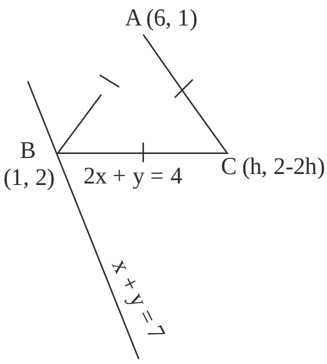
Sol.



Point B (1, 2)

Now let C be (h, 4 - 2h)

(As C lies on 2x + y = 4)

$\because \Delta$ is isosceles with base BC

$\therefore AB = AC$

$\sqrt{25+1} = \sqrt{(6-h)^2 + (2h-3)^2}$

$\sqrt{26} = \sqrt{36 + h^2 - 12h + 4h^2 + 9 - 12h}$

$26 = 5h^2 - 24h + 45 \Rightarrow 5h^2 - 24h + 19 = 0$

$\Rightarrow 5h^2 - 5h - 19h + 19 = 0$

$h = \frac{19}{5}$ or h = 1

Thus $C\left(\frac{19}{5}, \frac{-18}{5}\right)$

Centroid $\left(\frac{6+1+\frac{19}{5}}{3}, \frac{1+2-\frac{18}{5}}{3}\right)$

$\left(\frac{35+19}{15}, \frac{15-18}{15}\right)$

$\left(\frac{54}{15}, \frac{-3}{15}\right)$

$\alpha = \frac{54}{15}; \ \beta = \frac{-3}{15}$

$15(\alpha + \beta) = 51$

13. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \ a > b,$ be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal to :

(A) 29 (B) 31

(C) 32 (D) 34

Ans. (B)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b$

$e^2 = 1 - \frac{b^2}{a^2}$

$\frac{1}{16} = 1 - \frac{b^2}{a^2}$

$\frac{b^2}{a^2} = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow b^2 = \frac{15}{16}a^2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{16 \times \frac{2}{5}}{a^2} + \frac{9}{b^2} = 1$

$\frac{32}{5a^2} + \frac{9}{b^2} = 1$

$\frac{32}{5a^2} + \frac{9}{\frac{15}{16}a^2} = 1$

$\frac{80}{5a^2} = 1$

$16 = a^2$

$b^2 = 15$

14. If two straight lines whose direction cosines are given by the relations l + m – n = 0, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :

(A) 6 (B) 4

(C) 3 (D) 2

Ans. (A)

Sol. l + m – n = 0

$3l^2 + m^2 + cl(l + m) = 0$

n = l + m

$3l^2 + m^2 + cl^2 + clm = 0$

$(3 + c)l^2 + clm + m^2 = 0$

$$(3+c)\left(\frac{l}{m}\right)^2 + c\left(\frac{l}{m}\right) + 1 = 0 \quad .....(1)$$

$\because$ lies are parallel.

Roots of (1) must be equal

$\Rightarrow D = 0$

$c^2 - 4(3 + c) = 0$

$c^2 - 4c - 12 = 0$

$(c - 6)(c + 2) = 0$

c = 6 or c = - 2

+ve value of c = 6

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, ..., 10\}$ is :

(A) 0 (B) 1

(C) 2 (D) 3

Ans. (A)

Sol. $\vec{a} = i + j - k$

$\vec{c} = 2i - 3j + 2k$

$\vec{b} \times \vec{c} = \vec{a}$

$|\vec{b}| \in \{1, 2.......10\}$

$\because \vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{a}$ is perpendicular to $\vec{b}$ as well as $\vec{a}$ is perpendicular to $\vec{c}$

Now $\vec{a}.\vec{c} = 2 - 3 - 2 = -3 \neq 0$

This $\vec{b} \times \vec{c} = \vec{a}$ is not possible.

No. of vectors $\vec{b} = 0$

16. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3,……, 18 and are arranged in the increasing order $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_2 = 7$ and $x_4 = 11$ is :

(A) $\frac{1}{136}$ (B) $\frac{1}{72}$

(C) $\frac{1}{68}$ (D) $\frac{1}{34}$

Ans. (C)

Sol. No. of ways to select and arrange x_1, x_2, x_3, x_4, x_5 from 1, 2, 3……………….18

$n(s) = {}^{18}C_5$

$$\begin{matrix} x_1 & (x_2) & x_3 & (x_4) & x_5 \\ & 7 & & 11 & \end{matrix}$$

$n(E) = {}^6C_1 \times {}^3C_1 \times {}^7C_1$

$$P(E) = \frac{6 \times 3 \times 7}{{}^{18}C_5}$$

$$\frac{1}{17 \times 4} = \frac{1}{68}$$

17. Let X be a random variable having binomial distribution B(7, p). If P(X = 3) = 5P(X = 4), then the sum of the mean and the variance of X is :

(A) $\frac{105}{16}$ (B) $\frac{7}{16}$

(C) $\frac{77}{36}$ (D) $\frac{49}{16}$

Ans. (C)

Sol. B (7, p)

n = 7 p = p

given

$$P(x = 3) = 5P(x = 4)$$

$${}^7C_3 \times p^3(1-p)^4 = 5.{}^7C_4 p^4 (1-p)^3$$

$$\frac{{}^7C_3}{5\times {}^7C_4} = \frac{p}{1-p}$$

1 – p = 5p

6p = 1

$$p = \frac{1}{6} \Rightarrow q = \frac{5}{6}$$

n = 7

Mean = np $= 7\times\frac{1}{6} = \frac{7}{6}$

Var = npq $= 7\times\frac{1}{6}\times\frac{5}{6} = \frac{35}{36}$

Sum

$$= \frac{7}{6} + \frac{35}{36}$$

$$= \frac{42+35}{36}$$

$$= \frac{77}{36}$$

18. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to :

(A) -1 (B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$ (D) $-\frac{1}{4}$

Ans. (B)

Sol. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{\sin\left(3\times\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \times \cos\left(\frac{\frac{2\pi}{7}+\frac{6\pi}{7}}{2}\right)$$

$$= \frac{2\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} \times \cos\left(\frac{4\pi}{7}\right)$$

$$= \frac{\sin\left(\frac{7\pi}{7}\right) + \sin\left(\frac{-\pi}{7}\right)}{2\sin\frac{\pi}{7}}$$

$$= \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$= -\frac{1}{2}$$

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{17\pi}{12}$

(C) $\frac{31\pi}{12}$ (D) $-\frac{3\pi}{4}$

Ans. (A)

Sol. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\tan\left(\frac{3\pi}{4}\right)$

$$\sin^{-1}\sin\left(\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\cos^{-1}\left(\cos\frac{2\pi}{6}\right) = 2\pi - \frac{7\pi}{6} = \frac{5\pi}{6}$$

$$\tan^{-1}\tan\left(\frac{3\pi}{4}\right) = \frac{3\pi}{4} - \pi = \frac{-\pi}{4}$$

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\cos\frac{7\pi}{6} + \tan^{-1}\tan\frac{3\pi}{4}$$

$$= \frac{11\pi}{12}$$

20. The Boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to :

(A) $q \to (p \wedge q)$ (B) $p \to q$

(C) $p \to (p \to q)$ (D) $p \to (p \vee q)$

Ans. (D)

Sol. (~(p^q)) ∨ q

= (~p ∨ ~q) ∨ q

= ~p ∨ ~ q ∨ q

= ~ p ∨ t

= this statement is a tautology option D

p ⇒ (p ∨ q) is also a tautology.

OR

p	q	P^q	~(p^q)	~(p^q) ∨ q	p ∨ q	$p \to (p \vee q)$
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	F	T

SECTION-B

1. Let $f : R \to R$ be a function defined $f(x) = \frac{2e^{2x}}{e^{2x} + e}$.

Then $f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + ... + f\left(\frac{99}{100}\right)$ is equal to____________.

Ans. (99)

Sol.

$$f(x) + f(1-x) = \frac{2e^{2x}}{e^{2x} + e} + \frac{2e^{2-2x}}{e^{2-ex} + e} = \left[\frac{e^{2x}}{e^{2x} + e} + \frac{e^2}{e^2 + e^{2x+1}}\right]$$

$$= 2\left[\frac{e^{2x-1}}{e^{2x-1} + 1} + \frac{1}{1 + e^{2x-1}}\right] = 2$$

$$f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + f\left(\frac{3}{100}\right) + ... + f\left(\frac{99}{100}\right)$$

$$= \left\{f\left(\frac{1}{100}\right) + f\left(\frac{99}{100}\right)\right\} + \left\{f\left(\frac{2}{100}\right) + f\left(\frac{98}{100}\right)\right\} + ... + f\left\{\left(\frac{49}{100}\right) + f\left(\frac{51}{100}\right)\right\} + f\left(\frac{1}{2}\right)$$

$$= (2 + 2 + 2 + ----49\,times) + \frac{2e}{e + e}$$

$$= 98 + 1 = 99$$

2. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e$ P, then p is equal to _______.

Ans. (45)

Sol. $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$]

$(e^x)^3 - 11(e^x)^2 - 45 + \frac{81e^x}{2} = 0$]

$e^x = t$

$2t^3 - 22t^2 + 81t - 90 = 0$

$t_1 t_2 t_3 = 45$

$e^{x_1}.e^{x_2}.e^{x_3} = 45$

$e^{x_1 + x_2 + x_3} = 45$

$\log_e e^{x_1 + x_2 + x_3} = \log_e 45$

$x_1 + x_2 + x_3 = \log_e 45$

$\log_e P = \log_e 45$

P = 45

3. The positive value of the determinant of the matrix A, whose $Adj\left(Adj\left(A\right)\right)=\begin{pmatrix}14 & 28 & -14\\ -14 & 14 & 28\\ 28 & -14 & 14\end{pmatrix}$, is __________.

Ans. (14)

Sol. $Adj\left(AdjA\right)=\begin{bmatrix}14 & 18 & -14\\ -14 & 14 & 28\\ 28 & -14 & 14\end{bmatrix}$

$$\left|Adj\left(AdjA\right)\right|=\begin{bmatrix}14 & 28 & -14\\ -14 & 14 & 28\\ 28 & -14 & 14\end{bmatrix}=14\times14\times14\begin{vmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{vmatrix}$$

$$=\left(14\right)^3\left[3-2\left(-5\right)-1\left(-1\right)\right]=\left(14\right)^3\left[14\right]=\left(14\right)^4$$

$$\left|A\right|^4=\left(14\right)^4\Rightarrow\left|A\right|=14$$

4. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _______.

Ans. (56)

Sol. 16 cubes → 11 Blue, 5 Red

$$x_1+x_2+x_3+x_4+x_5+x_6=11$$

$$x_1,x_6\geq0,\qquad x_2,x_3,x_4,x_5\geq2$$

$$x_2=t_1+2$$

$$x_3=t_3+2$$

$$x_4=t_4+2$$

$$x_5=t_5+2$$

$$x_1,t_2,t_3,t_4,t_5,x_6\geq0$$

No. of solutions $={}^{6+3-1}C_3={}^{8}C_3=56$

5. If the coefficient of x^{10} in the binomial expansion of $\left(\dfrac{\sqrt{x}}{5^{\frac{1}{4}}}+\dfrac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k l$, where l, k ∈ N and l is co-prime to 5, then k is equal to _____.

Ans. (5)

Sol. $\left(\dfrac{\sqrt{x}}{5^{1/4}}+\dfrac{\sqrt{5}}{x^{1/3}}\right)^{60}$

$$T_{r+1}={}^{60}C_r\left(\frac{x^{1/2}}{5^{1/4}}\right)^{60-r}\left(\frac{5^{1/2}}{x^{1/3}}\right)r$$

$$={}^{60}C_r5\frac{3r-60}{4}.x\frac{180-5r}{6}$$

$$\frac{180-5r}{6}=10\Rightarrow r=24$$

Coeff. of $x^{10}={}^{60}C_{24}5^3=\dfrac{\lfloor60}{\lfloor24\lfloor36}5^3$

Powers of 5 in $={}^{60}C_{24}.5^3=\dfrac{5^{14}}{5^4\times5^8}\times5^3=5^5$

6. Let

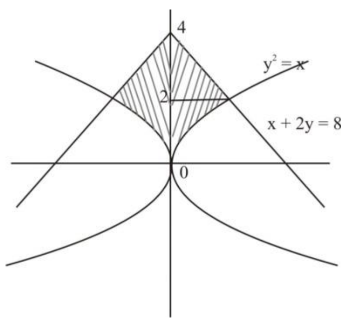
$A_1=\left\{\left(x,y\right):\left|x\right|\leq y^2,\left|x\right|+2y\leq8\right\}$ and

$A_2=\left\{\left(x,y\right):\left|x\right|+\left|y\right|\leq k\right\}$. If 27 (Area A_1) = 5 (Area A_2), then k is equal to :

Ans. (6)

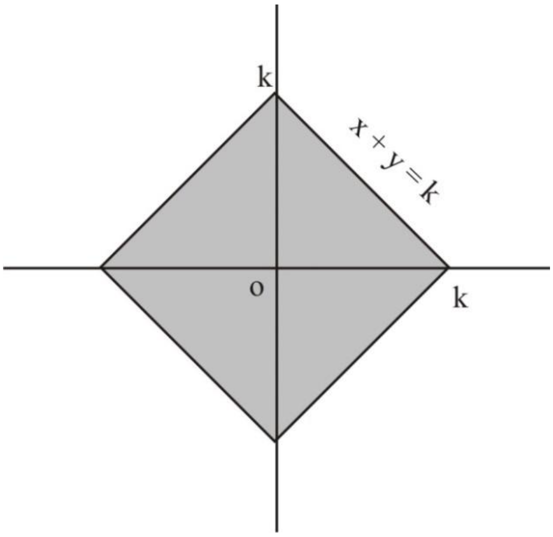
Sol. $A_1=\left\{\left(x,y\right):\left|x\right|\leq y^2,\left|x\right|+2y\leq8\right\}$ and

$A_2=\left\{\left(x,y\right):\left|x\right|+\left|y\right|\leq k\right\}$.



$$area(A_1) = 2\left[\int_0^2 y^2 dy + \int_2^4 (8-2y)dy\right]$$

$$= 2\left[\left(\frac{y^3}{3}\right)_0^2 + \left(8y - y^2\right)_2^4\right]$$



$$area(A_1) = 2\times\frac{20}{3} = \frac{40}{3}$$

Area $(A_2) = 4\times\frac{1}{2}k^2$

Area $(A_2) = 2k^2$

Now

27 (Area A_1) = 5 (Area A_2)

$9\times 4 = k^2$

k = 6

7. If the sum of the first ten terms of the series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+....$ is $\frac{m}{n}$, where m and n are co-prime numbers, then m + n is equal to ___________.

Ans. (276)

Sol. $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+.......$

$$T_n = \frac{n}{4n^4+1}$$

$$= \frac{n}{(2n^2+1)^2-(2n)^2} = \frac{n}{(2n^2+2n+1)(2n^2-2n+1)}$$

$$= \frac{1}{4}\left[\frac{1}{2n^2-2n+1} - \frac{1}{2n^2+2n+1}\right]$$

$$S_{10} = \sum_{n=1}^{10} T_n = \frac{1}{4}\left[\frac{1}{1}-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+........\frac{1}{200+20+1}\right]$$

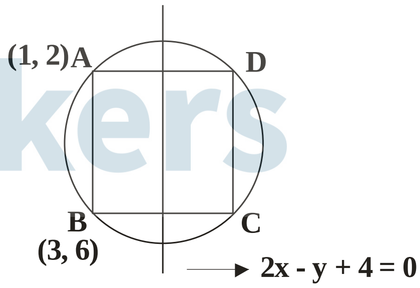
$$= \frac{1}{4}\left[1-\frac{1}{221}\right] = \frac{1}{4}\times\frac{220}{221} - \frac{55}{221} = \frac{m}{n}$$

m + n = 55 + 221 = 276

8. A rectangle R with end points of the one of its dies as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is 2x – y + 4 = 0, then the area of R is _________.

Ans. (16)

Sol.



Eq. of line AB

y = 2x

Slope of AB = 2

Slope of given diameter = 2

So the diameter is parallel to AB

Distance between diameter and line AB

$$= \left(\frac{4}{\sqrt{2^2+12}}\right) = \frac{4}{\sqrt{5}}$$

Thus BC = $2\times\frac{4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

$$AB = \sqrt{(1-3)^2+(2-6)^2} = \sqrt{20} = 2\sqrt{5}$$

Area = AB × BC = $\frac{8}{\sqrt{5}}\times 2\sqrt{5} = 16$ Ans.

9. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y=\left(x-\frac{1}{4}\right)^2+\alpha$, where $\alpha > 0$. Then $(4\alpha-8)^2$ is equal to ___________.

Ans. (63)

Sol. Vertex and focus of parabola $y^2 = 2x$

are V (0, 0) and $S\left(\frac{1}{2},0\right)$ resp.

Let equation of circle be

$(x-h)^2+(y-k)^2=4$

$\because$ Circle passes through (0, 0)

$\Rightarrow h^2+k^2=4$ ……(1)

$\because$ Circle passes through $\left(\frac{1}{2},0\right)$

$$\left(\frac{1}{2}-h\right)^2+k^2=4$$

$$\Rightarrow h^2+k^2-h=\frac{15}{4}\ \ldots\ldots(2)$$

On solving (1) and (2)

$$4-h=\frac{15}{4}$$

$$h=4-\frac{15}{4}=\frac{1}{4}$$

$$k=+\frac{\sqrt{63}}{4}$$

$k=-\frac{\sqrt{63}}{4}$ is rejected as circle with centre $\left(\frac{1}{4},-\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$$\left(x-\frac{1}{4}\right)^2+\left(k-\frac{\sqrt{63}}{4}\right)^2=4$$

From figure

$$\alpha=2+\frac{\sqrt{63}}{4}=\frac{8+\sqrt{63}}{4}$$

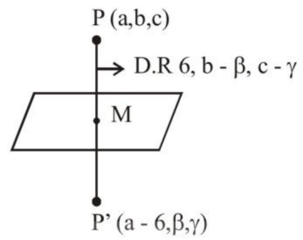
$$4\alpha-8=\sqrt{63}$$

$$(4\alpha-8)^2=63$$

10. Let the mirror image of the point (a, b, c) with respect to the plane 3x – 4y + 12z + 19 = 0 be (a- 6, β, γ). If a + b + c = 5, then 7 β - 9 γ is equal to ____________.

Ans. (137)

Sol.



$$M=\left(a-3,\frac{\beta+b}{2},\frac{\gamma+c}{2}\right)$$

Since M lies on $3x+4y+12z+19=0$

$$\Rightarrow 6a-4b+12c-4\beta+12\gamma+20=0\ \ldots.(1)$$

Since PP' is parallel to normal of the plane then

$$\frac{6}{3}=\frac{b-\beta}{-4}=\frac{c-\gamma}{12}$$

$$\Rightarrow \beta=b+8,\quad \gamma=c-24$$

$$a+b+c=5 \Rightarrow a+\beta-8+\gamma+24=5$$

$$\Rightarrow a=-\beta-\gamma-11$$

Now putting these values in (1) we get

$$6(-\beta-\gamma-11)-4(\beta-8)+12(\gamma+24)-4\beta+12\gamma+20=0$$

$$\Rightarrow 7\beta-9\gamma=170-33=137$$