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UNIT & DIMENSION, BASIC MATHS AND VECTOR

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PHYSICAL QUANTITIES AND UNITS

Physical quantities :

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationship.

Measurement :

Measurement is the comparison of a quantity with a standard of the same physical quantity.

Classification :

Physical quantities can be classified on the following bases :

I. Based on their directional properties

1. **Scalars:**The physical quantities which have only magnitude but no direction are called *scalar quantities*. **Ex.** mass, density, volume, time, etc.
2. **Vectors :** The physical quantities which have both magnitude and direction and obey laws of vector algebra are called *vector quantities*. **Ex.** displacement, force, velocity, etc.

II. Based on their dependency

1. **Fundamental or base quantities :** A set of physical quanties which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.
2. **Derived quantities :** The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities*. **Ex.** Speed (=distance/time), volume, acceleration, force, pressure, etc.

Physical quantities can also be classified as dimensional and dimensionless quantities or constants and variables.

Ex. Classify the quantities displacement, mass, force, time, speed, velocity, acceleration, moment of intertia, pressure and work under the following categories :

(a) base and scalar (b) base and vector (c) derived and scalar (d) derived and vector

Ans. (a) mass, time (b) displacement (c) speed, pressure, work
(d) force, velocity, acceleration

Units of Physical Quantities

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity. Four basic properties of units are :

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

System of Units :

1. **FPS or British Engineering system :**

In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

2. **CGS or Gaussian system :**

In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).

3. **MKS system :**

In this system also the fundamental quantities are length, mass and time but their fundamental units are metre (m), kilogram (kg) and second (s) respectively.

4. **International system (SI) of units :**

This system is modification over the MKS system and so it is also known as *Rationalised MKS system*.Besides the three base units of MKS system four fundamental and two supplementary units are also included in this system.

Classification of Units : The units of physical quantities can be classified as follows :

1. **Fundamental or base units :**

The units of fundamental quantities are called *base units*. In SI there are seven base units.

SI BASE QUANTITIES AND THEIR UNITS

S.No.	Physical quantity	SI unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	Kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

2. **Derived units :**

The units of derived quantities or the units that can be expressed in terms of the base units are called *derived* units

Ex. Unit of speed $= \dfrac{\text{unit of distance}}{\text{unit of time}} = \dfrac{\text{metre}}{\text{second}} = \text{ms}^{-1}$

Some derived units are named in honour of great scientists.

• Unit of force – newton (N) • Unit of frequency – hertz (Hz) etc.

UNITS OF SOME PHYSICAL QUANTITIES IN DIFFERENT SYSTEMS

Type of Physical Quantity	Physical Quantity	CGS (Originated in France)	MKS (Originated in France)	FPS (Originated in Britain)
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton(N)	poundal
	Work or Energy	erg	joule(J)	ft-poundal
	Power	erg/s	watt(W)	ft-poundal/s

Dimensions :

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. To make it clear, consider the physical quantity force. As we shall learn later, force is equal to mass times acceleration. Acceleration is change in velocity divided by time interval. Velocity is length divided by time interval. Thus,

force = mass × acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}} = \text{mass} \times \frac{\text{length / time}}{\text{time}} = \text{mass} \times \text{length} \times (\text{time})^{-2}$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. The dimensions in all other base quantities are zero.

1. Dimensional formula :

The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to remind that the equation is among the dimensions and not among the magnitudes. Thus above equation may be written as [force]= MLT^{-2}.

Such an expression for a physical quantity in terms of the base quantities is called the dimensional formula. Thus, the dimensional formula of force is MLT^{-2}. The two versions given below are equivalent and are used interchangeably.

(a) The dimensional formula of force is MLT^{-2}.

(b) The dimensions of force are 1 in mass, 1 in length and –2 in time.

The *dimensional formula* of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.

Ex. Dimensional formula of mass is $[M^1L^0\,T^0]$ and that of speed (= distance/time) is $[M^0L^1T^{-1}]$

2. Applications of dimensional analysis :

(i) To convert a physical quantity from one system of units to the other :

This is based on a fact that *magnitude of a physical quantity remains same whatever system is used for measurement*

i.e. magnitude = numeric value (n) × unit (u) = constant or $n_1u_1 = n_2u_2$

So if a quantity is represented by $[M^aL^bT^c]$

Then $$\boxed{n_2 = n_1\left[\frac{u_1}{u_2}\right] = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$$

n_2	=	numerical value in II system
n_1	=	numerical value in I system
M_1	=	unit of mass in I system
M_2	=	unit of mass in II system
L_1	=	unit of length in I system
L_2	=	unit of length in II system
T_1	=	unit of time in I system
T_2	=	unit of time in II system

Ex. 1m = 100 cm= 3.28 ft = 39.4 inch

(SI) (CGS) (FPS)

Ex. The acceleration due to gravity is 9.8 m s^{-2}. Give its value in ft s^{-2}

Sol. As 1m = 3.2 ft $\quad \therefore$ 9.8 m/s² = 9.8 × 3.28 ft/s² = 32.14 ft/s² $\approx$ 32 ft/s²

Ex. Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is $[F] = [M^1\,L^1\,T^{-2}]$

Therefore if n_1, u_1, and n_2, u_2 corresponds to SI & CGS units respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2} = 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1 \times 1000 \times 100 \times 1 = 10^5$$

$\therefore$ 1 newton = 10^5 dyne.

Q. The value of Gravitational constant G in MKS system is 6.67×10^{-11} N–m^2/kg^2. What will be its value in CGS system ?

Ans. 6.67×10^{-8} cm^3/g s^2

(ii) ***To check the dimensional correctness of a given physical relation :***

If in a given relation, the terms on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the *principle of homogeneity of dimensions*.

Ex. Check the accuracy of the relation $T = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis.

Sol. The dimensions of LHS = the dimension of T = $[M^0 L^0 T^1]$

The dimensions of RHS = $\left(\frac{\text{dimensions of length}}{\text{dimensions of acceleration}}\right)^{1/2}$ ($\because$ 2π is a dimensionless constant)

$$= \left(\frac{L}{LT^{-2}}\right)^{1/2} = (T^2)^{1/2} = [T] = [\, M^0 L^0 T^1]$$

Since the dimensions are same on both the sides, the relation is correct.

(iii) ***To derive relationship between different physical quantities :***

Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Ex. It is known that the time of revolution T of a satellite around the earth depends on the universal gravitational constant G, the mass of the earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

Sol. We have $[T] = [G]^a [M]^b [R]^c$

$$[M]^0 [L]^0 [T]^1 = [M]^{-a} [L]^{3a} [T]^{-2a} \times [M]^b \times [L]^c = [M]^{b-a} [L]^{c+3a} [T]^{-2a}$$

Comparing the exponents

For [T] : $1 = -2a \Rightarrow a = -\frac{1}{2}$

For [M] : $0 = b - a \Rightarrow b = a = -\frac{1}{2}$

For [L] : $0 = c + 3a \Rightarrow c = -3a = \frac{3}{2}$

Putting the values we get $T = G^{-1/2} M^{-1/2} R^{3/2} = \sqrt{\frac{R^3}{GM}}$

So the actual expression is $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Limitations of this method :

(i) In Mechanics the formula for a physical quantity depending on more than three physical quantities cannot be derived. It can only be checked.

(ii) This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + at^2/2$ also can't be derived.

(iii) The relation derived from this method gives no information about the dimensionless constants.

(iv) If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.

(v) It gives no information whether a physical quantity is a scalar or a vector.

Units and Dimensions of Physical Quantities

Quantity	Common Symbol	SI unit	Dimension
Displacement	s	METRE (m)	L
Mass	m, M	KILOGRAM (kg)	M
Time	t	SECOND (s)	T
Area	A	m^2	L^2
Volume	V	m^3	L^3
Density	ρ	kg/m^3	M/L^3
Velocity	v, u	m/s	L/T
Acceleration	a	m/s^2	L/T^2
Force	F	newton (N)	ML/T^2
Work	W	joule (J) (=N – m)	ML^2/T^2
Energy	E, U, K	joule (J)	ML^2/T^2
Power	P	watt (W) (=J/s)	ML^2/T^3
Momentum	p	kg-m/s	ML/T
Gravitational constant	G	$N\text{-}m^2/kg^2$	L^3/MT^2
Angle	θ, φ	radian	
Angular velocity	ω	radian/s	T^{-1}
Angular acceleration	α	$radian/s^2$	T^{-2}
Angular momentum	L	$kg\text{-}m^2/s$	ML^2/T
Moment of inertia	I	$kg\text{-}m^2$	ML^2
Torque	τ	N-m	ML^2/T^2
Angular frequency	ω	radian/s	T^{-1}
Frequency	ν	hertz (Hz)	T^{-1}
Period	T	s	T
Young's modulus	Y	N/m^2	M/LT^2
Bulk modulus	B	N/m^2	M/LT^2
Shear modulus	η	N/m^2	M/LT^2
Surface tension	S	N/m	M/T^2
Coefficient of viscosity	η	$N\text{-}s/m^2$	M/LT
Pressure	P, p	N/m^2, Pa	M/LT^2
Wavelength	λ	m	L
Intensity of wave	I	W/m^2	M/T^3
Temperature	T	KELVIN (K)	K
Specific heat capacity	c	J/kg–K	L^2/T^2K
Stefan's constant	σ	$W/m^2\text{–}K^4$	M/T^3K^4
Heat	Q	J	ML^2/T^2
Thermal conductivity	K	W/m–K	ML/T^3K

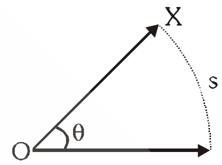
Basic Mathematics used in physics

Plane–angle

It is measure of change in direction.

If a line rotates in a plane about one of its ends, the other end sweeps an arc.

Angle (θ) between two orientations of the line is defined by ratio of the arc length(s) to length of the line(r) $\quad \theta = \frac{s}{r}$ radian

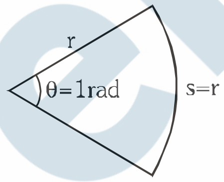


Angles measured in anticlockwise and clockwise directions are usually taken positive and negative respectively.

Angle is measured in radians (rad) or degrees. One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.

π rad= 180° $\qquad \pi = 3.1415 = \frac{22}{7}$

1°= 60' (minute), $\quad$ 1' (minute) = 60" (sec)

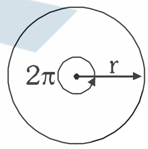


Example

Write expression for circumference of a circle of radius 'r'.

Solution

s = (Total angle about a point) r = $2\pi r$

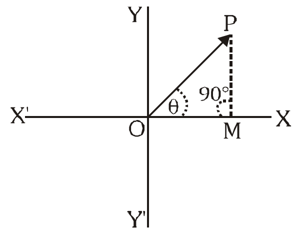


Trigonometrical ratios (or T–ratios)

Let two fixed lines XOX' and YOY' intersecting at right angles to each other at point O.

- Point O is called origin.
- Line XOX' is known as x–axis and YOY' as y–axis.
- Regions XOY, YOX', X'OY' and Y'OX are called I, II, III and IV quadrant respectively.

Consider a line OP making angle θ with OX as shown. Line PM is perpendicular drawn from P on OX. In the right angled triangle OPM, side OP is called hypotenuse, the side OM adjacent to angle θ is called base and the side PM opposite to angle θ is called the perpendicular.



Following ratios of the sides of a right angled triangle are known as trigonometrical ratios or T-ratio

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP} \quad \cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP} \quad \tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM}$$

$$\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP} \quad \sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM} \quad \text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$$

$$\text{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Some trigonometric identities

$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \text{cosec}^2\theta$

Example

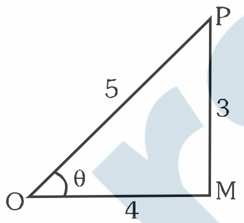
Given $\sin\theta = \frac{3}{5}$. Find all the other T–ratios, if θ lies in the first quadrant.

Solution

In Δ OMP , $\sin\theta = \frac{3}{5}$ So MP = 3 and OP = 5 $\because$ $OM = \sqrt{(5)^2 - (3)^2} = \sqrt{25-9} = \sqrt{16} = 4$

Now $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$ $\tan\theta = \frac{MP}{OM} = \frac{3}{4}$ $\cot\theta = \frac{OM}{MP} = \frac{4}{3}$

$\sec\theta = \frac{OP}{OM} = \frac{5}{4}$ $\operatorname{cosec}\theta = \frac{OP}{MP} = \frac{5}{3}$



T–ratios of some commonly used angles

Angle (θ)	0 rad	$\frac{\pi}{6}$ rad	$\frac{\pi}{4}$ rad	$\frac{\pi}{3}$ rad	$\frac{\pi}{2}$ rad
	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

sin (90°–θ)=cosθ cos (90°–θ)=sinθ tan (90°–θ)=cotθ	sin (180°–θ)=sinθ cos (180°–θ)=–cosθ tan (180°–θ)=–tanθ	sin (360°–θ)=–sinθ cos (360°–θ)= cosθ tan (360°–θ)=–tanθ
sin (90°+θ)=cosθ cos (90°+θ)=–sinθ tan (90°+θ)=–cotθ	sin (180°+θ)=–sinθ cos (180°+θ)=–cosθ tan (180°+θ)=tanθ	sin (–θ)=-sinθ cos (–θ)=cosθ tan (–θ)=–tanθ

- When θ is very small we can use following approximations : $\cos\theta \approx 1$

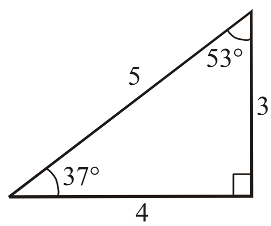
$$\left.\begin{array}{l}\sin\theta \simeq \theta \\ \tan\theta \simeq \theta\end{array}\right\} \text{If } \theta \text{ is in radians}$$

$\tan\theta \approx \sin\theta$.

- In the given right angled triangle we have very commonly used T-ratios

$\sin 37° = \frac{3}{5}$ $\cos 37° = \frac{4}{5}$ $\tan 37° = \frac{3}{4}$

$\sin 53° = \frac{4}{5}$ $\cos 53° = \frac{3}{5}$ $\tan 53° = \frac{4}{3}$



Example

Find the value of (i) $\cos(-60°)$ (ii) $\tan 210°$ (iii) $\sin 300°$ (iv) $\cos 120°$

Solution

(i) $\cos(-60°) = \cos 60° = \frac{1}{2}$

(ii) $\tan 210° = \tan(180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

(iii) $\sin 300° = \sin(270° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

VECTORS

Precise description of laws of physics and physical phenomena requires expressing them in form of mathematical equations. In doing so we encounter several physical quantities, some of them have only magnitude and other have direction in addition to magnitude. Quantities of the former kind are referred as scalars and the latter as vectors and mathematical operations with vectors are collectively known as vector analysis.

Vectors

A vector has both magnitude and sense of direction, and follows triangle law of vector addition. For example, displacement, velocity, and force are vectors.

Vector quantities are usually denoted by putting an arrow over the corresponding letter, as $\vec{A}$ or $\vec{a}$. Sometimes in print work (books) vector quantities are usually denoted by boldface letters as **A** or **a**.

Magnitude of a vector $\vec{A}$ is a positive scalar and written as $|\vec{A}|$ or A.

Geometrical Representation of Vectors.

A vector is represented by a directed straight line, having the magnitude and direction of the quantity represented by it.

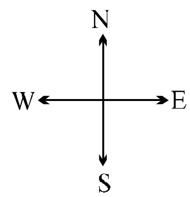
e.g. if we want to represent a force of 5 N acting 45° N of E

(i) We choose direction co-ordinates.

(ii) We choose a convenient scale like 1 cm ≡ 1N

(iii) We draw a line of length equal in magnitude and in the direction of vector to the chosen quantity.

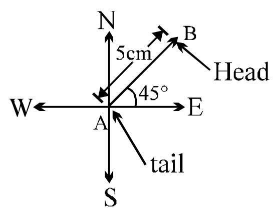
(iv) We put arrow in the direction of vector.



$\overrightarrow{AB}$

Magnitude of vector:

$$|\overrightarrow{AB}| = 5N$$

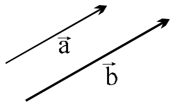


1 cm ≡ 1 N

By definition magnitude of a vector quantity is scalar and <u>is always positive</u>.

TERMINOLOGY OF VECTORS

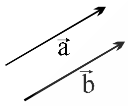
1. ***Parallel vector:*** If two vectors have same direction, they are parallel to each other. They may be located anywhere in the space.



Antiparallel vectors: When two vectors are in opposite direction they are said to be antiparallel vectors.

Equality of vectors: When two vectors have equal magnitude and are in same direction and represent the same physical quantity, they are equal.

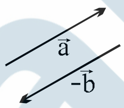
i.e. $\vec{a} = \vec{b}$



Thus when two parallel vectors have same magnitude they are equal. (Their initial point & terminal point may not be same)

Negative of a vector: When a vector have equal magnitude and is in opposite direction, it is said to be negative vector of the former.

i.e. $\vec{a} = -\vec{b}$ or $\vec{b} = -\vec{a}$

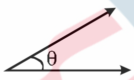


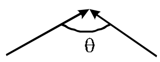
Thus when two antiparallel vectors have same magnitude they are negative of each other.

Remark : Vector shifting is allowed without change in their direction.

2. Angle Between two Vectors

It is the smaller angle formed when the initial points or the terminal points of the two vectors are brought together. It should be noted that $0^\circ \le \theta \le 180^\circ$.





3. Addition Of Vectors:

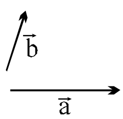
Parallelogram law of addition:

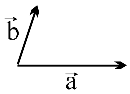
Steps:

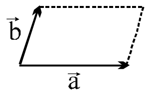
(i) Keep two vectors such that there tails coincide.

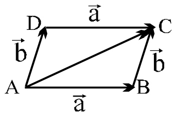
(ii) Draw parallel vectors to both of them considering both of them as sides of a parallelogram.

(iii) Then the diagonal drawn from the point where tails coincide represents the sum of two vectors, with its tail at point of coincidence of the two vectors.



(i) 

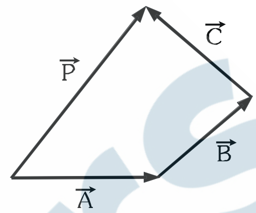
(ii) 

(iii) 

$\overrightarrow{AC} = \vec{a} + \vec{b}$

Addition of more than two Vectors

The triangle law can be extended to define addition of more than two vectors. Accordingly, if vectors to be added are drawn in head to tail fashion, resultant is defined by a vector drawn from the tail of the first vector to the head of the last vector. This is also known as the ***polygon rule for vector addition.***



Operation of addition of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ and their resultant $\vec{P}$ are shown in figure.

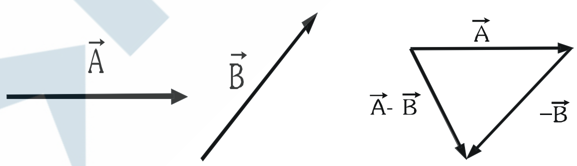
$\vec{A} + \vec{B} + \vec{C} = \vec{P}$

Here it is not necessary that three or more vectors and their resultant are coplanar. In fact, the vectors to be added and their resultant may be in different planes. However if all the vectors to be added are coplanar, their resultant must also be in the same plane containing the vectors.

Subtraction of Vectors

A vector opposite in direction but equal in magnitude to another vector $\vec{A}$ is known as negative vector of $\vec{A}$. It is written as $-\vec{A}$. Addition of a vector and its negative vector results a vector of zero magnitude, which is known as a null vector. A null vector is denoted by arrowed zero $(\vec{0})$.

The idea of negative vector explains operation of subtraction as addition of negative vector. Accordingly to subtract a vector from another consider vectors $\vec{A}$ and $\vec{B}$ shown in the figure. To subtract $\vec{B}$ from $\vec{A}$, the negative vector $-\vec{B}$ is added to $\vec{A}$ according to the triangle law as shown in figure-II.



If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\overline{OA}$ & $\overline{OB}$ then their sum $\vec{a} + \vec{b}$ is a vector represented by $\overline{OC}$, where OC is the diagonal of the parallelogram OACB.

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (commutative)
- $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ (associativity)
- $\vec{a} + \vec{0} = \vec{a} = \vec{0} + \vec{a}$
- $\vec{a} + (-\vec{a}) = \vec{0} = (-\vec{a}) + \vec{a}$
- $|\vec{a} + \vec{b}| \le |\vec{a}| + |\vec{b}|$
- $|\vec{a} - \vec{b}| \ge ||\vec{a}| - |\vec{b}||$
- $\left|\vec{a} \pm \vec{b}\right| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 \pm 2|\vec{a}||\vec{b}|\cos\theta}$ where θ is the angle between the vectors

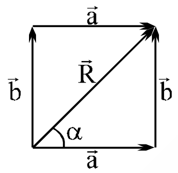
Some Important Results :

(1) If $\theta = 0° \Rightarrow \vec{a} \parallel \vec{b}$

then, $|\vec{R}| = |\vec{a}| + |\vec{b}|$ & $|\vec{R}|$ is maximum

(2) If $\theta = \pi \Rightarrow \vec{a}$ anti $\parallel \vec{b}$

then, $|\vec{R}| = \left||\vec{a}| - |\vec{b}|\right|$ & $|\vec{R}|$ is minimum



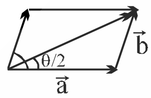
(3) If $\theta = \pi/2 \Rightarrow \vec{a} \perp \vec{b}$

$R = \sqrt{a^2 + b^2}$

& $\tan\alpha = b/a$ (α is angle made by $\vec{R}$ with $\vec{a}$)

(4) $|\vec{a}| = |\vec{b}| = a$

$|\vec{R}| = 2a\cos\theta/2$ & $\alpha = \theta/2$



(5) If $|\vec{a}| = |\vec{b}| = a$ & $\theta = 120°$

then $|\vec{R}| = a$

4. Multiplication Of A Vector By A Scalar:

If $\vec{a}$ is a vector & m is a scalar, then m $\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|m|$ times that of $\vec{a}$.

This multiplication is called SCALAR MULTIPLICATION. If $\vec{a}$ and $\vec{b}$ are vectors & m, n are scalars, then:

$m\ (\vec{a}) = (\vec{a})\ m = m\vec{a}$

$m\ (n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$

$(m+n)\ \vec{a} = m\vec{a} + n\vec{a}$

$m\ (a + \vec{b}) = m\vec{a} + m\vec{b}$

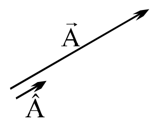
Resolution of a Vector into Components

Following laws of vector addition, a vector can be represented as a sum of two (in two-dimensional space) or three (in three-dimensional space) vectors each along predetermined directions. These directions are called axes and parts of the original vector along these axes are called components of the vector.

UNIT VECTOR:

A unit vector is a vector of magnitude of 1, with no units. Its only purpose is to point, i.e. to describe a direction in space.

A unit vector in direction of vector $\vec{A}$ is represented as $\hat{A}$



& $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

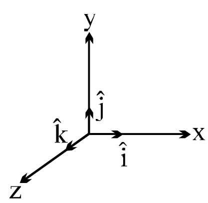
or $\vec{A}$ can be expressed in terms of a unit vector in its direction i.e. $\vec{A} = |\vec{A}|\hat{A}$

<ins>Unit Vectors along three coordinates axes</ins>:–

unit vector along x-axis is $\hat{i}$

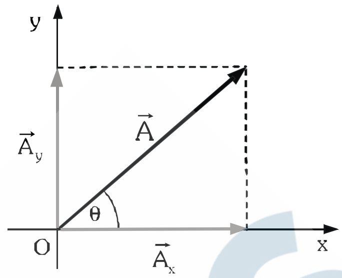
unit vector along y-axis is $\hat{j}$

unit vector along z-axis is $\hat{k}$



Cartesian components in two dimensions

If a vector is resolved into its components along mutually perpendicular directions, the components are called Cartesian or rectangular components.



In figure is shown, a vector $\vec{A}$ resolved into its Cartesian components $\vec{A}_x$ and $\vec{A}_y$ along the x and y-axis. Magnitudes A_x and A_y of these components are given by the following equation.

$A_x = A\cos\theta$ and $A_y = A\sin\theta$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

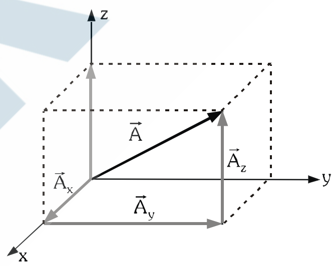
$$A = \sqrt{A_x^{\ 2} + A_y^{\ 2}}$$

Here $\hat{i}$ and $\hat{j}$ are the unit vectors for x and y coordinates respectively.

Mathematical operations e.g. addition, subtraction, differentiation and integration can be performed independently on these components. This is why in most of the problems use of Cartesian components becomes desirable.

Cartesian components in three dimensions

A vector $\vec{A}$ resolved into its three Cartesian components one along each of the directions x, y, and z-axis is shown in the figure.



$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

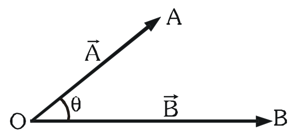
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Product of Vectors

In all physical situation, whose description involve product of two vectors, only two categories are observed. One category where product is also a vector involves multiplication of magnitudes of two vectors and sine of the angle between them, while the other category where product is a scalar involves multiplication of magnitudes of two vectors and cosine of the angle between them. Accordingly, we define two kinds of product operation. The former category is known as vector or cross product and the latter category as scalar or dot product.

Scalar or dot product of two vectors

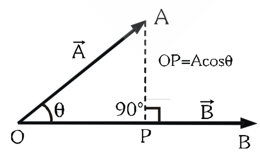
The scalar product of two vectors $\vec{A}$ and $\vec{B}$ equals to the product of their magnitudes and the cosine of the angle θ between them.

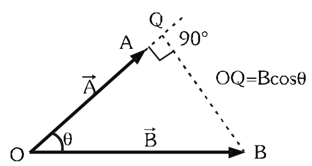


$$\vec{A}\cdot\vec{B} = AB\cos\theta = OA\cdot OB\cdot\cos\theta$$

The above equation can also be written in the following ways.

$$\vec{A}\cdot\vec{B} = (A\cos\theta)B = OP\cdot OB \qquad \vec{A}\cdot\vec{B} = A(B\cos\theta) = OA\cdot OQ$$

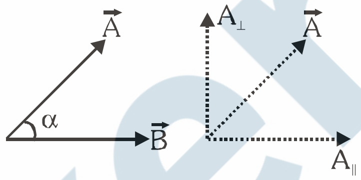




Above two equations and figures, suggest a scalar product as product of magnitude of the one vector and magnitude of the component of another vector in the direction of the former vector.

KEY POINTS

- Dot product of two vectors is commutative: $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$
- If two vectors are perpendicular, their dot product is zero. $\vec{A}\cdot\vec{B}=0$, if $\vec{A}\perp\vec{B}$
- Dot product of a vector by itself is known as self-product. $\vec{A}\cdot\vec{A}=A^2 \Rightarrow A=\sqrt{\vec{A}\cdot\vec{A}}$
- The angle between the vectors $\theta=\cos^{-1}\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right)$
- (a) Component of $\vec{A}$ in direction of $\vec{B}$



$$\vec{A}_{||}=\left(|\vec{A}|\cos\theta\right)\hat{B}=|\vec{A}|\left(\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\right)\hat{B}=\left(\frac{\vec{A}.\vec{B}}{|\vec{B}|}\right)\hat{B}=(\vec{A}.\hat{B})\hat{B}$$

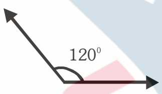
(b) Component of $\vec{A}$ perpendicular to $\vec{B}$: $\vec{A}_\perp=\vec{A}-\vec{A}_{||}$

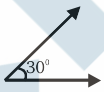
- Dot product of Cartesian unit vectors: $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$
  $\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$
- If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, their dot product is given by

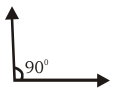
  $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$

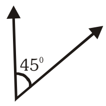
Solved Examples

1. Two displacement vectors of same magnitude are arranged in the following manner

(I) 

(II) 

(III) 

(IV) 

Magnitude of resultant is maximum for

(A) case I (B) case II (C) case III (D) case IV

Ans. (B)

Sol. Magnitude of Resultant of $\vec{A}$ and $\vec{B}$ = $\sqrt{A^2+B^2+2AB\cos\theta}$ which is maximum for $\theta=30^0$

2. Two vectors $\vec{P}$ and $\vec{Q}$ are added, the magnitude of resultant is 15 units. If $\vec{Q}$ is reversed and added to $\vec{P}$ resultant has a magnitude $\sqrt{113}$ units. The resultant of $\vec{P}$ and a vector perpendicular to $\vec{P}$ and equal in magnitude to $\vec{Q}$ has a magnitude

(A) 13 units (B) 17 units (C) 19 units (D) 20 units

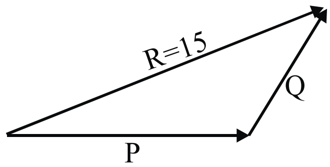
Ans. (A)

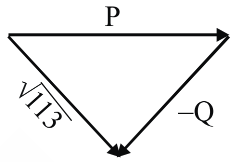
Sol. $P^2 + Q^2 + 2PQ\cos\theta = 225$ ...(i)

$P^2 + Q^2 - 2PQ\cos\theta = 113$ ...(ii)

By adding (i) & (ii) $2(P^2 + Q^2) = 338$

$P^2 + Q^2 = 169 \Rightarrow \sqrt{P^2 + Q^2} = 13$





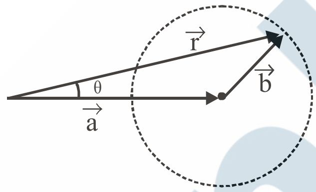
3. Three forces are acting on a body to make it in equilibrium, which set **can not** do it?

(A) 3 N, 3 N, 7 N (B) 2 N, 3 N, 6 N (C) 2 N, 1 N, 1 N (D) 8 N, 6 N, 1 N

Ans. **(A, B, D)**

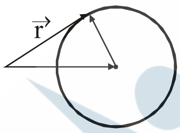
Sol. They must form a triangle. $(a + b \geq c)$

4. Keeping one vector constant, if direction of other to be added in the first vector is changed continuously, tip of the resultant vector describes a circle. In the following figure vector $\vec{a}$ is kept constant. When vector $\vec{b}$ added to $\vec{a}$ changes its direction, the tip of the resultant vector $\vec{r} = \vec{a} + \vec{b}$ describes circle of radius b with its center at the tip of vector $\vec{a}$. Maximum angle between vector $\vec{a}$ and the resultant $\vec{r} = \vec{a} + \vec{b}$ is



(A) $\tan^{-1}\left(\frac{b}{r}\right)$ (B) $\tan^{-1}\left(\frac{b}{\sqrt{a^2 - b^2}}\right)$ (C) $\cos^{-1}(r/a)$ (D) $\cos^{-1}(a/r)$

Ans. **(A,B,C)**

Sol. 

Angle between $\vec{r}$ and $\vec{b}$ is maximum when $\vec{r}$ is tangent to circle.

5. If $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = 10\hat{i} + 5\hat{j} + 5\hat{k}$, if the magnitude of component of $(\vec{B} - \vec{A})$ along $\vec{A}$ is $4\sqrt{x}$. Then x will be.

Ans. **6**

Sol. $r = \vec{B} - \vec{A} = 4(2\hat{i} + \hat{j} + \hat{k})$

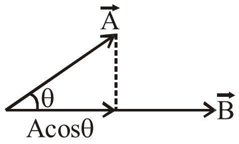
$$r\cos\theta = \frac{\vec{r}.\vec{A}}{|A|} = \frac{4(4+1+1)}{\sqrt{6}} = 4\sqrt{6}$$

$x = 6$

6. The component of $\vec{A} = \hat{i} + \hat{j} + 5\hat{k}$ perpendicular to $\vec{B} = 3\hat{i} + 4\hat{j}$ is

(A) $-\frac{4}{25}\hat{i} + \frac{3}{25}\hat{j} + 5\hat{k}$ (B) $-\frac{8}{25}\hat{i} - \frac{6}{25}\hat{j} + 5\hat{k}$ (C) $\frac{4}{25}\hat{i} - \frac{3}{25}\hat{j} + 5\hat{k}$ (D) $+\frac{8}{25}\hat{i} - \frac{6}{25}\hat{j} + 5\hat{k}$

Ans. **(C)**

Sol. 

$$\vec{A}_{\parallel} = A\cos\theta = A\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

$$= \frac{\vec{A}\cdot\vec{B}}{B} = \frac{3+4}{5} = \frac{7}{5}$$

$$\vec{A}_{\parallel} = \frac{7}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = \frac{7}{25}\left(3\hat{i}+4\hat{j}\right)$$

$$\vec{A}_{\parallel} = \frac{21}{25}\hat{i} + \frac{28}{25}\hat{j}$$

$$\vec{A}_{\perp} = \left(\hat{i}+\hat{j}+5\hat{k}\right) - \left(\frac{21}{25}\hat{i} + \frac{28}{25}\hat{j}\right)$$

$$= \frac{4}{25}\hat{i} - \frac{3}{25}\hat{j} + 5\hat{k}$$

ALGEBRA : SOME USEFUL FORMULAE

Quadratic equation and its solution

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equation. General quadratic equation is $ax^2 + bx + c = 0$. The general solution of the above quadratic equation or value of variable x is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Example

Solve $2x^2 + 5x - 12 = 0$

Solution

By comparison with the standard quadratic equation $a = 2$, $b = 5$ and $c = -12$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4\times 2\times(-12)}}{2\times 2} = \frac{-5\pm\sqrt{121}}{4} = \frac{-5\pm 11}{4} = \frac{+6}{4}, \frac{-16}{4} \Rightarrow x = \frac{3}{2}, -4$$

Binomial approximation

In case, x is very small, then terms containing higher powers of x can be neglected. In such a case, $(1 + x)^n = 1 + nx$

Also $(1 + x)^{-n} = 1 - nx$ and $(1 - x)^n = 1 - nx$ and $(1 - x)^{-n} = 1 + nx$

Exponential Expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... \text{ and } e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ...$$

Componendo and dividendo theorem :

If $\frac{p}{q} = \frac{a}{b}$ then by componendo and dividendo theorem $\frac{p+q}{p-q} = \frac{a+b}{a-b}$

Determinant

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc, \text{ For example } \begin{vmatrix} -3 & 3 \\ -5 & 1 \end{vmatrix} = 12, \begin{vmatrix} 2 & -4 \\ -3 & 3 \end{vmatrix} = -6$$

$$D = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Example

$$\begin{vmatrix} \overset{+}{5} & \overset{-}{4} & \overset{+}{3} \\ 2 & 1 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 5 \begin{vmatrix} 1 & 6 \\ 8 & 9 \end{vmatrix} - 4 \begin{vmatrix} 2 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 7 & 8 \end{vmatrix} = 5\,(9 - 48) - 4(18 - 42) + 3(16 - 7) = -72$$

Logarithm

$\log_e x = \ell n\, x$ (base e) $\log x = \log_{10} x$ (base 10)

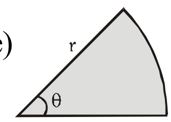
(a) Product formula $\log mn = \log m + \log n$ (b) Quotient formula $\log \frac{m}{n} = \log m - \log n$

(c) Power formula $\log m^n = n \log m$

<u>GEOMETRY : SOME USEFUL FORMULAE</u>

Formulae for determination of area :

- Area of a square = $(\text{side})^2$
- Area of rectangle = length × breadth
- Area of a triangle = (1/2) × base × height
- Area of a trapezoid =(1/2) × (distance between parallel sides) × (sum of parallel sides)
- Area enclosed by a circle = πr^2 (where r = radius)
- Area of a sector a circle = $\frac{1}{2}\theta r^2$ (where r = is radius and θ is angle subtended at a centre)

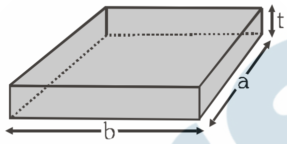


- Area of ellipse = π ab (where a and b are semi major and semi minor axis respectively)
- Surface area of a sphere = $4\pi r^2$ (where r = radius)
- Area of a parallelogram = base × height
- Area of curved surface of cylinder = $2\pi r\ell$ (where r = radius and ℓ = length)

- Area of whole surface of cylinder = $2\pi r(r + \ell)$ (where ℓ = length)
- Surface area of a cube = $6(\text{side})^2$
- Total surface area of a cone = $\pi r^2 + \pi r\ell$ [where $\pi r\ell = \pi r\sqrt{r^2 + h^2}$ = lateral area (h=height)]

Formulae for determination of volume :

- Volume of a rectangular slab = length × breadth × height = abt



- Volume of a cube = $(\text{side})^3$
- Volume of a sphere = $\frac{4}{3}\pi r^3$ (where r = radius)
- Volume of a cylinder = $\pi r^2\ell$ (where r = radius and ℓ = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$ (where r = radius and h = height)

EXERCISE (S-1)

Units & Dimensions

1. A particle is in a unidirectional potential field where the potential energy (U) of a particle depends on the x-coordinate given by $U_x = k(1 - \cos ax)$ & k and 'a' are constants. Find the physical dimensions of 'a' & k.

2. The equation for the speed of sound in a gas states that $v = \sqrt{\gamma k_B T / m}$. Speed v is measured in m/s, γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. Find the SI unit for the Boltzmann constant, k_B?

3. The time period (T) of a spring mass system depends upon mass (m) & spring constant (k) & length of the spring (ℓ) $\left[k = \frac{Force}{length}\right]$. Find the relation among T, m, ℓ & k using dimensional method.

4. The distance moved by a particle in time t from centre of a ring under the influence of its gravity is given by $x = a \sin\omega t$, where a & ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitational constant (G). Using dimensional analysis find an expression for ω in terms of r, m and G.

5. A satellite is orbiting around a planet. Its orbital velocity (v_0) is found to depend upon
(A) Radius of orbit (R)
(B) Mass of planet (M)
(C) Universal gravitation constant (G)
Using dimensional analysis find an expression relating orbital velocity (v_0) to the above physical quantities.

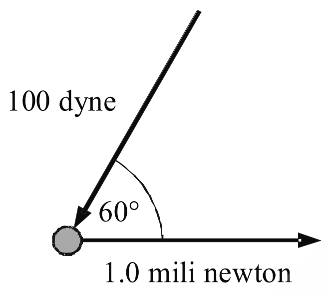
6. Assume that the largest stone of mass 'm' that can be moved by a flowing river depends upon the velocity of flow v, the density d & the acceleration due to gravity g . If 'm' varies as the K^{th} power of the velocity of flow, then find the value of K .

7. Given $\vec{F} = \frac{\vec{a}}{t}$ where symbols have their usual meaning. The dimensions of a is .

Addition of vectors

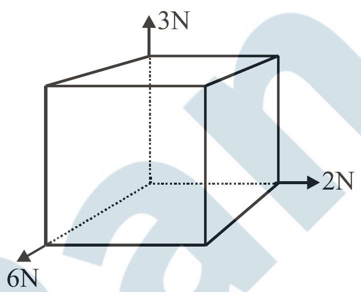
8. A block is applied two forces of magnitude 5N each. One force is acting towards East and the other acting along 60° North of East. The resultant of the two forces (in N) is of magnitude :-

9. Two forces act on a particle simultaneously as shown in the figure. Find net force in milli newton on the particle. [Dyne is the CGS unit of force]



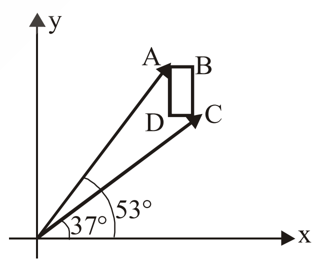
10. The maximum and minimum magnitudes of the resultant of two forces are 35 N and 5 N respectively. Find the magnitude of resultant force when act orthogonally to each other.

11. Three forces of magnitudes 2 N, 3 N and 6 N act at corners of a cube along three sides as shown in figure. Find the resultant of these forces in N.



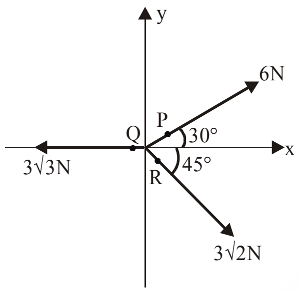
Resolution of vectors and unit vector

12. The farm house shown in figure has rectangular shape and has sides parallel to the chosen x and y axes. The position vector of corner A is 125 m at 53° and corner C is 100 m at 37° from x axis. Find the length of the fencing required in meter.



13. Vector B has x, y and z components of 4.00, 6.00 and 3.00 units, respectively. Calculate the magnitude of B and the angles that B makes with the coordinates axes.

14. Three ants P, Q and R are pulling a grain with forces of magnitude 6N, $3\sqrt{3}$N and $3\sqrt{2}$N as shown in the figure. Find the magnitude of resultant force (in N) acting on the grain.



15. Three boys are pushing horizontally a box placed on horizontal table. One is pushing towards north with a $5\sqrt{3}$N force. The second is pushing towards east and third pushes with a force 10 N such that the box is in equilibrium. Find the magnitude of the force, second boy is applying in newton.

Scalar product of vectors

16. Consider the two vectors : $\vec{L} = 1\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{l} = 4\hat{i} + 5\hat{j} + 6\hat{k}$. Find the value of the scalar α such that the vector $\vec{L} - \alpha\vec{l}$ is perpendicular to $\vec{L}$.

17. Find components of vector $\vec{a} = \hat{i} + \hat{j} + 3\hat{k}$ in directions parallel to and perpendicular to vector $\vec{b} = \hat{i} + \hat{j}$.

18. (a) Calculate $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ where $\vec{a} = 5\hat{i} + 4\hat{j} - 6\hat{k}$, $\vec{b} = -2\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = 4\hat{i} + 3\hat{j} + 2\hat{k}$.

(b) Calculate the angle between $\vec{r}$ and the z-axis.

(c) Find the angle between $\vec{a}$ and $\vec{b}$

19. If the velocity of a particle is $(2\hat{i} + 3\hat{j} - 4\hat{k})$ and its acceleration is $(-\hat{i} + 2\hat{j} + \hat{k})$ and angle between them is $\frac{n\pi}{4}$. The value of n is.

Method of approximation

20. Quito, a city in Ecuador and Kampala, a city situated in Uganda both lie on the Equator. The longitude of Quito is 82°30' W and that of Kampala is 37°30' E. What is the distance from Quito to Kampala.
(a) along the shortest surface path
(b) along a direct (through-the-Earth) path? (The radius of the Earth is 6.4×10^6 m)

21. Use the approximation $(1+x)^n \approx 1 + nx, |x| << 1$, to find approximate value for

(a) $\sqrt{99}$ (b) $\frac{1}{1.01}$

22. Use the small angle approximations to find approximate values for (A) sin 8° and (B) tan 5°

EXERCISE (S-2)

1. The equation of state for a real gas at high temperature is given by $P=\frac{nRT}{V-b}-\frac{a}{T^{1/2}V(V+b)}$ where n, P, V & T are number of moles, pressure, volume & temperature respectively & R is the universal gas constant. Find the dimensions of constant a in the above equation.

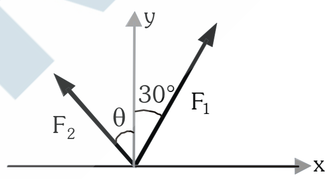
2. If Energy (E), velocity (v) and time (T) are fundamental units. What will be the dimension of surface tension?

3. In system called the star system we have 1 star kilogram = 10^{20} kg. 1 starmeter = 10^{8} m, 1 starsecond = 10^{3}s then calculate the value of 1 joule in this system.

4. A vector $\vec{A}$ of length 10 units makes an angle of 60° with the vector $\vec{B}$ of length 6 units. Find the magnitude of the vector difference $\vec{A}-\vec{B}$ & the angle it makes with vector $\vec{A}$.

5. A bird is at a point P (4, –1, –5) and sees two points P_1 (–1, –1, 0) and P_2 (3, –1, –3). At time t = 0, it starts flying with a constant speed of 10 m/s to be in line with points P_1 and P_2 in minimum possible time t. Find t, if all coordinates are in kilometers.

6. In the figure, F_1 and F_2, the two unknown forces give a resultant force of $80\sqrt{3}$ N along the y–axis. It is required that F_2 must have minimum magnitude. Find the magnitudes of F_1 and F_2.



7. A particle is displaced from A ≡ (2, 2, 4) to B≡ (5, –3, –1). A constant force of 34 N acts in the direction of $\vec{AP}$, where P ≡ (10, 2, –11). (Coordinates are in m).

(i) Find the $\vec{F}$. (ii) Find the work done by the force to cause the displacement.

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Units & Dimensions

1. The dimensional formula for which of the following pair is not the same

(A) impulse and momentum (B) torque and work

(C) stress and pressure (D) momentum and angular momentum

2. Which of the following can be a set of fundamental quantities

(A) length, velocity, time (B) momentum, mass, velocity

(C) force, mass, velocity (D) momentum, time, frequency

3. Which of the following functions of A and B may be performed if A and B possess different dimensions?

(A) $\frac{A}{B}$ (B) A + B (C) A – B (D) None

4. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are respectively :–

(A) LT^{-2}, L and T (B) L^2, T and LT^2 (C) LT^2, LT and L (D) L, LT and T^2

5. If area (A), velocity (v), and density (ρ) are base units, then the dimensional formula of force can be represented as :-

(A) $Av\rho$ (B) $Av^2\rho$ (C) $Av\rho^2$ (D) $A^2v\rho$

6. Density of wood is 0.5 g/ cc in the CGS system of units. The corresponding value in MKS units is :-

(A) 500 (B) 5 (C) 0.5 (D) 5000

7. In a book, the answer for a particular question is expressed as $b = \frac{ma}{k}\left[\sqrt{1+\frac{2k\ell}{ma}}\right]$ here m represents mass, a represents acceleration, ℓ represents length. The unit of b should be :-

(A) m/s (B) m/s^2 (C) meter (D) /sec

8. The frequency f of vibrations of a mass m suspended from a spring of spring constant k is given by $f=Cm^xk^y$, where C is a dimensionless constant. The values of x and y are, respectively

(A) $\frac{1}{2}, \frac{1}{2}$ (B) $-\frac{1}{2}, -\frac{1}{2}$ (C) $\frac{1}{2}, -\frac{1}{2}$ (D) $-\frac{1}{2}, \frac{1}{2}$

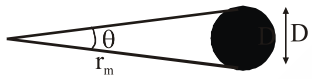
9. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:

(A) FT^2 (B) $F^{-1}A^2T^{-1}$ (C) FA^2T (D) AT^2

10. In a particular system the units of length, mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equal to :-
(A) 0.1 N (B) 1 N (C) 10 N (D) 100 N

11. The units of three physical quantities x, y and z are $gcm^2 s^{-5}$, gs^{-1} and cms^{-2} respectively. The relation among the units of x, y and z is :-
(A) $z = x^2y$ (B) $y^2 = xz$ (C) $x = yz^2$ (D) $x = y^2z$

12. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the fact that the moon is about 384000 km away to find the approximate diameter of the moon.



(A) 192000 km (B) 3350 km (C) 1600 km (D) 1920 km

13. **Statement 1 :** Method of dimensions cannot tell whether an equation is correct.
and
Statement 2 : A dimensionally incorrect equation may be correct.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

14. The equation of the stationary wave is $y = 2A\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$ which of the following statement is wrong?
(A) The unit of ct is same as that of λ. (B) The unit of x is same as that of λ.
(C) The unit of $\frac{2\pi c}{\lambda}$ is same as that of $\frac{2\pi x}{\lambda t}$ (D) The unit of $\frac{c}{\lambda}$ is same as that of $\frac{x}{\lambda}$

15. Due to some unknown interaction, force F experienced by a particle is given by the following equation.

$$\vec{F} = -\frac{A}{r^3}\vec{r}$$

Where A is positive constant and r distance of the particle from origin of a coordinate system. Dimensions of constant A are :-
(A) ML^2T^{-2} (B) ML^3T^{-2} (C) ML^4T^{-2} (D) ML^0T^0

Addition of vectors

16. Three concurrent forces of the same magnitude are in equilibrium. What is the angle between the force? Also name the triangle formed by the force as sides :-
(A) 60° equilateral triangle (B) 120° equilateral triangle
(C) 120°, 30°, 30° an isosceles triangle (D) 120° an obtuse angled triangle

17. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between two forces is :-
(A) 150° (B) 90° (C) 60° (D) 120°

18. The resultant of two forces acting at an angle of 120° is 10 kg wt and is perpendicular to one of the forces. That force is :

(A) $10\sqrt{3}$ kg wt (B) $20\sqrt{3}$ kg wt (C) 10 kg wt (D) $\frac{10}{\sqrt{3}}$ kg wt

19. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60° is $\sqrt{7}$ Q, then P/Q is :-

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4

20. There are two force vectors, one of 5N and other of 12N at what angle the two vectors be added to get resultant vector of 17N, 7N and 13N respectively.

(A) 0°, 180° and 90° (B) 0°, 90° and 180°
(C) 0°, 90° and 90° (D) 180°, 0° and 90°

21. A body placed in free space, is simultaneously acted upon by three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. The body is in equilibrium and the forces $\vec{F}_1$ and $\vec{F}_2$ are known to be 36 N due north and 27 N due east respectively. Which of the following best describes the force $\vec{F}_3$?

(A) 36 N due south. (B) 53 N due 60° south of east
(C) 45 N due 53° south of west (D) 45 N due 37° north of west

22. The ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{A}$ and $\vec{B}$ is 3 : 2. The relation between A and B is

(A) A = 5B (B) 5A = B (C) A = 3B (D) A = 4B

23. Find the resultant of the following two vectors $\vec{A}$ and $\vec{B}$.

$\vec{A}$: 40 units due east and ; $\vec{B}$: 25 units 37° north of west

(A) 25 units 37° north of west (B) 25 units 37° north of east
(C) 40 units 53° north of west (D) 40 units 53° north of east

24. Two vectors $\vec{a}$ and $\vec{b}$ add to give a resultant $\vec{c} = \vec{a} + \vec{b}$. In which of these cases angle between $\vec{a}$ and $\vec{b}$ is maximum: (a, b, c represent the magnitudes of respective vectors)

(A) c = a + b (B) $c^2 = a^2 + b^2$ (C) c = a – b (D) can not be determined

25. If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60°, then $|\hat{a} - \hat{b}|$ is :-

(A) 0 (B) 1 (C) 2 (D) 4

26. A man moves towards 3 m north then 4 m towards east and finally 5 m towards 37° south of west. His displacement from origin is :-

(A) $5\sqrt{2}$m (B) 0m (C) 1 m (D) 12 m

Resolution of vectors and unit vector

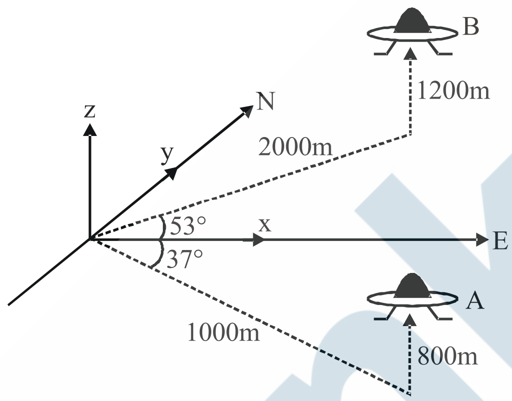
27. The projection of a vector, $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k}$, on the x–y plane has magnitude :–

(A) 3 (B) 4 (C) $\sqrt{14}$ (D) $\sqrt{10}$

28. A bird moves from point (1, –2) to (4, 2). If the speed of the bird is 10 m/s, then the velocity vector of the bird is

(A) $5(\hat{i} - 2\hat{j})$ (B) $5(4\hat{i} + 2\hat{j})$ (C) $0.6i + 0.8\hat{j}$ (D) $6\hat{i} + 8\hat{j}$

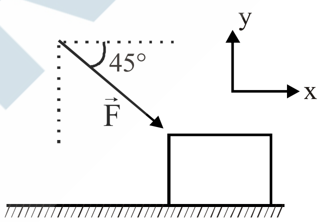
29. Personnel at an air post control tower track a UFO. At 11:02 am it was located at position A and at 11:12 am is was located at position B. Displacement vector of UFO is :



(A) $400\hat{i} + 2200\hat{j} + 400\hat{k}$ (B) $1200\hat{i} + 1000\hat{j} + 800\hat{k}$

(C) $2000\hat{i} + 2200\hat{j} + 2000\hat{k}$ (D) $400\hat{i} + 1000\hat{j} + 400\hat{k}$

30. A person pushes a box kept on a horizontal surface with force of 100 N. In unit vector notation force $\vec{F}$ can be expressed as :



(A) $100\ (\hat{i} + \hat{j})$ (B) $100\ (\hat{i} - \hat{j})$ (C) $50\sqrt{2}\ (\hat{i} - \hat{j})$ (D) $50\sqrt{2}\ (\hat{i} + \hat{j})$

31. For the given vector $\vec{A} = 3\hat{i} - 4\hat{j} + 10\hat{k}$, the ratio of magnitude of its component on the x-y plane and the component on z–axis is :-

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) None

32. After firing, a bullet is found to move at an angle of 37° to horizontal. Its acceleration is 10 m/s^2 downwards. Find the component of acceleration in the direction of the velocity.

(A) – 6 m/s^2 (B) – 4 m/s^2 (C) – 8 m/s^2 (D) – 5 m/s^2

Scalar product of vectors

33. In a methane (CH_4) molecule each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the centre. In coordinates where one of theC–H bonds is in the direction of $\hat{i}+\hat{j}+\hat{k}$, an adjacent C–H bond in the $\hat{i}-\hat{j}-\hat{k}$ direction. Then angle between these two bonds :-

(A) $\cos^{-1}\left(-\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{2}{3}\right)$ (C) $\cos^{-1}\left(-\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

34. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is :-

(A) 45° (B) 60° (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

35. The velocity of a particle is $\vec{v}=6\hat{i}+2\hat{j}-2\hat{k}$. The component of the velocity of a particle parallel to vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ is :-

(A) $6\hat{i}+2\hat{j}+2\hat{k}$ (B) $2\hat{i}+2\hat{j}+2\hat{k}$ (C) $\hat{i}+\hat{j}+\hat{k}$ (D) $6\hat{i}+2\hat{j}-2\hat{k}$

36. A particle moves from a position $3\hat{i}+2\hat{j}-6\hat{k}$ to a position $14\hat{i}+13\hat{j}+9\hat{k}$ in m and a uniform force of $4\hat{i}+\hat{j}+3\hat{k}$ N acts on it. The work done by the force is :-

(A) 200 J (B) 100 J (C) 300 J (D) 500 J

37. Which of the following is perpendicular to $\hat{i}-\hat{j}-\hat{k}$?

(A) $\hat{i}+\hat{j}+\hat{k}$ (B) $-\hat{i}+\hat{j}+\hat{k}$ (C) $\hat{i}+\hat{j}-\hat{k}$ (D) none of these

MULTIPLE CORRECT TYPE QUESTIONS

38. Which of the following statements about the sum of the two vectors $\vec{A}$ and $\vec{B}$, is/are correct?

(A) $|\vec{A}+\vec{B}|\le A+B$ (B) $|\vec{A}+\vec{B}|\ge A+B$ (C) $|\vec{A}+\vec{B}|\ge|\vec{A}-\vec{B}|$ (D) $|\vec{A}+\vec{B}|\ge|A-B|$

39. Priya says that the sum of two vectors by the parallelogram method is $\vec{R}=5\hat{i}$. Subhangi says it is $\vec{R}=\hat{i}+4\hat{j}$. Both used the parallelogram method, but one used the wrong diagonal. Which one of the vector pairs below contains the original two vectors?

(A) $\vec{A}=+3\hat{i}-2\hat{j}$; $\vec{B}=-2\hat{i}+2\hat{j}$ (B) $\vec{A}=-3\hat{i}-2\hat{j}$; $\vec{B}=+2\hat{i}+2\hat{j}$

(C) $\vec{A}=+3\hat{i}+2\hat{j}$; $\vec{B}=+2\hat{i}-2\hat{j}$ (D) $\vec{A}=+3\hat{i}+2\hat{j}$; $\vec{B}=-2\hat{i}+2\hat{j}$

40. For the equation x= AC sin (Bt) + D $e^{(BCt)}$, where x and t represent position and time respectively, which of the following is/are **CORRECT** :-

(A)Dimension of AC is LT^{-1} (B) Dimension of B is T^{-1}

(C)Dimension of AC and D are same (D) Dimension of C is T^{-1}

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 41 to 43

In a certain system of absolute units the acceleration produced by gravity in a body falling freely is denoted by 5, the kinetic energy of a 500 kg shot moving with velocity 400 metres per second is denoted by 2000 & its momentum by 100.

41. The unit of length is :-

(A) 15 m (B) 50 m (C) 25 m (D) 100 m

42. The unit of time is :-

(A) 10 s (B) 20 s (C) 5 s (D) 15 s

43. The unit of mass is :-

(A) 200 kg (B) 400 kg (C) 800 kg (D) 1200 kg

Paragraph for Question Nos. 44 and 45

For any particle moving with some velocity $(\vec{v})$ & acceleration $(\vec{a})$, tangential acceleration & normal acceleration are defined as follows

Tangential acceleration – The component of acceleration in the direction of velocity.

Normal acceleration – The component of acceleration in the direction perpendicular to velocity.

If at a given instant, velocity & acceleration of a particle are given by.

$$\vec{v} = 4\hat{i} + 3\hat{j}$$

$$\vec{a} = 10\hat{i} + 15\hat{j} + 20\hat{k}$$

44. Find the tangential acceleration of the particle at the given instant :-

(A) $17(4\hat{i}+3\hat{j})$ (B) $\frac{17}{5}(4\hat{i}+3\hat{j})$ (C) $17(4\hat{i}-3\hat{j})$ (D) $\frac{17}{5}(4\hat{i}-3\hat{j})$

45. Find the normal acceleration of the particle at the given instant :-

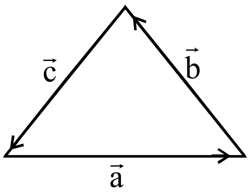
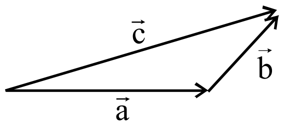
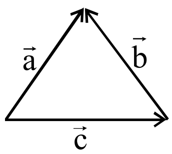
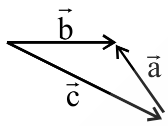
(A) $\frac{-9\hat{i}+12\hat{j}+50\hat{k}}{5}$ (B) $\frac{9\hat{i}-12\hat{j}-50\hat{k}}{5}$ (C) $\frac{-18\hat{i}+24\hat{j}+100\hat{k}}{5}$ (D) $\frac{18\hat{i}-24\hat{j}-100\hat{k}}{5}$

MATRIX MATCH TYPE QUESTION

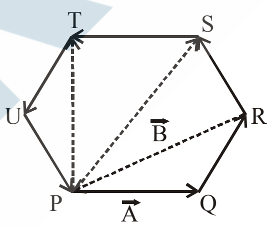
46. Two particles A and B start from origin of a coordinate system towards point P (10, 20) and Q (20, 10) respectively with speed $5\sqrt{5}$ each. Both continue their motion for 10 s and then stop. There after particle B moves towards particle A with speed $2\sqrt{2}$ and after particle B meets particle A, they both return to origin following a straight line path with speed $5\sqrt{5}$. Match the items of column-I with suitable items of Column-II.

Column-I		Column-II	
(A)	Initial velocity vector of A	(P)	$(-5\hat{i}-10\hat{j})$
(B)	Initial velocity of B	(Q)	$(5\hat{i}+10\hat{j})$
(C)	Velocity vector of B while it moves towards A	(R)	$(10\hat{i}+5\hat{j})$
(D)	Velocity vector of A and B while they return to origin	(S)	$(2\hat{i}-2\hat{j})$
		(T)	$(-2\hat{i}+2\hat{j})$

47. Column-I show vector diagram relating three vectors $\vec{a}, \vec{b}$ and $\vec{c}$. Match the vector equation in column-II, with vector diagram in column-I :

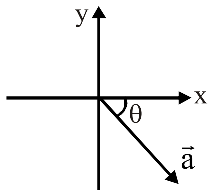
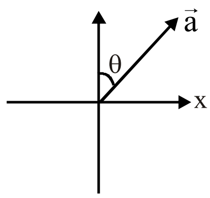
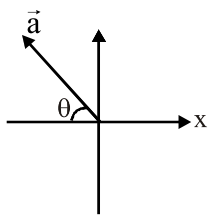
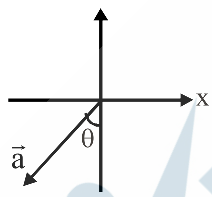
Column-I		**Column-II**	
(A)		(P)	$\vec{a}-(\vec{b}+\vec{c})=0$
(B)		(Q)	$\vec{b}-\vec{c}=\vec{a}$
(C)		(R)	$\vec{a}+\vec{b}=-\vec{c}$
(D)		(S)	$\vec{a}+\vec{b}=\vec{c}$

48. In a regular hexagon two vectors $\overrightarrow{PQ}=\vec{A}$, $\overrightarrow{RP}=\vec{B}$. Express other vector's in term of them :-



Column–I		**Column–II**	
(A)	$\overrightarrow{PS}$	(P)	$-2\vec{B}-3\vec{A}$
(B)	$\overrightarrow{PT}$	(Q)	$-\vec{B}-\vec{A}$
(C)	$\overrightarrow{RS}$	(R)	$-\vec{B}-2\vec{A}$
(D)	$\overrightarrow{TS}$	(S)	$-2(\vec{B}+\vec{A})$
		(T)	$\vec{A}$

49. Show a vector $\vec{a}$ at angle θ as shown in the figure column-II. Show its unit vector representation.

Column–I		**Column–II**	
(A)		(P)	$\vec{a} = a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j}$
(B)		(Q)	$\vec{a} = -a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$
(C)		(R)	$\vec{a} = -a\sin\theta\,\hat{i} - a\cos\theta\,\hat{j}$
(D)		(S)	$\vec{a} = a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j}$

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and 1 unit of length is 10 m. In this system, one unit of power will correspond to :-

(A) 16 watts (B) $\frac{1}{16}$ watts (C) 25 watts (D) none of these

2. If the unit of length be doubled then the numerical value of the universal gravitation constant G will become (with respect to present value)

(A) Double (B) Half (C) 8 times (D) 1/8 times

3. If in a system, the force of attraction between two point masses of 1 kg each situated 1 km apart is taken as a unit of force and is called notwen (newton written in reverse order) & if $G = 6.67\times 10^{-11}$ N–m^2 kg^{-2} in SI units then which of the following is true?

(A) 1 notwen = 6.67×10^{-11} newton (B) 1 newton = 6.67×10^{-17} notwen

(C) 1 notwen = 6.67×10^{-17} newton (D) 1 newton = 6.67×10^{-12} notwen

4. In two different systems of units an acceleration is represented by the same number, while a velocity is represented by numbers in the ratio 1:3. The ratios of unit of length and time are respectively

(A) $\frac{1}{3}, \frac{1}{9}$ (B) $\frac{1}{9}, \frac{1}{3}$ (C) 1, 1 (D) None of these

5. **Statement–1 :** Whenever the unit of measurement of a quantity is changed, its numerical value changes.

and

Statement–2 : Smaller the unit of measurement smaller is its numerical value.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

6. Forces proportional to AB, BC and 2CA act along the sides of triangle ABC in order. Their resultant represented in magnitude and direction as

(A) CA (B) AC (C) BC (D) CB

7. A man rows a boat with a speed of 18 km/hr in north–west direction. The shoreline makes an angle of 15° south of west. Obtain the component of the velocity of the boat along the shoreline.

(A) 9 km/hr (B) $18\frac{\sqrt{3}}{2}$ km/hr (C) 18 cos (15°) km/hr (D) 18 cos (75°) km/hr

8. **Statement 1** : Unit vector has a unit though its magnitude is one

and

Statement 2 : Unit vector is obtained by dividing a vector by its own magnitude.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

9. A vector of magnitude 10 m in the direction 37° south of west has its initial point at (5 m, 2 m). If positive x–axis represents the east and positive y–axis the north, the coordinates of its terminal point are

(A) (–3 m, –4 m) (B) (3 m, 4 m) (C) (–4 m, 6 m) (D) (–4 m,–6 m)

10. A plumber steps down 1 m out of his truck and walks 50 m east and then 25 m south, and then takes an elevator to the basement of the building 9 m below street level. If the east, the north and the upward direction are represented by the positive x, y and z–axes, which one of the following represents displacement (meters) of the plumber?

(A) $50\hat{i}-25\hat{j}-9\hat{k}$ (B) $50\hat{i}+25\hat{j}-9\hat{k}$ (C) $50\hat{i}-25\hat{j}-10\hat{k}$ (D) $50\hat{i}+25\hat{j}-10\hat{k}$

11. A body moves in anticlockwise direction on a circular path in the x-y plane. The radius of the circular path is 5 m and its centre is at the origin. In a certain interval of time, displacement of the body is observed to be 6 m in the positive y-direction. Which of the following is true?

(A) Its initial position vector is $5\hat{i}$ m. (B) Its initial position vector is $(-3\hat{i}+4\hat{j})$ m.

(C) Its final position vector is $\left(4\hat{i}+3\hat{j}\right)$ m. (D) Its final position vector is $6\hat{j}$ m.

12. A boy A is standing 20√3 m away in a direction 30° north of east from his friend B. Another boy C standing somewhere east of B can reach A, if he walks in a direction 60° north of east. In a Cartesian coordinate system with its x–axis towards the east, the position of C with respect to A is

(A) $\left(-20\hat{i}+-10\hat{j}\right)$ m (B) $\left(-10\hat{i}-10\sqrt{3}\hat{j}\right)$ m

(C) $\left(10\hat{i}+10\sqrt{3}\hat{j}\right)$ m (D) It depends on where we chose the origin.

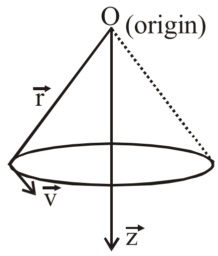
13. Find the component of $\vec{r}$ in the direction of $\vec{a}$:–

(A) $\frac{(\vec{r}.\vec{a})\vec{a}}{a^2}$ (B) $\frac{(\vec{r}.\vec{a})\vec{a}}{a}$ (C) $\frac{(\vec{r}\cdot\vec{a})\vec{r}}{r}$ (D) $\frac{(\vec{r}\cdot\vec{a})\vec{r}}{r^2}$

14. Consider three vectors $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{C}=2\hat{i}-3\hat{j}+4\hat{k}$. A vector $\vec{X}$ of the form $\alpha\vec{A}+\beta\vec{B}$ (α and β are numbers) is perpendicular to $\vec{C}$. The ratio of α and β is

(A) 1 : 1 (B) 2 : 1 (C) –1 : 1 (D) 3 : 1

15. A string connected with bob is suspended by the point (O) such that it sweeps out conical surface in horizontal plane. Here $\vec{r}$ is the position vector of bob, $\vec{v}$ is its velocity and $\vec{z}$ is the axis of swept cone as shown. Select **INCORRECT** statement :-



(A) $\vec{r}.\vec{z}$ is always zero
(B) $\vec{r}.\vec{v}$ is always zero
(C) $\vec{z}\cdot\vec{v}$ is always constant
(D) $\vec{r}\cdot\vec{z}$ is always non zero constant

16. x-component of a vector $\vec{A}$ is twice of its y-component and $\sqrt{2}$ times of its z-component. Find out the angle made by the vector from y-axis.

(A) $\cos^{-1}\left(\frac{2}{\sqrt{7}}\right)$
(B) $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$
(C) $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$
(D) $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

17. Given the vectors $\vec{A}=2\hat{i}+3\hat{j}-\hat{k}$; $\vec{B}=3\hat{i}-2\hat{j}-2\hat{k}$ & $\vec{C}=p\hat{i}+p\hat{j}+2p\hat{k}$. Find the angle between $(\vec{A}-\vec{B})$ & $\vec{C}$

(A) $\theta=\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$
(B) $\theta=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(C) $\theta=\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$
(D) none of these

MULTIPLE CORRECT TYPE QUESTIONS

18. Four forces acting on a particle keep it in equilibrium, then :-
(A) the force must be coplanar.
(B) the forces cannot be coplanar.
(C) the forces may or may not be coplanar.
(D) if three of these forces are coplanar, so must be the fourth.

19. A man is standing at point (x = 5 m, y = 0). Then he walks along straight line to (x = 0, y = 5m). A second man walks from the same initial position along the x-axis to the origin and then along the y-axis to (x = 0, y = 5m). Mark the **CORRECT** statement(s) :
(A) Displacement vector of first man and second man are equal
(B) Distance travelled by second man is greater
(C) Magnitude of displacement of second man is same that of first man but direction is different
(D) Magnitude of displacement is $\sqrt{50}$ m for 2nd man.

20. The vector i + xj + 3k is rotated through an angle θ and doubled in magnitude, then it becomes 4i + (4x – 2) j + 2k. The values of x are

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2

21. The value of $|\vec{A}+\vec{B}-\vec{C}+\vec{D}|$ can be zero if :–

(A) $|\vec{A}|=5, |\vec{B}|=3, |\vec{C}|=4; |\vec{D}|=13$
(B) $|\vec{A}|=2\sqrt{2}, |\vec{B}|=2, |\vec{C}|=2; |\vec{D}|=5$
(C) $|\vec{A}|=2\sqrt{2}, |\vec{B}|=2, |\vec{C}|=2; |\vec{D}|=10$
(D) $|\vec{A}|=5, |\vec{B}|=4, |\vec{C}|=3; |\vec{D}|=8$

22. The four pairs of force vectors are given, which pairs of force vectors cannot be added to give a resultant vector of magnitude 10 N?
(A) 2N, 13 N (B) 5N, 16 N (C) 7N, 8N (D) 100N, 105 N

23. Select **CORRECT** statement(s) for three vectors $\vec{a}=-3\hat{i}+2\hat{j}-\hat{k}, \vec{b}=\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{c}=2\hat{i}+\hat{j}-4\hat{k}$

(A) The above vectors can form triangle.
(B) Component of $\vec{a}$ along $\vec{c}$ is 3.
(C) $\vec{a}$ makes angle $\cos^{-1}\sqrt{\frac{2}{7}}$ with y-axis.
(D) A vector having magnitude twice the vector $\vec{a}$ and anti parallel to vector $\vec{b}$ is $\sqrt{\frac{2}{5}}(-2\hat{i}+6\hat{j}-10\hat{k})$

24. If a vector $\vec{P}$ makes an angle α, β, γ with x, y, z axis respectively then it can be represented as $\vec{P}=P\left[\cos\alpha\hat{i}+\cos\beta\hat{j}+\cos\gamma\hat{k}\right]$. Choose the CORRECT option(s) :-

(A) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$
(B) $\vec{P}.\vec{P}=P^2$
(C) $\vec{P}.(\hat{i}-\hat{k})=P(\cos\alpha-\cos\gamma)$
(D) $\vec{P}.\hat{i}=\cos\alpha$

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 25 to 27

A boy lost in a jungle finds a note. In the note was written the following things.

Displacements

1. 300 m 53° South of East.
2. 400 m 37° North of East
3. 500 m North
4. 500 √2 m North-West
5. 500 m South

He starts walking at constant speed 2 m/s following these displacements in the given order.

25. How far and in which direction is he from the starting point after 5 min. and 50 s?
(A) 500 m due East
(B) 500 m due West
(C) 700 m due South-West
(D) 700 m due North-East

26. How far and in which direction is he from the starting point after 10 minutes?

(A) 500 $\sqrt{2}$ m due North
(B) 1200 m due North-East
(C) 500$\sqrt{2}$ m due North-East
(D) 900 m due 37° North of East

27. How far and in which direction has he finally displaced after all the displacements in the note?

(A) 500$\sqrt{2}$ m due North-East
(B) 500 m due North
(C) 866 m due North-West
(D) 500$\sqrt{3}$ m due 60° North of West

Paragraph for Question Nos. 28 to 30

A physical quantity is a physical property of a phenomenon, body, or substance, that can be quantified by measurement.

The magnitude of the components of a vector are to be considered dimensionally distinct. For example, rather than an undifferentiated length unit L, we may represent length in the x direction as L_x, and so forth. This requirement stems ultimately from the requirement that each component of a physically meaningful equation (scalar or vector) must be dimensionally consistent. As an example, suppose we wish to calculate the drift S of a swimmer crossing a river flowing with velocity V_x and of width D and he is swimming in direction perpendicular to the river flow with velocity V_y relative to river, assuming no use of directed lengths, the quantities of interest are then V_x, V_y both dimensioned as $\frac{L}{T}$, S the drift and D width of river both having dimension L. With these four quantities, we may conclude that the equation for the drift S may be written: $S \propto V_x^{\,a} V_y^{\,b} D^c$

Or dimensionally $L = \left(\frac{L}{T}\right)^{a+b} \times (L)^c$ from which we may deduce that $a + b + c = 1$ and $a + b = 0$, which leaves one of these exponents undetermined. If, however, we use directed length dimensions, then V_x will be dimensioned as $\frac{L_x}{T}$, V_y as $\frac{L_y}{T}$, S as L_x and D as L_y. The dimensional equation becomes : $L_x = \left(\frac{L_x}{T}\right)^a \left(\frac{L_y}{T}\right)^b (L_y)^c$ and we may solve completely as $a = 1$, $b = -1$ and $c = 1$. The increase in deductive power gained by the use of directed length dimensions is apparent.

28. Which of the following is not a physical quantity

(A) Height of a boy
(B) Weight of a boy
(C) Fever of a boy
(D) Speed of a running boy

29. From the concept of directed dimension what is the formula for a range (R) of a cannon ball when it is fired with vertical velocity component V_y and a horizontal velocity component V_x assuming it is fired on a flat surface. [Range also depends upon acceleration due to gravity, g and k is numerical constant]

(A) $R = \frac{k(V_x V_y)}{g}$
(B) $R = \frac{k(V_x)^2}{g}$
(C) $R = \frac{k(V_x)^3}{V_y g}$
(D) $R = \frac{k(V_y)^3}{V_x g}$

30. A conveyer belt of width D is moving along x-axis with velocity V. A man moving with velocity U on the belt in the direction perpendicular to the belt's velocity with respect to belt wants to cross the belt. The correct expression for the drift (S) suffered by man is given by (k is numerical constant)

(A) $S = k\frac{UD}{V}$ (B) $S = k\frac{VD}{U}$ (C) $S = k\frac{U^2D}{V^2}$ (D) $S = k\frac{V^2D}{U^2}$

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT
(THREE COLUMNS AND FOUR ROWS)

Answer Q.31, Q.32 and Q.33 by appropriately matching the information given in the three columns of the following table.

L, M and T are units of length, Mass and Time respectively in a system of units.

	Coloumn-1		Column-2		Column-3
(I)	L = 10 m	(i)	M = 100 gm	(P)	T = 0.1 sec
(II)	L = 10 cm	(ii)	M = 10 kg	(Q)	T = 10 ms
(III)	L = 0.1 mm	(iii)	M = 10 gm	(R)	T = 10 sec
(IV)	L = 1 km	(iv)	M = 1 tonne	(S)	T = 0.01 sec

31. In which of the following combinations unit of force is 10^6 dyne.

(A) (IV) (i) (P) (B) (II) (iii) (S) (C) (III) (iv) (P) (D) (I) (ii) (Q)

32. In which of the following system, unit of energy is 10^9 erg

(A) (III) (i) (S) (B) (IV) (iii) (R) (C) (II) (iv) (Q) (D) (I) (iii) (P)

33. In which of the following system, unit for coefficient of viscosity is 100 poiseuille

(A) (III) (ii) (S) (B) (II) (i) (Q) (C) (III) (iii) (R) (D) (IV) (iv) (P)

MATRIX MATCH TYPE QUESTION

34. In a new system of units known as RMP, length is measured in 'retem', mass is measured in 'marg' and time is measured in 'pal'.

100 retem = 1.0 meter

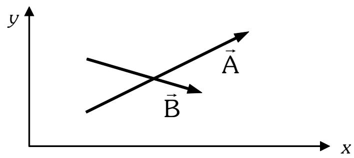
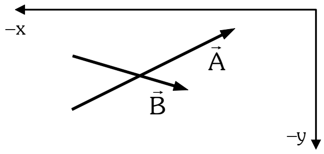
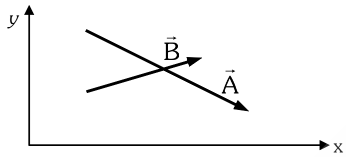
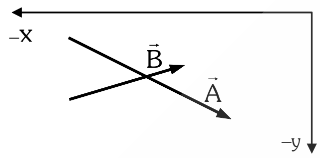
1.0 marg = 10^{-3} kilogram

10 pal = 1.0 second

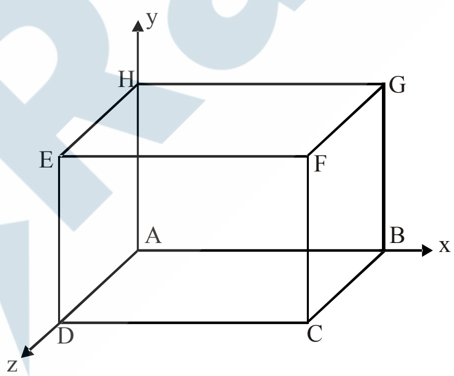
In the given table some unit conversion factors are given. Suggest suitable match.

	Column-I		Column-II
(A)	One SI unit of force	(P)	10^2 units of RMP
(B)	One SI unit of potential energy	(Q)	10^3 units of RMP
(C)	One SI unit of power	(R)	10^4 units of RMP
(D)	One SI unit of momentum	(S)	10^5 units of RMP
		(T)	10^6 units of RMP

35. Refer the following table, where in the first column four pairs of two vectors are shown and in the second column some possible outcomes of basic mathematical operation on these vectors are given. Suggest suitable match(s).

Column-I		Column - II	
(A)		(P)	X-component of $\vec{A}+\vec{B}$ is positive
(B)		(Q)	Y-component of $\vec{A}+\vec{B}$ is negative
(C)		(R)	X-component of $\vec{A}-\vec{B}$ is positive
(D)		(S)	Y-component of $\vec{A}-\vec{B}$ is negative
		(T)	$\vec{B}.\vec{A}$ is positive

36. Figure shows a cube of edge length a.



	Column-I		**Column-II**
(A)	The angle between AF and x-axis	(P)	60°
(B)	Angle between AF and DG	(Q)	$\cos^{-1}\frac{1}{3}$
(C)	Angle between AE and AG	(R)	$\cos^{-1}\frac{1}{\sqrt{3}}$
		(S)	$\cos^{-1}\sqrt{\frac{2}{3}}$

EXERCISE (J-M)

1. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency. **[JEE Main-2013]**

(1) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (2) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (3) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

EXERCISE (J-A)

1. Match List I with List II and select the correct answer using the codes given below the lists :

	List I		**List II**
P.	Boltzmann constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Planck constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

[JEE Advanced-2013]

Codes :

	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

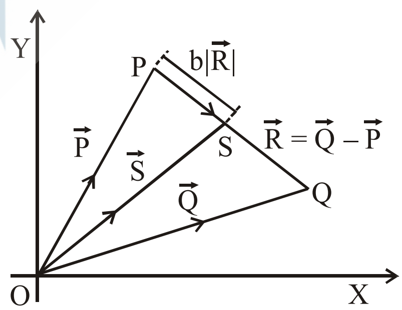
2. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is. **[JEE Advanced-2014]**

3. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are) **[JEE Advanced-2015]**

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 cV$ (D) $\mu_0 cI = \varepsilon_0 V$

4. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is :

[JEE Advanced-2017]



(A) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$ (C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (D) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. L^{-1}, ML^2T^{-2} **2. Ans.** $kg.m^2.s^{-2}.K^{-1}$ **3. Ans.** $T = a\sqrt{\frac{m}{k}}$ **4. Ans.** $\omega = K\sqrt{\frac{GM}{r^3}}$

5. Ans. $v_0 = k\sqrt{\frac{GM}{R}}$ **6. Ans.** K = 6 **7. Ans.** $[MLT^{-1}]$ **8. Ans.** $\sqrt{75}$

9. Ans. 1 **10. Ans.** 25 **11. Ans.** 7 **12. Ans.** 90 m

13. Ans. $\cos\alpha = \left(\frac{4}{\sqrt{61}}\right)$, $\cos\beta = \left(\frac{6}{\sqrt{61}}\right)$ $\cos\gamma = \left(\frac{3}{\sqrt{61}}\right)$, $magnitude = \sqrt{61}$

14. Ans. 3 **15. Ans.** 5 **16. Ans.** $\frac{7}{16}$ **17. Ans.** $\hat{i} + \hat{j}$, $3\hat{k}$

18. Ans. (a) $11\hat{i} + 5\hat{j} - 7\hat{k}$, (b) $\cos^{-1}\left(\frac{-7}{\sqrt{195}}\right)$, (c) $\cos^{-1}\left(\frac{-20}{\sqrt{1309}}\right)$

19. Ans. 2 **20. Ans.** (a) $\frac{2\pi}{3} \times 6.4 \times 10^6 m$, (b) $\sqrt{3} \times 6.4 \times 10^6$ m

21. Ans. (a) 9.95, (b) 0.99 **22. Ans.** 0.14, 0.09

EXERCISE (S-2)

1. Ans. $ML^5T^{-2}K^{1/2}$ **2. Ans.** $[S] = Ev^{-2}T^{-2}$ **3. Ans.** 10^{-30} star joule

4. Ans. $2\sqrt{19}$, $\cos^{-1}\frac{7}{2\sqrt{19}}$ or $\tan^{-1}\frac{3\sqrt{3}}{7}$ **5. Ans.** 100 s **6. Ans.** 120 N, $40\sqrt{3}$ N

7. Ans. $16\hat{i} - 30\hat{k}, 198J$

EXERCISE (O-1)

1. Ans. (D) **2. Ans. (C)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (B)** **6. Ans. (A)**
7. Ans. (C) **8. Ans. (D)** **9. Ans. (D)** **10. Ans. (A)** **11. Ans. (C)** **12. Ans. (B)**
13. Ans. (C) **14. Ans. (D)** **15. Ans. (B)** **16. Ans. (B)** **17. Ans. (D)** **18. Ans. (D)**
19. Ans. (C) **20. Ans. (A)** **21. Ans. (C)** **22. Ans. (A)** **23. Ans. (B)** **24. Ans. (C)**
25. Ans. (B) **26. Ans. (B)** **27. Ans. (D)** **28. Ans. (D)** **29. Ans. (A)** **30. Ans. (C)**
31. Ans. (B) **32. Ans. (A)** **33. Ans. (C)** **34. Ans. (B)** **35. Ans. (B)** **36. Ans. (B)**
37. Ans. (D) **38. Ans. (A,D)** **39. Ans. (C,D)** **40. Ans. (B, C)** **41. Ans. (B)** **42. Ans. (C)**
43. Ans. (A) **44. Ans. (B)** **45. Ans. (C)** **46. Ans. (A)-(Q); (B)-(R); (C)-(T); (D)-(P)**
47. Ans. (A)-R; (B)-S; (C)-P; (D)-Q **48. Ans. (A)-(S); (B)-(P); (C)-(R); (D)-(T)**
49. Ans. (A)-S; (B)-P; (C)-Q; (D)-R

EXERCISE (O-2)

1. Ans. (A) **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (B)** **5. Ans. (C)** **6. Ans. (A)**
7. Ans. (A) **8. Ans. (D)** **9. Ans. (A)** **10. Ans. (C)** **11. Ans. (C)** **12. Ans. (B)**
13. Ans. (A) **14. Ans. (A)** **15. Ans. (A)** **16. Ans. (B)** **17. Ans. (C)** **18. Ans. (C, D)**
19. Ans. (A,B,D) **20. Ans. (A, D)** **21. Ans. (B,D)** **22. Ans. (A,B)** **23. Ans. (A,C,D)**
24. Ans. (A, B, C) **25. Ans. (A)** **26. Ans. (C)** **27. Ans. (B)** **28. Ans. (C)**
29. Ans. (A) **30. Ans. (B)** **31. Ans. (B,C)** **32. Ans. (B,D)** **33. Ans. (B)**
34. Ans. (A) → (Q); (B) → (S); (C) → (R); (D) → (R)
35. Ans. (A) → (P,R,T); (B) → (P,R,T); (C) → (P,Q,R,S,T); (D) → (P,Q,R,S,T)
36. Ans. (A) R (B) Q (C) P

EXERCISE (J-M)

1. Ans. (3)

EXERCISE (J-A)

1. Ans. (C) **2. Ans.** 3 **3. Ans. (A,C)** **4. Ans. (C)**