$\therefore\ IF = e^{-\tan^{-1}(\sqrt{2}\cot 2x)}$

$$ye^{-\tan^{-1}(\sqrt{2}\cot 2x)} = \int x\,dx$$

$$ye^{-\tan^{-1}\left(\sqrt{2}\cot 2x\right)} = \frac{x^2}{2} + c$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32} + c \ \Rightarrow c = 0$$

$$y = \frac{x^2}{2}e^{\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{\tan^{-1}\left(\sqrt{2}\cot\frac{2\pi}{3}\right)}$$

$$= \frac{\pi^2}{18}e^{-\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)}$$

$$\alpha = \sqrt{\frac{2}{3}} \Rightarrow 3\alpha^2 = 2$$

3. Question ID: 101783

Let d be the distance between the foot of perpendiculars of the points P(1, 2 – 1) and Q(2, – 1, 3) on the plane – x + y + z = 1. Then d^2 is equal to _______.

Ans. (26)

Sol. Points P(1, 2, –1) and Q (2, –1, 3) lie on same side of the plane.

Perpendicular distance of point P from plane is

$$\left|\frac{-1+2-1-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

Perpendicular distance of point Q from plane is

$$= \left|\frac{-2-1+3-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow \overrightarrow{PQ}$ is parallel to given plane. So, distance between P and Q = distance between their foot of perpendiculars.

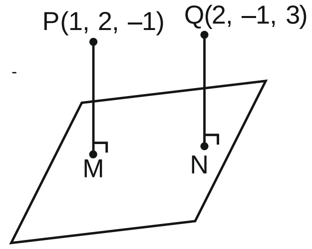
$$\Rightarrow \left|\overrightarrow{PQ}\right| = \sqrt{(1-2)^2 + (2+1)^2 + (-1-3)^2}$$

$$= \sqrt{26}$$

$$\left|\overrightarrow{PQ}\right|^2 = 26 = d^2$$

Alternate

$$-x + y + z - 1 = 0$$



$M(x_1, y_1, z_1)$

$$\frac{x_1 - 1}{-1} = \frac{y_1 - 2}{1} = \frac{z_1 + 1}{1} = \frac{1}{3}$$

$$x_1 = \frac{2}{3}, y_1 = \frac{7}{3}, z_1 = \frac{-2}{3}$$

$$M\left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$$

$N(x_2, y_2, z_2)$

$$\frac{x_2 - 2}{-1} = \frac{y_2 + 1}{1} = \frac{z_2 - 3}{1} = \frac{1}{3}$$

$$x_2 = \frac{5}{3}, y_2 = \frac{-2}{3},\ z_2 = \frac{10}{3}$$

$$N = \left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$$

$d^2 = 1^2 + 3^2 + 4^2 = 26$

4. Question ID: 101784

The number of elements in the set $S = \{\theta \epsilon\ [-4\pi, 4\pi]\ : 3\,cos^2\,2\theta + 6\,cos2\theta - 10cos^2\theta + 5 = 0\}$ is ________.

Ans. (32)

Sol. $3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$

$3\cos^2 2\theta + \cos 2\theta = 0$

$\cos 2\theta = 0$ OR $\cos 2\theta = -1/3$

$\theta \in [-4\pi, 4\pi]$

$2\theta = (2n + 1).\frac{\pi}{2}$

$\therefore\ \theta = \pm\pi/4. \pm 3\pi/4 .... \pm 15\pi/4$

Similarly $\cos 2\theta = -1/3$ gives 16 solution

5. Question ID: 101785

The number of solutions of the equation $2\theta - cos^2\theta + \sqrt{2} = 0$ is R is equal to ______.

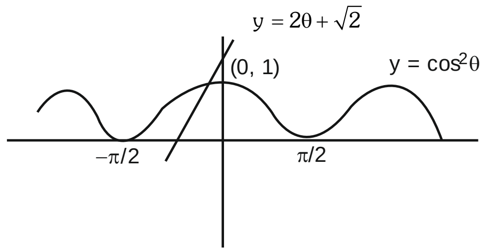
Official Ans. by NTA (1)

Ans. (1)

Sol. $2\theta - \cos^2\theta + \sqrt{2} = 0$

$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

$y = 2\theta + \sqrt{2}$



Both graphs intersect at one point.

6. Question ID: 101786

$50\ tan\left(3tan^{-1}\left(\frac{1}{2}\right) + 2cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) +$

$4\sqrt{2}\tan\left(\frac{1}{2}tan^{-1}(2\sqrt{2})\right)$ is equal to ______.

Ans. (29)

Sol. $50\tan\left(3\tan^{-1}\frac{1}{2} + 2\cos^{-1}\frac{1}{\sqrt{5}}\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2(\tan^{-1}\frac{1}{2} + \tan^{-1}2)\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2.\frac{\pi}{2})\right) + 4\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 50\left(\tan\ \tan^{-1}\frac{1}{2}\right) + 4$

= 25 +4 = 29

7. Question ID: 101787

Let c, k ∈ R. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all x, y ∈ R, then the value of $|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))|$ is equal to _______.

Ans. (3395)

Sol. $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ ....(1)

& $f(x + y) = f(x) + f(y) - xy \quad \forall xy \in R$

$$\lim_{y\to 0}\frac{f(x+y) - f(x)}{y} = \lim_{y\to 0}\frac{f(y) - xy}{y} \Rightarrow f'(x) = f'(0) - x$$

$$f(x) = -\frac{1}{2}x^2 + f'(0).x + \lambda \quad \text{but } f(0) = 0 \Rightarrow \lambda = 0$$

$$f(x) = -\frac{1}{2}x^2 + (1 - c^2).x \qquad ....(2)$$

∴ as $f'(0) = 1 - c^2$

Comparing equation (1) and (2)

We obtain, $c = -\frac{3}{2}$

∴ $f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$

Now $\left|2\sum_{x=1}^{20} f(x)\right| = \sum_{x=1}^{20} x^2 + \frac{5}{2}.\sum_{x=1}^{20} x$

= 2870 + 525

= 3395

8. Question ID: 101788

Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, a > 0, b > 0, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2} + \sqrt{14}\right)$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then value of $a^2 + b^2$ is equal to __________.

Ans. (88)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Given $e^2=1+\frac{b^2}{a^2}\Rightarrow\frac{11}{4}=1+\frac{b^2}{a^2}\ \Rightarrow b^2=\frac{7}{4}a^2$

$\therefore\ \frac{x^2}{(a)^2}-\frac{y^2}{\left(\frac{\sqrt{7}}{2}a\right)^2}=1$ Now given

$2a+2.\frac{\sqrt{7}a}{2}=4\left(2\sqrt{2}+\sqrt{14}\right)$

$a\left(2+\sqrt{7}\right)=4\sqrt{2}(2+\sqrt{7})$

$a=4\sqrt{2}\Rightarrow a^2=32$

$b^2=\frac{7}{4}\times16\times2=56$

9. Question ID: 101789

Let $P_1:\vec{r}.(2\hat{i}+\hat{j}-3\hat{k})=4$ be a plane. Let P_2 be another plane which passes through the points (2, -3, 2) (2, − 2, − 3) and (1, −4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of α + β is equal to ______.

Ans. (28)

Sol. $P_1:\vec{r}.(2\hat{i}+\hat{j}-3\hat{k})=4$

P_1: $2x+y-3z=4$

$$P_2\begin{vmatrix}x-2 & y+3 & z-2\\ 0 & 1 & -5\\ -1 & -1 & 0\end{vmatrix}=0$$

$\Rightarrow -5x+5y+z+23=0$

Let a, b, c be the d’rs of line of intersection

Then $a=\frac{16\lambda}{15};\ b=\frac{13\lambda}{15};\ c=\frac{15\lambda}{15}$

$\therefore\ \alpha=13:\ \beta=15$

10. Question ID: 101790

Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i\in\{1, 2, 3, \ldots\ldots\ldots, 100\}$ for $1\le i\le 4$ and $b_i\ne b_j$ for $i\ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to ________.

Ans. (18915)

Sol. $b_i\in\{1,2,3\ldots\ldots\ldots100\}$

Let A = set when b_1 b_2 b_3 are consecutive

$n(A)=\frac{97+97+\ldots\ldots+97}{98\text{times}}=97\times98$

Similarly when b_2b_3 b_4 are consecutive

N(A) = 97 × 98

$n(A\cap B)=\frac{97+97+-------97}{98\text{times}}=97\times98$

Similarly when b_2 b_3 b_4 are consecutive

n(B) = 97 × 98

n(A∩B) = 97

n(AUB) = n(A)+n(B) − n(A∩B)

Number of permutation = 18915