JEE Main 2023 (1st Attempted)
(Shift - 01 Mathematics Paper) 24.01.2023

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

61. The distance of the point (7, –3, –4) from the plane passing through the points (2,–3,1), (–1, 1, –2) and (3, –4, 2) is :

(1) 4

(2) 5

(3) $5\sqrt{2}$

(4) $4\sqrt{2}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. **Equation of Plane is**

$$=\begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix}=0$$

$x - z - 1 = 0$

Distance of P (7, –3, –4) from Plane is

$$d=\left|\frac{7+4-1}{\sqrt{2}}\right|=5\sqrt{2}$$

62. $\lim\limits_{t\to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+....+n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(1) $n^2 + n$

(2) n

(3) $\frac{n(n+1)}{2}$

(4) n^2

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\lim\limits_{t\to 0}\left(1^{\operatorname{cosec}^2 t}+2^{\operatorname{cosec}^2 t}+........+n^{\operatorname{cosec}^2 t}\right)^{\sin^2 t}$

$$=\lim_{t\to 0} n\left(\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t}+\left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t}+........+1\right)^{\sin^2 t}$$

$= n$

63. Let $\vec{u}=\hat{i}-\hat{j}-2k, \vec{v}=2\hat{i}+\hat{j}-k, \vec{v}.\vec{w}=2$ and $\vec{v}\times\vec{w}=\vec{u}+\lambda\vec{v}$. Then $\vec{u}.\vec{w}$ is equal to

(1) 1

(2) $\frac{3}{2}$

(3) 2

(4) $-\frac{2}{3}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\vec{u}=(1,-1,-2)$, $\vec{v}=(2,1,-1)$, $\vec{v}.\vec{w}=2$

$\vec{v}\times\vec{w}=\vec{u}+\lambda\vec{v}$ ....(1)

Taking dot with $\vec{w}$ in (1)

$\vec{w}.(\vec{v}\times\vec{w})=\vec{u}.\vec{w}+\lambda\vec{v}.\vec{w}$

$\Rightarrow 0=\vec{u}.\vec{w}+2\lambda$

Taking dot with $\vec{v}$ in (1)

$\vec{v}.(\vec{v}\times\vec{w})=\vec{u}.\vec{v}+\lambda\vec{v}.\vec{v}$

$\Rightarrow 0=(2-1+2)+\lambda(6)$

$\lambda=-\frac{1}{2}$

$\Rightarrow \vec{u}.\vec{w}=-2\lambda=1$

64. The value $\sum\limits_{r=0}^{22} {}^{22}C_r\,{}^{23}C_r$ is

(1) ${}^{45}C_{23}$

(2) ${}^{44}C_{23}$

(3) ${}^{45}C_{24}$

(4) ${}^{44}C_{22}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\sum\limits_{r=0}^{22} {}^{22}C_r.{}^{23}C_r=\sum\limits_{r=0}^{22} {}^{22}C_r.{}^{23}C_{23-r}$

$= {}^{45}C_{23}$

65. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

(1) directrix $4x = 3$

(2) directrix $4x = -3$

(3) Length of latus rectum $\frac{3}{2}$

(4) Length of latus rectum 2

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $y^2 = 24x$

$a = 6$

$xy = 2$

$AB \equiv ty = x + 6t^2$ …………..(1)

$AB \equiv T = S_1$

$kx + hy = 2hk$ ……………. (2)

From (1) and (2)

$\frac{k}{1} = \frac{h}{-t} = \frac{2hk}{-6t^2}$

$\Rightarrow$ then locus is $y^2 = -3x$

Therefore directrix is $4x = 3$

66. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$

$2x + Ny + 2z = 2$

$3x + 3y + Nz = 3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

(1) 18 (2) 19

(3) 20 (4) 21

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $x + y + z = 1$

$2x + Ny + 2z = 2$

$3x + 3y + Nz = 3$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}$$

$= (N-2)(N-3)$

For unique solution $\Delta \neq 0$

So $N \neq 2, 3$

$\Rightarrow$ P (system has unique solution) $= \frac{4}{6}$

So $k = 4$

Therefore sum $= 4 + 1 + 4 + 5 + 6 = 20$

67. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$

68. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

(1) 4 (2) 3

(3) 2 (4) $\frac{5}{2}$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$

A is $\frac{2\vec{q}+\vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Area of ΔPQR is $= \frac{1}{2}|\vec{q} \times \vec{r}|$

Area of ΔABC is $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$\overrightarrow{AB} = \frac{\vec{r}-2\vec{q}}{3}, \overrightarrow{AC} = \frac{-\vec{r}-\vec{q}}{3}$

Area of $\Delta ABC = \frac{1}{6}|\vec{q} \times \vec{r}|$

$\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)} = 3$

69. If A and B are two non-zero $n \times n$ matrics such that $A^2 + B = A^2 B$, then

(1) $AB = I$

(2) $A^2B = I$

(3) $A^2 = I$ or $B = I$

(4) $A^2B = BA^2$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $A^2 + B = A^2 B$

$(A^2 - I)(B - I) = I$ …….(1)

$A^2 + B = A^2 B$

$A^2(B - I) = B$

$A^2 = B(B - I)^{-1}$

$A^2 = B(A^2 - I)$

$A^2 = BA^2 - B$

$A^2 + B = BA^2$

$A^2B = BA^2$

70. Let y = y(x) be the solution of the differential equation $x^3\ dy + (xy - 1)\ dx = 0,\ x > 0$, $y\left(\frac{1}{2}\right) = 3 - e$. Then y(1) is equal to

(1) 1

(2) e

(3) 2–e

(4) 3

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\frac{dy}{dx} = \frac{1 - xy}{x^3} = \frac{1}{x^3} - \frac{y}{x^2}$

$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$

$\text{If} = e^{\int \frac{1}{x^2}dx} = e^{-\frac{1}{x}}$

$y.e^{-\frac{1}{x}} = \int e^{-\frac{1}{x}}.\frac{1}{x^3}dx\ \left(\text{put } -\frac{1}{x} = t\right)$

$y.e^{-\frac{1}{x}} = -\int e^t.t\,dt$

$y = \frac{1}{x} + 1 + Ce^{\frac{1}{x}}$

Where C is constant

Put $x = \frac{1}{2}$

$3 - e = 2 + 1 + Ce^2$

$C = -\frac{1}{e}$

y(1) = 1

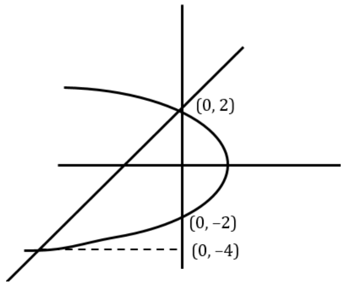
71. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is :

(1) $\frac{25}{3}$ (2) $\frac{22}{3}$ (3) 9 (4) $\frac{23}{3}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol.



$y^2 + 4x = 4$

$y^2 = -4(x - 1)$

$A = \int_{-4}^{2}\left(\frac{4 - y^2}{4} - \frac{y - 2}{2}\right)dy = 9$

72. Let α be a root of the equation $(a - c)x^2 + (b - a)x + (c - b) = 0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$

is singular. Then the value of $\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is

(1) 6 (2) 3

(3) 9 (4) 12

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\Delta = 0 = \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$

$\Rightarrow \alpha^2(c - b) - \alpha(c - a) + (b - a) = 0$

It is singular when $\alpha = 1$

$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$

$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$

$= 3\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

73. The distance of the point (–1, 9, –16) from the plane 2x + 3y – z = 5 measured parallel to the line $\frac{x+4}{3}=\frac{2-y}{4}=\frac{z-3}{12}$ is

(1) $13\sqrt{2}$ (2) 31

(3) 26 (4) $20\sqrt{2}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Equation of line

$\frac{x+1}{3}=\frac{y-9}{-4}=\frac{z+16}{12}$

G.P on line $(3\lambda-1, -4\lambda+9, 12\lambda-16)$

point of intersection of line & plane

$6\lambda-2-12\lambda+27-12\lambda+16=5$

$\lambda=2$

Point (5, 1, 8)

Distance = $\sqrt{36+64+576}=26$

74. For three positive integers p, q, r, $x^{pq^2}=y^{qr}=z^{p^2r}$ and r = pq + 1 such that 3, 3 $\log_y x$, 3$\log_z y$, 7$\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then r – p – q is equal to

(1) 2 (2) 6

(3) 12 (4) –6

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $pq^2=\log_x\lambda$

$qr=\log_y\lambda$

$p^2r=\log_z\lambda$

$\log_y x=\frac{qr}{pq^2}=\frac{r}{pq}$ ....(1)

$\log_x z=\frac{pq^2}{p^2r}=\frac{q^2}{pr}$ ....(2)

$\log_z y=\frac{p^2r}{qr}=\frac{p^2}{q}$ ....(3)

$3,\frac{3r}{pq},\frac{3p^2}{q},\frac{7q^2}{pr}$ in A.P

$\frac{3r}{pq}-3=\frac{1}{2}$

$r=\frac{7}{6}pq$ ....(4)

r = pq + 1

pq = 6 ....(5)

r = 7.....................(6)

$\frac{3p^2}{q}=4$

After solving p = 2 and q = 3

75. Let $p,q\in\mathbb{R}$ and $\left(1-\sqrt{3}i\right)^{200}=2^{199}(p+iq)$, $i=\sqrt{-1}$ Then p + q + q^2 and p – q + q^2 are roots of the equation.

(1) $x^2+4x-1=0$ (2) $x^2-4x+1=0$

(3) $x^2+4x+1=0$ (4) $x^2-4x-1=0$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\left(1-\sqrt{3}i\right)^{200}=2^{199}(p+iq)$

$2^{200}\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^{200}=2^{199}(p+iq)$

$2\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=p+iq$

$p=-1, q=-\sqrt{3}$

$\alpha=p+q+q^2=2-\sqrt{3}$

$\beta=p-q+q^2=2+\sqrt{3}$

$\alpha+\beta=4$

$\alpha.\beta=1$

equation $x^2-4x+1=0$

76. The relation R = $\{(a,b):\gcd(a,b)=1, 2a\neq b, a,b\in\mathbb{Z}\}$ is: ____

(1) transitive but not reflexive

(2) symmetric but not transitive

(3) reflexive but not symmetric

(4) neither symmetric nor transitive

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Reflexive : (a, a) ⇒ gcd of (a, a) = 1

Which is not true for every a ϵ Z.

Symmetric:

Take a = 2, b = 1 ⇒ gcd (2, 1) = 1

Also 2a = 4 ≠ b

Now when a = 1, b = 2 ⇒ gcd (1, 2) = 1

Also now 2a = 2 = b

Hence a = 2b

$\Rightarrow$ R is not Symmetric

Transitive:

Let a = 14, b = 19, c = 21

gcd (a, b) = 1

gcd (b, c) = 1

gcd (a, c) = 7

Hence not transitive

$\Rightarrow$ R is neither symmetric nor transitive.

77. The compound statement

$\left(\sim(P\wedge Q)\right)\vee\left((\sim P)\wedge Q\right)\Rightarrow\left((\sim P)\wedge(\sim Q)\right)$ is equivalent to

(1) $\left((\sim P)\vee Q\right)\wedge\left((\sim Q)\vee P\right)$

(2) $(\sim Q)\vee P$

(3) $\left((\sim P)\vee Q\right)\wedge(\sim Q)$

(4) $(\sim P)\vee Q$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. **Let r** = $\left(\sim(P\wedge Q)\right)\vee\left((\sim P)\wedge Q\right)$; s = $\left((\sim P)\wedge(\sim Q)\right)$

P	Q	$\sim(P\wedge Q)$	$(-P)\wedge Q$	r	s	r→s
T	T	F	F	F	F	T
T	F	T	F	T	F	F
F	T	T	T	T	F	F
F	F	T	F	T	T	T

Option (A) : $\left((\sim P)\vee Q\right)\wedge\left((\sim Q)\vee P\right)$

is equivalent to (not of only P) $\wedge$ (not of only Q)

= (Both P, Q) and (neither P nor Q)

78. Let $f(x)=\begin{cases}x^2\sin\left(\frac{1}{x}\right) & ,x\neq 0\\ 0 & ,x=0\end{cases}$; Then at x = 0

(1) f is continuous but not differentiable

(2) f is continuous but f ′ is not continuous

(3) f and f ′ both are continuous

(4) f ′ is continuous but not differentiable

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. Continuity of f(x) **:** $f\left(0^+\right) = h^2.\sin\frac{1}{h}=0$

$f(0^-)=(-h)^2.\sin\left(\frac{-1}{h}\right)=0$

f(0) = 0

f(x) is continuous

$f'\left(0^+\right)=\lim\limits_{h\to 0}\frac{f(0+h)-f(0)}{h}=\frac{h^2.\sin\left(\frac{1}{h}\right)-0}{h}=0$

$f'\left(0^-\right)=\lim\limits_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\frac{h^2.\sin\left(\frac{1}{-h}\right)-0}{-h}=0$

f(x) is differentiable.

$f'(x)=2x.\sin\left(\frac{1}{x}\right)+x^2.\cos\left(\frac{1}{x}\right).\frac{-1}{x^2}$

$f'(x)=\begin{cases}2x.\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$

$\Rightarrow$ $f'(x)$ is not continuous (as $\cos\left(\frac{1}{x}\right)$ is highly oscillating at x = 0)

79. The equation $x^2 - 4x + [x] + 3 = x[x]$, where [x] denotes the greatest integer function, has:

(1) exactly two solutions in $(-\infty,\infty)$

(2) no solution

(3) a unique solution in $(-\infty,1)$

(4) a unique solution in $(-\infty,\infty)$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $x^2-4x+[x]+3=x[x]$

$\Rightarrow x^2-4x+3=x[x]-[x]$

$\Rightarrow$ (x–1) (x –3) = [x]. (x –1)

$\Rightarrow$ x = 1 or x – 3 = [x]

$\Rightarrow$x –[x] = 3

$\Rightarrow$$\{x\}$ = 3 (Not Possible)

Only one solution x = 1 in $(-\infty,\infty)$

80. Let Ω be the sample space and $A\subseteq\Omega$ be an event. Given below are two statements :

(S1) : If P (A) = 0, then A = ϕ

(S2) : If P (A) = , then A = Ω

Then

(1) only (S1) is true

(2) only (S2) is true

(3) both (S1) and (S2) are true

(4) both (S1) and (S2) are false

Official Ans. by NTA (4)

Allen Ans. (3)

Sol. Ω = sample space
A = be an event
If P (A) = 0 $\Rightarrow$ A = ϕ
If P (A) = 1 $\Rightarrow$ A = Ω
Then both statement are true

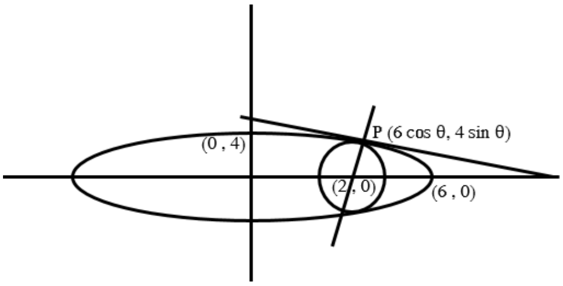
SECTION-B

81. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse = $\frac{x^2}{36}+\frac{y^2}{16}=1$.
If $(1, \alpha)$ lies on C, then $10\,\alpha^2$ is equal to________

Official Ans. by NTA (118)

Allen Ans. (118)

Sol.



Equation of normal of ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ at any point P $(6\cos\theta, 4\sin\theta)$ is

$3\sec\theta x - 2\,\text{cosec}\,\theta y = 10$ this normal is also the normal of the circle passing through the point (2, 0) So,
$6\sec\theta = 10$ or $\sin\theta = 0$ (Not possible)

$\cos\theta=\frac{3}{5}$ and $\sin\theta=\frac{4}{5}$ so point $P=\left(\frac{18}{5},\frac{16}{5}\right)$

So the largest radius of circle

$r=\frac{\sqrt{320}}{5}$

So the equation of circle $(x-2)^2+y^2=\frac{64}{5}$

Passing it through $(1,\alpha)$

Then $\alpha^2=\frac{59}{5}$

$10\alpha^2=118$

82. Suppose $\sum_{r=0}^{2023} r^2\,{}^{2023}C_r = 2023\times\alpha\times 2^{2022}$. Then the value of α is __________

Official Ans. by NTA (1012)

Allen Ans. (1012)

Sol. using result

$$\sum_{r=0}^{n} r^2\,{}^{n}C_r = n(n+1).2^{n-2}$$

Then $\sum_{r=0}^{2023} r^2\,{}^{2023}C_r = 2023\times 2024\times 2^{2021}$

$= 2023\times\alpha\times 2^{2022}$ So,

$\Rightarrow \alpha = 1012$

83. The value of $12\int_0^3 \left|x^2-3x+2\right|dx$ is _______

Official Ans. by NTA (22)

Allen Ans. (22)

Sol. $12\int_0^3 \left|x^2-3x+2\right|dx$

$= 12\int_0^3 \left|\left(x-\frac{3}{2}\right)^2-\frac{1}{4}\right|dx$

If $x-\frac{3}{2}=t$

dx = dt

$= 24\int_0^{3/2}\left|t^2-\frac{1}{4}\right|dt$

$= 24\left[-\int_0^{1/2}\left(t^2-\frac{1}{4}\right)dt+\int_{1/2}^{3/2}\left(t^2-\frac{1}{4}\right)dt\right]=22$

84. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is___

Official Ans. by NTA (60)

Allen Ans. (60)

Sol. **Even digits occupy at even places**

$$\frac{4!}{2!2!}\times\frac{5!}{2!3!}=\frac{24\times 120}{4\times 12}=60$$

85. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set S = $\{x + \lambda : x$ is an integer solution of E $\}$ is_______

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. $|x|^2 - 2|x| + |\lambda - 3| = 0$

$|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$

$(|x| - 1)^2 + |\lambda - 3| = 1$

At $\lambda = 3$, $x = 0$ and 2,

at $\lambda = 4$ or 2, then

$x = 1$ or -1

So maximum value of $x + \lambda = 5$

86. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

Official Ans. by NTA (546)

Allen Ans. (546)

Sol. For at most two language courses

$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$

87. Let a tangent to the Curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is________

Official Ans. by NTA (7)

Allen Ans. (7)

Sol. Equation of tangent at point $P(4\cos\theta, 3\sin\theta)$ is

$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{3} = 1$ So A is $(4\sec\theta, 0)$ and point B is $(0, 3\,\text{cosec}\,\theta)$

Length AB = $\sqrt{16\sec^2\theta + 9\,\text{cosec}^2\theta}$

$= \sqrt{25 + 16\tan^2\theta + 9\cot^2\theta} \geq 7$

88. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ is _______.

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ ....(1)

Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos x)^{2023}} dx$ ....(2)

Adding (1) & (2)

$2I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} 1\,dx$

$I = 2$

89. The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to _______

Official Ans. by NTA (14)

Allen Ans. (14)

Sol. **Shortest distance between the lines**

$$= \frac{\begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}}{\left|\begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}\right|}$$

$$= \frac{16 + 12 + 168}{|-4\hat{i} + 6\hat{j} - 12k|} = \frac{196}{14} = 14$$

90. The 4th term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is ______

Official Ans. by NTA (12)

Allen Ans. (12)

Sol. $T_4 = 500$ where a = first term,

$r = \text{common ratio} = \frac{1}{m}, m \in N$

$ar^3 = 500$

$\frac{a}{m^3} = 500$

$S_n - S_{n-1} = ar^{n-1}$

$S_6 > S_5 + 1$ and $S_7 - S_6 < \frac{1}{2}$

$S_6 - S_5 > 1$ $\quad$ $\frac{a}{m^6} < \frac{1}{2}$

$ar^5 > 1$ $\quad$ $m^3 > 10^3$

$\frac{500}{m^2} > 1$ $\quad$ $m > 10$ ……(2)

$m^2 < 500$ …….(1)

From (1) and (2)

m = 11, 12, 13…………..,22

So number of possible values of m is 12