coplanar, if:

(1) b= 1, a $\in$R – {0}
(2) a = 2, b = 3
(3) a = 2, b = 2
(4) a= 1, b $\in$R – {0}

Sol. **(1)**

$$\frac{x+1}{a} = y = \frac{z-1}{a}$$

$$\frac{x+2}{3} = y = \frac{z}{3/b}$$



lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b} - a\right) - 1(a-3) = 0$$

$$a - \frac{3}{b} - a + 3 = 0$$

b = 1, a $\in$ R – {0}

2. If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^\theta \sin x\,dx$ is equal to:

(1) 2
(2) –1
(3) 0
(4) 1

Sol. **(4)**

$$z = \frac{1}{1 - \cos\theta + 2i\sin\theta}$$

$$= \frac{2\sin^2\frac{\theta}{2} - 2i\sin\theta}{(1-\cos\theta)^2 + 4\sin^2\theta}$$

$$= \frac{\sin\frac{\theta}{2} - 2i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)}$$

$$\text{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}\right)}$$

$= \sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$

$= 1 - \cos^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$

$= 3\cos^2\frac{\theta}{2} = \frac{3}{2}$

$= \cos^2\frac{\theta}{2} = \frac{1}{2}$

$\frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$

$\theta = 2n\pi \pm \frac{\pi}{2}$

$\theta \in (0, \pi)$

$\boxed{\theta = \frac{\pi}{2}}$

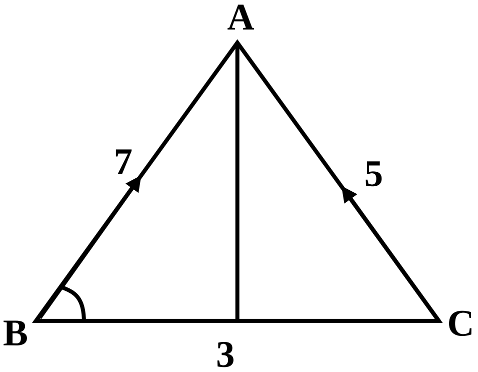
$\int_0^{\frac{\pi}{2}} \sin\theta d\theta - [-\cos\theta]_0^{\frac{\pi}{2}}$

$= -(0 - 1) = 1$

3. In a triangle ABC, if $|\overrightarrow{BC}| = 3, |\overrightarrow{CA}| = 5$ and $|\overrightarrow{BA}| = 7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to:

(1) $\frac{11}{2}$ (2) $\frac{13}{2}$ (3) $\frac{19}{2}$ (4) $\frac{15}{2}$

Sol. **(1)**



Projection of $\overrightarrow{BA}$
on $\overrightarrow{BC}$ is equal to

$= |\vec{BA}| \cos \angle ABC$

$\cos\theta = \frac{49 + 9 - 25}{2.7.3} = \frac{11}{14}$

$\therefore BA\cos\theta = \frac{11}{2}$

4. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1–k), the probability that exactly one of B and C occurs is (1–2k), the probability that exactly one of C and A occurs is (1–k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is:

(1) greater than $\frac{1}{2}$ (2) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(3)exactly equal to $\frac{1}{2}$ (4) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

Sol. **(1)**

$P(\vec{A} \cap B) + P(A \cap \vec{B}) = 1 - K$

$P(\vec{A} \cap C) + P(A \cap \vec{C}) = 1 - 2k$

$P(\vec{B} \cap C) + P(B \cap \vec{C}) = 1 - K$

$P(A \cap B \cap C) = k^2$

$P(A) + P(B) - 2P(A \cap B) = 1 - k$ ... (i)

$P(B) + P(C) - 2P(B \cap C) = 1 - k$ ... (ii)

$P(C) + P(A) - 2P(A \cap C) = 1 - 2k$ ... (iii)

(1) + (2) + (3)

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{-4k+3}{2}$

So

$P(A \cup B \cup C) = \frac{-4k+3}{2} + k^2$

$P(A \cup B \cup C) = \frac{2k^2 - 4K + 3}{2}$

$= \frac{2(k-1)^2 + 1}{2}$

$P(A \cup B \cup C) > \frac{1}{2}$

5. For the natural numbers m, n, if $(1-y)^m (1+y)^n = 1 + a_1y + a_2y^2 + ... + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of (m + n) is equal to:

(1) 88 (2) 64 (3) 100 (4) 80

Sol. **(4)**

$(1 - y)^m(1 + y)^n$

Coefficient of $y = 1.{}^nC_1 + {}^mC_1(-1)$

$= n - m = 10$ ... (1)

Coefficient of $y^2(a_2)$

$= 1.{}^nC_2 - {}^nC_1 . {}^nC_1 + 1. {}^mC_2 = 10$

$= \frac{n(n-1)}{2} - m.n + \frac{m(m-1)}{2} = 10$

$m^2 + n^2 - 2mn - (n + m) = 20$

$(n - m)^2 - (n + m) = 20$

$n + m = 80$ ... (2)

By equation (1) & (2)

$m = 35, n = 45$

6. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (–4, 1) and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then a + b is equal to:

(1) 3 (2) 11 (3) 5 (4) 7

Sol. (3)

Centre of smallest circle is A

Centre of largest circle is B

$r_2 = |CP - CA| = 3\sqrt{2} - 3$

$r_1 = CP + CB = 3\sqrt{2} + 3$

$\frac{r_1}{r_2} = \frac{(3\sqrt{2}+3)^2}{9} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$

a = 3, b = 2

7. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to:

(1) $\frac{-291}{76}$ (2) $\frac{-181}{69}$ (3) $\frac{151}{63}$ (4) $\frac{220}{21}$

Sol. (4)

$$\underbrace{\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{5}}_{x>0,\ y>0,\ xy<1} + \tan^{-1}\frac{5}{12}$$

$$\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}} = \underbrace{\tan^{-1}\frac{15}{8} + \tan^{-1}\frac{5}{12}}_{x>0,\ y>0,\ xy<1}$$

$$\tan^{-1}\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\cdot\frac{5}{12}} = \tan^{-1}\frac{220}{21}$$

$$\tan\left(\tan^{-1}\frac{220}{21}\right) = \frac{220}{21}$$

8. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of |a–b| is equal to:

(1) 7 (2) 11 (3) 9 (4) 1

Sol. (4)

$$10 = \frac{7+10+11+15+a+b}{6}$$

$\Rightarrow a + b = 17$ ... (i)

$$\frac{20}{3} = \frac{7^2+10^2+11^2+15^2+a^2+b^2}{6} - 10^2$$

$a^2 + b^2 = 145$ ... (ii)

Solve (i) and (ii) a = 9, b = 8 or a = 8, b = 9

$|a - b| = 1$

9. If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + ...,$ where x > 0 is 504, then x is equal to:

(1) 81 (2) 243 (3) 7 (4) 9

Sol. (1)

$s = 2\log_9 x + 3\log_9 x + ... + 22\log_9 x$

$s = \log_9 x\,(2 + 3 + ... + 22)$

$s = \log_9 x \left\{\frac{21}{2}(2+22)\right\}$

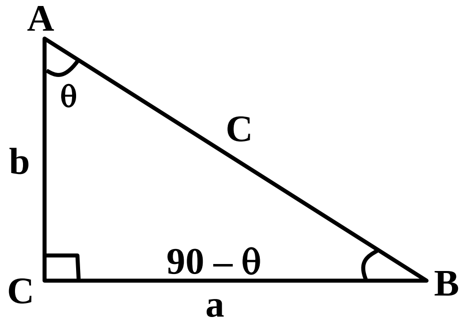
Given $252 \log_9 x = 504$

$\Rightarrow \log_9 x = 2 \Rightarrow x = 81$

10. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to:

(1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{5}-1}{2}$ (3) $\frac{\sqrt{2}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

Sol. (2)



$\angle A = \theta$

$\angle B = 90° - \theta$

$c^2 = a^2 + b^2$

$\frac{1}{a} \rightarrow$ largest side

$$\therefore \frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$$

$\frac{\quad}{a^2} = b^2 + c^2$

Use a = 2R sin A = 2R sin θ

b = 2R sin B = 2R sin (90 $-\theta$) = 2R cos θ

c = 2R sin B = 2 sin90º = 2R

$\frac{4R^2\cos^2\theta . 4R^2}{4R^2\sin^2\theta} = 4R^2\cos^2\theta + 4R^2$

$1 - \sin^2\theta = \sin^2\theta\,(1 - \sin^2\theta) + \sin^2\theta$

$\sin^2\theta = \frac{3-\sqrt{5}}{2}$

$\sin\theta = \frac{\sqrt{5}-1}{2}$

11. Let $f : R - \left\{\frac{\alpha}{6}\right\} \to R$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of α for which $(fof)(x) = x$, for all $x \in R - \left\{\frac{\alpha}{6}\right\}$, is:

(1) No such α exists (2) 5 (3) 6 (4) 8

Sol. (2)

$f(x) = \frac{5x+3}{6x-\alpha} = y$ ...(i)

$5x + 3 = 6xy - \alpha y$

$x(6y - 5) = \alpha y + 3$

$x = \frac{\alpha y + 3}{6y - 5}$

$f^{-1}(x) = \frac{\alpha x + 3}{6x - 5}$ ... (ii)

fo f(x) = x

$f(x) = f^{-1}(x)$

From eq^n (i) & (ii)

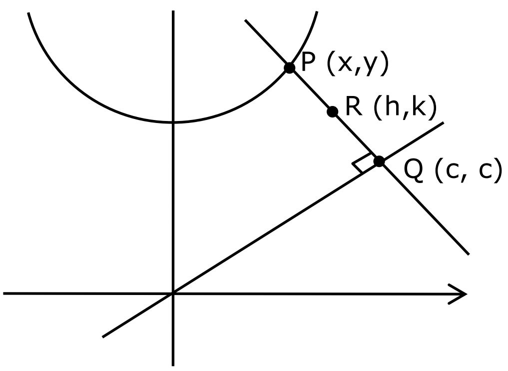
Clearly $\alpha = 5$

12. Let P be a variable point on the parabola $y = 4x^2+1$. Then the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line y = x is:

(1) $(3x - y)^2 + (x - 3y) + 2 = 0$ (2) $2(x - 3y)^2 + (3x - y) + 2 = 0$

(3) $2(3x - y)^2 + (x - 3y) + 2 = 0$ (4) $(3x - y)^2 + 2(x - 3y) + 2 = 0$

Sol. (3)



$$\frac{K-C}{h-C} = -1$$

$$C = \frac{h+k}{2} \qquad P(x, y)$$

$$R = \left(\frac{x+C}{2}, \frac{y+C}{2}\right)$$

$$R = \left(\frac{x}{2}+\frac{h}{4}+\frac{K}{4}, \frac{y}{2}+\frac{h}{4}+\frac{k}{4}\right)$$

$$\therefore x = \frac{3h}{2}-\frac{K}{2}, y = \frac{3K}{2}-\frac{h}{2} \text{ ,put in}$$

$Y = 4x^2 + 1$

$$\left(\frac{3K-h}{2}\right) = 4\left(\frac{3h-k}{2}\right)^2 + 1$$

13. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$ is equal to:

(1) 0 (2) π (3) 1 (4) $-\pi$

Sol. **(4)**

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} ([x]+[-\sin x] \quad \dots (i)$$

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} ([-x]+[\sin x]dx \dots (ii)$$

(King property)

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\underbrace{[-x]+[-x]}_{-1}\right) + \left(\underbrace{[\sin x]+[-\sin x]}_{-1}\right)dx$$

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (-2)dx = -2(\pi)$$

$I = -\pi$

14. Let y = y(x) satisfies the equation $\frac{dy}{dx} - |A| = 0$, for all x > 0, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$.If y(π) = π + 2, then the value of $y\left(\frac{\pi}{2}\right)$ is:

(1) $\frac{\pi}{2} - \frac{4}{\pi}$ (2) $\frac{3\pi}{2} - \frac{1}{\pi}$ (3) $\frac{\pi}{2} - \frac{1}{\pi}$ (4) $\frac{\pi}{2} + \frac{4}{\pi}$

Sol. (4)

$|A| = \frac{-y}{x} + 2\sin x + 2$

$\frac{dy}{dx} = |A|$

$\frac{dy}{dx} = \frac{y}{x} + 2\sin x + 2$

$\frac{dy}{dx} + \frac{-y}{x} = 2\sin x + 2$

I.F. = $e^{\int \frac{1}{x}dx}$ = x

$\Rightarrow yx = \int x(2\sin x + 2)dx$

$xy = x^2 - 2x\cos x + 2\sin x + c$ ... (i)

Now x = π , y = π + 2

Use in (i)

c = 0

Now (i) be comes

$xy = x^2 - 2x\cos x + 2\sin x$

put $x = \frac{\pi}{2}$

$\frac{\pi}{2} y = \frac{\pi^2}{4} + 2$

15. The sum of all the local minimum values of the twice differentiable function F: R → R defined by $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is:

(1) −22 (2) 0 (3) −27 (4) 5

Sol. (3)

$f(x) = x^3 - 3x^2 - \frac{3}{2}f''(2)x + f''(1)$ ... (i)

$f'(x) = 3x^2 - 6x - \frac{3}{2}f''(2)$ ... (ii)

$f''(x) = 6x - 6$ ... (iii)

$f''(2) = 12 - 6 = 6$

and $f''(1) = 0$

Use (ii)

$f'(x) = 3x^2 - 6x - \frac{3}{2}f''(2)$

$f'(x) = 3x^2 - 6x - \frac{3}{2} \times 6$

$f'(x) = 3x^2 - 6x - 9$

$f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$\Rightarrow x = -1 \ \& \ 3$

Use (iii)

$f''(x) = 6x - 6$

$f''(-1) = -12 < 0$ maxima

$f''(3) = 12 > 0$ minima

Use (i)

$f(x) = x^3 - 3x^2 - \frac{3}{2} f''(2)x + f''(1)$

$f(x) = x^3 - 3x^2 - \frac{3}{2} \times 6 \times x + 0$

$f(x) = x^3 - 3x^2 - 9x$

$f(3) = 27 - 27 - 9 \times 3 = -27$

16. The value of $k \in R$, for which the following system of linear equations

$3x - y + 4z = 3,$

$x + 2y - 3x = -2,$

$6x + 5y + kz = -3,$

has infinitely many solutions, is:

(1) 3 (2) -3 (3) 5 (4) -5

Sol. **(4)**

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & K \end{vmatrix} = 0$$

$\Rightarrow 3(2K + 15) + K + 18 - 28 = 0$

$\Rightarrow 7K + 35 = 0 \Rightarrow K - 5$

17. Consider the following three statements :

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 = 17.

Then, which of the following statements is correct?

(1) (A) and (C) are true while (B) is false

(2) (A) is true while (B) and (C) are false

(3) (A) is false, but (B) and (C) are true

(4) (A) and (B) are false while (C) is true

Sol. **(1)**

Truth Table

P	q	P $\to$q
T	T	T
T	F	F
F	T	T
F	F	T

18. Let g(t) = $\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$, where f(x) = $\log_e\left(x+\sqrt{x^2+1}\right), x\in R$. Then which one of the following is correct?

(1) g(1) + g(0) = 0 (2) $g(1)=\sqrt{2}g(0)$ (3) g(1) = g(0) (4) $\sqrt{2}g(1)=g(0)$

Sol. **(4)**

g(t) = $\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$,

g(t) = $\pi\cos\frac{\pi}{4}t+\int_{-\pi/2}^{\pi/2}f(x)dx$

g(t) = $\pi\cos\frac{\pi}{4}t$

g(1) = $\frac{\pi}{\sqrt{2}}, f(0)=\pi$

19. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, –1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P?

(1) (1, 2, 2) (2) (-1, 1, 2) (3) (1, 1, 1) (4) (1, 1, 2)

Sol. **(1)**

Plane p is $\perp$ to line

$\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

& passes through pt. (2, 3, –1) equation of plane p

2(x – 2) + 1(y– 3) + 1(z + 1) = 0

2x + y + z – 6 = 0

pt(1, 2, 2) satisfies above equation

20. If f : R $\to$ R is given by f(x) = x + 1, then the value of

$\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+....+f\left(\frac{5(n-1)}{n}\right)\right]$, is:

(1) $\frac{3}{2}$ (2) $\frac{7}{2}$ (3) $\frac{5}{2}$ (4) $\frac{1}{2}$

Sol. **(2)**

$I = \sum_{r=0}^{n-1} f(\frac{5r}{n})\frac{1}{n}$

$I = \int_0^1 f(5x)dx$

$I = \int_0^1 (5x+1)dx$

$I = \int_0^1 \left[\frac{5x^2}{2}+x\right]_0^1$

$I = \frac{5}{2}+1=\frac{7}{2}$

SECTION B

1. Let a curve y = y(x) be given by the solution of the differential equation

$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$

If it intersects y-axis at y = -1, and the intersection point of the curve with x-axis is (α, 0) the e^{α} is equal to ______.

Sol. **(2)**

$\text{Cos}\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$

Put $\cos^{-1}(e^{-x}) = \theta$, $\theta \in [0, \pi]$

$\text{Cos}\ \theta = e^{-x} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 = e^{-x}$

$\cos\frac{\theta}{2} = \sqrt{\frac{e^{-x}+1}{2}} = \sqrt{\frac{e^x+1}{2e^x}}$

$\sqrt{\frac{e^x+1}{2e^x}}\,dx = \sqrt{e^{2x}-1}\ dy$

0

$\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{e^x}\sqrt{e^x-1}} = \int dy$

Put $e^x = t$, $\frac{dt}{dx} = e^x$

$\int\frac{dt}{t\sqrt{t^2-t}} = \sqrt{2}\,y$

Put $t = \frac{1}{z}$, $\frac{dt}{dz} = -\frac{1}{z^2}$

$\int\frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}-\frac{1}{z}}} = \sqrt{2}\,y$

$$-\int \frac{dz}{\sqrt{1-z}} = \sqrt{2}\,y$$

$$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}\,y + c$$

$$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}\,y + c$$

$2(1 - e^{-x})^{1/2} = \sqrt{2}\,y + c$ put given condition .

$\therefore \boxed{c = \sqrt{2}}$

$2(1 - e^{-\alpha})^{1/2} = \sqrt{2}\,(y + 1)$, passes through $(\alpha, 0)$

$2(1-e^{-\alpha})^{1/2} = \sqrt{2}$

$$\sqrt{1-e^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1 - e^{-\alpha} = \frac{1}{2}$$

$$e^{-\alpha} = \frac{1}{2} \Rightarrow e^{\alpha} = 2$$

2. For $k \in N$, let $\dfrac{1}{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+20)} = \sum_{k=0}^{20} \dfrac{A_k}{a+k}$, where a > 0. Then the value of $100\left(\dfrac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to ________.

Sol. **(9)**

$$\frac{1}{\alpha(\alpha+1)\ldots.(\alpha+20)} = \sum_{k-0}^{20} \frac{A_k}{\alpha+k}$$

$$A_{14} = \frac{1}{(-14)(-13)\ldots.(-1)(1)\ldots(6)} = \frac{1}{14!\cdot 6!}$$

$$A_{15} = \frac{1}{(-15)(-14)\ldots.(-1)(1)\ldots(5)} = \frac{-1}{15!5!}$$

$$A_{13} = \frac{1}{(-13)\ldots.(-1)(1)\ldots(7)} = \frac{-1}{13!7!}$$

$$\frac{A_{14}}{A_{13}} = = \frac{-7}{14} = -\frac{1}{2}$$

$$\frac{A_{15}}{A_{13}} = = \frac{42}{15\times 14} = \frac{1}{5}$$

$$100\left(\frac{A_{14}}{A_{13}} + \frac{A_{15}}{A_{13}}\right)^2 = 100\left(-\frac{1}{2}+\frac{1}{5}\right)^2 = 9$$

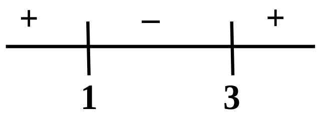
3. Let a function $g : [0, 4] \to R$ be defined as $g(x) = \begin{cases} \max\limits_{0\le t\le x}\{t^3-6t^2+9t-3\}, & 0 \le x \le 3 \\ 4-x, & 3 < x \le 4 \end{cases}$, then the number of points in the interval (0, 4) where g(x) is NOT differentiable, is _____.

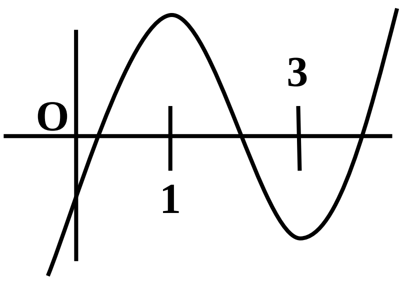
Sol. **(1)**

$f(X) = x^3 - 6x^2 + 9x - 3$

$f(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$

$f(1) = 1$ and $f(3) = -3$





$$g(x) = \begin{cases} f(x) & 0 \le x \le 1 \\ 1 & 1 \le x \le 3 \\ 4 - x & 3 < x \le 4 \end{cases}$$

g(x)is continuous

$$g'(x) = \begin{cases} 3(x-1)(x-3) & 0 \le x \le 1 \\ 0 & 1 \le x \le 3 \\ -1 & 3 < x \le 4 \end{cases}$$

g(x) is non-differentiable at x = 3

4. Let A= $\{a_{ij}\}$ be a 3 x 3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i.j, \end{cases}$$

then $\det(3\,\text{Adj}(2A^{-1}))$ is equal to ______.

Sol. **(108)**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$|A| = 4$

$|3\text{adj}(2A^{-1})| = |3.2^2\text{adj}(A^{-1})|$

$12^3|\text{adj}(A^{-1})| = 12^3|A^{-1}|^2 = \dfrac{12^3}{|A|^2} = \dfrac{12^3}{16} = 108$

5. For p > 0, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta = \dfrac{(\alpha\sqrt{3} - 2)}{4\sqrt{3} + 3}$, then the value of α is equal to ________.

Sol. **(6)**

$|\vec{V_1}| = |\vec{V_2}|$

$3P^2 + 1 = 4 + (P + 1)^2$

$2P^2 - 2P - 4 = 0$

$\Rightarrow P^2 - P - 2 = 0$

$P = 2, -1$ (rejected)

$\cos\theta = \dfrac{\vec{V_1}\cdot\vec{V_2}}{|\vec{V_1}|\cdot|\vec{V_2}|}$

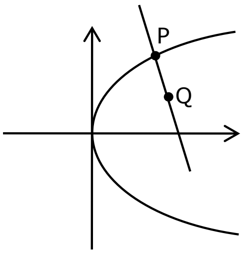
$\cos\theta = \dfrac{4\sqrt{3}+3}{\sqrt{13}\sqrt{13}} = \dfrac{4\sqrt{3}+3}{13}$

$\tan\theta = \dfrac{\sqrt{112-24\sqrt{3}}}{4\sqrt{3}+\sqrt{3}} = \dfrac{6\sqrt{3}-2}{4\sqrt{3}+3} = \dfrac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$

$\Rightarrow \alpha = 6$

6. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to ________.

Sol. **(9)**



Minimum distance is along the normal

$P \equiv \left(\frac{3}{2}t^2, 3t\right)$

Normal at point P

$tx + y = 3t + \frac{3}{2}t^3$

Passes through $\left(3, \frac{3}{2}\right)$

$\Rightarrow 3t + \frac{3}{2} = 3t + \frac{3}{2}t^3$

$P \equiv \left(\frac{3}{2}, 3\right) = 3t + \frac{3}{2}t^3$

$P \equiv \left(\frac{3}{2}, 3\right) = (\alpha, \beta)$

$\Rightarrow t^3 = 1 \Rightarrow t = 1$

$2(\alpha + \beta) = 2\left(\frac{3}{2}+3\right) = 9$

7. If $\lim\limits_{x\to 0}\frac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x\sin^2 x} = 10$, $\alpha, \beta, \gamma \in R$, then the value of $\alpha+\beta+\gamma$ is ______.

Sol. (3)

$$\lim_{x\to 0}\frac{\alpha x\left(1+x+\frac{x^2}{2}\right)-\beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)+\gamma x^2(1-x)}{x^3}$$

$$\lim_{x\to 0}\frac{x(\alpha-\beta)+x^2\left(\alpha+\frac{\beta}{2}+\gamma\right)+x^3\left(\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right)}{x^3}$$

For limit to exist

$\alpha - \beta = 0$, $\alpha + \frac{\beta}{2} + \gamma = 0$

$\frac{\alpha}{2}-\frac{\beta}{3}-\gamma = 10$... (i)

$\beta = \alpha$, $\gamma = -3\frac{\alpha}{2}$

Put in (i)

$\frac{\alpha}{2}-\frac{\alpha}{3}+\frac{3\alpha}{2} = 10$

$\frac{\alpha}{6}+\frac{3\alpha}{2} = 10 \Rightarrow \frac{\alpha+9\alpha}{6} = 10$

$\Rightarrow \alpha = 6$

$\alpha = 6, \beta = 6, \gamma = -9$

$\alpha + \beta + \gamma = 3$

8. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$, $x > 0$, is ______.

Sol. (1)

$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0$

$\log_{(x+1)}(2x + 5)(x + 1) + 2\log_{(2x+5)}(x + 1) = 4$

$\log_{(x+1)}(2x + 5) + 1 + 2\log_{(2x+5)}(x + 1) = 4$

Put $\log_{(x+1)}(2x + 5) = t$

$t + \frac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$

$t = 1, 2$

$\log_{(x+1)}(2x + 5) = 1$ & $\log_{(x+1)}(2x + 5) = 2$

$x + 1 = 2x + 3$ & $2x + 5 = (x + 1)^2$

$x = -4$ (rejected) $x^2 = 4 \Rightarrow x = 2, -2$ (rejected)

So, $x = 2$

No. of solution = 1

9. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then tha value of $47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to _______.

Sol. **(7)**

$a_{n+2} = 2a_{n+1} + a_n$, let $\sum_{n=1}^{\infty}\frac{a_n}{8^n} = P$

Divide by 8^n we get

$$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$$

$$\Rightarrow 64\frac{a_{n+2}}{8^{n+2}} = \frac{16a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$$

$$64\sum_{n=1}^{\infty}\frac{a_{n+2}}{8^{n+2}} = 16\sum_{n=1}^{\infty}\frac{a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty}\frac{a_n}{8^n}$$

$$64\left(P - \frac{a_1}{8} - \frac{a_2}{8^2}\right) = 16\left(P - \frac{a_1}{8}\right) + P$$

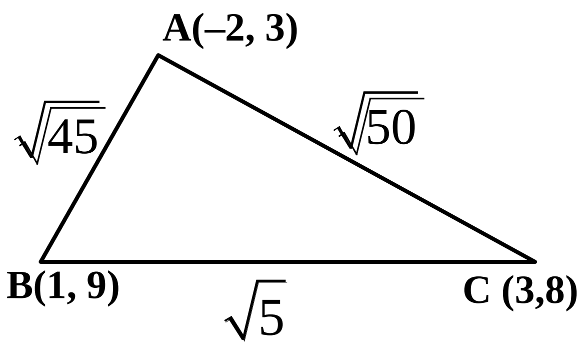
$$\Rightarrow 64\left(P - \frac{1}{8} - \frac{1}{64}\right) = 16\left(P - \frac{1}{8}\right) + P$$

$64P - 8 - 1 = 16P - 2 + P$

$47P = 7$

10. Consider a triangle having vertices A(–2, 3), B(1, 9) and C(3, 8). If a line L passing through the circum-center of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is _________.

Sol. **(9)**



$(\sqrt{50})^2 = (\sqrt{45})^2 + (\sqrt{5})^2$

$\angle B = 90^o$

Circum-center = $\left(\frac{1}{2}, \frac{11}{2}\right)$

Mid point of BC = $\left(2, \frac{17}{2}\right)$

Line : $\left(y - \frac{11}{2}\right) = 2\left(x - \frac{1}{2}\right) \Rightarrow y = 2x + \frac{9}{2}$

Passing through $\left(0, \frac{\alpha}{2}\right)$

$\frac{\alpha}{2} = \frac{9}{2} \Rightarrow \alpha = 9$