MATH
JEE-MAIN (April-Attempt) 8 April (Shift-1) Paper

SECTION - A

1. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6, (x>1)$ is equal to :

(1) 32 (2) 24 (3) 29 (4)26

Sol. **2**

$y=\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$

$y = 2[{}^6C_0x^6 + {}^6C_2x^4(x^3-1) + {}^6C_4x^2(x^3-1)^2 + {}^6C_6(x^3-1)^3]$

sum of coff. of all even powers in y

$= 2[{}^6C_0-{}^6C_2+{}^6C_4+{}^6C_4-{}^6C_6-3\,{}^6C_6]$

$= 2[1-15+15+15-1-3]$

$= 2[12]$

$= 24$

2. If $\alpha=\cos^{-1}\left(\frac{3}{5}\right), \beta=\tan^{-1}\left(\frac{1}{3}\right)$, where $0<\alpha,\beta<\frac{\pi}{2}$, then $\alpha-\beta$ is equal to :

(1) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\tan^{-1}\left(\frac{9}{14}\right)$ (3) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Sol. **1**

$\alpha=\cos^{-1}\left(\frac{3}{5}\right)$, $\beta=\tan^{-1}\left(\frac{1}{3}\right)$

$\cos(\alpha-\beta)=\frac{3}{5}\frac{3}{\sqrt{10}}+\frac{4}{5}\frac{1}{\sqrt{10}}$

$=\frac{9}{5\sqrt{10}}+\frac{4}{5\sqrt{10}}$

$=\frac{9+4}{5\sqrt{10}}$

$(\alpha-\beta)=\cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$

$(\alpha-\beta)=\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

3. The shortest distance between the line y = x and the curve y^2 = x-2 is :

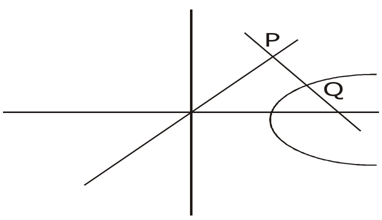
(1) $\frac{11}{4\sqrt{2}}$ (2) 2 (3) $\frac{7}{8}$ (4) $\frac{7}{4\sqrt{2}}$

Sol. **4**

Let Q: $(t^2 + 2, t)$

for SD $\Rightarrow$ slope of tangent at Q $=1$

$\frac{1}{2t}=1$



$\left(t = \frac{1}{2}\right)$

$Q : \left(\frac{9}{4}, \frac{1}{2}\right)$

Shortest distance $PQ = \left|\frac{\frac{1}{2} - \frac{9}{4}}{\sqrt{2}}\right|$

$PQ = \frac{7}{4\sqrt{2}}$

4. If α and β be the roots of the equation $x^2-2x+2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is:

(1) 5 (2) 2 (3) 4 (4) 3

Sol. 3

$x^2-2x+1+1 = 0$

$(x-1)^2 = -1$

$x = 1 \pm i$

$\Rightarrow \alpha = 1 + i$; $\Rightarrow \beta = 1 - i$

Now $\left(\frac{\alpha}{\beta}\right)^n = 1$

$\left(\frac{1+i}{1-i}\right)^n = 1$

$\left(\frac{(1+i)^2}{2}\right)^n = 1$

$\left(\frac{2i}{2}\right)^n = 1$

$(i)^n = 1$

n = 4

5. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

(1) P(A|B) = P (B)-P(A)

(2) P(A|B) $\geq$ P(A)

(3) P(A|B) $\leq$ P(A)

(4) P(A|B) =1

Sol. **2**

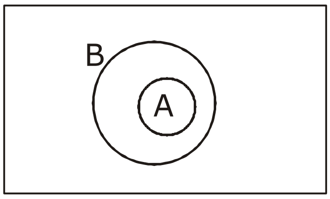
$A \subset B$

$\Rightarrow P(A \cap B) = P(A)$

$P(A\,|\,B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{P(A)}{P(B)} \geq P(A)$

$\Rightarrow P(A\,|\,B) \geq P(A)$



6. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4,10,12,14, then the product of the remaining two observations is:

(1) 45 (2) 40 (3) 48 (4) 49

Sol. **3**

$\dfrac{\sum x^2}{N} - \mu^2$ = variance & $S.D = \sqrt{\text{variance}}$

N = 7 ; variance = 16 , μ=8

$\therefore \dfrac{2^2+4^2+10^2+12^2+14^2+a^2+b^2}{7} - 64 = 16$

$\Rightarrow a^2+b^2 = 560-460$

$\Rightarrow a^2+b^2 = 100$

$\because$ Mean = 8

$\therefore \dfrac{2+4+10+12+14+a+b}{7} = 8$

$\Rightarrow$ a+b = 56-42

$\Rightarrow$ a + b = 14

$\Rightarrow$ ab = 48

7. If $f(x) = \log_e\left(\dfrac{1-x}{1+x}\right), |x| < 1$, then $f\left(\dfrac{2x}{1+x^2}\right)$ is equal to :

(1) $(f(x))^2$ (2) 2f(x) (3) -2f(x) (4) $2f(x^2)$

Sol. **2**

$f(x) = \ln\left(\dfrac{1-x}{1+x}\right)$

$f\left(\dfrac{2x}{1+x^2}\right) = \ln\left(\dfrac{1-\dfrac{2x}{1+x^2}}{1+\dfrac{2x}{1+x^2}}\right)$

$= \ln\left(\left(\dfrac{1-x}{1+x}\right)^2\right)$

$= 2\ln\left(\dfrac{1-x}{1+x}\right)$

= 2f(x)

8. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4+12x^3-36x^2+25, x \in R$, then :

(1) $S_1 = \{-2,1\}; S_2=\{0\}$
(2) $S_1 = \{-2\}; S_2=\{0,1\}$
(3) $S_1 = \{-2,0\}; S_2=\{1\}$
(4) $S_1 = \{-1\}; S_2=\{0,2\}$

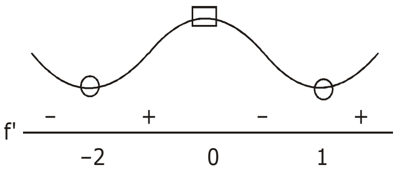
Sol. **1**

$f(X) = 9x^4 + 12x^3 - 36x^2 +25$

$f'(x) = 36x^3 + 36x^2-72x$

$= 36x(x^2+x-2)$

$= 36x(x+2)(x-1)$



local max. : $x \in \{0\} = S_2$

Local min. : $x \in \{-2,1\} = S_1$

9. Let y = y(x) be the solution of the differential equation, $(x^2+1)^2 \frac{dy}{dx} + 2x(x^2+1)y = 1$ such that y(0) = 0. If $\sqrt{a}y(1) = \frac{\pi}{32}$, then the value of 'a' is :

(1) 1
(2) $\frac{1}{4}$
(3) $\frac{1}{16}$
(4) $\frac{1}{2}$

Sol. **3**

$(x^2+1)^2 \frac{dy}{dx} + 2x(x^2+1) = 1$

$\frac{dy}{dx} + \frac{2x}{(x^2+1)} y = \frac{1}{(x^2+1)^2}$ I_{LDE}

IF = $= e^{\ln(x^2+1)} = x^2+1$

$y.(x^2+1) = \int \frac{1}{(x^2+1)^2}.(x^2+1)dx$

$y(x^2+1) = \tan^{-1}(x) + C$

For C : 0.1 = 0 + C $\Rightarrow$ C = 0

$y = \frac{\tan^{-1}(x)}{(x^2+1)}$

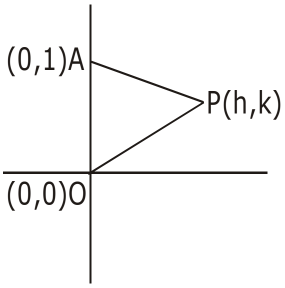
Now $\sqrt{a}y(1) = \frac{\pi}{32} \Rightarrow \sqrt{a}.\frac{\pi/4}{2} = \frac{\pi}{32} \Rightarrow \sqrt{a} = \frac{1}{4}$

$a = \frac{1}{16}$

10. Let O(0,0) and A(0,1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

(1) $8x^2-9y^2 + 9y = 18$
(2) $9x^2+8y^2-8y = 16$
(3) $9x^2-8y^2 + 8y = 16$
(4) $8x^2+9y^2 - 9y = 18$

Sol. **B**



PA+PO+OA = 4 $\Rightarrow$ PA+PO = 3

$\Rightarrow$ locus of P is ellipse

PA+PO = 3 $\Rightarrow$ 2b = 3 $\Rightarrow$ b = 3/2

AO = 2be $\Rightarrow$ e = 1/3

Now, $e^2 = 1- a^2/b^2 \Rightarrow \frac{1}{9} = 1 - \frac{4a^2}{9}$

$1 = 9-4a^2 \Rightarrow a^2 = 2$

$E : \frac{x^2}{2} + \frac{4(y-1/2)^2}{9} = 1$

$9x^2 + 8(y-1/2)^2 = 18$

$9x^2 + 8y^2 + 2 - 8y = 18$

$9x^2 + 8y^2 - 8y - 16 = 0$

11. Let $f:[0,2] \to R$ be a twice differentiable function such that $f''(x)>0$, for all $x \in (0,2)$. If $\phi(x) = f(x) + f(2-x)$, then ϕ is :

(1) decreasing on (0,1) and increasing on (1,2).
(2) increasing on (0,1) and decreasing on (1,2).
(3) decreasing on (0,2)
(4) increasing on (0,2)

Sol. **1**

$f:[0,2] \to R$, $f''(x)>0 \quad \forall\ x \in (0,2)$

$\Rightarrow f' \uparrow$

$\phi(x) = f(x) + f(2-x)$

$\phi'(x) = f'(x) - f'(2-x)$

$\phi' > 0 \Rightarrow f'(x) > f'(2-x) \Rightarrow x > 2-x \Rightarrow x > 1$

$\Rightarrow \phi \uparrow \Rightarrow 1 < x < 2$

$\Rightarrow \phi \downarrow \Rightarrow 0 < x < 1$

12. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, (\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is :

(1) $\frac{\pi}{64}$ (2) 0 (3) $\frac{\pi}{32}$ (4) $\frac{\pi}{16}$

Sol. **1**

$A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \alpha \in R$

$A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Now, $A^2 = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$

$A^3 = \begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{pmatrix}$

In gen. : $A^n = \begin{pmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{pmatrix}$

Now $A^{32} = \begin{pmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\Rightarrow \cos 32\alpha = 0$, $\sin 32\alpha = 1$

$\sin 32\alpha = 1$, $\cos 32\alpha = 0$

$\Rightarrow \alpha = \frac{\pi}{64}$

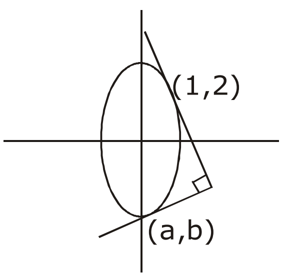
13. If the tangents on the ellipse $4x^2+y^2=8$ at the points (1,2) and (a,b) are perpendicular to each other, then a^2 is equal to :

(1) $\frac{4}{17}$ (2) $\frac{128}{17}$ (3) $\frac{64}{17}$ (4) $\frac{2}{17}$

Sol. **4**

E: $4x^2 + y^2 = 8$

$\frac{x^2}{2} + \frac{32}{8} = 1 \Rightarrow \frac{dy}{dx} = \frac{-4x}{y}$



Slope of tangent at (1,2) = m_1 = -2

Slope of tangent at (a,b) m_2 = -4a/b

$\because$ m_1 . m_2 = -1 $\Rightarrow \frac{8a}{b} = 1 \Rightarrow b = 8a$

Now, (a,b) on ellipse

$4a^2 + b^2 = 8$

$4a^2 + 64a^2 = 8$

$a^2 = 8/68 = 4/34 = 2/17$

14. $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ equals:

(1) 4 (2) $2\sqrt{2}$ (3) $4\sqrt{2}$ (4) $\sqrt{2}$

Sol. **3**

$$\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}} = \lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)\left(\frac{x^2}{1-\cos x}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$= 1.2.2\sqrt{2} = 4\sqrt{2}$

15. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vector $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is :

(1) $\sqrt{\frac{3}{2}}$ (2) $\sqrt{6}$ (3) $\frac{\sqrt{3}}{2}$ (4) $3\sqrt{6}$

Sol. **1**

$\vec{a} = 2\hat{i}+3\hat{j}+\hat{k}$ & $\hat{n} = \vec{b}\times\vec{c}$

$$\Rightarrow \hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = <\hat{i}-2\hat{j}+\hat{k}>$$

Proj. of $\vec{a}$ on $\hat{n}$ = $\left|\frac{\vec{a}.\hat{n}}{|\hat{n}|}\right|$

$= \left|\frac{2-6+1}{\sqrt{6}}\right|$

$= \left|\frac{3}{\sqrt{6}}\right| = \sqrt{\frac{3}{2}}$

16. The greatest value of $c \in R$ for which the system of linear equations, x-cy-cz = 0, cx-y+cz = 0 cx+cy-z = 0 has a non-trivial solution, is :

(1) 0 (2) 2 (3) $\frac{1}{2}$ (4) -1

Sol. **3**

For non-trinial solu. of homog. system of equation

$$\Delta = \begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$(1-c^2) + c(-c-c^2) - c(c^2+c) = 0$

$1-c^2-c^2-c^3-c^3-c^2 = 0$

$1-3c^2-2c^3=0$

$2c^3+3c^2-1= 0$

$(2c-1)(c^2+2c+1) = 0$

$(2c-1)(c+1)^2 = 0$

$c= 1/2$ or $c= -1$

Greatest value of $c = 1/2$

17. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :

(1) if you are not a citizen of India, then you are not born in India.

(2) if you are a citizen of India, then you are born in India.

(3) if you are not born in India, then you are not a citizen of India.

(4) if you are born in India, then you are not a citizen of India.

Sol. **1**

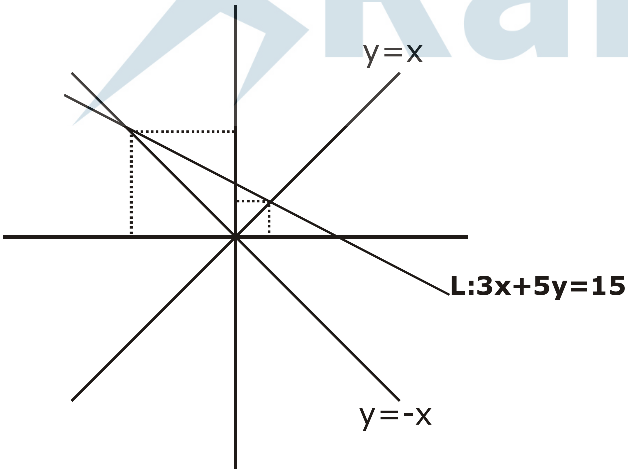
Contrapositive statement of $p \to q$ is $\sim q \to \sim p$

18. A point on the straight line, $3x+5y = 15$ which is equidistant from the coordinate axes will lie only in :

(1) 4^{th} quadrant (2) 1^{st} and 2^{nd} quadrants

(3) 1^{st}, 2^{nd} and 4^{th} quadrants (4) 1^{st} quadrant

Sol. **2**

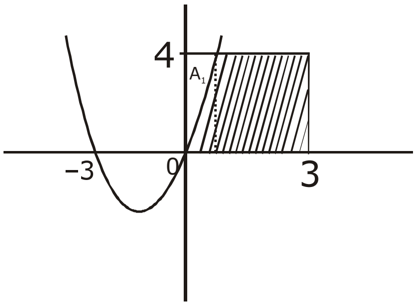


In 1^{st} & 2^{nd} quadrants according to figure then intersect in (1)& (2)

19. The area (in sq. units) of the region $A = \{(x,y) \in R \times R \mid 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is :

(1) 8 (2) $\frac{59}{6}$ (3) $\frac{26}{3}$ (4) $\frac{53}{6}$

Sol. **2**



Req area = 12-(A_1)

$= 12\text{-}(4\text{-}\int_0^1 (x^2 + 3x)\,dx\ )$

= 12-(4-(1/3 + 3/2))

$= 12 - \left(4 - \left(\frac{2+9}{6}\right)\right)$

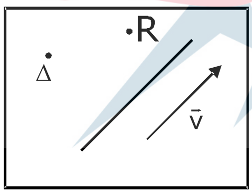
= 8 + 11/6

= 59/6

20. The equation of a plane containing the line of intersection of the planes 2x-y-4= 0 and y+2z-4=0 and passing through the point (1,1,0) is :

(1) x-3y-2z= -2 (2) X+3y+z=4 (3) x-y-z=0 (4) 2x-z=2

Sol. **3**



using family of plane

P : $P_1 + \lambda P_2 = 0$

P : (2)x + (-1+λ)+(2λ)-4-4λ= 0

it pass through (1,1,0)

2-1+λ-4-4λ= 0

$3\lambda = -3$

$\lambda = -1$

P: 2x-2y-2z = 0

P: x-y-z = 0

21. All possible numbers are formed using the digits 1,1,2,2,2,2,3,4,4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :

(1) 162 (2) 175 (3) 160 (4) 180

Sol. **4**

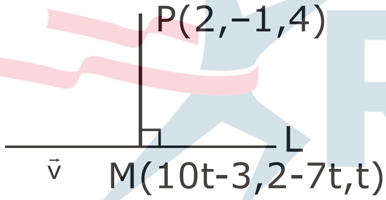
4 even place & 5 odd place

$= {}^4C_3 \cdot \frac{3!}{2!} \times \frac{6!}{4!2!}$
= 4 . 3 . 15
= 180

22. The length of the perpendicular from the point (2,-1,4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :
(1) greater than 3 but less than 4
(2) greater than 2 but less than 3
(3) less than 2
(4) greater than 4

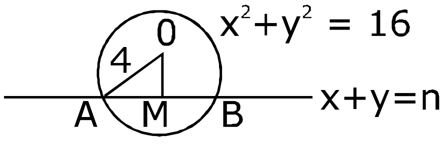
Sol. **1**
For t
$\overrightarrow{PM}.\vec{V} = 0$
(10t-5).10+(3-7t)(-7)+(t-4) = 0
100t-50-21+49t+t-4= 0
150t -75 = 0
t = 1/2
M : (2,-3/2, 1/2) $\Rightarrow \overrightarrow{PM} = (0,-1/2,-7,2)$

distance $|\overrightarrow{PM}| = \sqrt{\frac{1}{4}+\frac{49}{4}} = \sqrt{\frac{50}{4}} = \frac{5}{\sqrt{2}}$



23. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2+y^2=16$, by the lines, x+y=n, $n \in N$, where N is the set of all natural numbers, is :
(1) 105 (2) 160 (3) 210 (4) 320

Sol. **3**



$\because 0 < \frac{n}{\sqrt{2}} < 4 \Rightarrow 0 < n < 4\sqrt{2} \Rightarrow 0<n<5.6$

AB = 2AM = $2\sqrt{16-\left(\frac{n}{\sqrt{2}}\right)^2}$

sum of square of AB = $\sum 4\left(16-\frac{n^2}{2}\right)$

$= 4\sum\left(16-\frac{n^2}{2}\right) = 4\left(\sum_{n=1}^{5}16-\sum_{n=1}^{5}\frac{n^2}{2}\right)$

= 4(80-1/2 (1+4+9+16+25)

= 320 - 110

= 210

24. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2,(x>0)$ is equal to :

(1) 12 (2) 9 (3) 10 (4) 4

Sol. **3**

$|\sqrt{x}-2|+\sqrt{x}(x-4)+2=0$

(i) For $\sqrt{x}\ge 2 \Rightarrow \sqrt{x}-2+x-4\sqrt{x}+2=0$

$x-3\sqrt{x}=0$

$\sqrt{x}\left(\sqrt{x}-3\right)=0$

$x=0\,|\,x=9 \Rightarrow$ x = 9 is solution

(ii) For $\sqrt{x}<2 \Rightarrow 2-\sqrt{x}+x-4\sqrt{x}+2=0$

$x-5\sqrt{x}+4=0$

$\left(\sqrt{x-4}\right)\left(\sqrt{x}-1\right)=0 \Rightarrow x=1\,|\,x=16 \Rightarrow$ x = 1 is solution

$\therefore$ sum of solution = 1+9 = 10

25. If $\cos(\alpha+\beta)=\frac{3}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$, then tan(2α) is equal to :

(1) $\frac{33}{52}$ (2) $\frac{21}{16}$ (3) $\frac{63}{52}$ (4) $\frac{63}{16}$

Sol. **4**

tan2α = tan (α+β+α–β)

$= \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}$

$= \frac{48+15}{36-20}$

$= \frac{63}{16}$

26. The sum of the series $2.^{20}C_0+5.^{20}C_1+8.^{20}C_2+11.^{20}C_3+....+62.^{20}C_{20}$ is equal to :

(1) 2^{23} (2) 2^{24} (3) 2^{26} (4) 2^{25}

Sol. **4**

$2.\,^{20}C_0+5.^{20}C_1+8.^{20}C_2+11.^{20}C_3+....+60.^{20}C_{20}$

In Gen. $S_n = \sum_{r=0}^{20}(3r+2)^{10}C_r$

$S_n = 3\sum_{r=0}^{20} r\,{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$

$= 3.20\sum_{r=1}^{20}{}^{19}C_{r-1} + 2\left(2^{20}\right)$

$= 60.2^{19} + 2.2^{20}$
$= 2^{19}\ (4+60)$
$= 64.\ 2^{19}$
$= 2^{6}\ .\ 2^{19}$
$= 2^{25}$

27. The sum of all natural numbers 'n' such that 100<n<200 and H.C.F. (91,n)>1 is :
(1) 3203 (2) 3221 (3) 3121 (4) 3303

Sol. **3**

$91 = 13 \times 7 \quad \because \text{HCF}(91, n) > 1$

sum of n = multiple of 7 + multiple of 13 - multiple of 13×7
= (105 + ...+ 196) + (104+ ....195) -182
= 7(105+196) + 4 (104 + 195) - 182
= 2107+1196-182
= 3121

28. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to : (where c is a constant of integration.)

(1) x+2sinx+sin2x+c (2) 2x+sinx+sin2x+c
(3) x+2sinx+2sin2x+c (4) 2x+sinx+2sin2x+c

Sol. **1**

$\int \frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$

$= \int \frac{\sin(3x)+\sin(2x)}{\sin x}dx$

$= \int \frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x}dx$

$\int\left(3 - 4\sin^2 x + 2\cos x\right)dx$

$= 3x - 4\int\left(\frac{1-\cos 2x}{2}\right)dx + 2\int \cos x\,dx$

$= 3x - 2\left(x - \frac{\sin 2x}{2}\right) + 2\sin x$

= x + sin2x + 2sinx + C

29. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and g(x) = $\log_e x$, (x>0) then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is :

(1) $\log_e 3$ (2) $\log_e 1$ (3) $\log_e 2$ (4) $\log_e e$

Sol. 2

$f(x)=\frac{2-x\cos x}{2+x\cos x}$ & g(x) = lnx (x>0)

$I=\int_{-\pi/4}^{\pi/4} g(f(x))dx \Rightarrow I=\int_{-\pi/4}^{\pi/4} \ln\left(\frac{2-x\cos x}{x+x\cos x}\right)dx$

I = 0 ($\because$ g(f(x)) is an odd function)

30. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2$ $x\in\left(0,\frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to :

(1) $2x,-\frac{\pi}{3}$ (2) $\frac{\pi}{6}-x$ (3) $x-\frac{\pi}{6}$ (4) $\frac{\pi}{3}-x$

Sol. 3

$$2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2$$

$$2y=\left(\cot^{-1}\left(\frac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}\right)\right)^2$$

$$2y=\left(\cot^{-1}\left(\frac{\tan\left(\frac{\pi}{3}\right)+\tan x}{1-\tan\frac{\pi}{3}.\tan x}\right)\right)^2$$

$$2y=\left(\cot^{-1}\left(\tan\left(\frac{\pi}{3}+x\right)\right)\right)^2$$

$$2y=\left(\frac{\pi}{2}-\tan^{-1}\left(\tan\left(\frac{\pi}{3}+x\right)\right)\right)^2$$

$$2y=\left(\frac{\pi}{2}-(\frac{\pi}{3}+x)\right)^2 \quad (0<x<\pi/6)$$

$$2y = \left(\frac{\pi}{6} - x\right)^2$$

or

$$2y = \left(\frac{\pi}{2} - \left(\frac{\pi}{3} + x - \pi\right)\right)^2 \quad (\pi/6<x<\pi/2)$$

$$2y = \left(\frac{7\pi}{2} - x\right)^2$$

$$\frac{dy}{dx} = -\left(\frac{\pi}{6} - x\right) \text{ if } 0<x<\pi/6$$

Or

$$\frac{dy}{dx} = -\left(\frac{7\pi}{6} - x\right) \text{ if } \pi/6<x<\pi/2$$