JEE Main 2023 (2nd Attempt)
(Shift - 02 Mathematics Paper)

13.04.2023

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. If the system of equations

$2x + y - z = 5$

$2x - 5y + \lambda z = \mu$

$x + 2y - 5z = 7$

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to

(1) 916

(2) 912

(3) 920

(4) 904

Official Ans. By NTA (1)

Allen Ans. (1)

Sol.

$$\begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$2(25 - 2\lambda) - (-10 - \lambda) - (4 + 5) = 0$

$50 - 4\lambda + 10 + \lambda - 9 = 0$

$51 = 3\lambda \Rightarrow \lambda = 17$

$$\begin{vmatrix} 2 & 1 & 5 \\ 2 & -5 & \mu \\ 1 & 2 & 7 \end{vmatrix} = 0$$

$\Rightarrow 2(-35 - 2\mu) - (14 - \mu) + 5(4 + 5) = 0$

$-70 - 4\mu - 14 + \mu + 45 = 0$

$-3\mu = 39$

$-\mu = 13$

$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 2(\lambda^2 + \mu^2)$

$= 2(17^2 + 13^2) = 916$

2. The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

(1) 8

(2) 9

(3) $\frac{80}{9}$

(4) $\frac{26}{3}$

Official Ans. By NTA (3)

Allen Ans. (3)

Sol. $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$$T_{r+1} = {}^5C_r\,(2x^3)^{5-r}\left(\frac{-1}{3x^2}\right)^r = {}^5C_r\,\frac{(2)^{5-r}}{(-3)^r}(x)^{15-5r}$$

$\therefore 15 - 5r = 5$

$\therefore r = 2$

$T_3 = 10\left(\frac{8}{9}\right)x^5$

So, coefficient is $\frac{80}{9}$

3. The plane, passing through the points (0, –1, 2) and (–1, 2, 1) and parallel to the line passing through (5, 1, –7) and (1, –1, –1), also passes through the point.

(1) (1, –2, 1)

(2) (0, 5, –2)

(3) (–2, 5, 0)

(4) (2, 0, 1)

Official Ans. By NTA (3)

Allen Ans. (3)

Sol. Points (0, –1, 2) and (–1, 2, 1) parallel to the line of (5, 1, –7) and (1, –1, –1)

$$\text{Normal Vector} = \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ -1 & 3 & -1 \end{vmatrix}$$

$\vec{n} = 16\hat{i} + 10\hat{j} + 14\hat{k}$

$16x + 10y + 14z = d$

Point (0, –1, 2)

$0 - 10 + 28 = d \Rightarrow d = 18$

$8x + 5y + 7z = 9$ is equation of plane.

4. Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

(1) $-64\sqrt{2}$

(2) $-128\sqrt{2}$

(3) -64

(4) -128

Official Ans. By NTA (4)

Allen Ans. (4)

Sol. $x^2 - \sqrt{2}x + 2 = 0$

$$x = \frac{\sqrt{2} \pm \sqrt{2-8}}{2} = \frac{\sqrt{2} \pm \sqrt{6}i}{2}$$

$$\alpha = \frac{\sqrt{2} + \sqrt{6}i}{2} = \sqrt{2}e^{\frac{i\pi}{3}} \ \& \beta = \sqrt{2}e^{\frac{-i\pi}{3}}$$

$$\alpha^{14} = 2^7 e^{\frac{i14\pi}{3}} = 128\left[e^{\frac{i2\pi}{3}}\right]$$

$$\beta^{14} = 128\left[e^{\frac{-i2\pi}{3}}\right]$$

$$\alpha^{14} + \beta^{14} = 128(2)\cos\left(\frac{2\pi}{3}\right) = -128$$

5. Let a_1, a_2, a_3, …. be a G.P. of increasing positive numbers. Let the sum of its 6^{th} and 8^{th} terms be 2 and the product of its 3^{rd} and 5^{th} terms be $\frac{1}{9}$. Then 6 (a_2 + a_4) (a_4 + a_6) is equal to

(1) $2\sqrt{2}$

(2) 2

(3) $3\sqrt{3}$

(4) 3

Official Ans. By NTA (4)

Allen Ans. (4)

Sol.

$ar^5 + ar^7 = 2$

$(ar^2)(ar^4) = \frac{1}{9}$

$a^2 r^6 = \frac{1}{9}$

Now, $r > 0$

$ar^5(1 + r^2) = 2$

Now, $ar^3 = \frac{1}{3}$ or $-\frac{1}{3}$ (rejected)

$r^2 = 2$

$r = \sqrt{2}$

$a = \frac{1}{6\sqrt{2}}$

Now, $6(a_2 + a_4)(a_4 + a_6)$

$6(ar + ar^3)(ar^3 + ar^5)$

$6a^2r^4(1 + r^2)$

$6\left(\frac{1}{36.2}\right)(4)(9) = 3$

6. Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation

(1) $x^2 - 7x + 12 = 0$

(2) $x^2 - 13x + 42 = 0$

(3) $x^2 - 14x + 48 = 0$

(4) $x^2 - 10x + 25 = 0$

Official Ans. By NTA (2)

Allen Ans. (2)

Sol. upon solving we get coordinates as (6, 8), (1, 2) and (5, –7)

So centroid : (α, β) is

$$\alpha = \frac{6+1+5}{3} = 4$$

$$\beta = \frac{8+2-7}{3} = 1$$

$\alpha + 2\beta = 6$

$2\alpha - \beta = 7$

Ans. $x^2 - 13x + 42 = 0$

7. **Let** $|\vec{a}|=2$, $|\vec{b}|=3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $\left|\left(\vec{a}+2\vec{b}\right)\times\left(2\vec{a}-3\vec{b}\right)\right|^2$ is equal to

(1) 482

(2) 441

(3) 841

(4) 882

Official Ans. By NTA (4)

Allen Ans. (4)

Sol. $|\vec{a}|=2, |\vec{b}|=3$

$$\left|\left(\vec{a}+2\vec{b}\right)\times(2\vec{a}-3\vec{b})\right|^2$$

$$\left|-3\vec{a}\times\vec{b}+4\vec{b}\times\vec{a}\right|^2$$

$$\left|-3\vec{a}\times\vec{b}-4\vec{a}\times\vec{b}\right|^2$$

$$\left|-7\vec{a}\times\vec{b}\right|^2$$

$$\left(-7|\vec{a}|\times|\vec{b}|\sin\left(\frac{\pi}{4}\right)\right)^2$$

$$49\times4\times9\times\frac{1}{2}=882$$

8. Let N be the foot of perpendicular from the point P (1, –2, 3) on the line passing through the points (4, 5, 8) and (1, –7, 5). Then the distance of N from the plane $2x - 2y + z + 5 = 0$ is

(1) 6

(2) 9

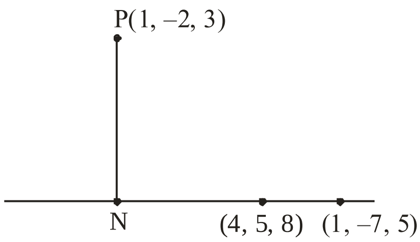
(3) 7

(4) 8

Official Ans. By NTA (3)

Allen Ans. (3)

Sol.



Equation of line

$$\frac{x-4}{4-1}=\frac{y-5}{5-(-7)}=\frac{z-8}{8-5}$$

$$\frac{x-4}{3}=\frac{y-5}{12}=\frac{z-8}{3}$$

Let point $N(3\lambda+4, 12\lambda+5, 3\lambda+8)$

$$\overrightarrow{PN}=(3\lambda+4-1)\hat{i}+(12\lambda+5-(-2))\hat{j}+(3\lambda+8-3)\hat{k}$$

$$\overrightarrow{PN}=(3\lambda+3)\hat{i}+(12\lambda+7)\hat{j}+(3\lambda+5)\hat{k}$$

And parallel vector to line (say $\vec{a}=3\hat{i}+12\hat{j}+3\hat{k}$)

Now, $\overrightarrow{PN}\cdot\vec{a}=0$

$(3\lambda+3)3+(12\lambda+7)12+(3\lambda+5)3=0$

$162\lambda+108=0\Rightarrow\lambda=\frac{-108}{162}=\frac{-2}{3}$

So point N is (2, –3, 6)

Now distance is $=\left|\frac{2(2)-2(-3)+6+5}{\sqrt{4+4+1}}\right|=7$

9. If $\lim_{x\to0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{1-\cos(2x)}=17$, then $5a^2+b^2$ is equal to

(1) 72

(2) 76

(3) 68

(4) 64

Official Ans. By NTA (3)

Allen Ans. (3)

Sol. $$\lim_{x\to 0}\frac{e^{ax}-\cos(bx)-\frac{cxe^{-cx}}{2}}{\frac{(1-\cos 2x)}{4x^2}\times 4x^2}=17$$

On expansion,

$$\lim_{x\to 0}\frac{\left(1+ax+\frac{a^2x^2}{2}\right)-\left(1-\frac{b^2x^2}{2}\right)-\frac{cx}{2}(1-cx)}{2x^2}=17$$

$$\lim_{x\to 0}\frac{\left(a-\frac{c}{2}\right)x+x^2\left(\frac{a^2}{2}+\frac{b^2}{2}+\frac{c^2}{2}\right)}{2x^2}=17$$

For limit to be exist $a-\frac{c}{2}=0$

$$a=\frac{c}{2}$$

and $\frac{a^2+b^2+c^2}{4}=17$

$a^2+b^2+4a^2=17\times 4$

$5a^2+b^2=68$

10. Let the centre of a circle C be (α, β) and its radius $r < 8$. Let $3x + 4y = 24$ and $3x - 4y = 32$ be two tangents and $4x + 3y = 1$ be a normal to C. Then $(\alpha - \beta + r)$ is equal to

(1) 7

(2) 9

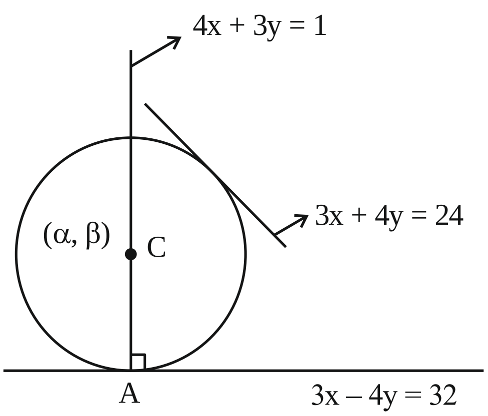
(3) 5

(4) 6

Official Ans. By NTA (1)

Allen Ans. (1)

Sol.



First find point A by solving $4x + 3y = 1$ and $3x - 4y = 32$

After solving, point A is $(4, -5)$

centre (α, β) lie on $4x + 3y = 1$

$$4\alpha+3\beta=1\Rightarrow\beta=\frac{1-4\alpha}{3}$$

Now distance from centre to line $3x - 4y - 32 = 0$ and $3x + 4y - 24 = 0$ are equal.

$$\left|\frac{3\alpha-4\left(\frac{1-4\alpha}{3}\right)-32}{5}\right|=\left|\frac{3\alpha+4\left(\frac{1-4\alpha}{3}\right)-24}{5}\right|$$

after solving $\alpha = 1$ and $\alpha=\frac{28}{3}$

For $\alpha = 1$, centre $(1, -1) \Rightarrow$ radius $= 5$

For $\alpha=\frac{28}{3}$, centre $\left(\frac{28}{3},\frac{-109}{2}\right)$

$\Rightarrow$ radius ≈ 49.78 (rejected)

Hence, $\alpha = 1, \beta = -1, r = 5$

$\alpha - \beta + r = 7$

11. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is

(1) 327

(2) 326

(3) 328

(4) 324

Official Ans. By NTA (1)

Allen Ans. (1)

Sol. First arrange in alphabetical order

i.e. ADMNOY

A _ _ _ _ _ = 5!

D _ _ _ _ _ = 5!

M A _ _ _ _ = 4!

M D _ _ _ _ = 4!

M N _ _ _ _ = 4!

M O A _ _ _ = 3!

M O D _ _ _ = 3!

M O N A _ _ = 2!

M O N D A Y = 1

= 327

12. The range of $f(x)=4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ is

(1) $[0, \pi]$

(2) $[0, 2\pi)$

(3) $[0, \pi)$

(4) $[0, 2\pi]$

Official Ans. By NTA (2)

Allen Ans. (2)

Sol. $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$

$$\frac{x^2+1-1}{x^2+1}=1-\frac{1}{x^2+1}\Rightarrow[0,1)$$

Range of $f(x) = [0, 2\pi)$

13. The statement $(p\wedge(\sim q)\vee((\sim p)\wedge q)\vee((\sim p)\wedge(\sim q))$ is equivalent to

(1) $(\sim p)\vee(\sim q)$

(2) $p\vee(\sim q)$

(3) $(\sim p)\vee q$

(4) $p\vee q$

Official Ans. By NTA (1)

Allen Ans. (1)

Sol. $(p\wedge(\sim q)\vee((\sim p)\wedge q)\vee((\sim p)\wedge(\sim q))$

$(p \wedge (\sim q)) \vee ((\sim p) \wedge (q \vee (\sim q)))$

$(p \wedge (\sim q)) \vee ((\sim p) \wedge t)$

$(p \wedge (\sim q)) \vee (\sim p)$

$(\sim p) \vee (p \wedge \sim q)$

$(\sim p \vee p) \wedge (\sim p \vee \sim q)$

$t \wedge (\sim p \vee \sim q)$

$= \sim p \vee \sim q$

14. The random valuable X follows binomial distribution B (n, p) for which the difference of the mean and the variance is 1. If $2P(X = 2) = 3P(X = 1)$, then $n^2 P(X > 1)$ is equal to

(1) 12

(2) 15

(3) 11

(4) 16

Official Ans. By NTA (3)

Allen Ans. (3)

Sol. $np - npq = 1$

$\Rightarrow np^2 = 1$

$2\,{}^nC_2p^2q^{n-2} = 3\,{}^nC_1pq^{n-1}$

$\Rightarrow np - p = 3q$ $(\therefore q = 1 - p)$

$\Rightarrow p = \frac{1}{2}$

Hence n = 4

$P(x > 1) = 1 - (p(x = 0) + p(x = 1))$

$$= 1-\left({}^4C_0\left(\frac{1}{2}\right)^4+{}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3\right)=\frac{11}{16}$$

15. Let for $A=\begin{bmatrix}1 & 2 & 3\\ a & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$, |A| = 2. If |2adj (2adj (2A))|

= 32^n, then 3n + α is equal to

(1) 10

(2) 9

(3) 12

(4) 11

Official Ans. By NTA (4)

Allen Ans. (4)

Sol. $A=\begin{bmatrix}1 & 2 & 3\\ a & 3 & 1\\ 1 & 1 & 2\end{bmatrix}$ |A| = 2,

1(6 – 1) – 2 (2α – 1) + 3(α – 3) = 2

5 – 4 α + 2 + 3 α – 9 = 2

– α – 4 = 0

α = –4

8|Adj(2Adj(2A))|

8|Adj(2×2^2 Adj (A))|

8|Adj(2^3AdjA)|

8|2^6Adj(AdjA)|

$2^3(2^6)^3$|Adj(Adj)|

$2^3 . 2^{18} |A|^4$

$2^{21} . 2^4 = 2^{25} = (2^5)^5 = (32)^5$

n = 5

α = –4

16. Let $S=\{Z\in C:\bar{z}=i(z^2+\text{Re}(\bar{z}))\}$. Then $\sum_{z\in S}|z|^2$ is equal to

(1) $\frac{7}{2}$

(2) 4

(3) $\frac{5}{2}$

(4) 3

Official Ans. By NTA (2)

Allen Ans. (2)

Sol. Let Z = x + iy, x ∈ R, y ∈ R

x –iy = i($x^2 - y^2$ + (2xy)i + x)

x = –2xy **…..(1)**

$-y = -y^2 + x^2 + x$ **…..(2)**

⇒ x = 0, y = $-\frac{1}{2}$ (from (1))

If x ≠ 0, then y = 0, 1

If y = $-\frac{1}{2}$, then x = $\frac{1}{2}, -\frac{3}{2}$

Z = 0 + i0, 0 + i, $\frac{1}{2} - \frac{i}{2}$, $-\frac{3}{2} - \frac{i}{2}$

17. The area of the region

$\{(x, y): x^2 \le y \le |x^2-4|, y\ge 1\}$ is

(1) $\frac{3}{4}(4\sqrt{2}-1)$

(2) $\frac{4}{3}(4\sqrt{2}-1)$

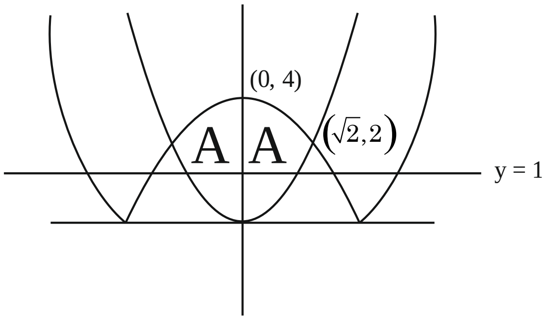
(3) $\frac{4}{3}(4\sqrt{2}+1)$

(4) $\frac{3}{4}(4\sqrt{2}+1)$

Official Ans. By NTA (2)

Allen Ans. (2)

Sol.



Required area = $2\left[\int_1^2 \sqrt{y}\,dy + \int_2^4 \sqrt{4-y}\,dy\right] = \frac{4}{3}\left[4\sqrt{2}-1\right]$

18. Let for a triangle ABC,

$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB}.\overrightarrow{CA}$ is equal to

(1) 60

(2) 120

(3) 108

(4) 54

Official Ans. By NTA (1)

Allen Ans. (1)

Sol. $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$

$\alpha = 2, \beta = 4, \gamma - \delta = 3$

$\frac{1}{2}\left|\overline{AB} \times \overline{AC}\right| = 5\sqrt{6}$

$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$

$\Rightarrow \delta = 5, \gamma = 8$

Hence $\overline{CB} \cdot \overline{CA} = 60$

19. The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is

(1) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$

(2) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

(3) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

(4) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

Official Ans. By NTA (2)

Allen Ans. (2)

Sol. Condition of co-planarity

$$\begin{vmatrix} x_2 - x_1 & a_1 & a_2 \\ y_2 - y_1 & b_1 & b_2 \\ z_2 - z_1 & c_1 & c_2 \end{vmatrix} = 0$$

Where a_1, b_1, c_1 are direction cosine of 1st line and a_2, b_2, c_2 are direction cosine of 2nd line.

Now, solving options

Point (–3, 1, 5) & point (–1, 2, 5)

(1) $\begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$

$= -3(5) - (10) + 5(-1 + 4)$

$= -15 - 10 + 15 = -10$

(2) Point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$$

$= 3(5) - (10) + 5(1 + 4)$

$-25 + 25 = 0$

(3) Point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$$

$-3(4) - (8) + 5(1 + 4)$

$-12 - 8 + 25 = 5$

(4) Point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

$-3(-5) - (-20) + 5(-1 - 8)$

$15 + 20 - 45 = -10$

20. The value of $\dfrac{e^{-\frac{\pi}{4}}+\int_{0}^{\frac{\pi}{4}} e^{-x}\tan^{50} x\,dx}{\int_{0}^{\frac{\pi}{4}} e^{-x}\left(\tan^{49} x+\tan^{51} x\right)dx}$ is

(1) 50

(2) 49

(3) 51

(4) 25

Official Ans. By NTA (1)

Allen Ans. (1)

Sol. $\int_{0}^{\pi/4} e^{-x}\tan^{50} x dx$

$$\left[-e^{-y}(\tan x)^{50}\right]_0^{\pi/4}+\int_0^{\pi/4} e^{-x}\,(50)\,(\tan x)^{49}\sec^2 x$$

$$=-e^{-\pi/4}+0+50\int_0^{\pi/4} e^{-x}(\tan x)^{49}(\tan^2 x+1)$$

$$=-e^{-\pi/4}+50\left(\int_0^{\pi/4} e^{-x}(\tan x)^{51}+(\tan x)^{49}\right)dx$$

Now, $\dfrac{-e^{-\pi/4}+\int_0^{\pi/4} e^{-x}(\tan x)^{50}dx}{\int_0^{\pi/4} e^{-x}(\tan^{49} x+\tan^{51} x)dx}$

$$\frac{50\int_0^{\pi/4} e^{-x}\left((\tan x)^{51}+(\tan x)^{49}\right)dx}{\int_0^{\pi/4} e^{-x}(\tan^{49} x+\tan^{51} x)dx}=50$$

SECTION-B

21. The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _______.

Official Ans. by NTA (269)

Allen Ans. (269)

Sol. $\overline{x} = 50$

$\sum x_i = 500$

$\sum x_{i\,correct} = 500 + 20 + 25 - 45 - 50 = 450$

$\sigma^2 = 144$

$\dfrac{\sum x_i^2}{10} - (50)^2 = 144$

$$\sum x_{i correct}^2 = \left(144+(50)^2\right)\times 10-(45)^2-(50)^2+(20)^2+(25)^2$$

$= 22940$

Correct variance $= \dfrac{\sum(x_{icorrect})^2}{10} - \left(\dfrac{\sum x_{icorrect}}{10}\right)^2$

$= 2294 - (45)^2$

$= 2294 - 2025 = 269$

22. Let A= {–4, –3, –2, 0, 1, 3, 4} and R = {(a, b) ∈ A × A : b = |a| or b^2 = a + 1} be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is______.

Official Ans. by NTA (7)

Allen Ans. (7)

Sol. R = [(–4, 4), (–3, 3), (3, –2), (0, 1), (0, 0), (1, 1), (4, 4), (3, 3)}

For reflexive, add ⇒ (–2, –2), (–4, –4), (–3, –3)

For symmetric, add ⇒ (4, –4), (3, –3), (–2, 3), (1, 0)

23. Let $f(x) = \sum_{k=1}^{10} kx^k$, $x \in R$. If $2f(2) + f'(2) = 119(2)^n + 1$ then n is equal to ______.

Official Ans. by NTA (10)

Allen Ans. (10)

Sol. $f(x) = \sum_{k=1}^{10} k\, x^k$

$f(x) = x + 2x^2 + \ldots\ldots\ldots + 10\,x^{10}$

$f(x).\ x = x^2 + 2x^3 + \ldots\ldots\ldots + 9\,x^{10} + 10x^{11}$

$f(x)\,(1-x) = x + x^2 + x^3 + \ldots\ldots.. + x^{10} - 10\,x^{11}$

$$f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(x) = \frac{x - x^{11} - 10x^{11} + 10x^{12}}{(1-x)^2} \Rightarrow \frac{10x^{12} - 11x^{11} + x}{(1-x)^2}$$

Hence $2f(2) + f'(2) = 119.2^{10} + 1$

$\Rightarrow$So, $n = 10$

24. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is____.

Official Ans. by NTA (16)

Allen Ans. (16)

Sol. For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3.

(2, 1, 3), (2, 3, 4), (2, 5, 5), (2, 2, 5), (2, 2, 2), (4, 1, 1), (4, 4, 1), (4, 4, 4), (4, 3, 5)

2, 1, 3 $\Rightarrow$ 312, 132

2, 3, 4 $\Rightarrow$ 342, 432, 234, 324

2, 5, 5 $\Rightarrow$ 552

2, 2, 5 $\Rightarrow$ 252, 522

2, 2, 2 $\Rightarrow$ 222

4, 1, 1 $\Rightarrow$114

4, 4, 1 $\Rightarrow$ 414, 144

4, 4, 4 $\Rightarrow$ 444

4, 3, 5 $\Rightarrow$354, 534

Total 16 numbers.

25. Let $[\alpha]$ denote the greatest integer $\le \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots.. + [\sqrt{120}]$ is equal to.

Official Ans. by NTA (825)

Allen Ans. (825)

Sol. $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots\ldots. [\sqrt{120}]$

$\Rightarrow$ 1 + 1 + 1 + 2 + 2 + 2 +2 + 2 + 3 + 3 + ……. + 3 = 7 times

+ 4 + 4 + …….. + 4 = 9 times + ……. 10 + 10 + ……. + 10 = 21 times

$$\Rightarrow \sum_{r=1}^{10} (2r+1)\,.r$$

$$\Rightarrow 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$$

$$\Rightarrow 2 \times \frac{10 \times 11 \times 21}{6} + \frac{10 \times 11}{2}$$

$\Rightarrow$770 + 55

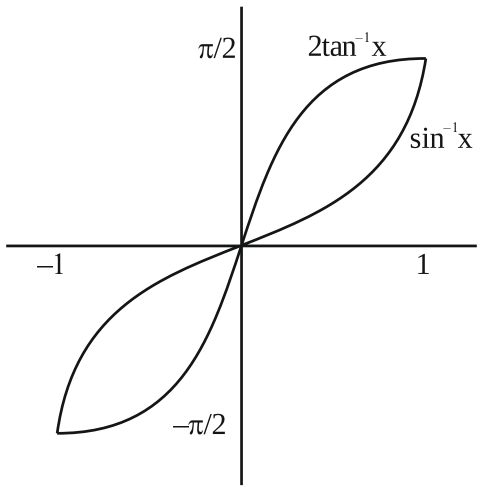
$\Rightarrow$ 825

26. For $x \in (-1, 1]$, the number of solutions of the equation $\sin^{-1}x = 2\tan^{-1}x$ is equal to

Official Ans. by NTA (2)

Allen Ans. (2)

Sol.



27. If y = y (x) is the solution of the differential equation $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, x > 1 such that $y(2)=\frac{2}{9}\log_e\left(2+\sqrt{3}\right)$ and $y\left(\sqrt{2}\right)$ = $\alpha\log_e\left(\sqrt{\alpha}+\beta\right)+\beta-\sqrt{\gamma},\alpha,\beta,\gamma\in N$, then $\alpha\beta\gamma$ is equal to ____.

Official Ans. by NTA (6)

Allen Ans. (6)

Sol. $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}, x>1$

I.F. = $e^{\int\frac{4x}{x^2-1}dx}$

I.F. = $(x^2-1)^2$

$\Rightarrow d\left(y.(x^2-1)^2\right)=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}.(x^2-1)^2$

$\Rightarrow\int d\left(y.(x^2-1)^2\right)=\int\frac{x+2}{(x^2-1)^{\frac{1}{2}}}dx$ ______ (1)

$y(x^2-1)^2=\sqrt{x^2-1}+2\ \ln\left(x+\sqrt{x^2-1}\right)+C$

$\Rightarrow C=-\sqrt{3}$

So $\left(x^2-1\right)^2=\sqrt{x^2-1}+2\ln\left(x+\sqrt{x^2-1}\right)-\sqrt{3}$

$\Rightarrow\alpha\beta\gamma=6$

28. The foci of a hyperbola are (±2,0) and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line 2x + 3y = 6, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x- and y-axes are a and b respectively, then |6a| + |5b| is equal to_____.

Official Ans. by NTA (12)

Allen Ans. (12)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

ae = 2 & e = $\frac{3}{2}\Rightarrow a=\frac{4}{3}$

also $b^2=a^2e^2-a^2\Rightarrow 4-\frac{16}{9}$

$\Rightarrow b^2=\frac{20}{9}$

Slope of tangent = $\frac{3}{2}$

So tangent equation will be

$y=mx\pm\sqrt{a^2m^2-b^2}$

$\Rightarrow y=\frac{3x}{2}\pm\sqrt{\frac{16}{9}.\frac{9}{4}-\frac{20}{9}}$

$\Rightarrow y=\frac{3x}{2}\pm\frac{4}{3}\Rightarrow|x_{intercept}|=\frac{8}{9}$

$|y_{intercept}|=\frac{4}{3}$

$\Rightarrow|6a|+|5b|=\frac{48}{9}+\frac{60}{9}=\frac{109}{9}=12$

29. Let $f_n=\int_0^{\frac{\pi}{2}}\left(\sum_{k=1}^{n}\sin^{k-1}x\right)\left(\sum_{k=1}^{n}(2k-1)\sin^{k-1}x\right)\cos x\,dx$, n ∈ N. Then $f_{21}-f_{20}$ is equal to____.

Official Ans. by NTA (41)

Allen Ans. (41)

Sol.

$f_n(x)=\int_0^{\frac{\pi}{2}}(1+\sin x+\sin^2x+\sin^3x+...+\sin^{n-1}(x))$

$(1+3\sin x+5\sin^2x+...+(2n-1))\sin^{n-1}x.\cos x dx$

Multiply & divide by $\sqrt{\sin x}$

$\int_0^{\frac{\pi}{2}}\left((\sin x)^{\frac{1}{2}}+(\sin x)^{\frac{3}{2}}+(\sin x)^{\frac{5}{2}}+(\sin x)^{\frac{7}{2}}+...(\sin x)^{\frac{2n-1}{2}}\right)$

$(1+3\sin x+5\sin^2x+...+(2n-1)\sin^{n-1}(x))\frac{\cos x}{\sqrt{\sin x}}dx$

Put $(\sin x)^{1/2}+(\sin x)^{3/2}+(\sin x)^{5/2}+...+(\sin x)^{n-1/2}=t$

$\frac{1}{2}\frac{(1+3\sin x+5\sin^2x+...(2n-1)\sin^{n-1}x)}{\sqrt{\sin x}}\cos x dx=dt$

$f_n=2\int_0^n t\,dt$

$f_n=n^2$

$f_{21}-f_{20}=(21)^2-(20)^2$

$=441-400$

$=41$

30. The remainder, when 7^{103} is divided by 17 is ___.

Official Ans. by NTA (12)

Allen Ans. (12)

Sol. $7^{103} = 7 \times 7^{102}$

$= 7 \times (49)^{51}$

$= 7 \times (51 - 2)^{51}$

Remainder :- $7 \times (-2)^{51}$

$\Rightarrow -7\,(2^3 \,.\, (16)^{12})$

$\Rightarrow -56\,(17 - 1)^{12}$

Remainder $= -56 \times (-1)^{12} = -56 + 68 = 12$