Q. Let $f(x) = \dfrac{x}{\left(1+x^4\right)^{\frac{1}{4}}}$ & $g(x) = f(f(f(f(x))))$ then $\displaystyle\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)dx$ is

(1) $\dfrac{13}{6}$ (2) $\dfrac{6}{13}$ (3) $\dfrac{2}{5}$ (4) $\dfrac{7}{2}$

$$f(x) = \frac{x}{(1+x^4)^{\frac{1}{4}}}$$

$$\boxed{g(x) = \frac{x}{(1+4x^4)^{\frac{1}{4}}}}$$

$$\int_0^{\sqrt{2\sqrt{5}}} \frac{x^2 \times x}{(1+4x^4)^{1/4}} dx$$

$$\int_1^3 \frac{1}{4} \frac{t^3 dt}{(t^{\cancel{4}})^{\cancel{\frac{1}{4}}}} =$$

$$1+4x^4 = t^4$$

$$16x^3 dx = 4t^3 dt$$

$$\Rightarrow x^3 dx = \frac{1}{4} t^3 dt$$

$$\int_1^3 \frac{1}{4} t^2 dt$$

$$= \frac{1}{4} \cdot \left.\frac{t^3}{3}\right|_1^3$$

$$\Rightarrow \frac{1}{12}\left(3^3 - 1\right)$$

$$= \frac{1}{12}(27-1) = \frac{26}{12} = \frac{13}{6} //$$

Q. $\bar{a}.\bar{b} = 3\sqrt{2}$ & $|\bar{b}|^2 = 6$ such that $\bar{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$. If angle between $\bar{a}$ & $\bar{b}$ is $\frac{\pi}{4}$ then value of $(\alpha^2+\beta^2)\,|\bar{a}\times\bar{b}|^2$ is ______.

$|\vec{b}| = \sqrt{6}$

$|\vec{a}| = \sqrt{1+\alpha^2+\beta^2}$

$*$ $\vec{a}\cdot\vec{b} = 3\sqrt{2}$

$\Rightarrow |\bar{a}||\bar{b}|\cdot\cos\theta = 3\sqrt{2}$

$\Rightarrow \sqrt{1+\alpha^2+\beta^2} \times \cancel{\sqrt{6}}^{\sqrt{3}} \times \frac{1}{\cancel{\sqrt{2}}} = \cancel{3}^{\sqrt{3}}\sqrt{2}$

$\Rightarrow \sqrt{1+\alpha^2+\beta^2} = \sqrt{6}$

$\Rightarrow 1+\alpha^2+\beta^2 = 6$

$\Rightarrow \boxed{\alpha^2+\beta^2 = 5}$

$|\bar{a}\times\bar{b}|^2 = ?$

$|\bar{a}\times\bar{b}|^2 = |\bar{a}|^2|\bar{b}|^2\sin^2\theta$

$\boxed{\bar{a}\times\bar{b} = |a||b|\sin\theta}$

$= (1+\alpha^2+\beta^2) \times \cancel{6}^{3} \times \frac{1}{\cancel{2}}$

$= (5+1)\times 3$

$= \underline{\underline{18}}$

Q. $R = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$, $x \sin\theta = y \sin\left(\theta + \frac{2\pi}{3}\right) = z \sin\left(\theta + \frac{4\pi}{3}\right) \neq 0$

Statement – 1 : Trace (R) = 0

Statement – 2 : Trace (adj(adj (R)) = 0

~~(1) Statement –1 is true and statement – 2 is false~~ (2) Statement –1 is false and statement – 2 is false

(3) Statement –1 is false and statement – 2 is true ~~(4) Statement –1 is true and statement – 2 is true~~

$R = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$Tr(R) = x + y + z$

(I) $x \sin\theta \neq 0 \Rightarrow x \neq 0,$

(II) $x \sin\theta = y \sin\left(\frac{2\pi}{3}\right)$

$\Rightarrow y = \frac{x \sin\theta}{\sin\left(\pi - \frac{\pi}{3}\right)} = \frac{2x \sin\theta}{\sqrt{3}}$ ✓

$\Rightarrow z = \frac{x \sin\theta}{\sin\left(\pi + \pi/3\right)} = -\frac{2x \sin\theta}{\sqrt{3}}$ ✓

$$x+y+z = x + \frac{2x\sin\theta}{\sqrt{3}} - \frac{2x\sin\theta}{\sqrt{3}}$$

$$\underline{\neq 0}$$

$$R = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}, \quad adj(R) = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}, \quad adj(adj R) = \begin{bmatrix} x^2yz & 0 & 0 \\ 0 & y^2xz & 0 \\ 0 & 0 & z^2xy \end{bmatrix}$$

$$tr(adj(adj(R))) = x^2yz + xy^2z + xyz^2 = xyz(x+y+z) \neq 0$$

Q. If $|\vec{a}\times\vec{b}| = 2$, and $|\vec{a}| = 1$, then $|(\vec{a}\times\vec{b})-\vec{a}|^2$ is

$|\vec{a}\times\vec{b}| = 2$

$|\vec{a}| = 1$

$|(\vec{a}\times\vec{b})-\vec{a}|^2 = |\vec{a}\times\vec{b}|^2 + |\vec{a}|^2 - \cancel{2(\vec{a}\times\vec{b})\cdot\vec{a}} \rightarrow 0$

$= 2^2 + 1$

$= \underline{\underline{5}}$

Q. If 11th term of G.P., whose 1st term is 'a' and 3rd term is 'b', is equal to pth term of G.P. whose first term is 'a' and 5th term is 'b', then value of p is

21

(Seqn & Series)

$GP_1 : a, \; ar_1^2 = b$

$GP_2 : a, \; ar_2^4 = b$

$T_{11} = T_p'$

$\cancel{a}r_1^{10} = \cancel{a}r_2^{p-1}$

$\cancel{a}r_1^2 = \cancel{a}r_2^4$

$\Rightarrow \boxed{r_1 = \pm(r_2^2)}$

$\Rightarrow (\pm r_2^2)^{10} = r_2^{p-1}$

$\Rightarrow r_2^{20} = r_2^{p-1} \Rightarrow p = 20+1 = 21$

Q. Let $f(x)=\begin{cases} x^2+3x+a\,; & x\le 1 \\ bx+2\,; & x>1 \end{cases}$ is differentiable everywhere. The value of $\int_{-2}^{2} f(x)dx$ is

(1) $\frac{37}{2}$ (2) $\frac{36}{2}$ (3) $\frac{37}{4}$ (4) $\frac{36}{4}$

* diff everywhere ⇒ cts at $x=1$ also,

LHL = RHL = f(1)

$\Rightarrow 1+3+a$

$\boxed{1+3+a=b+2}$

$\Rightarrow \boxed{a+2-b=0}$

diff ⇒ LHD = RHD at $x=1$,

$2x+3=b+0$

$2+3=b \Rightarrow \boxed{b=5}$

$\Rightarrow a+2-5=0 \Rightarrow a=3$

$a=3$
$b=5$

$$f(x)=\begin{cases} x^2+3x+3, & x\le 1 \\ 5x+2, & x>1 \end{cases}$$

$$\int_{-2}^{2} f(x)dx = \int_{-2}^{1} f(x)dx + \int_{1}^{2} f(x)dx$$

$$= \int_{-2}^{1} (x^2+3x+3)dx + \int_{1}^{2} (5x+2)dx \longrightarrow$$

Q. Bag A contains 3 white & 7 red balls and bag B contains 2 white & 3 Red balls. If a ball is picked up randomly then what is the probability that the ball picked is white from bag A

(1) $\frac{3}{20}$ (2) $\frac{2}{20}$ (3) $\frac{3}{10}$ (4) $\frac{4}{20}$

\# Bag A $\begin{cases} 3W \\ 7R \end{cases}$

Bag B $\begin{cases} 2W \\ 3R \end{cases}$

Bag A, Ball → W ⇒ ${}^3C_1 \times {}^{10}C_1$ ✓

Bag B, Ball → W ⇒ ${}^2C_1 \times {}^5C_1$

$$P(E) = \frac{\text{fav}}{\text{Total}}$$

$$= \frac{{}^3C_1 \times {}^{10}C_1}{{}^2C_1 \times {}^5C_1 + {}^3C_1 \times {}^{10}C_1}$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Q. Area bounded by $(y-2)^2 = x-1$, $x-2y+4=0$ and positive coordinate axes, is

$*\ (y-2)^2 = x-1$ $\quad$ $x=0$, $y=2$

$V(1,2)$, $\quad x-2y+4=0$

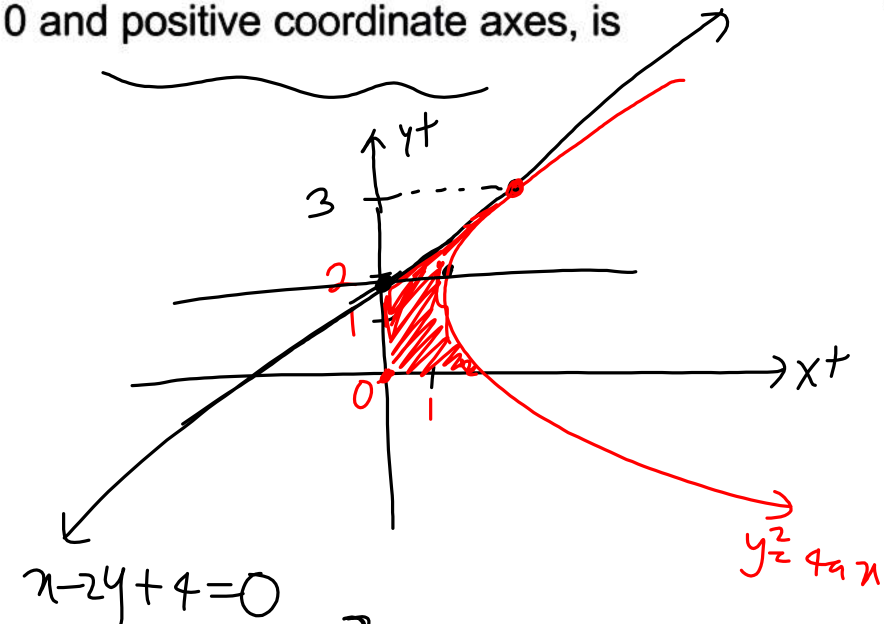
$\Rightarrow (y-2)^2+1 = x$

$\Rightarrow (y-2)^2+1 = 2y-4$

$\Rightarrow y^2+4-4y+1-2y+4=0$

$\Rightarrow y^2-6y+9=0$

$\Rightarrow (y-3)^2=0 \Rightarrow y=3,3$



$$\text{area} = \int_0^2 \left[(y-2)^2+1\right] dx + \int_2^3 \left((y-2)^2+1-(2y-4)\right) dy$$

Q. Let $f(x) = ae^{2x} + be^{x} + cx$, where $f(0) = -1$, $f'(\log_e 2) = 21$ and $\int_0^{\log_e 4} (f(x) - cx)dx = \frac{39}{2}$ then the value of $|a + b + c|$ is ________.

$$\boxed{\begin{aligned} a &= +5 \\ b &= -6 \\ c &= -24 \end{aligned}}$$

$f(x) = ae^{2x} + be^{x} + cx,$

$f(0) = -1$

$\boxed{-1 = a + b}$ — 1

$f'(x) = ae^{2x}(2) + be^{x} + c$

$f'(\log_e 2) = 2ae^{2(\ln 2)} + be^{\ln 2} + c$

$21 = 2a \times 4 + 2b + c$

$\Rightarrow \boxed{21 = 8a + 2b + c}$ — 2

$\int_0^{\log_e 4} (ae^{2x} + be^{x})\,dx = \frac{39}{2}$

$\Rightarrow \left. a\frac{e^{2x}}{2} + be^{x} \right|_0^{\log_e 4} = \frac{39}{2}$

$\Rightarrow \frac{a}{2}\left(e^{2\log 4}\right) + b\left(e^{\log 4}\right) - \frac{a}{2} - b = \frac{39}{2}$

$\Rightarrow \boxed{\frac{a}{2} \times 8 + 4b - \frac{a}{2} - b = \frac{39}{2}}$ — 3

Q. A question paper has three sections A,B, C having 8, 6, 4 questions respectively. If the student has to answer 15 questions attempting atleast four from each section. Find the number of ways the paper can be answered by student.

(1) 342 (2) 344 (3) 374 (4) 340

A → 8 B → 6 C → 4

Total = 15

Min → 4

	A	B	C
C–1	5	6	4
C–2	6	5	4
C–3	7	4	4

(C–1) $^8C_5 \times {}^6C_6 \times {}^4C_4$

(C–2) $^8C_6 \times {}^6C_5 \times {}^4C_4$ +

(C–3) $^8C_7 \times {}^6C_4 \times {}^4C_4$ +

Q. Let $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, $3\sin(\alpha + \beta) = 2\sin(\alpha - \beta)$ and $\tan\alpha = k \tan\beta$ then the value of k is

(1) – 5 (2) 5 (3) 3 (4) – 3

$\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$

$3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$

$\Rightarrow \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{2}{3}$

$\Rightarrow \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \frac{2}{3}$

$(C \& D) \Rightarrow \frac{\cancel{2}\sin\alpha\cos\beta}{\cancel{2}\cos\alpha\sin\beta} = \frac{2+3}{2-3}$

$(C \& D)$

$\frac{a}{b} \rightsquigarrow \frac{a+b}{a-b}$

$$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{5}{-1}$$

$$\Rightarrow \tan\alpha = -5 \tan\beta$$

$$\downarrow$$

$$\underline{\underline{k}}$$

Q. $L_1 \equiv \hat{i}-\hat{j}+2\hat{k}+\lambda(\hat{i}-\hat{j}+2\hat{k}) \quad \lambda \in R$

$L_2 \equiv \hat{j}-\hat{k}+\mu(3\hat{i}+\hat{j}+p\hat{k})$

$L_3 \equiv s(l\hat{i}+m\hat{j}+n\hat{k})$

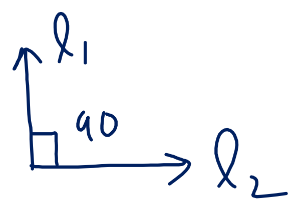
L_1 is perpendicular to L_2

L_3 is perpendicular to L_1 & L_2

then (ℓ, m, n) can be

~~(1) (–1, 7, 4)~~ (2) (4, –1, 7) (3) (7, 4, –1) (4) (7, –1, 4)

$L_1 \perp L_2 \Rightarrow$



$(\hat{i}-\hat{j}+2\hat{k})\cdot(3\hat{i}+\hat{j}+p\hat{k})=0 \Rightarrow 3-1+2p=0 \Rightarrow \boxed{p=-1}$

L_3 is $\perp^{ar}$ to both $L_1 \,\&\, L_2$.

L_3 $\qquad$ $L_1 \times L_2 \Rightarrow \perp^{ar}$ to $L_1 \,\&\, L_2$

$\Rightarrow L_3 \parallel (L_1 \times L_2)$

$$L_1 \times L_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix}$$

$l_1 \quad \hat{i} - \hat{j} + 2\hat{k}$

$l_2 : \quad 3\hat{i} + \hat{j} - \hat{k}$

$$= \hat{i}(1-2) - \hat{j}(-1-6) + \hat{k}(1+3)$$

$$l\hat{i} + m\hat{j} + n\hat{k} = -\hat{i} + 7\hat{j} + 4\hat{k}$$

$l = -1,\ m = 7,\ n = 4$

Q. Find number of real solutions of the equation $x(x^2 + |x| + 5|x-1| - 6|x-2|) = 0$

(1) 2 (2) 3 (3) 4 (4) 6

* $x = 0$ $x^2 + |x| + 5|x-1| + 6|x-2| = 0$

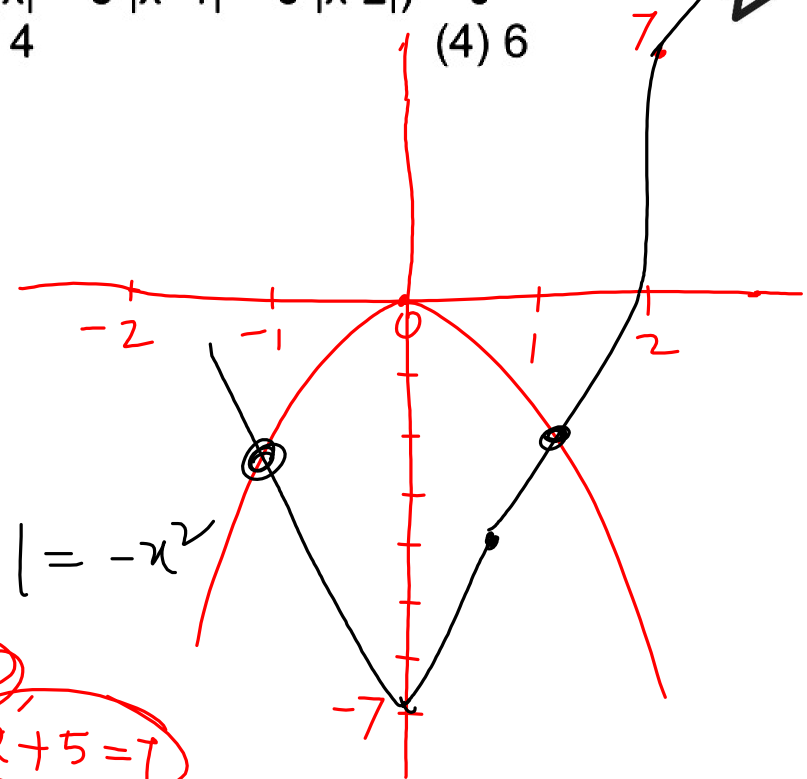
M1 lengthy → cases → simplify → 2

M2 graphical –

$3 + 10 - 6 = 7$

$\Rightarrow |x| + 5|x-1| - 6|x-2| = -x^2$

$x = 0$, $0 + 5 - 12 = -7$ $x = 2$ $2 + 5 = 7$



Q. If the domain of $f(x) = \ell n\left(\frac{2x+3}{4x^2-x-3}\right) + \cos^{-1}\left(\frac{2x+1}{x+2}\right)$ is (α, β) find $5\alpha - 4\beta$.

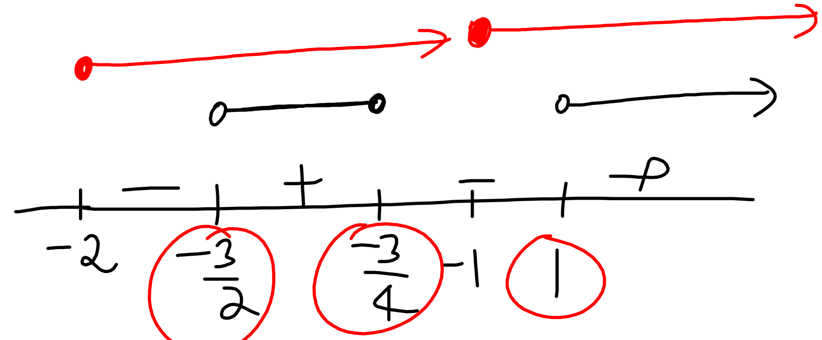
(1) – 2 (2) 2 (3) 4 (4) – 4

$\ln x,\ x>0$

$$\frac{2x+3}{4x^2-x-3} > 0$$

$$\Rightarrow \frac{2\left(x+3/2\right)}{4x^2-4x+3x-3} > 0$$

$$\Rightarrow \frac{2\left(x+3/2\right)}{(4x+3)(x-1)} > 0$$



$$-1 \le \frac{2x+1}{x+2} \le 1$$

$$\frac{2x+1}{x+2} + 1 \ge 0 \Rightarrow \frac{2x+1+x+2}{(x+2)} \ge 0 \qquad \frac{3(x+1)}{x+2} \ge 0$$

Q. If A = {1, 2,3,4} then find number of symmetric relation on A which is not reflexive is

$\boxed{n(A)=4}$

$$\# = 2^{\frac{n^2+n}{2}}$$

$$= 2^{\frac{16+4}{2}}$$

$$= 2^{10}$$