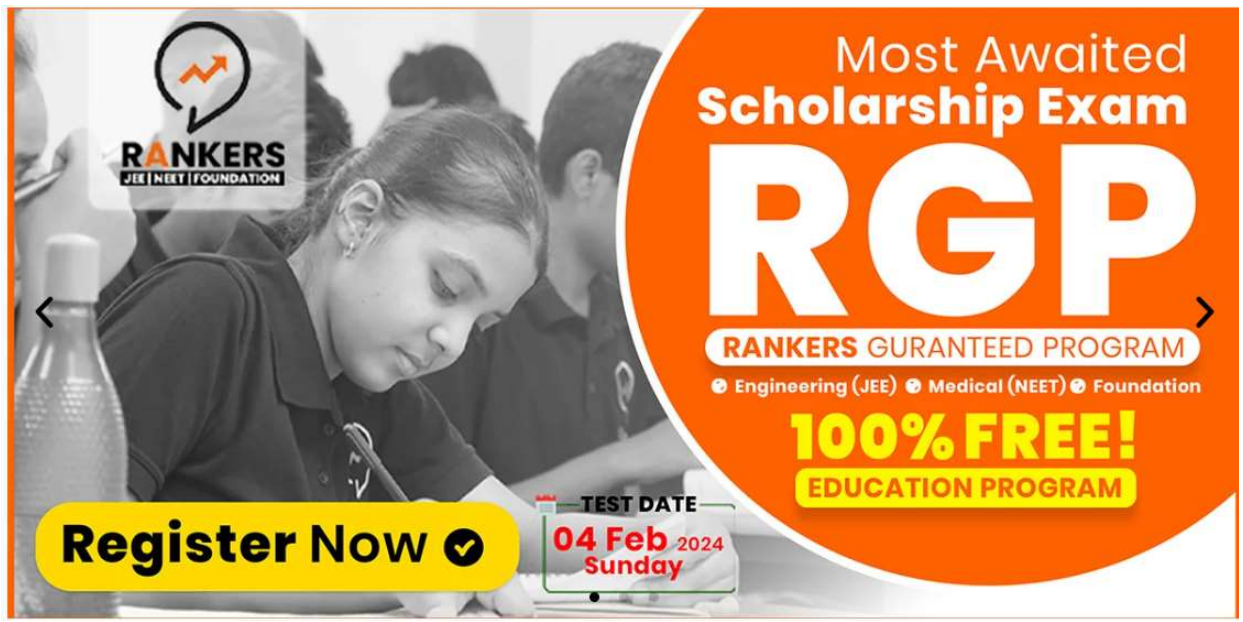
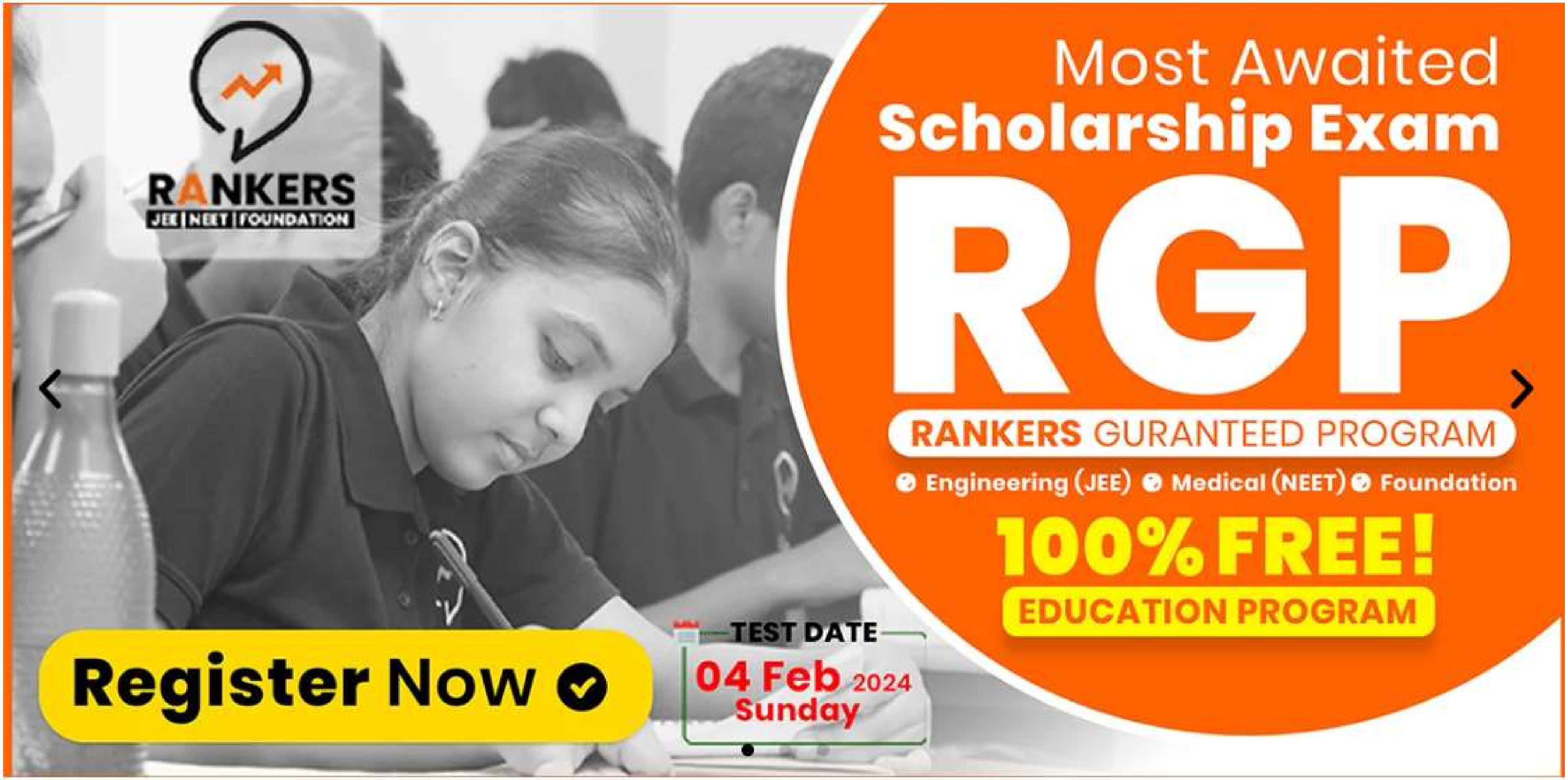
JEE Main (2024)

MEMORY BASED PAPER SOLUTION

29 JAN 2024 (S-01)





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Mathematics

1. What is probability of getting '2' in even number of throws of a die is ?

(1) $\frac{4}{11}$ (2) $\frac{5}{11}$ (3) $\frac{6}{11}$ (4) $\frac{3}{11}$

$P(2) = \frac{1}{6}$

$P(\bar{2}) = 1 - \frac{1}{6} = 5/6$

$\bar{2}2,\ \bar{2}\bar{2}\bar{2}2,\ \bar{2}\bar{2}\bar{2}\bar{2}\bar{2}2, \ldots$

$\frac{1}{6}\times\frac{5}{6},\ \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6},\ \left(\frac{5}{6}\right)^5\times\frac{1}{6}, \ldots$

$a_1 = \frac{5}{36},\ a_2 = \left(\frac{5}{6}\right)^3\times\frac{1}{6} = \frac{5}{36}\times\frac{25}{36},\ r = \frac{a_2}{a_1} = \frac{25}{36}, \ldots$

$S_\infty = \frac{a}{1-r}$

$= \frac{\frac{5}{36}}{1-\frac{25}{36}}$

$= \frac{5}{36-25}$

$= \frac{5}{11}$

2. A G.P. of 64 terms is such that S_n (total) = $7(S_n)_{\text{odd terms}}$ then common ratio of G.P is

(1) 2 (2) 6 (3) 10 (4) 14

$n = 64$, GP

Cond^n $\boxed{S_{64} = 7 S_{odd}}$

64

32 even 32 odd.

$a, ar, ar^2, ar^3, ar^4, \ldots, ar^{63}$

$S_n = \dfrac{a(r^n - 1)}{r-1} = \dfrac{\cancel{a}(\cancel{r^{64}} - 1)}{r-1}$

$S_{odd} = \dfrac{\cancel{a}(\cancel{(r^2)^{32}} - 1)}{r^2 - 1}$

$(r \to r^2)$

$\Rightarrow \dfrac{1}{\cancel{r-1}} = 7\left(\dfrac{1}{(r+1)\cancel{(r-1)}}\right)$

$\Rightarrow r + 1 = 7$

$r = 6$

3. Value of $\lim_{x \to \frac{\pi}{2}^-} \int_{x^3}^{(\pi/2)^3} \frac{\cos(t^{1/3})}{(x-\pi/2)^2} dt$ is

(1) $3\left(\frac{\pi}{3}\right)^3$ (2) $\frac{3}{2}\left(\frac{\pi}{2}\right)^2$ (3) $\left(\frac{\pi}{2}\right)^3$ (4) $\left(\frac{\pi}{2}\right)^2$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}^-} \frac{0 - \cos(x^3)^{\frac{1}{3}} \frac{d}{dx}(x^3)}{\left(x-\frac{\pi}{2}\right)^2}$$

$$\frac{\cos x}{(x-\pi/2)^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to\frac{\pi}{2}^-} \frac{-\cos x \, 3x^2}{\left(x-\frac{\pi}{2}\right)^2}$$

$$\Rightarrow -3x^2 \times \left(-\frac{1}{2}\right)$$

$$\Rightarrow \frac{3}{2} \times \frac{\pi^2}{4}$$

$$= \frac{-\sin x}{2(x-\pi/2)} = -\frac{1}{2}$$

4. If all word by using letters of word "GTWENTY" are arranged as in dictionary, then rank of the word "GTWENTY" is

(1) 552 (2) 553 (3) 554 (4) 551

G T W E N T Y

2 4 6 1 3 5 7

1 2 3 0 0 0 0

6! 5! 4! 3! 2! 1! 0!

2! 2!

$\frac{24 \times 3}{72}$

$360 + 120 + 72 + 0 + 0 + 0 + 0 + 1$

$6 \times 60 \Rightarrow 360$

360
120
73
553

5. If $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + \ldots\ldots \frac{{}^{11}C_9}{10} = \frac{n}{m}$, then value of $\boxed{n+m-8}$ is

(1) 2033 (2) 2034 (3) 2032 (4) 2034

$$\frac{{}^{11}C_1}{1+1} + \frac{{}^{11}C_2}{2+1} + \frac{{}^{11}C_3}{3+1} + \cdots + \frac{{}^{11}C_9}{9+1}$$

$$\Rightarrow \sum_{r=1}^{9} {}^{11}C_{r+1}$$

$$\Rightarrow \frac{1}{12} \times \sum_{r=1}^{9} 12 \times {}^{11}C_{r+1} \Rightarrow \boxed{\frac{1}{12} \sum_{r=1}^{9} {}^{12}C_{r+1}}$$

$$12 \times {}^{11}C_{r+1} \Rightarrow \frac{11!}{(r+1)!\,(10-r)!}$$

$$\Rightarrow \frac{12!}{(r+1)!}$$

$$\Rightarrow \frac{1}{12}\left[{}^{12}C_2 + {}^{12}C_3 + \cdots + {}^{12}C_{10}\right]$$

$$\Rightarrow \frac{1}{12}\Big[\left({}^{12}C_0 + {}^{12}C_1\right) - \left({}^{12}C_1 + {}^{12}C_0\right) + \left({}^{12}C_2 + {}^{12}C_3 + \cdots + {}^{12}C_{10}\right) +$$

$$+\left({}^{12}C_{11} + {}^{12}C_{12}\right) - \left({}^{12}C_{11} + {}^{12}C_{12}\right)\Big]$$

$$\Rightarrow \frac{1}{12}\left[2^{12} - \left({}^{12}C_1 + {}^{12}C_0 + {}^{12}C_{11} + {}^{12}C_{12}\right)\right]$$

$$\Rightarrow \frac{1}{12}\left[2^{12} - \left(12 + 1 + 12 + 1\right)\right]$$

$$\frac{1}{12}\left[2^{12} - 26\right]$$

$$\Rightarrow \frac{1}{6}\left[2^{11} - 13\right] = \frac{n}{m}$$

$$n + m - 8 = \square$$

$${}^{n}C_0 + {}^{n}C_1 + \cdots + {}^{n}C_n = 2^n$$

$${}^{12}C_0 = \frac{12!}{0!\,12!}$$

6. In an A.P $a_6 = 2$ then common difference for which $a_1.a_3.a_6$ is least is

(1) $\frac{16}{15}$ (2) $\frac{8}{15}$ (3) $\frac{15}{8}$ (4) $\frac{15}{16}$

greatest

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6$

$2-5d \quad 2-4d \quad 2-3d \quad 2-2d \quad 2-d \quad 2$

34

$a_1 a_4 a_5 = (2-5d)(2-2d)(2-d) \rightarrow$ cubic in d

$f(d) = -10d^3 + 34d^2 - 32d + 8$

$f'(d) = -30d^2 + 68d - 32 \Rightarrow -2(15d^2 + 34d + 16)$

factor

$+ \quad x_2 \quad - \quad x_1 \quad +$

7. $A = \begin{bmatrix} \cancel{1} & \cancel{0} & \cancel{0} \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}_{3\times 3}$ and $|2A|^3 = 2^{21}$ then, find α $(\alpha, \beta, \in I^+)$

(1) 3 (2) 4 (3) 5 (4) 6

$|A| = \alpha^2 - \beta^2$

$|kA| = k^n A$, order $A = n$

$|2A| = 2^3 |A|$

$|2A|^3 = (2^3|A|)^3 = 2^9 \times |A|^3 = 2^{21}$

$\Rightarrow |A|^3 = 2^{21-9} = 2^{12}$

$|A|^3 = 2^{12}$

$\Rightarrow |A| = 2^4$
$= 16$

$\alpha = 2$

$\alpha^2 - \beta^2 = 16$

$\alpha = 5, \beta = 3$

$25 - 9 = 16$

8. If $\boxed{AA^T = I}$, where A^T is transpose of matrix A then, $\frac{1}{2}A\left[(A+A^T)^2+(A-A^T)^2\right] = ?$

(1) A^2 (2) $A^3 + I$ (3) $A^2 + I$ (4) $A^3 + A^T$

$AA^T = I \Rightarrow A$ is OG

$\Rightarrow \boxed{AA^T = I = A^TA}$

$= \frac{1}{2}A\left[(A+A^T)^2+(A-A^T)^2\right]$

$\Rightarrow \frac{1}{2}A\left[A^2+(A^T)^2+\cancel{2AA^T}+A^2+(A^T)^2-\cancel{2AA^T}\right]$

$\Rightarrow \frac{1}{\cancel{2}}A\left[\cancel{2}(A^2+A^{T^2})\right]$

$A(A^2+A^{T^2})$

$\Rightarrow A^3+AA^{T^2}$

$\Rightarrow A^3+A(A^TA^T)$

$\Rightarrow A^3+IA^T$

$\Rightarrow A^3+A^T$

9. A relation R is defined on I x I such that $\boxed{(a, b)R\ (c,d)}$ if and only if ad – bc is divisible by 5, then relation R is a

(1) Reflexive and symmetric relation both
(2) symmetric and transitive relation both
(3) Transitive and reflexive relation both
(4) Equivalence relation

$(a,b)R(c,d) \Leftrightarrow ad-bc = 5m$

$bc-ad=5k$

$cb-da=5k$

$\Rightarrow (c,d)R(a,b)$

① Reflexive – $(a,b)R(a,b) \Rightarrow ab-ab=0=0\ 5$

② symmetric $(a,b)R(c,d) \Rightarrow ad-bc \in R$

$ad-bc=5r$

$bc-ad=-5r=5(-r)=5k \in R$

~~Transitive~~ → $(a,b)R(c,d) \Leftrightarrow ad-bc$ is div. by 5

$(1,7), (5,5) \rightarrow 1\times5 - 7\times5 = 5-35 = \frac{30}{5}$ ✓

$(5,5), (3,7) \rightarrow 35-15 = \frac{20}{5}$ ✓

$(1,7), (3,7)$

$\Downarrow$

$\frac{7-21}{5}$

$(a,b)R(c,d), (c,d)R(e,f) \Leftrightarrow (a,b)R(e,f)$ ✓

10. If $|z+1| = \alpha z + \beta(1+i)$ and $z = \frac{1}{2} - 2i$, then the value of $|\alpha+\beta|$ is equal to, where $\alpha, \beta \in R$

(1) 4 (2) 3 (3) 2 (4) 1

$|z+1| = \alpha z + \beta(1+i)$ — ①

$z = \frac{1}{2} - 2i$

$|\alpha+\beta| = ?$

$\left|\frac{1}{2} - 2i + 1\right| = \alpha\left(\frac{1}{2} - 2i\right) + \beta(1+i)$

$\left|\frac{3}{2} - 2i\right| \Rightarrow \sqrt{\frac{9}{4} + 4} = \left(\frac{\alpha}{2} + \beta\right) + i(-2\alpha + \beta)$

$|x+iy| = \sqrt{x^2+y^2}$

$\Rightarrow \sqrt{\frac{25}{4}} = \boxed{\frac{5}{2} = \left(\frac{\alpha}{2} + \beta\right) + i(-2\alpha + \beta)}$

$$\frac{\alpha}{2}+\beta=\frac{5}{2} \quad —(1)$$

$$-2\alpha+\beta=0$$

$$\beta=2\alpha \quad \Rightarrow \beta=2$$

$$\frac{\alpha}{2}+2\alpha=\frac{5}{2}$$

$$\Rightarrow \frac{\alpha+4\alpha}{\cancel{2}}=\frac{5}{\cancel{2}}$$

$$\Rightarrow \alpha=1$$

$$|\alpha+\beta|=|2+1|=3$$

11. If mean and variance of observations 60, 60, 44, 58, 68, 56, α, β are 58 and 66.2 respectively then $\alpha^2 + \beta^2$ is equal to

(1) ~~6150.2~~ (2) 7181.6 (3) 9532.8 (4) 3252.6

* mean & variance,

$$\text{variance} = \sigma^2 = \frac{\Sigma x_i^2}{N} - (\mu)^2 = 66.2$$

calculation

$N = 9$

$$\frac{\alpha^2 + \beta^2 + \ldots}{9} - (58)^2 = 66.2$$

$\mu = 58$

12. $\vec{a}, \vec{b}, \vec{c}$ are three pairwise non-collinear vectors. $\vec{a}+6\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+5\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+\alpha\vec{b}+\beta\vec{c}=\vec{0}$, then $\alpha+\beta=?$

(1) 36 (2) 30 (3) 25 (4) 39

$\bar{a}, \bar{b}, \bar{c}$ pairwise non-colli

$\bar{a}+6\bar{b} = \lambda\vec{c}$ — (1)

$6 \times [\bar{b}+5\bar{c} = \mu\vec{a}$ — (2) $]$

$\vec{a}-30\vec{c} = \lambda\vec{c}-6\mu\vec{a}$

$\Rightarrow \vec{a}(1+6\mu) = \vec{c}(\lambda+30)$

$1+6\mu \to 0$, $\lambda+30 \to 0$

$\vec{a}+\alpha\vec{b}+\beta\vec{c}=0$

$1+6\mu=0 \Rightarrow \mu=-\frac{1}{6}$

$\lambda+30=0 \Rightarrow \lambda=-30$

using in (1),

$\vec{a}+6\vec{b}=-30\vec{c}$

$\vec{a}+6\vec{b}+30\vec{c}=0$

$\vec{a}+\alpha\vec{b}+\beta\vec{c}=0$

$\alpha=6$

$\beta=30$

$\alpha+\beta=6+30$

$=36$