MATHEMATICS
JEE-MAIN (September-Attempt)
4 September (Shift-1) Paper

SECTION - A

1. Let y=y(x) be the solution of the differential equation, $xy'-y=x^2(x\cos x+\sin x), x>0$. if $y(\pi)=\pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to

(1) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$ (2) $2+\frac{\pi}{2}$ (3) $1+\frac{\pi}{2}$ (4) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$

Sol. **(2)**

$xy' - y = x^2(x\cos x + \sin x)\ \ x > 0,\ y(\pi) = \pi$

$y' - \frac{1}{x}y = x\{x\cos x + \sin x\}$

$\text{I.F.} = e^{-\int\frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$\therefore\ y.\frac{1}{x} = \int\frac{1}{x}.x(x\cos x+\sin x)dx$

$\frac{y}{x} = \int(x\cos x+\sin x)dx$

$\frac{y}{x} = \int\frac{d}{dx}(x\sin x)dx$

$\frac{y}{x} = x\sin x + C$

$\Rightarrow y = x^2 = \sin x + cx$

$x = \pi,\ y = \pi$

$\pi = \pi c \Rightarrow C = 1$

$y = x^2\sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}+\frac{\pi}{2}$

$y' = 2x\sin x + x^2\cos x + 1$

$y'' = 2\sin x + 2x\cos x + 2x\cos x - x^2\sin x$

$y''\left(\frac{\pi}{2}\right) = 2-\frac{\pi^2}{4} \Rightarrow y\left(\frac{\pi}{2}\right)+y''\left(\frac{\pi}{2}\right) = 2+\frac{\pi}{2}$

2. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:

(1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{51}C_7 + {}^{30}C_7$ (3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Sol. **(1)**

$\sum_{r=0}^{20} {}^{50-r}C_6$

$\Rightarrow {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{31}C_6 + {}^{30}C_6$

add and subtract ${}^{30}C_7$

Using
${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r} \Rightarrow {}^{30}C_{6} + {}^{30}C_{7} = {}^{31}C_{7}$
${}^{31}C_{6} + {}^{31}C_{7} = {}^{32}C_{7}$
Similarly solving
${}^{51}C_{7} - {}^{30}C_{7}$

3. Let [t] denote the greatest integer $\le$ t. Then the equation in x, $[x]^2+2[x+2]-7=0$ has :
(1) exactly four integral solutions.
(2) infinitely many solutions.
(3) no integral solution.
(4) exactly two solutions.

Sol. **(2)**
$[x]^2 + 2[x + 2] - 7 = 0$
$[x]^2 + 2[x] - 3 = 0$
let $[x] = y$
$y^2 + 3y - y - 3 = 0$
$(y - 1)(y + 3) = 0$
$[x] = 1$ or $[x] = -3$
$x \in [1, 2)$ & $x \in [-3, -2)$

4. Let P(3,3) be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9,0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to :
(1) (9,3)
(2) $\left(\frac{9}{2}, 2\right)$
(3) $\left(\frac{9}{2}, 3\right)$
(4) $\left(\frac{3}{2}, 2\right)$

Sol. **(3)**
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ P(3,3)

$\frac{9}{a^2} - \frac{9}{b^2} = 1$ ....(1)

Equation of normal $\Rightarrow \frac{a^2x}{3} + \frac{b^2y}{3} = a^2e^2$

at x – axis $\Rightarrow y = 0$

$\frac{a^2x}{3} = a^2e^2 \Rightarrow x = 3e^2 = 9$
$e^2 = 3$
$e = \sqrt{3}$

$e^2 = 1 + \frac{b^2}{a^2} = 3$
$b^2 = 2a^2$ ...(2)
put in equation 1

$\frac{9}{a^2} - \frac{9}{2a^2} = 1 \Rightarrow \frac{9}{2a^2} = 1 \Rightarrow a^2 = \frac{9}{2}$

$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$

5. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a>b) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then a^2+b^2 is equal to

(1) 135 (2) 116 (3) 126 (4) 145

Sol. **(3)**

$L.R = \frac{2b^2}{a} = 10$ ...(1)

$\phi(t) = \frac{5}{12} - \left(t - \frac{1}{2}\right)^2 + \frac{1}{4} = \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$

$\therefore \phi(t)_{max} = \frac{2}{3} = e$

$e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$

$\frac{b^2}{a.a} = \frac{5}{9}$ from (1)

$\frac{5}{a} = \frac{5}{9} \Rightarrow a = 9$

$\therefore b^2 = 45$

$a^2 + b^2 = 45 + 81 = 126$

6. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx \, (x \geq 0)$. Then f(3) – f(1) is eqaul to :

(1) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$ (3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Sol. **(4)**

$f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$

$x = \tan^2 t$

$dx = 2\tan t \sec^2 t \, dt$

$$f(x) = \int \frac{\tan t.\ 2\tan t \sec^2 t dt}{\sec^4 t}$$

$$= 2\int \sin^2 t\, dt$$

$$x = 3 \Rightarrow t = \frac{\pi}{3}$$

$$x = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\therefore f(3) - f(1) = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \cos 2t)\, dt \Rightarrow \left(t - \frac{1}{2}\sin 2t\right)_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

7. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+......+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:

(1) (10,97) (2) (11,103) (3) (11,97) (4) (10,103)

Sol. **(2)** 1 + Sn

$T_n = 1 - (2n)^2(2n - 1)$

$= 1 - 4n^2(2n - 1)$

$= 1 - 8n^3 + 4n^2$

$$S_n = \sum_{n=1}^{10} T_n = n - \sum 8n^3 + \sum 4n^2$$

$$= n - 8 \times \frac{n^2(n+1)^2}{4} + \frac{4n(n+1)(2n+1)}{6}$$

$$= 10 - 2 \times 100 \times 121 + \frac{2}{3} \times 10 \times 11 \times 21$$

$= 10 - 24200 + 1540$

$= 10 - 22660$

$\therefore$ Sum of series $= 11 - 22660 = \alpha - 220\beta$

$\alpha = 11, \beta = 103$

8. The integral $\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to

(where C is a constant of integration):

(1) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

(2) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(3) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

(4) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

Sol. **(1)**

$$\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$$

$$\int \underbrace{x\sec x}_{I}\cdot\underbrace{\frac{x\cos x}{(x\sin x+\cos x)^2}}_{II}dx$$

$$x\sec x\left(\frac{-1}{x\sin x+\cos x}\right)+\int\frac{\sec x+x\sec x\tan x}{(x\sin x+\cos x)}dx$$

$$\Rightarrow \frac{-x\sec x}{x\sin x+\cos x}+\int\frac{(\cos x+x\sin x)}{\cos^2 x\,(x\sin x+\cos x)}dx \Rightarrow \frac{-x\sec x}{x\sin x+\cos x}+\tan x+C$$

9. Let f(x)= |x-2| and g(x) = f(f(x)) ,$x\in[0,4]$. Then $\int_0^3 (g(x)-f(x))dx$ is equal to:

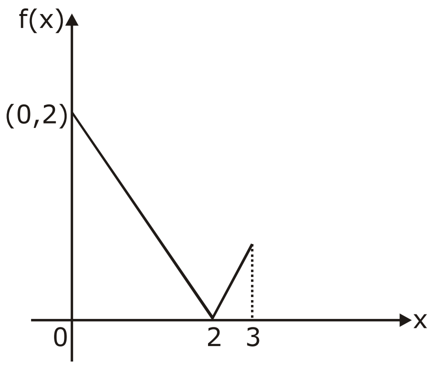
(1) $\frac{1}{2}$ (2) 0 (3) 1 (4) $\frac{3}{2}$

Sol. **(3)**

f(x) = |x – 2|

$$g(x) = ||x-2|-2| = \begin{cases} \text{if } x\ge 2 & \Rightarrow |x-4| \\ \text{if } x<2 & \Rightarrow |-x| \end{cases}$$

$$\therefore \int_0^3 (g(x)-f(x))dx$$



$= \int_0^3 g(x) - \int_0^3 f(x)\,dx$

$= \int_0^2 x dx + \int_2^3 (4-x) dx - \int_0^2 (2-x) dx - \int_2^3 (x-2) dx$

$\Rightarrow \left(\frac{x^2}{2}\right)_0^2 + \left(4x - \frac{x^2}{2}\right)_2^3 + \left(\frac{x^2}{2} - 2x\right)_0^2 - \left(\frac{x^2}{2} - 2x\right)_2^3$

$\Rightarrow 2 + \left\{12 - \frac{9}{2} - 8 + 2\right\} + \{2 - 4\} - \left(\frac{9}{2} - 6 - 2 + 4\right)$

$= 2 + \left\{6 - \frac{9}{2}\right\} -2 - \left\{\frac{9}{2} - 4\right\} = 2 + \frac{3}{2} - \left(2 + \frac{1}{2}\right) = \frac{7}{2} - \frac{5}{2} = 1$

10. Let x_0 be the point of Local maxima of $f(x) = \vec{a}.(\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ at $x=x_0$ is :

(1) -22 (2) -4 (3) -30 (4) 14

Sol. **(1)**

$\vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$

$\Rightarrow x\{x^2 - 2\} + 2\{-2x + 7\} + 3\{4 - 7x\}$

$= x^3 - 2x - 4x + 14 + 12 - 21x$

$f(x) = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

Max at $x_0 = -3$

$\therefore \vec{a} = (-3, -2, 3)$, $\vec{b} = (-2, -3, -1)$, $\vec{c} = (7, -2, -3)$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 6 + 6 - 3 - 14 + 6 + 3 - 21 + 4 - 9$

$= 25 - 47 = -22$

11. A triangle ABC lying in the first quadrant has two vertices as A(1,2) and B(3,1) If $\angle BAC = 90^0$, and ar(ΔABC) = $5\sqrt{5}$ s units, then the abscissa of the vertex C is :

(1) $1 + \sqrt{5}$ (2) $1 + 2\sqrt{5}$ (3) $2\sqrt{5} - 1$ (4) $2 + \sqrt{5}$

Sol. **(2)**

$AB = \sqrt{4+1} = \sqrt{5}$

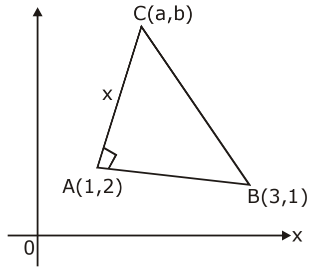
$\frac{1}{2} \times \sqrt{5} \times x = 5\sqrt{5}$

$x = 10$

$m_{AB} = \frac{1}{-2}$

$m_{AC} = 2 = \tan\theta$

$\therefore \sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$



by parametric co - ordinates $a = 1 + 10 \times \frac{1}{\sqrt{5}} = 1 + 2\sqrt{5}$

12. Let f be a twice differentiable function on (1,6). If f(2)=8, f′(2)=5, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1,6)$, then:

(1) $f(5)+f'(5) \geq 28$ (2) $f'(5)+f''(5) \leq 20$

(3) $f(5) \leq 10$ (4) $f(5)+f'(5) \leq 26$

Sol. **(1)**

$f(2) = 8, f'(2) = 5, f'(x) \geq 1, f''(x) \geq 4$

$x \in (1,6)$

$\int_2^5 f'(x) \geq \int_2^5 1\,dx$

$f(5) - f(2) \geq 3$

$f(5) \geq 11$ ...(1)

also $\int_2^5 f''(x)\,dx \geq \int_2^5 4dx$

$f(5) + f'(5) \geq 28$

$f'(5) - f'(2) \geq 12$

$f'(5) \geq 17$ ...(2)

13. Let α and β be the roots of $x^2-3x+p=0$ and γ and δ be the roots of $x^2-6x+q=0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression.Then ratio (2q+p): (2q-p) is:

(1) 33 :31 (2) 9 : 7 (3) 3 : 1 (4) 5 : 3

Sol. **(2)**

$x^2 - 3x + p = 0 \ (\alpha, \beta)$

$x^2 - 6x + q = 0 \ (\gamma, \delta)$

$\alpha + \beta = 3$

$\gamma + \delta = 6$

$\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$

$a(1 + r) = 3$ ...(1)

$ar^2(1 + r) = 6$ ...(2)

Divide (2) by (1)

$r^2 = 2, r = \sqrt{2} \Rightarrow a = \frac{3}{\sqrt{2}+1}$

$\alpha = \frac{3}{\sqrt{2}+1}, \beta = \frac{3\sqrt{2}}{\sqrt{2}+1}, \gamma = \frac{3.2}{\sqrt{2}+1}, \delta = \frac{3.2\sqrt{2}}{\sqrt{2}+1}$

$\alpha\beta = p = \frac{9\sqrt{2}}{\left(\sqrt{2}+1\right)^2}, \gamma\delta = \frac{36\sqrt{2}}{\left(\sqrt{2}+1\right)^2} \Rightarrow \frac{72+9}{72-9} = \frac{81}{63}$

= 9/7

14. Let $u = \frac{2z+i}{z-ki}$, z = x +iy and k>0. If the curve represented by Re(u) +Im(u) =1 intersects the y-axis at the points P and Q where PQ =5, then the value of k is :

(1) 4 (2) 1/2 (3) 2 (4) 3/2

Sol. **(3)**

$u = \frac{2z+i}{z-ki}$, z = x + iy

$= \frac{2x+i(2y+1)}{x+i(y-k)} \times \frac{x-i(y-k)}{x-i(y-k)}$

$\Rightarrow \frac{2x^2+(2y+1)(y-k)+i\{2xy+x-2xy+2xk\}}{x^2+(y-k)^2}$

Re(u) + Img(u) = 1

$2x^2 + (2y+1)(y-k) + x + 2xk = x^2 + (y-k)^2$

at y - axis, x = 0

$(2y + 1)(y - k) = (y - k)^2$

$2y^2 + y - 2yk - k = y^2 + k^2 - 2yk$

$y^2 + y - (k + k^2) = 0$ (y_1, y_2)

diff. of roots = 5

$\sqrt{1+4k+4k^2} = 5$

$4k^2 + 4k = 24$

$k^2 + k - 6 = 0$

$(k + 3)(k - 2) = 0$

k = 2

15. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?

(1) $a^2 - d^2 = 0$ (B) $a^2 - c^2 = 1$ (C) $0 \le a^2 + b^2 \le 1$ (D) $a^2 - b^2 = \frac{1}{2}$

Sol. **(4)**

$$\begin{bmatrix} c & is \\ is & c \end{bmatrix}\begin{bmatrix} c & is \\ is & c \end{bmatrix} = z\begin{bmatrix} c^2 - s^2 & 2ics \\ 2ics & c^2 - s^2 \end{bmatrix} = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix} \quad \text{(where } c = \cos\theta, s = \sin\theta\text{))}$$

$$A^5 = \begin{bmatrix} \cos(2^4\theta) & i\sin(2^4\theta) \\ i\sin(2^4\theta) & \cos(2^4\theta) \end{bmatrix}$$

$a = d = \cos(16\theta)$
$b = c = i\sin(16\theta)$
$a^2 - b^2 = \cos^2(16\theta) + \sin^2 16\theta = 1$

16. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is:
(1) 3 (2) 9 (3) 7 (4) 5

Sol. **3**

$$\frac{5+7+10+12+14+15+x+y}{8} = 10$$

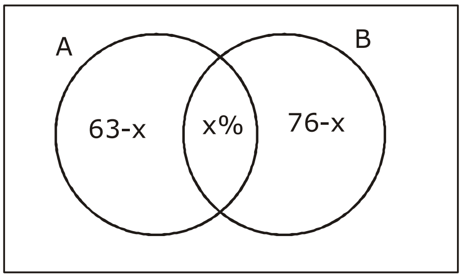
$x + y = 17$ ....(1)

$$\text{variance} = \frac{739 + x^2 + y^2}{8} - 100 = 13.5$$

$x^2 + y^2 = 169$ ...(2)
$\therefore x = 12, y = 5$
$|x - y| = 7$

17. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:
(1) 37 (2) 29 (3) 65 (4) 55

Sol. **(4)**



$A \cup B = 13 - x \le 100$
$x \ge 39$
also $x \le 63$

18. Given the following two statements:

$(S_1) : (q \vee p) \to (P \leftrightarrow \sim q)$ is a tautology

$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then:

(1) only (S_1) is correct.
(2) both (S_1) and (S_2) are correct.
(3) only (S_2) is correct
(4) both (S_1) and (S_2) are not correct.

Sol. **(4)**

S_1 =

p	q	$\sim q$	qvp	$p \leftrightarrow \sim q$	$(qvp) \to (p \leftrightarrow \sim q)$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T

S_1 is not correct

S_2 =

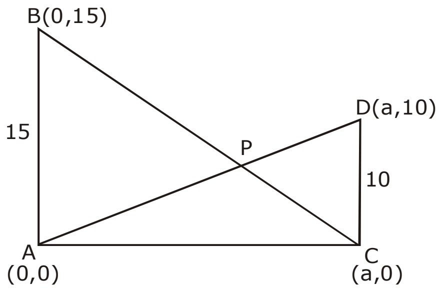
p	q	$\sim q$	$\sim p$	$\sim p \leftrightarrow q$	$\sim q \wedge (\sim p \leftrightarrow q)$
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	F	F

S_2 is false

19. Two vertical poles AB=15 m and CD=10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is:

(1) 5
(2) 20/3
(3) 10/3
(4) 6

Sol. **(4)**



equation of AD : $y = \frac{10x}{a}$

equation of BC : $\frac{x}{a} + \frac{y}{15} = 1$

$\Rightarrow \frac{a.y}{10a} + \frac{y}{15} = 1 \Rightarrow \frac{3y+2y}{30} = 1$

$5y = 30 \Rightarrow y = 6$

20. If $\left(a+\sqrt{2}\ b\ \cos x\right)\left(a-\sqrt{2}\ b\cos y\right) = a^2 - b^2$, where a>b>0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is:

(1) $\frac{a+b}{a-b}$ (2) $\frac{a-2b}{a+2b}$ (3) $\frac{a-b}{a+b}$ (4) $\frac{2a+b}{2a-b}$

Sol. **(1)**

$\left(a+\sqrt{2}\ b\ \cos x\right)\left(a-\sqrt{2}\ b\cos y\right) = a^2 - b^2$

diff both sides w.r.t y

$-\sqrt{2}\,b\sin x.\frac{dx}{dy}\left(a-\sqrt{2}b\cos y\right) + \left(a+\sqrt{2}b\cos x\right)\left(\sqrt{2}\,b\sin y\right) = 0$

$x = y = \frac{\pi}{4} \Rightarrow \frac{-bdx}{dy}(a-b) + (a+b)(b) = 0$

$\frac{dx}{dy} = \frac{a+b}{a-b}$

21. Suppose a differentiable function f(x) satisfies the identity $f(x+y)=f(x)+f(y)+xy^2+x^2y$,for all real x and y. If $\lim_{x\to 0}\frac{f(x)}{x} = 1$, then f'(3) is equal to............

Sol. $f(x + y) = f(x) + f(y) + xy^2 + x^2y$

$x = y = 0$

$f(0) = 2f(0) \Rightarrow f(0) = 0$

Partially diff. w.r.t. x
$f'(x + y) = f'(x) + y^2 + 2xy$
$x = 0 , y = x$

$f'(x) = f'(0) + x^2$ given $\lim_{x\to 0}\frac{f(x)}{x} = 1$

$f'(x) = 1 + x^2$ by L' hospital

$\therefore f(x) = x + \frac{x^3}{3} + c$ $\lim_{x\to 0}\frac{f'(x)}{1} = 1$

put $x = 0 \Rightarrow c = 0$ $f'(0) = 1$
$f'(3) = 10$

22. If the equation of a plane P, passing through the intersection of the planes, x+4y-z+7=0 and 3x+y+5z=8 is ax+by+6z=15 for some a, b $\in$ R, then the distance of the point (3,2,-1) from the plane P is..........

Sol. $p_1 + \lambda p_2 = 0$
$(x + 4y - z + 7) + \lambda (3x + y + 5z - 8) = ax + by + 6z - 15$

$$\frac{1-3\lambda}{a} = \frac{4+\lambda}{b} = \frac{-1+5\lambda}{6} = \frac{7-8\lambda}{-15}$$

$\therefore\ 15 - 75\lambda = 42 - 48\lambda$
$-27 = 27\lambda$
$\lambda = -1$
$\therefore$ plane is $-2x + 3y - 6z + 15 = 0$

$$d = \left|\frac{-6+6+6+15}{\sqrt{4+9+36}}\right| = 3$$

23. If the system of equations
x-2y+3z=9
2x+y+z=b
x-7y+az=24, has infinitely many solutions, then a-b is equal to.........

Sol. D = 0

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$1(a + 7) + 2(2a - 1) + 3(-14 - 1) = 0$
$a + 7 + 4a - 2 - 45 = 0$
$5a = 40$
$a = 8$

$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0$$

$\Rightarrow 9(8 + 7) + 2(8b - 24) + 3(-7b - 24) = 0$
$\Rightarrow 135 + 16b - 48 - 21b - 72 = 0$
$15 = 5b \Rightarrow b = 3$
$a - b = 5$

24. Let $(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ....

Sol. **8**

$(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$

a_7 = coeff of x^7
a_{13} = coeff of x^{13}

$\frac{10!}{p!q!r!}\left(2x^2\right)^p (3x)^q (4)^r$

for x^7

p	q	r
3	1	6
2	3	5
1	5	4
0	7	3

$a_7 = \frac{2^3.3.10!}{3!6!} + \frac{10!2^2.3^3}{2!3!5!} + \frac{10!2.3^5}{5!4!} + \frac{10!.3^7}{7!3!}$

for x^{13}

p	q	r
6	1	3
5	3	2
4	5	1
3	7	0

$a^{13} = \frac{2^6.3.10!}{6!3!} + \frac{2^5.3^3.10!}{5!3!2!} + \frac{2^4.3^5.10!}{4!5!} + \frac{2^3 10!}{3!7!} \therefore \frac{a_7}{a_{13}} = 8$

25. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ....

Sol. **3**

$P(H) = \frac{1}{10}$; $P(M) = \frac{9}{10}$

P(H) + P(M). P(H) + P(M). P(M). P(H) + ...

$= 1 - P(M)^n \geq \frac{1}{4}$

$= 1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$

$\left(\frac{9}{10}\right)^n \leq \frac{3}{4}$; $n \geq 3$