RANKERS

JEE Main (2024)

MEMORY BASED PAPER SOLUTION

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Mathematics

Q. $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $\quad |\vec{a}| = 1 \quad \vec{a}.\vec{b} = 2$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ $\quad |\vec{b}| = 4$

$\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$ then angle between $\vec{c}$ and $\vec{b}$

$|\vec{a}||\vec{b}|\cos\phi = 2$

$\cos\phi = \frac{1}{2}$

Sol^n

$\vec{b}\cdot\vec{c} = 0 - 3|\vec{b}|^2$

$\vec{b}\cdot\vec{c} = -48$

$|\vec{b}||\vec{c}|\cos\theta = -48$

$\cos\theta = \frac{-12}{|\vec{c}|}$

$= \frac{-12}{\sqrt{192}}$

$|\vec{c}|^2 = |2(\vec{a}\times\vec{b}) - 3\vec{b}|^2$

$= [2(\vec{a}\times\vec{b}) - 3\vec{b}]\cdot[2(\vec{a}\times\vec{b}) - 3\vec{b}]$

$= 4|\vec{a}\times\vec{b}|^2 - 0 - 0 + 9|\vec{b}|^2$

$\Rightarrow |\vec{c}|^2 = 4|\vec{a}\times\vec{b}|^2 + 144$

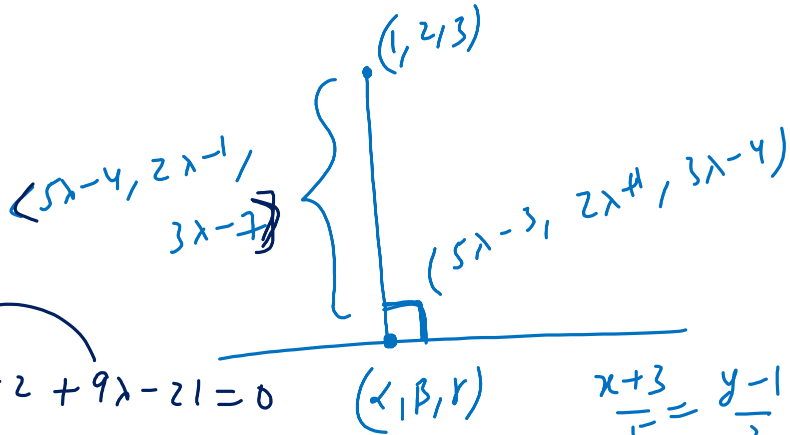
$= 4(|a||b|\sin\phi)^2 + 144$

$= 4\times(1\times 4\times \frac{\sqrt{3}}{2})^2 = 4\times 12 + 144$

$= 192$

Q.

Q. Let (α,β,γ) be the foot of perpendicular from the point $(1,2,3)$ on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$ then $19(\alpha+\beta+\gamma)$



$25\lambda-20+4\lambda-2+9\lambda-21=0$

$38\lambda-43=0$

$\boxed{\lambda=\frac{43}{38}}$ #

$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$

$\{5\lambda-3,\ 2\lambda+1,\ 3\lambda-4\}$ → α, β, γ

Q.

Q. If $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + \ldots\ldots$ up to 10 terms and $\beta = \sum_{N=1}^{10} N^4$ such that $4\alpha - \beta = 55k + 40$, then find k.

4–5 min #

lengthy

3 4 5

$t_n = an^2 + bn + c$ #

$t_1 = 1 = a + b + c$ #

$t_2 = 4 = 4a + 2b + c$ #

$t_3 = 8 = 9a + 3b + c$ #

$3a + b = 3$ — (1)

$5a + b = 4$

$-2a = -1$

$a = \frac{1}{2}$

$b = 3 - \frac{3}{2} = \frac{3}{2}$ #

$c = 1 - \frac{1}{2} - \frac{3}{2} = -1$

$\alpha = \sum_{n=1}^{10} \left(\frac{1}{2}n^2 + \frac{3}{2}n - 1\right)^2$

$\alpha =$ ______

Q.

Q.

Class	Frequency	cf
0-4	2	2
4-8	9	11
8-12	10	21
12-16	8	29
16-20	7	36
Total	**36**	

If median is M then find the value of 20M

(1) 208 (2) 104 (3) 52 (4) 216

M.C. → 8-12

$\frac{N}{2} = 18$

$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$

$= 8 + \left(\frac{18 - 11}{10}\right) \times 4 = \frac{108}{10} \times 20$

Q.

Q. sec x dy – {2(1–x) tan x + x(2–x) }dx = 0

$$dy - \{2(1-x)\sin x + x(2-x)\cos x\}dx = 0$$

Exact

$$\int dy = \int \{2(1-x)\sin x + (2x - x^2)\cos x\}dx$$

$y = (2x - x^2)\sin x$

Q.

Q. The value of $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is

(1) $\frac{\pi}{2\sqrt{2}}-\frac{\pi}{4}$ (2) $\frac{\pi}{2\sqrt{3}}-\frac{\pi}{8}$ (3) $\frac{\pi}{2\sqrt{3}}+\frac{\pi}{4}$ (4) $\frac{\pi}{\sqrt{3}}-\frac{\pi}{8}$

Sol^n

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1/n}{\left(1+\left(\frac{k}{n}\right)^2\right)\left(1+3\cdot\left(\frac{k}{n}\right)^2\right)}$$

$$\int_0^1\frac{dx}{(1+x^2)(1+3x^2)} = \frac{-1}{2}\int_0^1\frac{(1+3x^2)-3(1+x^2)}{(1+x^2)(1+3x^2)} = -\frac{1}{2}\int_0^1\frac{1}{1+x^2}dx - \int_0^1\frac{3}{1+3x^2}dx$$

$\tan^{-1}x$ $\quad\left[\frac{3}{\sqrt{3}}\tan^{-1}\sqrt{3}x\right]_0^1$

Q.

Q. If x, y ∈ {0,1,2,3, ….., 10} then the probability that $|x - y| > 5$ is

(1) $\frac{30}{121}$ (2) $\frac{31}{121}$ (3) $\frac{60}{121}$ (4) $\frac{62}{121}$

$n(S) = {}^{11}C_1 \times {}^{11}C_1 = 121$

n(E)

		y	
x = 0	$\|y\| > 5$	6, 7, 8, 9, 10	5
x = 1	$\|1-y\| > 5$	7, 8, 9, 10	4
x = 2	$\|2-y\| > 5$	8, 9, 10	3

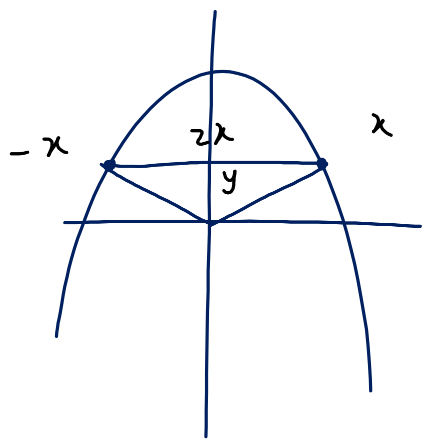
x = 3	$\|3-y\| > 5$	9, 10	2
x = 4	$\|4-y\| > 5$	10	1
x = 5	$\|5-y\| > 5$	×	
x = 6	$\|6-y\| > 5$	0	1
x = 7	$\|7-y\| > 5$	0, 1	2

x = 8 → 3
x = 9 → 4
x = 10 → 5

Q.

Q. A triangle is formed by vertices (0, 0), (x, y), (–x, y) on xy – plane. If the point (x, y) and (–x, y) lies on $y = -x^2 + 54$, then maximum area of triangle is

(1) $18\sqrt{2}$ (2) $108\sqrt{2}$ (3) $36\sqrt{2}$ (4) $54\sqrt{2}$



$$A = \frac{1}{2} \times 2x \times (54 - x^2)$$

$$\frac{dA}{dx} = 54 - 3x^2 = 0 \quad \Big| \quad x^2 = 18, \quad x = \pm\sqrt{18}$$

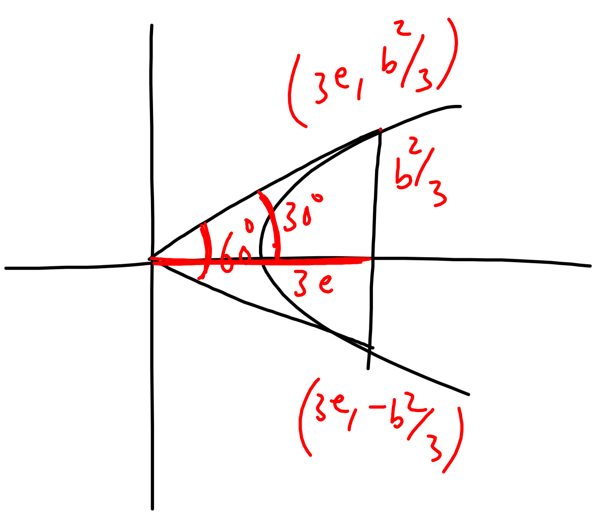
$$A_{max} = \sqrt{18} \times (54 - 18)$$

$$= 3\sqrt{2} \times 36 = 108\sqrt{2}$$

Q.

Q. If latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtends 60° at centre of hyperbola and $b^2 = \frac{\ell}{m}\left(1+\sqrt{n}\right)$, ℓ, m, n $\in$ N, ℓ, m being co-prime, then $\ell^2 + m^2 + n^2$ is

(1) 180 (2) 181 (3) 182 (4) 183



$\tan 30^\circ = \frac{b^2}{3\times 3e} = \frac{b^2}{9e} = \frac{1}{\sqrt{3}}$

$\Rightarrow b^2 = \frac{9e}{\sqrt{3}} = 3\sqrt{3}e$

$\Rightarrow 3\sqrt{3}e = 9e^2 - 9$

$\Rightarrow 3e^2 - \sqrt{3}e - 3 = 0$

$b^2 = a^2(e^2-1)$, $a^2 = 9$

$e = ?$

Q.

Q. Number of integral term in the expansion of $\left(7^{\frac{1}{2}} + 11^{\frac{1}{6}}\right)^{824}$

(1) 139 (2) 138 (3) 140 (4) 137

M1 proper Method –

$T_{r+1} = {}^{824}C_r \, 7^{\frac{1}{2}(824-r)} \, 11^{\frac{1}{6}r}$

$\frac{r}{6} \to$ integer

$\frac{r}{2} \to$ integer

$\Rightarrow \frac{r}{6} \in Z$

$r = 0, 6, 12, 18, \ldots, 6 \times 137$

$\frac{824}{6} = 137$

Total terms $= 137 + 1 = 138$

Q.

(M2) $n = 824$,

$$\begin{array}{c|c} 2 & 2, 6 \\ 3 & 1, 3 \\ & 1 \end{array}$$

$lcm(2, 3) = 6$

$$\left[\frac{824}{6}\right] + 1 \Rightarrow 137 + 1 = 138$$

Q. For any real number x, Let [x] denote the largest integer less than or equal to x. The value of

$$9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx$$

$[\ ] \to$ GIf

$y = \sqrt{\frac{10x}{x+1}}$ → at $x=0$, $y=0$; $x=9$, $\sqrt{9}=3$, $y=3$

$y_{max} = 3$, $y_{min} = 0$

$y=0$, $y=1$: $\sqrt{\frac{10x}{x+1}} = 1$, $\frac{10x}{x+1} = 1 \Rightarrow x = \frac{1}{9}$

$y=2$, $\sqrt{\frac{10x}{x+1}} = 2 \Rightarrow \frac{10x}{x+1} = 4$

$10x = 4x + 4$, $6x = 4$, $x = 2/3$

$$9\int_0^9 [y]\,dx$$

$$\Rightarrow 9\left[\int_0^{1/9} 0\,dx + \int_{1/9}^{2/3} 1\,dx + \int_{2/3}^{9} 2\,dx\right]$$

$$\Rightarrow 9\left[x\Big|_{1/9}^{2/3} + 2x\Big|_{2/3}^{9}\right] \Rightarrow \boxed{}$$

Q.

Q. If S_n denotes sum of first n terms of an A.P. such that, $S_{20} = 790$, $S_{10} = 145$ then $S_{15} - S_5$

(1) 540 (2) 395 (3) 555 (4) 575

$S_{20} = 790 \Rightarrow 10[2a+(n-1)d] = 790$

$\Rightarrow 2a+(n-1)d = 79$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = 145$

$5[2a+(n-1)d] = 145$ (29)

$\Rightarrow 2a+19d = 79$ —— (1)

$\Rightarrow 2a+9d = 29$ —— (2)

(1) − (2) $\Rightarrow 10d = 79 - 29 = 50$

$d = 5$

$2 \times a = 79 - 19 \times 5$

$2a = 79 - 95$

$\Rightarrow a = -\frac{16}{2} = -8$

Q.

$$\left.\begin{array}{l} a = -8 \\ d = 5 \end{array}\right\}$$

$$S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$$

$$= \boxed{}$$

Q. A line making an angle 30º with positive x-axis at (4, 0). Now it is rotated by an angle 15º in clockwise direction. The equation of line is

(1) $x + y - 4 = 0$

(2) $x - y - 4 = 0$

(3) $\left(\sqrt{3} - 2\right)x + y + 8 - 4\sqrt{3} = 0$

(4) $\left(2 - \sqrt{3}\right)x - y - 8 + 4\sqrt{3} = 0$

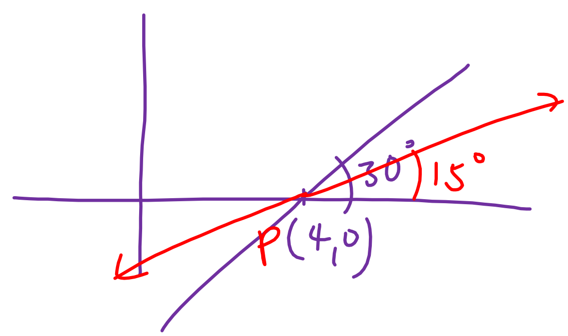
$P(4,0)$

$\theta = 15^\circ = 60 - 45^\circ$

$\Rightarrow \tan\theta = m = \dfrac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$

$\tan 15 = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}} = m$

(point-slope form)



Q.

$P(4,0) = (x_1, y_1)$

$m = \frac{\sqrt{3}-1}{1+\sqrt{3}}$

$y - y_1 = m(x - x_1)$

Q. Area bounded by curves $y^2 = 4(x-2)$ and $y = 2x - 8$ is

$y^2 = 4(x-2) \Rightarrow \frac{y^2}{4} + 2 = x$

$y = 2x - 8$

$\Rightarrow \frac{y+8}{2} = x$

$\Rightarrow \frac{y+8}{2} = \frac{y^2}{4} + 2$

$\Rightarrow \frac{y+8}{2} = \frac{y^2+8}{4}$

$\Rightarrow 2y + 16 = y^2 + 8$

$\Rightarrow y^2 + 8 - 2y - 16 = 0$

$\Rightarrow y^2 - 2y - 8 = 0$

$\Rightarrow y^2 - 4y + 2y - 8 = 0$

$\Rightarrow y(y-4) + 2(y-4) = 0$

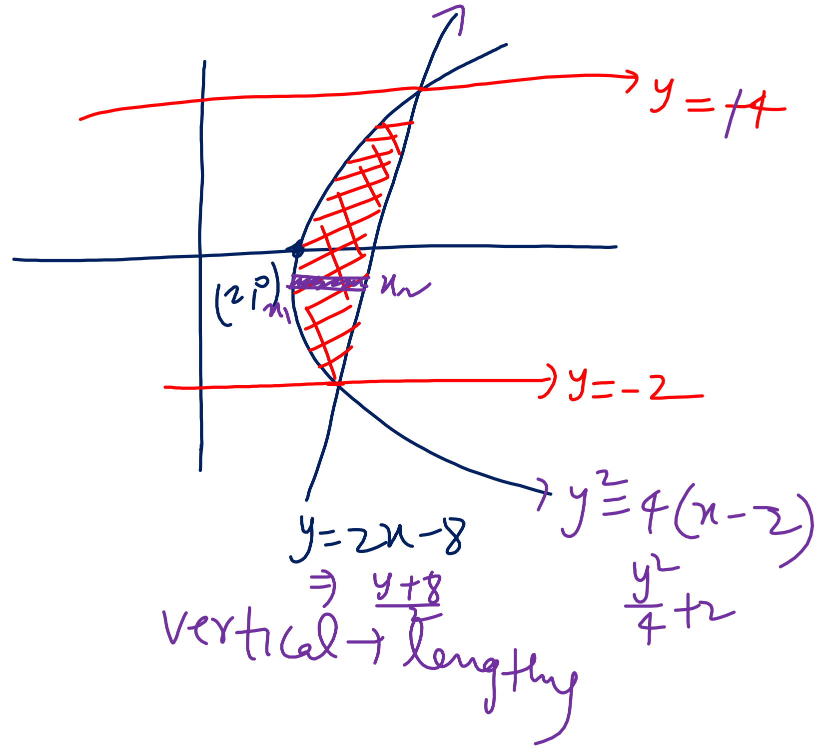
$\Rightarrow y = -2, 4$

Q.

horizonal strips -

$$\int_{y=-2}^{4}\left[\left(\frac{y+8}{2}\right)-\left(\frac{y^2}{4}+2\right)\right]dy$$

$\downarrow$



Q. Circle $(x+1)^2 + (y+2)^2 = r^2$ & $x^2 + y^2 - 4x - 4y + 4 = 0$
Cuts each other at two different points then value of r is

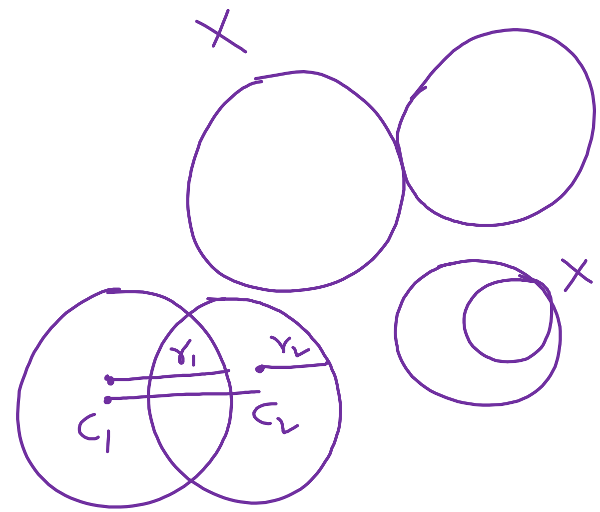
(1) $\frac{1}{2} < r < 7$ (2) $0 < r < 7$ (3) $3 < r < 7$ (4) $5 < r < 9$

cond^n for intersecting,

$|r_1 - r_2| < C_1C_2 < |r_1 + r_2|$

S_2: $x^2 + y^2 - 4x - 4y + 4 = 0$

$r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 4 - 4} = 2$



Q.

$r_1 = r$

$r_2 = 2$

$|r_1 - r_2| < C_1C_2 < |r_1 + r_2| \Rightarrow$ (i) $|r_1 - r_2| < C_1C_2$

$C_1(-1,-2),\ C_2(2,2)$

$C_1C_2 = \sqrt{9+16}$

$= 5$

$\Rightarrow |r-2| < 5$

$\Rightarrow -5 < r-2 < 5$

$\Rightarrow \boxed{-3 < r < 7}$ —— (1)

(ii) $5 < |2+r|$

$\boxed{r > 5-2 = 3}$ —— (2)

$\cap$

Q. $f(0)=\frac{1}{2}$, find $\lim\limits_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2}-1}=\alpha$ then find $8\alpha^2$.

(1) 3 (2) 1 (3) 2 (4) 0

$f(0)=\frac{1}{2}$ — ①

$\lim\limits_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2}-1}$ $\left(\frac{0}{0}\right)$

$\Rightarrow \lim\limits_{x\to 0}\frac{1\int_0^x f(t)dt + x\,f(x)\,(1-0)}{e^{x^2}\,2x}$ $\left(\frac{0}{0}\right)$

$\Rightarrow \frac{f(x)\cdot 1 + x f'(x) + f(x)}{e^{x^2}\cdot 2 + 2x \times e^{x^2}\,2x}$

$\Rightarrow \frac{2f(x)}{2e^{x^2}} = f(0)=\alpha$

$\alpha=\frac{1}{2}$ $\Rightarrow 8\times\alpha^2 = 8\times\frac{1}{4} = 2$

Q.

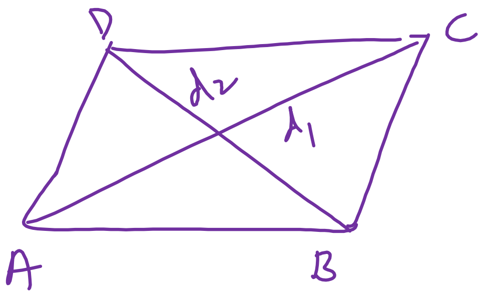
Q. Let A (2, 3, 5) and C (–3, 4, –2) be opposite vertices of a parallelogram ABCD. If the diagonal $\vec{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then area of parallelogram is equal to

$d_2 = \vec{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$ ✓

area(||gm) $= \frac{1}{2}|d_1 \times d_2|$

$d_1 = \vec{AC} = (2+3)\hat{i} + (3-4)\hat{j} + (5+2)\hat{k}$

$= 5\hat{i} - \hat{j} + 7\hat{k}$



area $= \frac{1}{2}|d_1 \times d_2| = \frac{1}{2}|-17\hat{i} - 8\hat{j} + 11\hat{k}|$

$= \frac{1}{2}\sqrt{17^2 + 8^2 + 11^2}$ //

$d_1 \times d_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}(-3-14) - \hat{j}(15-7) + \hat{k}(10+1)$

Q.

Q. If $z = x + iy$, $xy \neq 0$ satisfy the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ equal to

$(x \neq 0, y \neq 0, \Rightarrow z \neq 0 \Rightarrow |z| \neq 0)$

$z^2 + i\bar{z} = 0$

$|z^2| = |z|^2$

$z^2 + i\bar{z} = 0$

$\Rightarrow z^2 = -i\bar{z}$

$\Rightarrow |z^2| = |-i\bar{z}| = |\bar{z}| = |z|$

$\Rightarrow |z|^2 = |z|$

$\Rightarrow |z|(|z| - 1) = 0 \rightarrow |z| = 1 \Rightarrow |z^2| = 1$

Q.

Q. If the length of the minor axis of an ellipse is equal to half of the distance between the foci then the eccentricity of the ellipse is

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{7}}$ (4) $\frac{3}{\sqrt{7}}$

$e = ?$

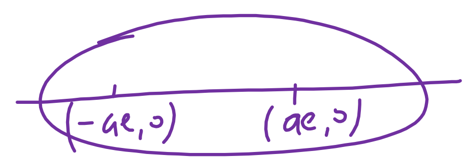
$2b = \frac{1}{2} 2ae$

$\Rightarrow \frac{b}{a} = \frac{e}{2}$

$\Rightarrow \left(\frac{b}{a}\right)^2 = \frac{e^2}{4}$ —— (1)

$b^2 = a^2(1-e^2)$

$\Rightarrow \left(\frac{b}{a}\right)^2 = 1 - e^2$



$\frac{e^2}{4} = 1 - e^2$

$\Rightarrow e^2 = 4 - 4e^2$

$\Rightarrow 5e^2 = 4$

$e = 2/\sqrt{5}$

Q.

Q. If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5} f'(0)$ is equal to

$f'(x) = \begin{vmatrix} 8\cos^3 x(-\sin x) & 8\sin^3 x \cos x & 2\sin 2x \cos 2x \cdot 2 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2' \\ R_2 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R_3' \end{vmatrix}$

$f'(0)$: $0 + 0 + 0$

$\frac{1}{5} f'(0) = 0$

Q.

Q. If $x^2 - 70x + \lambda = 0$ have roots $\alpha, \beta \in N$, $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. Find minimum value of λ

(1) 320 (2) 325 (3) 330 (4) 335

$x^2 - 70x + \lambda = 0$ ⟨ α, β,

$\alpha, \beta \in N$, $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$

$\alpha + \beta = 70$

$\alpha\beta = \lambda$

⇒ λ is not multiple of 2 or 3

$70, 0$ ✗ $1, 69 = \lambda$ ✗

$2 \times 68 = \lambda$ ✗

$3 \times 67 = \lambda$ ✗

$4 \times 66 = \lambda$ ✗

$5 \times 65 = \lambda$

$65 \times 5 =$ least

$\lambda_{min} = 325$

Q.

Q. g(x) is non constant differentiable functions $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$ and $f(x) = \frac{1}{2}[g(x) + g(2-x)]$

(1) $f'\left(\frac{1}{2}\right) + f'\left(\frac{3}{2}\right) = 1$

(2) f''(x) = 0, for at least 1 value of x ∈ (0, 2)

(3) f'''(x) = 0, for number of values of x ∈ (0, 1)

(4) f''(x) = 0, for exactly one value of x ∈ (0, 1)

$$f(x) = \frac{1}{2}[g(x) + g(2-x)]$$

$$f'(x) = \frac{1}{2}[g'(x) + g'(2-x)(-1)]$$

$$f'(x) = \frac{1}{2}[g'(x) - g'(2-x)]$$

$$f'\left(\tfrac{1}{2}\right) = \tfrac{1}{2}\left[g'\left(\tfrac{1}{2}\right) - g'\left(\tfrac{3}{2}\right)\right] = 0$$

$$f'\left(\tfrac{3}{2}\right) = \tfrac{1}{2}\left[g'\left(\tfrac{3}{2}\right) - g'\left(\tfrac{1}{2}\right)\right] = 0$$

$$f'(1) = \tfrac{1}{2}[g'(1) - g'(1)] = 0$$

$2 - \frac{1}{2} = \frac{3}{2}$

⇒ f' has 3 roots b/w (0,2)

⇒ f'' will have at least 2 roots b/w (0,2)

Q.

Q. $\lim_{x\to 0} \frac{ae^{x^2}+b\cos x}{x^2} = \frac{1}{2}$ then

(1) $a = \frac{1}{3}, b = \frac{1}{3}$

(2) $a = \frac{1}{2}, b = -\frac{1}{2}$

(3) $a = -\frac{1}{3}, b = -\frac{1}{2}$

(4) $a = \frac{1}{3}, b = -\frac{1}{3}$

$\lim_{x\to 0} \frac{ae^{x^2}+b\cos x}{x^2} = \frac{1}{2}$ (finite)

$\frac{a+b}{0} \Rightarrow \boxed{a+b=0}$ — (1)

$\Rightarrow \lim_{x\to 0} \frac{ae^{x^2}2x - b\sin x}{2x} \left(\frac{0}{0}\right)$

$\Rightarrow \lim_{x\to 0} \frac{ae^{x^2}\cdot 2 + 2xae^{x^2}(2x) - b\cos x}{2}$

$\Rightarrow \frac{2a-b}{2} = \frac{1}{2}$

Q.

$$a+b=0$$
$$2a-b=1$$
$$\Rightarrow 2a-(-a)=1$$
$$\Rightarrow 3a=1$$
$$\Rightarrow a=\frac{1}{3}$$
$$b=-\frac{1}{3}$$