

**MATHEMATICS**

**TEST PAPER WITH ANSWER**

**SECTION-A**

**61.** Let the six numbers  $a_1, a_2, a_3, a_4, a_5, a_6$  be in A.P. and  $a_1 + a_3 = 10$ . If the mean of these six numbers is  $\frac{19}{2}$  and their variance is  $\sigma^2$ , then  $8\sigma^2$  is equal to

- (1) 220
- (2) 210
- (3) 200
- (4) 105

**Official Ans. by NTA (2)**

**Allen Ans. (2)**

**Sol.**  $a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$$

$$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$$

$$\Rightarrow a_1 + a_6 = 19$$

$$\Rightarrow 2a_1 + 5d = 19 \text{ and } a_1 + d = 5$$

$$\Rightarrow a_1 = 2, d = 3$$

Numbers : 2, 5, 8, 11, 14, 17

Variance =  $\sigma^2 = \text{mean of squares} - \text{square of mean}$

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$$8\sigma^2 = 210$$

**62.** Let  $f(x)$  be a function such that  $f(x + y) = f(x) \cdot f(y)$

for all  $x, y \in \mathbb{N}$ . If  $f(1) = 3$  and  $\sum_{k=1}^n f(k) = 3279$ ,

then the value of  $n$  is

- (1) 6
- (2) 8
- (3) 7
- (4) 9

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

**Sol.**  $f(x + y) = f(x) \cdot f(y) \quad \forall x, y \in \mathbb{N}, f(1) = 3$

$$f(2) = f^2(1) = 3^2$$

$$f(3) = f(1) f(2) = 3^3$$

$$f(4) = 3^4$$

$$f(k) = 3^k$$

$$\sum_{k=1}^n f(k) = 3279$$

$$f(1) + f(2) + f(3) + \dots + f(k) = 3279$$

$$3 + 3^2 + 3^3 + \dots + 3^k = 3279$$

$$\frac{3(3^k - 1)}{3 - 1} = 3279$$

$$\frac{3^k - 1}{2} = 1093$$

$$3^k - 1 = 2186$$

$$3^k = 2187$$

$$\boxed{k = 7}$$

**63.** The number of real solutions of the equation

$$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0, \text{ is}$$

- (1) 4
- (2) 0
- (3) 3
- (4) 2

**Official Ans. by NTA (2)**

**Allen Ans. (2)**

**Sol.**  $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$$

$$\text{Let } x + \frac{1}{x} = t$$

$$3t^2 - 2t - 1 = 0$$

$$3t^2 - 3t + t - 1 = 0$$

$$3t(t - 1) + 1(t - 1) = 0$$

$$(t - 1)(3t + 1) = 0$$

$$t = 1, -\frac{1}{3}$$

$$x + \frac{1}{x} = 1, -\frac{1}{3} \Rightarrow \text{No solution.}$$

64. If  $f(x) = \frac{2^{2x}}{2^{2x} + 2}$ ,  $x \in \mathbb{R}$ ,  
 then  $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \dots + f\left(\frac{2022}{2023}\right)$  is  
 equal to  
 (1) 2011 (2) 1010  
 (3) 2010 (4) 1011

**Official Ans. by NTA (4)**

**Allen Ans. (4)**

- Sol.**  $f(x) = \frac{4^x}{4^x + 2}$   
 $f(x) + f(1-x) = \frac{4^x}{4^x + 2} + \frac{4^{1-x}}{4^{1-x} + 2}$   
 $= \frac{4^x}{4^x + 2} + \frac{4}{4 + 2(4^x)}$   
 $= \frac{4^x}{4^x + 2} + \frac{2}{2 + 4^x}$   
 $= 1$   
 $\Rightarrow f(x) + f(1-x) = 1$   
 Now  $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + \dots +$   
 $\dots + f\left(1 - \frac{3}{2023}\right) + f\left(1 - \frac{2}{2023}\right) + f\left(1 - \frac{1}{2023}\right)$   
 Now sum of terms equidistant from beginning and end is 1  
 Sum = 1 + 1 + 1 + ..... + 1 (1011 times)  
 = 1011

65. If  $f(x) = x^3 - x^2f'(1) + xf''(2) - f'''(3)$ ,  $x \in \mathbb{R}$ , then  
 (1)  $3f(1) + f(2) = f(3)$   
 (2)  $f(3) - f(2) = f(1)$   
 (3)  $2f(0) - f(1) + f(3) = f(2)$   
 (4)  $f(1) + f(2) + f(3) = f(0)$

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

- Sol.**  $f(x) = x^3 - x^2f'(1) + xf''(2) - f'''(3)$ ,  $x \in \mathbb{R}$   
 Let  $f'(1) = a$ ,  $f''(2) = b$ ,  $f'''(3) = c$   
 $f(x) = x^3 - ax^2 + bx - c$   
 $f'(x) = 3x^2 - 2ax + b$   
 $f''(x) = 6x - 2a$   
 $f'''(x) = 6$   
 $c = 6$ ,  $a = 3$ ,  $b = 6$   
 $f(x) = x^3 - 3x^2 + 6x - 6$   
 $f(1) = -2$ ,  $f(2) = 2$ ,  $f(3) = 12$ ,  $f(0) = -6$   
 $2f(0) - f(1) + f(3) = 2 = f(2)$

66. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is  
 (1) 120  
 (2) 168  
 (3) 220  
 (4) 48

**Official Ans. by NTA (2)**

**Allen Ans. (2)**

- Sol.** Four digit numbers greater than 7000  
 $= 2 \times 4 \times 3 \times 2 = 48$   
 Five digit number =  $5! = 120$   
 Total number greater than 7000  
 $= 120 + 48 = 168$

67. If the system of equations  
 $x + 2y + 3z = 3$   
 $4x + 3y - 4z = 4$   
 $8x + 4y - \lambda z = 9 + \mu$   
 has infinitely many solutions, then the ordered pair  $(\lambda, \mu)$  is equal to

- (1)  $\left(\frac{72}{5}, \frac{21}{5}\right)$  (2)  $\left(\frac{-72}{5}, \frac{-21}{5}\right)$   
 (3)  $\left(\frac{72}{5}, \frac{-21}{5}\right)$  (4)  $\left(\frac{-72}{5}, \frac{21}{5}\right)$

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

- Sol.**  $x + 2y + 3z = 3$  .....(i)  
 $4x + 3y - 4z = 4$  .....(ii)  
 $8x + 4y - \lambda z = 9 + \mu$  .....(iii)  
 (i)  $\times 4 -$  (ii)  $\Rightarrow 5y + 16z = 8$  ..... (iv)  
 (ii)  $\times 2 -$  (iii)  $\Rightarrow 2y + (\lambda - 8)z = -1 - \mu$  .....(v)  
 (iv)  $\times 2 -$  (iii)  $\times 5 \Rightarrow (32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$   
 For infinite solutions  $\Rightarrow 72 - 5\lambda = 0 \Rightarrow \lambda = \frac{72}{5}$   
 $21 + 5\mu = 0 \Rightarrow \mu = \frac{-21}{5}$   
 $\Rightarrow (\lambda, \mu) \equiv \left(\frac{72}{5}, \frac{-21}{5}\right)$

68. The value of  $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}}\right)^3$  is

(1)  $\frac{-1}{2}(1 - i\sqrt{3})$

(2)  $\frac{1}{2}(1 - i\sqrt{3})$

(3)  $\frac{-1}{2}(\sqrt{3} - i)$

(4)  $\frac{1}{2}(\sqrt{3} + i)$

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

**Sol.** Let  $\sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9} = z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3 = \left(\frac{1+z}{1+\frac{1}{z}}\right)^3 = z^3$$

$$\Rightarrow \left(i\left(\cos \frac{2\pi}{9} - i \sin \frac{2\pi}{9}\right)\right)^3$$

$$= -i \left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3}\right) = -i \left(\frac{-1}{2} - i \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{-1}{2}(\sqrt{3} - i)$$

69. The equations of the sides AB and AC of a triangle ABC are

$$(\lambda + 1)x + \lambda y = 4 \text{ and } \lambda x + (1 - \lambda)y + \lambda = 0$$

respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola  $y^2 = 6x$  in the first quadrant is

(1)  $\sqrt{6}$

(2)  $2\sqrt{2}$

(3) 2

(4) 4

**Official Ans. by NTA (2)**

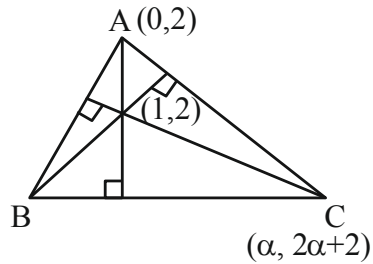
**Allen Ans. (2)**

**Sol.**  $AB: (\lambda + 1)x + \lambda y = 4$

$$AC: \lambda x + (1 - \lambda)y + \lambda = 0$$

Vertex A is on y-axis

$$\Rightarrow x = 0$$



$$\text{So } y = \frac{4}{\lambda}, y = \frac{\lambda}{\lambda - 1}$$

$$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda - 1}$$

$$\Rightarrow \lambda = 2$$

$$AB: 3x + 2y = 4$$

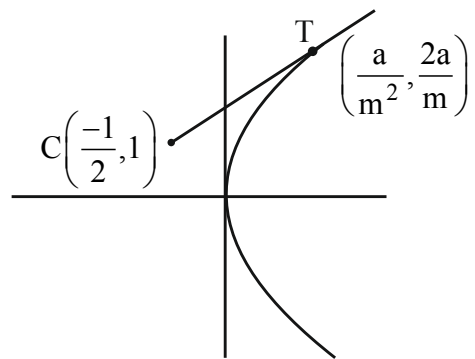
$$AC: 2x - y + 2 = 0$$

$$\Rightarrow A(0, 2) \text{ Let } C(\alpha, 2\alpha + 2)$$

$$\text{Now (Slope of Altitude through C)} \left(-\frac{3}{2}\right) = -1$$

$$\left(\frac{2\alpha}{\alpha - 1}\right)\left(-\frac{3}{2}\right) = -1 \Rightarrow \alpha = -\frac{1}{2}$$

$$\text{So } C\left(-\frac{1}{2}, 1\right)$$



$$\text{Let Equation of tangent be } y = mx + \frac{3}{2m}$$

$$m^2 + 2m - 3 = 0$$

$$\Rightarrow m = 1, -3$$

So tangent which touches in first quadrant at T is

$$T \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

$$\equiv \left(\frac{3}{2}, 3\right)$$

$$\Rightarrow CT = \sqrt{4 + 4} = 2\sqrt{2}$$

70. The set of all values of  $a$  for which  $\lim_{x \rightarrow a} ([x - 5] - [2x + 2]) = 0$ , where  $[\infty]$  denotes the

greater integer less than or equal to  $\infty$  is equal to

- (1)  $(-7.5, -6.5)$                       (2)  $(-7.5, -6.5]$   
 (3)  $[-7.5, -6.5]$                       (4)  $[-7.5, -6.5)$

**Official Ans. by NTA (1)**

**Allen Ans. (1)**

**Sol.**  $\lim_{x \rightarrow a} ([x - 5] - [2x + 2]) = 0$

$$\lim_{x \rightarrow a} ([x] - 5 - [2x] - 2) = 0$$

$$\lim_{x \rightarrow a} ([x] - [2x]) = 7$$

$$[a] - [2a] = 7$$

$$a \in I, a = -7$$

$$a \notin I, a = I + f$$

$$\text{Now, } [a] - [2a] = 7$$

$$-I - [2f] = 7$$

$$\text{Case-I: } f \in \left(0, \frac{1}{2}\right)$$

$$2f \in (0, 1)$$

$$-I = 7$$

$$I = -7 \Rightarrow a \in (-7, -6.5)$$

$$\text{Case-II: } f \in \left(\frac{1}{2}, 1\right)$$

$$2f \in (1, 2)$$

$$-I - 1 = 7$$

$$I = -8 \Rightarrow a \in (-7.5, -7)$$

$$\text{Hence, } a \in (-7.5, -6.5)$$

71. If  $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \dots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ , then  $\alpha$  is equal to

- (1) 30                                      (2) 60  
 (3) 15                                      (4) 10

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

**Sol.**  $S = 0 \cdot ({}^{30}C_0)^2 + 1 \cdot ({}^{30}C_1)^2 + 2 \cdot ({}^{30}C_2)^2 + \dots + 30 \cdot ({}^{30}C_{30})^2$

$$S = 30 \cdot ({}^{30}C_0)^2 + 29 \cdot ({}^{30}C_1)^2 + 28 \cdot ({}^{30}C_2)^2 + \dots + 0 \cdot ({}^{30}C_0)^2$$

$$2S = 30 \cdot ({}^{30}C_0)^2 + 30 \cdot ({}^{30}C_1)^2 + \dots + 30 \cdot ({}^{30}C_{30})^2$$

$$S = 15 \cdot {}^{60}C_{30} = 15 \cdot \frac{60!}{(30!)^2}$$

$$\frac{15 \cdot 10!}{(30!)^2} = \frac{\alpha \cdot 60!}{(30!)^2}$$

$$\Rightarrow \alpha = 15$$

72. Let the plane containing the line of intersection of the planes

$$P_1: x + (\lambda + 4)y + z = 1 \text{ and}$$

$P_2: 2x + y + z = 2$  pass through the points  $(0, 1, 0)$  and  $(1, 0, 1)$ . Then the distance of the point  $(2\lambda, \lambda, -\lambda)$  from the plane  $P_2$  is

- (1)  $5\sqrt{6}$   
 (2)  $4\sqrt{6}$   
 (3)  $2\sqrt{6}$   
 (4)  $3\sqrt{6}$

**Official Ans. by NTA (4)**

**Allen Ans. (4)**

**Sol.** Equation of plane passing through point of intersection of  $P_1$  and  $P_2$

$$P = P_1 + kP_2$$

$$(x + (\lambda + 4)y + z - 1) + k(2x + y + z - 2) = 0$$

Passing through  $(0, 1, 0)$  and  $(1, 0, 1)$

$$(\lambda + 4 - 1) + k(1 - 2) = 0$$

$$(\lambda + 3) - k = 0 \quad \dots(1)$$

Also passing  $(1, 0, 1)$

$$(1 + 1 - 1) + k(2 + 1 - 2) = 0$$

$$1 + k = 0$$

$$k = -1$$

put in (1)

$$\lambda + 3 + 1 = 0$$

$$\lambda = -4$$

Then point  $(2\lambda, \lambda, -\lambda)$

$$(-8, -4, 4)$$

$$d = \left| \frac{-16 - 4 + 4 - 2}{\sqrt{6}} \right|$$

$$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$$

73. Let  $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$  and  $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$ . Let  $\vec{\beta}_1$  be parallel to  $\vec{\alpha}$  and  $\vec{\beta}_2$  be perpendicular to  $\vec{\alpha}$ . If

$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ , then the value of  $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$  is

- (1) 6  
 (2) 11  
 (3) 7  
 (4) 9

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

**Sol.** Let  $\vec{\beta}_1 = \lambda \vec{\alpha}$

Now  $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$$= (\hat{i} + 2\hat{j} - 4\hat{k}) - \lambda(4\hat{i} + 3\hat{j} + 5\hat{k})$$

$$= (1 - 4\lambda)\hat{i} + (2 - 3\lambda)\hat{j} - (5\lambda + 4)\hat{k}$$

$$\vec{\beta}_2 \cdot \vec{\alpha} = 0$$

$$\Rightarrow 4(1 - 4\lambda) + 3(2 - 3\lambda) - 5(5\lambda + 4) = 0$$

$$\Rightarrow 4 - 16\lambda + 6 - 9\lambda - 25\lambda - 20 = 0$$

$$\Rightarrow 50\lambda = -10$$

$$\Rightarrow \boxed{\lambda = -\frac{1}{5}}$$

$$\vec{\beta}_2 = \left(1 + \frac{4}{5}\right)\hat{i} + \left(2 + \frac{3}{5}\right)\hat{j} - (-1 + 4)\hat{k}$$

$$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$$

$$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$$

$$5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$$

**74.** The locus of the mid points of the chords of the circle  $C_1: (x - 4)^2 + (y - 5)^2 = 4$  which subtend an angle  $\theta_1$  at the centre of the circle  $C_1$ , is a circle of radius  $r_1$ . If  $\theta_1 = \frac{\pi}{3}$ ,  $\theta_3 = \frac{2\pi}{3}$  and  $r_1^2 = r_2^2 + r_3^2$ , then  $\theta_2$  is equal to

(1)  $\frac{\pi}{4}$

(2)  $\frac{3\pi}{4}$

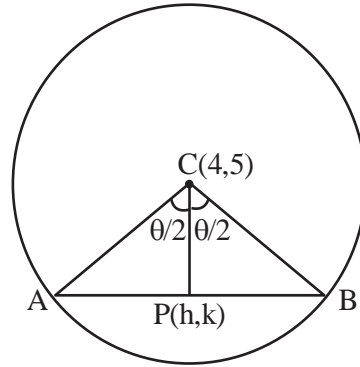
(3)  $\frac{\pi}{6}$

(4)  $\frac{\pi}{2}$

**Official Ans. by NTA (4)**

**Allen Ans. (4)**

**Sol.** In  $\Delta CPB$



$$\cos \frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2 \cos \frac{\theta}{2}$$

$$\Rightarrow (h - 4)^2 + (k - 5)^2 = 4 \cos^2 \frac{\theta}{2}$$

$$\text{Now } (x - 4)^2 + (y - 5)^2 = \left(2 \cos \frac{\theta}{2}\right)^2$$

$$\Rightarrow r_1 = 2 \cos \frac{\pi}{6} = \sqrt{3}$$

$$r_2 = 2 \cos \frac{\theta_2}{2}$$

$$r_3 = 2 \cos \frac{\pi}{3} = 1$$

$$\Rightarrow r_1^2 = r_2^2 + r_3^2$$

$$\Rightarrow 3 = 4 \cos^2 \frac{\theta_2}{2} + 1$$

$$\Rightarrow 4 \cos^2 \frac{\theta_2}{2} = 2$$

$$\Rightarrow \cos^2 \frac{\theta_2}{2} = \frac{1}{2}$$

$$\Rightarrow \boxed{\theta_2 = \frac{\pi}{2}}$$

**75.** If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes  $x + 2y + z = 0$  and  $3y - z = 3$  is  $(\alpha, \beta, \gamma)$ , then  $\alpha + \beta + \gamma$  is equal to

(1) -1

(2) 3

(3) 1

(4) 5

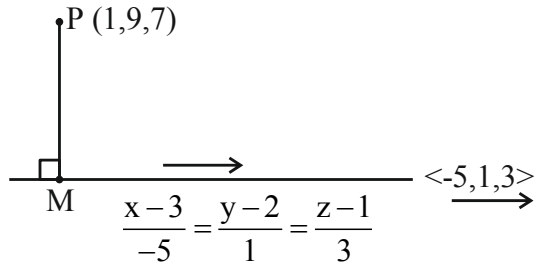
**Official Ans. by NTA (4)**

**Allen Ans. (4)**

**Sol.** Direction ratio of line =  $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$$

$$= -5\hat{i} + \hat{j} + 3\hat{k}$$



$$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$$

$$\overline{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$$

$$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$$

$$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$$

$$\Rightarrow \lambda = 1$$

$$\text{Point } M = (-2, 3, 4) = (\alpha, \beta, \gamma)$$

$$\alpha + \beta + \gamma = 5$$

- 76.** Let  $y = y(x)$  be the solution of the differential equation  $(x^2 - 3y^2)dx + 3xy dy = 0$ ,  $y(1) = 1$ . Then  $6y^2(e)$  is equal to

(1)  $3e^2$

(2)  $e^2$

(3)  $2e^2$

(4)  $\frac{3e^2}{2}$

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

**Sol.**  $(x^2 - 3y^2) dx + 3xy dy = 0$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3} \frac{x}{y} \quad (1)$$

Put  $y = vx$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$(1) \Rightarrow v + x \frac{dv}{dx} = v - \frac{1}{3} \frac{1}{v}$$

$$\Rightarrow v dv = \frac{-1}{3x}$$

Integrating both side

$$\frac{v^2}{2} = \frac{-1}{3} \ln x + c$$

$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3} \ln x + c$$

$$y(1) = 1$$

$$\Rightarrow \boxed{\frac{1}{2} = c}$$

$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3} \ln x + \frac{1}{2}$$

$$\Rightarrow y^2 = -\frac{2}{3} x^2 \ln x + x^2$$

$$y^2(e) = -\frac{2}{3} e^2 + e^2 = \frac{e^2}{3}$$

$$\Rightarrow \boxed{6y^2(e) = 2e^2}$$

- 77.** Let p and q be two statements.

Then  $\sim(p \wedge (p \Rightarrow \sim q))$  is equivalent to

(1)  $p \vee (p \wedge (\sim q))$

(2)  $p \vee ((\sim p) \wedge q)$

(3)  $(\sim p) \vee q$

(4)  $p \vee (p \wedge q)$

**Official Ans. by NTA (3)**

**Allen Ans. (3)**

**Sol.**  $\sim(p \wedge (p \rightarrow \sim q))$

$$\equiv \sim p \vee \sim(\sim p \vee \sim q)$$

$$\equiv \sim p \vee (p \wedge q)$$

$$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$$

$$\equiv t \wedge (\sim p \vee q)$$

$$\equiv \sim p \vee q$$

- 78.** The number of square matrices of order 5 with entries from the set  $\{0, 1\}$ , such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 225

(2) 120

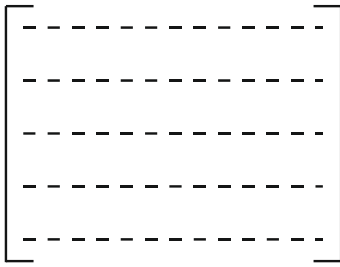
(3) 150

(4) 125

**Official Ans. by NTA (2)**

**Allen Ans. (2)**

Sol.



In each row and each column exactly one is to be placed –

$$\therefore \text{No. of such materials} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Alternate :

$$\begin{aligned} \left[ \begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \end{array} \right] &\rightarrow 5 \text{ ways} \\ \left[ \begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \end{array} \right] &\rightarrow 4 \text{ ways} \\ \left[ \begin{array}{ccccc} 0 & 0 & 0 & 1 & 0 \end{array} \right] &\rightarrow 3 \text{ ways} \\ \left[ \begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \end{array} \right] &\rightarrow 2 \text{ ways} \\ \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array} \right] &\rightarrow 1 \text{ ways} \end{aligned}$$

Step-1 : Select any 1 place for 1's in row 1.

Automatically some column will get filled with 0's.

Step-2 : From next row select 1 place for 1's.

Automatically some column will get filled with 0's.

$\Rightarrow$  Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

$$\text{Req. ways} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

79.  $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx$  is equal to

(1)  $\frac{\pi}{3}$

(2)  $\frac{\pi}{2}$

(3)  $\frac{\pi}{6}$

(4)  $2\pi$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol.  $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx$

We have  $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$

Hence  $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx = \frac{48}{2} \times \left[ \sin^{-1} \frac{2x}{3} \right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$$= 24 \times \left[ \sin^{-1} \left( \frac{2}{3} \times \frac{3\sqrt{3}}{4} \right) - \sin^{-1} \left( \frac{2}{3} \times \frac{3\sqrt{2}}{4} \right) \right]$$

$$= 24 \times \left[ \sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} \frac{1}{\sqrt{2}} \right]$$

$$= 24 \times \left( \frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= 24 \times \frac{\pi}{12} = 2\pi$$

80. Let A be a  $3 \times 3$  matrix such that  $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$ . Then  $|A^{-1}\text{adj}A|$  is equal to

(1)  $2\sqrt{3}$

(2)  $\sqrt{6}$

(3) 12

(4) 1

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Given  $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$

$$\Rightarrow |A|^{(n-1)^3} = 12^4$$

Given  $n = 3$

$$\Rightarrow |A|^8 = 12^4$$

$$\Rightarrow |A|^2 = 12$$

$$|A| = 2\sqrt{3}$$

We are asked

$$|A^{-1} \cdot \text{adj}A|$$

$$= |A^{-1}| \cdot |\text{adj}A|$$

$$= \frac{1}{|A|} \cdot |A|^{3-1}$$

$$= |A| = 2\sqrt{3}$$

81. The urns A, B and C contain 4 red, 6 black; 5 red, 5 black and  $\lambda$  red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola  $y^2 = \lambda x$  with one vertex at the vertex of the parabola is

**Official Ans. by NTA (432)**

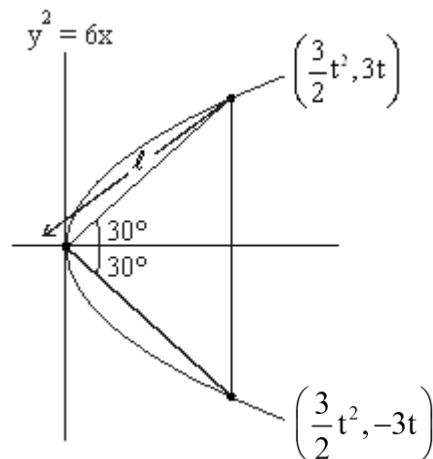
**Allen Ans. (432)**

Sol.	Urn A		Urn B		Urn C	
	Red	Black	Red	Black	Red	Black
	4	6	5	5	$\lambda$	4

$$P\left(\frac{C}{R}\right) = \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right) + P(B)P\left(\frac{R}{B}\right) + P(C)P\left(\frac{R}{C}\right)}$$

$$0.4 = \frac{\frac{1}{3} \times \frac{\lambda}{(\lambda+4)}}{\frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{\lambda}{(\lambda+4)}}$$

$$\Rightarrow \lambda = 6$$



$$\tan 30^\circ = 3t = \frac{3}{2}t^2$$

$$\frac{1}{\sqrt{3}} = \frac{2}{t}$$

$$t = 2\sqrt{3}$$

$$\left(\frac{3}{2}t^2, 3t\right) = (18, 6\sqrt{3})$$

$$l^2 = 18^2 + (6\sqrt{3})^2$$

$$= 324 + 108$$

$$= 432$$

82. If the area of the region bounded by the curves  $y^2 - 2y = -x$ ,  $x + y = 0$  is A, then  $8A$  is equal to

**Official Ans. by NTA (36)**

**Allen Ans. (36)**

**Sol.**  $y^2 - 2y = -x$

$$\Rightarrow y^2 - 2y + 1 = -x + 1$$

$$(y - 1)^2 = -(x - 1)$$

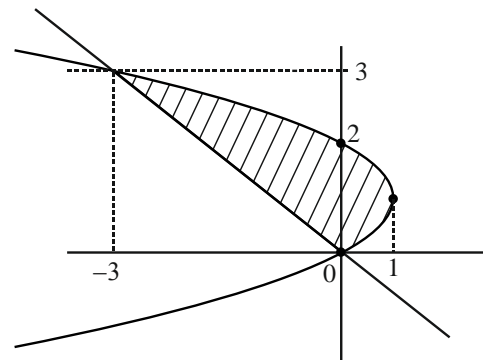
$$y = -x$$

Points of intersection

$$x^2 + 2x = -x$$

$$x^2 + 3x = 0$$

$$x = 0, -3$$



$$A = \int_0^3 (-y^2 + 2y + y) dy$$

$$= \left. \frac{3y^2}{2} - \frac{y^3}{3} \right|_0^3 = \frac{9}{2}$$

$$8A = 36$$

83. If  $\frac{1^3 + 2^3 + 3^3 + \dots \text{upto } n \text{ terms}}{1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots \text{upto } n \text{ terms}} = \frac{9}{5}$ , then

the value of n is

**Official Ans. by NTA (5)**

**Allen Ans. (5)**



**Sol.**  $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots + n \text{ terms} =$

$$\sum_{r=1}^n r(2r+1) = \sum_{r=1}^n (2r^2 + r)$$

$$= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6} (2(2n+1) + 3)$$

$$= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3}$$

$$= \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2} \times \frac{(4n+5)}{3}} = \frac{9}{5}$$

$$\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3}$$

$$\Rightarrow 15n(n+1) = 18(4n+5)$$

$$\Rightarrow 15n^2 + 15n = 72n + 90$$

$$\Rightarrow 15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0$$

$$\Rightarrow (n-5)(5n+6) = 0$$

$$\Rightarrow n = \frac{-6}{5} \text{ or } 5$$

$$\Rightarrow n = 5.$$

**84.** Let  $f$  be a differentiable function defined on  $\left[0, \frac{\pi}{2}\right]$  such that  $f(x) > 0$  and

$$f(x) + \int_0^x f(t) \sqrt{1 - (\log_e f(t))^2} dt = e, \forall x \in \left[0, \frac{\pi}{2}\right].$$

Then  $\left(6 \log_e f\left(\frac{\pi}{6}\right)\right)^2$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (27)**

**Allen Ans. (27)**

**Sol.**  $f(x) + \int_0^x f(t) \sqrt{1 - (\log_e f(t))^2} dt = e$

$$\Rightarrow f(0) = e$$

$$f'(x) + f(x) \sqrt{1 - (\ln f(x))^2} = 0$$

$$f(x) = y$$

$$\frac{dy}{dx} = -y \sqrt{1 - (\ln y)^2}$$

$$\int \frac{dy}{y \sqrt{1 - (\ln y)^2}} = -\int dx$$

Put  $\ln y = t$

$$\int \frac{dt}{\sqrt{1-t^2}} = -x + C$$

$$\sin^{-1} t = -x + C \Rightarrow \sin^{-1}(\ln y) = -x + C$$

$$\sin^{-1}(\ln f(x)) = -x + C$$

$$f(0) = e$$

$$\Rightarrow \frac{\pi}{2} = C$$

$$\Rightarrow \sin^{-1}(\ln f(x)) = -x + \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{-\pi}{6} + \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{3}$$

$$\Rightarrow \ln f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \text{ we need } \left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27.$$

**85.** The minimum number of elements that must be added to the relation  $R = \{(a, b), (b, c), (b, d)\}$  on the set  $\{a, b, c, d\}$  so that it is an equivalence relation, is \_\_\_\_\_.

**Official Ans. by NTA (13)**

**Allen Ans. (13)**

**Sol.** Given  $R = \{(a, b), (b, c), (b, d)\}$

In order to make it equivalence relation as per given set,  $R$  must be

$\{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)\}$

There already given so 13 more to be added.

86. Let  $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$ ,  $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$ ,  $\vec{a} \cdot \vec{c} = 7$ ,  $2\vec{b} \cdot \vec{c} + 43 = 0$ ,  $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$ . Then  $|\vec{a} \cdot \vec{b}|$  is equal to

**Official Ans. by NTA (8)**

**Allen Ans. (8)**

- Sol.**  $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$ ,  $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$ ,  $\vec{a} \cdot \vec{c} = 7$   
 $\vec{a} \times \vec{c} - \vec{b} \times \vec{c} = \vec{0}$ ,  
 $(\vec{a} - \vec{b}) \times \vec{c} = \vec{0} \Rightarrow (\vec{a} - \vec{b})$  is paralleled to  $\vec{c}$   
 $\vec{a} - \vec{b} = \mu\vec{c}$ , where  $\mu$  is a scalar  
 $-2\hat{i} + 7\hat{j} + 2\lambda\hat{k} = \mu \cdot \vec{c}$   
 Now  $\vec{a} \cdot \vec{c} = 7$  gives  $2\lambda^2 + 12 = 7\mu$   
 And  $\vec{b} \cdot \vec{c} = -\frac{43}{2}$  gives  $4\lambda^2 + 82 = 43\mu$   
 $\mu = 2$  and  $\lambda^2 = 1$   
 $|\vec{a} \cdot \vec{b}| = 8$

87. Let the sum of the coefficients of the first three terms in the expansion of  $\left(x - \frac{3}{x^2}\right)^n$ ,  $x \neq 0$ ,  $n \in \mathbb{N}$ , be 376. Then the coefficient of  $x^4$  is \_\_\_\_\_.

**Official Ans. by NTA (405)**

**Allen Ans. (405)**

- Sol.** Given Binomial  $\left(x - \frac{3}{x^2}\right)^n$ ,  $x \neq 0$ ,  $n \in \mathbb{N}$ ,  
 Sum of coefficients of first three terms  
 ${}^n C_0 - {}^n C_1 \cdot 3 + {}^n C_2 3^2 = 376$   
 $\Rightarrow 3n^2 - 5n - 250 = 0$   
 $\Rightarrow (n - 10)(3n + 25) = 0$   
 $\Rightarrow n = 10$   
 Now general term  ${}^{10} C_r x^{10-r} \left(\frac{-3}{x^2}\right)^r$   
 $= {}^{10} C_r x^{10-r} (-3)^r \cdot x^{-2r}$   
 $= {}^{10} C_r (-3)^r \cdot x^{10-3r}$   
 Coefficient of  $x^4 \Rightarrow 10 - 3r = 4$   
 $\Rightarrow r = 2$   
 ${}^{10} C_2 (-3)^2 = 405$

88. If the shortest between the lines

$$\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4} \text{ and}$$

$$\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5} \text{ is } 6, \text{ then the square}$$

of sum of all possible values of  $\lambda$  is

**Official Ans. by NTA (384)**

**Allen Ans. (384)**

- Sol.** Shortest distance between the lines

$$\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4}$$

$$\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5} \text{ is } 6$$

Vector along line of shortest distance

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}, \Rightarrow -\hat{i} + 2\hat{j} - \hat{k} \text{ (its magnitude is } \sqrt{6})$$

$$\text{Now } \frac{1}{\sqrt{6}} \begin{vmatrix} \sqrt{6} + \lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \pm 6$$

$$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$$

So, square of sum of these values is 384.

89. Let  $S = \{\theta \in [0, 2\pi) : \tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0\}$ .

Then  $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$  is equal to

**Official Ans. by NTA (2)**

**Allen Ans. (2)**

- Sol.**  $\tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0$

$$\tan(\pi \cos \theta) = -\tan(\pi \sin \theta)$$

$$\tan(\pi \cos \theta) = \tan(-\pi \sin \theta)$$

$$\pi \cos \theta = n\pi - \pi \sin \theta$$

$$\sin \theta + \cos \theta = n \text{ where } n \in \mathbb{I}$$

possible values are  $n = 0, 1$  and  $-1$  because

$$-\sqrt{2} \leq \sin \theta + \cos \theta \leq \sqrt{2}$$

$$\text{Now it gives } \theta \in \left\{0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{2}, \pi\right\}$$

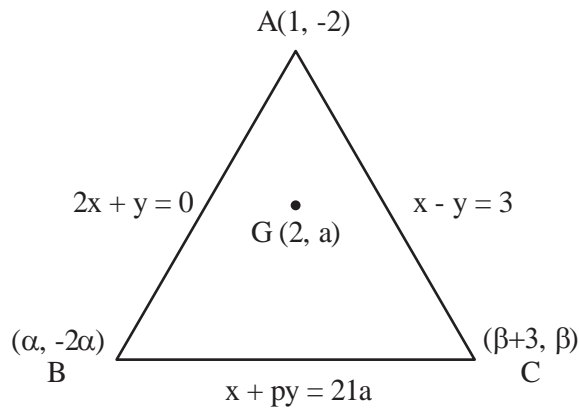
$$\text{So } \sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = 2(0) + 4\left(\frac{1}{2}\right) = 2$$

90. The equations of the sides AB, BC and CA of a triangle ABC are:  $2x + y = 0$ ,  $x + py = 21a$ , ( $a \neq 0$ ) and  $x - y = 3$  respectively. Let P (2, a) be the centroid of  $\Delta ABC$ . Then  $(BC)^2$  is equal to

**Official Ans. by NTA (122)**

**Allen Ans. (122)**

**Sol.**



Assume B( $\alpha$ ,  $-2\alpha$ ) and C( $\beta + 3$ ,  $\beta$ )

$$\frac{\alpha + \beta + 3 + 1}{3} = 2 \quad \text{also} \quad \frac{-2\alpha - 2 + \beta}{3} = a$$

$$\Rightarrow \alpha + \beta = 2 \quad -2\alpha - 2 + \beta = 3a$$

$$\Rightarrow \beta = 2 - \alpha \quad -2\alpha - 2 + 2 - \alpha = 3a \Rightarrow \alpha = -a$$

Now both B and C lies as given line

$$\alpha - p \cdot 2\alpha = 21a$$

$$\alpha(1 - 2p) = 21a \quad \dots (1)$$

$$-\alpha(1 - 2p) = 21a \Rightarrow p = 11$$

$$\beta + 3 + p\beta = 21a$$

$$\beta + 3 + 11\beta = 21a$$

$$21\alpha + 12\beta + 3 = 0$$

$$\text{Also } \beta = 2 - \alpha$$

$$21\alpha + 12(2 - \alpha) + 3 = 0$$

$$21\alpha + 24 - 12\alpha + 3 = 0$$

$$9\alpha + 27 = 0$$

$$\alpha = -3, \beta = 5$$

$$\text{So } BC = \sqrt{122} \quad \text{and } (BC)^2 = 122$$