JEE Main 2023 (2nd Attempted) (Shift - 01 Mathematics Paper)

06.04.2023

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3, x>0$. Then $18\int_1^2 f(x)dx$ is equal to:

(1) 10 $\log_e 2 - 6$
(2) 10 $\log_e 2 + 6$
(3) 5 $\log_e 2 + 3$
(4) 5 $\log_e 2 - 3$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3 \ldots\ldots..(1)$

replace $x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right)+4f(x)=x+3 \ldots\ldots(2)$

Eq. (1) × 5 – eq. (2) × 4

$f(x)=\frac{1}{9}\left(\frac{5}{x}-4x+3\right)$

$I = 18\int_1^2 \frac{1}{9}\left(\frac{5}{x}-4x+3\right)dx = 10\log_e 2-6$

2. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to

(1) 82
(2) 123
(3) 164
(4) 75

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. Probability of success $=\frac{1}{9}=p$

Probability of failure $q=\frac{8}{9}$

P(at least 4 success) = P (4 success) + P (5 success)

$= {}^5C_4\, p^4 q + {}^5C_5\, p^5 = \frac{41}{3^{10}} = \frac{123}{3^{11}}$

k = 123

3. If ${}^{2n}C_3 : {}^{n}C_3 = 10:1$, then the ratio $(n^2+3n):(n^2-3n+4)$ is

(1) 35: 16
(2) 65:37
(3) 27:11
(4) 2:1

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\frac{{}^{2n}C_3}{{}^{n}C_3}=10 \Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)}=10$

n = 8

So $(n^2+3n):(n^2-3n+4)=2$

4. If the ratio of the fifth term from the begining to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$, then the third term from the beginning is:

(1) $60\sqrt{2}$
(2) $60\sqrt{3}$
(3) $30\sqrt{2}$
(4) $30\sqrt{3}$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $\frac{{}^nC_4\, 2^{\frac{n-4}{4}}.\left(3^{\frac{-1}{4}}\right)^4}{{}^nC_4\, 3^{-\left(\frac{n-4}{4}\right)}.\left(2^{\frac{1}{4}}\right)^4}=\frac{\sqrt{6}}{1}$

$\Rightarrow n = 10$

So $T_3 = {}^{10}C_2\ 2^{\frac{1}{4}.8}.3^{-\frac{1}{4}.2} = \frac{45.4}{\sqrt{3}} = 60\sqrt{3}$

5. Let $\vec{a}=2\hat{i}+3\hat{j}+4k, \vec{b}=2\hat{i}-2\hat{j}-2k$ and $\vec{c}=-\hat{i}+4\hat{j}+3k$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a}.\vec{d}$ = 18, Then $|\vec{a}\times\vec{d}|^2$ is equal to

(1) 640
(2) 760
(3) 680
(4) 720

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\vec{a} = \lambda(\vec{b} \times \vec{c})$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} + 2k$$

$\vec{d} = \lambda(2\hat{i} - \hat{j} + 2k)$

$\vec{a}.\vec{d} = 18$

$\lambda = 2$

So $\vec{d} = 2(2\hat{i} - \hat{j} + 2k)$

$$\vec{d} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & -2 & 4 \\ 2 & 3 & 4 \end{vmatrix} = -20\hat{i} - 8\hat{j} + 16k$$

$|\vec{d} \times \vec{a}|^2 = 720$

6. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line L: $9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2, then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on

(1) $6x + y = 10$

(2) $6x - y = 15$

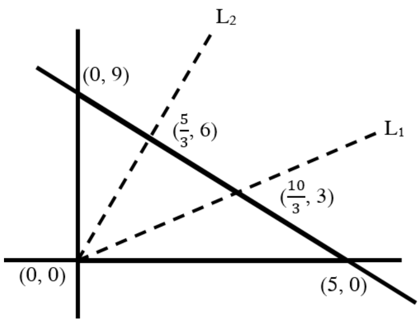
(3) $y - x = 5$

(4) $y - 2x = 5$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol.



$m_{L_1} = \frac{3.3}{10} = \frac{9}{10}$

$m_{L_2} = \frac{6.3}{5} = \frac{18}{5}$

$y = (m_1 + m_2)x$

$y = \frac{9}{2}x$

Point of intersection with L is $\left(\frac{10}{7}, \frac{45}{7}\right)$

7. From the top A of a vertical wall AB of height 30 m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively. B and Q are on the same horizontal level. If C is a point on AB such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to

(1) $600(\sqrt{3} - 1)$

(2) $300(\sqrt{3} + 1)$

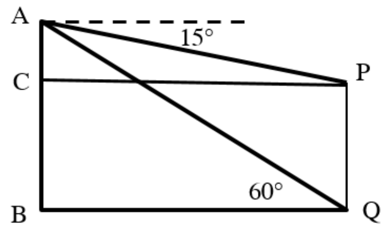
(3) $200(3 - \sqrt{3})$

(4) $300(\sqrt{3} - 1)$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol.



$\tan 60° = \sqrt{3} = \frac{30}{BQ}$

$BQ = 10\sqrt{3}m = CP$

$\tan 15° = 2 - \sqrt{3} = \frac{AC}{CP}$

$AC = 10\sqrt{3}(2 - \sqrt{3})$

Area $= 10\sqrt{3}(60 - 20\sqrt{3}) = 600(\sqrt{3} - 1)$

8. The sum of the first 20 terms of the series 5 + 11 + 19 + 29 + 41 + ... is

(1) 3450

(2) 3250

(3) 3420

(4) 3520

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $S_{20} = 5 + 11 + 19 + 29 + \ldots\ldots.$

Let $T_r = ar^2 + br + c$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 19$

$a = 1, b = 3, c = 1$

Hence $S_{20} = \sum_{r=1}^{20} r^2 + 3\sum_{r=1}^{20} r + \sum_{r=1}^{20} 1 = 3520$

9. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13, then σ^2 is equal to

(1) 9

(2) 12

(3) 11

(4) 10

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. **Combine var.** = $\frac{n_1\sigma^2+n_2\sigma^2}{n_1+n_2}+\frac{n_1n_2(m_1-m_2)^2}{(n_1+n_2)^2}$

$$13=\frac{15.14+15.\sigma^2}{30}+\frac{15.15(12-14)^2}{30\times 30}$$

$$13=\frac{14+\sigma^2}{2}+\frac{4}{4}$$

$\sigma^2=10$

10. Let A = $[a_{ij}]_{2\times 2}$ where $a_{ij}\neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and b = |A|, then $3a^2 + 4b^2$ is equal to

(1) 7

(2) 14

(3) 3

(4) 4

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Let $A=\begin{bmatrix} p & q \\ r & s\end{bmatrix}$

$$A^2=\begin{bmatrix} p^2+qr & pq+qs \\ pr+rs & qs+s^2\end{bmatrix}$$

$\Rightarrow p^2+qr=1\ (1)\ \ pq+qs=0\Rightarrow q(p+s)=0\ (3)$

$\Rightarrow s^2+qr=1\ (2)\ \ pr+rs=0\Rightarrow r(p+s)=0\ \ (4)$

Equation (1) – equation (2)

$p^2=s^2\Rightarrow p+s=0$

Now $3a^2 + 4b^2$

$=3(p+s)^2+4(ps-qr)^2$

$=3.0+4\ (-p^2-qr)^2=4(p^2+qr)^2=4$

11. Let $I(x)=\int\frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$. If I(0) = 0 the I $\left(\frac{\pi}{4}\right)$ is equal to

(1) $\log_e\frac{(x+4)^2}{16}-\frac{\pi^2}{4(\pi+4)}$

(2) $\log_e\frac{(x+4)^2}{16}+\frac{\pi^2}{4(\pi+4)}$

(3) $\log_e\frac{(x+4)^2}{32}-\frac{\pi^2}{4(\pi+4)}$

(4) $\log_e\frac{(x+4)^2}{32}+\frac{\pi^2}{4(\pi+4)}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $I(x)=\int\frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx$

Let xtan x + 1= t

$$I=x^2\left(\frac{-1}{x\tan x+1}\right)+\int\frac{2x}{x\tan x+1}dx$$

$$I=x^2\left(\frac{-1}{x\tan x+1}\right)+2\int\frac{x\cos x}{x\sin x+\cos x}dx$$

$$I=x^2\left(\frac{-1}{x\tan x+1}\right)+2\ln|x\sin x+\cos x|+C$$

As $I(0)=0\Rightarrow C=0$

$$I\left(\frac{\pi}{4}\right)=\ln\left(\frac{(\pi+4)^2}{32}\right)-\frac{\pi^2}{4(\pi+4)}$$

12. If the equation of the plane passing through the line of intersection of the planes 2x – y + z = 3, 4x– 3y +5z + 9 = 0 and parallel to the line $\frac{x+1}{-2}=\frac{y+3}{4}=\frac{z-2}{5}$ is ax + by + cz + 6 = 0. then a + b + c is equal to

(1) 14

(2) 12

(3) 13

(4) 15

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. **Equation of family of plane**

$(2x-y+z-3)\ +\ \lambda(4x-3y+5z+9)=0$

$x(2+4\lambda)-y(1+3\lambda)+z(1+5\lambda)-3+9\lambda=0$

Parallel to the line

$-2(2+4\lambda)-(1+3\lambda)4+(1+5\lambda)5=0$

$5\lambda=3$

$\lambda=\frac{3}{5}$

equation of plane

$11x-7y+10z+6=0$

a + b + c = 14

13. Statement $(P\Rightarrow Q)\wedge(R\Rightarrow Q)$ is logically equivalent to

(1) $(P\vee R)\Rightarrow Q$

(2) $(P\Rightarrow R)\wedge(Q\Rightarrow R)$

(3) $(P\Rightarrow R)\vee(Q\Rightarrow R)$

(4) $(P\wedge R)\Rightarrow Q$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $(P\Rightarrow Q)\wedge(R\Rightarrow Q)$

We known that $P\Rightarrow Q\ \equiv\ \sim P\vee Q$

$\Rightarrow(\sim P\vee Q)\wedge(\sim R\vee Q)$

$\Rightarrow(\sim P\wedge\sim R)\vee Q$

$\Rightarrow\sim(P\vee R)\vee Q$

$\Rightarrow(P\vee R)\Rightarrow Q$

14. The sum of all the roots of the equation $|x^2-8x+15|-2x+7=0$ is:

(1) $9+\sqrt{3}$

(2) $11+\sqrt{3}$

(3) $9-\sqrt{3}$

(4) $11-\sqrt{3}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. **For** $x\le 3 \text{ or } x\ge 5$

$x^2-8x+15-2x+7=0$

$x=5+\sqrt{3}$

For $3<x<5$, $x^2-8x+15+2x-7=0$

x = 4

Hence sum $=9+\sqrt{3}$

15. Let $a_1, a_2, a_3 \ldots a_n$ be n positive consecutive terms of an arithmetic progression. If d > 0 is its common difference, then

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

(1) 1

(2) $\sqrt{d}$

(3) $\frac{1}{\sqrt{d}}$

(4) 0

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$

On rationalising each term

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$$

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{(n-1)d}{\left(\sqrt{a_n}+\sqrt{a_1}\right)d}\right)=1$$

16. If the system of equations

x + y + az = b

2x + 5y + 2z = 6

x + 2y + 3z = 3

has infinitely many solutions, then 2a + 3b is equal to

(1) 23

(2) 28

(3) 25

(4) 20

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\Delta=\begin{vmatrix}1&1&a\\2&5&2\\1&2&3\end{vmatrix}=0\Rightarrow 11-4-a=0$

a = 7

$\Delta_1=\begin{vmatrix}b&1&a\\6&5&2\\3&2&3\end{vmatrix}=0\Rightarrow 11b-12-21=0$

b = 3

2a + 3b = 23

17. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to

(1) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$ (2) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

(3) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$ (4) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $2x^y + 3y^x = 20$

$$2x^y\left[\frac{y}{x}+(\ln x)\, y'\right]+3y^x\left[\frac{xy'}{y}+\ln y\right]=0$$

$$y'=\frac{-(12\ln 2+8)}{12+8\ln 2}=-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$$

18. One vertex of a rectangular parallelopiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3, 4, 5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is:

(1) $\frac{12}{\sqrt{5}}$ (2) $\frac{12}{5\sqrt{5}}$

(3) $12\sqrt{5}$ (4) $\frac{12}{5}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol Equation of OP is $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

$a_1 = (0, 0, 0)$ $a_2 = (3, 0, 5)$

$b_1 = (3, 4, 5)$ $b_2 = (0, 0, 1)$

Equation of edge parallel to z axis

$$\frac{x-3}{0}=\frac{y-0}{0}=\frac{z-5}{1}$$

$$S.D=\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}$$

$$\frac{\begin{vmatrix}3&0&5\\3&4&5\\0&0&1\end{vmatrix}}{\left\|\begin{matrix}\hat{i}&\hat{j}&k\\3&4&5\\0&0&1\end{matrix}\right\|}=\frac{3(4)}{|4\hat{i}-3\hat{j}|}=\frac{12}{5}$$

19. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda k,\ \hat{i}+2\hat{j}+3k,\ -2\hat{i}+\lambda\hat{j}+4k$ and $-\hat{i}+5\hat{j}+6k$. Let the set $S=\{\lambda\in\mathbb{R}:$ The points A, B, C and D are coplanar$\}$. Then $\sum_{\lambda\in S}(\lambda+2)^2$ is equal to

(1) 41 (2) 25

(3) 13 (4) $\frac{37}{2}$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Since A, B, C, D are coplanner

Hence $\left[\overrightarrow{BA}\ \ \overrightarrow{CA}\ \ \overrightarrow{DA}\right]=0$

$$\begin{vmatrix}4&3&2\lambda-3\\7&5-\lambda&2\lambda-4\\6&0&2\lambda-6\end{vmatrix}=0$$

$\lambda=2,3$ Hence $\sum_{\lambda\in S}(\lambda+2)^2=41$

20. Let $A=\{x\in\mathbb{R}:[x+3]+[x+4]\le 3\}$,

$B=\left\{x\in\mathbb{R}:3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3}<3^{-3x}\right\}$, where [t] denotes greatest integer function. Then,

(1) $A\cap B=\phi$

(2) $A = B$

(3) $B\subset C, A\ne B$

(4) $A\subset B, A\ne B$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $[x]+3+[x]+4\le 3$

$2[x]\le -4$

$[x]\le -2 \Rightarrow x\in(-\infty,-1)$ ....(A)

$$3^x\left(\frac{3.\frac{1}{10}}{1-\frac{1}{10}}\right)^{x-3}<3^{-3x}$$

$27<3^{-3x}$

$-3x>+3$

$x<-1$ ....(B)

$A = B$

SECTION-B

21. Let $a \in \mathbb{Z}$ and [t] be the greatest integer $\leq$ t. Then the number of points, where the function f(x) = [a + 13 sin x], x ∈ (0, π) is not differentiable, is ____

Official Ans. by NTA (25)

Allen Ans. (25)

Sol. f(x) = [a + 13 sin x], x ∈ (0, π)

For [n sin x]; Total number of non differentiable points are = 2n – 1 for $x \in (0, \pi)$

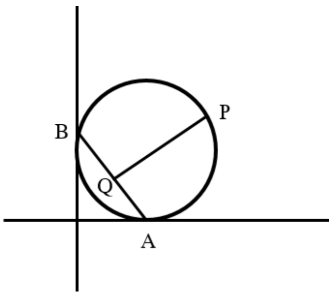
So number of non differentiable points for [13 sin x] ⇒ 25 Points

22. A circle passing through the point $P(\alpha, \beta)$ in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of $\alpha\beta$ is______

Official Ans. by NTA (121)

Allen Ans. (121)

Sol.



Let equation of circle is $(x-a)^2+(y-a)^2=a^2$

which is passing through P (α, β)

then $(\alpha-a)^2+(\beta-a)^2=a^2$

$\alpha^2+\beta^2-2\alpha a-2\beta a+a^2=0$

Here equation of AB is x + y = a

Let Q (α', β') be foot of perpendicular of P on AB

$$\frac{\alpha'-\alpha}{1}=\frac{\beta'-\beta}{1}=\frac{-(\alpha+\beta-a)}{2}$$

$$PQ^2=(\alpha'-\alpha)^2+(\beta'-\beta)=\frac{1}{4}(\alpha+\beta-a)^2+\frac{1}{4}(\alpha+\beta-a)^2$$

$$121=\frac{1}{2}(\alpha+\beta-a)^2$$

$$242=\alpha^2+\beta^2-2\alpha a-2\beta a+a^2+2\alpha\beta$$

$$242 = 2\alpha\beta$$

$$\Rightarrow \alpha\beta=121$$

23. The number of ways of giving 20 distinct oranges to 3 children such that each child gets atleast one orange is_________

Official Ans. by NTA (171)

Allen Ans. (Bonus)

Sol. 20 distinct oranges distributed among 3 children so that each child gets at least one orange

$= 3^{20} - {}^3C_1\, 2^{20} + {}^3C_2\, 1^{20}$

Bonus

24. If the area of the region

$S=\{(x,y):2y-y^2\leq x^2\leq 2y, x\geq y\}$ is equal to

$\frac{n+2}{n+1}-\frac{\pi}{n-1}$, then the natural number n is equal to ________

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. $x^2+y^2-2y\geq 0$ & $x^2-2y\leq 0$, $x\geq y$

Hence required area = $\frac{1}{2}\times 2\times 2-\int_0^2\frac{x^2}{2}dx-\left(\frac{\pi}{4}-\frac{1}{2}\right)$

$=\frac{7}{6}-\frac{\pi}{4} \Rightarrow n = 5$

25. Let the point (p, p + 1) lie inside the region

$E=\left\{(x,y):3-x\leq y\leq\sqrt{9-x^2}, 0\leq x\leq 3\right\}$ If the set of all values of p is the interval (a, b). then $b^2 + b - a^2$ is equal to _________

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $3-x \le y \le \sqrt{9-x^2}$

Points (p, p + 1) lies on y = x + 1

So point of intersection between

y = x + 1 & y = 3 – x is x = 1, y = 2

and point of intersection between

$x+1=\sqrt{9-x^2}$ is $x=\frac{-1+\sqrt{17}}{2}$

Hence $p \in \left(1, \frac{-1+\sqrt{17}}{2}\right)$

Hence $b^2 + b - a^2 = 3$

26. Let y = y(x) be a solution of the differential equation (xcos x)dy + (xysinx + ycos x – 1)dx = 0, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3} y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to ______

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. (xcos x)dy + (xysinx + ycos x – 1)dx = 0, $0 < x < \frac{\pi}{2}$

$$\frac{dy}{dx} + \left(\frac{x\sin x + \cos x}{x\cos x}\right) y = \frac{1}{x\cos x}$$

IF = x secx

$$y.x\sec x = \int \frac{x\sec x}{x\cos x} dx = \tan x + c$$

Since $y\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{\pi}$

Hence $c = \sqrt{3}$

Hence $\left|\frac{\pi}{6} y''\left(\frac{\pi}{6}\right) + y'\left(\frac{\pi}{6}\right)\right| = |-2| = 2$

27. The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is _____

Official Ans. by NTA (5005)

Allen Ans. (5005)

Sol. $\left(x^4 - \frac{1}{x^3}\right)^{15}$

$$T_{r+1} = {}^{15}C_r \left(x^4\right)^{15-r} \left(\frac{-1}{x^3}\right)^r$$

60 –7r = 18

r = 6

Hence coeff. of $x^{18} = {}^{15}C_6 = 5005$

28. Let A={1, 2, 3, 4,.....10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation R = {(a, b) ∈ A × A: $2(a-b)^2 + 3(a-b) \in B$} is______

Official Ans. by NTA (18)

Allen Ans. (18)

Sol. $A = \{1, 2, 3, ...10\}$

$B = \{0, 1, 2, 3, 4\}$

R = {(a, b) ∈ A × A: $2(a-b)^2 + 3(a-b) \in B$}

Now $2(a-b)^2 + 3(a-b) = (a-b)(2(a-b)+3)$

⇒ a = b or a– b = –2

When a = b ⇒ 10 order pairs

When a– b = –2 ⇒8 order pairs

Total = 18

29. Let the image of the point P(l, 2, 3) in the plane 2x – y + z = 9 be Q. If the coordinates of the point R are (6, 10, 7), then the square of the area of the triangle PQR is_

Official Ans. by NTA (594)

Allen Ans. (594)

Sol. Let Q (α, β, γ) be the image of P, about the plane

2x – y + z = 9

$$\frac{\alpha-1}{2} = \frac{\beta-2}{-1} = \frac{\gamma-3}{1} = 2$$

$\Rightarrow \alpha = 5, \beta = 0, \gamma = 5$

Then area of triangle PQR is $= \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$

$= \left|-12\hat{i} - 3\hat{j} + 21\hat{k}\right| = \sqrt{144+9+441} = \sqrt{594}$

Square of area = 594

30. Let the tangent to the curve $x^2 + 2x - 4y + 9 = 0$ at the point P(1, 3) on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$. then $(AB)^2$ is equal to

Official Ans. by NTA (292)

Allen Ans. (292)

Sol. Equation of tangent at P (1, 3) to the curve

$x^2 + 2x - 4y + 9 = 0$ is $y - x = 2$

Then the point A is (0, 2)

Equation of line passing through P and parallel to the line $x - 3y = 6$.

The possible coordinate of B are (4, 4) or (16, 8)

But (4, 4) does not satisfy $2x - 3y = 8$

Thus the point B is (16, 8)

Then $(AB)^2 = 292$