

8. The sum $1 + 2 \cdot 3 + 3 \cdot 3^2 + \dots + 10 \cdot 3^9$ is equal to

- (A) $\frac{2 \cdot 3^{12} + 10}{4}$ (B) $\frac{19 \cdot 3^{10} + 1}{4}$
 (C) $5 \cdot 3^{10} - 2$ (D) $\frac{9 \cdot 3^{10} + 1}{2}$

Ans. (B)

Sol. $S = 1 \cdot 3^0 + 2 \cdot 3^1 + 3 \cdot 3^2 + \dots + 10 \cdot 3^9$
 $3S = 1 \cdot 3^1 + 2 \cdot 3^2 + \dots + 9 \cdot 3^9 + 10 \cdot 3^{10}$
 $-2S = (1 \cdot 3^0 + 3^1 + 3^2 + \dots + 3^9) - 10 \cdot 3^{10}$

$$S = 5 \times 3^{10} - \left(\frac{3^{10} - 1}{4} \right)$$

$$S = \frac{20 \cdot 3^{10} - 3^{10} + 1}{4} = \frac{19 \cdot 3^{10} + 1}{4}$$

9. Let P be the plane passing through the intersection of the planes

$\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 5$ and $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 3$, and the point $(2, 1, -2)$. Let the position vectors of the points X and Y be $\hat{i} - 2\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 2\hat{k}$ respectively. Then the points

- (A) X and X + Y are on the same side of P
 (B) Y and Y - X are on the opposite sides of P
 (C) X and Y are on the opposite sides of P
 (D) X + Y and X - Y are on the same side of P

Ans. (C)

$$P + \lambda P = 0$$

Sol. $(x + 3y - z - 5) + \lambda(2x - y + z - 3) = 0$

$(2, 1, -2)$ lies on this plane

$$\therefore \lambda = 1 \Rightarrow \text{plane is } 3x + 2y - 8 = 0$$

10. A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is

- (A) $y = \sqrt{2}x$ (B) $x = \sqrt{2}y$
 (C) $y^2 - x^2 = 2xy$ (D) $x^2 - y^2 = 2xy$

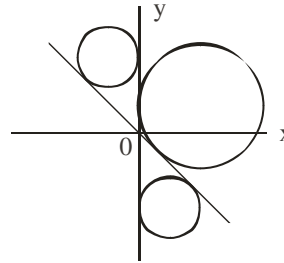
Ans. (D)

Sol. Let (h, k) is centre of circle

$$\left| \frac{h - k}{\sqrt{2}} \right| = |h|$$

$$k^2 - h^2 + 2hk = 0$$

\therefore Equation of locus is $y^2 - x^2 + 2xy = 0$

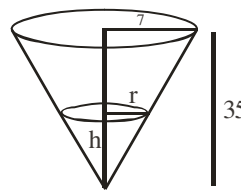


11. Water is being filled at the rate of $1 \text{ cm}^3 / \text{sec}$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2 / sec) at which the wet conical surface area of the vessel increases is

- (A) 5 (B) $\frac{\sqrt{21}}{5}$
 (C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Ans. (C)

Sol. From figure $\frac{r}{h} = \frac{7}{35} \Rightarrow h = 5r$



$$\text{Given } \frac{dV}{dt} = 1 \Rightarrow \frac{d}{dt} \left(\frac{\pi r^2 h}{3} \right) = 1$$

$$\Rightarrow \frac{d}{dt} \left(\frac{5\pi}{3} r^3 \right) = 1 \Rightarrow r^2 \frac{dr}{dt} = \frac{1}{5\pi}$$

Let wet conical surface area = S

$$= \pi r \ell = \pi r \sqrt{h^2 + r^2}$$

$$= \sqrt{26} \pi r^2 \Rightarrow \frac{dS}{dt} = 2\sqrt{26} \pi r \frac{dr}{dt}$$

$$\text{When } h = 10 \text{ then } r = 2 \Rightarrow \frac{dS}{dt} = \frac{2\sqrt{26}}{10}$$

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx$, $n \in \mathbb{N}$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference -2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference -2

Ans. (D)

Sol. $b_n = \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2nx}{\sin x} dx$

$$b_{n+1} - b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2(n+1)x - \cos^2 nx}{\sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{-\sin(2n+1)x \sin x}{\sin x} dx$$

$$= \left(\frac{\cos(2n+1)x}{2n+1} \right)_0^{\frac{\pi}{2}} = \frac{-1}{2n+1}$$

$\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with c.d. = -2

13. If $y = y(x)$ is the solution of the differential equation $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$ such that

$y(e) = \frac{e}{3}$, then $y(1)$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

Ans. (B)

Sol. $\frac{dy}{dx} - \frac{y}{x} = -\frac{3}{2} \left(\frac{y}{x} \right)^2$ $y = vx$

$$\frac{dv}{v^2} = -\frac{3dx}{2x}$$

$$-\frac{1}{v} = -\frac{3}{2} \ln|x| + C$$

$$-\frac{x}{y} = -\frac{3}{2} \ln|x| + C$$

$$x = e, y = \frac{e}{3}$$

$$C = -\frac{3}{2}$$

$$\text{When } x = 1, y = \frac{2}{3}$$

14. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$,

$y = 12(1 + \sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis

is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6(3 + 2\sqrt{2})$ (B) $3(7 + 4\sqrt{3})$

(C) 27 (D) 48

Ans. (3)

Sol. $\frac{dy}{dx} = \frac{2(1 + \sin t) \times \cos t}{1 + \cos 2t}$

$$\Rightarrow \frac{2(1 + \sin t) \cos t}{2 \cos^2 t} = \sqrt{3}$$

$$\Rightarrow t = \frac{\pi}{6}, y_0 = 27$$

15. The value of $2\sin(12^\circ) - \sin(72^\circ)$ is :

(A) $\frac{\sqrt{5}(1 - \sqrt{3})}{4}$ (B) $\frac{1 - \sqrt{5}}{8}$

(C) $\frac{\sqrt{3}(1 - \sqrt{5})}{2}$ (D) $\frac{\sqrt{3}(1 - \sqrt{5})}{4}$

Ans. (D)

Sol. $\sin 12^\circ + \sin 12^\circ - \sin 72^\circ$
 $= \sin 12^\circ - 2 \cos 42^\circ \sin 30^\circ$
 $= \sin 12^\circ - \sin 48^\circ$
 $= -2 \cos 30^\circ \sin 18^\circ$
 $= -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4}$
 $= \frac{\sqrt{3}}{4} (1 - \sqrt{5})$

16. A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is

- (A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$
 (C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Ans. (D)

Sol. $P(n) = \frac{1}{n}$
 $P(2) = \frac{1}{2}$ $P(8) = \frac{1}{8}$
 $P(4) = \frac{1}{4}$ $P(16) = \frac{1}{16}$
 $P(32) = \frac{2}{32}$

Possible cases

16, 16, 16 and 32, 8, 8

Probability = $\frac{1}{16^3} + \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8} \times 3 = \frac{13}{16^3}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to

- (A) $p \Rightarrow q$ (B) $q \Rightarrow p$
 (C) $\sim(p \Rightarrow q)$ (D) $\sim(q \Rightarrow p)$

Ans. (C)

Sol. $\sim p \vee q \equiv p \rightarrow q$

$\sim q \wedge p \equiv \sim(p \rightarrow q)$

Negation of $\sim(p \rightarrow q) \rightarrow (p \rightarrow q)$

is $\sim(p \rightarrow q) \wedge (\sim(p \rightarrow q))$ i.e. $\sim(p \rightarrow q)$

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

- (A) $\frac{3}{2}$ (B) $\frac{26}{9}$
 (C) $\frac{5}{2}$ (D) $\frac{23}{6}$

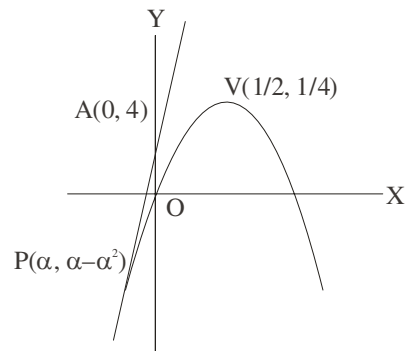
Ans. (C)

Sol. Slope of tangent at P = Slope of line AP

$y'|_P = 1 - 2\alpha = \frac{\alpha - \alpha^2 - 4}{\alpha}$

Solving $\alpha = -2 \Rightarrow P(-2, -6)$

Slope of PV = $\frac{5}{2}$



19. The value of $\tan^{-1} \left(\frac{\cos\left(\frac{15\pi}{4}\right) - 1}{\sin\left(\frac{\pi}{4}\right)} \right)$ is equal to

- (A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$
 (C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Ans. (B)

Sol. $\frac{12}{3} \left[\int_3^b \left(\frac{1}{x^2-4} - \frac{1}{x^2-1} \right) dx \right] = \log \frac{49}{40}$

$$\frac{12}{3} \left[\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| - \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \right]_3^b = \log \frac{49}{40}$$

$$\ln \frac{(b-2)(b+1)^2}{(b+2)(b-1)^2} = \ln \frac{49}{50}$$

$b = 6$

4. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of

$$\left(2x^3 + \frac{3}{x} \right)^{10} \text{ is } 5^{10} - \beta \cdot 3^9, \text{ then } \beta \text{ is equal to } \underline{\hspace{2cm}}$$

Ans. (83)

Sol. $T_{r+1} = {}^{10}C_r (2x^3)^{10-r} \left(\frac{3}{x} \right)^r$

$$= {}^{10}C_r 2^{10-r} 3^r x^{30-4r}$$

Put $r = 0, 1, 2, \dots, 7$ and we get $\beta = 83$

5. If the mean deviation about the mean of the numbers $1, 2, 3, \dots, n$, where n is odd, is $\frac{5(n+1)}{n}$,

then n is equal to _____

Ans. (21)

Sol. Mean deviation about mean of first n natural

numbers is $\frac{n^2-1}{4n}$

$\therefore n = 21$

6. Let $\vec{b} = \hat{i} + \hat{j} + \lambda \hat{k}, \lambda \in \mathbb{R}$. If \vec{a} is a vector such that

$$\vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k} \quad \text{and} \quad \vec{a} \cdot \vec{b} + 21 = 0, \quad \text{then}$$

$$(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k}) \text{ is equal to}$$

Ans. (14)

Sol. $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$

$$\Rightarrow 13 - 1 - 4\lambda = 0 \Rightarrow \lambda = 3$$

$$\Rightarrow \vec{b} = \hat{i} + \hat{j} + 3\hat{k} \Rightarrow \vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$$

$$\Rightarrow (\vec{a} \times \vec{b}) \times \vec{b} = (13\hat{i} - \hat{j} - 4\hat{k}) \times (\hat{i} + \hat{j} + 3\hat{k})$$

$$\Rightarrow -21\vec{b} - 11\vec{a} = \hat{i} - 43\hat{j} + 14\hat{k}$$

$$\Rightarrow \vec{a} = -2\hat{i} + 2\hat{j} - 7\hat{k}$$

Now $(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k}) = 14$

7. The total number of three-digit numbers, with one digit repeated exactly two times, is

Ans. (243)

Sol. If 0 taken twice then ways = 9

If 0 taken once then ${}^9C_1 \times 2 = 18$

If 0 not taken then ${}^9C_1 {}^8C_1 \cdot 3 = 216$

Total = 243

8. Let $f(x) = |(x-1)(x^2-2x-3)| + x - 3, x \in \mathbb{R}$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval $(0, 4)$, then $m + M$ is equal to _____

Ans. (3)

Sol. $f(x) = \begin{cases} (x^2-1)(x-3) + (x-3), & x \in (0,1] \cup [3,4) \\ -(x^2-1)(x-3) + (x-3), & x \in [1,3] \end{cases}$

$$\Rightarrow f'(x) = \begin{cases} 3x^2 - 6x, & x \in (0,1) \cup (3,4) \\ -3x^2 + 6x + 2, & x \in (1,3) \end{cases}$$

$f(x)$ is non-derivable at $x = 1$ and $x = 3$

also $f'(x) = 0$ at $x = 1 + \sqrt{\frac{5}{3}} \Rightarrow m + M = 3$

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

be $\frac{5}{4}$. If the equation of the normal at the point

$\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x + \beta y = \lambda$, then

$\lambda - \beta$ is equal to

Ans. (85)

Sol. $e^2 = 1 + \frac{b^2}{a^2} = \frac{25}{16} \Rightarrow \frac{b^2}{a^2} = \frac{9}{16} \dots\dots(1)$

A $\left(\frac{8}{\sqrt{5}}, \frac{12}{5}\right)$ satisfies $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{64}{5a^2} - \frac{144}{25b^2} = 1 \dots\dots(2)$

Solving (1) & (2) $b = \frac{6}{5}$ $a = \frac{8}{5}$

Normal at A is $\frac{\sqrt{5}a^2x}{8} + \frac{5b^2y}{12} = a^2 + b^2$

Comparing it $8\sqrt{5}x + \beta y = \lambda$

Gives $\lambda = 100, \beta = 15$

$\lambda - \beta = 85$

10. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the

line in zx-plane with x and z intercepts $-\frac{1}{8}$ and

$-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance

between the line l_1 and l_2 , then d^{-2} is equal to

Ans. (51)

Sol. $8x + 4\sqrt{2}y = 1, z = 0$

$\Rightarrow \frac{x - \frac{1}{8}}{1} = \frac{y - 0}{-\sqrt{2}} = \frac{z - 0}{0} = \lambda$

$-8x - 6\sqrt{3}z = 1, y = 0$

$\Rightarrow \frac{x + \frac{1}{8}}{3\sqrt{3}} = \frac{y - 0}{0} = \frac{z - 0}{-4}$

$\begin{vmatrix} \frac{1}{4} & 0 & 0 \\ 1 & -\sqrt{2} & 0 \\ 3\sqrt{3} & 0 & -4 \end{vmatrix} = \sqrt{2}$

$d = \frac{1}{\sqrt{51}}$

$\frac{1}{d^2} = 51$