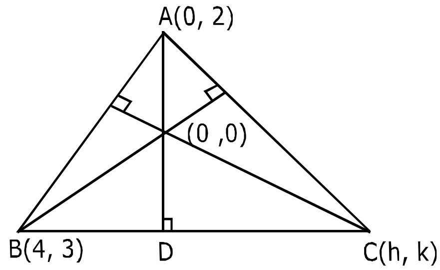
MATHEMATICS
JEE-MAIN (January-Attempt) 10 January (Shift-2) Paper

SECTION - A

1. Two vertices of a triangle are (0, 2) and (4, 3) If its orthocentre is at the origin, then its third vertex lies in which quadrant?

(A) fourth (B) third (C) first (D) second

Sol. **D**



Equation of line BC is y = 3

$\Rightarrow$ k = 3

Eqaution of line AC is $y - 2 = -\frac{4}{3}(x-0)$

$\Rightarrow$ 3y + 4x = 6

Passes through (h, 3)

$9 + 4h = 6 \Rightarrow h = -\frac{3}{4}$

orthocentre is = $(-3/4, 3)$ lie in second quadrent

2. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(A) $\frac{19}{21}$ (B) $\frac{23}{22}$ (C) $\frac{22}{23}$ (D) $\frac{21}{19}$

Sol. **D**

$$\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right) = \sum_{n=1}^{19}\cot^{-1}[1+n(n+1)]$$

$$= \sum_{n=1}^{19}\tan^{-1}\left[\frac{1}{1+n(n+1)}\right]$$

$$= \sum_{n=1}^{19}\tan^{-1}\left[\frac{(n+1)-n}{1+n(n+1)}\right]$$

$$= \sum_{n=1}^{19}\tan^{-1}(n+1) - \tan^{-1}n$$

$$= \left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+\ldots\ldots\ldots+\left(\tan^{-1}20-\tan^{-1}19\right)$$

$$= \left(\tan^{-1}20-\frac{\pi}{4}\right)$$

Now,

$$\cot[\tan^{-1}20 - \pi/4] = \frac{1}{\tan[\tan^{-1}20-\pi/4]} = \frac{1+(20)(1)}{20-1} = \frac{21}{19}$$

3. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1, e) also passes through the point:

(A) $\left(\frac{5}{3}, 2e\right)$ (B) (3, 6e) (C) (2, 3e) (D) $\left(\frac{4}{3}, 2e\right)$

Sol. **D**

$$\frac{dy}{dx} = e^{x^2}1 + xe^{x^2}.2x$$

$$= e^{x^2}[1+2x^2]$$

$\left(\frac{dy}{dx}\right)_{(1,e)} = 3e$

Equation of tagent y – e = 3e (x – 1) which passes though $\left(\frac{4}{3}, 2e\right)$

4. Consider the following three statements:
P : 5 is prime number.
Q : 7 is a factor of 192.
R : L.C.M. of 5 and 7 is 35.
Then the truth value of which one of the following statements is true?
(A) $(\sim P) \wedge (\sim Q \wedge R)$ (B) $(\sim P) \vee (Q \wedge R)$ (C) $(P \wedge Q) \vee (\sim R)$ (D) $P \vee (\sim Q \wedge R)$

Sol. **D**

P	Q	~Q	R	$\sim Q \wedge R$	$P \vee (\sim Q \wedge R)$
T	F	T	T	T	T

5. If $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K\left({}^{50}C_{25}\right)$ then K is equal to:
(A) 2^{24} (B) 2^{25} (C) 2^{25-1} (D) $(25)^2$

Sol. **B**

$\sum_{r=0}^{25} {}^{50}C_r \cdot {}^{50-r}C_{25-r} = k\,{}^{50}C_{25}$

$\sum_{r=0}^{25} \frac{\lfloor 50}{\lfloor r \lfloor 50-r} \cdot \frac{\lfloor 50-r}{\lfloor 25-r \lfloor 25} = k\,{}^{50}C_{25}$

$\sum_{r=0}^{25} \frac{\lfloor 50 \lfloor 25}{\lfloor r \lfloor 25-r} \cdot \frac{}{(\lfloor 25)(\lfloor 25)} = k.{}^{50}C_{25}$

${}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = k\,{}^{50}C_{25} \Rightarrow {}^{25}C_0 + {}^{25}C_1 + \ldots\ldots + {}^{25}C_{25} = k$

$\Rightarrow K = 2^{25}$

6. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$, where [t] denotes the greatest integer less than or equal to t, is:

(A) $\frac{3}{20}(4\pi - 3)$ (B) $\frac{1}{12}(7\pi + 5)$ (C) $\frac{3}{10}(4\pi - 3)$ (D) $\frac{1}{12}(7\pi - 5)$

Sol. **A**

$\int_{-\frac{\pi}{2}}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^{0} \frac{dx}{-1-1+4} + \int_{0}^{1} \frac{dx}{4} + \int_{1}^{\pi/4} \frac{dx}{1+0+4}$

$\Rightarrow (x)_{\pi/2}^{-1} + \frac{1}{2}(x)_{-1}^{0} + \frac{1}{4}(x)_{0}^{1} + \frac{1}{5}(x)_{1}^{\pi/2}$

$\Rightarrow -1 + \frac{\pi}{2} + \frac{1}{2}(1) + \frac{1}{4}(1) + \frac{1}{5}\left(\frac{\pi}{2} - 1\right)$

$\Rightarrow \frac{-1}{1} + \frac{\pi}{2} + \frac{1}{2} + \frac{1}{4} + \frac{\pi}{10} - \frac{1}{5}$

$$\Rightarrow \frac{-20+10\pi+10+5+2\pi-4}{20}$$

$$\Rightarrow \frac{12\pi-9}{20}$$

7. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\ldots\ldots\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}\cdot$ is :

(A) $\frac{1}{256}$ (B) $\frac{1}{2}$ (C) $\frac{1}{1024}$ (D) $\frac{1}{512}$

Sol. D

Let $\frac{\pi}{2^{10}} = \theta$

$\frac{\pi}{2^9} = 2\theta$

$(\cos\theta \cos 2\theta - \cos 2^8 \theta)\sin(\pi/2^{10})$

$$\frac{\sin\left(2^9\frac{\pi}{2^{10}}\right)}{2^9\sin\left(\frac{\pi}{2^{10}}\right)}.\sin\left(\frac{\pi}{2^{10}}\right)$$

$$\Rightarrow \frac{\sin\left(\frac{\pi}{2}\right)}{2^9} \Rightarrow \frac{1}{2^9}$$

8. Let N be the set of natural numbers and two functions f and g be defined as f, g: N $\to$ N such that

$f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ and $g(n) = n - (-1)^n$. Then fog is:

(A) both one-one and onto (B) neither one-one nor onto

(C) one-one but not onto. (D) onto but not one-one

Sol. D

$g(n) = \begin{cases} n+1, & \text{n odd} \\ n-1, & \text{n even} \end{cases}$

n=1 f(g(1)) = f(2) = 1

n=2 f(g(2)) = f(1) = 1 → Many one

n=3 f(g(3)) = f(4) = 2

n=4 f(g(4)) = f(3) = 2

This will give all values of the n

function is many one, onto functions

9. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$ then f'(1\2) is :

(A) $\frac{24}{25}$ (B) $\frac{4}{5}$ (C) $\frac{6}{25}$ (D) $\frac{18}{25}$

Sol. **A**

Diff. both sides

$f(x) = 2x + [0 - x^2 f(x)]$

$\Rightarrow$ $f(x) = \frac{2x}{1+x^2}$

$f'(x) = \frac{(1+x^2)2 - (2x)(2x)}{(1+x^2)^2}$

put $x = \frac{1}{2}$

$f'\left(\frac{1}{2}\right) = \frac{2\left(\frac{5}{4}\right) - (4)\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2}$

$= \frac{\left(\frac{5}{2} - 1\right)}{\left(\frac{5}{4}\right)^2}$

$= \frac{24}{25}$

10. the positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x} + \frac{\lambda}{x^2}\right)^{10}$ is, 720, is:

(A) $\sqrt{5}$ (B) $2\sqrt{2}$ (C) 4 (D) 3

Sol. **C**

$x^2 \ [\sqrt{x} + \lambda / x^2]^{10}$

$x^2 \ {}^{10}C_r(\sqrt{x})^{10-r}\left(\lambda / x^2\right)^r$

${}^{10}C_r(x)^{(5-r/2)}\lambda^r \ x^{2-2r}$

${}^{10}C_r\lambda^r \ x^{(7-5r/2)}$ for coff. of x^2 = $7 - \frac{5r}{2} = 2 \Rightarrow r = 2$

$\Rightarrow {}^{10}C_2\lambda^2 = 720$

$\Rightarrow 45\lambda^2 = 720 \Rightarrow \lambda^2 = 16 \Rightarrow \lambda = 4$

11. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations x + 3y + 7z = 0, −x + 4y + 7z = 0, $(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$ has a non-trivial solution, is:

(A) four (B) three (C) two (D) one

Sol. **C**

$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2$

$$(7)\begin{vmatrix} 0 & 1 & 2 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$(7)\ [(2+7\sin 3\theta)+2(-\cos 2\theta - 4\sin 3\theta)] = 0$

$\Rightarrow$ $2 + 7\sin 3\theta - 2\cos 2\theta - 8\sin 3\theta = 0$

$\Rightarrow$ $2(1-\cos 2\theta) = \sin 3\theta$

$\Rightarrow$ $2 \cdot 2\sin^2\theta = 3\sin\theta - 4\sin^3\theta$

$\Rightarrow$ $\sin\theta\,[3-4\sin^2\theta - 4\sin\theta] = 0$

$\sin\theta \neq 0\ (\theta \in (0, \pi))$ $\Rightarrow 4\sin^2\theta + 4\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta + 3)(2\sin\theta - 1)$

$\Rightarrow \sin\theta = -\frac{3}{2}, \frac{1}{2}$

$\sin\theta = -\frac{3}{2}$ is not possible

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Number of values of θ is 2.

12. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z, then :

(A) $R(z) = -3$
(B) $R(z) < 0$ and $I(z) > 0$
(C) $I(z) = 0$
(D) $R(z) > 0$ and $I(z) > 0$

Sol. **C**

$$Z = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5 + \left[\cos\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{\pi}{6}\right)\right]^5$$

$$= \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} + \cos\left(\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)$$

$$= \cos\frac{5\pi}{6} + i\sin\frac{\pi}{6} + \cos\frac{5\pi}{6} + i\sin\left(-\frac{5\pi}{6}\right)$$

$\text{Re}(Z) = -2\frac{\sqrt{3}}{2}$ $\quad I_m(z) = 0$

13. The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)dx + 2xy\,dy = 0$ which passes through (1, 1), is:

(A) a circle with centre on the x-axis
(B) a circle with centre on the y-axis
(C) a hyperbola with transverse axis along the x-axis
(D) an ellipse with major axis along the y-axis

Sol. **A**

M-I $(x^2 - y^2)\,dx = -2xy\,dy$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

$\frac{dy}{dx} = \frac{(y/x)^2 - 1}{2(y/x)}$

put $y/x = v$

$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$

$\Rightarrow \int \frac{-2v}{v^2 + 1} dv = \int \frac{dx}{x}$

$\Rightarrow -\ell n\,(v^2 + 1) = \ell nx + c$

$\Rightarrow -\ell n(y^2/x^2 + 1) = \ell nx + c$ —(1)

passes through (1, 1)

$-\ell n2 = c$

From (1)

$\ell n(2) - \ell n\left(\frac{y^2}{x^2} + 1\right) = \ln x$

$\ell n\left(\frac{2}{\frac{y^2}{x^2} + 1}\right) = \ln x$

$\frac{2x^2}{y^2 + x^2} = x \quad \Rightarrow x^2 + y^2 = 2x$

circle which centre = (1, 0)

M-II

$x^2\,dx = y^2\,dx - 2xy\,dy$

$\Rightarrow -dx = d(y^2/x)$

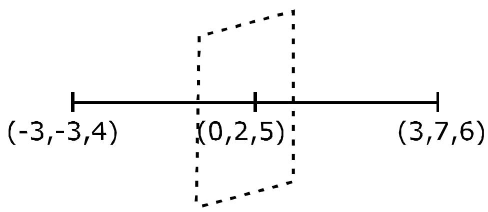
$\Rightarrow -x = y^2/x + c$

passes though (1, 1) $\Rightarrow c = -2$

14. The plane which bisects the line segment joining the points (–3, –3, 4) and (3, 7, 6) at right angles, passes through which one of the following points?

(A) (4, –1, 7) (B) (–2, 3, 5) (C) (2, 1, 3) (D) (4, 1, – 2)

Sol. **D**



Direction ratios of Normal = (6, 10, 2)

equation of plane $\Rightarrow \bar{r} \cdot (6, 10, 2) = (0, 2, 5)\,(6, 10, 2)$

$\Rightarrow 6x + 10y + 2z = 20 + 10$

$\Rightarrow 3x + 5y + z = 15$

Which satisfy by point (4, 1, –2)

15. Let $a_1, a_2, a_3 \ldots.., a_{10}$ be in G.P. with $a_i > 0$ for i= 1,2 .... , 10 and S be the set of pairs (r, k), r, k∈N (the set of natural numbers) for which $\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix}$

Then the number of elements in S, is:

(A) 10 (B) 2 (C) infinitely many (D) 4

Sol. **C**

Let common ratios is R ⇒ $a_3 = a_4 R$, $a_3 = a_2 R$ Apply $C_3 \to C_3 - C_2, C_2 \to C_2 - C_1$

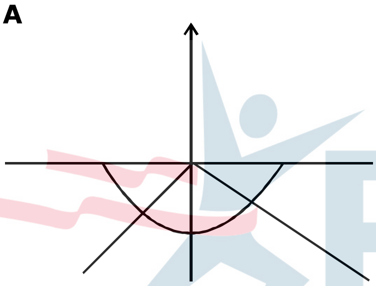
$$\Delta = \begin{vmatrix} \ell n\, a_1^r a_2^k & \ell n (R)^{r+k} & \ell n\, R^{(r+k)} \\ \ell n\, a_4^r a_5^k & \ell n (R)^{r+k} & \ell n\, R^{(r+k)} \\ \ell n\, a_7^r a_8^k & \ell n (R)^{r+k} & \ell n\, R^{(r+k)} \end{vmatrix}$$

$\Delta = 0 \to$ Infinite value satisfy this

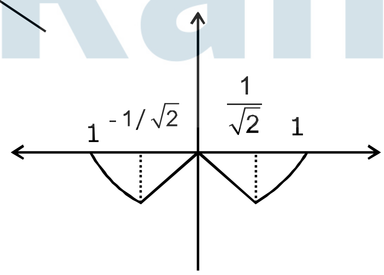
16. Let f: (–1, 1) → R be a function defined by f(x) = max $\left\{-|x|, \sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly:

(A) three elements (B) one element (C) five elements (D) two elements

Sol. **A**



$\max^m \left(-|x|, -\sqrt{1-x^2}\right) =$



not diff at $x = \pm\frac{1}{\sqrt{2}}, 0$

17. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is:

(A) $2\sqrt{11}$ (B) $8\sqrt{2}$ (C) $6\sqrt{3}$ (D) $3\sqrt{2}$

Sol. **C**

For parabola $x^2 = 4ay$

length of chord is = $4\sqrt{a(1+m^2)(am^2+c)}$

From given chord $y = \frac{x}{\sqrt{2}} + 4$ | $x^2 = 4y$

$m = \frac{1}{\sqrt{2}}, C = 4$ | $a = 1$

Centre of chord $= 4\sqrt{(1)\left(1+\frac{1}{2}\right)\left(\frac{1}{2}+4\right)}$

$= 4\sqrt{\frac{3}{2}\left(\frac{9}{2}\right)}$ $= 6\sqrt{3}$

18. let f be a differentiable function such thast $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}, (x > 0)$ and $f(1) \neq 4$. Then $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$:

(A) exists and equals 0 .
(B) exists and equals $\frac{4}{7}$
(C) does not exist.
(D) exists and equals 4.

Sol. **D**

$\frac{dy}{dx} + \frac{3}{4x}y = 7$

$P = \frac{3}{4x}$, $Q = 7$

$I.f = e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ell nx} = x^{3/4}$

$y(x^{3/4}) = \int 7x^{3/4}dx$

$y\, x^{3/4} = 7\frac{x^{7/4}}{7/4} + C$

$\Rightarrow y = 4x + C\, x^{-3/4}$

$\lim_{x\to 0^+} x.f(1/x)$

$\lim_{x\to 0^+}(x)[\frac{4}{x} + cx^{\frac{3}{4}}] = 4$

19. If mean and standard deviation of 5 observation x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2 ......x_5$, and – 50 is equal to:
(A) 582.5
(B) 509.5
(C) 586.5
(D) 507.5

Sol. **D**

$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 10 \Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 50$ —(1)

$\frac{\sum x_1^2}{5} - (\bar{x})^2 = 9 \Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 545$

$\bar{x}_{new} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 - 50}{6} = 0$

Variance Now $\frac{\sum_{i=1}^{6} x_1^2}{6} - (\bar{x}_{new})^2$

$\Rightarrow \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + 2500}{6}$

$= 507.5$

20. If $\int x^5 e^{-4x^3}dx = \frac{1}{48}e^{-4x^3}$ f(x) + C where C is a constant of integration, then f(x) is equal to:

(A) $-2x^3 + 1$ (B) $-2x^3 - 1$ (C) $4x^3 + 1$ (D) $-4x^3 - 1$

Sol. **D**

$\int x^2 \cdot x^3 e^{-4x^3} dx$ $4x^3 = t$

$x^2\, dx = \frac{1}{12} dt$

$\frac{1}{12}\int\left(\frac{t}{4}\right)e^{-t}dt$

$\frac{1}{48}\int t\, e^{-t}dt \Rightarrow \frac{1}{48}[t\left(-e^{-t}\right) - \int(1)\left(-e^{-t}\right)dt]$

$\Rightarrow \frac{-te^{-t}}{48} - \frac{e^{-t}}{48} + c$ replace t

$\Rightarrow \frac{e^{-4x^3}}{48}\left[-\left(4x^3+1\right)\right] + C$

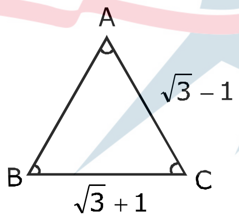
$\Rightarrow \frac{\left(-4x^3\right)e^{-4x^3} - e^{-4x^3}}{48} + c$

21. With the usual notation, in ΔABC, if $\angle A + \angle B = 120^o$, a = $\sqrt{3}+1$ and b = $\sqrt{3}$ – 1, then the ratio $\angle A : \angle B$, is :

(A) 3 : 1 (B) 9 : 7 (C) 5 : 3 (D) 7 : 1

Sol. **D**

A + B = 120º



$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$

$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2\left(\sqrt{3}\right)}\cot(30^o) = \frac{1}{\sqrt{3}}.\sqrt{3} = 1$

$\frac{A-B}{2} = 45^o$

$A - B = 90^o$
$A + B = 120^o$

$2A = 210^o$

$A = 105^o$
$B = 15^o$

22. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target atleast once is greater than $\frac{5}{6}$, is:

(A) 3 (B) 6 (C) 5 (D) 4

Sol. **C**

$p(x) = \frac{1}{3}, p(\bar{x}) = \frac{2}{3}$

at least are hit = 1–(no hit)

$\Rightarrow 1 - \left(\frac{2}{3}\right)^n$

$1 - \left(\frac{2}{3}\right)^n > 5/6 \qquad \Rightarrow \frac{1}{6} > \left(\frac{2}{3}\right)^n$

min value of n is 5

23. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, x + y + z = 2?

(A) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$ (B) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

(C) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$ (D) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

Sol. **D**

Let the point one the line is $(2\lambda + 4, 2\lambda + 5, \lambda + 3)$ lie on plane

$(2\lambda+4) + (2\lambda + 5) + (\lambda + 3) - 2 = 0$

$5\lambda + 10 = 0 \Rightarrow \ = -2$

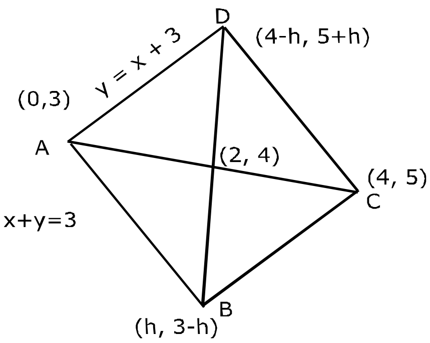
$\Rightarrow$ point of intersection (0, 1, 1)

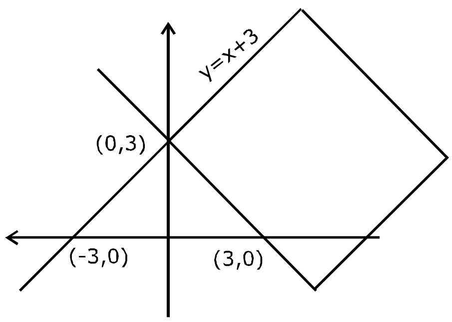
Which lie on line D

24. Two sides of a parallelogram are along the lines, x + y = 3 and x – y + 3 = 0 . If its diagonals intersect at (2, 4) then one of its vertex is:

(A) (2, 1) (B) (3, 6) (C) (2, 6) (D) (3, 5)

Sol. **B**





(4-h, 5+h) lie on line y = x + 3

$\Rightarrow 5 + h = 4 - h + 3 \Rightarrow 2h = 2$

$h = 1$

vertex B is = (1, 2)

vertex D is = (3, 6)

25. Let S = $\left\{(x, y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$ where $r \neq \pm 1$. Then S represents:

(A) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$ when r > 1.

(B) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$ \

(C) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when r > 1.

(D) a hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when 0 < r < 1.

Sol. **C**

$r \in (0, 1)$ than $\frac{x^2}{1-r} - \frac{y^2}{1+r} = -1$

$e = \sqrt{1 + \frac{1-r}{1+r}} = \sqrt{\frac{2}{1+r}} \rightarrow$ hyperbola

r > 1 $\qquad \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1 \rightarrow$ represent ellipse

$a^2 = b^2 (1-e^2) \Rightarrow r - 1 = (r + 1)(1 - e^2)$

$\Rightarrow e^2 = -1\left(\frac{r-1}{r+1}\right)$

$e = \sqrt{\frac{2}{1+r}}$

26. Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\,\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is:

(A) 3 (B) 4 (C) – 4 (D) – 3

Sol. **C**

$\vec{\alpha}$ and $\vec{\beta}$ are collinear

$\frac{\lambda - 2}{4\lambda - 1} = \frac{1}{3}$

$\Rightarrow$ = –4

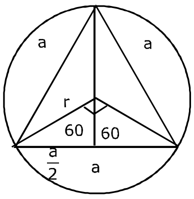
27. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2$ 10x + 12y + c = 0 is $27\sqrt{3}$ sq. units then c is equal to:

(A) 13 (B) – 25 (C) 25 (D) 20

Sol. **C**

Let side of equilatrial Δ = a

$\sin 60^o = \frac{a/2}{r}$



$= \Rightarrow \frac{\sqrt{3}}{2} = \frac{a}{2r} \Rightarrow a = \sqrt{3}r$

r = $\sqrt{25+36-C} = \sqrt{61-C}$

area $\Rightarrow \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

$\frac{\sqrt{3}}{4}(3)(61-C) = 27\sqrt{3}$

$\Rightarrow$ 61 – C = 36

$\Rightarrow$ C = 25

28. Let A = $\begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where b > 0. Then the minimum value of $\frac{\det(A)}{b}$ is

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $-2\sqrt{3}$ (D) $2\sqrt{3}$

Sol. **D**

A = $\begin{vmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{vmatrix}$

det (A) = b^2 +3

$\min^m\left(\frac{\det A}{b}\right) = \min^m(b+3/b)$

$\frac{b+3/b}{2} \geq \sqrt{b.(3/b)}$

b + 3/b $\geq 2\sqrt{3}$

$\min^m$ value = $2\sqrt{3}$

29. A helicopter is flying along the curve given by $y - x^{3/2} = 7, (x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

(A) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{5}}{6}$ (D) $\frac{1}{3}\sqrt{\frac{7}{3}}$

Sol. **A**

Let point p on curve is = $(t, 7+t^{3/2})$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$$

$$\left(\frac{dy}{dx}\right)\left(t_1 7 + t^{3/2}\right) = \frac{3}{2}t^{1/2}$$

slope of normal at P is = $-\frac{2}{3}t^{-1/2}$

slope of PQ is = $\frac{-t^{3/2}}{\frac{1}{2}-t}$

$$\therefore \frac{-2}{3}t^{-1/2} = \frac{-t^{3/2}}{\frac{1}{2}-t} \Rightarrow \frac{2}{3\sqrt{t}} = \frac{t\sqrt{t}}{\frac{1}{2}-t}$$

$\Rightarrow 3t^2 + 2t - 1 = 0$

$\Rightarrow t = 1/3$

Point P = $\left[\frac{1}{3}, 7 + \left(\frac{1}{3}\right)^{3/2}\right]$

distane = $\frac{1}{6}\sqrt{\frac{7}{3}}$

30. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is :

(A) 1 (B) 2 (C) $\frac{15}{8}$ (D) $\frac{4}{9}$

Sol. **B**

$S = \alpha^2 + \beta^2$

$S = (\alpha + \beta)^2 - 2\alpha\beta$

$S = (3-\lambda)^2 - 2(2 - \lambda)$

$S = \lambda^2 - 6\lambda + 9 - 4 + 2\lambda$

$S = \lambda^2 - 4\lambda + 5$

$S = (\lambda - 2)^2 + 1$

Minimum value occur when $\lambda = 2$