WB JEE Engineering Entrence Exam

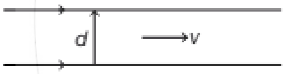
SOLVED PAPER – 2022

Physics

Category-I (Q. Nos. 1 to 30)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

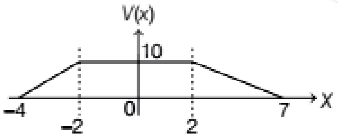
1. Two infinite line-charges parallel to each other are moving with a constant velocity v in the same direction as shown in the figure below.



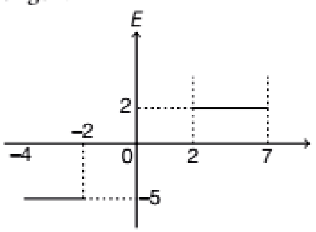
The separation between two line-charges is d. The magnetic attraction balances the electric repulsion when, [c = speed of light in free space].

(a) $v=\sqrt{2}c$ (b) $v=\frac{c}{\sqrt{2}}$ (c) $v=c$ (d) $v=\frac{c}{2}$

2. The electric potential for an electric field directed parallel to X-axis is shown in the figure.



Choose the correct plot of electric field strength.

(a)


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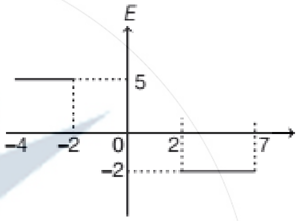

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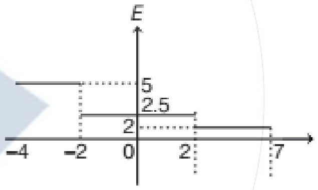
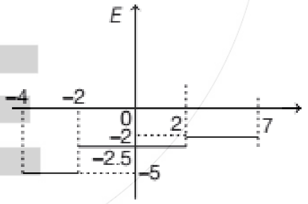


(d)


3. An electron revolves around the nucleus in a circular path with angular momentum **L**. A uniform magnetic field **B** is applied perpendicular to the plane of its orbit. If the electron experiences a torque **T**, then

(a) **T**||**L**
(b) **T** is anti-parallel to **L**
(c) $\mathbf{T}\cdot\mathbf{L}=0$
(d) Angle between **T** and **L** is 45°

4. A straight wire is placed in a magnetic field that varies with distance x from origin as $\mathbf{B}=B_0\left(2-\frac{x}{a}\right)\hat{\mathbf{k}}$. Ends of wire are at $(a,0)$ and

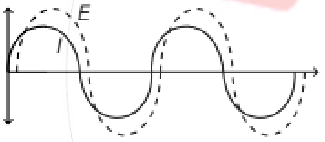
$(2a, 0)$ and it carries a current I. If force on wire is $\mathbf{F} = IB_0\left(\frac{ka}{2}\right)\hat{\mathbf{j}}$, then value of k is

(a) 1 (b) 5
(c) – 1 (d) $\frac{1}{2}$

5. In a closed circuit, there is only a coil of inductance L and resistance $100\,\Omega$. The coil is situated in a uniform magnetic field. All on a sudden, the magnetic flux linked with the circuit changes by 5 Wb. What amount of charge will flow in the circuit as a result?

(a) 500 C
(b) 0.05 C
(c) 20 C
(d) Value of L is to be known to find the charge flown

6. When an AC source of emf E with frequency $\omega = 100$ rad/s is connected across a circuit, the phase difference between E and current I in the circuit is observed to be $\frac{\pi}{4}$ as shown in the figure. If the circuit consist of only RC or RL in series, then



(a) $R = 1\ \text{k}\,\Omega,\ C = 5\mu\text{F}$
(b) $R = 1\ \text{k}\,\Omega,\ L = 10\,\text{H}$
(c) $R = 1\text{k}\,\Omega,\ L = 1\,\text{H}$
(d) $R = 1\text{k}\,\Omega,\ C = 10\mu\text{F}$

7.

A battery of emf E and internal resistance r is connected with an external resistance R as shown in the figure. The battery will act as a constant voltage source, if

(a) $r << R$
(b) $r >> R$
(c) $r = R$
(d) It will never act as a constant voltage source

8. If the kinetic energies of an electron, an alpha particle and a proton having same de-Broglie wavelengths are ε_1, ε_2 and ε_3 respectively, then

(a) $\varepsilon_1 > \varepsilon_3 > \varepsilon_2$ (b) $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$
(c) $\varepsilon_1 < \varepsilon_3 < \varepsilon_2$ (d) $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$

9. In a Young's double slit experiment, the intensity of light at a point on the screen where the path difference between the interfering waves is λ, (λ being the wavelength of light used) is 1. The intensity at a point where the path difference is $\lambda/4$ will be (assume two waves have same amplitude)

(a) zero (b) I (c) $\frac{I}{2}$ (d) $\frac{I}{4}$

10. In Young's double slit experiment with a monochromatic light, maximum intensity is 4 times the minimum intensity in the interference pattern. What is the ratio of the intensities of the two interfering waves?

(a) 1/9 (b) 1/3 (c) 1/16 (d) 1/2

11. The human eye has an approximate angular resolution of $\theta = 5.8 \times 10^{-4}$ rad and typical photo printer prints a minimum of 300 dpi (dots per inch, 1 inch = 2.54 cm). At what minimal distance d should a printed page be held, so that one does not see the individual dots?

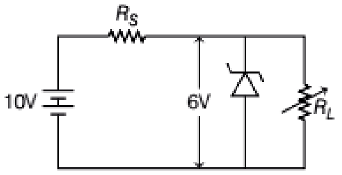
(a) 20.32 cm (b) 29.50 cm
(c) 14.59 cm (d) 6.85 cm

12. Suppose in a hypothetical world the angular momentum is quantised to be even integral multiples of $\frac{h}{2\pi}$. The largest possible wavelength emitted by hydrogen atoms in visible range in a world according to Bohr's model will be

(Consider $hc = 1242$ MeV-nm)

(a) 153 nm (b) 409 nm (c) 121 nm (d) 487 nm

13. A Zener diode having breakdown voltage $V_Z = 6$ V is used in a voltage regulator circuit as shown in the figure. The minimum current required to pass through the Zener to act as a voltage regulator is 10 mA and maximum allowed current through Zener is 40 mA. The maximum value of R_S for Zener to act as a voltage regulator is



(a) 100 Ω (b) 400 Ω (c) 0.4 Ω (d) 950 Ω

14. The expression $\overline{A}(A+B)+(B+AA)(A+\overline{B})$ simplifies to

(a) $A+B$ (b) AB (c) $\overline{A+B}$ (d) $\overline{A}+\overline{B}$

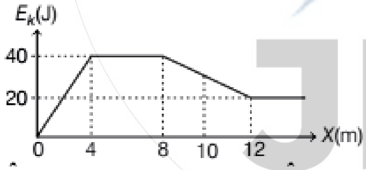
15. Given, the percentage error in the measurements of A, B, C and D are respectively, 4% 2%, 3% and 1%. The relative error in $Z = \dfrac{A^4 B^{\frac{1}{3}}}{CD^{\frac{3}{2}}}$ is

(a) $\frac{127}{2}\%$ (b) $\frac{127}{5}\%$ (c) $\frac{127}{6}\%$ (d) $\frac{127}{7}\%$

16. The entropy S of a black hole can be written as $S = \beta k_B A$, where k_B is the Boltzmann constant and A is the area of the black hole. Then, β has dimension of

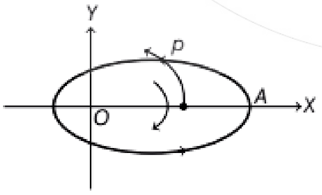
(a) $[L^2]$ (b) $[ML^2T^{-1}]$
(c) $[L^{-2}]$ (d) Dimensionless

17. The kinetic energy (E_k) of a particle moving along X-axis varies with its position (x) as shown in the figure. The force acting on the particle at $x = 10$ m is



(a) $5\hat{i}$ N (b) zero (c) $97.5\,\hat{i}$ N (d) $-5\hat{i}$ N

18.



A particle is moving in an elliptical orbit as shown in figure. If **p**, **L** and **r** denote the linear momentum, angular momentum and position vector of the particle (from focus O) respectively at a point A, then the direction of $\alpha = \mathbf{p} \times \mathbf{L}$ is along

(a) + ve X-axis (b) − ve X-axis
(c) + ve Y-axis (d) − ve Y-axis

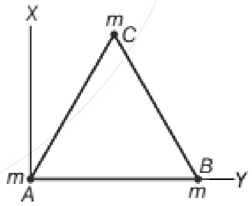
19. A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motion, the phase difference (δ) between the two motions is

(a) $\delta = \frac{\pi}{3}$ (b) $\delta = \frac{2\pi}{3}$
(c) $\delta = \pi$ (d) $\delta = \frac{\pi}{2}$

20. A body of mass m is thrown with velocity u from the origin of a co-ordinate axes at an angle θ with the horizon. The magnitude of the angular momentum of the particle about the origin at time t when it is at the maximum height of the trajectory is proportional to

(a) u (b) u^2
(c) u^3 (d) independent of u

21. Three particles, each of mass m grams situated at the vertices of an equilateral ΔABC of side a cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC in g-cm^2 units will be



(a) $2ma^2$ (b) $\frac{3}{2}ma^2$
(c) $\frac{3}{4}ma^2$ (d) $\frac{5}{4}ma^2$

22. A body of mass m is thrown vertically upward with speed $\sqrt{3}\, v_e$, where v_e is the escape velocity of a body from earth surface. The final velocity of the body is

(a) zero (b) $2v_e$

(c) $\sqrt{3}\, v_e$ (d) $\sqrt{2}\, v_e$

23. If a string, suspended from the ceiling is given a downward force F_1, its length becomes L_1, its length is L_2, if the downward force is F_2. What is its actual length?

(a) $\frac{L_1 + L_2}{2}$ (b) $\sqrt{L_1 L_2}$

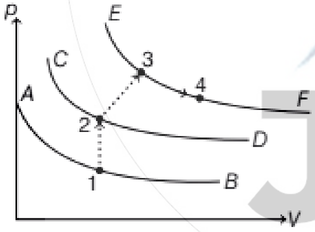
(c) $\frac{F_2 L_1 + F_1 L_2}{F_2 + F_1}$ (d) $\frac{F_2 L_1 - F_1 L_2}{F_2 - F_1}$

24. 27 drops of mercury coalese to form a bigger drop. What is the relative increase in surface energy?

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$

(c) $-\frac{2}{3}$ (d) 8

25. Certain amount of an ideal gas is taken from its initial state 1 to final state 4 through the paths $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ as shown in figure. AB, CD, EF are all isotherms. If v_p is the most probable speed of the molecules. Then,



(a) v_p at $3 = v_p$ at $4 > v_p$ at $2 > v_p$ at 1
(b) v_p at $3 > v_p$ at $1 > v_p$ at $2 > v_p$ at 4
(c) v_p at $3 > v_p$ at $2 > v_p$ at $4 > v_p$ at 1
(d) v_p at $2 = v_p$ at $3 > v_p$ at $1 > v_p$ at 4

26. Consider a thermodynamic process, where internal energy $U = Ap^2V$ ($A =$ constant). If the process is performed adiabatically, then

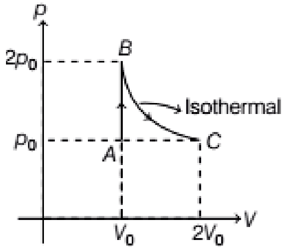
(a) $Ap^2(V + 1) =$ constant

(b) $(Ap + 1)^2 V =$ constant

(c) $(Ap + 1)V^2 =$ constant

(d) $\frac{V}{(Ap + 1)^2} =$ constant

27. One mole of a diatomic ideal gas undergoes a process shown in p-V diagram. The total heat given to the gas ($\ln 2 = 0.7$) is

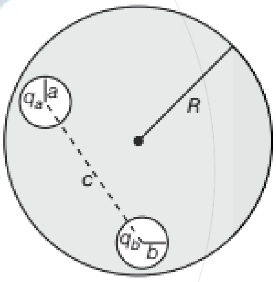


(a) $2.5 p_0 V_0$ (b) $3.9 p_0 V_0$ (c) $1.1 p_0 V_0$ (d) $1.4\, p_0 V_0$

28. Two charges, each equal to $-q$ are kept at $(-a, 0)$ and $(a, 0)$. A charge q is placed at the origin. If q is given a small displacement along y-direction, the force acting on q is proportional to

(a) y (b) $-y$ (c) $\frac{1}{y}$ (d) $-\frac{1}{y}$

29.



A neutral conducting solid sphere of radius R has two spherical cavities of radius a and b as shown in the figure above. Centre to centre distance between two cavities is c. q_a and q_b charges are placed at the centres of cavities, respectively. The force between q_a and q_b is

(a) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_a q_b}{c^2}$ (b) $\frac{1}{4\pi\varepsilon_0} q_a q_b \left(\frac{1}{a^2} + \frac{1}{b^2}\right)$

(c) zero (d) Insufficient data

30. Consider two concentric conducting sphere of radii R and $2R$, respectively. The inner sphere is given a charge $+Q$. The other sphere is grounded. The potential at $r = \frac{3R}{2}$ is

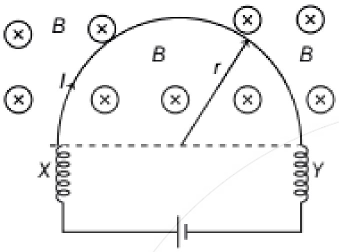
(a) $\frac{1}{4\pi\varepsilon_0} \frac{Q}{6R}$ (b) zero

(c) $\frac{1}{4\pi\varepsilon_0} \frac{2Q}{3R}$ (d) $\frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$

Category-II (Q. Nos. 31 to 35)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

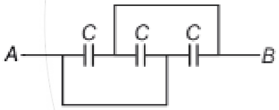
31. A horizontal semi-circular wire of radius r is connected to battery through two similar springs X and Y to an electric cell, which sends current I through it. A vertically downward uniform magnetic field B is applied on the wire, as shown in the figure.



What is the force acting on each spring?

(a) $2\pi rBI$
(b) $\frac{1}{2}\pi rBI$
(c) BIr
(d) $2BIr$

32. Find the equivalent capacitance between A and B of the following arrangement.

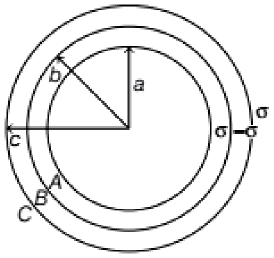


(a) C
(b) $3C$
(c) $\frac{2C}{3}$
(d) $\frac{3C}{2}$

33. A golf ball of mass 50 gm placed on a tee, is struck by a golf-club. The speed of the golf ball as it leaves the tee is 100 m/s, the time of contact on the ball is 0.02 s. If the force decreases to zero linearly with time, then the force at the beginning of the contact is

(a) 100 N
(b) 200 N
(c) 250 N
(d) 500 N

34. Three concentric metallic shells A, B and C of radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma, -\sigma$ and $+\sigma$, respectively. The potential of shell B is



(a) $(a+b+c)\frac{\sigma}{\varepsilon_0}$
(b) $\frac{\sigma c}{\varepsilon_0}$
(c) $\left(\frac{a^2}{c}-\frac{b^2}{c}+c\right)\frac{\sigma}{\varepsilon_0}$
(d) $\left(\frac{a^2}{b}-b+c\right)\frac{\sigma}{\varepsilon_0}$

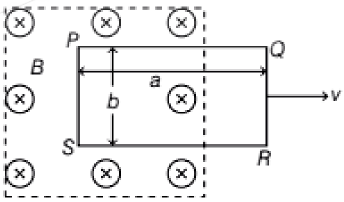
35. One mole of an ideal monoatomic gas expands along the polytrope $pV^3 = \text{constant}$ from V_1 to V_2 at a constant pressure p_1. The temperature during the process is such that molar specific heat $C_V = \frac{3R}{2}$. The total heat absorbed during the process can be expressed as

(a) $p_1V_1\left(\frac{V_1^2}{V_2^2}+1\right)$
(b) $p_1V_1\left(\frac{V_1^2}{V_2^2}-1\right)$
(c) $p_1V_1\left(\frac{V_1^3}{V_2^2}-1\right)$
(d) $p_1V_1\left(\frac{V_1}{V_2^2}-1\right)$

Category-III (Q. Nos. 36 to 40)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked, then score $= 2 \times$ number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong but there is no negative marking for the same and zero marks will be awarded.

36.



As shown in figure, a rectangular loop of length *a* and width *b* and made of a conducting material of uniform cross-section is kept in a horizontal plane, where a uniform magnetic field of intensity *B* is acting vertically downward. Resistance per unit length of the loop is λ Ω/m. If the loop is pulled with uniform velocity *v* in horizontal direction, which of the following statement is/are true?

(a) Current in the loop $I = \dfrac{Bbv}{\lambda(2b + 2a)}$.

(b) Current will be in clockwise direction, looking from the top.

(c) $V_p - V_S = V_Q - V_R$, where *V* is the potential.

(d) There cannot be any induction in part *SR*.

37. A sample of hydrogen atom in its ground state is radiated with photons of 10.2 eV energies. The radiation from the sample is absorbed by excited ionised He^+. Then, which of the following statement/s is/are true?

(a) He^+ electron moves from $n = 2$ to $n = 4$.

(b) In the He^+ emission spectra, there will be 6 lines.

(c) Smallest wavelength of He^+ spectrum is obtained when transition taken place from $n = 4$ to $n = 3$.

(d) He^+ electron moves from $n = 2$ to $n = 3$.

38. A particle is moving in *x*-*y* plane according to $\mathbf{r} = b\cos\omega t\,\hat{\mathbf{i}} + b\sin\omega t\,\hat{\mathbf{j}}$, where ω is constant. Which of the following statement(s) is/are true?

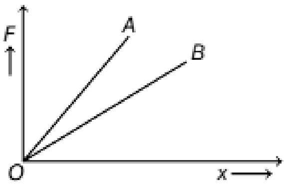
(a) $\dfrac{E}{\omega}$ is a constant, where *E* is the total energy of the particle.

(b) The trajectory of the particle in *x*-*y* plane is a circle.

(c) In a_x - a_y plane, trajectory of the particle is an ellipse (a_x, a_y denotes the components of acceleration)

(d) $\mathbf{a} = \omega^2 \mathbf{v}$

39.



Two wires *A* and *B* of same length are made of same material. Load (*F*) *versus* elongation (*x*) graph for these two wires is shown in the figure. Then, which of the following statement(s) is/are true?

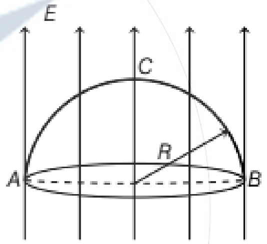
(a) The cross-sectional area of *A* is greater than that of *B*.

(b) Young's modulus of *A* is greater than Young's modulus of *B*.

(c) The cross-sectional area of *B* is greater than that of *A*.

(d) Young's modulus of both *A* and *B* are same.

40.



A hemisphere of radius *R* is placed in a uniform electric field *E*, so that its axis is parallel to the field. Which of the following statement(s) is/are true?

(a) Flux through the curved surface of hemisphere is $\pi R^2 E$.

(b) Flux through the circular surface of hemisphere is $\pi R^2 E$.

(c) Total flux enclosed is zero.

(d) Work done in moving a point charge *q* from *A* to *B* via the path *ACB* depends upon *R*.

Chemistry

Category-I (Q. Nos. 41 to 70)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

41. A sample of $MgCO_3$ is dissolved in dil.HCl and the solution is neutralised with ammonia and buffered with NH_4Cl / NH_4OH. Disodium hydrogen phosphate reagent is added to the resulting solution. A white precipitate is formed. What is the formula of the precipitate?

(a) $Mg_3(PO_4)_2$ (b) $Mg(NH_4)PO_4$
(c) $MgHPO_4$ (d) $Mg_2P_2O_7$

42. $XeF_2, NO_2, HCN, ClO_2, CO_2$

Identify the non-linear molecule-pair from the above mentioned molecules.

(a) XeF_2, ClO_2 (b) CO_2, NO_2
(c) HCN, NO_2 (d) ClO_2, NO_2

43. The number of atoms in body centred and face centred cubic unit cell respectively are

(a) 2 and 4 (b) 4 and 3
(c) 1 and 2 (d) 4 and 6

44. The number of unpaired electrons in Mn^{2+} ion is

(a) 2 (b) 3 (c) 5 (d) 6

45. The average speed of H_2 at T_1K is equal to that of O_2 at T_2K. The ratio $T_1 : T_2$ is

(a) 1 : 6 (b) 16 : 1 (c) 1 : 4 (d) 1 : 1

46. Sodium nitroprusside is

(a) $Na_4[Fe(CN)_5NO_2]$ (b) $Na_2[Fe(CN)_5NO]$
(c) $Na_3[Fe(CN)_5NO]$ (d) $Na_4[Fe(CN)_5NO_3]$

47. Choose the correct statement for the $[Ni(CN)_4]^{2-}$ complex ion (Atomic number of Ni = 28).

(a) The complex is square planar and paramagnetic.
(b) The complex is tetrahedral and diamagnetic.
(c) The complex is square planar and diamagnetic.
(d) The complex is tetrahedral and paramagnetic.

48. The boiling point of the water is higher than liquid HF. The reason is that

(a) hydrogen bonds are stronger in water
(b) hydrogen bonds are stronger in HF
(c) hydrogen bonds are larger in number in HF
(d) hydrogen bonds are larger in number in water

49. The metal-pair that can produce nascent hydrogen in alkaline medium is

(a) Zn, Al (b) Fe, Ni
(c) Al, Mg (d) Mg, Zn

50. The correct bond order of B—F bond in BF_3 molecule is

(a) 1 (b) $1\frac{1}{2}$
(c) 2 (d) $1\frac{1}{3}$

51. Which of the following is radioactive?

(a) Hydrogen
(b) Deuterium
(c) Tritium
(d) None of the above

52. The correct order of acidity of the following hydra acids is

(a) HF > HCl > HBr > HI
(b) HF < HCl < HBr < HI
(c) HF < HCl > HBr > HI
(d) HF > HCl < HBr > HI

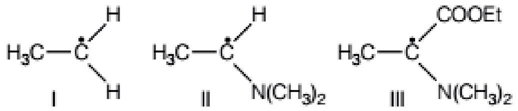
53. To a solution of colourless sodium salt, a solution of lead nitrate was added to have a white precipitate which dissolves in warm water and reprecipitates on cooling. Which of the following acid radical is present in the salt?

(a) Cl^- (b) SO_4^{2-}
(c) S^{2-} (d) NO_3^-

54. Oxidation states of Cr in $K_2Cr_2O_7$ and CrO_5 are, respectively

(a) + 6, + 5
(b) + 6, + 10
(c) + 6, + 6
(d) None of the above

55. The correct order of relative stability for the given free radicals is



(a) II > I > III (b) II > III > I
(c) III > I > II (d) III > II > I

56. 

$\overset{\ominus}{\ddot{C}}H_3$

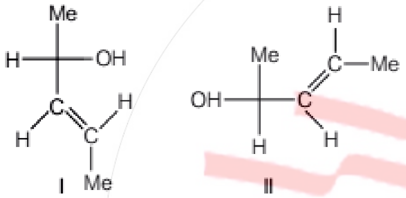
(1)

$H_2\overset{\ominus}{\underset{\cdot\cdot}{C}}-CHOCH_3$

(2)

Hybridisation of the negative carbons in (1) and (2) are

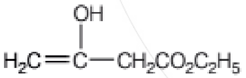
(a) sp^2 and sp^3 (b) sp^3 and sp^2
(c) both sp^2 (d) both sp^3

57. 

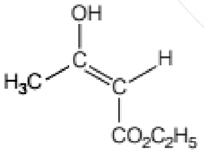
The correct relationship between molecules I and II is

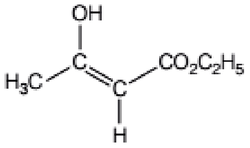
(a) enantiomer (b) homomer
(c) diastereomer (d) constitutional isomer

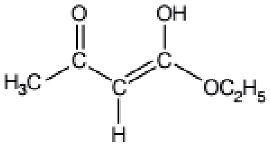
58. The enol form in which ethyl-3-oxobutanoate exists is

(a) 

$H_2C=\overset{OH}{\overset{|}{C}}-CH_2CO_2C_2H_5$

(b) 

(c) 

(d) 

59. How many monobrominated product(s) (including stereoisomers) would form in the free radical bromination of *n*-butane?

(a) 2 (b) 1 (c) 3 (d) 4

60. What is the correct order of acidity of salicylic acid, 4-hydroxybenzoic acid, and 2, 6-dihydroxybenzoic acid?

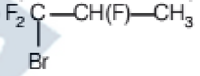
(a) 2, 6-dihydroxybenzoic acid > Salicylic acid > 4-hydroxybenzoic acid
(b) 2, 6-dihydroxybenzoic acid > 4-hydroxybenzoic acid > salicylic acid
(c) Salicylic acid > 2, 6-dihydroxybenzoic acid > 4-hydroxybenzoic acid
(d) Salicylic acid > 4-hydroxybenzoic acid > 2, 6-dihydroxybenzoic acid

61. How much solid oxalic acid (molecular weight = 126) has to be weighed to prepare 100 mL exactly 0.1 N oxalic acid solution in water?

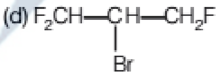
(a) 1.26 g (b) 0.126 g (c) 0.63 g (d) 0.063 g

62. The major product of the following reaction is

$$F_3C—CH = CH_2 + HBr \longrightarrow$$

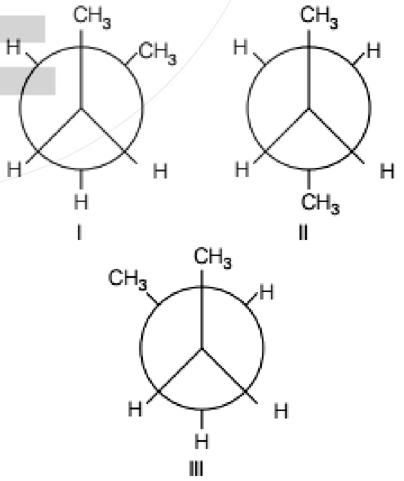
(a) $F_3C—CH_2—CH_2Br$
(b) $F_3C—CH(Br)—CH_3$
(c) 

$F_2\underset{\underset{Br}{|}}{C}—CH(F)—CH_3$

(d) 

$F_2CH—\underset{\underset{Br}{|}}{CH}—CH_2F$

63. The correct order of relative stability of the given conformers of *n*-butane is



(a) II > I = III (b) II > III > I
(c) II > I > III (d) I = III > II

64. $C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$

Benzene burns in oxygen according to the above equation. What is the volume of oxygen (at STP) needed for complete combustion of 39 g of liquid benzene?

(a) 11.2 L (b) 22.4 L
(c) 84 L (d) 168 L

65. Avogadro's law is valid for

(a) all gases
(b) ideal gas
(c) van der Waals' gas
(d) real gas

66. A metal (M) forms two oxides. The ratio M : O (by weight) in the two oxides are 25 :4 and 25 : 6. The minimum value of atomic mass of M is

(a) 50 (b) 100 (c) 150 (d) 200

67. The de-Broglie wavelength (λ) for electron (e), proton (p) and He^{2+} ion (α) are in the following order. Speed of e, p and α are the same.

(a) $\alpha > p > e$ (b) $e > p > \alpha$
(c) $e > \alpha > p$ (d) $\alpha < p > e$

68. 1 mL of water has 25 drops. Let N_0 be the Avogadro number. What is the number of molecules present in 1 drop of water? (Density of water = 1 g/mL)

(a) $\frac{0.02}{9}N_0$ (b) $\frac{18}{25}N_0$
(c) $\frac{25}{18}N_0$ (d) $\frac{0.04}{25}N_0$

69. In Bohr model of atom, radius of hydrogen atom in ground state is r_1 and radius of He^+ ion in ground state is r_2. Which of the following is correct?

(a) $\frac{r_1}{r_2} = 4$
(b) $\frac{r_1}{r_2} = \frac{1}{2}$
(c) $\frac{r_2}{r_1} = \frac{1}{4}$
(d) $\frac{r_2}{r_1} = \frac{1}{2}$

70. Which one of the following is the correct set of four quantum numbers (n, l, m, s)?

(a) $\left(3, 0, -1, +\frac{1}{2}\right)$ (b) $\left(4, 3, -2, -\frac{1}{2}\right)$
(c) $\left(3, 1, -2, -\frac{1}{2}\right)$ (d) $\left(4, 2, -3, +\frac{1}{2}\right)$

Category-II (Q. Nos. 71 to 75)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

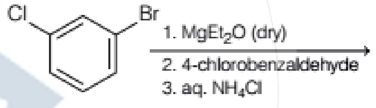
71. Let $(C_{rms})_{H_2}$ is the r.m.s. speed of H_2 at 150 K. At what temperature, the most probable speed of helium $[(C_{mp})_{He}]$ will be half of $(C_{rms})_{H_2}$?

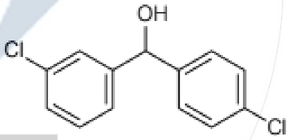
(a) 75 K (b) 112.5 K
(c) 225 K (d) 900 K

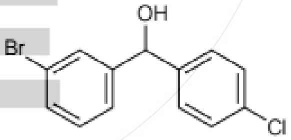
72. The correct pair of electron affinity order is

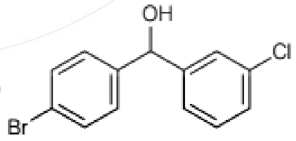
(a) O > S, F > Cl (b) O < S, Cl > F
(c) S > O, F > Cl (d) S < O, Cl > F

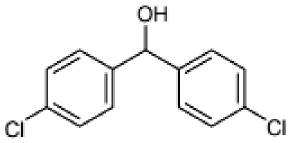
73. The product of the following reaction is



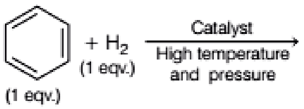
(a)


(b)


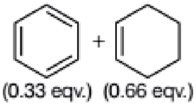
(c)


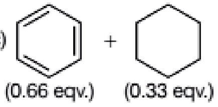
(d)


74. The product of the following hydrogenation reaction is



(a)


(b)


(c)


(d)


75. Pick the correct statement.

(a) Relative lowering of vapour pressure is independent of *T*.

(b) Osmotic pressure always depends on the nature of solute.

(c) Elevation of boiling point is independent of nature of the solvent.

(d) Lowering of freezing point is proportional to the molar concentration of solute.

Category-III (Q.Nos. 76 to 80)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and no incorrect answer is marked, then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will be considered wrong, but there is no negative marking for the same and zero mark will be awarded.

76. During the preparation of NH_3 in Haber's process,the promoter(s) used is/are

(a) PtO_2

(b) Mo

(c) Mix of Al_2O_3 and K_2O

(d) Fe and Mn

77. The correct statement(s) about B_2H_6 is/are

(a) all B atoms are sp^3 hybridised.

(b) it is paramagnetic

(c) it contains 3C - $4e^-$ bonding

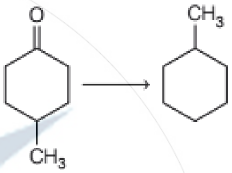
(d) there are two types of H present

78. Which of the following would produce enantiomeric products when reacted with methyl magnesium iodide?

(a) Benzaldehyde (b) Propiophenone

(c) Acetone (d) Acetaldehyde

79.



The above conversion can be carried out by,

(a) Zn-Hg/conc. HCl

(b) i. H_2NNH_2 ii. NaOH in ethylene glycol, Δ

(c) i. $HSCH_2CH_2SH/H^{\oplus}$ ii. H_2 /Ni

(d) bromine water

80. Which of the statement are incorrect?

(a) pH of a solution of salt of strong acid and weak base is less than 7.

(b) pH of solution of a weak acid and weak base is basic, if $K_b < K_a$.

(c) pH of an aqueous solution of 10^{-8}M HCl is 8.

(d) Conjugate acid of NH_2^- is NH_3.

Mathematics

Category-I (Q. Nos. 1 to 50)

Carry 1 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/4 mark will be deducted.

1. The values of a, b, c for which the function

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \dfrac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{1/2}}, & x>0 \end{cases}$$ is continuous

at $x = 0$, are

(a) $a = \frac{3}{2}, b = -\frac{3}{2}, c = \frac{1}{2}$

(b) $a = -\frac{3}{2}, c = \frac{3}{2}$, b is arbitrary non-zero real number

(c) $a = -\frac{5}{2}, b = -\frac{3}{2}, c = \frac{3}{2}$

(d) $a = -2, b \in R - \{0\}, c = 0$

2. Domain of $y = \sqrt{\log_{10} \frac{3x - x^2}{2}}$ is

(a) $x < 1$ (b) $2 < x$ (c) $1 \le x \le 2$ (d) $2 < x < 3$

3. Let $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$, where a_0, a_1, a_2, a_3 are real constants. Then, $f(x)$ is differentiable at $x = 0$

(a) whatever be a_0, a_1, a_2, a_3

(b) for no values of a_0, a_1, a_2, a_3

(c) only if $a_1 = 0$

(d) only if $a_1 = 0, a_3 = 0$

4. If $y = e^{\tan^{-1} x}$, then

(a) $(1+x^2)y_2 + (2x-1)y_1 = 0$

(b) $(1+x^2)y_2 + 2xy = 0$

(c) $(1-x^2)y_2 - y_1 = 0$

(d) $(1+x^2)y_2 + 3xy_1 + 4y = 0$

5. $\lim\limits_{x \to 0} \left(\frac{1}{x} \ln \sqrt{\frac{1+x}{1-x}} \right)$ is

(a) 1/2 (b) 0

(c) 1 (d) does not exist

6. Let $f : [a, b] \to R$ be continuous in $[a, b]$, differentiable in (a, b) and $f(a) = 0 = f(b)$. Then,

(a) there exists at least one point $c \in (a, b)$ for which $f'(c) = f(c)$

(b) $f'(x) = f(x)$ does not hold at any point of (a, b)

(c) at every point of (a, b), $f'(x) > f(x)$

(d) at every point of (a, b), $f'(x) < f(x)$

7. $I = \int \cos(\ln x) dx$. Then, I is equal to

(a) $\frac{x}{2}\{\cos(\ln x) + \sin(\ln x)\} + c$

(b) $x^2\{\cos(\ln x) - \sin(\ln x)\} + c$

(c) $x^2 \sin(\ln x) + c$

(d) $x \cos(\ln x) + c$

(c denotes constant of integration)

8. Let f be derivable in $[0, 1]$, then

(a) there exists $c \in (0, 1)$ such that $\int_0^c f(x) dx = (1-c) f(c)$

(b) there does not exist any point $d \in (0, 1)$ for which $\int_0^d f(x) dx = (1-d) f(d)$

(c) $\int_0^c f(x) dx$ does not exist, for any $c \in (0, 1)$

(d) $\int_0^c f(x) dx$ is independent of $c, c \in (0, 1)$

9. Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}} dx = \frac{2}{3} g(f(x)) + c$, then

(a) $f(x) = \sqrt{x}, g(x) = x^{3/2}$

(b) $f(x) = x^{3/2}, g(x) = \sin^{-1} x$

(c) $f(x) = \sqrt{x}, g(x) = \sin^{-1} x$

(d) $f(x) = \sin^{-1} x, g(x) = x^{3/2}$

10. The value of $\int_0^{\pi/2} \frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x} + (\sin x)^{\cos x}} dx$ is

(a) $\frac{\pi}{4}$ (b) 0

(c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$

11. Let $\lim_{\varepsilon \to 0^+} \int_{\varepsilon}^{x} \frac{bt\cos 4t - a\sin 4t}{t^2}\, dt = \frac{a\sin 4x}{x} - 1$, $\left(0 < x < \frac{\pi}{4}\right)$. Then, a and b are given by

(a) $a = 2, b = 2$
(b) $a = \frac{1}{4}, b = 1$
(c) $a = -1, b = 4$
(d) $a = 2, b = 4$

12. Let $f(x) = \int_{\sin x}^{\cos x} e^{-t^2} dt$. Then, $f'\left(\frac{\pi}{4}\right)$ equals

(a) $\sqrt{\frac{1}{e}}$
(b) $-\sqrt{\frac{2}{e}}$
(c) $\sqrt{\frac{2}{e}}$
(d) $-\sqrt{\frac{1}{e}}$

13. If $x\frac{dy}{dx} + y = x\frac{f(xy)}{f'(xy)}$, then $|f(xy)|$ is equal to

(a) $ce^{x^2/2}$
(b) ce^{x^2}
(c) ce^{2x^2}
(d) $ce^{x^2/3}$

where, c is the constant of integration.

14. A curve passes through the point (3, 2) for which the segment of the tangent line contained between the coordinate axes is bisected at the point of contact. The equation of the curve is

(a) $y = x^2 - 7$
(b) $x = \frac{y^2}{2} + 2$
(c) $xy = 6$
(d) $x^2 + y^2 - 5x + 7y + 11 = 0$

15. The solution of $\cos y\frac{dy}{dx} = e^{x + \sin y} + x^2 e^{\sin y}$ is $f(x) + e^{-\sin y} = C$ (C is arbitrary real constant), where $f(x)$ is equal to

(a) $e^x + \frac{1}{2}x^3$
(b) $e^{-x} + \frac{1}{3}x^3$
(c) $e^{-x} + \frac{1}{2}x^3$
(d) $e^x + \frac{1}{3}x^3$

16. The point of contact of the tangent to the parabola $y^2 = 9x$ which passes through the point (4, 10) and makes an angle θ with the positive side of the axis of the parabola where $\tan\theta > 2$, is

(a) $\left(\frac{4}{9}, 2\right)$
(b) (4, 6)
(c) (4, 5)
(d) $\left(\frac{1}{4}, \frac{1}{6}\right)$

17. Let $f(x) = (x-2)^{17}(x+5)^{24}$. Then,

(a) f does not have a critical point at $x = 2$
(b) f has a minimum at $x = 2$
(c) f has neither a maximum nor a minimum at $x = 2$
(d) f has a maximum at $x = 2$

18. If $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{c}}$ is unit vector perpendicular to $\mathbf{a}$ and coplanar with $\mathbf{a}$ and $\mathbf{b}$, then unit vector $\mathbf{d}$ perpendicular to both $\mathbf{a}$ and $\mathbf{c}$ is

(a) $\pm\frac{1}{\sqrt{6}}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$
(b) $\pm\frac{1}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$
(c) $\pm\frac{1}{\sqrt{6}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$
(d) $\pm\frac{1}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{k}})$

19. If the equation of one tangent to the circle with centre at $(2, -1)$ from the origin is $3x + y = 0$, then the equation of the other tangent through the origin is

(a) $3x - y = 0$
(b) $x + 3y = 0$
(c) $x - 3y = 0$
(d) $x + 2y = 0$

20. Area of the figure bounded by the parabola $y^2 + 8x = 16$ and $y^2 - 24x = 48$ is

(a) $\frac{11}{9}$ sq units
(b) $\frac{32}{3}\sqrt{6}$ sq units
(c) $\frac{16}{3}$ sq units
(d) $\frac{24}{5}$ sq units

21. A particle moving in a straight line starts from rest and the acceleration at any time t is $a - kt^2$, where a and k are positive constants. The maximum velocity attained by the particle is

(a) $\frac{2}{3}\sqrt{\frac{a^3}{k}}$
(b) $\frac{1}{3}\sqrt{\frac{a^3}{k}}$
(c) $\sqrt{\frac{a^3}{k}}$
(d) $2\sqrt{\frac{a^3}{k}}$

22. If a, b, c are in GP and $\log a - \log 2b$, $\log 2b - \log 3c$, $\log 3c - \log a$ are in AP, then a, b, c are the lengths of the sides of a triangle which is

(a) acute angled
(b) obtuse angled
(c) right angled
(d) equilateral

23. Let $a_n = (1^2 + 2^2 + \ldots .\, n^2)^n$ and $b_n = n^n(n!)$. Then,

(a) $a_n < b_n, \forall n$
(b) $a_n > b_n, \forall n$
(c) $a_n = b_n$ for infinitely many n
(d) $a_n < b_n$ if n is even and $a_n > b_n$ if n is odd

24. The number of zeroes at the end of $100!$ is

(a) 21 (b) 22

(c) 23 (d) 24

25. If $|z-25i|\le 15$, then Maximum $\arg(z)$ – Minimum $\arg(z)$ is equal to

(a) $2\cos^{-1}\left(\frac{3}{5}\right)$

(b) $2\cos^{-1}\left(\frac{4}{5}\right)$

(c) $\frac{\pi}{2}+\cos^{-1}\left(\frac{3}{5}\right)$

(d) $\sin^{-1}\left(\frac{3}{5}\right)-\cos^{-1}\left(\frac{3}{5}\right)$

(arg z is the principal value of argument of z).

26. If $z=x-iy$ and $z^{1/3}=p+iq\ (x,y,p,q\in R)$, then $\dfrac{\left(\frac{x}{p}+\frac{y}{q}\right)}{(p^2+q^2)}$ is equal to

(a) 2 (b) –1

(c) 1 (d) –2

27. If a,b are odd integers, then the roots of the equation $2ax^2+(2a+b)x+b=0,\ a\neq 0$ are

(a) rational (b) irrational

(c) non-real (d) equal

28. There are n white and n black balls marked 1, 2, 3, ... n. The number of ways in which we can arrange these balls in a row so that neighbouring balls are of different colours is

(a) $(n!)^2$ (b) $(2n)!$

(c) $2(n!)^2$ (d) $\dfrac{(2n)!}{(n!)^2}$

29. Let $f(n)=2^{n+1}$, $g(n)=1+(n+1)2^n$ for all $n\in N$. Then,

(a) $f(n)>g(n)$

(b) $f(n)<g(n)$

(c) $f(n)$ and $g(n)$ are not comparable

(d) $f(n)>g(n)$ if n is even and $f(n)<g(n)$ if n is odd

30. A is a set containing n elements. P and Q are two subsets of A. Then, the number of ways of choosing P and Q so that $P\cap Q=\phi$ is

(a) $2^{2n-2n}C_n$ (b) 2^n

(c) 3^n-1 (d) 3^n

31. Under which of the following condition(s) does(do) the system of equations $\begin{bmatrix}1&2&4\\2&1&2\\1&2&(a-4)\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\4\\a\end{bmatrix}$ possesses (posses) unique solution?

(a) $\forall a\in R$

(b) $a=8$

(c) for all integral values of a

(d) $a\neq 8$

32. If $\Delta(x)=\begin{vmatrix}x-2&(x-1)^2&x^3\\x-1&x^2&(x+1)^3\\x&(x+1)^2&(x+2)^3\end{vmatrix}$, then coefficient of x in $\Delta(x)$ is

(a) 2 (b) – 2

(c) 3 (d) – 4

33. If $p=\begin{bmatrix}1&\alpha&3\\1&3&3\\2&4&4\end{bmatrix}$ is the adjoint of the 3×3 matrix A and $\det A=4$, then α is equal to

(a) 4 (b) 11

(c) 5 (d) 0

34. If $A=\begin{bmatrix}1&1\\0&i\end{bmatrix}$ and $A^{2018}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$, then $(a+d)$ equals

(a) $1+i$ (b) 0 (c) 2 (d) 2018

35. Let S,T,U be three non-void sets and $f:S\to T$, $g:T\to U$ and composed mapping $g\cdot f:S\to U$ be defined. Let $g\cdot f$ be injective mapping. Then,

(a) f, g both are injective

(b) neither f nor g is injective

(c) f is obviously injective

(d) g is obviously injective

36. For the mapping $f:R-\{1\}\to R-\{2\}$, given by $f(x)=\dfrac{2x}{x-1}$, which of the following is correct?

(a) f is one-one but not onto

(b) f is onto but not one-one

(c) f is neither one-one nor onto

(d) f is both one-one and onto

37. A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. Then, the set of possible values of x are in

(a) $[0, 1]$
(b) $\left[\frac{1}{3}, \frac{1}{2}\right]$
(c) $\left[\frac{1}{3}, \frac{2}{3}\right]$
(d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

38. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen is non-zero is

(a) $\frac{3}{16}$
(b) $\frac{3}{8}$
(c) $\frac{1}{4}$
(d) $\frac{5}{8}$

39. If $(\cot\alpha_1)(\cot\alpha_2) \dots (\cot\alpha_n) = 1$, $0 < \alpha_1, \alpha_2, \dots \alpha_n < \pi/2$, then the maximum value of $(\cos\alpha_1)(\cos\alpha_2) \dots (\cos\alpha_n)$ is given by

(a) $\frac{1}{2^{n/2}}$
(b) $\frac{1}{2^n}$
(c) $\frac{1}{2n}$
(d) 1

40. If the algebraic sum of the distances from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero, then the line passes through fixed point.

(a) $(-1, 1)$
(b) $(1, -1)$
(c) $(-1, -1)$
(d) $(1, 1)$

41. The side AB of ΔABC is fixed and is of length $2a$ units. The vertex moves in the plane such that the vertical angle is always constant and is α. Let X-axis be along AB and the origin be at A. Then, the locus of the vertex is

(a) $x^2 + y^2 + 2ax\sin\alpha + a^2\cos\alpha = 0$
(b) $x^2 + y^2 - 2ax - 2ay\cot\alpha = 0$
(c) $x^2 + y^2 - 2ax\cos\alpha - a^2 = 0$
(d) $x^2 + y^2 - ax\sin\alpha - ay\cos\alpha = 0$

42. If the sum of the distances of a point from two perpendicular lines in a plane is 1 unit, then its locus is

(a) a square
(b) a circle
(c) a straight line
(d) two intersecting lines

43. A line passes through the point $(-1, 1)$ and makes an angle $\sin^{-1}\left(\frac{3}{5}\right)$ in the positive direction of X-axis. If this line meets the curve $x^2 = 4y - 9$ at A and B, then $|AB|$ is equal to

(a) $\frac{4}{5}$ unit
(b) $\frac{5}{4}$ units
(c) $\frac{3}{5}$ unit
(d) $\frac{5}{3}$ units

44. Two circles $S_1 = px^2 + py^2 + 2g'x + 2f'y + d = 0$ and $S_2 = x^2 + y^2 + 2gx + 2fy + d' = 0$ have a common chord PQ. The equation of PQ is

(a) $S_1 - S_2 = 0$
(b) $S_1 + S_2 = 0$
(c) $S_1 - pS_2 = 0$
(d) $S_1 + pS_2 = 0$

45. Let $P(3\sec\theta, 2\tan\theta)$ and $Q(3\sec\phi, 2\tan\phi)$ be two points on $\frac{x^2}{9} - \frac{y^2}{4} = 1$ such that $\theta + \phi = \frac{\pi}{2}, 0 < \theta, \phi < \frac{\pi}{2}$. Then, the ordinate of the point of intersection of the normals at P and Q is

(a) $\frac{13}{2}$
(b) $-\frac{13}{2}$
(c) $\frac{5}{2}$
(d) $-\frac{5}{2}$

46. Let P be a point on (2, 0) and Q be a variable point on $(y-6)^2 = 2(x-4)$. Then, the locus of mid-point of PQ is

(a) $y^2 + x + 6y + 12 = 0$
(b) $y^2 - x + 6y + 12 = 0$
(c) $y^2 + x - 6y + 12 = 0$
(d) $y^2 - x - 6y + 12 = 0$

47. AB is a chord of a parabola $y^2 = 4ax$, $(a > 0)$ with vertex A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

(a) a unit
(b) $2a$ units
(c) $8a$ units
(d) $4a$ units

48. AB is a variable chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If AB subtends a right angle at the origin O, then $\frac{1}{OA^2} + \frac{1}{OB^2}$ equals

(a) $\frac{1}{a^2} + \frac{1}{b^2}$
(b) $\frac{1}{a^2} - \frac{1}{b^2}$
(c) $a^2 + b^2$
(d) $a^2 - b^2$

49. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to the X-axis is

(a) $y + 3z + 6 = 0$
(b) $y + 3z - 6 = 0$
(c) $y - 3z + 6 = 0$
(d) $y - 3z - 6 = 0$

50. The line $x - 2y + 4z + 4 = 0$, $x + y + z - 8 = 0$ intersect the plane $x - y + 2z + 1 = 0$ at the point

(a) $(-2, 5, 1)$
(b) $(2, -5, 1)$
(c) $(2, 5, -1)$
(d) $(2, 5, 1)$

Category-II (Q.Nos. 51 to 65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 1/2 mark will be deducted.

51. If I is the greatest of

$$I_1 = \int_0^1 e^{-x} \cos^2 x \, dx, I_2 = \int_0^1 e^{-x^2} \cos^2 x \, dx,$$

$$I_3 = \int_0^1 e^{-x^2} dx, I_4 = \int_0^1 e^{-x^2/2} dx, \text{ then}$$

(a) $I = I_1$
(b) $I = I_2$
(c) $I = I_3$
(d) $I = I_4$

52. $\lim_{x \to \infty} \left(\dfrac{x^2 + 1}{x + 1} - ax - b \right), (a, b \in R) = 0$. Then,

(a) $a = 0, b = 1$
(b) $a = 1, b = -1$
(c) $a = -1, b = 1$
(d) $a = 0, b = 0$

53. If the transformation $z = \log \tan \dfrac{x}{2}$ reduces the differential equation $\dfrac{d^2y}{dx^2} + \cot x \dfrac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$ into the form $\dfrac{d^2y}{dz^2} + ky = 0$, then k is equal to

(a) -4
(b) 4
(c) 2
(d) -2

54. From the point $(-1, -6)$, two tangents are drawn to $y^2 = 4x$. Then, the angle between the two tangents is

(a) $\pi/3$
(b) $\pi/4$
(c) $\pi/6$
(d) $\pi/2$

55. If α is a unit vector, $\beta = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\gamma = \hat{\mathbf{i}} + \hat{\mathbf{k}}$, then the maximum value of $[\alpha \; \beta \; \gamma]$ is

(a) 3
(b) $\sqrt{3}$
(c) 2
(d) $\sqrt{6}$

56. The maximum value of $f(x) = e^{\sin x} + e^{\cos x}$; $x \in R$ is

(a) $2e$
(b) $2\sqrt{e}$
(c) $2e^{1/\sqrt{2}}$
(d) $2e^{-1/\sqrt{2}}$

57. A straight line meets the coordinate axes at A and B. A circle is circumscribed about the ΔOAB, O being the origin. If m and n are the distances of the tangent to the circle at the origin from the points A and B respectively, the diameter of the circle is

(a) $m(m + n)$
(b) $m + n$
(c) $n(m + n)$
(d) $\dfrac{1}{2}(m + n)$

58. Let the tangent and normal at any point $P(at^2, 2at)$, $(a > 0)$, on the parabola $y^2 = 4ax$ meet the axis of the parabola at T and G respectively. Then, the radius of the circle through P, T and G is

(a) $a(1 + t^2)$
(b) $(1 + t^2)$
(c) $a(1 - t^2)$
(d) $(1 - t^2)$

59. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assumes the least value is

(a) 0
(b) 1
(c) 2
(d) 3

60. If x satisfies the inequality $\log_{25} x^2 + (\log_5 x)^2 < 2$, then x belongs to

(a) $\left(\dfrac{1}{5}, 5\right)$
(b) $\left(\dfrac{1}{25}, 5\right)$
(c) $\left(\dfrac{1}{5}, 25\right)$
(d) $\left(\dfrac{1}{25}, 25\right)$

61. The solution of $\det(A - \lambda I_2) = 0$ be 4 and 8 and $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$. Then,

(a) $x = 4, y = 10$
(b) $x = 5, y = 8$
(c) $x = 3, y = 9$
(d) $x = -4, y = 10$

(I_2 is identity matrix of order 2)

62. If P_1P_2 and P_3P_4 are two focal chords of the parabola $y^2 = 4ax$ then the chords P_1P_3 and P_2P_4 intersect on the

(a) directrix of the parabola
(b) axis of the parabola
(c) latusrectum of the parabola
(d) Y-axis

63. $f : X \to R,\ X = \{x \mid 0 < x < 1\}$ is defined as $f(x) = \dfrac{2x-1}{1-|2x-1|}$. Then,

(a) f is only injective
(b) f is only surjective
(c) f is bijective
(d) f is neither injective nor surjective

64. Let f be a non-negative function defined in $[0, \pi/2]$, f' exists and be continuous for all x and $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt$ and $f(0) = 0$.

Then,

(a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
(b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
(c) $f\left(\frac{4}{3}\right) < \frac{4}{3}$ and $f\left(\frac{2}{3}\right) < \frac{2}{3}$
(d) $f\left(\frac{4}{3}\right) > \frac{4}{3}$ and $f\left(\frac{2}{3}\right) > \frac{2}{3}$

65. PQ is a double ordinate of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ such that ΔOPQ is an equilateral triangle, O being the centre of the hyperbola. Then, the eccentricity e of the hyperbola satisfies.

(a) $1 < e < \dfrac{2}{\sqrt{3}}$
(b) $e = \dfrac{2}{\sqrt{3}}$
(c) $e = 2\sqrt{3}$
(d) $e > \dfrac{2}{\sqrt{3}}$

Category-III (Q. Nos. 66 to 75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and also no incorrect answer is marked, then score $= 2 \times$ number of correct answers marked $\div$ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero marks will be awarded.

66. From a balloon rising vertically with uniform velocity v ft/sec a piece of stone is let go. The height of the balloon above the ground when the stone reaches the ground after 4 sec is $[g = 32 \text{ ft/sec}^2]$

(a) 220 ft
(b) 240 ft
(c) 256 ft
(d) 260 ft

67. Let $f(x) = x^2 + x\sin x - \cos x$. Then,

(a) $f(x) = 0$ has at least one real root
(b) $f(x) = 0$ has no real root
(c) $f(x) = 0$ has at least one positive root
(d) $f(x) = 0$ has at least one negative root

68. Let z_1 and z_2 be two non-zero complex numbers. Then,

(a) Principal value of $\arg(z_1z_2)$ may not be equal to Principal value of $\arg z_1$ + Principal value of $\arg z_2$
(b) Principal value of $\arg(z_1z_2)$ = Principal value of $\arg z_1$ + Principal value of $\arg z_2$
(c) Principal value of $\arg(z_1/z_2)$ = Principal value of $\arg z_1$ − Principal value of $\arg z_2$
(d) Principal value of $\arg(z_1/z_2)$ may not be $\arg z_1 - \arg z_2$

69. Let $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$. Then,

(a) Δ is independent of θ
(b) Δ is independent of ϕ
(c) Δ is a constant
(d) $\left(\dfrac{d\Delta}{d\theta}\right)_{\theta=\frac{\pi}{2}} = 0$

70. Let R and S be two equivalence relations on a non-void set A. Then,

(a) $R \cup S$ is equivalence relation
(b) $R \cap S$ is equivalence relation
(c) $R \cap S$ is not equivalence relation
(d) $R \cup S$ is not equivalence relation

71. Chords of an ellipse are drawn through the positive end of the minor axis. Their mid-point lies on

(a) a circle
(b) a parabola
(c) an ellipse
(d) a hyperbola

72. Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then

(a) the lines will pass through a fixed point
(b) there will be a set of parallel lines
(c) all lines intersect the line $x = x_1$
(d) all lines will be parallel to the line $y = x_1$

73. Let $p(x)$ be a polynomial with real coefficients, $p(0) = 1$ and $p'(x) > 0$ for all $x \in R$. Then,

(a) $p(x)$ has at least two real roots
(b) $p(x)$ has only one positive real root
(c) $p(x)$ may have negative real root
(d) $p(x)$ has infinitely many real roots

74. Twenty metres of wire is available to fence off a flower bed in the form of a circular sector. What must the radius of the circle be, if the area of the flower bed be greatest?

(a) 10 m
(b) 4 m
(c) 5 m
(d) 6 m

75. The line $y = x + 5$ touches

(a) the parabola $y^2 = 20x$
(b) the ellipse $9x^2 + 16y^2 = 144$
(c) the hyperbola $\frac{x^2}{29} - \frac{y^2}{4} = 1$
(d) the circle $x^2 + y^2 = 25$

Answers

Physics

1. (c)	2. (b)	3. (a)	4. (a)	5. (b)	6. (b)	7. (a)	8. (a)	9. (c)	10. (a)
11. (c)	12. (d)	13. (b)	14. (a)	15. (c)	16. (c)	17. (d)	18. (d)	19. (b)	20. (c)
21. (d)	22. (d)	23. (d)	24. (c)	25. (a)	26. (b)	27. (b)	28. (b)	29. (a)	30. (c)
31. (b)	32. (b)	33. (c)	34. (d)	35. (b)	36. (a,c,d)	37. (a,b)	38. (a,b)	39. (a,d)	40. (a,c)

Chemistry

41. (b)	42. (d)	43. (a)	44. (c)	45. (*)	46. (b)	47. (c)	48. (d)	49. (a)	50. (d)
51. (c)	52. (b)	53. (a)	54. (c)	55. (d)	56. (d)	57. (b)	58. (c)	59. (c)	60. (a)
61. (c)	62. (a)	63. (a)	64. (c)	65. (b)	66. (b)	67. (b)	68. (a)	69. (d)	70. (b)
71. (b)	72. (b)	73. (a)	74. (c)	75. (a)	76. (b,c)	77. (a,c,d)	78. (a,b)	79. (a,b,c)	80. (b,c)

Mathematics

1. (d)	2. (c)	3. (c)	4. (a)	5. (c)	6. (a)	7. (a)	8. (a)	9. (b)	10. (a)
11. (b)	12. (b)	13. (a)	14. (c)	15. (d)	16. (a)	17. (c)	18. (b)	19. (c)	20. (b)
21. (a)	22. (b)	23. (b)	24. (d)	25. (b)	26. (d)	27. (a)	28. (c)	29. (b)	30. (d)
31. (d)	32. (b)	33. (b)	34. (b)	35. (d)	36. (d)	37. (b)	38. (b)	39. (a)	40. (d)
41. (b)	42. (c)	43. (b)	44. (c)	45. (b)	46. (d)	47. (d)	48. (a)	49. (c)	50. (d)
51. (d)	52. (b)	53. (b)	54. (d)	55. (d)	56. (c)	57. (b)	58. (a)	59. (b)	60. (b)
61. (d)	62. (a)	63. (c)	64. (c)	65. (d)	66. (c)	67. (a,c,d)	68. (a,d)	69. (b,d)	70. (b)
71. (c)	72. (b,c)	73. (c)	74. (c)	75. (a,b,c)					

Note (*) *None of the option is correct.*

Answer *with* Explanations

Physics

1. (c) Electric field due to line charge is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 d}$$

where, λ = linear charge density.

$$\lambda = \frac{q}{l} \Rightarrow q = \lambda l$$

Electric force due to one wire on another wire,

$$F_E = q \cdot E$$

$$= \lambda l \cdot \frac{\lambda}{2\pi\varepsilon_0 d}$$

$$\Rightarrow \quad F_E = \frac{\lambda^2 l}{2\pi\varepsilon_0 d}$$

Now, current due to moving charged wire is given as

$$I = \frac{dq}{dt} = \frac{d}{dt}(\lambda dl) \qquad \left[\because v = \frac{dl}{dt}\right]$$

$$= \lambda v$$

Magnetic force on one wire due to another wire,

$$F_B = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Since, both forces balance each other, thus

$$F_E = F_B$$

$$\Rightarrow \quad \frac{\lambda^2 l}{2\pi\varepsilon_0 d} = \frac{\mu_0(\lambda v)(\lambda v)l}{2\pi d} \qquad [\because I_1 = I_2 = I]$$

$$\Rightarrow \quad \frac{1}{\varepsilon_0 \mu_0} = v^2$$

$$\text{or} \quad v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

2. (b) We know that, $E = -\frac{dV}{dr}$

Thus, electric field from (− 4 to − 2),

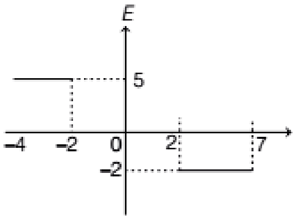
$$E = -\left(\frac{10 - 0}{4 - (-2)}\right) = \frac{10}{2} = 5\text{N/C}$$

Electric field from (− 2 to 2),

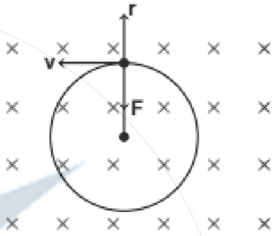
$$E = -\left(\frac{10 - 10}{2 - (-2)}\right) = 0$$

Electric field from (2 to 7),

$$E = -\left(\frac{10 - 0}{2 - 7}\right) = -2\text{N/C}$$



3. (a) According to the question, given situation can be represented by the figure below.



From the Lorentz force, we have

$$F = q(v \times B)$$

Hence, force is acting directly towards the centre.

Thus, both angular momentum (**L**) and torque (**T**) will have the same direction. i.e. **T** || **L**.

4. (a) Given, $\mathbf{B} = B_0\left(2 - \frac{x}{a}\right)\hat{\mathbf{k}}$

Force on a current carrying wire is given as

$$dF = I B \cdot dl$$

$$\Rightarrow \quad \int dF = \int IB_0\left(2 - \frac{x}{a}\right) dx$$

$$\Rightarrow \quad F = IB_0 \int_a^{2a} \left(2 - \frac{x}{a}\right) dx$$

$$= IB_0 \left|2x - \frac{x^2}{2a}\right|_a^{2a} = IB_0\left(\frac{a}{2}\right)$$

Therefore, the value of k is 1.

5. (b) Given, change in flux, $d\phi = 5\,\text{Wb}$

Emf due to change in flux is given as

$$E = \frac{d\phi}{dt} \quad \Rightarrow \quad E \cdot dt = 5$$

Now, it is given that, $R = 100\ \Omega$

Thus, current, $I = \frac{V}{R} = \frac{E}{R}$ ... (i)

Also, $I = \frac{dq}{dt}$ ... (ii)

where, dq is the charge flowing per unit time.

Thus, from Eqs. (i) and (ii), we have

$$\frac{E}{R} = \frac{dq}{dt} \Rightarrow \frac{E \cdot dt}{R} = dq$$

$$\Rightarrow \quad \frac{5}{100} = dq \Rightarrow dq = 0.05\,\text{C}$$

6. (b) At any moment, the phase difference between current and voltage is given by

$$\tan\phi = \frac{X_L}{R}$$

Since, voltage leads current by $\frac{\pi}{4}$, thus given circuit will be RL series circuit

$$\therefore \quad \tan\frac{\pi}{4} = \frac{X_L}{R} \Rightarrow R = X_L = \omega L$$

From option (b), $R = 1\,\text{k}\Omega = 1000\,\Omega$

$$\therefore \quad X_L = \omega L$$
$$= 100 \times 10 \qquad [\because \omega = 100 \text{ rad/s}]$$
$$= 1000\,\Omega$$

Thus, the condition is satisfied. Hence option (b) is correct.

7. (a) The constant voltage source provides a constant voltage to the load resistance regardless of variations or changes, in the load resistance. For this to happen, the source must have an internal resistance which is very low as compared to the resistance of the load.

i.e., $r << R$

8. (a) de-Broglie wavelength of a charged particle having energy ε is given by

$$\lambda = \frac{h}{\sqrt{2m\varepsilon}}$$

Thus, $\lambda_e = \frac{h}{\sqrt{2m_e\varepsilon_1}}$

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha\varepsilon_2}}$$

$$\lambda_p = \frac{h}{\sqrt{2m_p\varepsilon_3}}$$

Since, $\lambda_e = \lambda_\alpha = \lambda_p$ and $m_\alpha > m_p > m_e$

Thus, we can say

$$\frac{1}{\sqrt{m_\alpha\varepsilon_2}} = \frac{1}{\sqrt{m_p\varepsilon_3}} = \frac{1}{\sqrt{m_e\varepsilon_1}}$$

$$\therefore \quad \varepsilon_1 > \varepsilon_3 > \varepsilon_2$$

9. (c) Intensity at any point on the screen is given by

$$I = 4I_0\cos^2\frac{\phi}{2}$$

where, I_0 is the intensity of either wave and ϕ is the phase difference between two waves.

Phase difference $(\phi) = \frac{2\pi}{\lambda} \times$ path difference

$$= \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

$$\therefore \quad I = 4I_0\cos^2\left(\frac{2\pi}{2}\right)$$

$$= 4I_0\cos^2(\pi) = 4I_0 = I$$

$$\Rightarrow \quad I_0 = \frac{I}{4} \qquad \ldots\text{(i)}$$

When path difference is $\frac{\lambda}{4}$, then

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\therefore \quad I = 4I_0\cos^2\left(\frac{\pi}{4}\right) = 2I_0 = 2 \times \frac{I}{4} = \frac{I}{2}$$

10. (a) Given, $\frac{I_{\max}}{I_{\min}} = \frac{4}{1}$

Let I_1 and I_2 be the intensities of interfering waves,

then $\frac{(\sqrt{I_2} + \sqrt{I_1})^2}{(\sqrt{I_2} - \sqrt{I_1})^2} = \frac{I_{\max}}{I_{\min}}$

$$\Rightarrow \quad \left(\frac{\sqrt{\frac{I_2}{I_1}} + 1}{\sqrt{\frac{I_2}{I_1}} - 1}\right)^2 = \frac{4}{1}$$

$$\Rightarrow \quad \frac{\sqrt{\frac{I_2}{I_1}} + 1}{\sqrt{\frac{I_2}{I_1}} - 1} = \frac{2}{1} \Rightarrow \sqrt{\frac{I_2}{I_1}} = \frac{3}{1}$$

or $\frac{I_1}{I_2} = \frac{1}{9}$

11. (c) Given, angular resolution of human eye,

$$\theta = 5.8 \times 10^{-4} \text{ rad}$$

The linear distance between two successive dots in a printer,

$$l = \frac{2.54}{300} = 0.846 \times 10^{-2} \text{ cm}$$

At a distance d cm, the gap l will subtend an angle which is given by

$$d = \frac{l}{\theta} = \frac{0.846 \times 10^{-2}}{5.8 \times 10^{-4}} = 14.59 \text{ cm}$$

12. (d) The angular momentum is quantised to even multiple of $\frac{h}{2\pi}$. Hence, the quantum numbers that are allowed are $n = 2$ and $n = 4$ ……. . So,

$$E = 136\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= 136\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= 136\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$= 255 \text{ eV}$$

Now, given that, $hc = 1242$ MeV-nm

Hence, $\lambda = \frac{hc}{255 \text{ eV}} = \frac{1242 \text{ eV - nm}}{2.55 \text{ eV}}$

$$= 487 \text{ nm}$$

13. (b) Given, $V_s = 10$ V

$$V_z = 6 \text{ V}$$

Maximum current $(I_{z\max}) = 40$ mA

$$= 40 \times 10^{-3} \text{A}$$

Minimum current $(I_{z\min}) = 10 \text{ mA} = 10 \times 10^{-3}$ A

Maximum value of series resistance R_S in the voltage regulator circuit is given as

$$R_{\max} = \frac{V_s - V_z}{I_{z\min}}$$

$$= \frac{10 - 6}{10 \times 10^{-3}}$$

$$= 400 \ \Omega$$

14. (a) Given expression, $\overline{A}(A + B) + (B + AA)(A + \overline{B})$

$= \overline{A}(A + B) + (B + A)(A + \overline{B})$

$= A\overline{A} + \overline{A}B + AB + B\overline{B} + AA + A\overline{B}$

$= A + A\overline{B} + B(\overline{A} + A)$ $[\because A\overline{A} = B\overline{B} = 0 \text{ and } A.A = A]$

$= A + A\overline{B} + B(\overline{A} + A)$

$= A(1 + \overline{B}) + B(1)$ $[\because A + \overline{A} = 1]$

$= A + B$ $[\because 1 + \overline{B} = 1]$

15. (c) The percentage error in Z is given as

$$\frac{\Delta Z}{Z}\% = 4\frac{\Delta A}{A} + \frac{1}{3}\frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{3}{2}\frac{\Delta D}{D}$$

$$= 4 \times 4 + \frac{1}{3} \times 2 + 3 + \frac{3}{2} \times 1$$

$$= 16 + \frac{2}{3} + 3 + \frac{3}{2}$$

$$= \frac{127}{6}\%$$

16. (c) Given, $S = \beta k_B A$

where, dimensional formula of $S = [\text{M}^1\text{L}^2\text{T}^{-2}\text{K}^{-1}]$

Dimensional formula of $k_B = [\text{M}^1\text{L}^2\text{T}^{-2}\text{K}^{-1}]$

Dimensional formula of $A = [\text{L}^2]$

Thus, $[\text{M}^1\text{L}^2\text{T}^{-2}\text{K}^{-1}] = \beta\,[\text{M}^1\text{L}^2\text{T}^{-2}\text{K}^{-1}][\text{L}^2]$

$\Rightarrow \quad \beta = [\text{L}^{-2}]$

17. (d) According to work-energy theorem, change in kinetic energy = work done

= force × distance

Thus, force $= \frac{\text{change in energy}}{\text{change in distance}}$

At $x = 10$ m, we have

$$\mathbf{F} = \frac{20 - 40}{12 - 8}\hat{\mathbf{i}} = -5\hat{\mathbf{i}} \text{ N}$$

18. (d) At position A, direction of $\mathbf{p}$ is along + ve Y-axis.

At position A, direction of $\mathbf{L}$ is along + ve Z-axis because $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and $\mathbf{r}$ is along + ve X-axis.

Thus, $\alpha = \mathbf{p} \times \mathbf{L}$

$= \hat{\mathbf{i}} \times \hat{\mathbf{k}}$

$= -\hat{\mathbf{j}}$ or – ve Y-axis

19. (b) According to the question,

$$A = \sqrt{A^2 + A^2 + 2AA\cos\delta}$$

$\Rightarrow \quad \cos\delta = -\frac{1}{2}$

or $\quad \delta = 120°$

or $\quad = \frac{2\pi}{3}$

20. (c) Maximum height of projectile,

$$h = \frac{u^2\sin^2\theta}{2g} \quad \text{... (i)}$$

At maximum height velocity of projectile,

$$v = u\cos\theta$$

∴ Momentum at highest point = $mu\cos\theta$

∴ Angular momentum about origin,

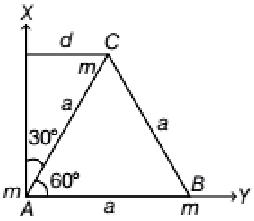
$$p = mu\cos\theta \times h$$

$$= mu\cos\theta \times \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{mu^3\cos\theta\sin^2\theta}{2g}$$

$\Rightarrow \quad p \propto u^3$

21. (d) Moment of inertia about line $AX = ma^2 + md^2$



We know that,

$$\sin\theta = \frac{d}{a}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{d}{a} \quad (\because \theta = 30°)$$

$$\Rightarrow \quad d = \frac{a}{2}$$

$$\therefore \quad I_{Ax} = ma^2 + m\left(\frac{a}{2}\right)^2$$

$$= ma^2 + \frac{ma^2}{4} = \frac{5}{4}ma^2$$

22. (d) Given, initial speed, $v = \sqrt{3}\, v_e$

We know that, $v_e = \sqrt{\frac{2GM}{R}}$

At the surface of earth,

Total energy = PE + KE

$$= -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{R} + \frac{1}{2}m\left(\sqrt{3}\sqrt{\frac{2GM}{R}}\right)^2$$

$$= -\frac{GMm}{R} + \frac{1}{2}3m \times \frac{2GM}{R}$$

$$= -\frac{GMm}{R} + \frac{3GMm}{R} = \frac{2GMm}{R}$$

At infinity, total energy = PE + KE

$$= 0 + \frac{1}{2}mv^2$$

From the principle of conservation of energy,

$$\frac{2GMm}{R} = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{4GM}{R} = v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{4GM}{R}}$$

$$= \sqrt{2 \times \frac{2GM}{R}} = \sqrt{2}\, v_e$$

23. (d) According to the Hooke's law, for deformation of string of length l,

$$\frac{F_1}{A} \propto \frac{\Delta l_1}{l} \qquad \text{... (i)}$$

and in second case,

$$\frac{F_2}{A} \propto \frac{\Delta l_2}{l} \qquad \text{... (ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{F_1}{F_2} = \frac{\Delta l_1}{\Delta l_2} = \frac{l_1 - l}{l_2 - l}$$

$$\Rightarrow \quad (F_2 - F_1)l = F_2 l_1 - F_1 l_2$$

$$\Rightarrow \quad l = \frac{F_2 l_1 - F_1 l_2}{F_2 - F_1}$$

24. (c) Let r be the radius of small mercury drop.

Thus, total volume of 27 drops $= 27 \times \frac{4}{3}\pi r^3$

Total volume of bigger drop $= \frac{4}{3}\pi R^3$

$\therefore$ According to given situation,

$$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R = 3r$$

Now, surface energy = surface tension × area

$$= T \times A$$

Surface energy of 27 drops $= T \times 27 \times (4\pi r^2)$

$$= 27 \times 4\pi r^2 T$$

Surface energy of bigger drop $= T \times 4\pi R^2$

$$= T \times 4\pi(3r)^2$$

$$= 9 \times 4\pi r^2 T$$

Increase in surface energy $= 9 \times 4\pi r^2 T - 27 \times 4\pi r^2 T$

$$= -18 \times 4\pi r^2 T$$

$\therefore$ Relative increase in surface energy

$$= \frac{-18 \times 4\pi r^2 T}{27 \times 4\pi r^2 T} = -\frac{2}{3}$$

25. (a) For the given isotherm, the temperature of the curve EF is greater than the curve CD and AB.

As we know that, most probable speed of the molecule is given as

$$v_p = \sqrt{\frac{2RT}{M}}$$

Thus, the v_p at 3 and 4 will be same.

Hence, v_p at $3 = v_p$ at $4 > v_p$ at $2 > v_p$ at 1.

26. (b) $U = Ap^2V$, A = constant

For adiabatic process, $dQ = 0$

∴ From first law of thermodynamics,

$$dQ = dU + dW$$

$$\therefore \quad dU + dW = 0 \qquad (\because dQ = 0)$$

$$\Rightarrow \quad d(Ap^2V) + pdV = 0$$

$$\Rightarrow \quad A(2p \cdot Vdp + p^2dV) + pdV = 0$$

$$\Rightarrow \quad (Ap^2 + p)dV + 2ApVdp = 0$$

$$\Rightarrow \quad (Ap^2 + p)dV = -2ApVdp$$

$$\Rightarrow \quad \frac{dV}{-2AV} = \frac{pdp}{p(Ap+1)}$$

$$\Rightarrow \quad \frac{dp}{(Ap+1)} + \frac{1}{2A} \cdot \frac{dv}{v} = 0$$

Integrating on both sides,

$$\Rightarrow \quad \int \frac{dp}{(Ap+1)} + \frac{1}{2A}\int \frac{dV}{V} = \int 0$$

$$\Rightarrow \quad \frac{\ln(Ap+1)}{A} + \frac{1}{2A}\ln V = \ln C$$

$$\Rightarrow \quad \ln(Ap+1) + \ln V^{1/2} = \ln C$$

$$\Rightarrow \quad \ln[V^{1/2}(Ap+1)] = \ln C$$

$$\Rightarrow \quad V^{1/2}(Ap+1) = C$$

$$\Rightarrow \quad V(Ap+1)^2 = C \text{ (constant)}$$

27. (b) The given process BC is isothermal and AB is isochoric.

From A to B, we have

$$p_0 \longrightarrow 2p_0$$

as $\qquad p \propto T$

Hence, $T_B = 2T_A = 2T_0$

$$Q_{AB} = \Delta U_{AB} + W_{AB} = C_V \Delta T + 0$$

$$\Rightarrow \quad Q_{AB} = \frac{5}{2}R(2T_0 - T_0) = \frac{5}{2}p_0V_0$$

$(\because C_V = \frac{5}{2}R$ for diatomic gas and $pV = RT)$

From B to C, we have

$$Q_{BC} = \Delta U_{BC} + W_{BC} = 0 + R(2T_0)\ln\frac{2V_0}{V_0}$$

$$= 2RT_0 \ln 2$$

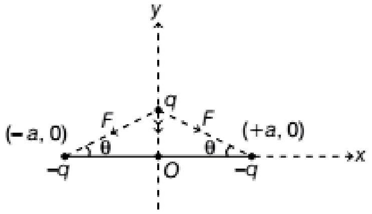
$$= 2\ln 2 p_0V_0$$

Hence, $Q_{total} = Q_{AB} + Q_{BC}$

$$= (2.5 + 2\ln 2)p_0V_0$$

$$= 3.9\, p_0V_0$$

28. (b) Consider the diagram given below.



When the charge q is displaced momentarily, it starts oscillating. The net force acting on q is given by $F_{net} = 2F\sin\theta$

$$= -\frac{2kq \times q}{(\sqrt{y^2+a^2})^2} \cdot \frac{y}{\sqrt{y^2+a^2}} = -\frac{2kq^2y}{(\sqrt{y^2+a^2})^{\frac{3}{2}}}$$

For $a >> y$, we have

$$F_{net} = -\frac{2kq^2y}{a^3} \text{ or } F_{net} \propto -y$$

29. (a) We can find the force between q_a and q_b by calculating electric field first.

From Gauss's law,

$$\oint E \cdot ds = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E \cdot 4\pi c^2 = \frac{q_a}{\varepsilon_0}$$

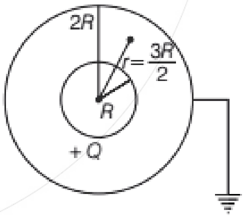
(∵ For cavity a electric field is to be calculated upto q_b)

$$\Rightarrow \quad E_a = \frac{q_a}{4\pi\varepsilon_0 c^2}$$

Now, force on q_b due to q_a is given by

$$F = E_a \cdot q_b = \frac{q_a q_b}{4\pi\varepsilon_0 c^2}$$

30. (c) Consider the figure given below.



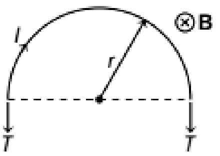
Due to grounding, charge on outer sphere= 0

Now, potential due to inner sphere,

$$V = \frac{kQ}{r} = \frac{kQ}{\frac{3R}{2}}$$

$$= \frac{2kQ}{3R} \text{ or } \frac{1}{4\pi\varepsilon_0}\frac{2Q}{3R}$$

31. (b) Considering the semi-circular wire of radius r. Force acting on it is shown below



Let T be the force exerted by each spring.

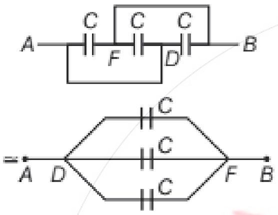
$\therefore \quad T + T = IBl$

$2T = IBl$

$2T = IB \cdot \pi r \quad (\because l = \pi r)$

$\therefore \quad T = \frac{IB\pi r}{2}$

32. (b)



Here, all the three capacitors are connected in parallel. So, equivalent capacitance will be

$C_{eq} = C + C + C = 3C$

33. (c) Given, initial velocity, $u = 0$ m/s

Final velocity, $v = 100$ m/s

Mass of ball, $m = 50 \times 10^{-3}$ kg

Time of contact = 0.02 s

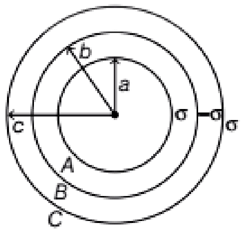
Impulse, $F = \frac{m(v-u)}{t}$

$= \frac{50 \times 10^{-3}(100-0)}{0.02}$

$= \frac{5 \times 100}{2}$

$= 250$ N

34. (d) Net potential at any point $(r = b)$ of shell B



= potential due to shell A + potential due to shell B + potential due to shell C.

$$V = V_{A,\text{ out}} + V_{B,\text{surface}} + V_{C,\text{in}}$$

$$= V_{A,\text{ out}} + V_{B,\text{ surface}} + V_{C,\text{ surface}}$$

(for shell, $V_{\text{in}} = V_{\text{surface}}$)

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_a}{b} + \frac{q_b}{b} + \frac{q_c}{c}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{b} + \frac{(-\sigma)\, 4\pi b^2}{b} + \frac{(\sigma) 4\pi c^2}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

35. (b) Given $pV^3 = C$, $C_V = \frac{3R}{2}$, $n = 1$ mol

For polytropic process,

$pV^x = C$

$\therefore \quad x = 3$

$$C = C_V + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{1-3} = R$$

Heat supplied $(Q) = nC\Delta T$

$= 1 \times R \times (T_2 - T_1) = R(T_2 - T_1)$

Now, As $\quad pV = nRT \quad$ ...(i)

$\therefore \quad pV = RT \quad (\because n = 1)$

$\therefore \quad R = \frac{pV}{T} = \frac{p_1 V_1}{T_1}$

Also, $\quad pV^3 = C$

$\Rightarrow \quad \left(\frac{RT}{V}\right)V^3 = C \quad$ [From Eq. (i)]

$\Rightarrow \quad TV^2 = C \Rightarrow T_1 V_1^2 = T_2 V_2^2$

$\therefore \quad T_2 = \frac{T_1 V_1^2}{V_2^2}$

Heat supplied $(Q) = \frac{p_1 V_1}{T_1}\left[\frac{T_1 V_1^2}{V_2^2} - T_1\right] = p_1 V_1\left[\frac{V_1^2}{V_2^2} - 1\right]$

36. (a, c, d) Given, resistance per unit length of loop $= \lambda\ \Omega$/m

(a) Current in the loop, $I = \frac{E}{R}$

$I = \frac{Blv}{R} \quad (\because \text{emf } E = Blv) \quad$ ... (i)

We know that,

$\frac{R}{2(a+b)} = \lambda \Rightarrow R = \lambda\,(2a + 2b) \quad$ ... (ii)

Using Eqs. (i) and (ii), we get

$$I = \frac{Bbv}{\lambda(2a + 2b)}$$

So, option (a) is correct.

(b) Using right hand thumb rule, direction of current in the loop will be clockwise, looking from the top.

But as the flux is changing, so according to Lenz's law, direction of current will be anti-clockwise.

So, option (b) is incorrect.

(c) As the arm PS and QR has same potential, then

$V_P - V_S = V_Q - V_R$ is also same.

So, option (c) is correct.

(d) As the arm SR is parallel to the velocity. So, there will not be any current or induction.

So, option (d) is correct.

37. (a, b,)(a) Given, energy of photons = 10.2 eV

Energy for nth state,

$$E_n = -13.6\, Z^2 \times \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$= -13.6 \times (2)^2 \left(\frac{1}{2^2} - \frac{1}{4^2}\right) \qquad (\because Z = 2)$$

$$= -13.6 \times 4 \times \left(\frac{4-1}{16}\right)$$

$$= -3.4 \times \frac{3}{4}$$

$$= -10.2\,\text{eV}$$

or $\quad E = 10.2\,\text{eV}$

So, option (a) is correct.

(b) Number of lines in emission spectra

$$= \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6 \qquad [\because n = 4]$$

So, option (b) is correct.

(c) For smallest wavelength of He^+ spectrum the final state should be $n_f = \infty$

So, option (c) is incorrect.

(d) He^+ electron cannot jump from $n = 2$ to $n = 3$ as the energy of photon is less than required.

So, option (d) is incorrect.

38. (a, b) Given, trajectory of the particle,

$$\mathbf{r} = b\cos\omega t\,\hat{\mathbf{i}} + b\sin\omega t\,\hat{\mathbf{j}}$$

It's component can be written as

$$x = b\cos\omega t \qquad \ldots \text{(i)}$$

$$y = b\sin\omega t \qquad \ldots \text{(ii)}$$

Both Eqs. (i) and (ii) together represent the circle in x-y plane. Thus, the trajectory of the particle in x-y plane is circle.

Now, $\quad \dfrac{d\mathbf{r}}{dt} = \omega b(-\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}})$

$$\frac{d^2\mathbf{r}}{dt^2} = -b\omega^2(\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}})$$

$$= -\omega^2(\mathbf{r})$$

Therefore, $\mathbf{a} = -\omega^2\mathbf{r}$

Hence, option (c) and (d) are incorrect.

Energy of the particle in circular motion is given as $\frac{1}{2}m\omega^2A^2$ (where, A is the maximum displacement from the centre), which is constant.

Hence, $\dfrac{E}{\omega}$ is also constant.

So, option (a) and (b) are correct.

39. (a,d) Load F can be given as

$$F = \left(\frac{AY}{L}\right)\Delta x$$

where, A = cross-sectional area

Y = Young's modulus

L = length

Δx = elongation in length.

i.e., F *versus* Δx graph is a straight line of slope $\frac{YA}{L}$.

$(\text{slope})_A > (\text{slope})_B$

$$\left(\frac{YA}{L}\right)_A > \left(\frac{YA}{L}\right)_B$$

or $\quad (A)_A > (A)_B$

As we know that, they are of same material.

Hence, $Y_B = Y_A$

So, option (a) and (d) are correct.

40. (a, c) According to Gauss' theorem

Net flux through the surface $= \dfrac{q_{\text{enclosed}}}{\varepsilon_0}$

Here, $q_{\text{enclosed}} = 0$

So, $\quad \phi_{\substack{\text{curved}\\\text{surface}}} + \phi_{\substack{\text{circular}\\\text{surface}}} = 0$

$$\phi_{\substack{\text{curved}\\\text{surface}}} = -\phi_{\substack{\text{circular}\\\text{surface}}}$$

$$= -\mathbf{E}\cdot\mathbf{S} = -ES\cos 180^\circ$$

$$= ES \qquad (\because \cos 180^\circ = -1)$$

$$= E \cdot \pi R^2 = \pi R^2 E$$

As, electrostatic field is conservative in nature. So, work done is path independent.

So, option (a) and (c) are correct.

Chemistry

41. (b) The given reaction take place as follows:

$$\underset{\text{Magnesium carbonate}}{MgCO_3} \xrightarrow{\text{dil. 2 HCl}} \underset{\text{Magnesium chloride}}{MgCl_2} + H_2O + CO_2$$

$$\underset{\text{Magnesium chloride}}{MgCl_2} + 2NH_3 + 2H_2O \longrightarrow Mg(OH)_2 + 2NH_4Cl$$

$$\underset{\text{Magnesium hydroxide}}{Mg(OH)_2} + \underset{\text{Disodium hydrogen phosphate}}{Na_2HPO_4} + NH_4Cl \longrightarrow \underset{\text{White ppt.}}{Mg(NH_4)PO_4} + NH_3 + H_2O + NaCl$$

$\therefore$ The formula of white ppt. is $Mg(NH_4)PO_4$.

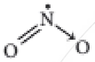
42. (d) The non-linear molecule pair is ClO_2 and NO_2.

The structure of the given molecules are as follows :

(i) XeF_2

Linear

(ii) N_2O



Bent

(iii) HCN

$H-C\equiv N$

Linear

(iv) ClO_2

Bent

(v) CO_2

$O=C=O$

Linear

43. (a) Number of atoms in body centred unit cell = 8 atoms are present at 8 corners and 1 atom is present at body centre.

So, total contribution $= 8 \times \frac{1}{8} + 1 \times \frac{1}{1}$

$$= 1 + 1 = 2$$

Number of atoms in face centred cubic unit cell = 8 atoms are present at 8 corners each and 6 atoms are present at 6 faces each atom contributes one half to unit cell.

$$= 8 \times \frac{1}{8} + 6 \times \frac{1}{2}$$

Hence, total number of atoms in one face unit cell $= 1 + 3 = 4$

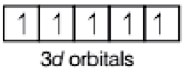
44. (c) Atomic number of Mn is 25.

Electronic configuration of Mn $= [Ar]3d^5 4s^2$

Electronic configuration of $Mn^{2+} = [Ar]3d^5$

There are five 3*d* orbitals, each orbital occupied by single electron. Thus, number of unpaired electrons is 5.

$$Mn^{2+} = [Ar]3d^5$$



45. (*) $(V_{avg})_{O_2} = \sqrt{\frac{8RT_2}{\pi M_{O_2}}}$

$$(V_{avg})_{H_2} = \sqrt{\frac{8RT_1}{\pi M_{H_2}}}$$

$$(V_{avg})_{O_2} = (V_{avg})_{H_2} \qquad \{\text{given}\}$$

$$\therefore \quad \sqrt{\frac{8RT_2}{\pi M_{O_2}}} = \sqrt{\frac{8RT_1}{\pi M_{H_2}}}$$

$$\therefore \quad \sqrt{\frac{T_1}{2}} = \sqrt{\frac{T_2}{32}}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{16}$$

Hence no option is matched with the answer.

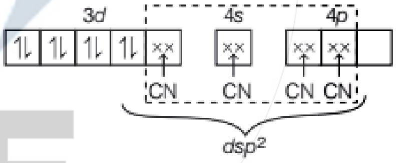
46. (b) The formula of sodium nitroprusside in $Na_2[Fe(CN)_5NO]$.

Hence option (c) is correct.

47. (c) Ni^{2+} ion has $3d^8$ outer electronic configuration.

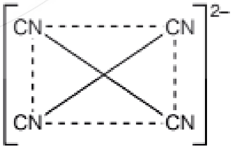
CN^- is a strong field ligand. So, pairing of electron takes place.

Electronic configuration of $[Ni(CN)_4]^{2-}$



Therefore, $[Ni(CN)_4]^{2-}$ is dsp^2 hybridisation. As all the electrons are paired, hence it is diamagnetic.

Geometry of $[Ni(CN)_4]^{2-}$ is square planar.



Hence, the option (c) is correct.

48. (d) Extensive hydrogen bonding is present in water molecules as compare to HF molecules. It is due to the presence of twice the hydrogen bonding than HF. Number of hydrogen bonds,

despite these being individually weaker, thus H_2O has high boiling point than HF.

Hence, option (d) is correct.

49. (a) Only Zn and Al produce nascent hydrogen with alkaline medium.

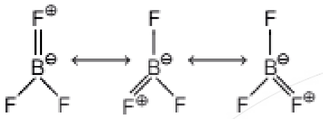
$$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + 2[H]$$

$$Al + 2NaOH \longrightarrow Na_2AlO_2 + 2[H]$$

50. (d) Bond order

$$= \frac{\text{Number of bonds present between atoms}}{\text{Total number of resonance structures}}$$

$\because BF_3$ has three resonance structures.



So, bond order $= \frac{4}{3} = 1\frac{1}{3}$

So, the correct option is (d).

51. (c) Tritium is the heaviest and only radioactive isotope of hydrogen.

52. (b) The stability of these halides decreases down in the group due to decrease in bond dissociation enthalpy.

Thus, the correct order of acidity is

$$HF < HCl < HBr < HI.$$

53. (a) The given reaction take place as follows

$$\underset{\text{Sodium salt}}{2NaCl} + \underset{\text{Lead nitrate}}{Pb(NO_3)_2} \longrightarrow \underset{\substack{\text{(White ppt.)}\\ \text{lead chloride}}}{PbCl_2\downarrow} + \underset{\text{Sodium nitrate}}{2NaNO_3}$$

$PbCl_2$ is soluble in hot water and forms white ppt. in cold condition.

This show that Cl^- acid radical is present in the salt.

54. (c) Oxidation state of Cr in $K_2Cr_2O_7$.

Let the 0.5 of Cr be x.

So, $2(+1) + 2(x) + 7(-2) = 0$

$$2 + 2x - 14 = 0$$

$$2x = 14 - 2$$

$$x = \frac{12}{2} \Rightarrow x = +6$$

Oxidation of Cr is + 6.

Oxidation state of Cr in CrO_5.

In this molecule four oxygen atom present as peroxide and one oxygen as oxide.

Let the oxidation state of Cr be x in CrO_5.

So, $x + 4(-1) + 1 \times (-2) = 0$

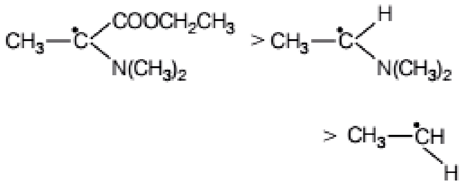
$$x - 4 - 2 = 0$$

$$x = +6$$

$\therefore$ Oxidation state of Cr is + 6.

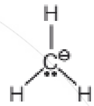
Hence, the correct option is (c).

55. (d) The stability of free radical increases with increases is order as:



$\therefore$ III > II > I

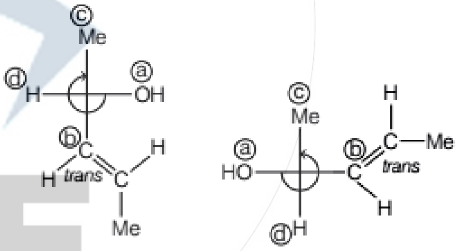
56. (d) $\overset{\ominus}{\underset{\bullet\bullet}{C}}H_3$ carbanion has 3 bonding pairs and 1 lone pair. Its structure is as follows



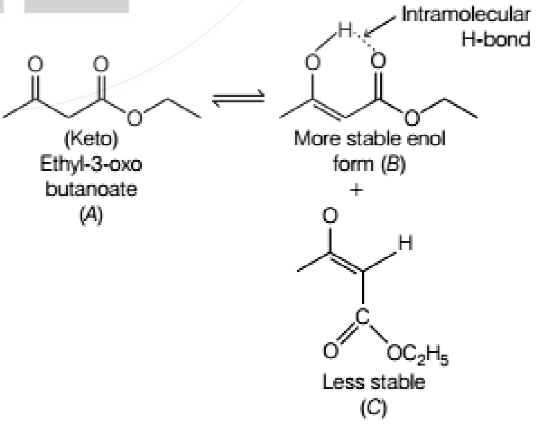
Hybridisation in sp^3

In $H_2\overset{\ominus}{\underset{\bullet\bullet}{C}}$—$CHOCH_3$ molecule C is also sp^3 hybridisation, because it contain 3 bond pairs and 1 lone pair.

57. (b) They both have *S*-configuration. So, they are homomers.

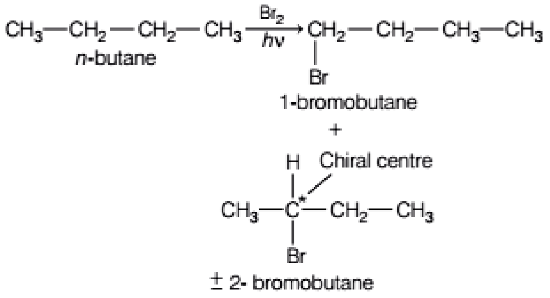


58. (c)



∴ (*B*) form is more stable than (*C*) form due the presence of intramolecular hydrogen bonding.

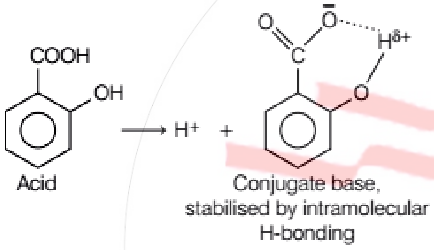
59. (c) Monobromination of *n*-butane in form of free radical take place as follows:



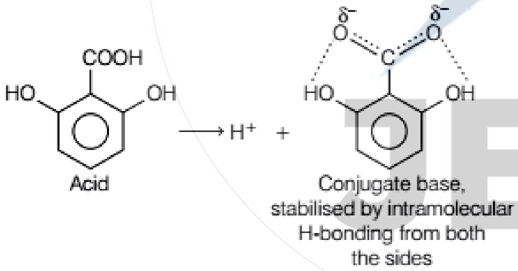
∴ Total product = 3

60. (a) The acidic strength of an acid increases with the increase in stability of its conjugate base by intramolecular hydrogen bonding.

(i) Salicylic acid



(ii) 2, 6-dihydroxybenzoic acid

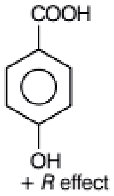


Also, pK_a value of 2, 6-dihydroxybenzoic acid is greater than salicylic acid.

∴ It is a stronger acid.

(iii) In 4-hydroxybenzoic acid —OH group show $+R$ effect.

Thus, it decrease the acidic character 4-hydroxybenzoic acid.



∴ The acidity order is 2, 6-dihydroxybenzoic acid > salicylic acid > 4-hydroxybenzoic acid.

61. (c) Given, molecular weight of oxalic acid = 126

Volume = 100 mL

Normality = 0.1 N

Mass = ?

We know,

$$\text{Normality} = \frac{\dfrac{\text{Mass}}{\text{Equivalent weight}}}{\text{Volume (Litre)}}$$

$$\left(\therefore \text{Equivalent weight} = \frac{\text{Molecular weight}}{\eta\text{-factor}}\right.$$

$$\left. = \frac{126}{2} [\eta\text{-factor} = 2 \text{ for oxalic acid}]\right)$$

$$\therefore \quad 0.1 = \frac{\text{Mass}}{63 \times \dfrac{100}{1000}} \Rightarrow \text{Mass} = \frac{0.1 \times 63 \times 100}{1000}$$

∴ Mass = 0.63 g

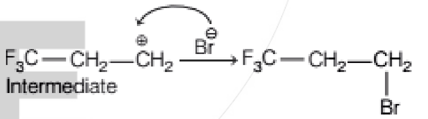
62. (a) The given reaction take place as follows:

$$F_3C—CH{=}CH_2 \xrightarrow{H^+} F_3C—\overset{+}{C}H—CH_2$$

(Less stable due to $-I$ effect by $-F$)

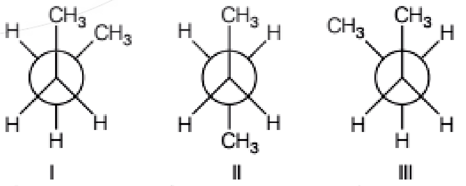
$$+ F_3C—CH—\overset{\oplus}{C}H_2$$

(More stable)



3-bromo-1, 1, 1-trifluoro propane (Major)

63. (a) The given conformers of *n*-butane are



Staggered form is the most stable form as bulky methyl group are at 180° from each other and hence have minimum angular strain. While gauche from (I and III) are less stable form as bulky methyl group are closer.

Hence, the order of stability is

II > I = III

64. (c) $C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$

78 g of C_6H_6 required $= \frac{15}{2}$ mole of O_2

$39 \text{ g}\left(\frac{1}{2}\text{ mole}\right) C_6H_6$ requires $= \frac{15}{2} \times \frac{1}{2}$ mole of O_2

$= \frac{15}{4}$ mole of O_2

$\because \quad \text{Moles} = \frac{\text{Volume (L)}}{22.4}$

$$\frac{15}{4} = \frac{\text{Volume}}{22.4}$$

$\therefore$ Volume = 84 L (O_2)

65. (b) Avagadro's law is valid for ideal gas.

According to Avagadro's law, volume of ideal gas is directly proportional to number of moles of gas at constant temperature and pressure.

66. (b) Let the two oxides be M_2O_x and M_2O_y.

Let atomic weight of metal, $M = a$

$\therefore$ For MO_x,

ratio of weight

$$2a : 16x = 25 : 4 \qquad \text{...(i)}$$

For MO_y,

ratio of weight

$$2a : 16y = 25 : 6 \qquad \text{...(ii)}$$

From (i) and (ii), we get

$\therefore \quad 16x : 16y = 4 : 6$

$\therefore \quad x : y = 2 : 3$

$\therefore$ The two oxides may be M_2O_2 or M_2O_3.

Now, $2a : 16x^2 = 25 : 4$

$\therefore \quad a = \frac{25}{4} \times 16 = 100$

67. (b) de-Broglie equation,

$$\lambda = \frac{h}{mv}$$

where, λ = wavelength

m = mass

v = velocity

h = Plank's constant

$\therefore \quad \lambda \propto \frac{1}{m}$

We know,

$m_{He^{2+}} > m_p > m_e$

So, $\quad \lambda_e > \lambda_p > \lambda_{He^{2+}}$

or $\quad e > p > \alpha$

68. (a) Number of drops in 1 mL water = 25

$\therefore$ Volume of 1 drop $= \frac{1}{25}$ mL

$\therefore$ Mass of water = Density × Volume

$= 1 \times \frac{1}{25}$

$= \frac{1}{25}$ g

$\therefore$ Moles of water $= \frac{\text{Mass}}{\text{Molecular mass}}$

$= \frac{1}{25 \times 18}$

$\therefore$ Molecules of water $= N_0 \times$ Moles of water

$= \frac{N_0}{25 \times 18} = \frac{0.02}{9} N_0$

69. (d) Radius of 'H' atom in ground state $= r_1$

Radius of He^+ ion in ground state $= r_2$

$\because \quad r \propto \frac{n^2}{Z}$

where, n = principal quantum number

Z = atomic number

$\therefore \quad \frac{r_1}{r_2} = \frac{Z_2}{Z_1} = \frac{2}{1} \qquad \{\because n_1 = n_2 = 1\}$

$\Rightarrow \quad \frac{r_2}{r_1} = \frac{1}{2}$

70. (b) Let principal quantum number $= n$

The value of azimuthal quantum number

$l = 0$ to $(n - 1)$.

The value of magnetic quantum number

$m = -l$ to $+l$.

Spin quantum number $(s) = +\frac{1}{2}, -\frac{1}{2}$

$\therefore$ The correct set of quantum number (n, l, m, s) is

$\left(4, 3, -2, -\frac{1}{2}\right)$

For $n = 4$,

$l = 0, 1, 2, 3; \quad l = 3$

and $\quad m = -2, -1, 0, 1, +2$

71. (b) $(C_{rms})_{H_2} = \sqrt{\frac{3RT}{2}} = \sqrt{\frac{3\times R\times 150}{2}}$

$$= \sqrt{225R}$$

$$(C_{mp})_{He} = \frac{(C_{rms})_{H_2}}{2}$$

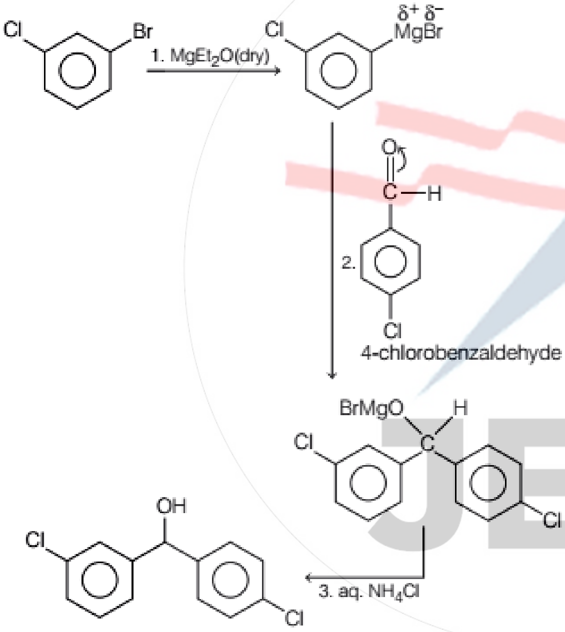
$$(C_{mp})_{He} = \sqrt{\frac{2RT}{4}}$$

$$\therefore \quad \sqrt{\frac{2RT}{4}} = \frac{\sqrt{225R}}{2}$$

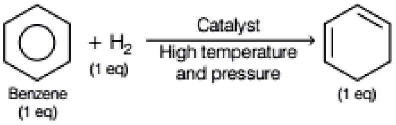
$$\therefore \quad T = 1125\,K$$

72. (b) Electron affinity of S > O and Cl > F. This is so because of the small size of oxygen and fluorine (in respective case), due to which they got higher charge density and thus electronic repulsion increases when new electron is added.

73. (a) The given reaction take place as follows:



74. (c) The hydrogenation reaction of benzene take place as follows



75. (a) (a) Relative lowering of vapour pressure

$$\frac{p^{\circ} - p}{p^{\circ}} = x_{\beta}$$

∴ It is independent of temperature.

∴ It is a correct statement.

(b) Osmotic pressure does not depends on nature of solute.

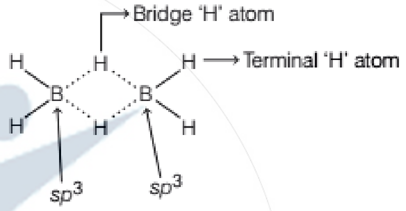
(c) Boiling point is a property of solvent.

∴ ΔT_p is also dependent on nature of solvent.

∴ b, c and d are incorrect statements.

76. (b, c) During preparation of NH_3 in Haber's process Mo as well as mixture of Al_2O_3 and K_2O are used as promoters.

77. (a,c,d) The structure of B_2H_6 is as follows:



∴ All B atoms are sp^3 hybridised.

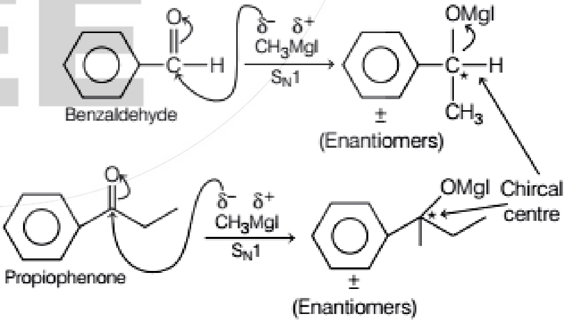
It contains two $3C-2e^-$ bonds and four $2C-2e^-$ bonds.

In B_2H_6, two types of H atoms are present i.e. bridge and terminal H atom.

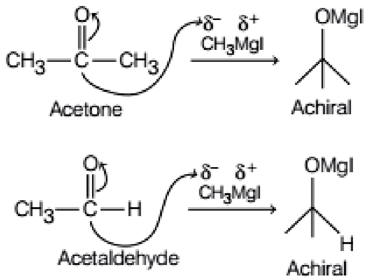
It is diamagnetic as all the electrons are paired.

∴ Statement a, c and d are correct statements.

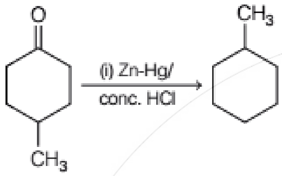
78. (a, b) The given compound reacts with methyl magnesium iodide as follows:

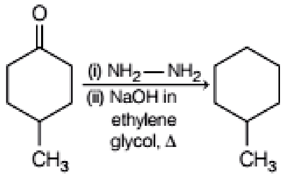


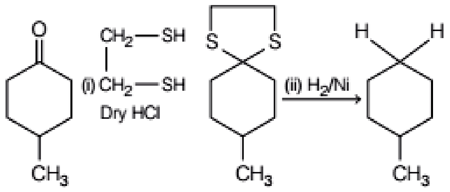
As products in a and b reactions have chiral centre, therefore they produce enantiomeric products.



79. (a, b, c) The given reaction is a reduction reaction. It take place in following ways:







80. (b, c) For a weak acid and weak base salt solution. If $K_a > K_b$ i.e. $pK_b > pK_a$ than $pH = 1/2$ $(pK_w + pK_a - pK_p)$ value is less than 7.

$\therefore$ (b) is an incorrect statement.

The pH of 10^{-8} M HCl is less than 7.

$\therefore$ Statement (c) is also incorrect.

Mathematics

1. (d) LHL

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin(a+1)x + \sin x}{x}$$

$$= \lim_{h\to 0} \frac{\sin(a+1)\,(0-h) + \sin(0-h)}{(0-h)}$$

[Put $x = 0 - h$; when $x \to 0^-$, then $h \to 0$]

$$= \lim_{h\to 0} \frac{\sin(a+1)h + \sin h}{h}$$

$$= \lim_{h\to 0} (a+1)\,\frac{\sin(a+1)h}{(a+1)h} + \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (a+1)\cdot 1 + 1$$

$$= a + 2$$

$$\text{RHL} = \lim_{x\to 0^+} f(x)$$

$$= \lim_{x\to 0^+} \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{1/2}}$$

$$= \lim_{h\to 0} \frac{(h+bh^2)^{1/2} - h^{1/2}}{bh^{1/2}}$$

$$= \lim_{h\to 0} \frac{h^{1/2}\left[\left(1 + \frac{1}{2}bh + \ldots\right) - 1\right]}{bh^{1/2}}$$

$$= 0$$

and $f(0) = C$ for continuity

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \text{RHL}$$

$$\lim_{x\to 0^+} f(x) = f(0)$$

$\therefore \quad a + 2 = 0 = c$

$a = -2$, $c = 0$, b is arbitrary non-zero real.

2. (c) Given, $y = \sqrt{\log_{10} \frac{3x - x^2}{2}}$

To find-Domain

$$\frac{3x - x^2}{2} > 0$$

$$x(3-x) > 0$$

$$x \in (0, 3)$$

Now,

$$\log_{10}\left[\frac{3x - x^2}{2}\right] \geq 0$$

$$\Rightarrow \quad \frac{3x - x^2}{2} \geq 1$$

$$\Rightarrow \quad 3x - x^2 \geq 2$$

$$\Rightarrow \quad x^2 - 3x + 2 \leq 0$$

$$(x-1)(x-2) \leq 0$$

$$\Rightarrow \quad x \in [1, 2]$$

3. (c) We know

$|x|$ is not differentiable at $x = 0$

where as,

$|x|^3$ is differentiable at $x = 0$

Also $|x|^2 = x^2$ is differentiable for all real x.

If $a_1 = 0$, then

$$f(x) = a_3|x|^3 + a_2|x|^2 + a_0$$

is differentiable at $x = 0$ which is possible when $a_1 = 0$.

4. (a) $y = e^{\tan^{-1} x}$

$$\frac{dy}{dx} = \frac{e^{\tan^{-1} x}}{1 + x^2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(1 + x^2) \cdot \frac{e^{\tan^{-1} x}}{1 + x^2} - e^{\tan^{-1} x}(2x)}{(1 + x^2)^2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(1 - 2x)e^{\tan^{-1} x}}{(1 + x^2)^2}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2}(1 + x^2) = (1 - 2x)\frac{dy}{dx}$$

$$\therefore \quad (1 + x^2)y_2 + (2x - 1)y_1 = 0$$

5. (c) $\lim\limits_{x \to 0} \dfrac{\ln\sqrt{\dfrac{1 + x}{1 - x}}}{x}$

which is indeterminant form.

By substitution method,

Put $x = \cos 2\theta$

$$2\theta = \cos^{-1} x$$

$$\theta = \frac{1}{2}(\cos^{-1} x)$$

When $x \to 0$, then $\left(\theta \to \dfrac{\pi}{4}\right)$

$$\Rightarrow \quad \lim_{\theta \to \pi/4} \frac{\ln(\cot^2\theta)^{1/2}}{\cos 2\theta}$$

$$\Rightarrow \quad \lim_{\theta \to \pi/4} \frac{\ln(\cot\theta)}{\cos 2\theta}$$

$$\Rightarrow \quad \lim_{\theta \to \pi/4} \frac{\ln|\cot\theta|}{\cos 2\theta}$$

$$\lim_{\theta \to \pi/4} \frac{\frac{1}{\cot\theta} \times -\mathrm{cosec}^2\theta}{-(\sin 2\theta)(2)} = \frac{2}{2} = 1$$

6. (a) Let us assume $g(x) = e^{-x} f(x)$

$\because \quad f'(c) - f(c)$ or

$f'(x) - f(x)$ expression gives us following

$$f'(x) - f(x) = 0$$

$$\Rightarrow \quad e^{-x} f'(x) + e^{-x}(-1) f(x) = 0$$

$$\frac{d}{dx}[e^{-x} f(x)] = 0$$

From the condition and nature of e^{-x}, we can say that

(i) $g(x)$ is continuous in $[a, b]$

(ii) $g(x)$ is differentiable in (a, b)

(iii) $g(a) = e^{-a} f(a) = 0$ and $g(b) = e^{-b} f(b) = 0$

$[\because f(a) = 0 = f(b)]$

From Rolle's theorem,

$g'(c) = 0$ for at least one $c \in (a, b)$

$\Rightarrow \quad e^{-c} f'(c) - e^{-c} f(c) = 0$

$\Rightarrow \quad f'(c) - f(c) = 0 \qquad [\because e^{-c} \neq 0]$

$\Rightarrow \quad f'(c) = f(c)$ for at least one $c \in (a, b)$

7. (d) Let $I = \int \cos(\ln x)\, dx$

$$I = x\cos\ln x + \int x\sin(\ln x)\frac{1}{x}dx$$

$$\Rightarrow \quad I = x\cos\ln(x) + \int \sin(\ln x)dx$$

$$\Rightarrow \quad x\cos(\ln x) + [x\sin(\ln x) - \int x\cos(\ln x)\cdot\frac{1}{x}dx$$

$$\Rightarrow \quad 2I = x[\cos(\ln x) + \sin(\ln x)]$$

$$\Rightarrow \quad I = \frac{x}{2}[(\cos\ln x) + \sin\ln x] + c$$

8. (a) Let $f(x) = x$

Solve by options,

Alternate (A),

$$\int_0^c f(x)\,dx = (1 - c)\,f(c)$$

$$\int_0^c x\,dx = (1 - c)\,c$$

$$\left[\frac{x^2}{2}\right]_0^c = (1 - c)\,c$$

$$\frac{c^2}{2} - 0 = (1 - c)c \text{ or } \frac{c}{2} = (1 - c)$$

$$c = 2 - 2c$$

$$3c = 2 \quad \Rightarrow \quad c = \frac{2}{3}$$

$\therefore$ Which is true.

If alternate A is true.

$\because$ B is false vice-versa.

For alternate (D), $\int_0^c x\,dx = \dfrac{c^2}{2}$

which dependent on c.

Hence, (D) is false.

9. (b) $\int \sqrt{x}\,(1 - x^3)^{-1/2}dx$

$$\int \sqrt{x}\,[1 - (x^{3/2})^2]^{-1/2}\,dx$$

Let $x^{3/2} = t$

$$\frac{3}{2}x^{1/2}dx = dt$$

$$I = \frac{2}{3}\int (1 - t^2)^{-1/2} dt$$

$$\Rightarrow \quad I = \frac{2}{3}\int \frac{dt}{\sqrt{1 - t^2}}$$

$$\Rightarrow \quad I = \frac{2}{3}\sin^{-1} t + c$$

$$\Rightarrow \quad I = \frac{2}{3}\sin^{-1}(x^{3/2}) + c$$

$$\Rightarrow \quad = \frac{2}{3} g[f(x)] + c$$

$\therefore$ $g(x) = \sin^{-1} x$ and $f(x) = x^{3/2}$

10. (a) $I_1 = \int_0^{\pi/2} \frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x} + (\sin x)^{\cos x}} dx$ ... (i)

Put $x = \left(\frac{\pi}{2} - x\right)$

$$I_2 = \int_0^{\pi/2} \frac{(\sin x)^{\cos x}}{(\sin x)^{\cos x} + (\cos x)^{\sin x}} \quad \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1\, dx$$

$$\Rightarrow \quad 2I = [x]_0^{\pi/2} \Rightarrow 2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

11. (b) Given,

$$\int_0^x \frac{bt\cos 4t - a\sin 4t}{t^2} dt = \frac{a\sin 4x}{x} - 1$$

Differentiating both side w.r.t. x,

$$\frac{bx\cos 4x - a\sin 4x}{x^2} = \frac{a\{4x\cos 4x - \sin 4x\}}{x^2}$$

On comparing, we get

$$b = 4a$$

$$\therefore \quad a = \frac{1}{4} \text{ and } b = 1$$

12. (b) Given, $\int_{\sin x}^{\cos x} e^{-t^2} dt$

$$\Rightarrow \quad g(\cos x)\,\frac{d}{dx}(\cos x) - g(\sin x)\,\frac{d}{dx}(\sin x)$$

$$\because \quad f(x)\int_{a(x)}^{b(x)} g(t)\,dx$$

$$f'(x) = g(b)\,\frac{d}{dx}(b) - g(a)\frac{d(a)}{dx}$$

$$\Rightarrow \quad e^{-\cos^2 x}(-\sin x) - e^{-\sin^2 x}(\cos x)$$

At $x = \frac{\pi}{4}$

$$\Rightarrow \quad -e^{-1/2}\left(\frac{1}{\sqrt{2}}\right) - e^{-1/2}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad -\sqrt{2}e^{-1/2}$$

$$\Rightarrow \quad -\sqrt{\frac{2}{e}}$$

13. (a) $x\frac{dy}{dx} + y = x\,\frac{f(xy)}{f'(xy)}$

i.e. $\frac{d}{dx}(xy) = x\frac{f(x,y)}{f'(x,y)} \Rightarrow \frac{f'(xy)}{f(xy)} d(xy) = xdx$

$$\Rightarrow \quad \int \frac{f'(xy)}{f(xy)}\, d(xy) = \int x\, dx$$

$$\Rightarrow \quad \log[f(xy)] = \frac{x^2}{2} + c$$

$$\Rightarrow \quad f(xy) = e^{\left(\frac{x^2}{2} + c\right)}$$

$$= e^{\frac{x^2}{2}} \cdot e^c = ce^{\frac{x^2}{2}}$$

14. (c) Let the slope of tangent at point $P(x, y)$ be m.

Equation of tangent will be $(Y - y) = m(X - x)$

If $Y = 0$, then $X = x - \frac{y}{m}$ and if $X = 0$, then $Y = y - mx$. Since, $P(x, y)$ is mid-point of $A\left(x - \frac{y}{m}, 0\right)$ and $B\,(0, y - mx)$.

$$\therefore \quad x - \frac{y}{m} = 2x \quad \text{and} \quad y = -mx$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \quad \int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \log y = -\log x + \log c$$

$$\log\ xy = \log\ c$$

$$xy = c$$

Since, the line passing through the point (3, 2).

$\therefore$ $c = 6$

The equation of curve is $xy = 6$

15. (d) $\cos y\,\frac{dy}{dx} = e^{x + \sin y} + x^2 e^{\sin y}$

On separating the variables, we get

$$\cos ye^{-\sin y}\ dy = (e^x + x^2)dx$$

On integrating both side,

$$\int \cos\ ye^{-\sin y}\ dy = \int (e^x + x^2)\ dx$$

Let $e^{-\sin y} = t$

$$\therefore \quad e^{-\sin y}\,(-\cos y)\ dy = dt$$

$$-\int dt = e^x + \frac{x^3}{3}$$

$\Rightarrow \quad -t = e^x + \frac{x^3}{3} + c$

$\Rightarrow \quad -e^{-\sin y} = e^x + \frac{x^3}{3} + c$

$\Rightarrow \quad -e^x - \frac{x^3}{3} - e^{-\sin y} = c$

$$e^x + \frac{x^3}{3} + e^{-\sin y} = c$$

Hence, $\quad f(x) = \left(e^x + \frac{x^3}{3}\right)$

16. (a) Equation of the tangent at any point $\left[\frac{9t^2}{4}, \frac{9t}{2}\right]$

On the parabola is

$ty = x + \frac{9t^2}{4}$ and it passes through (4, 10)

$$y^2 = 9x$$

$\therefore \quad a = \frac{9}{4}$

$$y = mx + \frac{a}{m}$$

$\Rightarrow \quad y = mx + \frac{9}{4m} \qquad \left[\because a = \frac{9}{4}\right]$

Since, it passes through (4, 10)

$\Rightarrow \quad 10 = 4m + \frac{9}{4m}$

$\Rightarrow \quad 16m^2 - 40m + 9 = 0$

$\Rightarrow \quad m = \frac{1}{4}, \frac{9}{4}$

Equation the tangents are

$$Y = \frac{1}{4}x + 9 \quad \text{and} \quad y = \frac{9}{4}x + 1$$

$$x - 4y + 36 = 0$$

$$9x - 4y + 4 = 0$$

$$y = mx + \frac{a}{m}$$

touches at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

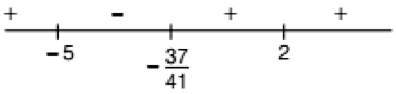
$\therefore$ Point of contact are (36, 18) and $\left[\frac{4}{9}, 2\right]$

17. (c) $f(x) = (x-2)^{17}(x+5)^{24}$

$f'(x) = 17(x-2)^{16}(x+5)^{24} + (x-2)^{17}(24)(x+5)^{23}$

$= (x-2)^{16}(x+5)^{23}\{17x + 85 + 24x - 48\}$

$= (x-2)^{16}(x+5)^{23}(41x + 37) = 0$



$\therefore f(x)$ has neither a maximum nor a minimum at $x = 2$

18. (b) Since **c** is a unit vector coplanar with **a** and **b**.

Let $\mathbf{c} = x\mathbf{a} + y\mathbf{b}$

$$x(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + y(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$(x+y)\hat{\mathbf{i}} + (x-y)\hat{\mathbf{j}} - (x-y)\hat{\mathbf{k}}$$

Since, **c** is perpendicular to **a**.

$\therefore \quad x + y + x - y + x - y = 0$

$$3x = y$$

Since $|\mathbf{c}| = 1$

$\therefore \quad (x+y)^2 + (x-y)^2 + (x-y)^2 = 1$

$\Rightarrow \quad (4x)^2 + (-2x)^2 + (-2x)^2 = 1$

$\Rightarrow \quad 24x^2 = 1 \Rightarrow x = \frac{1}{2\sqrt{6}}$

$$\mathbf{c} = 4x\,\hat{\mathbf{i}} - 2x\hat{\mathbf{j}} + 2x\hat{\mathbf{k}}$$

$$= \frac{2}{\sqrt{6}}\hat{\mathbf{i}} - \frac{1}{\sqrt{6}}\hat{\mathbf{j}} + \frac{1}{\sqrt{6}}\hat{\mathbf{k}}$$

i.e. $\quad \mathbf{c} = \frac{1}{\sqrt{6}}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$$\mathbf{a} = d_1\hat{\mathbf{i}} + d_2\hat{\mathbf{j}} + d_3\hat{\mathbf{k}}$$

Since, **d** is perpendicular to **a** and **c**.

$\therefore \quad \mathbf{d} \cdot \mathbf{a} = 0$

$$d_1 + d_2 - d_3 = 0$$

and $\mathbf{d} \cdot \mathbf{c} = 0$

$$2d_1 - d_2 + d_3 = 0 \quad \Rightarrow \quad 3d_1 = 0$$

$$d_1 = 0$$

$\therefore \quad d_2 = d_3$

Since $|\mathbf{d}| = 1$

$\therefore \quad d_1^2 + d_2^2 + d_3^2 = 1$

$$d_2 = d_3 = \frac{1}{\sqrt{2}}$$

Hence, $d = \frac{1}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$

19. (c) Length of perpendicular from the centre of circle on tangent = radius of circle.

$$r = \left|\frac{3x_1 + y_1}{\sqrt{(3)^2 + (1)^2}}\right| \Rightarrow r = \frac{5}{\sqrt{10}} = \sqrt{\frac{5}{2}}$$

Equation of circle $(x-h)^2 + (y-k)^2 = r^2(h, b)$

Centre points

$$(x-2)^2+(y+1)^2=\left(\sqrt{\frac{5}{2}}\right)^2$$

$$x^2+4-4x+y^2+1+2y=\frac{5}{2}$$

Let the equation of other tangent which is passing through origin is $y=mx$

$$x^2+4-4x+m^2x^2+1+2mx=\frac{5}{2} \quad [\because\ y=mx]$$

$$\Rightarrow\ x^2(1+m^2)+(2m-4)x+\frac{10-5}{2}=0$$

$$\Rightarrow\ (1+m^2)x^2+(2m-4)x+\frac{5}{2}=0$$

Compare with $ax^2+bx+c=0$, where

$$a=(1+m^2),\ b=2m-4$$

and $c=\frac{5}{2}$

$$\Rightarrow\ (2m-4)^2-4(1+m^2)\times\frac{5}{2}=0$$

$$\Rightarrow\ 4m^2+16-16m-4\times\frac{5}{2}-4m^2\times\frac{5}{2}=0$$

$$\Rightarrow\ 4m^2-16m+16-10-10m^2=0$$

$$\Rightarrow\ -6m^2-16m+6=0$$

$$\Rightarrow\ -2(3m^2+8m-3)=0$$

$$\Rightarrow\ (3m-1)(m+3)=0$$

$$\Rightarrow\ 3m-1=0,\quad m+3=0$$

$$m=\frac{1}{3};\ m=-3$$

(i) $m=-3,\ y=-3x,\ 3x+y=0$

(ii) $m=\frac{1}{3},\ y=\frac{1}{3}x,\ 3y-x=0$

Other tangent $3y-x=0$

$$x-3y=0$$

20. (b) The given curves are

$y^2+8x=16$ and $y^2-24x=48$

On solving both equations, we get

$x=-1$ and $y=\pm\sqrt{24}$

$\therefore$ Required area

$$=\int_{-\sqrt{24}}^{\sqrt{24}}(x_1-x_2)dy$$

$$=\int_{-\sqrt{24}}^{\sqrt{24}}\left\{\left(2-\frac{y^2}{8}\right)-\left(\frac{y^2}{14}-2\right)\right\}dy$$

$$=\int_{-\sqrt{24}}^{\sqrt{24}}\left(4-\frac{4y^2}{24}\right)dy$$

$$=\int_{-\sqrt{24}}^{\sqrt{24}}\left(4-\frac{y^2}{6}\right)dy$$

$$=\left(4y-\frac{y^3}{18}\right)_{\sqrt{-24}}^{\sqrt{24}}$$

$$=\left(4\sqrt{24}-\frac{24\sqrt{24}}{18}\right)-\left(-4\sqrt{24}+\frac{24\sqrt{24}}{18}\right)$$

$$=8\sqrt{24}-\frac{48\sqrt{24}}{18}$$

$$=8\sqrt{24}-\frac{8\sqrt{24}}{3}$$

$$=\frac{16\sqrt{24}}{3}$$

$$=\frac{16\times2\sqrt{6}}{3}$$

$$=\frac{32\sqrt{6}}{3}\ \text{sq units}$$

21. (a) $V=a-kt^2$

$$\frac{dV}{dt}=a-kt^2$$

$$\int_0^V dV=\int_0^t(a-kt^2)dt$$

$$V=\text{at}-\frac{kt^3}{3} \qquad \dots\text{(i)}$$

When V is max $a=0$

$\therefore\quad a-Kt^2=0$

$\Rightarrow\quad t=\sqrt{\frac{a}{k}}$

Using this in Eq. (i)

$$V_{\max}=a\sqrt{\frac{a}{k}}-\frac{k}{3}.\frac{a}{k}\sqrt{\frac{a}{k}}=\frac{2}{3}a\sqrt{\frac{a}{k}}$$

22. (b) By given condition $2[\log 2b-\log 3c]$

$=\log a-\log 2b+\log 3c-\log a$

$\log 3c-\log 2b$

$\Rightarrow\quad 3[\log 2b-\log 3c]=0$

$2b=3c$

Also, $b^2=ac$

$\therefore\quad \frac{9c^2}{4}=ac$

$\Rightarrow\quad c=\frac{4a}{9}$

$$b=\frac{3c}{2}=\frac{3}{2}.\frac{4}{9}a=\frac{2a}{3}$$

29. (b) $f(n) = 2^{n+1}$ ... (i)

$g(n) = 1 + (n+1)2^n$ ... (ii)

$g(n) = 1 + (n+1)\dfrac{f(n)}{2}$ $[f_n = 2^{n+1}]$

$\therefore \quad 2^n = \dfrac{f(n)}{2}$

$g(1) = f(1) + 1$

$n > 1, \dfrac{n+1}{2} > 1$

$g(n) > \left(\dfrac{n+1}{2}\right) f(n) > f(n)$

$f(n) < g(n)$

30. (d) Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$

$1 \le i \le n$

(i) $a_i \in P$ and $a_i \in Q$ (ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$ (iv) $a_i \notin P$ and $a_i \notin Q$

$P \cap Q = \phi$

Since, case in favour 3 i.e. (ii), (iii) and (iv).

Total element in $A = n$

$\therefore$ Required number of ways = 3^n

31. (d) $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & (a-4) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ a \end{bmatrix}$

$x + 2y + 4z = 6$

$2x + y + 2z = 4$

$x + 2y + (a-4)z = a$

For unique solution, $|A| \neq 0$, then the system is consistant independent and have unique solution.

$\therefore \quad a \neq 8$

32. (b) $\begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ (x-1) & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, then

$$\Delta x = \begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ 1 & (2x-1) & (3x^2+3x+1) \\ 2 & 4x & 6x^2+12x+8 \end{vmatrix}$$

Applying $R_3 \to R_3 - 2R_2$, then

$$\begin{vmatrix} (x-2) & (x-1)^2 & x^3 \\ 1 & (2x-1) & 3x^2+3x+1 \\ 0 & 2 & 6x+6 \end{vmatrix}$$

Expand w.r.t. C, then

$\Delta(x) = (x-2)\{(2x-1)(6x-6) - 2(3x^2+3x+1)\}$
$-1\{(x-1)^2(6x+6) - 2x^3\}$

$\therefore$ Coefficient of x in $\Delta(x) = -8 + 6 = -2$

33. (b) $P \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix}$

For matrix 3×3 matrix,

$|\text{Adj } A| = |A|^2$

$|\text{Adj} A| = 16$

$\Rightarrow \quad 1(12-12) - \alpha(4-6) + 3(4-6) = 16$

$\Rightarrow \quad 2\alpha - 6 = 16$

$\Rightarrow \quad 2\alpha = 16 + 6$

$\alpha = \dfrac{22}{2}$

$\therefore \quad \alpha = 11$

34. (b) $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$

$A^{2018} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 1+i \\ 0 & i^2 \end{bmatrix}$

Then,

$A^3 = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 & 1+i \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} 1 & i \\ 0 & i^3 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & i \\ 0 & i^3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+i \\ 0 & i^4 \end{bmatrix}$

So, $A^{2018} = \begin{bmatrix} 1 & 1+i \\ 0 & i^{2018} \end{bmatrix}$

On comparing with $A^{2018} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

So, $a = 1$

$d = i^{2018} = (i^2)^{1009} = (-1)^{1009} = -1$

Hence, $a + d = 1 - 1 = 0$

35. (d) $gof : S \to U$

gof is injective or (one-one) function:

So, $g(fx_1) = g(fx_2)$

$f(x_1) = f(x_2)$

$\therefore$ g is obviously injective.

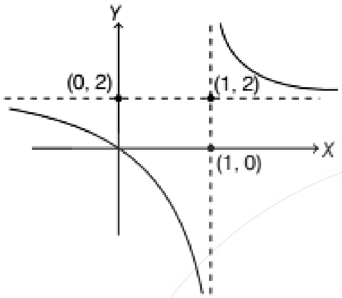
36. (d) $f(x) = \frac{2x}{x-1}$

$$y = \frac{2x}{x-1} = \frac{2x-2+2}{x-1} = \frac{2(x-1)+2}{(x-1)}$$

$$= \frac{2(x-1)}{(x-1)} + \frac{2}{(x-1)}$$

$\therefore$ $(y-2)(x-1) = 2$

onto → co-domain = range



Thus, f is both one-one and onto.

37. (b) Given that the probability of three events are

$P(A) = \frac{3x+1}{3}$ $\Rightarrow$ $P(B) = \frac{1-x}{4}$ $\Rightarrow$ $P(C) = \frac{1-2x}{2}$

As we know that probability of event lies between 0 and 1.

This can be represented as $0 \le P(x) \le 1$. So let us use this theorem for each probability one by one.

For given event A, we have $0 \le P(A) \le 1$

Let us now substitute the value given

$$0 \le \left[P(A) = \frac{3x+1}{3}\right] < 1 \qquad \dots \text{(i)}$$

We get the interval of x from Eq. (i),

$$0 \le \frac{3x+1}{3} \le 1$$

$$\Rightarrow \quad \frac{-1}{3} \le x \le \frac{2}{3} \qquad \dots \text{(ii)}$$

Similarly, for the given event B, we have

$$0 \le P(B) \le 1$$

Let us now substitute the value given.

Let us simplify further to find the interval of x

$$0 \le \frac{1-x}{4} \le 1$$

$$-3 \le x \le 1 \qquad \dots \text{(iii)}$$

For event c

$$0 \le p(c) \le 1$$

$$0 \le \frac{1-2x}{2} \le 1$$

$$\Rightarrow \quad 1 - 2x \le 2$$

$$\Rightarrow \quad -\frac{1}{2} \le x \le \frac{1}{2} \qquad \dots \text{(iv)}$$

Now, three mutually exclusive events are

$$0 \le P(A \cup B \cup C) \le 1$$

$\therefore$ [Probability must be 0 to 1]

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$$
$$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

So, probability of intersection any of these event is 0

$$P(A \cap B) = P(B \cap C) = P(C \cap A)$$
$$= P(A \cap B \cap C) = 0$$

So, the union of events become

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - 0 - 0 - 0$$

$$\Rightarrow \quad P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$0 < \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$$

$$0 \le 13 - 3x \le 12$$

$$\frac{1}{3} \le x \le \frac{13}{3} \qquad \dots \text{(v)}$$

Finally, we have four inequalities

$-\frac{1}{3} \le x \le \frac{2}{3} \dots$ [from Eq.(ii)]

$-3 \le x \le 1 \dots$ [from Eq.(iii)]

$-\frac{1}{2} \le x \le \frac{1}{2} \dots$ [from Eq. (iv)]

$\frac{1}{3} \le x \le \frac{13}{3} \dots$ [from Eq. (v)]

Hence, the common of each inequality is $\frac{1}{3} \le x \le \frac{1}{2}$.

38. (b) Determinant of order 2 will be of the form

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\Delta = ad - bc$$

The total number of ways of choosing a, b, c and d is

$$2^4 = 16$$

Now, $\Delta \ne 0$ if and only if either $ad = 1$

$bc = 0$ or $ad = 0$, $bc = 1$ for $ad = 1$ $(a, d) = (1, 1)$

For $bc = 0$ $(b, c) = (0, 1), (1, 0), (0, 0)$

So, three cases are there.

Similarly, for $ad = 0$ and $bc = 1$

There will be three cases.

$\therefore$ Number of favourable cases $= 3 + 3 = 6$

So, required probability $= \frac{6}{16} = \frac{3}{8}$

39. (a) We know,

$(\cot\alpha_1)(\cot\alpha_2)(\cot\alpha_3)\ ....\ (\cot\alpha_n) = 1$

$\Rightarrow \cos\alpha_1 \cos\alpha_2 \ ...\ \cos\alpha_n = \sin\alpha_1 \sin\alpha_2 \sin\alpha_3 ... \sin\alpha_n$

Multiply $(\cos\alpha_1\ \cos\alpha_2 \cos\alpha_3\ ....\ \cos\alpha_n)$ both side

$(\cos\alpha_1\ \cos\alpha_2\ ....\ \cos\alpha_n)^2$

$= (\cos\alpha_1\ \sin\alpha_1)(\cos\alpha_2 \sin\alpha_2)\ ....\ (\cos\alpha_n \sin\alpha_n)$

$\Rightarrow (\cos\alpha_1\ \cos\alpha_2\ ...\ \cos\alpha_n)^2 = \frac{1}{2^n}$

$(\sin 2\alpha_1)(\sin 2\alpha_2)\ ....\ (\sin 2\alpha_n)$

$\Rightarrow (\cos\alpha_1\ \cos\alpha_2\ ....\ \cos\alpha_n)^2 \le \frac{1}{2^n}$

$\Rightarrow \cos\alpha_1\ \cos\alpha_2\ ....\ \cos\alpha_n \le \frac{1}{2^{n/2}}$

Thus, the maximum value of

$(\cos\alpha_1, \cos\alpha_2\ ....\ \cos\alpha_n)$ is $\frac{1}{2^{n/2}}$

40. (d) Let the equation of the line be $ax + by + c = 0$

According to the given condition, we have

$$\left|\frac{2a+c}{\sqrt{a^2+b^2}}\right| + \left|\frac{2b+c}{\sqrt{a^2+b^2}}\right| + \left|\frac{a+b+c}{\sqrt{a^2+b^2}}\right| = 0$$

$$\Rightarrow \pm\frac{2a+c}{\sqrt{a^2+b^2}} \pm \left(\frac{2b+c}{\sqrt{a^2+b^2}}\right) \pm \left(\frac{a+b+c}{\sqrt{a^2+b^2}}\right) = 0$$

$$\Rightarrow \quad 2a + c + 2b + c + a + b + c = 0$$

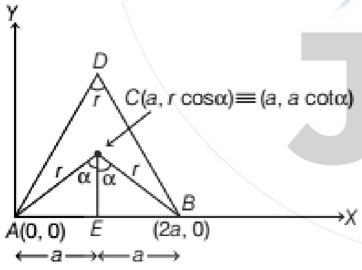
$$\Rightarrow \quad 3a + 3b + 3c = 0$$

$$\Rightarrow \quad a + b + c = 0$$

Comparing the line $ax + by + c = 0$ and $a + b + c = 0$ and get the fixed point.

$\therefore$ The required fixed point is (1, 1).

41. (b) Let $AC = BC = r$



Now, from ΔACE

$$r\sin\alpha = a$$

$$r = a\ \text{cosec}\,\alpha$$

$\Rightarrow$ Now, again in ΔACE

$CE = r\cos\alpha = (a\text{cosec}\alpha)\cos\alpha = a\cot\alpha$

Now locus of the vertex will be a circle with radius $r = a\text{cosec}\alpha$ and cetnre $C(a, a\cot\alpha)$

$\therefore$ Required locus is

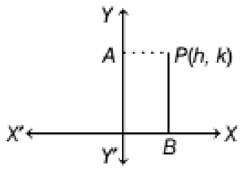
$$(x-a)^2 + (y - a\cot\alpha)^2 = a^2\text{cosec}^2\alpha$$

$$x^2 + a^2 - 2ax + y^2 + a^2\cot^2\alpha - 2ay\cot\alpha$$

$$= a^2\text{cosec}^2\alpha$$

$$\Rightarrow \quad x^2 + y^2 - 2ax - 2ay\cot\alpha = 0$$

42. (c) Let the locus of a point in a plane be $P(h, k)$.



According to the question,

$$|PA| + |PB| = 1$$

$$\Rightarrow \quad |h| + |k| = 1$$

Hence, locus of a point is $|x| + |y| = 1$ which represents the straight line.

43. (b) Let $\theta = \sin^{-1}\frac{3}{5}$

$$\Rightarrow \quad \sin\theta = \frac{3}{5} \Rightarrow \tan\theta = \frac{3}{4}$$

So, equation of the line passes through $(-1, 1)$ with slope $\frac{3}{4}$ is $y - 1 = \left(\frac{3}{4}\right)(x+1)$

$$3x - 4y + 7 = 0$$

Which meets the curve $x^2 = 4y - 9$ at points for which $3x - (x^2 + 9) + 7 = 0$ or $x^2 - 3x + 2 = 0$

$\Rightarrow x = 1, 2$ and the coordinates of A are $\left(1, \frac{10}{4}\right)$ and of B are $\left(2, \frac{13}{4}\right)$.

So, that $|AB| = \sqrt{(2-1)^2 + \left(\frac{13}{4} - \frac{10}{4}\right)^2} = \frac{5}{4}$ units

44. (c) We know that,

The equation of common chord of two circles $S = x^2 + y^2 + 2gx + 2fy + c = 0$

and $S' = x^2 + y^2 + 2g'x + 2f'y + c' = 0$ is

$2x(g - g') + 2y(f - f') + c - c' = 0$

i.e. $S - S' = 0$

45. (b) An equation of the normal at P is

$3x\cos\theta + 2y\cot\theta = 9 + 4$

$$\Rightarrow \quad 3x + xy\,\text{cosec}\theta = 13\sec\theta \quad ...\text{(i)}$$

An equation of normal at Q is

$$3x + 2y\ \text{cosec}\phi = 13\sec\phi \quad ...\text{(ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$2y\,[\operatorname{cosec}\theta - \operatorname{cosec}\phi] = 13\,(\sec\theta - \sec\phi)]$$

$$\Rightarrow \quad 2y\,[\operatorname{cosec}\theta - \sec\theta] = 13(\sec\theta - \operatorname{cos}\operatorname{ec}\phi)]$$

$$\left[\because \phi = \frac{\pi}{2} - \theta\right]$$

$$y = -\frac{13}{2}$$

46. (d) Let the coordinates of Q be (h, k)

and α and β be the mid-point of PQ.

Then,

$$\alpha = \frac{2+h}{2},$$

$$h = 2\alpha - 2$$

and $$\beta = \frac{0+k}{2}$$

$$k = 2\beta$$

Since, $Q(h, k)$ lies on $(y-6)^2 = 2(x-4)$

$$\Rightarrow \quad (k-6)^2 = 2(h-4) \qquad \ldots \text{(i)}$$

Put the value of h and k in Eq. (i)

$$(2\beta - 6)^2 = 2(2\alpha - 2 - 4)$$

$$\Rightarrow \quad 4(\beta - 3)^2 = 2(2\alpha - 6)$$

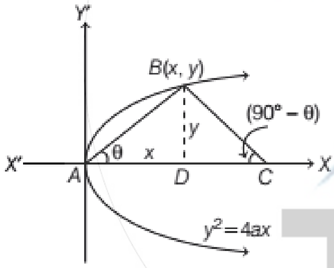
$$\Rightarrow \quad 2(\beta - 3)^2 = 2(\alpha - 3)$$

$$\Rightarrow \quad \beta^2 + 9 - 6\beta = \alpha - 3$$

$$\alpha - \beta^2 + 6\beta - 12 = 0$$

$\therefore$ The locus of (h, k) is $x - y^2 + 6y - 12 = 0$

47. (d) We know that,



In ΔABD, we have

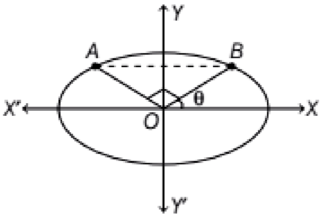
$$\tan\theta = \frac{y}{x} \qquad \ldots \text{(i)}$$

In ΔBCD, we have $\tan(90° - \theta) = \dfrac{y}{CD}$

$$CD = y\tan\theta = \frac{y^2}{x} \qquad \text{[by using Eq. (i)]}$$

$$CD = \frac{4ax}{x} = 4a \qquad [\because y^2 = 4ax]$$

48. (a) Suppose OB makes an angle θ with the positive direction of the X-axis, then OA makes an angle of $\theta + \dfrac{\pi}{2}$ with the positive direction of the X-axis.



Let $OA = r_1$ and $OB = r_2$ coordinates of A and B are $(-r_1\sin\theta, r_1\cos\theta)$ and $(r_2\cos\theta, r_2\sin\theta)$ respectively.

As B lies on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{r_2^2\cos^2\theta}{a^2} + \frac{r_2^2\sin^2\theta}{b^2} = 1$$

$$\Rightarrow \quad \frac{1}{OB^2} = \frac{1}{r_2^2} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} \qquad \ldots \text{(i)}$$

Similarly, as A lies on the ellipse

$$\frac{1}{OA^2} = \frac{1}{r_1^2} = \frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2} \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{OA^2} + \frac{1}{OB^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

49. (c) The equation of the given first plane is

$$x + y + z - 1 = 0 \qquad \ldots \text{(i)}$$

where (1, 1, 1) are direction ratio of normal to the plane.

The equation of second plane is

$$2x + 3y - z + 4 = 0 \qquad \ldots \text{(ii)}$$

where (2, 3, −1) are direction ratio of normal to the second plane.

We know that, the equation of any plane passes through the intersection of two given planes

$ax + by + cz + d = 0$ and $a_1x + b_1y + c_1z + d_1 = 0$ is

$ax + by + cz + d + \lambda(a_1x + b_1y + c_1z + d_1) = 0 \ldots$ (iii)

where λ is arbitrary constant.

Here, $a = 1$, $b = 1$, $c = 1$, $d = -1$

$$a_1 = 2,\ b_1 = 3,\ c_1 = -1,\ d_1 = 4$$

Putting these values in Eq. (iii), we get

$$x + y + z - 1 + \lambda(2x + 3y - z + 4) = 0$$

$$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 - \lambda) - 1 + 4\lambda = 0 \qquad \ldots \text{(iv)}$$

It is given that the plane (iv) is parallel to the X-axis, then the plane (iv) is perpendicular to the YZ, plane. Therefore, the normal of the plane (iv) and the YZ-plane are also perpendicular to each other.

Equation of YZ plane is $x = 0$ i.e.

$$1 \cdot x + 0 \cdot y + 0 \cdot z = 0$$

$$\Rightarrow \quad 1(1 + 2\lambda) + 0(1 + 3\lambda) + 0(1 - \lambda) = 0$$

$$1 + 2\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$$

Put the value of $\lambda = -\frac{1}{2}$ in Eq. (iv), we get

$$x\left(1 - \frac{2}{2}\right) + y\left(1 - \frac{3}{2}\right) + z\left(1 + \frac{1}{2}\right) - 1 + 4\left(-\frac{1}{2}\right) = 0$$

$$x \cdot 0 + y\left(-\frac{1}{2}\right) + \frac{3z}{2} - 1 - 2 = 0$$

$$-y + 3z - 6 = 0$$

$\therefore$ Required equation of plane $= y - 3z + 6 = 0$

50. (d) Given, the equations of the line,

$$x - 2y + 4z + 4 = 0 \quad \ldots\text{(i)}$$

$$x + y + z - 8 = 0 \quad \ldots\text{(ii)}$$

and equation of the plane

$$x - y + 2z + 1 = 0 \quad \ldots\text{(iii)}$$

Solve Eqs. (i), (ii) and (iii), we get

$$x = 2,\ y = 5 \text{ and } z = 1$$

Hence, the required point is (2, 5, 1).

51. (d) For $0 < x < 1$, we have $\frac{1}{2}x^2 < x^2 < x$

i.e. $-x^2 > -x$. So that, $e^{-x^2} > e^{-x}$

Hence, $\int_0^1 e^{-x^2} \cos^2 x\, dx > \int_0^1 e^{-x} \cos^2 x\, dx$.

Also $\cos^2 x \le 1$, therefore

$$\int_0^1 e^{-x^2} \cos^2 x\, dx \le \int_0^1 e^{-x^2} dx < \int_0^1 e^{-x^2/2} dx = I_4$$

Hence, I_4 is the greatest integral.

52. (b) $\lim\limits_{x \to \infty} \left(\frac{x^2 + 1 + 2x - 2x}{x + 1} - ax - b\right) = 0$

$$\Rightarrow \lim_{x \to \infty} \left(\frac{(x+1)^2 - 2x}{x + 1} - ax - b\right) = 0$$

$$\Rightarrow \lim_{x \to \infty} \left((x + 1) - \frac{2x}{x + 1} - ax - b\right) = 0$$

$$\Rightarrow \lim_{x \to \infty} \left[(x + 1) - \left(\frac{2x + 2 - 2}{x + 1}\right)\right] - \lim_{x \to \infty} (ax + b) = 0$$

$$\Rightarrow \lim_{x \to \infty} \left[x + 1 - \left(\frac{2x + 2 - 2}{x + 1}\right)\right] = \lim_{x \to \infty} (ax + b)$$

$$\Rightarrow \lim_{x \to \infty} \left[x + 1 - 2 + \frac{2}{x + 1}\right] = \lim_{x \to \infty} (ax + b)$$

$$\Rightarrow \lim_{x \to \infty} (x - 1) + 0 = a \lim_{x \to \infty} x + b$$

$$\Rightarrow \lim_{x \to \infty} x - 1 = a \lim_{x \to \infty} x + b$$

On comparing the above equation, we get

$a = 1$, $b = -1$

53. (b) We have, $z = \log \tan\left(\frac{x}{2}\right)$

Now, $\frac{dz}{dx} = \frac{1}{2}\frac{\sec^2(x/2)}{\tan(x/2)} = \frac{1}{\sin x} = \operatorname{cosec} x$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \operatorname{cosec} x \frac{dy}{dz}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dz}\left(\frac{dz}{dx}\right)\frac{dy}{dx}$$

$$= \frac{d}{dz}\left(\operatorname{cosec} x \frac{dy}{dz}\right) \operatorname{cosec} x$$

$$= \left[\operatorname{cosec} x \frac{d^2y}{dz^2} - \operatorname{cosec} x \cot x \frac{dx}{dz} \frac{dy}{dz}\right] \operatorname{cosec} x$$

Putting these values in the given differential equation, we have

$$\frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$$

$$\Rightarrow \operatorname{cosec}^2 x \frac{d^2y}{dz^2} - \operatorname{cosec} x \cot x \frac{dy}{dz} + \cot x$$

$$\operatorname{cosec} x \frac{dy}{dz} + 4y \operatorname{cosec}^2 x = 0$$

$$\Rightarrow \operatorname{cosec}^2 x\left(\frac{d^2y}{dz^2} + 4y\right) = 0 \Rightarrow \frac{d^2y}{dz^2} + 4y = 0$$

On comparing $\frac{d^2}{dx^2} + ky = 0$, then

$$k = 4$$

54. (d) Equation of tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$

Since, tangent passes through $(-1, -6)$

$$-6 = -m + \frac{1}{m}$$

$$\Rightarrow \quad m^2 - 6m - 1 = 0$$

On solving above equation, we get

$$m_1 = 6.16$$

$$m_2 = -0.16$$

$$m_1 m_2 = 6.16 \times (-0.16)$$

$$m_1 m_2 \cong -1$$

$\therefore$ The angle between them is 90° (approx).

55. (d) Given, $\beta = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$$\gamma = \hat{\mathbf{i}} + \hat{\mathbf{k}}$$

So, $[\alpha \quad \beta \quad \gamma] = \alpha \cdot [\beta \times \gamma]$

$$= \alpha \cdot \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix} = \alpha \cdot [\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}]$$

$$= |\alpha| \sqrt{1 + 4 + 1} \cos\theta$$

$$= |\alpha| \sqrt{6} \cos\theta$$

$$= \sqrt{6} \cos\theta$$

Max $[\alpha \quad \beta \quad \gamma] = \sqrt{6} \times 1 = \sqrt{6}$

Hence, max value $[\alpha \cdot \beta \cdot \gamma] = \sqrt{6}$

56. (c) We have $f(x) = e^{\sin x} + e^{\cos x}$

$$\therefore \quad \frac{e^{\sin x} + e^{\cos x}}{2} \geq e^{\sqrt{\sin x + \cos x}}$$

$$\frac{e^{\sin x} + e^{\cos x}}{2} \geq e^{-\frac{\sqrt{2}}{2}}$$

$$e^{\sin x} + e^{\cos x} \geq 2e^{-\frac{\sqrt{2}}{2}}$$

$$\geq 2e^{-\frac{1}{\sqrt{2}}}$$

Thus, maximum value of $f(x)$ is $2e^{1/\sqrt{2}}$.

57. (b) Let the line be $\frac{x}{a} + \frac{y}{b} = 1$

$\therefore$ A is $(a, 0)$, B is $(0, b)$.

Also, O is $(0, 0)$.

$\therefore$ Equation of circumcircle to ΔOAB is

$$x^2 + y^2 - ax - by = 0$$

Tangent at (0, 0) to the circle is

$$x \cdot 0 + y \cdot 0 + \frac{a}{2}(x + 0) + \frac{b}{2}(y + 0) = 0$$

$$\Rightarrow \quad ax + by = 0$$

$$m = \frac{a \cdot a + b \cdot 0}{\sqrt{a^2 + b^2}} = \frac{a^2}{\sqrt{a^2 + b^2}}$$

$$n = \frac{a \cdot 0 + b \cdot b}{\sqrt{a^2 + b^2}} = \frac{b^2}{\sqrt{a^2 + b^2}}$$

$$m + n = \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} = \sqrt{a^2 + b^2}$$

Radius of circle $= \frac{\sqrt{a^2 + b^2}}{2}$

Hence, diameter of circle is $m + n$.

58. (a) Tangent at P is $ty = x + at^2$, which meets axis at $T(-at^2, 0)$. Normal at P is $tx + y = 2at + at^3$, which meets axis at $G(2a + at^2, 0)$

$\therefore$ $\angle TPG = \frac{\pi}{2}$, so TG is diameter of the circle.

Equation of circle, is

$$(x + at^2)(x - 2a - at^2) + (y - 0)(y - 0) = 0$$

$$x^2 + y^2 - 2ax - at^2(2a + at^2) = 0$$

Here, $\quad g = -a$

$$c = at^2(2a + at^2)$$

$$f = 0$$

So, radius $(r) = \sqrt{g^2 + f^2 - c}$

$$\Rightarrow \quad r = \sqrt{(-a)^2 + 0 + at^2(2a + at^2)}$$

$$r = a\sqrt{1 + 2t^2 + t^4}$$

$$= a(1 + t^2)$$

$$r = a(1 + t^2)$$

59. (b) Suppose α, β be the roots of the given equation

Then,

$\alpha + \beta = a - 2$ and $\alpha\beta = -(a + 1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a - 2)^2 + 2(a + 1)$

$\alpha^2 + \beta^2 = a^2 - 2a + 6$

$= (a - 1)^2 + 5$

$\alpha^2 + \beta^2 \geq 5.$

So, the minimum value of $\alpha^2 + \beta^2$ is 5,

which attains at $a = 1$

60. (b) $\frac{\log x^2}{\log 25} + \left[\frac{\log x}{\log 5}\right]^2 < 2$

$$\Rightarrow \quad \frac{2\log x}{2\log 5} + \left[\frac{\log x}{\log 5}\right]^2 - 2 < 0 \quad \left[\text{Here, } y = \frac{\log x}{\log 5}\right]$$

$$\Rightarrow \quad y^2 + y - 2 < 0$$

$$\Rightarrow \quad -2 < y < 1$$

$$\Rightarrow \quad -2 < \frac{\log x}{\log 5} < 1$$

$$\Rightarrow \quad -2\log 5 < \log x < \log 5$$

$$\Rightarrow \quad \frac{1}{25} < x < 5, \text{ i.e. } x \in \left(\frac{1}{25}, 5\right)$$

61. (d) Given, $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$ and $|A - \lambda I_2| = 0$

Now, $A - \lambda I_2 = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{bmatrix}$$

$\therefore \quad |A - \lambda I_2| = 0$

$$\Rightarrow \quad \begin{vmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (2-\lambda)(y-\lambda) - 3x = 0$

$\Rightarrow \quad 2y - 2\lambda - \lambda y + \lambda^2 - 3x = 0$

When $\lambda = 4$, then $2y - 8 - 4y + 16 - 3x = 0$

$\Rightarrow \quad 3x + 2y - 8 = 0 \quad \dots \text{(i)}$

and when $\lambda = 8$ then $2y - 16 - 8y + 64 - 3x = 0$

$\Rightarrow \quad 3x + 6y - 48 = 0 \quad \dots \text{(ii)}$

On solving Eqs. (i) and (ii), we get

$$x = -4, \ y = 10$$

62. (a) Let coordinates of P_1 be $(at_1^2, 2at_1)$, then coordinates of P_2 are $\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$.

Similarly, if coordinates of P_3 are $(at_2^2, 2at_2)$.

Then, coordinates of P_4 are $\left(\frac{a}{t_2^2}, \frac{-2a}{t_2}\right)$.

An equation of the chord P_1P_3 is

$$\frac{x - at_1^2}{at_2^2 - at_1^2} = \frac{y - 2at_1}{2at_2 - 2at_1}$$

or $\quad 2(x + at_1t_2) = (t_1 + t_2)y \quad \dots \text{(i)}$

Similarly, the equation of the chord P_2P_4 is

$$2\left(x + \frac{a}{t_1t_2}\right) = -\left(\frac{1}{t_1} + \frac{1}{t_2}\right)y$$

$2(t_1t_2x + a) = -(t_1 + t_2)y \quad \dots \text{(ii)}$

Adding Eqs. (i) and (ii), we get

$$2(1 + t_1t_2)(x + a) = 0$$

$$x + a = 0$$

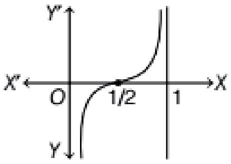
Therefore, intersection of point P_1P_3 and P_2P_4 lies on directrix.

63. (c) Given, $f(x) = \frac{2x-1}{1 - |2x-1|}$

$$\therefore \quad f(x) = \begin{cases} \frac{2x-1}{1 + 2x - 1}, & 0 < x < \frac{1}{2} \\ \frac{2x}{1 - 2x + 1}, & \frac{1}{2} \le x < 1 \end{cases}$$

$$= \begin{cases} \frac{2x-1}{2x}, & 0 < x < \frac{1}{2} \\ \frac{2x-1}{2-2x}, & \frac{1}{2} \le x < 1 \end{cases}$$

Now, let us draw the graph of above function.



From the above graph it is clear that $f(x)$ is one-one and since the range of $f(x)$ is real number, which implies that codomain is equal to range.

$\therefore$ $f(x)$ is one-one and onto both and hence $f(x)$ is bijective.

64. (c) Given, $\int_0^x \sqrt{1 - (f'(t))^2}\, dt = \int_0^x f(t)\, dt, \ 0 \le x \le \frac{\pi}{2}$

On differentiating both sides w.r.t. x by using Leibnitz rule, we get

$$\sqrt{1 - [f'(x)]^2} = f(x)$$

$\Rightarrow \quad f'(x) = \pm \sqrt{1 - [f(x)]^2}$

$\Rightarrow \quad \int \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}\, dx = \pm \int dx$

$\Rightarrow \quad \sin^{-1}[f(x)] = \pm x + c$

Put $x = 0 \Rightarrow \sin^{-1}[f(0)] = \pm 0 + c$

$\Rightarrow \quad c = \sin^{-1}(0) = 0 \quad [\because f(0) = 0]$

$\therefore \quad f(x) = \pm \sin x$

$\because$ f is a non-negative function defined in $\left[0, \frac{\pi}{2}\right]$.

$\therefore \quad f(x) = \sin x$

And we know that, $\sin x < x, \ \forall \ x > 0$

$\therefore \sin\left(\frac{1}{2}\right) < \frac{1}{2}$ and $\sin\left(\frac{1}{3}\right) < \frac{1}{3} \Rightarrow f\left(\frac{1}{2}\right) < \frac{1}{2}$

and $\quad f\left(\frac{1}{3}\right) < \frac{1}{3}$

Similarity $f\left(\frac{4}{3}\right) < \frac{4}{3}$ and $f\left(\frac{2}{3}\right) < \frac{2}{3}$

65. (d) Let the coordinates of P be (h, k).

$\therefore \quad PQ = 2k$ and $OP = OQ = PQ$

$\Rightarrow \quad \sqrt{h^2 + k^2} = 2k \ \Rightarrow h^2 + k^2 = 4k^2$

$\Rightarrow \quad h^2 = 3k^2$

$\therefore$ (h, k) lies on hyperbola $\Rightarrow \frac{h^2}{a^2} - \frac{k^2}{b^2} = 1$

$\Rightarrow \quad \frac{3k^2}{a^2} - \frac{k^2}{b^2} = 1$

$$\Rightarrow \quad \frac{3b^2 - a^2}{a^2b^2} = \frac{1}{k^2} > 0$$

$$\Rightarrow \quad \frac{3}{a^2} - \frac{1}{b^2} > 0 \Rightarrow \frac{3}{a^2} > \frac{1}{b^2}$$

$$\Rightarrow \quad \frac{b^2}{a^2} > \frac{1}{3} \Rightarrow e^2 - 1 > \frac{1}{3}$$

$$\Rightarrow \quad e^2 > \frac{4}{3} \Rightarrow e > 2/\sqrt{3}$$

66. (c) If h is the height of the balloon when stone is dropped. Then, using equation

$$h = -ut + \frac{1}{2}gt^2$$

Here $u = v$ ft/sec, $t = 4$ sec

$$h = -vt + \frac{1}{2}32t^2 = -4v + 256$$

$$h = 256 - 4v \qquad \ldots \text{(i)}$$

If balloon rises h' ft after dropping the balloon and height attained by balloon is 4 sec .

h' = velocity of balloon × time taken

$$= v \times 4$$

$$h' = 4v$$

Total height of the balloon from the ground

$$h + h' = 256 + 4v - 4v$$

$$= 256 \text{ ft}$$

67. (a, c, d) Given $f(x) = x^2 + x\sin x - \cos x$

Now $f'(x) = \frac{d}{dx}(x^2) + \frac{d}{dx}(x\sin x) - \frac{d}{dx}(\cos x)$

$$f'(x) = 2x + x\cos x + \sin x + \sin x$$

$$\Rightarrow \quad f'(x) = 2x + x\cos x + 2\sin x$$

$\Rightarrow f(x)$ is an increasing function.

Now, $f(0) = -1$

$$f(\infty) = +\infty$$

$$f(-\infty) = +\infty$$

$\because \quad f(-\infty) = +$ ve and $f(0) = -$ ve

$\therefore$ There must be atleast one negative root in $(-\infty, 0)$.

$\because \quad f(0) = -$ ve and $f(\infty) = +$ ve

$\therefore$ Then, must be atleast one positive root in $(0, \infty)$.

68. (a,d) We use properties of argument of complex numbers.

(i) $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2k\pi$ $(K = 0$ or 1 or $-1)$

(ii) $\arg(z_1z_2) = \arg(z_1) + \arg(z_2) + 2k\pi$ $(K = 0$ or 1 or $-1)$

69. (b,d) Let

$$\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$$

Apply $C_1 \to C_1 - \cot\phi C_2$, we get

$$\Delta = \begin{vmatrix} 0 & \sin\theta\sin\phi & \cos\theta \\ 0 & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta/\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$$

On expanding along C_1, we get

$$\Delta = -\frac{\sin\theta}{\sin\phi}[-\sin\phi\sin^2\theta - \cos^2\theta\sin\phi]$$

$$= \sin\theta, \text{ this is independent of } \phi$$

Also, $\frac{d\Delta}{d\theta} = \cos\theta$

$$\Rightarrow \quad \left.\frac{d\Delta}{d\theta}\right|_{\theta = \frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) = 0$$

70. (b) **Reflexivity** Let a be an arbitrary element of A.

Then, $a \in A$

$(a, a) \in R$ and $(a, a) \in S$ [$\because R$ and S are reflexive]

$(a, a) \in R \cap S$

Thus, $(a, a) \in R \cap S$ for all $a \in A$. So, $R \cap S$ is a reflexive relation on A.

Symmetry Let $a, b \in A$ such that $(a, b) \in R \cap S$

Then $(a, b) \in R \cap S$

$(a, b) \in R$ and $(a, b) \in S\, b\, (b, a) \in R$

and $(b, a) \in S$ [$\because S$ and R are symmetric]

$(b, a) \in R \cap S$ for all $(a, b) \in R \cap S$

So, $R \cap S$ in symmetric on A.

Transitive Let $a, b, c \in A$ such that $(a, b) \in R \cap S$ and $(b, c) \in R \cap S$

Then, $(a, b) \in R \cap S$ and $(b, c) \in R \cap S$

$\Rightarrow \{(a, b) \in R$ and $(b, c) \in S\}$

$\Rightarrow \{(a, b) \in R$ and $(b, c) \in R\}$

and $\{(a, b) \in S, (b, c) \in S\}$

$\Rightarrow (a, c) \in R\ (a, c) \in S$

$\therefore R$ and S are transitive so

$(a, b) \in R\ (b, c) \in R \Rightarrow (a, c) \in R$

$(a, b) \in S$ and $(b, c) \in S \Rightarrow (a, c) \in S$

$(a, c) \in R \cap S$

Thus, $(a, b) \in R \cap S$

$(a, c) \in R \cap S$

So, $R \cap S$ is transitive on A.

Hence, $R \cap S$ is an equivalence relation on A.

71. (c) Let (h, k) be the mid-point of a chord passing through the positive end of the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Then, the equation of the chord is

$$\frac{hx}{a^2}+\frac{ky}{b^2}-1=\frac{h^2}{a^2}+\frac{k^2}{b^2}-1$$

$$\Rightarrow \quad \frac{hx}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

Thus, curve passes through the minor axis $(0, b)$.

$$\therefore \frac{k}{b}=\frac{h^2}{a^2}+\frac{k^2}{b^2}.$$

Hence, the locus of (h, k) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{y}{b}$, which represent an ellipse.

72. (b, c) The above equation $(y-y_1)=m(x-x_1)$ is given. $y-y_1-m(x-x_1)=0$ is a family of lines

$$y-y_1=0 \Rightarrow y=y_1$$

$$x-x_1=0 \Rightarrow x=x_1$$

(b) There will be a set of parallel lines.

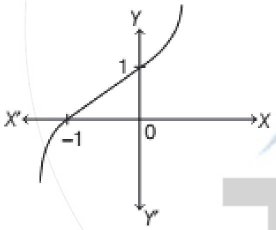
(c) all lines intersect the line $x = x_1$.

73. (c) $P(0)=1$

$$P'(x)>0;\ \forall x \in R$$

Let $P(x)=x^3+1$

$$P'(x)=3x^2$$



$\therefore$ $P(x)$ may have always be increasing as shown in above.

$P(x)$ may have negative real root.

74. (c) Perimeter of sector = 20 m

$$\frac{\theta}{360°}2\pi r+2r=20$$

$$\Rightarrow \quad \frac{\theta}{360°}\pi r+r=10$$

$$\Rightarrow \quad \frac{\theta}{360°}=\frac{10-r}{\pi r}$$

We know that , area of sector $A=\frac{\theta}{360°}\pi r^2$

$$A=\frac{10-r}{\pi r}.\ \pi r^2$$

$$A=(10-r)r$$

$$\frac{dA}{dr}=\frac{d}{dr}(10r-r^2)$$

$$=10-2r$$

For maximum area

$$\frac{dA}{dr}=0$$

$$\Rightarrow \quad 10-2r=0$$

$$\Rightarrow \quad r=5$$

$$\Rightarrow \quad \frac{d^2A}{dr^2}=\frac{d}{dr}(10-2r)=-2$$

$$\Rightarrow \quad \frac{d^2A}{dr^2}<0$$

Maximum value of $r = 5$ m

75. (a, b, c) Given $y = x + 5$

Comparing with $y = mx + c$

$$m=1 \text{ and } c=5$$

Solve by option

Option (a) Condition of tangency

$$c=\frac{a}{m} \quad \Rightarrow \quad 5=\frac{5}{1}$$

which is true.

Option (b) $9x^2+16y^2=144$

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

Condition of tangency $c^2=a^2m^2+b^2$

$$25=16\times1+9=25$$

which is true.

Option (c) $\frac{x^2}{29}-\frac{y^2}{4}=1$

$\because$ Condition of tengency

$$c^2=a^2m^2-b^2$$

$$25=29\times1-4=25$$

which is true.

Option (d) Now length of perpendicular from centre (0, 0) to the line $y = x + 5$ is $\frac{|5|}{\sqrt{2}}$

i.e. $\frac{5}{\sqrt{2}} \neq$ radius (5)