JEE Main 2023 (2nd Attempted)
(Shift - 02 Mathematics Paper)

10.04.2023

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let f be a continuous function satisfying $\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3,\ \forall\ t>0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to :

(1) $\pi\left(1-\frac{\pi^3}{16}\right)$

(2) $-\pi^2\left(1+\frac{\pi^2}{16}\right)$

(3) $-\pi\left(1+\frac{\pi^3}{16}\right)$

(4) $\pi^2\left(1-\frac{\pi^2}{16}\right)$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3,\ \forall\ t>0$

$\left(f(t^2)+t^4\right)=2t$

$f(t^2)=2t-t^4$

$t=\frac{\pi}{2}\Rightarrow f\left(\frac{\pi^2}{4}\right)=\frac{2\pi}{2}-\frac{\pi^4}{16}$

$=\pi-\frac{\pi^4}{16}=\pi\left(1-\frac{\pi^3}{16}\right)$

2. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is:

(1) 3360

(2) 1680

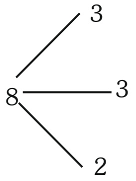
(3) 560

(4) 1120

Official Ans. by NTA (1)

Allen Ans. (2)

Sol.



Ways = $\frac{8!}{3!3!2!2!}\times 3!$

$=\frac{8\times7\times6\times5\times4}{4}$

$= 56 \times 30$

$= 1680$

3. For, $\alpha,\beta,\gamma,\delta\in\mathbb{N}$, if

$$\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx=\frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x}-\frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x}+C,$$

Where $e=\sum_{n=0}^{\infty}\frac{1}{n!}$ and C is constant of integration, then $\alpha+2\beta+3\gamma-4\delta$ is equal to:

(1) 1

(2) –4

(3) –8

(4) 4

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\left(x=e^{\ln x}\right)$

$\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx=\int\left[e^{2(x\ln x-x)}+e^{-2(x\ln x-x)}\right]\ln x\,dx$

$x\ln x - x = t$

$\ln x.dx = dt$

$\int\left(e^{2t}+e^{-2t}\right)dt$

$\frac{e^{2t}}{2}-\frac{e^{-2t}}{2}+C$

$=\frac{1}{2}\left(\frac{x}{e}\right)^{2x}-\frac{1}{2}\left(\frac{e}{x}\right)^{2x}+C$

$\alpha=\beta=\gamma=\delta=2$

$\alpha+2\beta+3\gamma-4\delta=4$

4. Let the image of the point P(1, 2, 6) in the plane passing through the points A(1, 2, 0), B(1, 4, 1) and C(0, 5, 1) be Q (α, β, γ). Then $(\alpha^2+\beta^2+\gamma^2)$ is equal to :

(1) 65
(2) 70
(3) 76
(4) 62

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Equation of plane $A(x-1)+B(y-2)+C(z-0)=0$

Put (1, 4, 1) $\Rightarrow 2B + C = 0$

Put (0, 5, 1) $\Rightarrow -A + 3B + C = 0$

Sub : $B - A = 0 \Rightarrow A = B,\ C = -2B$

$1(x-1)+1(y-2)-2(z-0)=0$

$x+y-2z-3=0$

Image is (α, β, γ) pt$\equiv(1, 2, 6)$

$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=\frac{-2(1+2-12-3)}{6}$$

$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=4$$

$\alpha=5, \beta=6, \gamma=-2 \Rightarrow \alpha^2+\beta^2+\gamma^2$

$=25+36+4=65$

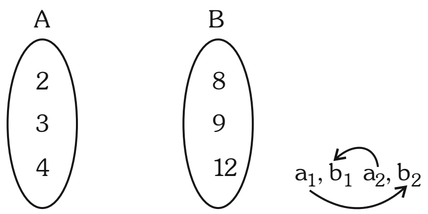
5. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation R = $\{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is :

(1) 36
(2) 12
(3) 18
(4) 24

Official Ans. by NTA (1)

Allen Ans. (1)

Sol.



a_1 divides b_2

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

a_2 divides b_1

Each element has 2 choices

$\Rightarrow 3 \times 2 = 6$

Total = 6 × 6 = 36

6. If $A=\frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|\text{adj}(\text{adj}(2A))|$ is equal to :

(1) 2^8
(2) 2^{12}
(3) 2^{20}
(4) 2^{16}

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $|\text{adj}\,\text{adj}(2A)|=|2A|^{(n-1)^2}$

$=|2A|^4$

$=(2^3|A|)^4$

$= 2^{12}|A|^4 \Rightarrow 2^{16}$

$$|A| = \frac{1}{5!6!7!}5!6!\begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$R_3 \to R_3 \to R_2$

$R_2 \to R_2 \to R_1$

$$|A|=\begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix}=2$$

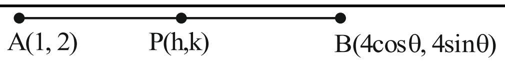
7. Let A be the point (1, 2) and B be any point on the curve $x^2+y^2=16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C (α, β), then the length of the line segment AC is

(1) $\frac{6\sqrt{5}}{5}$
(2) $\frac{4\sqrt{5}}{5}$
(3) $\frac{2\sqrt{5}}{5}$
(4) $\frac{3\sqrt{5}}{5}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol.



$\frac{12\cos\theta + 2}{5} = h \quad \Rightarrow \quad 12 \cos \theta = 5h - 2$

$\frac{12\sin\theta + 4}{5} = k \quad \Rightarrow \quad 12 \sin \theta = 5k \;\; 4$

Sq & add :

$144 = (5h - 2)2 + (5k - 4)2$

$$\left(x - \frac{2}{5}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{144}{25}$$

$$\text{Centre} \equiv \left(\frac{2}{5}, \frac{4}{5}\right) \equiv (\alpha, \beta)$$

$$AC = \sqrt{\left(1 - \frac{2}{5}\right)^2 + \left(2 - \frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{36}{25}} = \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$$

8. Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to :

(1) 30

(2) 90

(3) 15

(4) 60

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. P(odd number 7 times) = P(odd number 9 times)

$${}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$${}^nC_7 = {}^nC_9$$

$\Rightarrow \; n = 16$

Required

$$P = {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$$

$$= \frac{16 \cdot 15}{2} \times \frac{1}{2^{16}} = \frac{15}{2^{13}}$$

$$\Rightarrow \; \frac{60}{2^{15}} \; \Rightarrow \; k = 60$$

9. Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0, x \in (0,1)$. If g is decreasing in the interval (0, α) and increasing in the interval (α, 1), then $\tan^1(2\alpha) + \tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to :

(1) $\frac{3\pi}{2}$

(2) π

(3) $\frac{5\pi}{4}$

(4) $\frac{3\pi}{4}$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $g(x) = f(x) + f(1 - x) \;\&\; f''(x) > 0, x \in (0, 1)$

$g'(x) = f'(x) - f'(1 - x) = 0$

$\Rightarrow \; f'(x) = f'(1 - x)$

$x = 1 - x$

$x = \frac{1}{2}$

$g'(x) = 0$

at $x = \frac{1}{2}$

$g''(x) = f''(x) + f''(1 - x) > 0$

g is concave up

hence $\alpha = \frac{1}{2}$

$$\tan^{-1} 2\alpha + \tan^{-1}\frac{1}{\alpha} + \tan^{-1}\frac{\alpha+1}{\alpha}$$

$$\Rightarrow \; \tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$$

10. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle.

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{3}$

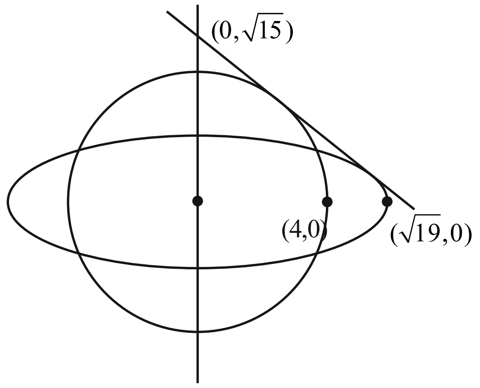
(3) $\frac{\pi}{12}$

(4) $\frac{\pi}{6}$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\frac{x^2}{19}+\frac{y^2}{15}=1$



Let tang be

$$y = mx \pm \sqrt{19m^2+15}$$

$$mx - y \pm \sqrt{19m^2+15} = 0$$

Parallel from (0, 0) = 4

$$\left|\frac{\pm\sqrt{19m^2+15}}{\sqrt{m^2+1}}\right| = 4$$

$19m^2 + 15 = 16m^2 + 16$

$3m^2 = 1$

$$m = \pm\frac{1}{\sqrt{3}}$$

$\theta = \frac{\pi}{6}$ with x-axis

Required angle $\frac{\pi}{3}$.

11. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d} = 12$. Then $\left(-\hat{i} + \hat{j} - \hat{k}\right)\cdot(\vec{c}\times\vec{d})$ is equal to

(1) 48

(2) 42

(3) 44

(4) 24

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} + 5\hat{k}$

$\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$

$$\vec{d} = \lambda(\vec{a}\times\vec{b}) = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 3 & 0 & 5 \end{vmatrix}$$

$\vec{d} = \lambda(35\hat{i} - 13\hat{j} - 21\hat{k})$

$\lambda(35 + 13 - 42) = 12$

$\lambda = 2$

$\vec{d} = 2(35\hat{i} - 13\hat{j} - 21\hat{k})$

$(\hat{i} + \hat{j} - \hat{k})(\vec{c}\times\vec{d})$

$$= \begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 70 & -26 & -42 \end{vmatrix} = 44$$

12. If $S_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to

(1) 226 (2) 220

(3) 223 (4) 227

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $S_n = 4 + 11 + 21 + 34 + 50 + \ldots + n$ terms

Difference are in A.P.

Let $T_n = an^2 + bn + c$

$T_1 = a + b + c = 4$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 21$

By solving these 3 equations

$$a = \frac{3}{2}, b = \frac{5}{2}, c = 0$$

So $T_n = \frac{3}{2}n^2 + \frac{5}{2}n$

$S_n = \Sigma T_n$

$$= \frac{3}{2}\Sigma n^2 + \frac{5}{2}\Sigma n$$

$$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} = \frac{5}{2}\frac{(n)(n+1)}{2}$$

$$= \frac{n(n+1)}{4}\left[2n + 1 + 5\right]$$

$$S_n = \frac{n(n+1)}{4}(2n+6) = \frac{n(n+1)(n+3)}{2}$$

$$\frac{1}{60}\left(\frac{29\times30\times32}{2} - \frac{9\times10\times12}{2}\right) = 223$$

13. If the points P and Q are respectively the circumcentre and the orthocentre of a ΔABC, then $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is equal to

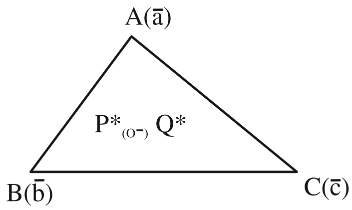
(1) $2\overrightarrow{QP}$ (2) $\overrightarrow{QP}$

(3) $2\overrightarrow{PQ}$ (4) $\overrightarrow{PQ}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol.



$\overline{PA}+\overline{PB}+\overline{PC}=\bar{a}+\bar{b}+\bar{c}$

$\overline{PG}=\dfrac{\bar{a}+\bar{b}+\bar{c}}{3}$

$\Rightarrow \bar{a}+\bar{b}+\bar{c}=3\overline{PG}=\overline{PQ}$

Ans. (4)

14. The statement ~[p ∨ (~ (p ∧ q))] is equivalent to

(1) (~(p ∧ q)) ∧ q

(2) ~ (p ∧ q)

(3) ~(p ∨ q)

(4) (p ∧ q) ∧ (~p)

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\sim[pv(\sim(p\wedge q))]$

$\sim p\wedge(p\wedge q)$

15. Let $S=\left\{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right):9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and $\beta=\sum_{x\in s}\tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta-14)^2$ is equal to

(1) 32

(2) 8

(3) 64

(4) 16

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Let $9^{\tan^2 x}=P$

$\frac{9}{P}+P=10$

$P^2-10P+9=0$

$(P-9)(P-1)=0$

$P=1,9$

$9^{\tan^2 x}=1,\ 9^{\tan^2 x}=9$

$\tan^2 x=0,\tan^2 x=1$

$x=0,\pm\frac{\pi}{4}\quad \therefore x\in\left(-\frac{\pi}{2},\frac{p}{2}\right)$

$\beta=\tan^2(0)+\tan^2\left(+\frac{\pi}{12}\right)+\tan^2\left(-\frac{\pi}{12}\right)$

$=0+2(\tan 15°)^2$

$2(2-\sqrt{3})^2$

$2(7-4\sqrt{3})$

Than $\frac{1}{6}(14-8\sqrt{3}-14)^2=32$

16. If the coefficients of x and x^2 in $(1+x)^p(1-x)^q$ are 4 and –5 respectively, then 2p + 3q is equal to

(1) 63

(2) 69

(3) 66

(4) 60

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $(1+x)^P(1-x)^q$

$\left(1+px+\frac{p(p-1)}{2!}x^2+\ldots\right)$

$\left(1-qx+\frac{q(q-1)}{2!}x^2-\ldots\right)$

$p-q=4$

$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$

$p^2+q^2-p-q-2pq=-10$

$(q+4)^2+q^2-(q+4)-q-2(4+q)q=-10$

$q^2+8q+16-q^2-q-4-q-8q-2q^2=-10$

$-2q=-22$

$q=11$

$p=15$

$2(15)+3(11)$

$30+33=63$

17. Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x - 2y + z = 14$ is

(1) 4 (2) $\frac{10}{3}$

(3) 3 (4) $\frac{11}{3}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda$ …. (1)

$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu$ ….(2)

$\frac{x+3}{4}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma$ …. (3)

Intersection of (1) & (2) "A"

$(\lambda,-2\lambda+6,5\lambda-8)$ & $(4\mu+5,3\mu+7,\mu-2)$

$\lambda = 1, \mu = -1$

A(1, 4, –3)

Intersection of (1) & (3) "B"

$(\lambda,-2\lambda+6,5\lambda-8)$ & $(6\gamma-3,-3\gamma+3,\gamma+6)$

$\lambda = 3$

$\gamma = 1$

B(3, 0, 7)

Mid point of A & B ⇒ (2, 2, 2)

Perpendicular distance from the plane

$2x - 2y + z = 14$

$$\Rightarrow \left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right|=4$$

18. Let $S=\left\{z=x+iy:\frac{2z-3i}{4z+2i} \text{ is a real number}\right\}$.

Then which of the following is <u>NOT</u> correct?

(1) $y+x^2+y^2\neq-\frac{1}{4}$

(2) $x = 0$

(3) $(x, y)=\left(0,-\frac{1}{2}\right)$

(4) $y\in\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\frac{2z-3i}{qz+2i}\in R$

$$\frac{2(x+iy)-3i}{4(x+it)+2i}=\frac{2x+(2y-3)i}{4x+(4y+2)i}\times\frac{4x-(4y+2)i}{4x-(4y+2)i}$$

$4x(2y - 3) - 2x(4y + 2) = 0$

$x = 0$ $y\neq-\frac{1}{2}$

Ans. = 3

19. Let the number $(22)^{2022}$ + $(2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to

(1) 10

(2) 5

(3) 20

(4) 13

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $(22)^{2022} + (2022)^{22}$

divided by 3

$(21 + 1)^{2022} + (2022)^{22}$

$= 3k + 1$ $(\alpha = 1)$

Divided by 7

$(21 + 1)^{2022} + (2023 - 1)^{22}$

$7k + 1 + 1$ $(\beta = 2)$

$7k + 2$

So $\alpha^2 + \beta^2 \Rightarrow 5$

20. Let μ be the mean and σ be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. if [x] denotes the greatest integer ≤ x, then $[\mu^2 + \sigma^2]$ is equal

(1) 8

(2) 7

(3) 6

(4) 9

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\sum f_i = 62$

$\Rightarrow$ $3k^2 + 16k - 12k - 64 = 0$

$\Rightarrow$ $k = \text{or } -\frac{16}{3}$ (rejected)

$$\mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{8+2(15)+3(15)+4(17)+5}{62} = \frac{156}{62}$$

$$\sigma^2 = \sum f_i x_i^2 - \left(\sum f_i x_i\right)^2$$

$$= \frac{8\times1^2+15\times13+17\times16+25}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 + \mu^2 = \frac{500}{62}$$

$$\left[\sigma^2 + \mu^2\right] = 8$$

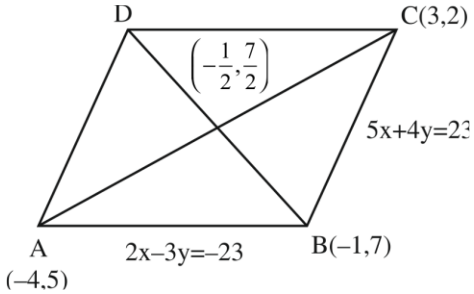
SECTION-B

21. Let the equations of two adjacent sides of a parallelogram ABCD be 2x – 3y = –23 and 5x + 4y = 23. If the equation of its one diagonal AC is 3x + 7y = 23 and the distance of A from the other diagonal is d, then 50 d^2 is equal to _____ .

Official Ans. by NTA (529)

Allen Ans. (529)

Sol.



A & C point will be (–4, 5) & (3, 2)

mid point of AC will be $\left(-\frac{1}{2},\frac{7}{2}\right)$

equation of diagonal BD is

$$y - \frac{7}{2} = \frac{\frac{7}{2}}{-\frac{1}{2}} \quad \left(x + \frac{1}{2}\right)$$

$\Rightarrow$ $7x + y = 0$

Distance of A from diagonal BD

$= d = \frac{23}{\sqrt{50}}$

$\Rightarrow$ $50d^2 = (23)^2$

$50d^2 = 529$

22. Let S be the set of values of λ, for which the system of equations

$6\lambda x - 3y + 3z = 4\lambda^2$,

$2x + 6\lambda y + 4z = 1$,

$3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12\sum_{\lambda\in S}|\lambda|$ is equal to _________ .

Official Ans. by NTA (24)

Allen Ans. (24)

Sol. $\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$ (For No Solution)

$2\lambda(9\lambda^2 - 4) + (3\lambda - 6) + (2 - 9\lambda) = 0$

$18\lambda^3 - 14\lambda - 4 = 0$

$(\lambda - 1)(3\lambda + 1)(3\lambda + 2) = 0$

$\Rightarrow \lambda = 1, -1/3, -2/3$

For each λ, $\Delta_1 = \begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \neq 0$

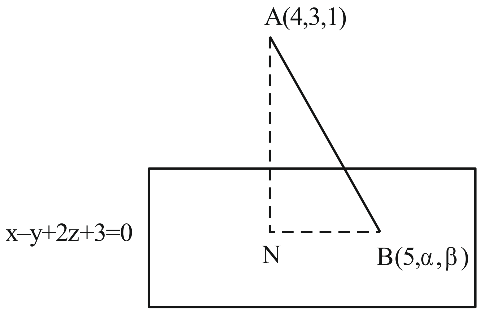
Ans. $12\left(1 + \frac{1}{3} + \frac{2}{3}\right) = 24$

23. Let the foot of perpendicular from the point A(4, 3, 1) on the plane P : x – y + 2z + 3 = 0 be N. If B(5, α, β), $\alpha, \beta \in \mathbb{Z}$ is a point on plane P such that the area of the triangle ABN in $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to _________ .

Official Ans. by NTA (7)

Allen Ans. (7)

Sol.



$AN = \sqrt{6}$

$5 - \alpha + 2\beta + 3 = 0$

$\Rightarrow$ $\alpha = 8 + 2\beta$ …. (1)

N is given by

$$\frac{x-4}{1}=\frac{y-3}{-1}=\frac{z-1}{2}=\frac{-(4-3+2+3)}{1+1+4}$$

⇒ $x = 3, y = 4, z = -1$

⇒ N is $(3, 4, -1)$

$$BN = \sqrt{4+(\alpha-4)^2+(\beta+1)^2}$$

$$= \sqrt{4+(2\beta+4)^2+(\beta+1)^2}$$

Area of $\Delta ABN = \frac{1}{2} AN \times BN = 3\sqrt{2}$

⇒ $\frac{1}{2} \times \sqrt{6} \times BN = 3\sqrt{2}$

$BN = 2\sqrt{3}$

⇒ $4 + (2\beta + 4)^2 + (\beta + 1)^2 = 12$

$(2\beta + 4)^2 + (\beta + 1)^2 - 8 = 0$

$5\beta^2 + 18\beta + 9 = 0$

$(5\beta + 3)(\beta + 3) = 0$

$\beta = -3$

⇒ $\alpha = 2$

⇒ $\alpha^2 + \beta^2 + \alpha\beta = 9 + 4 - 6 = 7$

24. Let quadratic curve passing through the point $(-1, 0)$ and touching the line $y = x$ at $(1, 1)$ be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is _____ .

Official Ans. by NTA (11)

Allen Ans. (11)

Sol. $f(x) = (x + 1)(ax + b)$

$1 = 2a + 2b$ ____(1)

$f'(x) = (ax + b) + a(x + 1)$

$1 = (3a + b)$ _____(2)

⇒ $b = 1/4, a = 1/4$

$$f(x)=\frac{(x+1)^2}{4}$$

$$f'(x)=\frac{x}{2}+\frac{1}{2} \qquad \alpha+1=\frac{(\alpha+1)^2}{4}, \alpha>-1$$

$\alpha + 1 = 4$

$\alpha = 3$

normal at (3, 4)

$$y-4=-\frac{1}{2}(x-3)$$

$y = 0 \qquad x = 8 + 3$

Ans. 11

25. Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10}, 100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If PA : PB = 1 : k and $y = y(x)$ is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$, $y(0) = k$, then $4y(1) - 5\log_e 3$ is equal to ______ .

Official Ans. by NTA (6)

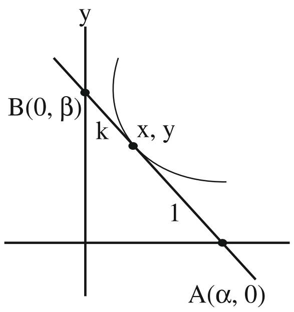
Allen Ans. (5) (answer is 4+ℓn3)

Sol. equation of tangent at P (x, y)

$$Y - y = \frac{dy}{dx}(X - x)$$

$Y = 0$

$$X = \frac{-ydx}{dy} + x$$



$$\frac{k\alpha+0}{k+1} = x, \ \alpha = \frac{k+1}{k}x$$

$$\frac{k+1}{k}x = -y\frac{dx}{dy} + x$$

$$x + \frac{x}{k} = -y\frac{dx}{dy} + x$$

$$x\frac{dy}{dx} + ky = 0 \qquad \frac{dy}{dx} + \frac{k}{x}y = 0$$

$y. x^k = C$

$C = 1$

$$100.\left(\frac{1}{10}\right)^k = 1$$

$K = 2$

$$\frac{dy}{dx} = \ell n(2x+1)$$

$$y = \frac{2x+1}{2}(\ell n(2x+1) - 1) + c$$

$$2 = \frac{1}{2}(0-1) + C$$

$$C = 2 + \frac{1}{2} = \frac{5}{2}$$

$$y(1) = \frac{3}{2}(\ell n3 - 1) + \frac{5}{2}$$

$$= \frac{3}{2}\ell n3 + 1$$

$4y(1) = 6\ell n3 + 4$

$4y(1) - 5\,\ell n3 = 4 + \ell n3$

26. Suppose a_1, a_2, 2, a_3, a_4 be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to ________ .

Official Ans. by NTA (16)

Allen Ans. (16)

Sol. $\frac{(a-2d)}{4}, \frac{(a-d)}{2}$, a, 2(a + d), 4(a + 2d)

a = 2

$$\left(\frac{1}{4}+\frac{1}{2}+1+6\right)\times 2+(-1+2+8)d=\frac{49}{2}$$

$$2\left(\frac{3}{4}+7\right)+9d=\frac{49}{2}$$

$$9d=\frac{49}{2}-\frac{62}{4}=\frac{98-62}{4}=9$$

d = 1

$\Rightarrow a_4 = 4\ (a + 2d)$

$= 16$

27. If the domain of the function $f(x)=\sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha,\beta)\cup(\gamma,\delta]$, then $|3\alpha+10(\beta+\gamma)+21\delta|$ is equal to ________ .

Official Ans. by NTA (24)

Allen Ans. (24)

Sol. $f(x) = \sec^{-1}\frac{2x}{5x+3}$

$$\left|\frac{2x}{5x+3}\right|$$

$$\left|\frac{2x}{5x+3}\right|\ge 1\Rightarrow |2x|\ge|5x+3|$$

$(2x)^2 - (5x + 3)^2 \ge |5x + 3|$

$(7x + 3)\ (-3x - 3) \ge 0$

$$\begin{array}{ccc} - & + & - \\ \hline & -1 & -\frac{3}{7} \end{array}$$

∴ domain $\left[-1,\frac{-3}{5}\right)\cup\left(\frac{-3}{5},\frac{-3}{7}\right]$

$\alpha=-1,\ \beta=\frac{-3}{5},\ \gamma=\frac{-3}{5},\ \delta=\frac{-3}{7}$

$3\alpha + 10\ (\beta + \gamma) + 21\delta = -3$

$$-3+10\left(\frac{-6}{5}\right)+\left(\frac{-3}{7}\right)21=-24$$

28. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to ________ .

Official Ans. by NTA (26664)

Allen Ans. (26664)

Sol. **2, 1, 2, 3**

$\_\_\quad\_\_\quad\_\_\quad\underline{1}\quad \frac{3!}{2!}=3$

$\_\_\quad\_\_\quad\_\_\quad\underline{2}\quad 3!=6$

$\_\_\quad\_\_\quad\_\_\quad\underline{3}\quad \frac{3!}{2!}=3$

Sum of digits of unit place = 3 × 1 + 6 × 2 + 3 × 3

= 24

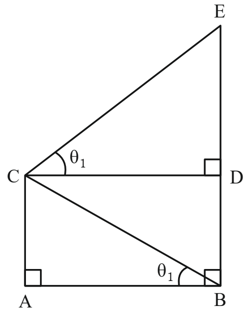
∴ required sum

= 24 ×1000 + 24 × 100 + 24 × 10 + 24 × 1

= 24 ×1111

Ans ; 26664

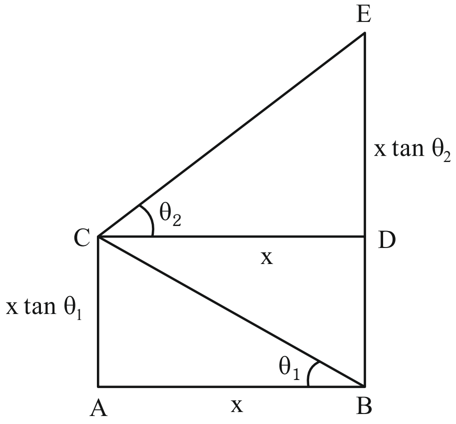
29. In the figure, $\theta_1+\theta_2=\frac{\pi}{2}$ and $\sqrt{3}(BE)=4(AB)$. If the area of ΔCAB is $2\sqrt{3}-3$ unit², when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of ΔCED is equal to ___________.



Official Ans. by NTA (6)

Allen Ans. (6)

Sol.



$\sqrt{3}$ BE = 4 AB

Ar (ΔCAB) = $2\sqrt{3}-3$

$\frac{1}{2}x^2\tan\theta_1 = 2\sqrt{3}-3$

BE = BD + DE

= x (tan θ_1 + tan θ_2)

BE = AB (tan θ_1 + cot θ_1)

$\frac{4}{\sqrt{3}}$ tan θ_1 + cot $\theta_1 \Rightarrow$ tan θ_1 = $\sqrt{3}, \frac{1}{\sqrt{3}}$

$\theta_1 = \frac{\pi}{6}$ $\qquad$ $\theta_2 = \frac{\pi}{3}$

$\theta_1 = \frac{\pi}{3}$ $\qquad$ $\theta_2 = \frac{\pi}{6}$

as $\frac{\theta_2}{\theta_1}$ is largest $\therefore$ $\theta_1 = \frac{\pi}{6}$ $\theta_2 = \frac{\pi}{3}$

$$\therefore x^2 = \frac{\left(2\sqrt{3}-3\right)\times 2}{\tan\theta_1} = \frac{\sqrt{3}\left(2-\sqrt{3}\right)\times 2}{\tan\frac{\pi}{6}}$$

$$x^2 = 12-6\sqrt{3} = \left(3-\sqrt{3}\right)^2$$

$x = 3-\sqrt{3}$

Perimeter of ΔCED

= CD + DE + CE

$= 3\sqrt{3}+\left(3-\sqrt{3}\right)\sqrt{3}+\left(3-\sqrt{3}\right)\times 2 = 6$

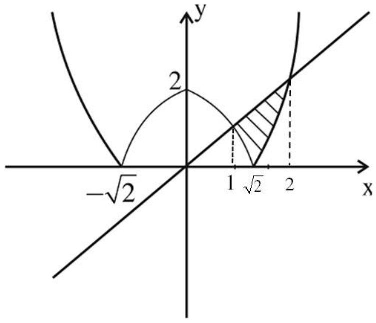
Ans : 6

30. If the area of the region $\left\{(x,y):\left|x^2-2\right|\le y\le x\right\}$ is A, then $6A+16\sqrt{2}$ is equal to ________ .

Official Ans. by NTA (27)

Allen Ans. (27)

Sol. $|x^2-2|\le y\le x$



$$A=\int_1^{\sqrt{2}}(x-(2-x^2))dx+\int_{\sqrt{2}}^{2}(x-(x^2-2))dx$$

$$=\left(1-2\sqrt{2}+\frac{2\sqrt{2}}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)+\left(2-\frac{8}{3}+4\right)-\left(1-\frac{2\sqrt{2}}{3}+2\sqrt{2}\right)$$

$$=-4\sqrt{2}+\frac{4\sqrt{2}}{3}+\frac{7}{6}+\frac{10}{3}=\frac{-8\sqrt{2}}{3}+\frac{9}{2}$$

$6A = -16\sqrt{2}+27$ $\therefore$ $6A+16\sqrt{2}=27$

Ans : 27