

**MATHEMATICS**  
**JEE-MAIN (January-Attempt) 12**  
**January (Shift-2) Paper**

**SECTION - A**

1. The total number of irrational terms in the binomial expansion of  $(7^{1/5} - 3^{1/10})^{60}$  is :  
 (A) 49                      (B) 55                      (C) 54                      (D) 48

**Sol.** C

$$= {}^{60}C_r (7^{1/5})^{60-r} (-3^{1/10})^r$$

$$= {}^{60}C_r (7^{12-r/5}) (-3^{1/10})$$

In  $r/5$ ,  $r = 0, 5, 10 \dots 60$   
 $r = 0, 10, 20, 30, 40, 50, 60$   
 no of rational term = 7  
 Irrational term's =  $61 - 7 = 54$

2. The intergral  $\int_1^e \left\{ \left( \frac{x}{e} \right)^{2x} - \left( \frac{e}{x} \right)^x \right\} \log_e x \, dx$  is equal to:

- (A)  $\frac{1}{2} - e - \frac{1}{e^2}$               (B)  $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$               (C)  $\frac{3}{2} - e - \frac{1}{2e^2}$               (D)  $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

**Sol.** C

$$\int_1^e \left( \frac{x}{e} \right)^{2x} \ln x \, dx - \int_1^e (e/x)^x \ln x \, dx$$

$$\left( \frac{x}{e} \right)^{2x} = t$$

$$\Rightarrow \left( \frac{x}{e} \right)^{2x} \ln x \, dx = \frac{1}{2} dt$$

$$\frac{1}{2} \int_{(1/e)^2}^1 dt + \int_e^1 dt_1$$

$$\Rightarrow \frac{1}{2} [1 - (1/e)^2] + [1 - e] \Rightarrow \frac{3}{2} - \frac{1}{2e^2} - e$$

3.  $\lim_{x \rightarrow 1^-} \frac{\sqrt{\pi} - \sqrt{2 \sin^{-1} x}}{\sqrt{1-x}}$  is equal to:

- (A)  $\sqrt{\frac{2}{\pi}}$               (B)  $\frac{1}{\sqrt{2\pi}}$               (C)  $\sqrt{\frac{\pi}{2}}$               (D)  $\sqrt{\pi}$

$$\lim_{x \rightarrow 5} \frac{\sqrt{\pi} - \sqrt{2 \sin^{-1} x}}{\sqrt{1-x}}$$

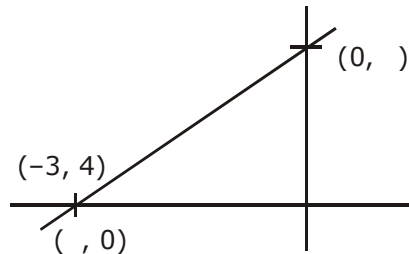
Apply L - H theorem

$$\lim_{x \rightarrow 5} \frac{-\frac{1}{2\sqrt{2 \sin^{-1} x}} \cdot \frac{2}{\sqrt{1-x^2}}}{\frac{1}{2}\sqrt{1-x}(-1)} \Rightarrow \lim_{x \rightarrow 5} \frac{2}{\sqrt{2 \sin^{-1} x} \sqrt{1+x}} \Rightarrow \sqrt{2/\pi}$$

4. If a straight line passing through the point  $P(-3, 4)$  is such that its intercepted portion between the coordinate axes is bisected at  $P$ , then its equation is :

(A)  $4x + 3y = 0$  (B)  $3x - 4y + 25 = 0$   
 (C)  $4x - 3y + 24 = 0$  (D)  $x - y + 7 = 0$

Sol. C



$$\frac{x}{\alpha} + \frac{y}{\beta} = 1$$

$$\text{Now } \frac{\alpha}{2} = -3 \Rightarrow \alpha = -6$$

$$\text{Equation } \frac{x}{-6} + \frac{y}{\beta} = 1$$

$$\beta/2 = 4 \Rightarrow \beta = 8$$

$$\Rightarrow -4x + 3y = 24 \text{ or } 4x - 3y + 24 = 0$$

5. There are  $m$  men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of  $m$  is:

(A) 12 (B) 9 (C) 7 (D) 11

Sol. A

M Men, 2 W

$$\text{Total games} = \binom{m+2}{2}(2)$$

$$\text{given between mens is} = \binom{m}{2}(2)$$

$$\text{games between men and women is} = \binom{m}{1} \binom{2}{1} (2)$$

$$2 \cdot \binom{m}{2} - 4 \cdot \binom{m}{1} = 84$$

$$\Rightarrow m(m-1) - 4m = 84$$

$$\Rightarrow m^2 - 5m - 84 = 0 \Rightarrow (m-12)(m+7) = 0$$

$$m = 12$$

6. Let  $f$  be a differentiable function such that  $f(1) = 2$  and  $f'(x) = f(x)$  for all  $x \in \mathbb{R}$ . If  $h(x) = f(f(x))$ , then  $h'(1)$  is equal to:

(A)  $4e$  (B)  $2e$   
 (C)  $2e^2$  (D)  $4e^2$

Sol. A

$$f'(x) = f(x) \Rightarrow f(x) = ke^x$$

$$x = 1$$

$$\Rightarrow 2 = ke \Rightarrow k = 2/e$$

$$f(x) = 2e^{x-1}$$

$$h(x) = f_1 f(x) = 2 \cdot e^{(2e^{x-1})} - 1$$

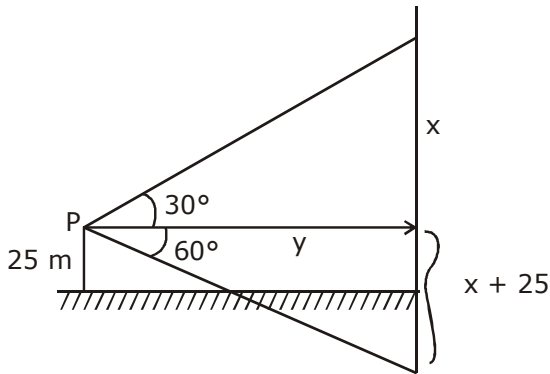
$$h'(x) = 2e^{(2e^{x-1})-1} : (2e^{x-1})$$

$$h'(1) = 2e^{(1)} \cdot 2e^0 = 4e$$

7. If the angle of elevation of a cloud from a point  $P$  which is 25m above a lake be  $30^\circ$  and the angle of depression of reflection of the cloud in the lake from  $P$  be  $60^\circ$ , then the height of the cloud (in meters) from the surface of the lake is:

(A) 60 (B) 45 (C) 42 (D) 50

Sol. D



$$\tan 30^\circ = \frac{x}{y} \Rightarrow y = \sqrt{3} x$$

$$\tan 60^\circ = \frac{25 + x + 25}{y}$$

$$\Rightarrow \sqrt{3} y = x + 50$$

$$\Rightarrow x = 25$$

Height of cloud from surface = 50 m

8. If the sum of the first 15 terms of the series  $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \dots$  is equal to  $225k$ , then  $k$  is equal to :
- (A) 9 (B) 27  
(C) 108 (D) 54

Sol.  $(3/4)^3 + (3/2)^3 + (9/4)^3 + (3)^3 + \dots$   
 $\Rightarrow (3/4)^3 + (6/2)^3 + (9/4)^3 + (12/4)^3 + \dots$

$$\Rightarrow \frac{3^3 + 6^3 + 9^3 + 12^3 + \dots + 15^3}{4^3}$$

$$\Rightarrow \frac{3^3 [1^3 + 2^3 + 3^3 + \dots + 15k]}{4^3}$$

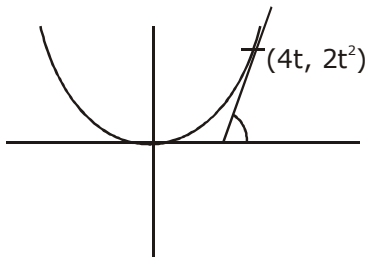
$$\Rightarrow \frac{3^3 \cdot \left(\frac{(15)(16)}{2}\right)^2}{4^3} = 225k$$

$$\Rightarrow \frac{27}{64} \cdot (25)(64) = 225k$$

$$k = 27$$

9. The equation of a tangent to the parabola,  $x^2 = 8y$ , which makes an angle  $\theta$  with the positive direction of x-axis, is:
- (A)  $x = y \cot\theta + 2 \tan\theta$  (B)  $x = y \cot\theta - 2 \tan\theta$   
(C)  $y = x \tan\theta + 2 \cot\theta$  (D)  $y = x \tan\theta - 2 \cot\theta$

Sol. A



$$\frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4}$$

$$\left(\frac{dy}{dx}\right)_{(4t, 2t^2)} = t$$

$$\tan \theta = t$$

eq. of tangent

$$y - 2 \tan^2 \theta = \tan \theta (x - 4 \tan \theta)$$

$$y = x \tan \theta - 2 \tan^2 \theta$$

$$y \cot \theta = x - 2 \tan \theta$$

$$\Rightarrow x = y \cot \theta + 2 \tan \theta$$

- 10.** In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses R.50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loos (in rupees) is :

- (A) 0 (B)  $\frac{400}{3}$  gain (C)  $\frac{400}{9}$  loss (D)  $\frac{400}{3}$  loss

**Sol. A**

$$P(w) = \frac{2}{6} = \frac{1}{3}, P(L) = 2/3$$

$$\text{Expected gain / loss} = (w) (100) + (L)(w) (-50 + 100) + (L^2) w (-50 - 50 + 100) + L^3 (-150)$$

$$= \frac{1}{3} \times 100 + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)(50) + 0 + \left(\frac{2}{3}\right)^3 (-150)$$

$$= \frac{100}{3} + \frac{100}{9} - \frac{(150)(8)}{27}$$

$$= \frac{900 + 300 - 1200}{27} = 0$$

- 11.** The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observation, is :

- (A) 5 (B) 1 (C) 3 (D) 7

**Sol. D**

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 4$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 20$$

$$\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = 5.2$$

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - 4^2 = 5.2$$

$$\frac{9 + 16 + 16 + x_4^2 + x_5^2}{5} = 21.2$$

$$\Rightarrow x_4^2 + x_5^2 = 106 - 41$$

$$\Rightarrow x_4^2 + x_5^2 = 65$$

$$\Rightarrow (x_4 + x_5)^2 - 2 \times 4 = 65$$

$$x_4 x_5 = 8$$

$$|x_4 - x_5|^2 = (x_4 + x_5)^2 - 4x_4 x_5 = 81 - 4 \times 8 = 49$$

$$|x_4 - x_5| = 7$$

12. The expression  $\sim(\sim p \rightarrow q)$  is logically equivalent to:

- (A)  $p \wedge q$                       (B)  $\sim p \wedge q$                       (C)  $p \wedge \sim q$                       (D)  $\sim p \wedge \sim q$

Sol. D

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$	$p \wedge q$	$\sim p \wedge \sim q$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge \sim q$
T	T	F	T	F	T	F	F	F	F
T	F	F	T	F	F	F	T	T	F
F	T	T	T	F	F	T	F	F	F
F	F	T	F	T	F	F	T	F	T

$$\sim(\sim p \rightarrow q) = (\sim p) \wedge (\sim q)$$

13. if an angle between the line,  $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$  and the plane,  $x - 2y - kz = 3$  is  $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ , then a value of k is:

- (A)  $-\frac{3}{5}$                       (B)  $\sqrt{\frac{3}{5}}$                       (C)  $\sqrt{\frac{5}{3}}$                       (D)  $-\frac{5}{3}$

Sol. C

$$\text{For line } \vec{b} = (2, 1, -2)$$

$$\text{For plane } \vec{n} = (1, -2, -k)$$

$$\text{angle bet line and plane is } \theta = \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right) \Rightarrow \cos \theta = \frac{2\sqrt{2}}{3}$$

$$\text{angle both normal and line is } (90^\circ - \theta) \quad \sin \theta = \frac{1}{3}$$

$$\cos(90^\circ - \theta) = \frac{2 - 2 + 2k}{(3)\sqrt{5+k^2}} \Rightarrow \sqrt{5+k^2} = 2k$$

$$5+k^2 = 4k^2 \Rightarrow k = \sqrt{\frac{5}{3}}$$

- 14.** The number of integral values of  $m$  for which the quadratic expression,  $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$ ,  $x \in \mathbb{R}$ , is always positive is:  
 (A) 8 (B) 7 (C) 6 (D) 3

**Sol.**  $(1+2m)x^2 - 2(1+3m)x + 4(1+m) > 0$

$$1 + 2m = 0$$

$$\text{at } m = -1/2$$

$$2\left(-\frac{1}{2}\right)x + 4\left(\frac{1}{2}\right) > 0$$

$$-x + 2 > 0$$

$$x - 2 < 0$$

$$x < 2$$

not possible

Now

$$a > 0$$

$$1 + 2m > 0,$$

$$m > -\frac{1}{2},$$

$$D < 0$$

$$4(1+3m)^2 - 4(1+2m) + 4(1+m) < 0$$

$$(9m^2 + 6m + 1) - 4(1+3m+2m^2) < 0$$

$$m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$$

$$\Rightarrow m^2 - 6m - 3 < 0$$

Integral value is =  $\{0, 1, 2, 3, 4, 5, 6\}$

- 15.** If a curve passes through the point  $(1, -2)$  and has slope of the tangent at any point  $(x, y)$  on it as  $\frac{x^2 - 2y}{x}$  then the curve also passes through the point:

(A)  $(3, 0)$

(B)  $(-1, 2)$

(C)  $(\sqrt{3}, 0)$

(D)  $(-\sqrt{2}, 1)$

**Sol.** C

$$\frac{dy}{dx} = \frac{x^2 - 2y}{x}$$

$$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$$

$$\text{I.F} = e^{\int \frac{2}{x} dx} = x^2$$

$$y(x^2) = \int x^3 dx$$

$$y(x^2) = \frac{x^4}{4} + C$$

Pass  $(1, -2)$

$$\Rightarrow -2 = \frac{1}{4} + C \Rightarrow C = -9/4$$

$$\text{curve is } y(x^2) = \frac{x^4}{4} - \frac{9}{4} \Rightarrow 4y x^2 = x^4 - 9$$

passes through  $(\sqrt{3}, 0)$

- 16.** If  ${}^nC_4, {}^nC_5$ , and  ${}^nC_6$  are in A.P, then n can be:  
 (A) 12 (B) 11 (C) 14 (D) 9

**Sol.**  $2({}^nC_5) = {}^nC_4 + {}^nC_6$   
 $\Rightarrow n = 14$

- 17.** If  $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ ; then for all  $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ , det (A) lies in the interval :

- (A)  $\left[\frac{5}{2}, 4\right)$  (B)  $\left(\frac{3}{2}, 3\right]$  (C)  $\left(1, \frac{5}{2}\right]$  (D)  $\left(0, \frac{3}{2}\right]$

**Sol. B**

$$|A| = \begin{vmatrix} 2 & \sin\theta & 1 \\ 0 & 1 & \sin\theta \\ 0 & -\sin\theta & 1 \end{vmatrix}$$

$$|A| = 2(1 + \sin^2\theta)$$

$$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \sin^2\theta \in \left[0, \frac{1}{2}\right)$$

$$\det(A) \text{ Range is } = 2\left[1, \frac{3}{2}\right) = [2, 3)$$

$$\det(A) \text{ range is } = 1\left(\frac{3}{2}, 3\right]$$

- 18.** Let  $\vec{a}, \vec{b}$  and  $\vec{c}$  be three unit vectors, out of which vectors  $\vec{b}$  and  $\vec{c}$  are non-parallel. If  $\alpha$  and  $\beta$  are the angles which vector  $\vec{a}$  makes with vectors  $\vec{b}$  and  $\vec{c}$  respectively and  $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$ , then  $|\alpha - \beta|$  is equal to :

- (A)  $60^\circ$  (B)  $30^\circ$  (C)  $90^\circ$  (D)  $45^\circ$

**Sol. B**

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{2}\vec{b}$$

$$\vec{a} \cdot \vec{c} = \frac{1}{2} \quad \vec{a} \cdot \vec{b} = 0$$

$$\cos\beta = \frac{1}{2} \quad \alpha = 90^\circ$$

$$\beta = 60^\circ$$

$$|\alpha - \beta| = 30^\circ$$

19. The integral  $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$  is equal to: (where C is a constant of integration)

(A)  $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$

(B)  $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$

(C)  $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$

(D)  $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$

Sol. B

$$\int \frac{3x^{13} + 2x^{11}}{(2 + 3/x^2 + 1/x^4)^4 x^{16}} dx$$

$$\int \frac{3/x^3 + 2/x^5}{(2 + 3/x^2 + 1/x^4)^4} dx$$

Let  $2 + 3/x^2 + 1/x^4 = t$

$$\Rightarrow (3/x^3 + 2/x^5) dx = \left(\frac{dt}{-2}\right)$$

$$\therefore \frac{-1}{2} \int \frac{dt}{t^4} \Rightarrow \frac{-1}{2} \left[ \frac{-1}{3t^3} \right] + c \Rightarrow \frac{1}{6(2 + 3/x^2 + 1/x^4)} + c$$

$$\Rightarrow \frac{x^{12}}{6[2x^4 + 3x^2 + 1]^3} + c$$

20. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the students selected has opted neither for NCC nor for NSS is:

(A)  $\frac{5}{6}$

(B)  $\frac{2}{3}$

(C)  $\frac{1}{6}$

(D)  $\frac{1}{3}$

Sol. C

$$P(\text{NCC}) = \frac{40}{60} = \frac{2}{3}$$

$$P(\text{NSS}) = \frac{30}{60} = \frac{1}{2}$$

$$P(\text{NCC} \cap \text{NSS}) = \frac{1}{3}$$

$$P(\overline{\text{NCC}} \cap \overline{\text{NSS}}) = P(\overline{\text{NCC} \cup \text{NSS}})$$

$$= 1 - [P(\text{NCC} \cup \text{NSS})]$$

$$= 1 - [P(\text{NCC}) + P(\text{NSS})] - P(\text{NCC} \cap \text{NSS})]$$

$$= 1 - \left(\frac{2}{3} + \frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{6 - 4 - 3 + 2}{6}$$

$$= \frac{1}{6}$$

21. The tangent to the curve  $y = x^2 - 5x + 5$ , parallel to the line  $2y = 4x + 1$ , also, passes through the point:

- (A)  $\left(-\frac{1}{8}, 7\right)$       (B)  $\left(\frac{1}{8}, -7\right)$       (C)  $\left(\frac{1}{4}, \frac{7}{2}\right)$       (D)  $\left(\frac{7}{2}, \frac{1}{4}\right)$

Sol.  $\frac{dy}{dx} = 2x - 5 = 2$ (given)

$$\Rightarrow x = \frac{7}{2}$$

$$\text{at } x = \frac{7}{2} \Rightarrow y = -\frac{1}{4}$$

equation tangent

$$y + \frac{1}{4} = 2 \left(x - \frac{7}{2}\right)$$

$$\Rightarrow 2x - y - 7 - \frac{1}{4} = 0$$

$$\Rightarrow 2x - y - \frac{29}{4} = 0$$

passes true  $\left(\frac{1}{8}, -7\right)$

22. If  $\sin^4 \alpha + 4 \cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$ ;  $\alpha, \beta \in [0, \pi]$ , then  $\cos(\alpha + \beta) - \cos(\alpha - \beta)$  is equal to:

- (A)  $-1$       (B)  $0$       (C)  $\sqrt{2}$       (D)  $-\sqrt{2}$

Sol. A.M  $\geq$  G.M

$$\frac{\sin^4 \alpha + 4 \cos^4 \beta + 1 + 1}{4} \geq \sqrt[4]{(\sin^4 \alpha)(4 \cos^4 \beta)(1)(1)}$$

$$\Rightarrow \sin^4 \alpha + 4 \cos^4 \beta + 2 \geq 4\sqrt{2} \sin \alpha \cos \beta$$

$$\text{given } \sin^4 \alpha + 4 \cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$$

$$\therefore \text{A.M} = \text{G.M} \Rightarrow \sin^4 \alpha = 1 = 4 \cos^4 \beta$$

$$\sin \alpha = 1, \cos \beta = \pm \frac{1}{\sqrt{2}}$$

$$\sin \beta = \frac{1}{\sqrt{2}}$$

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = 2 \sin \alpha \sin(-\beta)$$

$$= -2(1) \left(\frac{1}{\sqrt{2}}\right) = -\sqrt{2}$$

23. Let Z be the set of integers. If  $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$  and  $B = \{x \in Z : -3 < 2x - 1 < 9\}$ , then the number of subsets of the set  $A \times B$ , is.

- (A)  $2^{10}$       (B)  $2^{12}$       (C)  $2^{18}$       (D)  $2^{15}$

Sol. D

$$A = \{x \in \mathbb{Z}, \quad 2^{(x+2)(x^2-5x+6)} = 2^0$$

$$(x+2)(x^2-5x+6) = 0$$

$$x = -2, 2, 3$$

$$B = \{x \in \mathbb{Z}, \quad -3 < 2x - 1 < 9$$

$$\Rightarrow -2 < 2x < 10$$

$$\Rightarrow -1 < x < 5$$

$$x = 0, 1, 2, 3, 4$$

$$A \times B = \{(-2, 0), (-2, 1), (-2, 2), (-2, 3), (-2, 4), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)\}$$

$$A \times B \text{ elements} = 15$$

$$\text{total subsets} = 2^{15}$$

- 24.** If a circle of radius  $R$  passes through the origin  $O$  and intersects the coordinate axes at  $A$  and  $B$ , then the locus of the foot of perpendicular from  $O$  on  $AB$  is :

(A)  $(x^2 + y^2)^2 = 4R^2x^2y^2$

(B)  $(x^2 + y^2)^3 = 4R^2x^2y^2$

(C)  $(x^2 + y^2)(x + y) = R^2xy$

(D)  $(x^2 + y^2)^2 = 4Rx^2y^2$

**Sol. B**

$$y - k = \frac{-h}{k}(x - h)$$

$$hx + ky = h^2 + k^2$$

$$AB = 2R$$

$$\Rightarrow \frac{(h^2 + k^2)^2}{h^2} + \frac{h^2 + k^2}{k^2} = 4R^2$$

$$\Rightarrow (h^2 + k^2)^3 = 4h^2k^2R^2$$

$$(x^2 + y^2)^3 = 4x^2y^2R^2$$

- 25.** The set of all values of  $\lambda$  for which the system of linear equations  $x - 2y - 2z = \lambda x$

$$x + 2y + z = \lambda y$$

$$-x - y = \lambda z$$
 has a non-trivial solution:

(A) is a singleton

(B) contains exactly two elements

(C) is an empty set

(D) contains more than two elements

**Sol. A**

$$\begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)[(2-\lambda)(-\lambda)+1] + 2[-\lambda + 1] - 2[-1 + (2-\lambda)] = 0$$

$$\Rightarrow \lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$$

$$\Rightarrow (\lambda-1)^3 = 0 \Rightarrow \lambda = 1$$

- 26.** If the function  $f$  given by  $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$ , for some  $a \in \mathbb{R}$  is increasing in  $(0, 1]$  and

decreasing in  $[1, 5)$ , then a root of the equation,  $\frac{f(x)-14}{(x-1)^2} = 0$  ( $x \neq 1$ ) is :

(A) 7

(B) 5

(C) -7

(D) 6

**Sol.**  $f(x) = 3x^2 - 6|a-2|x + 3a$

$$f(x) \geq x \in (0, 1]$$

$$f(x) \leq 0 \quad x \in [1, 5]$$

$$f(x) = 0 \text{ at } x = 1 \Rightarrow 3 - 6(a-2) + 3a = 0$$

$$\Rightarrow a = 5$$

$$f(x) = x^3 - 9x^2 + 15x + 7$$

$$\frac{f(x) - 14}{(x-1)^2} \Rightarrow \frac{x^3 - 9x^2 + 15x - 7}{(x-1)^2} = 0$$

$$\Rightarrow \frac{(x-7)(x-1)^2}{(x-1)^2} = 0$$

$$\Rightarrow x - 7 = 0$$

$$x = 7$$

27.  $\lim_{n \rightarrow \infty} \left( \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \frac{n}{n^2 + 3^2} + \dots + \frac{1}{5n} \right)$  is equal to:

(A)  $\frac{\pi}{4}$

(B)  $\frac{\pi}{2}$

(C)  $\tan^{-1}(3)$

(D)  $\tan^{-1}(2)$

Sol. D

$$\lim_{n \rightarrow \infty} \left( \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \frac{n}{n^2 + 3^2} + \dots + \frac{n}{n^2 + (2n)^2} \right)$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^{2n} \frac{n}{n^2 + r^2}$$

$$\lim_{n \rightarrow \infty} \sum_{r=1}^{2n} \frac{1}{n \left[ 1 + \left( \frac{r}{n} \right)^2 \right]}$$

$$\Rightarrow \int_0^2 \frac{dx}{1+x^2}$$

$$= \left( \tan^{-1} x \right)_0^2$$

$$= \tan^{-1} 2$$

28. Let S and S' be foci of an ellipse and B be any one of the extremities of its minor axis. If  $\Delta S'BS$  is a right angled triangle with right angle at B and area  $(\Delta S'BS) = 8$  sq. units, then the length of a latus rectum of the ellipse is :

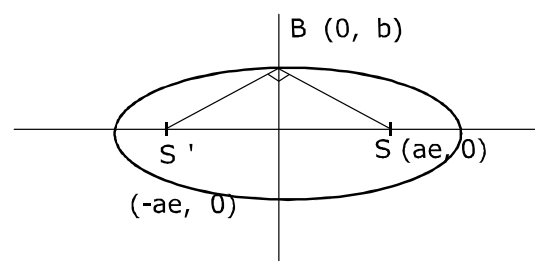
(A)  $4\sqrt{2}$

(B)  $2\sqrt{2}$

(C) 2

(D) 4

Sol. D



$$\text{area of } \Delta = \frac{1}{2} \sqrt{a^2 e^2 + b^2} \sqrt{a^2 e^2 + b^2}$$

$$\Rightarrow \frac{1}{2}(a^2e^2 + b^2) = 8$$

$$\Rightarrow a^2 e^2 + a^2 - a^2 e^2 = 16$$

$$\Rightarrow a^2 = 16$$

$$m_{Bs} \cdot m_{Bs'} = -1 \Rightarrow \left(\frac{-b}{ae}\right)\left(\frac{b}{ae}\right) = -1$$

$$\Rightarrow b^2 = a^2e^2$$

$$a^2 e^2 = 8 \Rightarrow e^2 = \frac{1}{2}$$

$$b^2 = a^2 e^2 = (16) \left(\frac{1}{2}\right) = 8$$

$$\text{Centre of L.L.R} = \frac{2b^2}{a}$$

$$= \frac{2(8)}{4}$$

$$= 4$$

29. Let  $z_1$  and  $z_2$  be two complex numbers satisfying  $|z_1| = 9$  and  $|z_2 - 3 - 4i| = 4$ . Then the minimum value of  $|z_1 - z_2|$  is :

- (A)  $\sqrt{2}$  (B) 2  
(C) 1 (D) 0

Sol. **D**

$|z_1| = 9$  Circle with centre = (0, 0), radius = 9  
 $|z_2 - 3 - 4i| = 4$  Circle with centre = (3, 4) radius = 4  
 $c_1c_2 = 5$   
 $|r_1 - r_2| = 5$   
 Circle touches Internally  
 min<sup>m</sup> value of  $|z_1 - z_2|$  is = 0

30. Let S be the set of all real values of  $\lambda$  such that a plane passing through the points  $(-\lambda^2, 1, 1)$ ,  $(1, -\lambda^2, 1)$  and  $(1, 1, -\lambda^2)$  also passes through the point  $(-1, -1, 1)$ . Then S is equal to:

- (A)  $\{\sqrt{3}, -\sqrt{3}\}$  (B)  $\{\sqrt{3}\}$   
(C)  $\{3, -3\}$  (D)  $\{1, -1\}$

Sol. **A**

$$\bar{n} = \overline{AB} \times \overline{AC} \qquad \overline{AB} = (1 + \lambda^2, -\lambda^2 - 1, 0)$$

$$\qquad \qquad \qquad \overline{AC} = (1 + \lambda^2, 0, -\lambda^2 - 1)$$

$$\bar{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 + \lambda^2 & -\lambda^2 - 1 & 0 \\ 1 + \lambda^2 & 0 & -\lambda^2 - 1 \end{vmatrix}$$

$$= \hat{i}(\lambda^2 + 1)^2 + \hat{j}(\lambda^2 + 1)^2 + \hat{k}(\lambda^2 + 1)^2$$

$$\bar{n} = (\lambda^2 + 1)^2 [\hat{i} + \hat{j} + \hat{k}]$$

equation of plane

$$\vec{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$$

$$\Rightarrow (\lambda^2 + 1)(x + y + z) = -(\lambda^2 + 1)^2 [-\lambda^2 + 1 + 1]$$

$$\Rightarrow x + y + z = -\lambda^2 + 2$$

pass through  $(-1, -1, 1)$

$$\Rightarrow -1 = -\lambda^2 + 2 \Rightarrow |\lambda = \pm\sqrt{3}$$

M-II All 4 pts. are coplaner

$$[\overline{AB} \ \overline{AC} \ \overline{AD}]$$

$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & 1-\lambda^2 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$$\Rightarrow \lambda = \pm\sqrt{3}$$

