Rankers

CONTENTS

KINETIC THEORY OF GASES & THERMODYNAMICS

KINETIC THEORY OF GASES

The properties of the gases are entirely different from those of solid and liquid. In case of gases, thermal expansion is very large as compared to solids and liquids .To state the conditions of a gas, its volume, pressure and temperature must be specified.

Intermolecular force   Solid > liquid > real gas > ideal gas (zero)

Potential energy   Solid < liquid < real gas < ideal gas (zero)

Internal energy, internal kinetic energy, internal potential energy

At a given temperature for solid, liquid and gas:

(i)   Internal kinetic energy : Same for all

(ii)   Internal potential Energy : Maximum for ideal gas (PE = 0) and Minimum for solids (PE = –ve)

(iii) Internal Energy : Maximum for Ideal gas and   Minimum for solid

At a given temperature for rared and compressed gas :

(i)   Internal kinetic energy $\rightarrow$ Same

(ii)   Internal potential energy $\rightarrow$ $(PE)_{Rared} > (PE)_{compressed}$

(iii) Internal Energy $\rightarrow$ $(U)_{Rared} > (U)_{compressed}$

	N.T.P.	S.T.P.
Temperature	0° C = 273.15 K	0.01° C = 273.16K
Pressure	1 atm = 1.01325×10^5 N/m²	1 atm
	= 1.01325×10^5 pascal	
Volume	22.4 litre	22.4 litre

IDEAL GAS CONCEPT

•   A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.

•   Volume of gas molecules is negligible as compared to volume of container so volume of gas = volume of container (Except 0 K)

•   No intermoleculer force act between gas molecules.

•   Potential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of gas. It is directly proportional to absolute temperature.
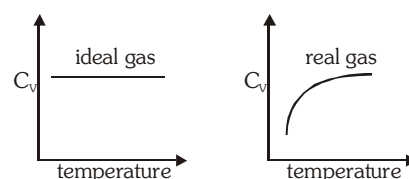
So, internal energy depends only and only on its temperature.

$E_{trans} \propto T$

For a substance $U = U_{KE} + U_{PE}$

U_{KE} : depends only on T, U_{PE} : depends upon intermolecular forces (Always negative)

•   Specific heat of ideal gas is constant quantity and it does not change with temperature

•   All real gases behaves as ideal gas at high temperature and low pressure.

•   Volume expansion coefficient (α) and pressure expansion coefficient (β) is same for a ideal gas and value of each is $\dfrac{1}{273}$ per °C $\alpha = \beta = \dfrac{1}{273}$ per °C

•   Gas molecule have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

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EQUATION OF STATE FOR IDEAL GAS

$PV = \mu RT$ where μ = number of moles of gas $\Rightarrow PV = \dfrac{M}{M_w}RT = \left[\dfrac{mN}{mN_0}\right]RT = \left[\dfrac{R}{N_0}\right]NT = NkT$

Ex. By increasing temperature of gas by 5° C its pressure increases by 0.5% from its initial value at constant volume then what is initial temperature of gas ?

Sol. $\because$ At constant volume $T \propto P$ $\therefore \dfrac{\Delta T}{T} \times 100 = \dfrac{\Delta P}{P} \times 100 = 0.5 \Rightarrow T = \dfrac{5 \times 100}{0.5} = 1000K$

Ex. Calculate the value of universal gas constant at STP.

Sol. Universal gas constant is given by $R = \dfrac{PV}{T}$

One mole of all gases at S.T.P. occupy volume $V = 22{\cdot}4$ litre $= 22{\cdot}4 \times 10^{-3}$ m³

$P = 760$ mm of Hg $= 760 \times 10^{-3} \times 13{\cdot}6 \times 10^3 \times 9.80$ N m⁻² $\quad T = 273$ K

$\therefore R = \dfrac{760 \times 10^{-3} \times 13.6 \times 10^3 \times 9.80 \times 22.4 \times 10^{-3}}{273} = 8{\cdot}31$ J mol⁻¹ K⁻¹

Ex. A closed container of volume 0.02 m³ contains a mixture of neon and argon gases at a temperature of 27°C and pressure of 1×10^5 Nm². The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal. Given : R = 8.314 J/mol/K.

Sol. Let m gram be the mass of neon. Then, the mass of argon is (28 – m)g.

Total number of moles of the mixture, $\quad \mu = \dfrac{m}{20} + \dfrac{28-m}{40} = \dfrac{28+m}{40}$ ...(i)

Now, $\quad \mu = \dfrac{PV}{RT} = \dfrac{1 \times 10^5 \times 0.02}{8.314 \times 300} = 0.8$ ...(ii)

By (i) and (ii), $\dfrac{28+m}{40} = 0.8 \Rightarrow 28 + m = 32 \Rightarrow m = 4$ gram or mass of argon $= (28 - 4)g = 24$ g

Ex. Calculate the temperature of the Sun if density is 1.4 g cm⁻³, pressure is 1.4×10^9 atmosphere and average molecular weight of gases in the Sun in 2 g/mole. [Given R = 8.4 J mol⁻¹K⁻¹]

Sol. $PV = \mu RT \Rightarrow T = \dfrac{PV}{\mu R}$ ...(i) $\qquad$ But $\mu = \dfrac{M}{M_w}$ and $\rho = \dfrac{M}{V}$ $\therefore \mu = \dfrac{\rho V}{M_w}$

From equation (i) $T = \dfrac{PVM_w}{\rho VR} = \dfrac{PM_w}{\rho R} = \dfrac{1.4 \times 10^9 \times 1.01 \times 10^5 \times 2 \times 10^{-3}}{1.4 \times 1000 \times 8.4} = 2.4 \times 10^7$ K

Ex. At the top of a mountain a thermometer reads 7°C and barometer reads 70 cm of Hg. At the bottom of the mountain they read 27°C and 76 cm of Hg respectively. Compare the density of the air at the top with that at the bottom.

Sol. By gas equation $PV = \dfrac{M}{M_w}RT \Rightarrow \dfrac{P}{\rho T} = \dfrac{R}{M_w}$ $\left[\because \mu = \dfrac{M}{M_w} \text{ and } \dfrac{M}{V} = \rho \right]$

Now as M_W and R are same for top and bottom $\left[\dfrac{P}{\rho T} \right]_T = \left[\dfrac{P}{\rho T} \right]_B$ So $\dfrac{\rho_T}{\rho_B} = \dfrac{P_T}{P_B} \times \dfrac{T_B}{T_T} = \dfrac{70}{76} \times \dfrac{300}{280} = \dfrac{75}{76} = 0.9868$

Ex. During an experiment an ideal gas is found to obey an additional law $VP^2 = $ constant. The gas is initially at temperature T and volume V. What will be the temperature of the gas when it expands to a volume 2V.

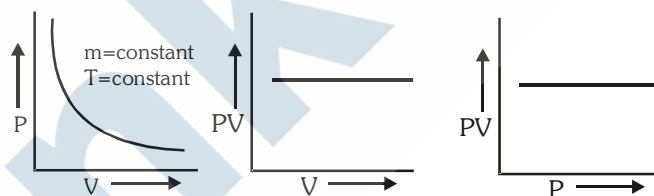**Sol.** By gas equation $PV = \mu RT$ and $VP^2 = $ constant on eliminating P

$\left[\dfrac{A}{\sqrt{V}} \right] V = \mu RT \Rightarrow \sqrt{V} = \dfrac{\mu R}{A}T \quad \therefore \dfrac{\sqrt{V_1}}{\sqrt{V_2}} = \left[\dfrac{T_1}{T_2} \right] \Rightarrow \dfrac{\sqrt{V}}{\sqrt{2V}} = \dfrac{T}{T'} \Rightarrow T' = \left(\sqrt{2} \right) T$

GAS LAWS

• **Boyle's Law**

According to it for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its

pressure, i.e., $V \propto \dfrac{1}{P}$ if m and T =

Constant

Ex. A sample of oxygen with volume of 500 cc at a pressure of 2 atm is compressed to a volume of 400 cc. What pressure is needed to do this if the temperature is kept constant ?

Sol. Temperature is constant, so $P_1 V_1 = P_2 V_2$

$\therefore \quad P_2 = P_1 \dfrac{V_1}{V_2} = 2 \left[\dfrac{500}{400} \right] = 2.5 \text{ atm}$

Ex. An air bubble doubles in radius on rising from bottom of a lake to its surface. If the atmosphere pressuer is equal to that due to a column of 10 m of water, then what will be the depth of the lake.

(Assuming that surface tension is negligible) ?

Sol. Given that constant temperature, we use $P_1 V_1 = P_2 V_2$

$P_2 = (10) \, dg$ (for water column) $\qquad P_1 = (10+h) \, dg$ (where h=depth of lake)

$V_1 = \dfrac{4\pi}{3}r^3 , \ V_2 = \dfrac{4\pi}{3}(2r)^3 = 8 \left(\dfrac{4\pi}{3}r^3 \right) = 8V_1$ Thus for $P_2 V_2 = P_1 V_1$,

We have $10 \, dg \, (8V_1) = (10 + h) \, dg \, V_1 \Rightarrow 80 = 10 + h \Rightarrow h = 70 \text{ m}$

Ex. A vessel of volume 8.0×10^{-3} m³ contains an ideal gas at 300 K and 200 k Pa. The gas is allowed to leak till the Pressure falls to 125 kPa. Calculate the amount of the gas leaked assuming that the temperature remains constant.

Sol. As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel in given by $n = \dfrac{PV}{RT}$.

The number of moles in the vessel before the leakage is $n_1 = \dfrac{P_1 V}{RT}$ and that after the leakage is $n_2 = \dfrac{P_2 V}{RT}$.
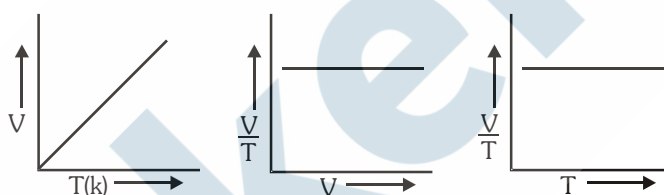
The amount leaked is $n_1 - n_2 = \dfrac{(P_1 - P_2)V}{RT} = \dfrac{(200-125) \times 10^3 \times 8.0 \times 10^{-3}}{8.3 \times 300} = 0.24$ mole

- **Charle's Law**

  According to it for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature, i.e.

  $V \propto T$ if m and P = Constant

Ex. 1500 ml of a gas at a room temperature of 23°C is inhaled by a person whose body temperature is 37°C, if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person ?
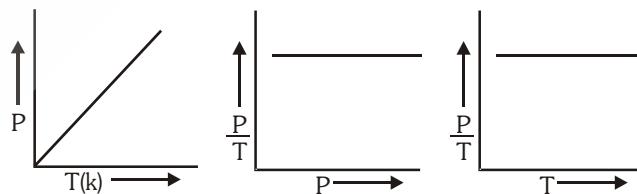
Sol. $T_1 = 273 + 37 = 310$ K; $T_2 = 273 + 23 = 296$ K. Pressure and amount of the gas are kept constant,

So $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$ ∴ $V_2 = V_1 \times \dfrac{T_2}{T_1} = 1500 \times \dfrac{293}{310} = 1417.74$ ml

Gay–Lussac's Law

According to it, for a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$ if m and V = constant

Ex. A sample of O_2 is at a pressure of 1 atm when the volume is 100 ml and its temperature is 27°C. What will be the temperature of the gas if the pressure becomes 2 atm and volume remains 100 ml.

Sol. $T_1 = 273 + 27 = 300$ K

For constant volume $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$ ⟹ $T_2 = T_1 \times \dfrac{P_2}{P_1} = 300 \times \dfrac{2}{1} = 600$ K = 600 – 273 = 327°C

Avogadro's Law

According to it, at same temperature and pressure of equal volumes of all gases contain equal number of molecules, i.e., $N_1 = N_2$ if P, V and T are same.

The kinetic theory of gases

- Rudolph Claussius (1822–88) and James Clark Maxwell (1831–75) developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules. The theory is based on following assumptions as regards to the motion of molecules and the nature of the gases.

Basic postulates of Kinetic theory of gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given as are all identical but are different than those another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparision to inter molecular distance (10^{-9} m)

Assumptions regarding motion :

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speeds is maximum.

Assumptions regarding collision:

- The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collisions.)

Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Assumptions regarding density:

- The density of gas is constant at all points of the container.

PROPERTIES/ASSUMPTIONS OF IDEAL GAS

- The molecules of a gas are in a state of continuous random motion. They move with all possible velocities in all possible directions. They obey Newton's law of motion.
- Mean momentum = 0; Mean velocity = 0. $< \vec{v} > = 0$; $< v^2 > \neq 0$ (Non zero); $< v^3 > = < v^5 > = 0$
- The average distance travelled by a molecule between two successive collisions is called as mean free path (λ_m) of the molecule.
- The time during which a collision takes place is negligible as compared to time taken by the molecule to cover the mean free path so NTP ratio of time of collision to free time of motion $10^{-8} : 1$.
- When a gas taken into a vessel it is uniformly distributed in entire volume of vessel such that its density, moleculer density, motion of molecules etc. all are identical for all direction, therefore root mean velocity

  $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 \rightarrow$ equal Pressure exerted by the gas in all direction $P_x = P_y = P_z = P \rightarrow$ equal

- All those assumptions can be justified, if number of gas molecules are taken very large

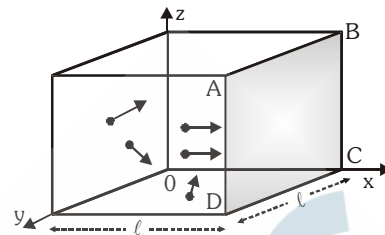## EXPRESSION FOR PRESSURE OF AN IDEAL GAS

Consider an ideal gas enclosed in a cubical vessel of length ℓ. Suppose there are 'N' molecules in a gas which are moving with velocities $\vec{v}_1, \vec{v}_2 \ldots \ldots \vec{v}_N$.

If we consider any single molecule than its instantaneous velocity

$\vec{v}$ can be expressed as $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

Due to random motion of the molecule $\quad v_x = v_y = v_z$

$$|v| = v_x \sqrt{3} = v_y \sqrt{3} = v_z \sqrt{3} = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Suppose a molecule of mass m is moving with a velocity v_x towards the face ABCD. It strikes the face of the cubical vessel and returns back to strike the opposite face.

Change in momentum of the molecule per collision $\Delta p = -mv_x - mv_x = -2mv_x$

Momentum transferred to the wall of the vessel per molecule per collision $\Delta p = 2mv_x$

The distance travelled by the molecule in going to face ABCD and coming back is 2ℓ.

So, the time between two successive collision is $\Delta t = \dfrac{2\ell}{v_x}$

Number of collision per sec per molecule is $f_c = \dfrac{v_x}{2\ell} = \dfrac{\text{molecule velocity}}{\text{mean free path}}$, $f_c = \dfrac{v_{rms}}{\lambda_m}$ or $f_c = \dfrac{v_m}{\lambda_m}$

Hence momentum transferred in the wall per second by the molecule is = force on the wall

$$\text{force } F = (2mv_x)\,\dfrac{v_x}{2\ell} = \dfrac{mv_x^2}{\ell} = \dfrac{mv^2}{3\ell}$$

Pressure exerted by gas molecule $P = \dfrac{F}{A} = \dfrac{1}{3}\dfrac{mv^2}{\ell \times A} \Rightarrow P = \dfrac{1}{3}\dfrac{mv^2}{V}$ $[\because A \times \ell = V]$

Pressure exerted by gas $P = \sum \dfrac{1}{3}\dfrac{mv^2}{V} = \sum \dfrac{1}{3}\dfrac{mv^2}{V} \times \dfrac{N}{N} = \dfrac{1}{3}\dfrac{mN}{V}\dfrac{\sum v^2}{N} = \dfrac{1}{3}\dfrac{mN}{V}v_{rms}^2$

$$v_{rms}^2 = \dfrac{3PV}{M} = \dfrac{3\mu RT}{\mu M_w} \Rightarrow v_{rms} = \sqrt{\dfrac{3RT}{M_w}}, \quad P = \dfrac{1}{3}\dfrac{M}{V}v_{rms}^2 = \dfrac{1}{3}\rho v_{rms}^2$$

- Average number of molecules for each wall $= \dfrac{N}{6}$. No. of molecules along each axis $= \dfrac{N}{3}$ $(N_x = N_y = N_z)$

- $\overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2 = \dfrac{\overline{v}_{rms}^2}{3}$ Root mean square velocity along any axis for gas molecule is $(v_{rms})_x = (v_{rms})_y$

$= (v_{rms})_z = \dfrac{v_{rms}}{\sqrt{3}}$

All gas laws and gas equation can be obtained by expression of pressure of gas (except Joule's law)

Ex. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} molecules are colliding per second on a stationary wall of area 2 cm² at an angle of 45° to the normal to the wall and reflected elastically with a speed 10^3 m/s. Find the pressure exerted on the wall will be (in N/m²)

Sol. As the impact is elastic $\quad |\vec{p}_1| = |\vec{p}_2| = p = mv = 3.32 \times 10^{-24}$ kg m/s

∴ The change in momentum along the normal $\Delta p = |\vec{p}_2 - \vec{p}_1| = 2p \cos 45° = \sqrt{2}p$

If f is the collision frequency then force applied on the wall $\quad F = \dfrac{\Delta p}{\Delta t} = \Delta p \times f = \sqrt{2}pf$

∴ $\quad$ Pressure $P = \dfrac{F}{A} = \dfrac{\sqrt{2}pf}{A} = \dfrac{\sqrt{2} \times 3.32 \times 10^{-24} \times 10^{23}}{2 \times 10^{-4}} = 2.347 \times 10^3 \, N/m^2$
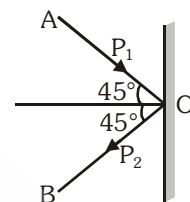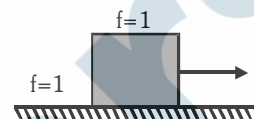
DEGREE OF FREEDOM (f)

- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called it's degrees of freedom.
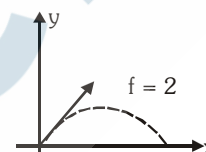
For example

(a) Block has one degree of freedom, because it is confined to move in a straight line and has only one transistional degree of freedom.
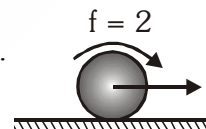
(b) The projectile has two degrees of freedom becomes it is confined to move

in a plane and so it has two translational degrees of freedom.

(c) The sphere has two degrees of freedom one rotational and another translational. Similarly a particle free to move in space will have three translational degrees of freedom.

Note : In pure rolling sphere has one degree of freedom as $KE = \dfrac{1}{2}mv^2 \left(1 + \dfrac{K^2}{R^2}\right) = \dfrac{7}{10}mv^2$

- The degrees of freedom are of three types :
  - **(a)** **Translational Degree of freedom :** Maximum three degree of freedom are there corresponding to translational motion.
  - **(b)** **Rotational Degree of freedom** : The number of degrees of freedom in this case depends on the structure of the molecule.
  - **(c)** **Vibrational Degree of freedom :** It is exhibited at high temperatures.

Degree of freedom for different gases according to atomicity of gas at low temperature

Atomicity of gas	Translational	Rotational	Total	
Monoatomic **Ex.** Ar, Ne, Ideal gas etc	3	0	3	
Diatomic **Ex.** O_2, Cl_2, N_2 etc.	3	2	5	

At high temperatures a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

Ex. Calculate the total number of degrees of freedom possessed by the molecules in one cm³ of H_2 gas at NTP.

Sol. 22400 cm³ of every gas constains 6.02×10^{23} molecules.

∴   Number of molecules in 1 cm³ of H_2 gas $= \dfrac{6.02 \times 10^{23}}{22400} = 0.26875 \times 10^{20}$

Number of degrees of freedom of a H_2 gas molecule = 5

∴   Total number of degrees of freedom of $0.26875 \times 10^{20} \times 5 = 1.34375 \times 10^{20}$.

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and the energy associated with each degree of freedom at absolute temperature T is $\dfrac{1}{2}kT$

For one molecule of gas

Energy related with each degree of freedom $= \dfrac{1}{2}kT$

Energy related with all degree of freedom $= \dfrac{f}{2}kT$ $\because \bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \dfrac{\bar{v}_{rms}^2}{3} \Rightarrow \dfrac{1}{2}mv_{rms}^2 = \dfrac{3}{2}kT$

So energy related with one degree of freedom $= \dfrac{1}{2}m\dfrac{v_{rms}^2}{3} = \dfrac{3}{2}\dfrac{kT}{3} = \dfrac{1}{2}kT$

Ex. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m². During an observation time of 1 second, an atom travelling with the root–mean–square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = \dfrac{25}{3}$ J/mol–K and $k = 1.38 \times 10^{-23}$ J/K.
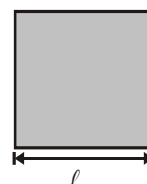
(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box.

Sol. Volume of the box = 1m³, Pressure of the gas = 100 N/m². Let T be the temperature of the gas

(a)   Time between two consecutive collisions with one wall $= \dfrac{1}{500}$ sec

This time should be equal to $\dfrac{2I}{v_{rms}}$, where ℓ is the side of the cube.

$2\ell v_{rms} = \dfrac{1}{500} \Rightarrow v_{rms} = 1000$ m/s $\therefore \sqrt{\dfrac{3RT}{M}} = 1000 \Rightarrow T = \dfrac{(1000)^2 M}{3R} = \dfrac{(10)^6(3 \times 10^{-3})}{3\left(\dfrac{25}{3}\right)} = 160K$

(b)   Average kinetic energy per atom $= \dfrac{3}{2}kT = \dfrac{3}{2}$ [(1.38 × 10⁻²³) = 160] J = 3.312 ×⁻²¹ J

(c)   From $PV = nRT = \dfrac{m}{M}RT$,   Mass of helium gas in the box $m = \dfrac{PVM}{RT}$

Substituting the values, $m = \dfrac{(100)(1)(4 \times 10^{-3})}{\left(\dfrac{25}{3}\right)(160)} = 3.0 \times 10^{-4}$ kg

DIFFERENT K.E. OF GAS (INTERNAL ENERGY)

- **Translatory kinetic energy (E_T)** $\quad E_T = \dfrac{1}{2}Mv_{rms}^2 = \dfrac{3}{2}PV$

  Kinetic energy of volume V is $= \dfrac{1}{2}Mv_{rms}^2$ Note : Total internal energy of ideal gas is kinetic

- **Energy per unit volume or energy density (E_V)**

  $E_V = \dfrac{\text{Total energy}}{\text{Volume}} = \dfrac{E}{V}$; $E_V = \dfrac{1}{2}\left[\dfrac{M}{V}\right]v_{rms}^2 = \dfrac{1}{2}\rho v_{rms}^2 \because P = \dfrac{2}{3}\left[\dfrac{1}{2}\rho v_{rms}^2\right] \therefore E_V = \dfrac{3}{2}P$

- **Molar K.E. or Mean Molar K.E. (E)**

  $E = \dfrac{1}{2}M_w v_{rms}^2$ for N_0 molecules or M_w (gram) $\qquad E = \dfrac{3}{2}RT = \dfrac{3}{2}N_0 kT$

- **Molecular kinetic energy or mean molecular K.E. ($\bar{E}$)**

  $E = \dfrac{1}{2}M_w v_{rms}^2$, $\bar{E} = \dfrac{E}{N_0} = \dfrac{3}{2}\dfrac{RT}{N_0} \Rightarrow \bar{E} = \dfrac{3}{2}kT$

GOLDEN KEY POINT

- Except 0 K, at any temperature T , $E > E_m > \bar{E}$
- At a common temperature, for all ideal gas

  E and $\bar{E}$ are same while E_m is different and depends upon nature of gas (M_w or m)
- For thermal equilibrium of gases, temperature of each gas is same and this temperature called as temperature of mixture (T_m) which can be find out on basis of conservation of energy (All gases are of same atomicity). $T_m = \dfrac{\sum NT}{\sum N} = \dfrac{N_1 T_1 + N_2 T_2 + \dots\dots + N_n T_n}{N_1 + N_2 \dots\dots N_n}$

- **1 mole gas :** Mean kinetic energy $= \dfrac{3}{2}RT$ $\qquad$; $\qquad$ Total kinetic energy $= \dfrac{f}{2}RT$

  1 molecule of gases : Mean kinetic energy $= \dfrac{3}{2}kT$; Total kinetic energy $= \dfrac{f}{2}kT$

  $f \to$ Degree of freedom

Ex. Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. Find the temperature of the mixture.

Sol. Total energy of molecules of first gas $= \dfrac{3}{2}n_1 kT_1$, Total energy of molecules of second gas $= \dfrac{3}{2}n_2 kT_2$

Let temperature of mixture be T then total energy of molecules of mixture $= \dfrac{3}{2}k(n_1 + n_2)T$

$\therefore \dfrac{3}{2}(n_1 + n_2)kT = \dfrac{3}{2}k(n_1 T_1 + n_2 T_2) \Rightarrow T = \dfrac{n_1 T_1 + n_2 T_2}{(n_1 + n_2)}$

Ex. The first excited state of hydrogen atom is 10.2 eV above its ground state. What temperature is needed to excite hydrogen atoms to first excited level.

Sol. K.E. of the hydrogen atom $\dfrac{3}{2}kT = 10.2 \text{ eV} = 10.2 \times (1.6 \times 10^{-19}) \text{ J}$

$\Rightarrow \quad T = \dfrac{2}{3} \times \dfrac{10.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 7.88 \times 10^4 \text{ K}$

DIFFERENT SPEEDS OF GAS MOLECULES

- **Average velocity**

  Because molecules are in random motion in all possible direction in all possible velocity. Therefore,

  the average velocity of the gas in molecules in container is zero. $<\vec{v}> = \dfrac{\vec{v}_1 + \vec{v}_2 + \ldots\ldots\ldots \vec{v}_N}{N} = 0$

  rms speed of molecules $v_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3RT}{M_w}} = \sqrt{\dfrac{3kT}{m}} = 1.73\sqrt{\dfrac{kT}{m}}$

  Mean speed of molecules : By maxwell's velocity distribution law $\quad v_M$ or $<|\vec{v}|> = v_{mean}$

  $<|\vec{v}|> = v_{mean} = \dfrac{|\vec{v}_1| + |\vec{v}_2| + \ldots\ldots\ldots |\vec{v}_n|}{N} = \sqrt{\dfrac{8}{\pi}\dfrac{P}{\rho}} = \sqrt{\dfrac{8RT}{\pi M_w}} = \sqrt{\dfrac{8kT}{\pi m}} = 1.59\sqrt{\dfrac{kT}{m}}$

  Most probable speed of molecules (v_{mp})

  At a given temperature, the speed to which maximum number of molecules belongs is called as most

  probable speed (v_{mp}) $\quad v_{mp} = \sqrt{\dfrac{2P}{\rho}} = \sqrt{\dfrac{2RT}{M_w}} = \sqrt{\dfrac{2kT}{m}} = 1.41\sqrt{\dfrac{kT}{m}}$

MAXWELL'S LAW OF DISTRIBUTION OF VELOCITIES
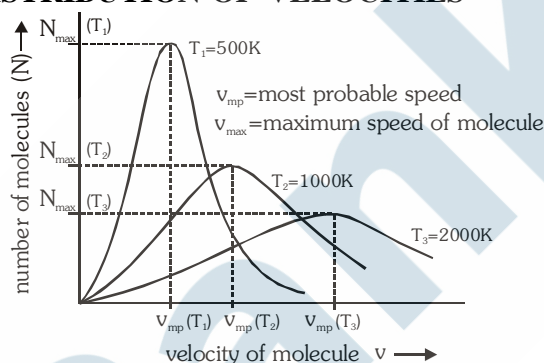
GOLDEN KEY POINT

- At any given temperature graph drawn in between molecular velocity and number of molecules is known as velocity distribution curve.
- The velocities of molecules of a gas are in between zero and infinity $(0 - \infty)$
- With the increase in the temperature, the most probable velocity and maximum molecule velocity both increases.
- The number of molecules within certain velocity range is constant although the velocity of molecule changes continuously at particular temperature.
- The area enclosed between the $(N - v)$ curve and the velocity axis presents the total number of molecules.

  On the basis of velocity distribution Maxwell established gives the law of equipartition of energy for gases of any temperature.

Ex. The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate
(i) average speed and (ii) rms speed (iii) most probable speed.

Sol. (i) average speed, $v_{av} = \dfrac{0+2+3+4+4+4+5+5+6+9}{10} = \dfrac{42}{10} = 4\cdot2$ ms^{-1}

(ii) rms speed, $v_{rms} = \left[\dfrac{(0)^2 + (2)^2 + (3)^2 + (4)^2 + (4)^2 + (4)^2 + (5)^2 + (5)^2 + (6)^2 + (9)^2}{10}\right]^{1/2} = \left[\dfrac{228}{10}\right]^{1/2} = 4.77$ ms^{-1}

(iii) most probable speed $v_{mp} = 4$ m/s

Ex. At what temperature, will the root mean square velocity of hydrogen be double of its value at S.T.P., pressure remaining constant ?

Sol. Let v_1 be the r.m.s. velocity at S.T.P. and v_2 be the r.m.s. velocity at unknown temperature T_2.

$$\therefore \quad \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2} \quad \text{or} \quad T_2 = T_1 \left[\frac{v_2}{v_1}\right]^2 = 273 \times (2)^2 = 273 \times 4 = 1092 \text{ K} = (1092 - 273) = 819°C$$

Ex. Calculate rms velocity of oxygen molecule at 27°C

Sol. Temperature, $T = 27°C \Rightarrow 273 + 27 = 300$ K,

Molecular weight of oxygen $= 32 \times 10^{-3}$ kg and $R = 8·31$ J mol^{-1} K^{-1}

rms velocity is $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3 \times 8·31 \times 300}{32 \times 10^{-3}}} = 483·5$ ms^{-1}

Ex. Calculate the kinetic energy of a gram moelcule of argon at 127°C.

Sol. Temperature, $T = 127°C = 273 + 127 = 400$ K, $R = 8.31$ J/mol K

K.E. per gram molecule of argon $= \dfrac{3}{2}$ R T $= \dfrac{3}{2} \times 8.31 \times 400 = 4986$ J

THERMODYNAMICS

Branch of physics which deals with the inter–conversion between heat energy and any other form of energy is known as thermodynamics. In this branch of physics we deals with the processes involving heat, work and internal energy. In this branch of science the conversion of heat into mechanical work and vice versa is studied.

- **Thermodynamical System**

  The system which can be represented in of pressure (P), volume (V) and temperature (T), is known thermodynamic system. A specified portion of matter consisting of one or more substances on which the effects of variables such as temperature, volume and pressure are to be studied, is called a system. e.g. A gas enclosed in a cylinder fitted with a piston is a system.

- **Surroundings**

  Anything outside the system, which exchanges energy with the system and which tends to change the properties of the system is called its surroundings.

- **Heterogeneous System**

  A system which is not uniform throughout is said to be heterogeneous. e.g. A system consisting of two or more immiscible liquids.

- **Homogeneous System**

  A system is said to be homogeneous if it is completely uniform throughout. e.g. Pure solid or liquid.

- **Isolated System**

  A system in which there can be no exchange of matter and energy with the surroundings is said to be an isolated system.

- **Universe**

  The system and its surroundings are together known as the universe.

- **Thermodynamic variables of the system**

  (i) Composition (μ) (ii) Temperature (T) (iii) Volume (V) (iv) Pressure (P)

- **Thermodynamic state**

  The state of a system can be described completely by composition, temperature, volume and pressure. If a system is homogeneous and has definite mass and composition, then the state of the system can be described by the remaining three variables namely temperature, pressure and volume. These variables are interrelated by equation $PV = \mu RT$ The thermodynamic state of the system is its condition as identified by two independent thermodynamic variables (P, V or P, T or V, T).

- **Zeroth law of thermodynamics**

  If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduce the concept of temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are the same.

  In measuring the temperature of a body, it is important that the thermometer be in the thermal equilibrium with the body whose temperature is to be measured.

- **Thermal equilibrium**

  Thermal equilibrium is a situation in which two objects in thermal contact cease to exchange energy by the process of heat. Heat is the transfer of energy from one object to another object as a result of a difference in temperature between them.

- **Internal Energy**

  Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy (U_k) and that due to molecular configuration is called internal potential energy (U_p). $dU = dU_k + dU_p$

  If there no intermolecular forces, then $dU_p = 0$ and $dU = dU_k = m\, c_v\, dT$

  c_v = Specific heat at constant volume and dT = Infinitesimal change in temperature

  m = Mass of system   M = Molecular weight

  Molar heat capacity $C_v = Mc_v$ For μ–moles of ideal gas $dU = \mu C_v dT = \dfrac{m}{M} C_v dT$

  Internal energy in the absence of inter–molecular forces is simply the function of temperature and state only, it is independent of path followed. $\Delta U = U_f - U_i$

  U_i = Internal energies in initial state and U_f = Internal energies in final state

- **Thermodynamic Processes**

  In the thermodynamic process pressure, volume, temperature and entropy of the system change with time.

  Thermodynamic process is said to take place if change occurs in the state of a thermodynamic system.

- **Sign convention used for the study of thermodynamic processes**

Heat gained by a system	Positive
Heat lost by a system	Negative
The work done by a system	Positive
Work done on the system	Negative
Increase in the internal energy of system	Positive
Decrease in the internal energy of system	Negative

- **Indicator Diagram or P–V Diagram**

  In the equation of state of a gas $PV = \mu RT$

  Two thermodynamic variables are sufficient to describe the behavior of a thermodynamic system.

  If any two of the three variables P, V and T are known then the third can be calculated.

  P–V diagram is a graph between the volume V and the pressure P of the system.

  The volume is plotted against X–axis while the pressure is plotted against Y–axis.

  The point A represents the initial stage of the system. Initial pressure of the system is P_i and initial volume of the system V_i.

  The point B represents the final state of the system. P_f and V_f are the final pressure and final volume respectively of the system. The points between A and B represent the intermediate states of the system. With the help of the indicator diagram we calculate the amount of work done by the gas or on the gas during expansion or compression.
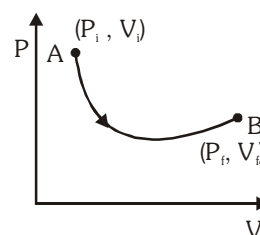
WORK DONE BY THERMODYNAMIC SYSTEM

One of the simple example of a thermodynamic system is a gas in a cylinder with a movable piston.

- **If the gas expands against the piston**

  Gas exerts a force on the piston and displace it through a distance and does work on the piston.

- **If the piston compresses the gas**

  When piston moved inward, work is done on the gas.

- **The work associated with volume changes**

  If pressure of gas on the piston = P.

  Then the force on the piston due to gas is F = PA

  When the piston is pushed outward an infinitesimal distance dx,

  the work done by the gas is dW = F × dx = PA dx

  The change in volume of the gas is dV = Adx, ∴ dW = PdV

  For a finite change in volume from V_i to V_f, this equation is then integrated between V_i to V_f to find the

$$\text{net work done } W = \int_{V_i}^{V_f} dW = \int_{V_i}^{V_f} PdV$$

Hence the work done by a gas is equal to the area under P–V graph.

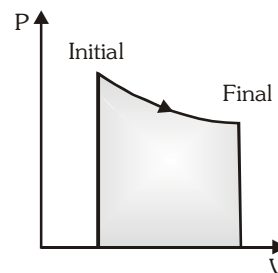Following different cases are possible.

(i) Volume is constant
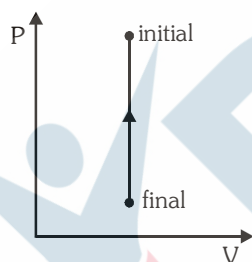
or
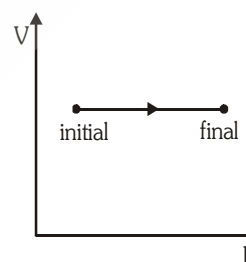
$$V = \text{constant} \quad \text{and} \quad W_{AB} = 0$$

(ii) Volume is increasing
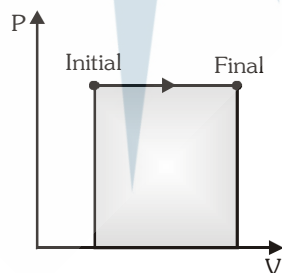
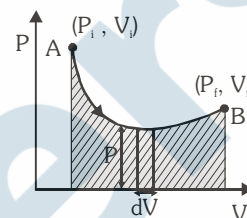or

V is increasing $W_{AB} > 0$ W_{AB} = Shaded area
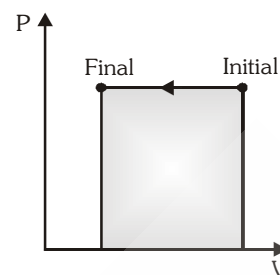
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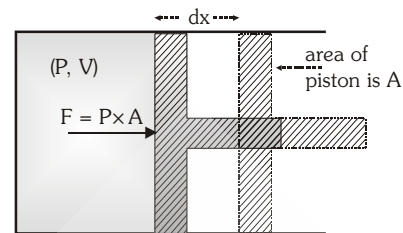
(iii) **Volume is decreasing**
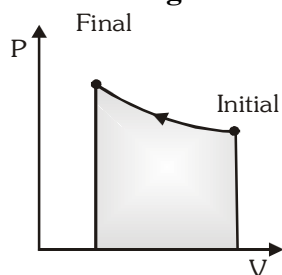
$$V \text{ is decreasing} \qquad W_{AB} < 0 \qquad\qquad W_{AB} = - \text{ Shaded area}$$

FIRST LAW OF THERMODYNAMICS

Let a gas in a cylinder with a moveable piston changes from an initial equilibrium state to a final equilibrium state.

System change its state through path **'a'** :

The heat absorbed by the system in this process = δQ_a

The work done by the system = δW_a

• **Again for path 'b' :**

Heat absorbed by the system = δQ_b , Work done by the system = δW_b.

It is experimental fact that the $\delta Q_a - \delta W_a = \delta Q_b - \delta W_b$

Both δQ and δW depend on the thermodynamic path taken between two equilibrium states, but difference ($\delta Q - \delta W$) does not depends on path in between two definite states of the system.

So, there is a function (internal energy) of the thermodynamic coordinates (P, V and T) whose final value (U_f) minus its initial value (U_i) equals the change $\delta Q - \delta W$ in the process.

• **dU = $\delta Q - \delta W$.** This is the first law of thermodynamics.

Heat supplied to the system and work done by the system are path dependent so they are denoted by δQ and δW respectively. Change in internal energy $\Delta U = U_f - U_i$ does not depends on path it depends only on initial and final positions of the system. So, it is denoted by dU (or ΔU)

• **First Law of Thermodynamics**

If some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system. $\delta Q = dU + \delta W$ or $Q = W + \Delta U$

* This law is applicable to every process in nature

* The first law of thermodynamics introduces the concept of internal energy.

* The first law of thermodynamics is based on the law of conservation of energy.

* δQ, dU and δW must be expressed in the same units (either in units of work or in units of heat).

* This law is applicable to all the three phases of matter, i.e., solid, liquid and gas.

* dU is a characteristic of the state of a system, it may be any type of internal energy–translational kinetic energy, rotational kinetic energy, binding energy etc.

• **Limitations of first law of thermodynamics** :

It does not explain the direction of heat flow and it does not explain how much amount of heat given will be converted into work.

• **Significance of the first law of thermodynamics** :

The first law of thermodynamics tells us that it is impossible to get work from any machine without giving it an equivalent amount of energy.

Ex. The pressure in monoatomic gas increases linearly from 4×10^5 Nm^{-2} to $8 \times 10^{+5}$ Nm^{-2} when its volume increases from 0.2m^3 to 0.5 m^3. Calculate.
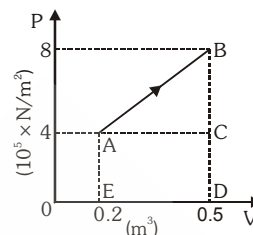
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(i) Work done by the gas,   (ii) Increase in the internal energy,

(iii) Amount of heat supplied,   (iv) Molar heat capacity of the gas R = 8.31 J mol^{-1} K^{-1}

Sol. $P_1 = 4 \times 10^5$ Nm^{-2} $P_2 = 8 \times 10^{+5}$ Nm^{-2}, $V_1 = 0.2$ m^3, $V_2 = 0.5$ m^3

(i)   Work done by the gas = Area under P–V graph (Area ABCDEA)

$$= \frac{1}{2}(AE + BD) \times AC \quad = \frac{1}{2}(4 \times 10^5 + 8 \times 10^5) \times (0.5 - 0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5 \text{ J}$$

(ii)   Increase in internal energy $\Delta U = C_v (T_2 - T_1) = \frac{C_v}{R} R(T_2 - T_1) = \frac{C_v}{R}(P_2 V_2 - P_1 V_1)$

For monoatomic gas

$$C_v = \frac{3}{2}R \quad \therefore \Delta U = \frac{3}{2}[(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)] = \frac{3}{2}[4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5 \text{ J}$$

(iii) $Q = \Delta U + W = 4.8 \times 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5$ J

(iv) $C = \dfrac{Q}{\eta \Delta T} = \dfrac{QR}{\eta R \Delta T} \ = \dfrac{QR}{\eta(P_2 V_2 - P_1 V_1)} = \dfrac{6.6 \times 10^5 \times 8.31}{1 \times 3.2 \times 10^5} = 17.14$ J/mole K

Ex. When a system is taken from state a to state b, in figure along the path

a $\to$ c $\to$ b, 60 J of heat flow into the system, and 30 J of work is

done :

(i) How much heat flows into the system along the path a $\to$

d $\to$ b if the work is 10 J.

(ii) When the system is returned from b to a along the curved path,

the work done by the system is –20 J. Does the system absorb or
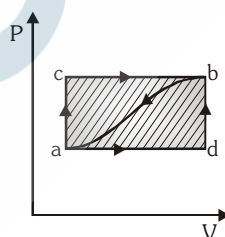
liberate heat, and how much?

(iii) If, $U_a = 0$ and $U_d = 22$ J, find the heat absorbed in the process a $\to$ d and d $\to$ b.

Sol. For the path a, c, b, $\Delta U = Q - W = 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path a, d, b, $Q = \Delta U + W = 30 + 10 = 40$ J

(ii) Along the curved path b, a, $Q = (U_a - U_b) + W = (-30) + (-20) = -50$ J, heat flows out the system

(iii) $Q_{ad} = 32$ J; $Q_{db} = 8$ J

ISOMETRIC OR ISOCHORIC PROCESS

•   Isochoric process is a thermodynamic process that takes place at constant volume of the system, but pressure and temperature varies for change in state of the system.
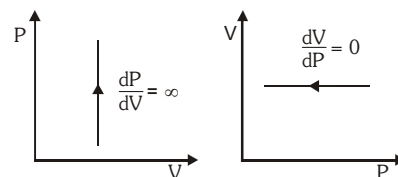
Equation of state   P = constant × T (P and T are variable, V is constant)

Work done   In this process volume remains constant $\Delta V = 0$ or $dV = 0 \Rightarrow W = \int_{V_i}^{V_f} P dV = 0$

Form of first Law $Q = \Delta U$

It means whole of the heat supplied is utilized for change

in internal energy of the system. $Q = \Delta U = \mu\, C_v \Delta T$

Slope of the P–V curve $\dfrac{dP}{dV} = \infty$

Specific heat at constant volume (C_V)

The quantity of heat required to raise the temperature of 1 gram mole gas through 1 °C at constant volume is equal to the specific heat at constant volume.

•   A gas enclosed in a cylinder having rigid walls and a fixed piston. When heat is added to the gas, there would be no change in the volume of the gas.

- When a substance melts, the change in volume is negligibly small. So, this may be regarded as a nearly isochoric process.
- Heating process in pressure cooker is an example of isometric process.

Ex. An ideal gas has a specific heat at constant pressure $C_p = \dfrac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m³ at a temperature of 300K and a pressure of 1.6×10^6 Nm⁻². An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

Sol. $C_V = C_P - R = \dfrac{5R}{2} - R = \dfrac{3R}{2}$, $\Delta V = 0$, $T_1 = 300$ K, V = 0.0083 m³, $P_1 = 1.6 \times 10^6$ Nm⁻²

From first law of thermodynamics $Q = \Delta U + P\Delta V \Rightarrow \Delta U = Q = 2.49 \times 10^4$ J

From gas equation $n = \dfrac{PV}{RT} = \dfrac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \dfrac{16}{3}$

$\therefore \Delta U = nC_V \Delta T \Rightarrow \Delta T = \dfrac{\Delta U}{nC_v} = \dfrac{2.49 \times 10^4 \times 6}{3 \times 8.3 \times 16} = 375$ K

Final temperature = 300 + 375 = 675K

According to pressure law $P \propto T \Rightarrow \dfrac{P_2}{P_1} = \dfrac{T_2}{T_1} \Rightarrow P_2 = \dfrac{T_2}{T_1} \times P_1 = \dfrac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6$ Nm⁻²

Ex. 5 moles of oxygen is heated at constant volume from 10°C to 20°C. What will be change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure.
$C_p = 8$ cal/mole and R = 8.36 joule/mole °C.

Sol. $\because C_V = C_P - R = 8 - 2 = 6$ cal/mole °C

$\therefore$ Heat absorbed by 5 moles of oxygen at constant volume

$Q = nC_V \Delta T = 5 \times 6 (20 - 10) = 30 \times 10 = 300$ cal

At constant volume $\Delta V = 0$. $\therefore \Delta W = 0$

$\therefore$ From first law of thermodynamics $Q = \Delta U + W \Rightarrow 300 = \Delta U + 0 \Rightarrow \Delta U = 300$ cal

ISOBARIC PROCESS

Isobaric process is a thermodynamic process that takes place at constant pressure, but volume and temperature varies for change in state of the system.

- **Equation of state** $\qquad$ V = constant × T or V ∝ T
- **Work done** $\qquad$ In this process pressure remains constant $\qquad \Delta P = 0$

    Work done $W = \int_{V_i}^{V_f} PdV = P(V_f - V_i)$

- **Form of first Law** $\qquad Q = \Delta U + P(V_f - V_i)$

    $\mu C_p\, dT = \mu C_v\, dT + P(V_f - V_i)$

It is clear that heat supplied to the system is utilized for :

(i) Increasing internal energy and

(ii) Work done against the surrounding atmosphere.

- **Slope of the PV curve :** $\left(\dfrac{dP}{dV}\right)_{isobaric} = 0$

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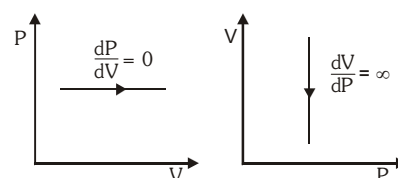
- **Specific heat at constant pressure (C_P)**

  The quantity of heat required to raise the temperature of 1 gram mole gas through 1°C at constant pressure is equal to the specific heat. Heating of water at atmospheric pressure. • Melting of solids and boiling of liquids at atmospheric pressure.

Ex. At normal pressure and 0°C temperature the volume of 1 kg of ice is reduced by 91 cm³ on melting. Latent heat of melting of ice is 3.4×10^5 J/kg. Calculate the change in the internal energy when 2kg of ice melts at normal pressure and 0°C. ($P = 1.01 \times 10^5$ Nm⁻²)

Sol. Heat energy absorbed by 2 kg of ice for melting $Q = mL = 2 \times 3.4 \times 10^5 = 6.8 \times 10^5$ J

Change in volume of 2 kg of ice = $2 \times 91 = 182$ cm³ = 182×10^{-6} m³

$\therefore W = P\Delta V = 1.01 \times 10^5 \times (-182 \times 10^{-6}) = -18.4$ J

Since, work is done on ice so work W is taken –ve. Now from first law of thermodynamics

$$Q = \Delta U + W \implies \Delta U = Q - W \qquad = 6.8 \times 10^5 - (-18.4) = (6.8 \times 10^5 + 18.4)J$$

Ex. What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise the temperature by 45°C at constant pressure. Molecular mass of N_2 = 28, R = 8.3 J mol⁻¹ K⁻¹.

Sol. Here $m = 2 \times 10^{-2}$ kg, $\Rightarrow n = \dfrac{m}{M} = \dfrac{2 \times 10^{-2}}{28 \times 10^{-3}} = \dfrac{5}{7}$ & $C_P = \dfrac{7}{2}R$

$$\therefore Q = nC_P\Delta T = \dfrac{5}{7} \times \dfrac{7}{2} \times 8.3 \times 45 = 933.75 \text{ J}$$

ISOTHERMAL PROCESS

In this process pressure and volume of system change but temperature remains constant.

In an isothermal process, the exchange of heat between the system and the surroundings is allowed. Isothermal process is carried out by either supplying heat to the substance or by extracting heat from it. A process has to be extremely slow to be isothermal.

- **Equation of state**

  $$P V = \text{constant } (\mu RT) \ (T \text{ is constant})$$

- **Work Done**

  Consider μ moles of an ideal gas, enclosed in a cylinder, at absolute temperature T, fitted with a frictionless piston. Suppose that gas undergoes an isothermal expansion from the initial state (P_1, V_1) to the final state (P_2, V_2).

  $$\therefore \text{Work done}: W = \int_{V_1}^{V_2} \dfrac{\mu RT}{V} dV = \mu RT \int_{V_1}^{V_2} \dfrac{dV}{V} = \mu RT \left[\log_e V\right]_{V_1}^{V_2}$$

  $$= \mu RT \left[\log_e V_2 - \log_e V_1\right] = \mu RT \log_e \left[\dfrac{V_2}{V_1}\right]$$

  $$\implies W = 2.303 \mu RT \log_{10}\left[\dfrac{P_1}{P_2}\right] \quad [\because P_1V_1 = P_2V_2]$$

Form of First Law

There is no change in temperature and internal energy of the system depends on temperature only

So $\quad \Delta U = 0, \ Q = 2.303 \ \mu RT \log_{10}\left[\dfrac{V_2}{V_1}\right]$
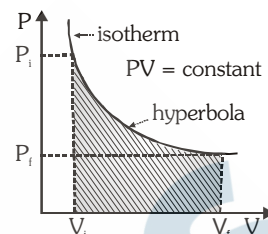
It is clear that Whole of the heat energy supplied to the system is utilized by the system in doing external work. There is no change in the internal energy of the system.

Slope of the isothermal curve

For an isothermal process, PV = constant

Differentiating, $PdV + VdP = 0$ $\Rightarrow VdP = -PdV \Rightarrow \dfrac{dP}{dV} = -\dfrac{P}{V}$

Slope of isothermal curve, $\left[\dfrac{dP}{dV}\right]_{isothermal} = -\dfrac{P}{V}$

For a given system :

•   The product of the pressure and volume of a given mass of a perfect gas remains constant in an isothermal process.

•   Boyle's law is obeyed in an isothermal process.

•   A graph between pressure and volume of a given mass of a gas at constant temperature is known as isotherm or isothermal of the gas.

•   Two isotherms for a given gas at two different temperatures T_1 and T_2 are shown in figure.

•   The curves drawn for the same gas at different temperatures are mutually parallel and do not cut each other.

•   If two isotherms intersect each other at a single point we get same value of P and V at intersection point.

• $PV = \mu RT_1$ for temperature T_1 and $PV = \mu RT_2$ for temperature T_2.

    It means $T_1 = T_2$ which is not possible.
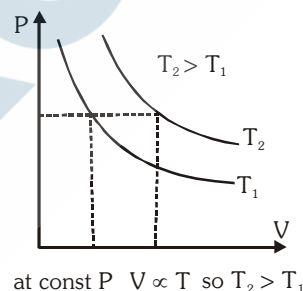
•   An ideal gas enclosed in a conducting cylinder fitted with a conducting piston. Let the gas be allowed to expand very–very slowly.

•   This shall cause a very slow cooling of the gas, but heat will be conducted into the cylinder from the surrounding. Hence the temperature of the gas remains constant. If the gas is compressed very–very slowly, heat will be produced, but this heat will be conducted to the surroundings and the temperature of the gas shall remain constant.

•   The temperature of a substance remains constant during melting. So, the melting process is an isothermal process.

•   Boiling is an isothermal process, when a liquid boils, its temperature remains constant.

•   If sudden changes are executed in a vessel of infinite conductivity then they will be isothermal.

Ex.   Two moles of a gas at 127°C expand isothermally until its volume is doubled. Calculate the amount of work done.

Sol.   n = 2,   T = 127 + 273 = 400K, $\dfrac{V_2}{V_1} = 2$

From formula $\quad W = 2.3026\, nRT \log_{10} \dfrac{V_2}{V_1} = 2.3026 \times 2 \times 8.3 \times 400 \times \log_{10} 2$

$$= 2.3026 \times 2 \times 8.3 \times 400 \times 0.3010 \approx 4.6 \times 10^3 J$$

Ex. Figure shows a process ABCA performed on an ideal gas.
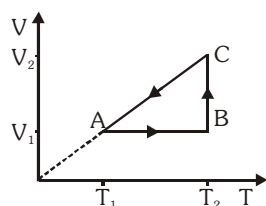
Find the net heat given to the system during the process.

Sol. Since the process is cyclic, hence the change in internal energy is zero.

The heat given to the system is then equal to the work done by it.

The work done in part AB is $W_1 = 0$ (the volume remains constant). The part BC represents an

isothermal process so that the work done by the gas during this part is $W_2 = nRT_2 \ln \dfrac{V_2}{V_1}$

During the part CA $\quad V \propto T \quad$ So, V/T is constant and hence, $P = \dfrac{nRT}{V}$ is constant

The work done by the gas during the part CA is $W_3 = P(V_1 - V_2) = nRT_1 - nRT_2 = - nR(T_2 - T_1)$.

The net work done by the gas in the process ABCA is $W = W_1 + W_2 + W_3 = nR\left[T_2 \ln \dfrac{V_2}{V_1} - (T_2 - T_1)\right]$

The same amount of heat is given to the gas.

ADIABATIC PROCESS

It is that thermodynamic process in which pressure, volume and temperature of the system change but there is no exchange of heat between the system and the surroundings.

A sudden and quick process will be adiabatic since there is no sufficient time available for exchange of heat so process adiabatic.

Equation of state : $PV = \mu RT$

Equation for adiabatic process $PV^{\gamma} = $ constant

Work done

Let initial state of system is (P_1, V_1, T_1) and after adiabatic change final state of system is (P_2, V_2, T_2) then we can write $P_1 V_1^{\gamma} = P_2 V_2^{\gamma} = K$ (here K is const.)

So $\quad W = \int_{V_1}^{V_2} PdV = K\int_{V_1}^{V_2} V^{-\gamma} dV = K\left(\dfrac{V^{-\gamma+1}}{-\gamma+1}\right)_{V_1}^{V_2} = \dfrac{K}{(-\gamma+1)}\left[V_2^{-\gamma+1} - V_1^{-\gamma+1}\right] \quad \left(\because K = P_1 V_1^{\gamma} = P_2 V_2^{\gamma}\right)$

$\Rightarrow W = \dfrac{1}{(\gamma-1)}\left[P_1 V_1^{\gamma} V_1^{-\gamma}.V_1 - P_2 V_2^{\gamma} V_2^{-\gamma}.V_2\right] = \dfrac{1}{(\gamma-1)}\left[P_1 V_1 - P_2 V_2\right]$

$\Rightarrow W = \dfrac{\mu R}{(\gamma-1)}(T_1 - T_2) \quad (\because PV = \mu RT)$

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Form of first law : $dU = -\delta W$

It means the work done by an ideal gas during adiabatic expansion (or compression) is on the cost of change in internal energy proportional to the fall (or rise) in the temperature of the gas.

If the gas expands adiabatically, work is done by the gas. So, W_{adia} is positive.

The gas cools during adiabatic expansion and $T_1 > T_2$.

If the gas is compressed adiabatically, work is done on the gas. So, W_{adia} is negative.

The gas heats up during adiabatic compression and $T_1 < T_2$.

Slope of the adiabatic curve

For an adiabatic process, $PV^\gamma = $ constant

Differentiating, $P^\gamma V^{\gamma-1} dV + V^\gamma dP = 0$

$$\Rightarrow V^\gamma dP = -\gamma PV^{\gamma-1} dV \Rightarrow \frac{dP}{dV} = -\frac{\gamma PV^{\gamma-1}}{V^\gamma} = -\gamma \frac{P}{V} = \gamma\left(-\frac{P}{V}\right)$$

Slope of adiabatic curve, $\left[\dfrac{dP}{dV}\right]_{adiabatic} = -\dfrac{\gamma P}{V}$

(Adiabatic expansion of gas)

monoatomic $\gamma = 1.33$
diatomic $\gamma = 1.4$
polyatomic $\gamma = 1.67$
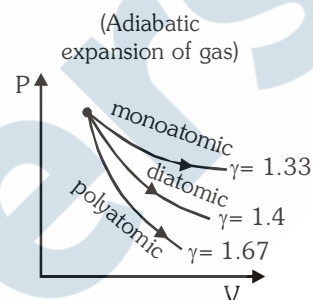
Slope of adiabatic is greater than the slope of isotherm

$$\left[\frac{dP}{dV}\right]_{adia} = \gamma\left[-\frac{P}{V}\right] = \gamma\left[\frac{dP}{dV}\right]_{iso} \Rightarrow \frac{\text{slope of adiabatic changes}}{\text{slope of isothermal changes}} = \gamma$$

Since γ is always greater than one so an adiabatic is steeper than an isotherm

(expansion of gas)

isothermal
adiabatic
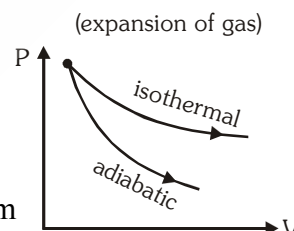
Examples of adiabatic process

• A gas enclosed in a thermally insulated cylinder fitted with a non–conducting piston. If the gas is compressed suddenly by moving the piston downwards, some heat is produced. This heat cannot escape the cylinder. Consequently, there will be an increase in the temperature of the gas.

• If a gas is suddenly expanded by moving the piston outwards, there will be a decrease in the temperature of the gas.

• Bursting of a cycle tube.

• Propagation of sound waves in a gas.

• In diesel engines burning of diesel without spark plug is done due to adiabatic compression of diesel vapour and air mixture

Ex. Why it is cooler at the top of a mountain than at sea level?

Sol. Pressure decreases with height. Therefore if hot air rises, it suffers adiabatic expansion.

From first law of thermodynamics $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$ [$\because \Delta Q = 0$]

This causes a decrease in internal energy and hence a fall of temperature.

Ex. $2m^3$ volume of a gas at a pressure of 4×10^5 Nm^{-2} is compressed adiabatically so that its volume becomes $0.5m^3$. Find the new pressure. Compare this with the pressure that would result if the compression was isothermal. Calculate work done in each process. $\gamma = 1.4$

Sol. $V_1 = 2m^3$, $P_1 = 4 \times 10^5$ Nm^{-2}, $V_2 = 0.5m^3$

In adiabatic process $\quad P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_2 = 4 \times 10^5 \left[\dfrac{2}{0.5}\right]^{1.4} = 4 \times 10^5 (4)^{1.4} = 2.8 \times 10^6$ Nm^{-2}

In isothermal process $\quad P_1 V_1 = P_2 V_2 \Rightarrow P_2 = \dfrac{P_1 V_1}{V_2} = \dfrac{4 \times 10^5 \times 2}{0.5} = 1.6 \times 10^6$ Nm^{-2}.

Now work done in adiabatic process $W = \dfrac{P_2 V_2 - P_1 V_1}{\gamma - 1} = \dfrac{(2.8 \times 10^6 \times 0.5) - (4 \times 10^5 \times 2)}{1.4 - 1} = 1.48 \times 10^6$ J.

Work done in isothermal process $W = 2.3026 RT \log \dfrac{V_2}{V_1} = 2.3026 P_1 V_1 \log \dfrac{V_2}{V_1}$

$= 2.3026 \times 4 \times 10^5 \times 2 \times \log\left[\dfrac{0.5}{2.0}\right] = 2.3026 \times 4 \times 10^5 \times 2\log\left(\dfrac{1}{4}\right) = -1.1 \times 10^6$ J

Ex. Two samples of a gas initially at same temperature and pressure are compressed from a volume V to $\dfrac{V}{2}$. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

Sol. Let initial volume, $V_1 = V$ and pressure, $P_1 = P$, final volume, $V_2 = \dfrac{V}{2}$ and final pressure, $P_2 = ?$

For isothermal compression $P_2 V_2 = P_1 V_1$ or $P_2 = \dfrac{P_1 V_1}{V_2} = \dfrac{PV}{\dfrac{V}{2}} = 2P$

For adiabatic compression $P_2' = P_1 \left[\dfrac{V_1}{V_2}\right]^\gamma \Rightarrow P_2' = P\left[\dfrac{V}{V/2}\right]^\gamma = 2^\gamma P$

$\Rightarrow P_2' = 2^\gamma P \quad \gamma > 1 \quad \therefore 2^\gamma > 2$ and $P_2' > P_2$

Pressure during adiabatic compression is greater than the pressure during isothermal compression.

Ex. Calculate the work done when 1 mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litre respectively. The final volume of the gas is 2 liters. Molar specific heat of the gas at constant volume is $\dfrac{3R}{2}$ [$(3)^{5/3} = 6.19$]

Sol. For an adiabatic change $PV^\gamma =$ constant $P_1 V_1^\gamma = P_2 V_2^\gamma$

As molar specific heat of gas at constant volume $C_v = \dfrac{3}{2}R$

$C_P = C_V + R = \dfrac{3}{2}R + R = \dfrac{5}{2}R \Rightarrow \gamma = \dfrac{C_P}{C_V} = \dfrac{\dfrac{5}{2}R}{\dfrac{3}{2}R} = \dfrac{5}{3}$

$$\therefore P_2 = \left[\frac{V_1}{V_2}\right]^\gamma P_1 = \left[\frac{6}{2}\right]^{5/3} \times 10^5 = (3)^{5/3} \times 10^5 = 6.19 \times 10^5 \ N/m^2$$

Work done $\quad W = \dfrac{1}{1-\gamma}[P_2V_2 - P_1V_1] = \dfrac{1}{1-(5/3)}[6.19 \times 10^5 \times 2 \times 10^{-3} - 10^{-5} \times 6 \times 10^{-3}]$

$$= -\left[\frac{2 \times 10^2 \times 3}{2}(6.19 - 3)\right] = -3 \times 10^2 \times 3.19 = -957 \ joules$$

– ive sign shows external work done on the gas

Ex. A motor tyre pumped to a pressure of 3 atm. It suddenly bursts. Calculate the fall in temperature due to adiabatic expansion. The temperature of air before expansion is 27°C. Given γ=1.4.

Sol. We know that $\ T_2{}^\gamma P_2{}^{1-\gamma} = T_1{}^\gamma P_1{}^{1-\gamma} \Rightarrow \left[\dfrac{T_2}{T_1}\right]^\gamma = \left[\dfrac{P_1}{P_2}\right]^{1-\gamma} \Rightarrow \left[\dfrac{T_2}{300}\right]^{1.4} = \left[\dfrac{3}{1}\right]^{1-1.4}$

$$\Rightarrow \left[\frac{T_2}{300}\right]^{1.4} = \left[\frac{1}{3}\right]^{0.4} \Rightarrow T_2 = 219.2 \ K \Rightarrow T_1 - T_2 = (300 - 219.2) \ K = 80.8 \ K$$

GOLDEN KEY POINT

- When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.
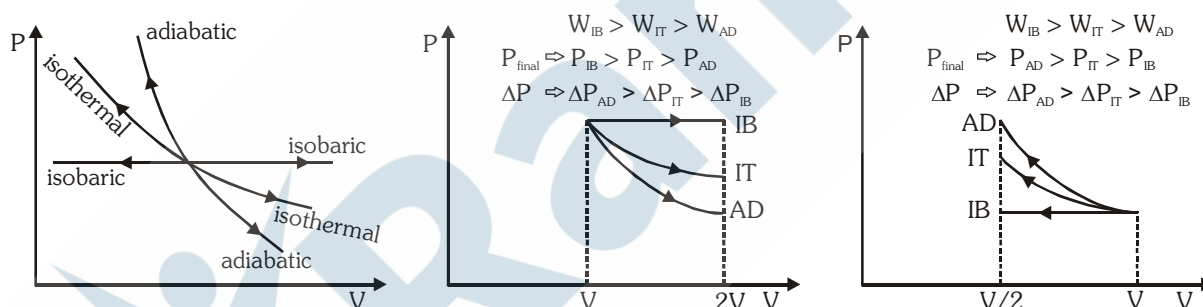
First Law of Thermodynamics Applied to Different Processes

Process	Q	ΔU	W
Cyclic	W	0	Area of the closed curve
Isochoric	ΔU	$\mu C_v \Delta T$ (μ mole of gas)	0
Isothermal	W	0	$\mu RT \log_e\left[\dfrac{V_f}{V_i}\right] = \mu RT \log_e\left[\dfrac{P_i}{P_f}\right]$
Adiabatic	0	−W	$\dfrac{\mu R(T_f - T_i)}{1 - \gamma}$
Isobaric	$\mu C_P \Delta T$	$\mu C_V \Delta T$	$P(V_f - V_i) = \mu R(T_f - T_i)$

Ex. Plot P – V , V – T graph corresponding to the P–T graph for an ideal gas shown in figure. Explain your answers.

Sol. For process AB   T = constant so $P \propto \dfrac{1}{V}$

For process BC   P = constant so $V \propto T$

For process CD   T = constant so $V \propto \dfrac{1}{P}$

For process DA   P = constant so $V \propto T$

(P - V curve)

(V - T curve)

FREE EXPANSION

Take a thermally insulated bottle with ideal gas at some temperature T_1 and, by means of a pipe with a stopcock, connect this to another insulated bottle which is evacuated. If we suddenly open the stopcock, the gas will rush from the first bottle into the second until the pressures are equalized.

Experimentally, we find that this process of free expansion does not change the temperature of the gas – when the gas attains equilibrium and stops flowing, the final temperature of both botles are equal to the initial temperature T_1.

free expansion of a gas

This process is called a free expansion.

The change in the internal energy of the gas can be calculated by applying the first law of thermodynamics to the free–expansion process.

The process is adiabatic because of the insulation, So Q = 0.

No part of the surroundings moves so the system does no work on its surroundings.

- **For ideal gas** $(\delta W)_{ext.}$ = Work done against external atmosphere

    = P dV = 0 (because P = 0)

    $(\delta W)_{int.}$ = Work done against internal molucular forces = 0

    $\delta Q = dU + \delta W \Rightarrow$ $0 = dU + 0$

The internal energy does not change dU = 0   So U and T are constant.

The initial and final states of this gas have the same internal energy.

Which implies that the internal energy of an ideal gas does not depend on the volume at all.

The free–expansion process has led us to the following conclusion :

The internal energy U(T) of an ideal gas depends only on the temperature.

- **For real gas**

In free expansion of real gases, measurements show that the temperature changes slightly in a free expansion. Which implies that the internal energy of a real gas depends on the volume also.

δQ = 0 $(\delta W)_{ext.} = 0$ $(\because P = 0)$

$(\delta W)_{int.} \neq 0$ (Intermolucular forces are present in real gases)

δQ = dU + δW $\Rightarrow 0 = dU + (\delta W)_{int.} \Rightarrow dU = - (\delta W)_{int.}$

$\Rightarrow$ U decreases. So T decreases.
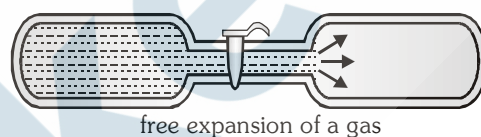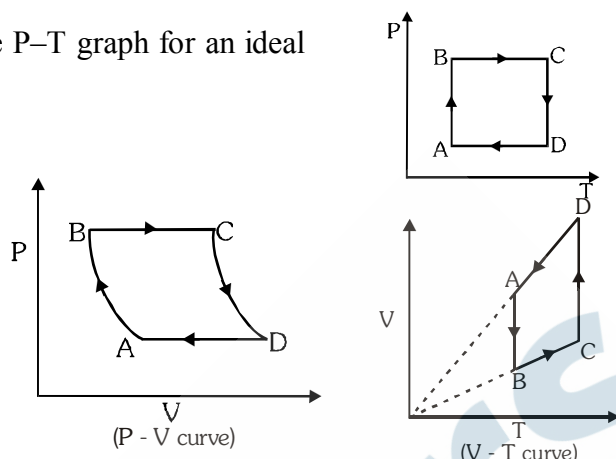
RELATION BETWEEN DEGREE OF FREEDOM AND SPECIFIC HEAT OF GAS

Energy related with each degree of freedom $= \dfrac{1}{2}kT$, Energy related with all degree of freedom $= \dfrac{f}{2}kT$

Internal energy of one mole of ideal gas (total K.E.) $U = \dfrac{f}{2}RT$ for Isometric process (volume constant) $\delta W = 0$

By first law of thermodynamics $\quad \delta Q = \delta W + dU \quad \Rightarrow \quad C_V \, dT = dU \quad \Rightarrow \quad C_V = \dfrac{dU}{dT}$

$$C_V = \dfrac{dU}{dT} = \dfrac{f}{2}R = \dfrac{R}{\gamma - 1} \quad . \; C_P = C_V + R = \left[\dfrac{f}{2} + 1\right]R = \dfrac{\gamma R}{\gamma - 1} \quad \text{and} \quad \gamma = \dfrac{C_P}{C_V} = 1 + \dfrac{2}{f}$$

$$C_V = \dfrac{R}{\gamma - 1} \quad , \qquad C_P = \dfrac{\gamma R}{\gamma - 1} \quad \text{and} \quad \gamma = 1 + \dfrac{2}{f}$$

General expression for C (C_P or C_V) in the process PV^x = constant $C = \dfrac{R}{\gamma - 1} + \dfrac{R}{1 - x}$

For isobaric process P = constant	so	x = 0	$\therefore$	$C = C_P = \dfrac{R}{\gamma - 1} + R = C_V + R$
For isothermal process, PV = constant	so	x = 1	$\therefore$	$C = \infty$
For adiabatic process PV^γ = constant	so	x = γ	$\therefore$	$C = 0$

Values of f, U, C_V, C_P and γ for different gases are shown in table below.

Atomicity of gas	f	C_V	C_P	γ
Monoatomic	3	$\dfrac{3}{2}R$	$\dfrac{5}{2}R$	$\dfrac{5}{3} = 1.67$
Diatomic	5	$\dfrac{5}{2}R$	$\dfrac{7}{2}R$	$\dfrac{7}{5} = 1.4$
Triatomic and Triatomic linear	7	$\dfrac{7}{2}R$	$\dfrac{9}{2}R$	$\dfrac{9}{7} = 1.28$
Poly atomic Triangular Non-linear	6	$\dfrac{6}{2}R = 3R$	$\dfrac{8}{2}R = 4R$	$\dfrac{4}{3} = 1.33$

- $1 < \gamma < 2$
- If atomicity of gases is same U, C_p, C_V and γ is same for gas mixture.
- If in a gas mixture gases are of different atomicity, then for gas mixture γ changes according to following condition. Diatomic $\gamma_1 \leq$ mixture $\leq \gamma_2$ mono atomic where $\gamma_1 < \gamma_2$
- If 'f' is the degree of freedom per molecule for a gas, then

Total energy of each molecule $= \dfrac{fkT}{2}$

Total energy per mole of gas $= N_0 \dfrac{f}{2}kT = \dfrac{f}{2}RT$

- According to kinetic theory of gases, the molecule are not interacting with each other. So potential energy is zero and internal energy of gas molecules is only their kinetic energy.

- For 'μ' mole of a gas : Internal energy at temperature T is $U = \dfrac{\mu f R T}{2} = \mu C_V T$

- Change in internal energy is given by $dU = \dfrac{\mu f R}{2}(dT) = \mu C_V dT$

  This change is process independent.

C_P is greater than C_V

If a gas is heated at constant volume, the gas does no work against external pressure. In this case, the whole of the heat energy supplied to the gas is spend in raising the temperature of the gas.

If a gas is heated at constant pressure, its volume increases. In this case, heat energy is required for the following two purpose :

(i) To increase the volume of the gas against external pressure.

(ii) To increase the temperature of 1 mole of gas through 1 K.

Thus, more heat energy is required to raise the temperature of 1 mole of gas through 1 K when it is heated at constant pressure than when it is heated at constant volume. $\therefore C_P > C_V$

The difference between C_P and C_V is equal to thermal equivalent of the work done by the gas in expanding against external pressure.

Mayer's formula : $C_P - C_V = R$

$\because$ At constant pressure $dQ = \mu C_P dT$, $dU = \mu C_V dT$ & $dW = PdV = \mu R dT$

Now from first law of thermodynamics $dQ = dW + dU$

$\Rightarrow \mu C_P dT = \mu R dT + \mu C_V dT \Rightarrow C_P = R + C_V \Rightarrow C_P - C_V = R$

Ex. Calculate the difference between two specific heats of 1 g of helium gas at NTP. Molecular weight of helium = 4 and J = 4.186×10^7 erg cal^{-1}.

Sol. Gas constant for 1 g of helium, $r = \dfrac{R}{M_w} = \dfrac{PV}{T \times M_w} = \dfrac{76 \times 13.6 \times 981 \times 22400}{273 \times 4} = 2.08 \times 10^7$ erg g^{-1} K^{-1}

$C_P - C_V = \dfrac{r}{J} = \dfrac{2.08 \times 10^7}{4.186 \times 10^7} = 0.5$ cal g^{-1} K^{-1}

Ex. Calculate the molar specific heat at constant volume. Given : specific heat of hydrogen at constant pressure is 6.85 cal mol^{-1} K^{-1} and density of hydrogen = 0.0899 g cm^{-3}. One mole of gas = 2.016 g, J = 4.2×10^7 erg cal^{-1} and 1 atmosphere = 10^6 dyne cm^{-2}.

Sol. Since the density of hydrogen is 0.0899 g cm^{-3} therefore volume occupied by 0.0899 g of hydrogen at NTP is 1000 cm^3. So, volume of 1 mole (2.016 g) of gas, $V = \dfrac{1000}{0.0899} \times 2.016$ cm^3

$C_P - C_V = \dfrac{R}{J} = \dfrac{PV}{T \times J} = \dfrac{10^6 \times 1000 \times 2.016}{0.0899 \times 273 \times 4.2 \times 10^7} = 1.96$ cal mol^{-1} K^{-1}

$\therefore C_V = C_P - 1.96 = (6.85 - 1.96) = 4.89$ cal mol^{-1} K^{-1}

Ex. The specific heat of argon at constant volume is 0.075 kcal/kg K. Calculate its atomic weight, [R = 2cal/mol K]

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Sol. As argon is monoatomic, its molar specific heat at constant volume will be

$$C_V = \frac{3}{2}R = \frac{3}{2} \times 2 = 3 \text{ cal/mol K}, \quad C_V = M_w c_V \text{ and } c_V = 0.075 \text{ cal/g K}$$
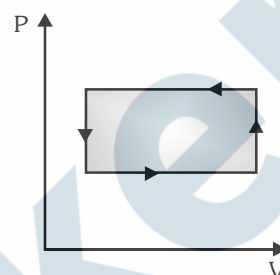
So $\quad 3 = M_w \times 0.075 \Rightarrow M_w = \dfrac{3}{0.075} = 40 \text{ gram/mole}$

Cyclic process

Cyclic process is that thermodynamic process in which the system returns to its initial stage after undergoing a series of changes. A typical cyclic process is represented on PV diagram as shown in figure.
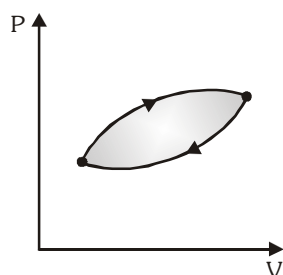
$$W_{\text{clockwise cycle}} = + \text{ Shaded area}$$

$$W_{\text{anticlockwise cycle}} = - \text{ Shaded area}$$

WORK DONE IN CLOCKWISE CYCLE
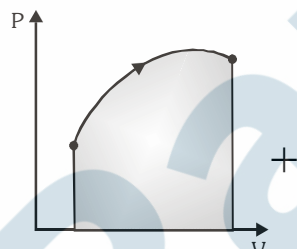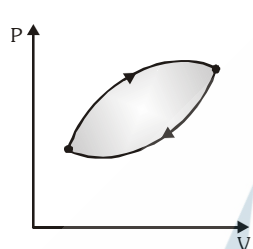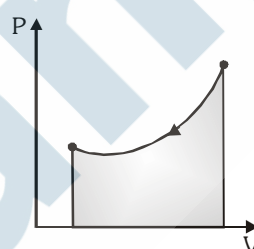
$W_{\text{AB I}}$ Positive $\qquad$ $W_{\text{AB II}}$ Negative

$$W_{\text{cyclic}} = W_{\text{AB I}} \text{(positive)} + W_{\text{AB II}} \text{ (negative)} = \text{area of closed path}$$

Similarly work done in anticlociwise cyclic process is negative.

8. CARNOT CYCLE

Carnot devised an ideal engine which is based on a reversible cycle of four operations in succession :

(i) isothermal expansion, $\qquad$ A → B

(ii) adiabatic expansion, $\qquad$ B → C

(iii) isothermal compression $\quad$ C → D

(iv) adiabatic compression. $\quad$ D → A
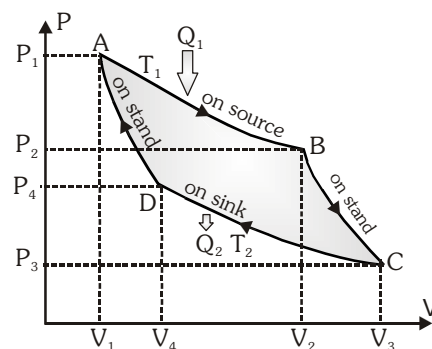
Main parts of Carnot's engine are

Source of heat

It is a hot body of very large heat capacity kept at a constant high temperature T_1.

Mechanical arrangements and working substance

It is a cylinder whose walls are perfectly non-conducting and its base is perfectly conducting fitted with non-conducting piston. This piston move without any friction. Ideal gas enclosed in cylinder as a working substance.

Heat sink

It is a cold body at low temperature T_2. It is a body of large heat capacity.

Stand

It is of two types : conducting and nonconducting.

Working

Through a set of reversible processes the working substance is taken back to initial condition to get maximum work from this type of ideal engine.

Processes of Carnot's cycle can be denoted by an indicator diagram.

Isothermal expansion A → B

Initially the cyclinder is taken to be in thermal equilibrium with the high temperature T_1, this is initial state of working substance denoted by point A (P_1, V_1, T_1).

After that the piston is allowed to move outward slowly. With the movement of the piston the process is very slow so that it is isothermal.

Heat from reservoir flows through the base of cylinder into the gas so temperature of the gas remains T_1.

Gas expand and receive heat Q_1 from source and gets state $B(P_2, V_2, T_1)$

This heat input Q_1 to the gas from path A to B is utilized for doing work W_1.

By path A to B the heat input to the gas = the work done against the external pressure.
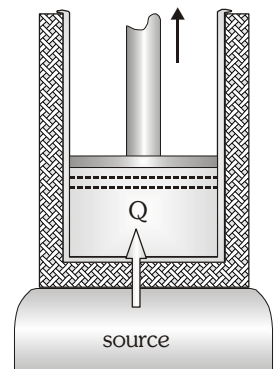
$$W_1 = Q_1 = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{\mu R T_1}{V} dV = \mu R T_1 \, \ell n \frac{V_2}{V_1}$$

Adiabatic expansion B → C

Now cylinder is put in contact with a non-conducting stand and piston is allowed to move outward, because no heat can enter in or leave out so the expansion of gas is adiabatic. The tempeature falls to $T_2 K$ and gas describes the adiabatic process from B to **point C** (P_3, V_3, T_2) during this expansion more work is done (W_2) at the expense of the internal energy.
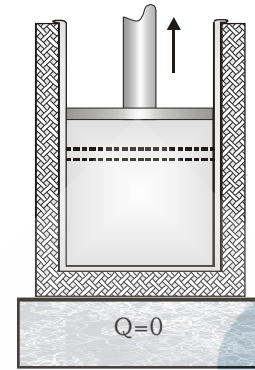
Work done in adiabatic path BC is $W_2 = \dfrac{\mu R}{\gamma - 1}(T_1 - T_2)$

Isothermal compression C → D

Now the gas cylinder is placed in contact with sink at temperature T_2. The piston is moved slowly inward so that heat produced during compression passes to the sink. The gas is isothermally compressed from C to **point D**. (P_4, V_4, T_2)
The heat rejected Q_2 to the cold reservoir (sink) at T_2 occurs over this path.

Amount of work done on gas W_3 = amount of heat rejected to the sink
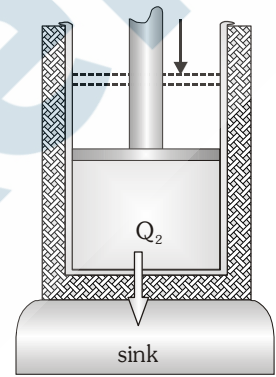
$$Q_2 = W_3 = \mu R T_2 \ell n \left(\frac{V_4}{V_3}\right) \Rightarrow Q_2 = \mu R T_2 \ell n\left(\frac{V_4}{V_3}\right)$$

Adiabatic compression D → A

The cylinder is removed from the sink and is put in contact with insulating stand now piston moves inward. Heat is not allowed to go out and it increases the internal energy of the system. Now work is done on the gas during adiabatic compression from state D to initial point A (P_1, V_1, T_1).
No heat exchanges occur over the adiabatic path.

Work done on the system $W_4 = \dfrac{\mu R}{\gamma - 1}(T_2 - T_1)$
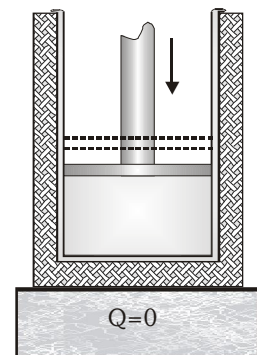
This cycle of operations is called a Carnot cycle.
In first two steps work is done by engine W_1 and W_2 are positive
In last two steps work is done on gas W_3 and W_4 are negative
The work done in complete cylce W = the area of the closed part of the P–V cycle.

$$W = W_1 + W_2 + W_3 + W_4$$

$$\therefore \quad W = \mu R T_1 \ell n \frac{V_2}{V_1} + \frac{\mu R}{\gamma - 1}(T_1 - T_2) + \mu R T_2 \ell n \frac{V_4}{V_3} + \frac{\mu R}{\gamma - 1}(T_2 - T_1) \quad = \mu R T_1 \ell n \frac{V_2}{V_1} + \mu R T_2 \ell n \frac{V_4}{V_3}$$

Efficiency of Carnot Engine, $\eta = \dfrac{W}{Q_1} = \dfrac{\mu RT_1 \ell n \dfrac{V_2}{V_1} + \mu RT_2 \ell n \dfrac{V_4}{V_3}}{\mu RT_1 \ell n \dfrac{V_2}{V_1}}$

B to C and D to A are adiabatic paths

so $T_1 V_2^{(\gamma-1)} = T_2 V_3^{(\gamma-1)}$ and $T_1 V_1^{(\gamma-1)} = T_2 V_4^{(\gamma-1)} \Rightarrow \dfrac{V_2}{V_1} = \dfrac{V_3}{V_4}$

$\eta = \dfrac{T_1 - T_2}{T_1} = \dfrac{Q_1 - Q_2}{Q_1} \Rightarrow \eta = 1 - \dfrac{Q_2}{Q_1} = 1 - \dfrac{T_2}{T_1}$ $\qquad \dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$

$\eta = \dfrac{T_1 - T_2}{T_1} \times 100\% \Rightarrow \eta = \dfrac{Q_1 - Q_2}{Q_1} \times 100\%$

The efficiency for the Carnot engine is the best that can be obtained for any heat engine.

The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink $T_2 = 0\,k$ and $T_1 = \infty$ which is impossible.

CARNOT THEOREM

No irreversible engine (I) can have efficiency greater than Carnot reversible engine (R) working

between same hot and cold reservoirs. $\eta_R > \eta_I \Rightarrow 1 - \dfrac{T_2}{T_1} > 1 - \dfrac{Q_2}{Q_1}$

9.   SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy to include heat energy. It tells us that heat and mechanical work are mutually interconvertible.

Second law of thermodynamics tells us in what conditions heat can be converted into useful work.

The following three conditions must be fulfilled to utilize heat for useful work :

(i)   A device called engine with a working substance is essential.

(ii)   The engine must work in a reversible cyclic process.

(iii)   The engine must operate between two temperatures. It will absorb heat from a hot body (called source), and converts a part of it into useful work and reject the rest to a cold body (called sink).

There are two conventional statements of second law :

Kelvin-Planck Statement

It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work. In other words, 100% conversion of heat into work is impossible.

Clausius Statement

It is impossible for a self-acting machine, unaided by any external agency to transfer heat from a cold to hot reservoir. In other words heat can not in itself flow from a colder to a hotter body.
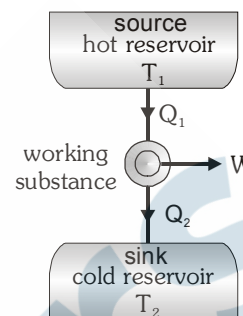
10. **HEAT ENGINE**

Heat engine is a device which converts heat into work.

Three parts of a heat engine:

(i)   Source of high temperature reservoir at temperature T_1

(ii)   Sink or low temperature reservoir at temperature T_2

(iii) Working substance.

In a cycle of heat engine the working substance extracts heat Q_1 from source, does some work W and rejects remaining heat Q_2 to the sink.

Efficiency of heat engine $\eta = \dfrac{\text{work done (W)}}{\text{heat taken from source}\,(Q_1)}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

Ex.   A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

Sol.   W = 1200 J, T_1 = 800 K, T_2 = 400 K

$$\therefore \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \Rightarrow 1 - \frac{400}{800} = \frac{1200}{Q_1} \Rightarrow 0.5 = \frac{1200}{Q_1}$$

Heat energy supplied by source $Q_1 = \dfrac{1200}{0.5} = 2400$ joule per cycle

Ex.   The temperatures T_1 and T_2 of the two heat reservoirs in an ideal carnot engine are 1500°C and 500°C respectively. Which of the following : increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine?

Sol. T_1 = 1500°C = 1500 + 273 = 1773 K and T_2 = 500°C = 500 + 273 = 773 K.

The efficiency of a carnot's engine $\eta = 1 - \dfrac{T_2}{T_1}$

When the temperature of the source is increased by 100°C, keeping T_2 unchanged, the new temperature of the source is T'_1 = 1500 + 100 = 1600°C = 1873 K. The efficiency becomes

$$\eta' = 1 - \frac{T_2}{T_1'} = 1 - \frac{773}{1873} = 0.59$$

On the other hand, if the temperature of the sink is decreased by 100°C, keeping T_1 unchanged, the new temperature of the sink is T'_2 = 500 − 100 = 400°C = 673 K. The efficiency now becomes

$$\eta'' = 1 - \frac{T'_2}{T_1} = 1 - \frac{673}{1773} = 0.62$$

Since η'' is greater than η', decreasing the temperature of the sink by 100°C results in a greater efficiency than increasing the temperatue of the source by 100°C.

Ex. A heat engine operates between a cold reservoir at temperature $T_2 = 300$ K and a hot reservoir at temperature T_1. It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of hot reservoir?

Sol. Work done by the engine in a cycle is $W = 200 - 120 = 80$ J. $\quad \eta = \dfrac{W}{Q} = \dfrac{80}{200} = 0.4$

From carnot's Theorem $\quad 0.4 \leq 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{300}{T_1} \quad$ or $\dfrac{300}{T_1} \leq 0.6$ or $T_1 \geq \dfrac{300}{0.6} \quad$ or $T_1 \geq 500$

11. REFRIGERATOR

It is inverse of heat engine. It extracts heat (Q_2) from a cold reservoir, External work W is done on it and rejects heat (Q_1) to hot reservoir. The coefficient of performance of a refrigerator.

$$\beta = \frac{\text{heat extracted from cold reservoir}}{\text{work done on refrigerator}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\dfrac{Q_1}{Q_2} - 1}$$

For Carnot reversible refrigerator $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$

$$\therefore \beta = \frac{Q_2}{W} = \frac{1}{\left[\dfrac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\dfrac{T_1}{T_2} - 1\right]} \Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$
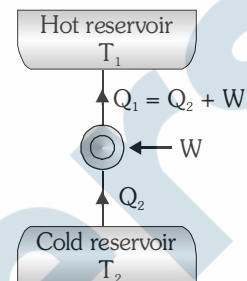
Ex. A carnot engine works as a refrigrator between 250 K and 300 K. If it receives 750 cal of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

Sol. $T_1 = 300$ K $\qquad T_2 = 250$ K $\qquad Q_2 = 750 \qquad Q_1 = ?$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad Q_1 = \frac{300}{250} \times 750 = 900 \text{ cal}$$

Ex. The temperature insides & outside of refrigerator are 260 K and 315 K respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

Sol. $T_2 = 260$ K, $T_1 = 315$ K, $W = 1$ joule

Coefficient of performance of Carnot refrigerator $\beta = \dfrac{Q_2}{W} = \dfrac{T_2}{T_1 - T_2}$

$$\therefore \frac{Q_2}{1} = \frac{260}{315 - 260} = \frac{260}{55} \Rightarrow Q_2 = \frac{260}{55} = 4.7 \text{ J}$$

Ex. A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the workdone. (Latent heat of ice is 3.4×10^5 J/kg)

Sol. Coefficient of performance (COP) $= \dfrac{T_2}{T_1 - T_2} = \dfrac{273}{300 - 273} = \dfrac{273}{27}$

$$W = \frac{Q_2}{\text{COP}} = \frac{mL}{\text{COP}} = \frac{100 \times 3.4 \times 10^5}{273 / 27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^6 \text{ J}$$

EXERCISE (S-1)

1. V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.
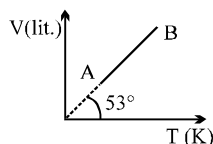
2. An empty pressure cooker of volume 10 litres contains air at atmospheric pressure 10^5 Pa and temperature of 27°C. It contains a whistle which has area of 0.1 cm² and weight of 100 g. What should be the temperature of air inside so that the whistle is just lifted up?
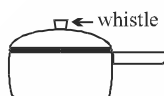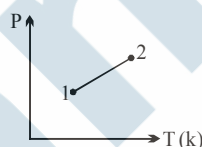

3. Examine the following plots and predict whether in
   (i) $P_1 < P_2$ and $T_1 > T_2$, in (ii) $T_1 = T_2 < T_3$, in (iii) $V_1 > V_2$, in (iv) $P_1 > P_2$ or otherwise.
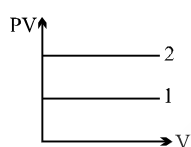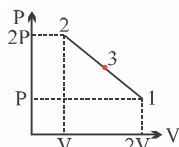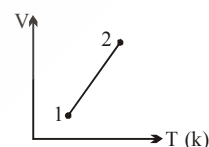

   (i)   (ii)   (iii)   (iv)

4. Find the molecular mass of a gas if the specific heats of the gas are C_P = 0.2 cal/gm°C and C_V = 0.15 cal/gm°C. [Take R = 2 cal/mole°C]

5. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is 1 m². Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas. You can leave the answer in fraction.

[JEE' 2004]
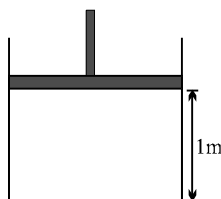
6. A mixture of 4 gm helium and 28 gm of nitrogen is enclosed in a vessel of constant volume 300 K. Find the quantity of heat absorbed by the mixture to doubled the root mean velocity of its molecules. (R = Universal gas constant)

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7. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **[JEE 2003]**

8. If heat is added at constant volume, 6300 J of heat are required to raise the temperature of an ideal gas by 150 K. If instead, heat is added at constant pressure, 8800 joules are required for the same temperature change. When the temperature of the gas changes by 300 K, determine the change is the internal energy of the gas.

9. Ideal diatomic gas is taken through a process $\Delta Q = 2\Delta U$. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

10. A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume ?

11. One mole of a gas mixture is heated under constant pressure, and heat required ΔQ is plotted against temperature difference acquired. Find the value of γ for mixture.
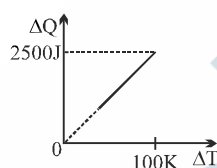

12. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in figure. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F
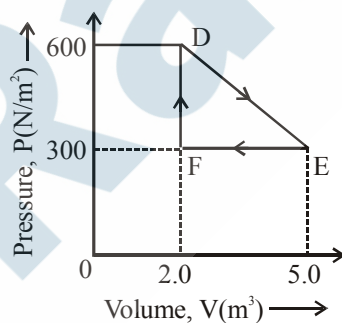
13. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case ? (Take 1 cal = 4.19 J)

14. One mole of an ideal monoatomic gas undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.
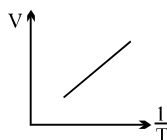
15. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following :

(a) What is the final pressure of the gas in A and B ?

(b) What is the change in internal energy of the gas ?

(c) What is the change in the temperature of the gas ?

(d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface ?

16. A cycle followed by an engine (made of one mole of perfect gas in a cylinder with a piston) is shown in figure.
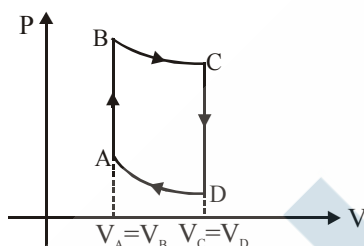
A to B : volume constant

B to C : adiabatic

C to D : volume constant

D to A : adiabatic

$V_C = V_D = 2V_A = 2V_B$

(a) In which part of the cycle heat is supplied to the engine from outside?

(b) In which part of the cycle heat is being given to the surrounding by the engine?

(c) What is the work done by the engine in one cycle? Write your answer in term of P_A, P_B, V_A

(d) What is the efficiency of the engine?

$[\gamma = \dfrac{5}{3}$ for the gas$]$, $(C_V = \dfrac{3}{2} R$ for one mole$)$

EXERCISE (S-2)

1. A barometer is faulty. When the true barometer reading are 73 and 75 cm of Hg, the faulty barometer reads 69 cm and 70 cm respectively.
   (i) What is the total length of the barometer tube?
   (ii) What is the true reading when the faulty barometer reads 69.5 cm ?
   (iii) What is the faulty barometer reading when the true barometer reads 74 cm?

2. A vessel of volume V = 30l is separated into three equal parts by stationary semipermeable thin membranes as shown in the Figure. The left, middle and right parts are filled with m_{H_2} = 30g of hydrogen, m_{O_2} =160g of oxygen, and m_{N_2} = 70g of nitrogen respectively. The left partition lets through only hydrogen, while the right partition lets through hydrogen and nitrogen. What will be the pressure in each part of the vessel after the equilibrium has been set in if the vessel is kept at a constant temperature T = 300K?
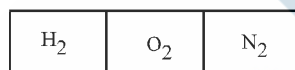
3. An ideal gas at NTP is enclosed in a adiabatic vertical cylinder having area of cross section A = 27cm², between two light movable pistons as shown in the figure. Spring with force constant k = 3700 N/m is in a relaxed state initially. Now the lower piston is moved upwards a height h/2, h being the initial length of gas column. It is observed that the upper piston moves up by a distance h/16. Find h taking γ for the gas to be 1.5. Also find the final temperature of the gas.
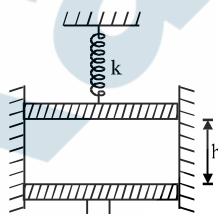

4. Consider one mole of perfect gas in a cylinder of unit cross section with a piston attached (Fig.). A spring (spring constant k) is attached (unstretched length L) to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas causing an increase of volume from V_0 to V_1.
   (a) What is the initial pressure of the system?
   (b) What is the final pressure of the system?
   (c) Using the first law of thermodynamics, write down a relation between Q, P_a, V, V_0 and k.
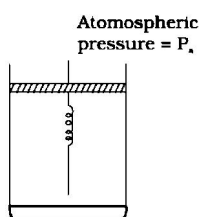
5. The figure shows an insulated cylinder divided into three parts A, B and C. Pistons I and II are connected by a rigid rod and can move without friction inside the cylinder. Piston I is perfectly conducting while piston II is perfectly insulating. The initial state of the gas ($\gamma = 1.5$) present in each compartment A, B and C is as shown. Now, compartment A is slowly given heat through a heater H such that the final volume of C becomes $\dfrac{4V_0}{9}$. Assume the gas to be ideal and find.

(a) Final pressures in each compartment A, B and C
(b) Final temperatures in each compartment A, B and C
(c) Heat supplied by the heater
(d) Work done by gas in A and B.
(e) Heat flowing across piston I.
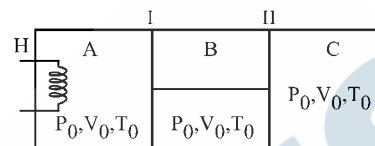
6. A thermally insulated vessel is divided into two equal parts by a heat-insulating piston which can move in the vessel without the friction. The left part of the vessel contains one mole of an ideal monatomic gas, & the right part is empty. The piston is connected to the right wall of the vessel through a spring whose length in free state is equal to the length of the vessel as shown in the figure. Determine the heat capacity C of the system, neglecting the heat capacities of the vessel, piston and spring.
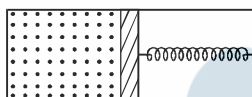
7. A weightless piston divides a thermally insulated cylinder into two parts of volumes V and 3V. 2 moles of an ideal gas at pressure P = 2 atmosphere are confined to the part with volume V = 1 litre. The remainder of the cylinder is evacuated. The piston is now released and the gas expands to fill the entire space of the cylinder. The piston is then pressed back to the initial position. Find the increase of internal energy in the process and final temperature of the gas. The ratio of the specific heat of the gas $\gamma = 1.5$.

8. Two moles of an ideal monoatomic gas are confined within a cylinder by a massless & frictionless spring loaded piston of cross-sectional area 4×10^{-3} m². The spring is, initially in its relaxed state. Now the gas is heated by an electric heater, placed inside the cylinder, for some time. During this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 K. Calculate the spring constant & the heat supplied by the heater. [$P_0 = 10^5$ Pa]

9. A gas takes part in two processes in which it is heated from the same initial state 1 to the same final temperature. The processes are shown on the P-V diagram by the straight line 1-2 and 1-3. 2 and 3 are the points on the same isothermal curve. Q_1 and Q_2 are the heat transfer along the two processes. Then in which case will the heat transfer be more.
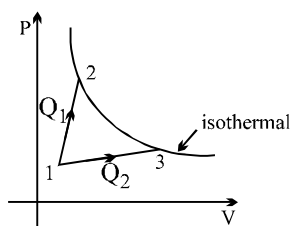
10. A sample of 2 kg of monoatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as in figure. Given, molecular mass of Helium = 4
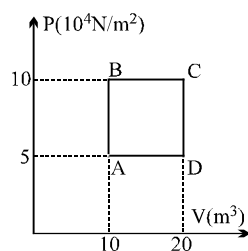
(i) What is the temperature of Helium in each of the states A, B, C & D ?

(ii) Is there any way of telling afterwards which sample of Helium went through the process ABC and which went through the process ADC? Write Yes or No.

(iii) How much is the heat involved in each of the processes ABC & ADC.

11. In a cyclic process ABCA consisting of isothermal expansion AB, isobaric compression BC and adiabatic compression CA, find the efficiency of cycle (Given : $T_A = T_B = 400$ K, $\gamma = 1.5$)
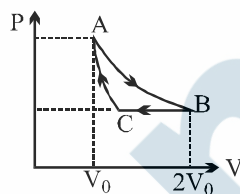
12. An adiabatic cylinder has 8 gram of helium. A light smooth adiabatic piston is connected to a light spring of force constant 300 N/m. The other end of the spring is connected with a block of mass 1 kg kept on a rough horizontal surface of coefficient of friction $\mu = 0.3$. Area of cross-section of cylinder is A = 25 cm^2. Initially the spring is in a relaxed position and the temperature of the gas is 400 K. The gas is heated slowly for some time by means of an electric heater so as to bring the block M on the verge of motion. Take $P_{atm} = 10^5$ N/m^2. Find

(a) the work done by the gas

(b) the final temperature

(c) heat supplied by the heater
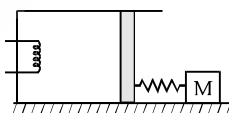
EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

KTG

1. Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3×10^5 pascal

(A) 2.4×10^{23} (B) 3×10^{23} (C) 6×10^{23} (D) 4.8×10^{23}

2. An ideal gas mixture filled inside a balloon expands according to the relation $PV^{2/3}$ = constant. The temperature inside the balloon is

(A) increasing   (B) decreasing   (C) constant   (D) can't be said

3. At a temperature T K, the pressure of 4.0g argon in a bulb is p. The bulb is put in a bath having temperature higher by 50K than the first one. 0.8g of argon gas had to be removed to maintained original pressure. The temperature T is equal to

(A) 510 K   (B) 200 K   (C) 100 K   (D) 73 K

4. 28 gm of N_2 gas is contained in a flask at a pressure of 10 atm and at a temperature of 57°. It is found that due to leakage in the flask, the pressure is reduced to half and the temperature reduced to 27°C. The quantity of N_2 gas that leaked out is :-

(A) 11/20 gm   (B) 20/11 gm   (C) 5/63 gm   (D) 63/5 gm

5. A container X has volume double that of container Y and both are connected by a thin tube. Both contains same ideal gas. The temperature of X is 200K and that of Y is 400K. If mass of gas in X is m then in Y it will be:

(A) m/8   (B) m/6   (C) m/4   (D) m/2

6. When 2 gms of a gas are introduced into an evacuated flask kept at 25°C the pressure is found to be one atmosphere. If 3 gms of another gas added to the same flask the pressure becomes 1.5 atmospheres. The ratio of the molecular weights of these gases will be :-

(A) 1 : 3   (B) 3 : 1   (C) 2 : 3   (D) 3 : 2

7. A rigid tank contains 35 kg of nitrogen at 6 atm. Sufficient quantity of oxygen is supplied to increase the pressure to 9 atm, while the temperature remains constant. Amount of oxygen supplied to the tank is :

(A) 5 kg   (B) 10 kg   (C) 20 kg   (D) 40 kg

8. During an experiment an ideal gas obeys an addition equation of state P^2V = constant. The initial temperature and pressure of gas are T and V respectively. When it expands to volume 2V, then its temperature will be:

(A) T   (B) $\sqrt{2}$ T   (C) 2 T   (D) $2\sqrt{2}$ T

9. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature 2T has a pressure of

(A) P/8   (B) P   (C) 2P   (D) 8P

10. The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line B. Then the expansion of the same ideal gas of mass 2 m at a pressure 2P is given by the straight line
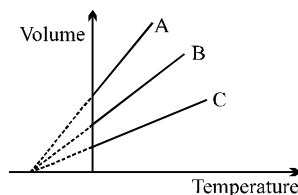
(A) C   (B) A   (C) B   (D) none

11. A cylindrical tube of cross-sectional area A has two air tight frictionless pistons at its two ends. The pistons are tied with a straight piece of metallic wire. The tube contains a gas at atmospheric pressure P_0 and temperature T_0. If temperature of the gas is doubled then the tension in the wire is


(A) $4 P_0 A$ (B) $P_0 A/2$ (C) $P_0 A$ (D) $2 P_0 A$

12. One mole of an ideal diatomic gas is taken through the cycle as shown in the figure.
$1 \rightarrow 2$: isochoric process
$2 \rightarrow 3$: straight line on P-V diagram
$3 \rightarrow 1$: isobaric process
The average molecular speed of the gas in the states 1, 2 and 3 are in the ratio
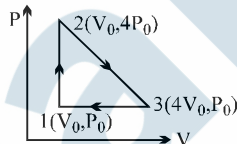
(A) $1 : 2 : 2$ (B) $1 : \sqrt{2} : \sqrt{2}$ (C) $1 : 1 : 1$ (D) $1 : 2 : 4$

13. One mole of an ideal gas at STP is heated in an insulated closed container until the average speed of its molecules is doubled. Its pressure would therefore increase by factor.

(A) 1.5   (B) $\sqrt{2}$ (C) 2   (D) 4

14. Three particles have speeds of 2u, 10u and 11u. Which of the following statements is correct?
(A) The r.m.s. speed exceeds the mean speed by about u.
(B) The mean speed exceeds the r.m.s. speed by about u.
(C) The r.m.s. speed equals the mean speed.
(D) The r.m.s. speed exceeds the mean speed by more than 2u.

15. One mole of an ideal gas is contained with in a cylinder by a frictionless piston and is initially at temperature T. The pressure of the gas is kept constant while it is heated and its volume doubles. If R is molar gas constant, the work done by the gas in increasing its volume is :-
(A) RT $ln2$ (B) 1/2 RT   (C) RT   (D) 3/2 RT

16. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :-
(A) 4 RT   (B) 15 RT   (C) 9 RT   (D) 11 RT

40

First Law of Thermodynamics :

17. In thermodynamic process pressure of a fixed mass of gas is changed in such a manner that the gas releases 30 joule of heat and 18 joule of work was done on the gas. If the initial internal energy of the gas was 60 joule, then, the final internal energy will be :

(A) 32 joule   (B) 48 joule   (C) 72 joule   (D) 96 joule

18. Two monoatomic ideal gas at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. The temperature of the mixture will be :

(A) $\dfrac{T_1 + T_2}{n_1 + n_2}$ (B) $\dfrac{T_1}{n_1} + \dfrac{T_2}{n_2}$ (C) $\dfrac{n_2 T_1 + n_1 T_2}{n_1 + n_2}$ (D) $\dfrac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$

19. An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then

[JEE' 2004 (Scr)]

(A) $P_3 > P_1$, W > 0   (B) $P_3 < P_1$, W < 0

(C) $P_3 > P_1$, W < 0   (D) $P_3 = P_1$, W = 0

20. Pressure versus temperature graph of an ideal gas is shown in figure

(A) During the process AB work done by the gas is positive

(B) During the process CD work done by the gas is negative

(C) During the process BC internal energy of the gas is increasing

(D) None

21. A polyatomic gas with six degrees of freedom does 25J of work when it is expanded at constant pressure. The heat given to the gas is :-

(A) 100J   (B) 150J   (C) 200J   (D) 250J

22. A reversible adiabatic path on a P-V diagram for an ideal gas passes through state A where $P = 0.7 \times 10^5$ N/m^{-2} and V = 0.0049 m^3. The ratio of specific heat of the gas is 1.4. The slope of path at A is :

(A) 2.0×10^7 Nm^{-5} (B) 1.0×10^7 Nm^{-5} (C) -2.0×10^7 Nm^{-5} (D) -1.0×10^7 Nm^{-5}

23. The adiabatic Bulk modulus of a diatomic gas at atmospheric pressure is

(A) 0 Nm^{-2} (B) 1 Nm^{-2} (C) 1.4×10^4 Nm^{-2} (D) 1.4×10^5 Nm^{-2}

24. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is :

(A) 2P/3   (B) P   (C) 3P/2   (D) 2P

25. One mole of an ideal gas at temperature T_1 expends according to the law $\dfrac{P}{V^2} = a$ (constant). The work done by the gas till temperature of gas becomes T_2 is :
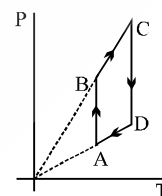
(A) $\dfrac{1}{2} R(T_2 - T_1)$ (B) $\dfrac{1}{3} R(T_2 - T_1)$ (C) $\dfrac{1}{4} R(T_2 - T_1)$ (D) $\dfrac{1}{5} R(T_2 - T_1)$

26. The first law of thermodynamics can be written as $\Delta U = \Delta Q + \Delta W$ for an ideal gas. Which of the following statements is correct?

(A) ΔU is always zero when no heat enters or leaves the gas

(B) ΔW is the work done by the gas in this written law.

(C) ΔU is zero when heat is supplied and the temperature stays constant

(D) $\Delta Q = -\Delta W$ when the temperature increases very slowly.

27. 2 moles of a monoatomic gas are expanded to double its initial volume, through a process $P/V =$ constant. If its initial temperature is 300 K, then which of the following is not true.

(A) $\Delta T = 900$ K   (B) $\Delta Q = 3200$ R   (C) $\Delta Q = 3600$ R   (D) $W = 900$ R

28. A student records ΔQ, ΔU & ΔW for a thermodynamic cycle $A \rightarrow B \rightarrow C \rightarrow A$. Certain entries are missing. Find correct entry in following options.

	AB	BC	CA
ΔW	40J		30J
ΔU		50J	
ΔQ	150J	10J	

(A) $W_{BC} = -70$ J   (B) $\Delta Q_{CA} = 130$ J

(C) $\Delta U_{AB} = 190$ J   (D) $\Delta U_{CA} = -160$ J

29. A given mass of a gas expands from a state A to the state B by three paths 1, 2 and 3 as shown in T-V indicator diagram. If W_1, W_2 and W_3 respectively be the work done by the gas along the three paths, then
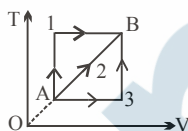
(A) $W_1 > W_2 > W_3$ (B) $W_1 < W_2 < W_3$ (C) $W_1 = W_2 = W_3$ (D) $W_1 < W_2, W_1 > W_3$

30. An ideal gas is taken from point A to point C on P-V diagram through two process AOC and ABC as shown in the figure. Process AOC is isothermal
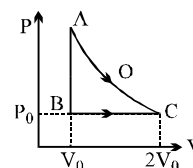
(A) Process AOC requires more heat than process ABC.

(B) Process ABC requires more heat than process AOC.

(C) Both process AOC & ABC require same amount of heat.

(D) Data is insufficient for comparison of heat requirement for the two processes.

31. Monoatomic, diatomic and triatomic gases whose initial volume and pressure are same, are compressed till their volume becomes half the initial volume.

(A) If the compression is adiabatic then monoatomic gas will have maximum final pressure.

(B) If the compression is adiabatic then triatomic gas will have maximum final pressure.

(C) If the compression is adiabatic then their final pressure will be same.

(D) If the compression is isothermal then their final pressure will be different.

32. Three processes form a thermodynamic cycle as shown on P-V diagram for an ideal gas. Process $1 \rightarrow 2$ takes place at constant temperature (300K). Process $2 \rightarrow 3$ takes place at constant volume. During this process 40J of heat leaves the system. Process $3 \rightarrow 1$ is adiabatic and temperature T_3 is 275K. Work done by the gas during the process $3 \rightarrow 1$ is :-
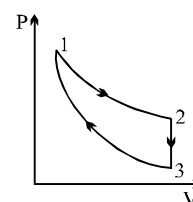

(A) -40J   (B) -20J   (C) $+40$J   (D) $+20$J

33. A closed container is fully insulated from outside. One half of it is filled with an ideal gas X separated by a plate P from the other half Y which contains a vacuum as shown in figure. When P is removed, X moves into Y. Which of the following statements is correct?
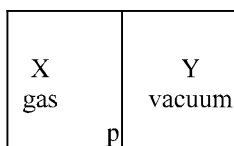

(A) No work is done by X   (B) X decreases in temperature

(C) X increases in internal energy   (D) X doubles in pressure

34. Water is heated in an open pan where the air pressure is 10^{+5} Pa. The water remains a liquid, which expands by a small amount as it is heated. Determine the ratio of the heat absorbed by the water to the work done by water. (γ for water $= 10^{-3}$/C, S = 1 cal/gm°C)

(A) 4.2×10^3 (B) 4.2×10^5 (C) 4.2×10^2 (D) 4.2×10^4

35. Two identical vessels A & B contain equal amount of ideal monoatomic gas. The piston of A is fixed but that of B is free. Same amount of heat is absorbed by A & B. If B's internal energy increases by 100 J the change in internal energy of A is :-
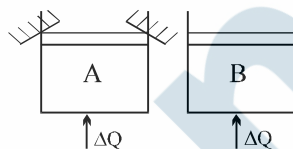
(A) 100 J   (B) $\dfrac{500}{3}$ J   (C) 250 J   (D) none of these

36. An ideal gas expands from volume V_1 to V_2. This may be achieved by either of the three processes: isobaric, isothermal and adiabatic. Let ΔU be the change in internal energy of the gas, Q be the quantity of heat added to the system and W be the work done by the gas. Identify which of the following statements is false for ΔU?

(A) ΔU is least under adiabatic process.

(B) ΔU is greatest under adiabatic process.

(C) ΔU is greatest under the isobaric process.

(D) ΔU in isothermal process lies in-between the values obtained under isobaric and adiabatic processes.

37. A cyclic process ABCA is shown in PT diagram. When presented on PV, it would
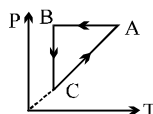
38. Suppose that two heat engines are connected in series, such that the heat exhaust of the first engine is used as the heat input of the second engine as shown in figure. The efficiencies of the engines are η_1 and η_2, respectively. The net efficiency of the combination is given by
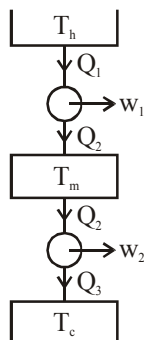
(A) $\eta_{net} = \eta_2 + (1 - \eta_1)\eta_2$

(B) $\eta_{net} = \dfrac{\eta_1}{(1 - \eta_1)\eta_2}$

(C) $\eta_{net} = \eta_1 + (1 - \eta_1)\eta_2$

(D) $\eta_{net} = \dfrac{1 - \eta_1}{(1 - \eta_2)\eta_2}$

39. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is- **[AIEEE - 2003]**

(A) 4/3   (B) 2   (C) 5/3   (D) 3/2

40. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of- **[AIEEE - 2003]**

(A) second law of thermodynamics   (B) conservation of momentum

(C) conservation of mass   (D) first law of thermodynamics

41. A carnot engine takes 3×10^6 cal of heat from a reservoir at 627 °C and gives it to a sink at 27 °C. The work done by the engine is- **[AIEEE - 2003]**

(A) 4.2×10^6 J   (B) 8.4×10^6 J   (C) 16.8×10^6 J   (D) zero

MULTIPLE CORRECT TYPE QUESTIONS

42. Two gases have the same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally

(A) The final temperature is greater for the isothermal process

(B) The final pressure is greater for the isothermal process

(C) The work done by the gas is greater for the isothermal process

(D) All the above options are incorrect

43. On the P–T graph of an ideal gas,

(A) adiabatic process will be a straight line

(B) isochoric process will be a straight line passing through the origin

(C) adiabatic curve will have a positive slope

(D) the slope of adiabatic curve will decrease with increase in T

44

44. During the melting of a slab of ice at 273 K at atmospheric pressure
(A) Positive work is done by the ice-water system on the atmosphere
(B) Positive work is done on the ice-water system by the atmosphere
(C) The internal energy of the ice-water increases
(D) The internal energy of ice-water system decreases

Paragraph for question nos. 45 and 46

A sample of ideal gas is taken through the cyclic process shown in the figure. The temperature of the gas at state A is $T_A = 200$ K. At states B and C the temperature of the gas is the same.
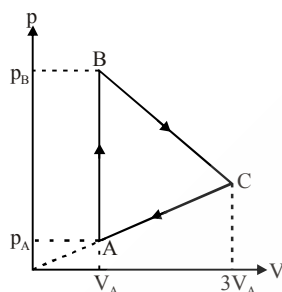

45. Net work done by the gas in the process is
(A) $2P_A V_A$ (B) $4P_A V_A$ (C) $6P_A V_A$ (D) $8P_A V_A$

46. Which of the following graphs best represent the cyclic process in T-V diagram.
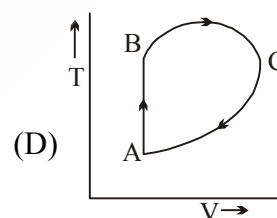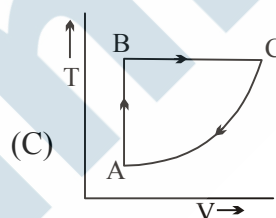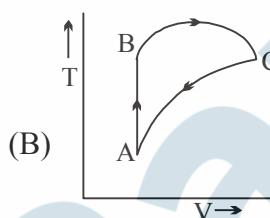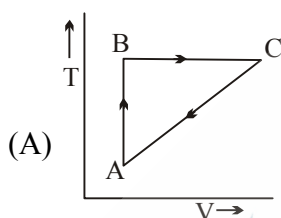
Paragraph for Question Nos. 47 to 49

The walls of the two connecting cylinders shown in the figure are adiabatic (thermally insulating). The cross-sectional areas of the parts are A_1 and A_2. There is a well-fitting but freely moving, thermally insulating piston in each cylinder, at a distance $\ell_1 = \ell_2 = \ell$ from the point where the cross-sectional area changes. The pistons are connected to each other by a thin and rigid rod. The enclosed volume contains air. The temperature and air pressure are T_0 and p_0 both inside and outside. The heater filament inside is operated for time t at a power of P.
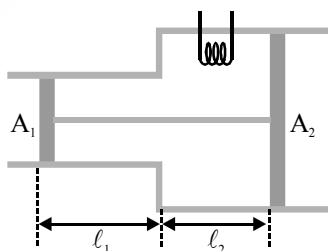
47. Mark the **CORRECT** statement :-
(A) Process performed is adiabatic   (B) Piston shifts towards left
(C) Process performed is polytropic   (D) Piston shifts towards right

48. What is change in temperature (Q is heat energy supplied, v_0 is initial volume) :-

(A) $\Delta T = \dfrac{2}{5}\dfrac{QT_0}{p_0 V_0}$

(B) $\Delta T = \dfrac{2}{7}\dfrac{QT_0}{p_0 V_0}$

(C) $\Delta T = \dfrac{5}{7}\dfrac{QT_0}{p_0 V_0}$

(D) $\Delta T = \dfrac{QT_0}{p_0 V_0}$

49. What is displacement (x) of piston till equilibrium is reached :-

(A) $x = \dfrac{2Pt}{(A_2 - A_1)7p_0}$

(B) $x = \dfrac{2Pt}{(A_2 - A_1)5p_0}$

(C) $x = \dfrac{2Pt}{(A_2 - A_1)3p_0}$

(D) $x = \dfrac{2Pt}{(A_2 - A_1)9p_0}$

MATRIX MATCH TYPE

50. For an ideal gas a process PV diagram is a circle. An adiabatic from A passes through C. An isotherm from A passes through B. We take a part of the circular cyclic process. Comment on the sign of the quantity of column-I.
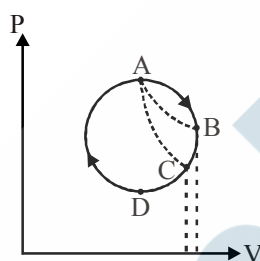
|	Column-I		Column-II
(A)	Heat given to the gas in going from A to C along circle	(P)	Positive
(B)	Heat given to the gas in going from B to C along circle	(Q)	Negative
(C)	Heat given to the gas in going from C to D along circle	(R)	Zero
		(S)	can't be said

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43m. The length of air column above mercury level in the tube will be : Take P_{atm} = 76 cm of mercury

   (A) 0.215 m   (B) 0.2 m   (C) 0.1 m   (D) 0.4 m

2. A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube ?

   (A) 33.3 kPa   (B) 66.7 kPa   (C) 3.33 MPa   (D) 6.67 MPa

3. At temperature T, N molecules of gas A each having mass m and at the same temperature 2N molecules of gas B each having mass 2m are filled in a container. The mean square velocity of molecules of gas B is v^2 and mean square of x component of velocity of molecules of gas A is w^2. The ratio of w^2/v^2 is :

   (A) 1   (B) 2   (C) 1/3   (D) 2/3

4. The figure, shows the graph of logarithmic reading of pressure and volume for two ideal gases A and B undergoing adiabatic process. From figure it can be concluded that
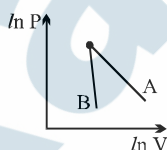

   (A) gas B is diatomic   (B) gas A and B both are diatomic

   (C) gas A is monoatomic   (D) gas B is monoatomic & gas A is diatomic

5. When unit mass of water boils to become steam at 100^0C, it absorbs Q amount of heat. The densities of water and steam at 100^0C are ρ_1 and ρ_2 respectively and the atmospheric pressure is p_0. The increase in internal energy of the water is

   (A) Q   (B) $Q + p_0\left(\dfrac{1}{\rho_1} - \dfrac{1}{\rho_2}\right)$ (C) $Q + p_0\left(\dfrac{1}{\rho_2} - \dfrac{1}{\rho_1}\right)$ (D) $Q - p_0\left(\dfrac{1}{\rho_1} + \dfrac{1}{\rho_2}\right)$

6. A cylinder made of perfectly non conducting material closed at both ends is divided into two equal parts by a heat proof piston. Both parts of the cylinder contain the same masses of a gas at a temperature $t_0 = 27°$ C and pressure $P_0 = 1$ atm. Now if the gas in one of the parts is slowly heated to t = 57°C while the temperature of first part is maintained at t_0 the distance moved by the piston from the middle of the cylinder will be (length of the cylinder = 84 cm)

   (A) 3 cm   (B) 5 cm   (C) 2 cm   (D) 1 cm

7. 1 kg of a gas does 20 kJ of work and receives 16 kJ of heat when it is expanded between two states. A second kind of expansion can be found between the initial and final state which requires a heat input of 9 kJ. The work done by the gas in the second expansion is :

(A) 32 kJ   (B) 5 kJ   (C) – 4 kJ   (D) 13 kJ

**8.** A process is shown in the diagram. Which of the following curves may represent the same process?

(A)   (B)   (C)   (D)
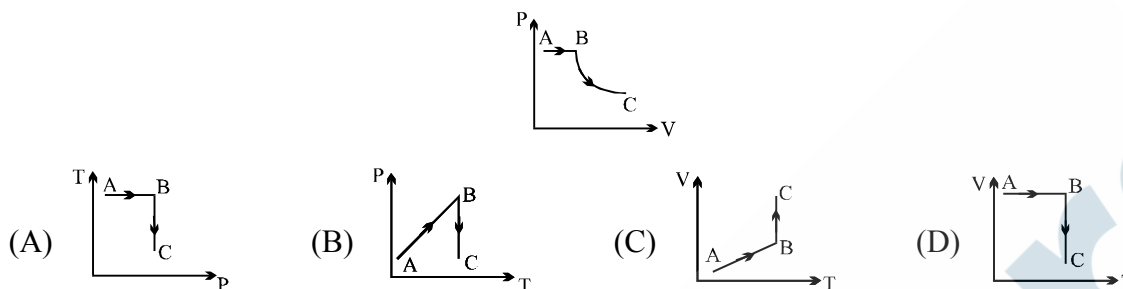
9. An ideal gas expands in such a way that PV^2 = constant throughout the process.

(A) The graph of the process of T-V diagram is a parabola.

(B) The graph of the process of T-V diagram is a straight line.

(C) Such an expansion is possible only with heating.

(D) Such an expansion is possible only with cooling.

10. Figure shows the pressure P versus volume V graphs for two different gas sample at a given temperature. M_A and M_B are masses of two samples, n_A and n_B are numbers of moles. Which of the following *must be incorrect.*
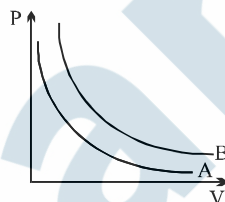

(A) $M_A > M_B$ (B) $M_A < M_B$ (C) $n_A > n_B$ (D) $n_A < n_B$

11. A vertical cylinder with heat-conducting walls is closed at the bottom and is fitted with a smooth light piston. It contains one mole of an ideal gas. The temperature of the gas is always equal to the surrounding's temperature, T_0. The piston is moved up slowly to increase the volume of the gas to η times. Which of the following is incorrect?

(A) Work done by the gas is $RT_0 \ln \eta$.

(B) Work done against the atmosphere is $RT_0(\eta - 1)$.

(C) There is no change in the internal energy of the gas.

(D) The final pressure of the gas is $\dfrac{1}{(\eta - 1)}$ times its initial pressure.

12. A gas is enclosed in a vessel at a constant temperature at a pressure of 5 atmosphere and volume 4 litre. Due to a leakage in the vessel, after some time, the pressure is reduced to 4 atmosphere. As a result, the

(A) volume of the gas decreased by 20%

(B) average K.E. of gas molecule decreases by 20%

(C) 20% of the gas escaped due to the leakage

(D) 25% of the gas escaped due to the leakage

13. An enclosed ideal gas is taken through a cycle as shown in the figure. Then

(A) Along AB, temperature decreases while along BC temperature increases

(B) Along AB, temperature increases while along BC the temperature decreases.

(C) Along CA work is done by the gas and the internal energy remains constant.

(D) Along CA work is done on the gas and internal energy of the gas increases.
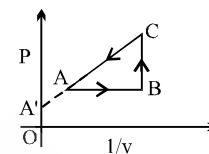
14. One mole of monoatomic ideal gas undergoes a cyclic process ABCA as shown in figure. Process BC is adiabatic. The temperatures at A, B and C are 300, 600 and 450K respectively. Choose the correct statement(s).

(A) In process CA change in internal energy is 225R.

(B) In process AB change in internal energy is –150R.

(C) In process BC change in internal energy is –225R.

(D) Change in internal energy during the whole cyclic process is +150R.
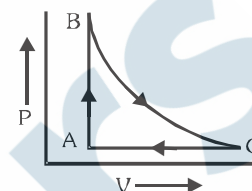
## MULTIPLE CORRECT TYPE QUESTIONS

15. Let v_{av}, v_{rms} and v_p respectively denote mean speed, root mean square speed and the most probable speed of the molecule in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m then :

(A) no molecule can have speed greater than $\sqrt{2}\, v_{rms}$

(B) no molecule can have speed less than $v_p / \sqrt{2}$

(C) $v_p < v_{av} < v_{rms}$

(D) the average kinetic energy of a molecule is $3/4\, mv_p^2$

16. A gas expands such that its initial and final temperature are equal. Also, the process followed by the gas traces a straight line on the P-V diagram :

(A) The temperature of the gas remains constant throughout.

(B) The temperature of the gas first increases and then decreases

(C) The temperature of the gas first decreases and then increases

(D) The straight line has a negative slope.

17. A cyclic process ABCD is shown in the p-V diagram. Which of the following curves represents the same process if BC & DA are isothermal processes
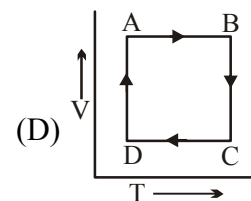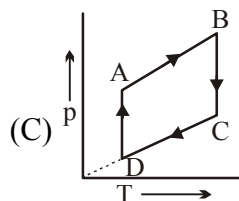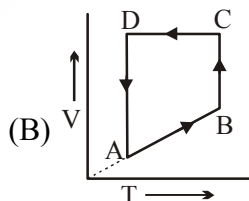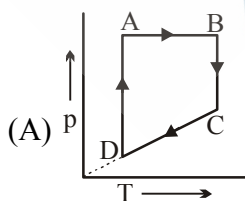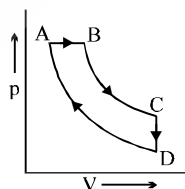
18. Two moles of helium gas is taken through the cycle ABCDA as shown in the figure. If $T_A = 1000$ K, $2P_A = 3P_B = 6P_C$.
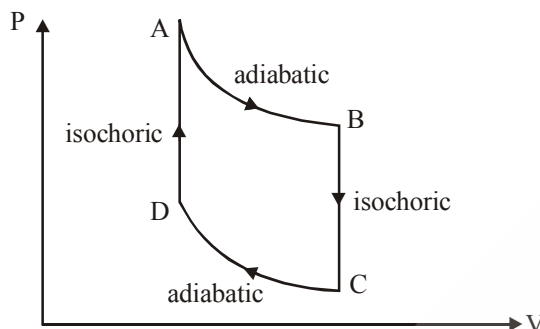
(A) work done by the gas in the process A to B is 3741 J.
(B) heat lost by the gas in the process B to C is 10600 J.
(C) temperature T_D is 2000 K.
(D) none of these

19. Figure shows the *P-V* diagram of an ideal gas undergoing a change of state from A to B. Four different parts I, II, III and IV as shown in the figure may lead to the same change of state.
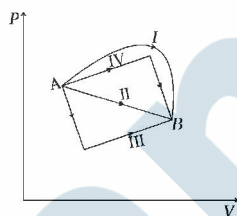
(A) Change in internal energy is same in IV and III cases, but not in I and II.
(B) Change in internal energy is same in all the four cases.
(C) Work done is maximum in case I
(D) Work done is minimum in case II.

COMPREHENSION TYPE QUESTIONS
Paragraph for Question Nos. 20 to 22

One mole of a monoatomic ideal gas occupies two chambers of a cylinder partitioned by means of a movable piston. The walls of the cylinder as well as the piston are thermal insulators. Initially equal amounts of gas fill both the chambers at (P_0, V_0, T_0). A coil is burnt in the left chamber which absorbs heat and expands, pushing the piston to the right. The gas on the right chamber is compressed until to pressure becomes $32 P_0$.
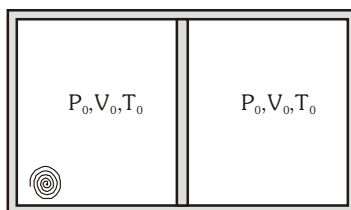
20. The final volume of left chamber is :-

(A) $\dfrac{V_0}{8}$ 　　　　(B) $\dfrac{15}{8}V_0$ 　　　　(C) $\dfrac{7}{8}V_0$ 　　　　(D) $\dfrac{9}{8}V_0$

21. The work done on the gas in the right chamber is :-

(A) $\dfrac{9}{2}P_0V_0$ (B) $-\dfrac{9}{2}P_0V_0$ (C) $\dfrac{13}{2}P_0V_0$ (D) $\dfrac{17}{2}P_0V_0$

22. The change in internal energy of the gas in the left chamber is :-

(A) $\dfrac{186}{4}RT_0$ (B) $\dfrac{177}{4}RT_0$ (C) $\dfrac{59}{2}RT_0$ (D) $\dfrac{131}{4}RT_0$
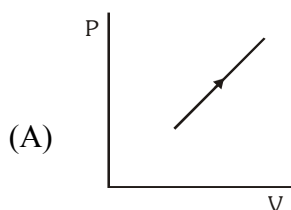
MATRIX MATCH TYPE QUESTION
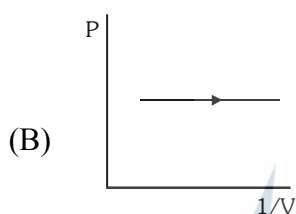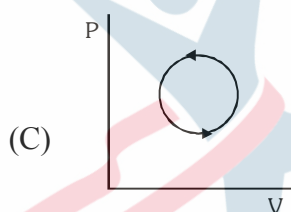
23.

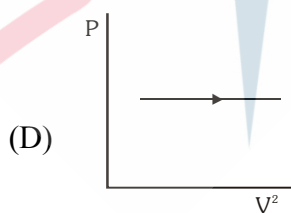| Column I (Pressure volume graph)	Column II (W is work done by gas, Q heat supply to gas)

(A)

(P) $W > 0$

(B)

(Q) $W < 0$

(C)

(R) $Q > 0$

(D)

(S) $Q < 0$

SUPPLEMENT FOR JEE-MAINS

1. Two engines A and B have their sources at 400 K and 350 K and sinks at 350 K and 300 K respectively. Which engine is more efficient and by how much ?

2. A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

3. A carnot engine works as a refrigrator between 250 K and 300 K. If it receives 750 cal of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

4. The temperature insides & outside of refrigerator are 260 K and 315 K respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

5. A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the workdone. (Latent heat of ice is 3.4×10^5 J/kg)

6. The coefficient of performance of a refrigerator working between 10° C and 20° C is :–
   (A) 1   (B) 28.3   (C) 29.3   (D) 3.53

7. A refrigerator freezes 5 kg of water at 0°C into ice at 0°C in a time interval of 20 minutes. Assume that room temperature is 20°C. Calculate the minimum power needed to accomplish it ?
   (A) 24.4 Watt   (B) 0.1025 Watt   (C) 0.0244 Watt   (D) 102.5 Watt

8. A carnot engine whose source temperature is at 400 K takes 100 kcal of heat at this temperature in each cycle and gives 70 kcal to the sink. Calculate (i) the temperature of the sink and (ii) the efficiency of the engine.
   (A) 280 K, 30 %   (B) 270 K, 30%   (C) 280 K, 10 %   (D) None of these

9. An ionization gauge installed in the artificial satellite showed that 1 cm^3 of the atmosphere contained about a thousand million particles of gas at a height of 300 km from the Earth's surface. Find the mean free path of the gas particles at this height. Take the diameter of the particles equal to 2×10^{-10} m.
   (A) ℓ = 5.6 km   (B) ℓ = 2.6 km   (C) ℓ = 8.4 km   (D) ℓ = 1.1 km

10. What maximum number of molecules of a gas should be contained in 1 cm^3 of a spherical vessels with a diameter of 15 cm so that the molecules do not collide with each other? The diameter of a gas molecule is 3×10^{-8} cm.
    (A) 1.7×10^{18}/CC   (B) 1.7×10^{16}/CC   (C) 2.5×10^{19}/CC   (D) 1.1×10^{14}/CC

11. An ideal refrigerator operates according to the reverse Carnot cycle and transmits heat from a cold source with water at a temperature of 0°C to a boiler with water at a temperature of 100 °C. What amount of water must be frozen in the cooler to convert 1 kg of water into vapour in the boiler?
    (A) 4.94 kg   (B) 3.24 kg   (C) 5.63 kg   (D) 2.12 kg

12. Oxygen (10 grammes) is heated from t_1 = 50°C to t_2 = 150°C. Find the change in entropy if the oxygen is heated: (1) isochorically, (2) isobarically.
    (A) ΔS_V = 1.76 J/K; ΔS_P = 2.46 J/K   (B) $\Delta S_V = \Delta S_P$ = 1.76 J/K
    (C) $\Delta S_V = \Delta S_P$ = 2.46 J/K   (D) ΔS_V = 2.46 J/K; ΔS_P = 1.76 J/K

13. An ideal gas expands into a vacuum in a rigid vessel. As a result there is :
    (A) a change in entropy   (B) an increase of pressure
    (C) a change in temperature   (D) a decrease of internal energy
    (E) a change in phase

14. An ideal gas is to taken reversibly from state i, at temperature T_1, to any of the other states labeled, I, II, III, IV and V on the p-V diagram below. All are at the same temperature T_2. Rank the five processes according to the change in entropy of the gas, least to greatest :
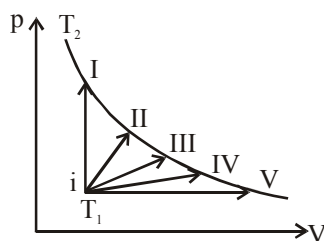
(A) I, II, III, IV, V   (B) V, IV, III, II, I

(C) I, then II, III, IV, and V tied   (D) I, II, III and IV tied, then V

(E) I and V tied, then II, III, IV

15. According to the second law of thermodynamics:

(A) Heat energy cannot be completely converted to work

(B) Work cannot be completely converted to heat energy

(C) For all cyclic processes we have dQ/T < 0

(D) The reason all heat engine efficiencies are less than 100% is friction, which is unavoidable

(E) All of the above are true

16. A reversible refrigerator operates between a low temperature reservoir at T_C and a high temperature reservoir at T_H. Its coefficient of performance is given by :

(A) $(T_H - T_C)/T_C$ (B) $T_C/(T_H - T_C)$ (C) $(T_H - T_C)/T_H$ (D) $T_H/(T_H - T_C)$

(E) $T_H(T_H + T_C)$

17. An inventor claims to have a heat engine that has an efficiency of 40% when it operates between a high temperature reservoir of 150°C and a low temperature reservoir of 30°C. This engine:

(A) Must violate the zeroth law of thermodynamics

(B) Must violate the first law of thermodynamics

(C) Must violate the second law of thermodynamics

(D) Must violate the third law of thermodynamics

(E) Does not necessarily violate any of the laws of thermodynamics

18. A carnot heat engine runs between a cold reservoir at temperature T_C and a hot reservoir at temperature T_H. You want to increase its efficiency. Of the following, which change results in the greatest increase in efficiency? The value of ΔT is the same for all changes.

(A) Raise the temperature of the hot reservoir by ΔT

(B) Raise the temperature of the cold reservoir by ΔT

(C) Lower the temperature of the hot reservoir by ΔT

(D) Lower the temperature of the cold reservoir by ΔT

(E) Lower the temperature of the hot reservoir by $1/2\Delta T$ and raise the temperature of the cold reservoir by $1/2\Delta T$.

19. A Carnot engine takes 3×10^6 cal of heat from reservoir at 627° and gives it to a sink at 27°C. Then work done by the engine is :-

(A) 4.2×10^6 J   (B) 8.4×10^6 J   (C) 16.8×10^6 J   (D) zero

EXERCISE (J-M)

1. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and tempeature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be- **[AIEEE-2008]**

   (1) $\dfrac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$ (2) $\dfrac{P_1 V_1 T_1 + P_2 V_2 T_2}{P_1 V_1 + P_2 V_2}$ (3) $\dfrac{P_1 V_1 T_2 + P_2 V_2 T_1}{P_1 V_1 + P_2 V_2}$ (4) $\dfrac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_1 + P_2 V_2 T_2}$

2. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion ? **[AIEEE-2009]**

   (1) 6×10^4 J   (2) 7×10^4 J   (3) 3×10^4 J   (4) 5×10^4 J

   Directions : Question number 3, 4 and 5 are based on the following paragraph.

   Two moles of helium gas are taken over the cycle ABCDA, as shown in the P–T diagram.
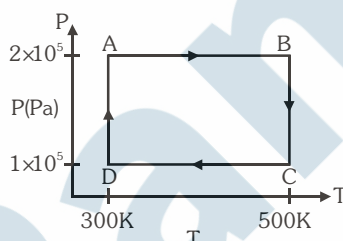
3. Assuming the gas to be ideal the work done on the gas in taking it from A to B is :- **[AIEEE-2009]**

   (1) 400 R   (2) 500 R   (3) 200 R   (4) 300

4. The work done on the gas in taking it from D to A is :- **[AIEEE-2009]**

   (1) –690 R   (2) +690 R   (3) –414 R   (4) +414 R

5. The net work done on the gas in the cycle ABCDA is :- **[AIEEE-2009]**

   (1) 1076 R   (2) 1904 R   (3) Zero   (4) 276 R

6. A diatomic ideal gas is used in a carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is :- **[AIEEE-2010]**

   (1) 0.25   (2) 0.5   (3) 0.75   (4) 0.99

7. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly broght to rest. Assuming no heat is lost to the surroundings, its temperature increases by :- **[AIEEE-2011]**

   (1) $\dfrac{\gamma M v^2}{2R}$ (2) $\dfrac{(\gamma-1)}{2R} M v^2$ (3) $\dfrac{(\gamma-1)}{2(\gamma+1)R} M v^2$ (4) $\dfrac{(\gamma-1)}{2\gamma R} M v^2$

8. A Carnot engine operating between temperatures T_1 and T_2 has efficientcy $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively:-   **[AIEEE-2011]**

(1) 330 K and 268 K   (2) 310 K and 248 K   (3) 372 K and 310 K   (4) 372 K and 330 K

9. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2, and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, then final temperature of the mixture is :-   **[AIEEE-2011]**

(1) $\dfrac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$ (2) $\dfrac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$ (3) $\dfrac{T_1 + T_2 + T_3}{3}$ (4) $\dfrac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$

10. The specific heat capacity of a metal at low temperautre (T) is given as $C_p(kJk^{-1}kg^{-1}) = 32\left(\dfrac{T}{400}\right)^3$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is:-   **[AIEEE-2011]**

(1) equal to 0.002 kJ   (2) greater than 0.148 kJ

(3) between 0.148 kJ and 0.028 kJ   (4) less than 0.028 kJ

11. A container with insulating walls is divided into two equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be :-
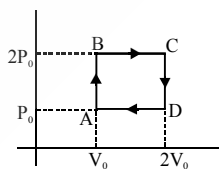
[AIEEE-2011]

(1) $\dfrac{P}{2}, T$ (2) $\dfrac{P}{2}, \dfrac{T}{2}$ (3) P, T   (4) $P, \dfrac{T}{2}$

12. Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas) :-

[AIEEE-2012]

(1) 12.5%
(2) 15.4%
(3) 9.1%
(4) 10.5%

13. A Carnot engine, whose efficiency is 40% takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :-   **[AIEEE-2012]**

(1) 600 K

(2) efficiency of Carnot engine cannot be made larger than 50%

(3) 1200 K

(4) 750 K

14. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is : **[JEE-Mains-2013]**
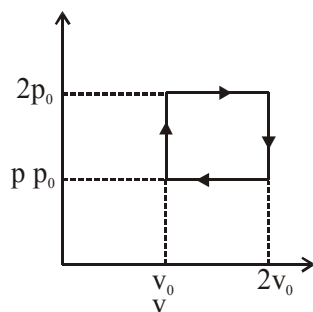
(1) $p_0 v_0$ 　　(2) $\left(\dfrac{13}{2}\right) p_0 v_0$ 　　(3) $\left(\dfrac{11}{2}\right) p_0 v_0$ 　　(4) $4 p_0 v_0$

15. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open of the tube is then closed and sealed and the tube is raised vertically up by addition 46 cm. What will be length of the air column above mercury in the tube now ? (Atmospheric pressure = 76 cm of Hg) **[JEE-Mains-2014]**

(1) 38 cm 　　(2) 6 cm 　　(3) 16 cm 　　(4) 22 cm

16. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement : **[JEE-Mains-2014]**

(1) The change in internal energy in the process AB is –350 R.

(2) The change in internal energy in the process BC is –500R

(3) The change in internal energy in whole cyclic process is 250 R.

(4) The change in internal energy in the process CA is 700 R.
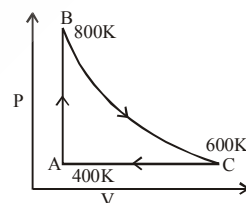
17. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways - **[JEE-Mains-2015]**
(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is -

(1) ln2, 2ln2 　　(2) 2ln2, 8ln2 　　(3) ln2, 4ln2 　　(4) ln2, ln2

18. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \dfrac{U}{V} \propto T^4$ and pressure $p = \dfrac{1}{3}\left(\dfrac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is -

[JEE-Mains-2015]

(1) $T \propto \dfrac{1}{R}$ 　　(2) $T \propto \dfrac{1}{R^3}$ 　　(3) $T \propto e^{-R}$ 　　(4) $T \propto e^{-3R}$

19. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is :- $\left(\gamma = \dfrac{C_p}{C_v} \right)$ **[JEE-Mains-2015]**

(1) $\dfrac{\gamma + 1}{2}$ 　　(2) $\dfrac{\gamma - 1}{2}$ 　　(3) $\dfrac{3\gamma + 5}{6}$ 　　(4) $\dfrac{3\gamma - 5}{6}$

20. 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be : **[JEE-Mains-2016]**
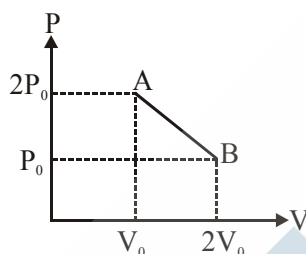
(1) $\dfrac{9P_0V_0}{nR}$ 　　(2) $\dfrac{9P_0V_0}{4nR}$ 　　(3) $\dfrac{3P_0V_0}{2nR}$ 　　(4) $\dfrac{9P_0V_0}{2nR}$

21. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :- **[JEE-Mains-2016]**

(1) $n = \dfrac{C - C_V}{C - C_P}$ 　(2) $n = \dfrac{C_P}{C_V}$ 　(3) $n = \dfrac{C - C_P}{C - C_V}$ 　(4) $n = \dfrac{C_P - C}{C - C_V}$

22. The temperature of an open room of volume 30 m³ increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :- **[JEE-Main 2017]**

(1) 2.5×10^{25} 　(2) -2.5×10^{25} 　(3) -1.61×10^{23} 　(4) 1.38×10^{23}

23. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is : **[JEE-Main 2017]**

(1) $a = 14\,b$ 　(2) $a = 28\,b$ 　(3) $a = \dfrac{1}{14}b$ 　(4) $a = b$

24. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

[JEE-Main 2018]

(1) (a) 195 K (b) –2.7 kJ 　　　　(2) (a) 189 K (b) –2.7 kJ
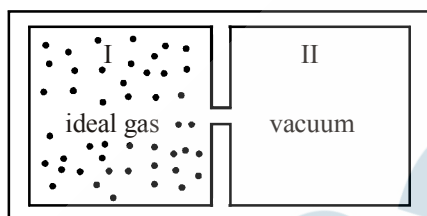(3) (a) 195 K (b) 2.7 kJ 　　　　(4) (a) 189 K (b) 2.7 kJ

EXERCISE (JA)

1. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is : **[JEE 2008]**

   (A) $\dfrac{1}{T}$ (B) $\dfrac{2}{T}$ (C) $\dfrac{3}{T}$ (D) $\dfrac{4}{T}$

2. **Column I** contains a list of processes involving expansion of an ideal gas. Match this with **Column II**. describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4×4 matrix given in the ORS. **[JEE 2008]**

 | Column I | Column II |
   |---|---|
   | (A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the chamber II has vacuum. The valve is opened.. | (P) The temperature of the gas decreases |

 | | |
   |---|---|
   | (B) An ideal monoatomic gas expands to twice its original volume such that pressure $P \propto \dfrac{1}{V^2}$, where V is the volume of the gas. | (Q) The temperature of the gas increases or remains constant |
   | (C) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \dfrac{1}{V^{4/3}}$, where V is its volume. | (R) The gas loses heat |
   | (D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph. | (S) The gas gains heat |
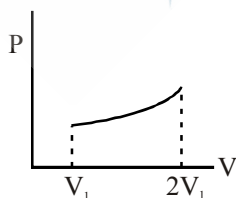
3. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then **[JEE-2009]**
   (A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
   (B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas
   (C) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas
   (D) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

4. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semicircle and CDA is half of an ellipse. Then, **[JEE-2009]**
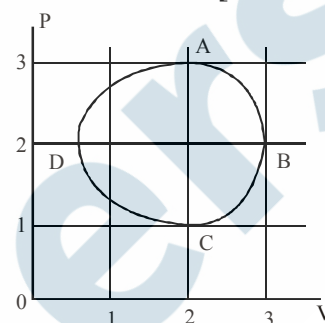
   (A) the process during the path $A \to B$ is isothermal

   (B) heat flows out of the gas during the path $B \to C \to D$

   (C) work done during the path $A \to B \to C$ is zero

   (D) positive work is done by the gas in the cycle ABCDA

5. A real gas behaves like an ideal gas if its
   (A) pressure and temperature are both high
   (B) pressure and temperature are both low
   (C) pressure is high and temperature is low
   (D) pressure is low and temperature is high

6. One mole of an ideal gas in initial state A undergoes 1 a cyclic process ABCA, as shown in the figure. its pressure at A is P_0. Choose the correct option(s) from the following. **[JEE-2010]**
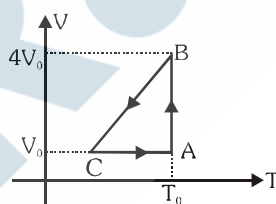
   (A) Internal energies at A and B are the same   (B) Work done by the gas in process AB is $P_0V_0 \ln 4$

   (C) Pressure at C is $\dfrac{P_0}{4}$ (D) Temperature at C is $\dfrac{T_0}{4}$

7. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. In the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is **[JEE-2010]**

8. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is **[JEE-2011]**

   (A) $\dfrac{9}{8}RT_1$ (B) $\dfrac{3}{2}RT_1$ (C) $\dfrac{15}{8}RT_1$ (D) $\dfrac{9}{2}RT_1$

9. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in P-V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**. **[JEE-2011]**

	Column I		Column II
(A)	Process A→B	(P)	Internal energy decreases.
(B)	Process B→C	(Q)	Internal energy increases.
(C)	Process C→D	(R)	Heat is lost.
(D)	Process D→A	(S)	Heat is gained.
		(T)	Work is done on the gas.
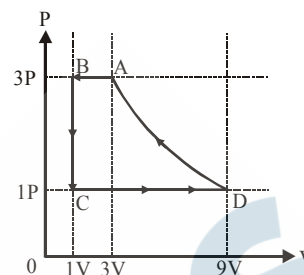
10. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\dfrac{v_{rms}(helium)}{v_{rms}(\arg on)} \right)$ is

[JEE-2012]

(A) 0.32   (B) 0.45   (C) 2.24   (D) 3.16

11. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to required no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly
(Take : R = 8.31 J/mol.K) **[JEE-2012]**
(A) 62 J   (B) 104 J   (C) 124 J   (D) 208 J

12. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is :- **[JEE-2013]**
(A) 1 : 4   (B) 1 : 2   (C) 6 : 9   (D) 8 : 9

13. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation :- **[JEE-2013]**
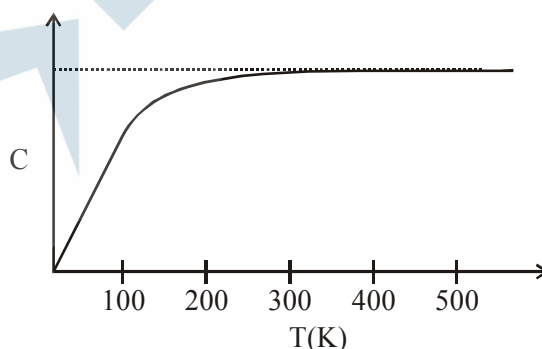
(A) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.
(B) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.
(C) there is no change in the rate of heat absorption in the range 400–500 K
(D) the rate of heat absorption increases in the range 200–300 K

14. One mole of a monatomic ideal gas is taken along two cyclic processes E→F→G→E and E→F→H→E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. **[JEE-2013]**
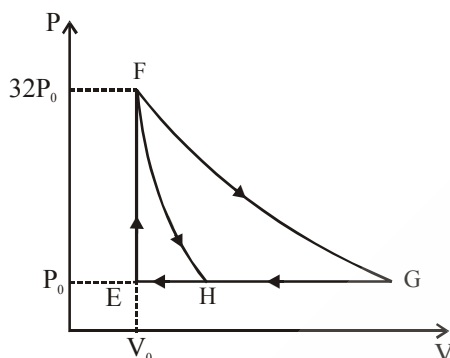
Match the paths in List I with the magnitudes of the work done in the List II and select the correct answer using the codes given blow the lists.

List I		List II	
P.	G → E	1.	$160\, P_0 V_0\, \ln 2$
Q.	G → H	2.	$36\, P_0 V_0$
R.	F → H	3.	$24\, P_0 V_0$
S.	F → G	4.	$31\, P_0 V_0$

Codes :

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

15. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf}/Q_{ib} is. **[JEE-Advance-2014]**
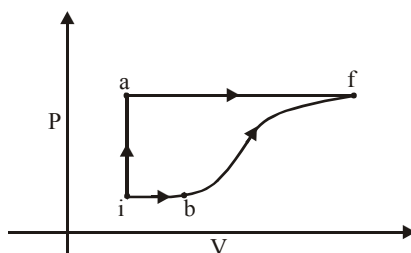
In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \dfrac{3}{2}R$, $C_P = \dfrac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \dfrac{5}{2}R, C_P = \dfrac{7}{2}R$.

[JEE-Advance-2014]
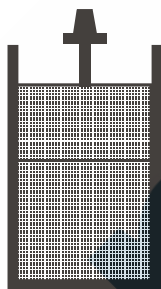
16. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(A) 550 K   (B) 525 K   (C) 513 K   (D) 490 K

17. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) 250 R   (B) 200 R   (C) 100 R   (D) –100 R

18. A container of fixed volume has a mixutre of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are) :-

[JEE-Advance-2015]

(A) The average energy per mole of the gas mixture is 2RT.

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

19. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) :- **[JEE-Advance-2015]**
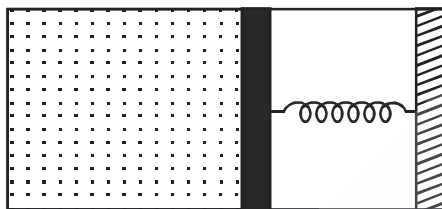
(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\dfrac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3 P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\dfrac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\dfrac{17}{6}P_1V_1$

20. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = \left(\dfrac{1}{32}\right) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that $P^3V^5 = $ constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumes V_f. The amount of heat supplied to the system in the two step process is approximately

[JEE-Advance 2016]

(A) 112 J   (B) 294 J   (C) 588 J   (D) 813 J

21. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ? **[JEE-Advance 2017]**

(A) The resistive force experienced by the plate is proportional to v

(B) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force.

Answer Q.22, Q.23 and Q.24 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamics process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamics processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n. **[JEE-Advance 2017]**
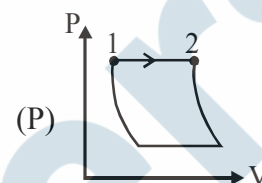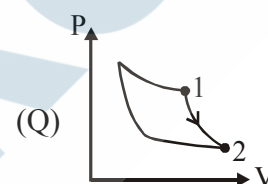
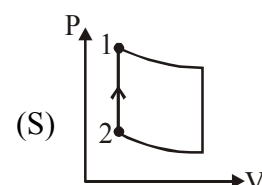| Column-1	Column-2	Column-3
(I) $W_{1\to 2} = \dfrac{1}{\gamma - 1}(P_2 V_2 - P_1 V_1)$	(i) Isothermal	(P)
(II) $W_{1\to 2} = -PV_2 + PV_1$	(ii) Isochoric	(Q)
(III) $W_{1\to 2} = 0$	(iii) Isobaric	(R)
(IV) $W_{1\to 2} = -nRT \ln \dfrac{V_2}{V_1}$	(iv) Adiabatic	(S)
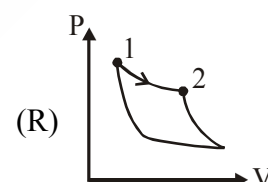
22. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iv) (R)   (B) (II) (iii) (P)   (C) (II) (iii) (S)   (D) (III) (iii) (P)

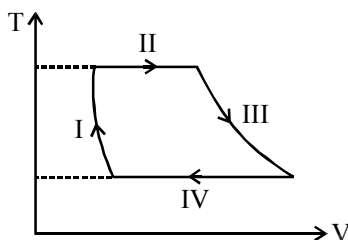**23.** Which one of the following options is the correct combination ?

(A) (III) (ii) (S)   (B) (II) (iv) (R)   (C) (II) (iv) (P)   (D) (IV) (ii) (S)

24. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ?

(A) (III) (iv) (R)   (B) (I) (ii) (Q)   (C) (IV) (ii) (R)   (D) (I) (iv) (Q)

25. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (whre V is the volume and T is the temperature). Which of the statements below is (are) true ?

(A) Process I is an isochoric process　　　　(B) In process II, gas absorbs heat

(C) In process IV, gas releases heat　　　　(D) Processes I and II are not isobaric

26. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0$ J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is............ . **[JEE-Advance 2018]**

27. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II. **[JEE-Advance 2018]**
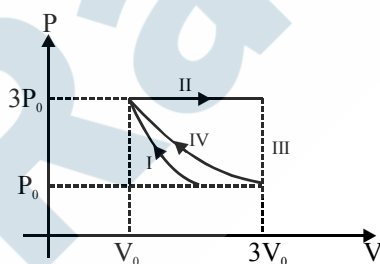
List–I		List–II	
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is $6 P_0 V_0$

(A) P $\rightarrow$ 4 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 1 ; S $\rightarrow$ 2

(B) P $\rightarrow$ 1 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 2 ; S $\rightarrow$ 4

(C) P $\rightarrow$ 3 ; Q $\rightarrow$ 4 ; R $\rightarrow$ 1 ; S $\rightarrow$ 2

(D) P $\rightarrow$ 3 ; Q $\rightarrow$ 4 ; R $\rightarrow$ 2 ; S $\rightarrow$ 1

ANSWER KEY

EXERCISE (S-1)

1. Ans. 1.25×10^4 N/m^2 **2. Ans.** 327 °C

3. Ans. (i) $P_1 < P_2$, $T_1 < T_2$; (ii) $T_1 = T_2 < T_3$; (iii) $V_2 > V_1$; (iv) $P_1 > P_2$ **4. Ans.** 40

5. Ans. (a) 4/3 m, (b) $T_3 = 400\left(\dfrac{4}{3}\right)^{0.4}$ K   **6. Ans.** 3600 R   **7. Ans.** $\Delta T = \dfrac{mv_0^{\,2}}{3R}$

8. Ans. 12600 J   **9. Ans.** 5R   **10. Ans.** 2.64   **11. Ans.** 1.5

12. Ans. 450 J   **13. Ans.** 16.9 J   **14. Ans.** R/2

15. Ans. (a) 0.5 atm (b) zero (c) zero (d) no

16. Ans. (a) A to B   (b) C to D

(c) $W_{AB} = \dfrac{1}{1-\gamma}(2^{1-\gamma} - 1)(P_B - P_A)V_A = \dfrac{3}{2}\left(1 - \left(\dfrac{1}{2}\right)^{2/3}\right)(P_B - P_A)V_A$

(d) $\left[1 - \left(\dfrac{1}{2}\right)^{2/3}\right]$

EXERCISE (S-2)

1. Ans. (i) 74 cm, (ii) 73.94 cm, (iii) 69.52 cm

2. Ans. $p_1 = p_{H_2} \simeq 1.25 \times 10^6$ Pa ; $p_2 = p_{H_2} + p_{O_2} + p_{N_2} \simeq 2.8125 \times 10^6$ Pa ; $p_3 = p_{H_2} + p_{N_2} \simeq 1.5625 \times 10^6$ Pa

3. Ans. 1.6 m, 364 K

4. Ans. (a) $P_i = P_a$

(b) $P_f = P_a + k(V - V_0)/A^2$ $V_0 = \ell A$

(c) $\Delta Q = P_a(V - V_0) + \dfrac{1}{2}k(V - v_0)^2 + C_v(T - T_0)$ $A = 1$

5. Ans. (a) Final pressure in A = $\dfrac{27}{8}P_0$ = Final pressure in C, Final pressure in B = $\dfrac{21}{4}P_0$

(b) Final temperature in A (and B) = $\dfrac{21}{4}T_0$, Final temperature in C = $\dfrac{3}{2}T_0$,

(c) 18 P_0V_0

(d) work done by gas in A = $+ P_0V_0$, work done by gas in B = 0,

(e) $\dfrac{17}{2}P_0V_0$

6. Ans. C = 2R   **7. Ans.** 400 J, $2T_0$ **8. Ans.** 2000 N/m, 1295 J

9. Ans. $Q_1 < Q_2$

10. Ans. (i) $T_A = 120.33$ K, $T_B = 240.66$ K, $T_C = 481.32$ K, $T_D = 240.66$ K, (ii) No

(iii) $\Delta Q_{ABC} = 3.25 \times 10^6$ J; $\Delta Q_{ADC} = 2.75 \times 10^6$ J

11. Ans. $1 - \dfrac{3\left(1 - \dfrac{1}{2^{1/3}}\right)}{\ln 2}$ **12. Ans.** 2.515, 404.8 K,

EXERCISE (O-1)

1. Ans. (A)	**2.** Ans. (A)	**3.** Ans. (B)	**4.** Ans. (D)	**5.** Ans. (C)	**6.** Ans. (A)
7. Ans. (C)	**8.** Ans. (B)	**9.** Ans. (C)	**10.** Ans. (C)	**11.** Ans. (C)	**12.** Ans. (A)
13. Ans. (D)	**14.** Ans. (A)	**15.** Ans. (C)	**16.** Ans. (D)	**17.** Ans. (B)	**18.** Ans. (D)
19. Ans. (C)	**20.** Ans. (C)	**21.** Ans. (A)	**22.** Ans. (C)	**23.** Ans. (D)	**24.** Ans. (B)
25. Ans. (B)	**26.** Ans. (C)	**27.** Ans. (B)	**28.** Ans. (D)	**29.** Ans. (A)	**30.** Ans. (A)
31. Ans. (A)	**32.** Ans. (A)	**33.** Ans. (A)	**34.** Ans. (D)	**35.** Ans. (B)	**36.** Ans. (B)
37. Ans. (C)	**38.** Ans. (C)	**39.** Ans. (D)	**40.** Ans. (A)	**41.** Ans. (B)	**42.** Ans. (A,B,C)
43. Ans. (B,C)		**44.** Ans. (B,C)	**45.** Ans. (D)	**46.** Ans. (D)	**47.** Ans. (C, D)
48. Ans. (B)	**49.** Ans. (A)	**50.** Ans. (A) P (B) Q (C) Q			

EXERCISE (O-2)

1. Ans. (C)	**2.** Ans. (A)	**3.** Ans. (D)	**4.** Ans. (D)	**5.** Ans. (B)	**6.** Ans. (C)
7. Ans. (D)	**8.** Ans. (C)	**9.** Ans. (D)	**10.** Ans. (C)	**11.** Ans. (D)	**12.** Ans. (C)
13. Ans. (A)	**14.** Ans. (C)	**15.** Ans. (C,D)	**16.** Ans. (B,D)	**17.** Ans. (A,B)	**18.** Ans. (A,B)
19. Ans. (B,C)		**20.** Ans. (B)	**21.** Ans. (A)	**22.** Ans. (B)	

23. Ans. (A) $\to$ (P,R) ; (B) $\to$ (Q, S) ; (C) $\to$ (Q, S) ; (D) $\to$ (P, R)

SUPPLEMENT FOR JEE-MAINS

1. Ans. B & 1.8% more	**2.** Ans. 2400 joule per cycle	**3.** Ans. 900 cal	**4.** Ans. 5.7 J
5. Ans. 3.36×10^6 J	**6.** Ans. (B)	**7.** Ans. (D)	**8.** Ans. (A)	**9.** Ans. (A)
10. Ans. (A)	**11.** Ans. (A)	**12.** Ans. (A)	**13.** Ans. (A)	**14.** Ans. (A)	**15.** Ans. (A)
16. Ans. (B)	**17.** Ans. (C)	**18.** Ans. (D)	**19.** Ans. (B)	

EXERCISE (J-M)

1. Ans. (1)	**2.** Ans. (3)	**3.** Ans. (1)	**4.** Ans. (4)	**5.** Ans. (4)	**6.** Ans. (3)
7. Ans. (2)	**8.** Ans. (3)	**9.** Ans. (4)	**10.** Ans. (3)	**11.** Ans. (1)	**12.** Ans. (2)
13. Ans. (4)	**14.** Ans. (2)	**15.** Ans. (3)	**16.** Ans. (2)	**17.** Ans. (Bonus)	**18.** Ans. (1)
19. Ans. (1)	**20.** Ans. (2)	**21.** Ans. (3)	**22.** Ans. (2)	**23.** Ans. (1)	**24.** Ans. (2)

EXERCISE (JA)

1. Ans. (C)	**2.** Ans. (A) q (b) p,r (c) p,s (d) q,s		**3.** Ans. (B,D)	**4.** Ans. (B,D)
5. Ans. (D)	**6.** Ans. (A,B,C,D)	**7.** Ans. 4	**8.** Ans. (A)	

9. Ans. (A) $\to$ (P,R,T) ; (B) $\to$ (P,R) ; (C) $\to$ (Q,S) ; (D) $\to$ (R,T)   **10.** Ans. (D)

11. Ans. (D)	**12.** Ans. (D)	**13.** Ans. (A, B, C, D) or (B, C, D)	**14.** Ans. (A)	**15.** Ans. 2
16. Ans. (D)	**17.** Ans. (D)	**18.** Ans. (A,B,D)	**19.** Ans. (A,B,C)	
20. Ans. (C)	**21.** Ans. (A,B,D)	**22.** Ans. (B)	**23.** Ans. (A)	**24.** Ans. (D)
25. Ans. (B,C,D)	**26.** Ans. 900 [899.95, 900.05]	**27.** Ans. (C)		