JEE Main 2023 (1st Attempted)
(Shift - 02 Mathematics Paper)

25.01.2023

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

61. Let the function $f(x)=2x^3 + (2p-7)x^2+3(2p-9)x-6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is

(1) $\left(\frac{9}{2},\infty\right)$ (2) $\left(0,\frac{9}{2}\right)$

(3) $\left(-\infty,\frac{9}{2}\right)$ (4) $\left(-\frac{9}{2},\frac{9}{2}\right)$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$

$f'(x) = 6x^2 + 2(2p-7)x + 3(2p-9)$

$f'(0) < 0$

$\therefore 3(2p-9) < 0$

$p < \frac{9}{2}$

$p \in \left(-\infty,\frac{9}{2}\right)$

62. Let z be a complex number such that $\left|\frac{z-2i}{z+i}\right| = 2, z \neq -i$. Then z lies on the circle of radius 2 and centre

(1) (2, 0) (2) (0, 0)

(3) (0, 2) (4) (0, –2)

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $(z-2i)\,(\bar{z}+2i) = 4(z+i)\,(\bar{z}-i)$

$z\bar{z} + 4 + 2i(z-\bar{z}) = 4(z\bar{z}+1+i(\bar{z}-z))$

$3z\bar{z} - 6i(z-\bar{z}) = 0$

$x^2 + y^2 - 2i(2iy) = 0$

$x^2 + y^2 + 4y = 0$

63. If the function

$$f(x) = \begin{cases} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}}, & 0 < x < \frac{\pi}{2} \\ \mu, & x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}}, & \frac{\pi}{2} < x < \pi \end{cases}$$

is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to

(1) 11 (2) 8

(3) $2e^4+8$ (4) 10

Official Ans. by NTA (4)

Allen Ans. BONUS

Sol. $\Rightarrow \lim\limits_{x\to\frac{\pi^+}{2}} e^{\frac{\cot 6x}{\cot 4x}} = \lim\limits_{x\to\frac{\pi^+}{2}} e^{\frac{\sin 4x.\cos 6x}{\sin 6x.\cos 4x}} = e^{2/3}$

$\Rightarrow \lim\limits_{x\to\frac{\pi^-}{2}} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} = e^{\lambda}$

$\Rightarrow f(\pi/2) = \mu$

For continuous function $\Rightarrow e^{2/3} = e^{\lambda} = \mu$

$\lambda = \frac{2}{3}, \mu = e^{2/3}$

Now, $9\lambda + 6\log_e\mu + \mu^6 - e^{6\lambda} = 10$

64. Let $f(x) = 2x^n + \lambda, \lambda \in \mathbb{R}, n \in \mathbb{N}$, and $f(4)=133$, $f(5)=255$. Then the sum of all the positive integer divisors of $(f(3)-f(2))$ is

(1) 61 (2) 60

(3) 58 (4) 59

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $f(x) = 2x^n + \lambda$

$f(4) = 133$

$f(5) = 255$

$133 = 2 \times 4^n + \lambda$ (1)

$255 = 2 \times 5^n + \lambda$ (2)

(2) – (1)

$122 = 2(5^n - 4^n)$

$\Rightarrow 5^n - 4^n = 61$

$\therefore n = 3$ & $\lambda = 5$

Now, $f(3) - f(2) = 2(3^3 - 2^3) = 38$

Number of Divisors is 1, 2, 19, 38 ; & their sum is 60

65. If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}, \hat{i} + 2\hat{j} - \hat{k}, -2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

(1) $\frac{73}{17}$ (2) $-\frac{107}{17}$

(3) $-\frac{73}{17}$ (4) $\frac{107}{17}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Let A : (3, –4, 2) C : (–2, –1, 3)

B : (1, 2, –1) D : (5, –2α, 4)

A, B, C, D are coplanar points, then

$$\Rightarrow \begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix} = 0$$

$$\Rightarrow \alpha = \frac{73}{17}$$

66. Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$. If $M = A^TBA$, then the inverse of the matrix $AM^{2023}A^T$ is

(1) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

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$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$M = A^TBA$

$M^2 = M.M = A^TBA\,A^TBA = A^TB^2A$

$M^3 = M^2.M = A^TB^2AA^TBA = A^TB^3A$

.

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$M^{2023} = \ldots\ldots\ldots\ldots\ldots A^TB^{2023}A$

$AM^{2023}A^T = \underbrace{AA^T}\, B^{2023}\, \underbrace{AA^T} = B^{2023}$

$$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

67. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $(p \to q)\Delta(p \nabla q)$ is a tautology. Then

(1) $\Delta = \wedge, \nabla = \vee$ (2) $\Delta = \vee, \nabla = \wedge$

(3) $\Delta = \vee, \nabla = \vee$ (4) $\Delta = \wedge, \nabla = \wedge$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Given $(p \to q)\ \Delta\ (p\ \nabla\ q)$

Option I $\Delta = \wedge$, $\nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Option 2 $\Delta = \vee$, $\nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Option 3 $\Delta = \vee$, $\nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Hence, it is tautology.

Option 4 $\Delta = \wedge$, $\nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \wedge (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

68. The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1,3,5,7,9 without repetition, is

(1) 6 (2) 12

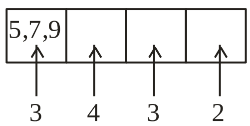
(3) 120 (4) 72

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Numbers between 5000 & 10000

Using digits 1, 3, 5, 7, 9



Total Numbers $= 3 \times 4 \times 3 \times 2 = 72$

69. The number of functions $f : \{1,2,3,4\} \to \{a \in \mathbb{Z} : |a| \le 8\}$ satisfying $f(n) + \frac{1}{n} f(n+1) = 1, \forall n \in \{1,2,3\}$ is

(1) 3 (2) 4

(3) 1 (4) 2

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $f : \{1, 2, 3, 4\} \to \{a \in \mathbb{Z} : |a| \le 8\}$

$f(n) + \frac{1}{n} f(n+1) = 1,\ \forall\ n \in \{1, 2, 3\}$

$f(n + 1)$ must be divisible by n

$f(4) \Rightarrow -6, -3, 0, 3, 6$

$f(3) \Rightarrow -8, -6, -4, -2, 0, 2, 4, 6, 8$

$f(2) \Rightarrow -8, \ldots\ldots\ldots\ldots\ldots, 8$

$f(1) \Rightarrow -8, \ldots\ldots\ldots\ldots\ldots, 8$

$\frac{f(4)}{3}$ must be odd since $f(3)$ should be even therefore 2 solution possible.

$f(4)$	$f(3)$	$f(2)$	$f(1)$
–3	2	0	1
3	0	1	0

70. The equations of two sides of a variable triangle are x = 0 and y = 3, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is :

(1) $4y^2-18y-3x-18=0$ (2) $4y^2+18y+3x+18=0$

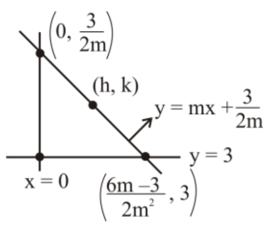
(3) $4y^2-18y+3x+18=0$ (4) $4y^2-18y-3x+18=0$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $y^2 = 6x$ & $y^2 = 4ax$

$\Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$



$y = mx + \frac{3}{2m}$; $(m \neq 0)$

$h = \frac{6m-3}{4m^2}$, $k = \frac{6m+3}{4m}$, Now eliminating m and we get

$\Rightarrow 3h = 2(-2k^2 + 9k - 9)$

$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$

71. Let f: $\mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\left\{\sqrt{2}(\sin x - \cos x) + m - 2\right\}$, for some m, such that the range of f is [0, 2]. Then the value of m is ________

(1) 5 (2) 3 (3) 2 (4) 4

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Since,

$-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2}$

$\therefore -2 \leq \sqrt{2}(\sin x - \cos x) \leq 2$

(Assume $\sqrt{2}(\sin x - \cos x) = k$)

$-2 \leq k \leq 2$ ...(i)

$f(x) = \log_{\sqrt{m}}(k + m - 2)$

Given,

$0 \leq f(x) \leq 2$

$0 \leq \log_{\sqrt{m}}(k + m - 2) \leq 2$

$1 \leq k + m - 2 \leq m$

$-m + 3 \leq k \leq 2$ ...(ii)

From eq. (i) & (ii), we get $-m + 3 = -2$

$\Rightarrow m = 5$

72. Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric.

Consider the statements

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then,

(1) Only S2 is true

(2) Only S1 is true

(3) Both S1 and S2 are false

(4) Both S1 and S2 are true

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Given, $A^T = A$, $B^T = -B$, $C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= B^{26}A^{13} - A^{13}B^{26} = -M$

Hence, M is skew symmetric

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$

$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$

Hence, N is symmetric.

$\therefore$ Only S2 is true.

73. Let y=y(t) be a solution of the differential equation

$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

Where, $\alpha > 0$, $\beta > 0$ and $\gamma > 0$. Then $\lim_{t\to\infty} y(t)$

(1) is 0 (2) does not exist

(3) is 1 (4) is –1

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

I.F. $= e^{\int \alpha dt} = e^{\alpha t}$

Solution $\Rightarrow y.e^{\alpha t} = \int \gamma e^{-\beta T}.e^{\alpha t} dt$

$\Rightarrow ye^{\alpha t} = \gamma \frac{e^{(\alpha-\beta)t}}{(\alpha-\beta)} + c$

$\Rightarrow y = \frac{\gamma}{e^{\beta t}(\alpha-\beta)} + \frac{c}{e^{\alpha t}}$

So, $\lim_{t\to\infty} y(t) = \frac{\gamma}{\infty} + \frac{c}{\infty} = 0$

74. $\sum_{k=0}^{6} {}^{51-k}C_3$ is equal to

(1) ${}^{51}C_4 - {}^{45}C_4$ (2) ${}^{51}C_3 - {}^{45}C_3$

(3) ${}^{52}C_4 - {}^{45}C_4$ (4) ${}^{52}C_3 - {}^{45}C_3$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\sum_{k=0}^{6} {}^{51-k}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ... + {}^{45}C_3$

$= {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3$

$= {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3 - {}^{45}C_4$

$({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r)$

$= {}^{52}C_4 - {}^{45}C_4$

75. The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is

(1) 2 (2) 3

(3) $\frac{5}{2}$ (4) $\frac{3}{2}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{-1}{12}}$ and $\frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$

$\Rightarrow$ Shortest distance $= \frac{(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

S.D. $=(-\hat{i}+2\hat{j}-\hat{k}).\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

$$\left\{\vec{p}\times\vec{q}\equiv\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & \frac{-1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}=\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k} \text{ or } 2\hat{i}-3\hat{j}+6\hat{k}\right\}$$

S.D. $=\frac{(-\hat{i}+2\hat{j}-\hat{k}).(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+3^2+6^2}}=\left|\frac{-14}{7}\right|=2$

76. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2, \sqrt{3N}, N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is

(1) 2 (2) 4

(3) 16 (4) 8

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $n(s) = 36$

Given : $N-2, \sqrt{3N}, N+2$ are in G.P.

$3N = (N-2)(N+2)$

$3N = N^2 - 4$

$\Rightarrow N^2 - 3N - 4 = 0$

$(N-4)(N+1) = 0 \Rightarrow \boxed{N=4}$ or $N=-1$ rejected

$(\text{Sum} = 4) \equiv \{(1, 3), (3,1), (2,2)\}$

$n(A) = 3$

$P(A)=\frac{3}{36}=\frac{1}{12}=\frac{4}{48}\Rightarrow\boxed{k=4}$

77. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to

(1) $\frac{11}{6}+\log_e 4$ (2) $\frac{11}{12}+\log_e 4$

(3) $\frac{11}{12}-\log_e 4$ (4) $\frac{11}{6}-\log_e 4$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $I = 16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$

$= 16\int_1^2 \frac{dx}{x^3x^4\left(1+\frac{2}{x^2}\right)^2}$

Let, $1+\frac{2}{x^2}=t\Rightarrow\frac{-4}{x^3}dx=dt$

$I=-4\int_3^{\frac{3}{2}}\frac{dt}{\left(\frac{2}{t-1}\right)^2 t^2}$

$I=-4\int_3^{\frac{3}{2}}\left(\frac{t-1}{2}\right)^2\frac{dt}{t^2}$

$I=-\frac{4}{4}\int_3^{\frac{3}{2}}\left(1-\frac{2}{t}+\frac{1}{t^2}\right)dt$

$$I = -1\left[t - 2\ell n|t| - \frac{1}{t}\right]_3^{\frac{3}{2}}$$

$$I = -1\left[\left(\frac{3}{2} - 2\ell n\frac{3}{2} - \frac{2}{3}\right) - \left(3 - 2\ell n3 - \frac{1}{3}\right)\right]$$

$$I = -1\left[2\ell n2 - \frac{11}{6}\right]$$

$$I = \frac{11}{6} - \ell n4$$

78. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:

(1) $4\sqrt{6} + \frac{44}{3}$ (2) $4\sqrt{6} + \frac{28}{3}$

(3) $4\sqrt{6} - \frac{44}{3}$ (4) $4\sqrt{6} - \frac{28}{3}$

Official Ans. by NTA (2)

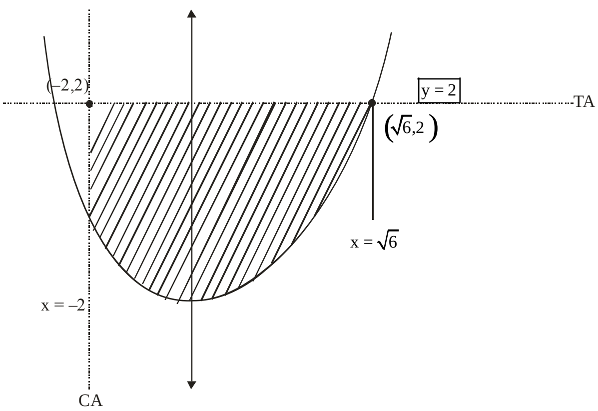
Allen Ans. (2)

Sol. $16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$

$16(x + 2)^2 - 64 - (y - 2)^2 + 4 + 44 = 0$

$16(x + 2)^2 - (y - 2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$$



$$A = \int_{-2}^{\sqrt{6}} \left(2 - \left(x^2 - 4\right)\right)dx$$

$$A = \int_{-2}^{\sqrt{6}} \left(6 - x^2\right)dx = \left(6x - \frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$$

$$A = \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-12 + \frac{8}{3}\right)$$

$$A = \frac{12\sqrt{6}}{3} + \frac{28}{3}$$

$$A = 4\sqrt{6} + \frac{28}{3}$$

79. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to

(1) $3\left(\hat{i} - \hat{j} - \hat{k}\right)$ (2) $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

(3) $3\left(\hat{i} - \hat{j} + \hat{k}\right)$ (4) $3\left(\hat{i} + \hat{j} - \hat{k}\right)$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\vec{a} \times \vec{b} = (\hat{i} - \hat{j})$

Taking cross product with $\vec{a}$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\hat{i} - \hat{j})$

$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow 2\vec{a} - 6\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

$\Rightarrow \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

80. The foot of perpendicular of the point (2, 0, 5) on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then. Which of the following is NOT correct?

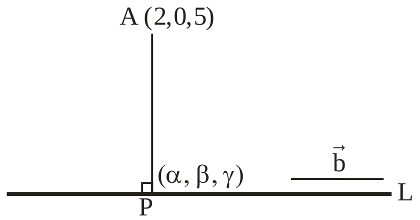
(1) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$ (2) $\frac{\alpha}{\beta} = -8$

(3) $\frac{\beta}{\gamma} = -5$ (4) $\frac{\gamma}{\alpha} = \frac{5}{8}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $L : \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = \lambda$ (let)



Let foot of perpendicular is

P(2λ –1, 5λ + 1, –λ –1)

$\overrightarrow{PA} = (3-2\lambda)\hat{i} - (5\lambda+1)\hat{j} + (6+\lambda)\hat{k}$

Direction ratio of line $\Rightarrow \vec{b} = 2\hat{i} + 5\hat{j} - \hat{k}$

Now, $\Rightarrow \overrightarrow{PA} \cdot \vec{b} = 0$

$\Rightarrow 2(3 - 2\lambda) - 5(5\lambda + 1) - (6 + \lambda) = 0$

$\Rightarrow \lambda = \frac{-1}{6}$

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1) \equiv P(\alpha, \beta, \gamma)$

$\Rightarrow \alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3} \Rightarrow \boxed{\alpha = -\frac{4}{3}}$

$\Rightarrow \beta = 5\left(-\frac{1}{6}\right) + 1 = \frac{1}{6} \Rightarrow \boxed{\beta = \frac{1}{6}}$

$\Rightarrow \gamma = -\lambda - 1 = \frac{1}{6} - 1 \Rightarrow \boxed{\gamma = -\frac{5}{6}}$

∴ Check options

SECTION-B

81. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then, 16a + 12b is equal to _____.

Official Ans. by NTA 3

Allen Ans. 3

Sol. $a, b, \frac{1}{18} \to GP$

$\frac{a}{18} = b^2$ ….. (i)

$\frac{1}{a}, 10, \frac{1}{b} \to AP$

$\frac{1}{a} + \frac{1}{b} = 20$

$\Rightarrow a + b = 20ab$, from eq. (i) ; we get

$\Rightarrow 18b^2 + b = 360b^3$

$\Rightarrow 360b^2 - 18b - 1 = 0 \quad \{\because b \neq 0\}$

$\Rightarrow b = \frac{18 \pm \sqrt{324 + 1440}}{720}$

$\Rightarrow b = \frac{18 + \sqrt{1764}}{720} \quad \{\because b > 0\}$

$\Rightarrow b = \frac{1}{12}$

$\Rightarrow a = 18 \times \frac{1}{144} = \frac{1}{8}$

Now, $16a + 12b = 16 \times \frac{1}{8} + 12 \times \frac{1}{12} = 3$

82. Points P(–3,2),Q(9,10) and R(α,4)lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line 2x – ky = 1, then k is equal to _____.

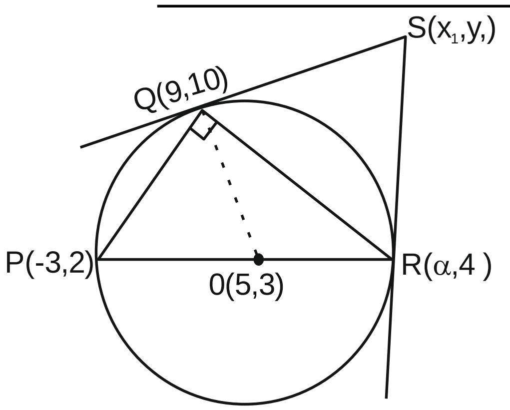
Official Ans. by NTA 3

Allen Ans. 3

Sol. $m_{PQ} \cdot m_{QR} = -1$

$\Rightarrow \frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1 \Rightarrow \boxed{\alpha = 13}$

$m_{0P} \cdot m_{QS} = -1 \Rightarrow m_{QS} = -\frac{4}{7}$



Equation of QS

$y - 10 = -\frac{4}{7}(x-9)$

$\Rightarrow$ 4x + 7y = 106 ….(1)

$m_{0R} \cdot m_{RS} = -1 \Rightarrow m_{RS} = -8$

Equation of RS

$y - 4 = -8(x-13)$

$\Rightarrow 8x + y = 108$ ….(2)

Solving eq. (1) & (2)

$x_1 = \frac{25}{2}$ $y_1 = 8$

$S(x_1, y_1)$ lies on $2x - ky = 1$

$25 - 8k = 1$

$\Rightarrow 8k = 24$

$\Rightarrow \boxed{k = 3}$

83. Let a $\in$ R and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is _____.

Official Ans. by NTA 45

Allen Ans. 45

Sol. $x^2 + 60^{\frac{1}{4}}x + a = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$\alpha + \beta = -60^{\frac{1}{4}}$ & $\alpha\beta = a$

Given $\alpha^4 + \beta^4 = -30$

$\Rightarrow (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = -30$

$\Rightarrow \left\{(\alpha+\beta)^2 - 2\alpha\beta\right\}^2 - 2a^2 = -30$

$\Rightarrow \left\{60^{\frac{1}{2}} - 2a\right\}^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4a \times 60^{\frac{1}{2}} - 2a^2 = -30$

$\Rightarrow 2a^2 - 4.60^{\frac{1}{2}}a + 90 = 0$

Product $= \frac{90}{2} = 45$

84. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 orange, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is____

Official Ans. by NTA 6860

Allen Ans. 6860

Sol. 7 Red apple(RA),5 white apple(WA),8 oranges (O)

5 fruits to be selected (Note:- fruits taken different)

Possible selections :- (2O, 1RA, 2WA) or (2O, 2RA, 1WA) or (3O, 1RA, 1WA)

$\Rightarrow {}^8C_2\, {}^7C_1\, {}^5C_2 + {}^8C_2\, {}^7C_2\, {}^5C_1 + {}^8C_3\, {}^7C_1\, {}^5C_1$

$\Rightarrow 1960 + 2940 + 1960$

$\Rightarrow$ 6860

85. If m and n respectively are the numbers of positive and negative value of θ in the interval [–π, π]that satisfy the equation $\cos 2\theta \cos\frac{\theta}{2} = \cos 3\theta \cos\frac{9\theta}{2}$, then mn is equal to _____.

Official Ans. by NTA 25

Allen Ans. 25

Sol. $\cos 2\theta \cdot \cos \frac{\theta}{2} = \cos 3\theta \cdot \cos \frac{9\theta}{2}$

$\Rightarrow 2\cos 2\theta \cdot \cos \frac{\theta}{2} = 2\cos \frac{9\theta}{2} \cdot \cos 3\theta$

$\Rightarrow \cos \frac{5\theta}{2} + \cos \frac{3\theta}{2} = \cos \frac{15\theta}{2} + \cos \frac{3\theta}{2}$

$\Rightarrow \cos \frac{15\theta}{2} = \cos \frac{5\theta}{2}$

$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$

$5\theta = 2k\pi$ or $10\theta = 2k\pi$

$\theta = \frac{2k\pi}{5} \qquad \theta = \frac{k\pi}{5}$

$\therefore \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$

$m = 5, n = 5$

$\therefore m.n = 25$

86. If $\int_{1/3}^{3} |\log_e x| dx = \frac{m}{n} \log_e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to ______.

Official Ans. by NTA 20

Allen Ans. 20

Sol. $\int_{1/3}^{3} |\ell nx| dx = \int_{1/3}^{1} (-\ell nx) dx + \int_{1}^{3} (\ell nx) dx$

$= -[x \ell nx - x]_{1/3}^{1} + [x \ell nx - x]_{1}^{3}$

$= -\left[-1 - \left(\frac{1}{3}\ell n\frac{1}{3} - \frac{1}{3}\right)\right] + [3\ell n3 - 3 - (-1)]$

$= \left[-\frac{2}{3} - \frac{1}{3}\ell n\frac{1}{3}\right] + [3\ell n3 - 2]$

$= -\frac{4}{3} + \frac{8}{3}\ell n3$

$= \frac{4}{3}(2\ell n3 - 1)$

$= \frac{4}{3}\left(\ell n\frac{9}{e}\right)$

$\therefore \quad m = 4, n = 3$

Now, $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

87. The remainder when $(2023)^{2023}$ is divided by 35 is______.

Official Ans. by NTA 7

Allen Ans. 7

Sol. $(2023)^{2023}$

$= (2030 - 7)^{2023}$

$= (35\text{ K} - 7)^{2023}$

$= {}^{2023}C_0(35\text{ K})^{2023}(-7)^0 + {}^{2023}C_1(35\text{ K})^{2022}(-7) + \ldots.. + \ldots\ldots. + {}^{2023}C_{2023}(-7)^{2023}$

$= 35\text{ N} - 7^{2023}$.

Now, $-7^{2023} = -7 \times 7^{2022} = -7(7^2)^{1011}$

$= -7(50 - 1)^{1011}$

$= -7({}^{1011}C_0\, 50^{1011} - {}^{1011}C_1(50)^{1010} + \ldots..\ {}^{1011}C_{1011})$

$= -7(5\lambda - 1)$

$= -35\lambda + 7$

$\therefore$ when $(2023)^{2023}$ is divided by 35 remainder is 7

88. If the shortest distance between the line joining the points(1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to_____.

Official Ans. by NTA 18

Allen Ans. 18

Sol. $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}) \qquad \vec{r} = \vec{a} + \lambda\vec{p}$

$\vec{r} = (+\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j}) \qquad \vec{r} = \vec{b} + \mu\vec{q}$

$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 3\hat{k}$

$d = \left|\frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$

$d = \left|\frac{(-3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{14}}\right|$

$= \left|\frac{-6+3}{\sqrt{14}}\right| = \frac{3}{\sqrt{14}}$

$\alpha = \frac{3}{\sqrt{14}}$

Now, $28\alpha^2 = \cancel{28}^{\,2} \times \frac{9}{\cancel{14}} = 18$

89. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer then a non-smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is ______.

Official Ans. by NTA 9

Allen Ans. 9

Sol. E_1 : Smokers

$P(E_1) = \frac{1}{4}$

E_2 : non-smokers

$P(E_2) = \frac{3}{4}$

E : diagnosed with lung cancer

$P(E/E_1) = \frac{27}{28}$

$P(E/E_2) = \frac{1}{28}$

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E)}$$

$$= \frac{\frac{1}{4}\times\frac{27}{28}}{\frac{1}{4}\times\frac{27}{28}+\frac{3}{4}\times\frac{1}{28}} = \frac{\cancel{27}^{9}}{\cancel{30}_{10}} = \frac{9}{10}$$

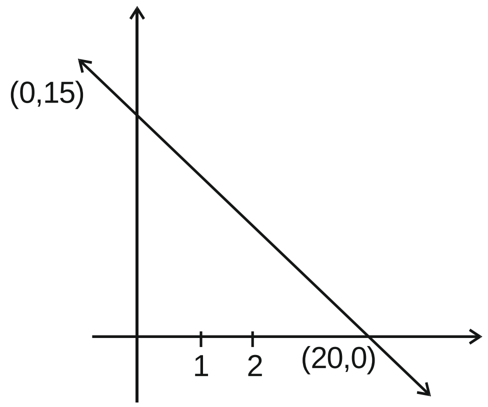
K = 9

90. A triangle is formed by X – axis, Y– axis and the line 3x + 4y = 60. Then the number of points P(a, b)which lie strictly inside the triangle, where a is an integer and b is a multiple of a, is_____.

Official Ans. by NTA 31

Allen Ans. 31

Sol. If x = 1, y = $\frac{57}{4} = 14.25$



(1, 1) (1, 2) – (1, 14) $\Rightarrow$ 14 pts.

If x = 2, $y = \frac{27}{2} = 13.5$

(2, 2) (2, 4) …. (2, 12) $\Rightarrow$ 6 pts.

If x = 3, $y = \frac{51}{4} = 12.75$

(3, 3) (3, 6) – (3, 12) $\Rightarrow$ 4 pts.

If x = 4, y = 12

(4, 4) (4, 8) $\Rightarrow$ 2 pts.

If x = 5. $y = \frac{45}{4} = 11.25$

(5, 5), (5, 10) $\Rightarrow$ 2 pts.

If x = 6, $y = \frac{21}{2} = 10.5$

(6, 6) $\Rightarrow$ 1 pt.

If x = 7, $y = \frac{39}{4} = 9.75$

(7, 7) $\Rightarrow$ 1 pt.

If x = 8, y = 9

(8, 8) $\Rightarrow$ 1 pt.

If x = 9 $y = \frac{33}{4} = 8.25$ $\Rightarrow$ no pt.

Total = 31 pts.