Solved Paper 2020*

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

		No. of Questions
Part I	Physics	1-40
Part II	Chemistry	41-80
Part III	a. English Proficiency	81-95
	b. Logical Reasoning	96-105
Part IV	Mathematics	106-150

- All questions are Multiple Choice Questions having four options out of which **only one** is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.
- Time allotted to complete this paper is 3 hrs.

PART I

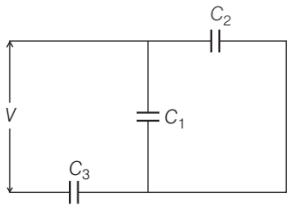
Physics

1. Three solenoid coils of same dimensions, same number of turns and same number of layers of windings are taken. Coil 1 has inductance L_1 wounded by Mn wire of resistance $6\,\Omega m^{-1}$, coil 2 with inductance L_2 wounded by similar wire but in reverse direction in each layer. Coil 3 with inductance L_3 wounded by a superconducting wire. The relation between their self inductances will be

a. $L_1 = L_2, L_3 = 0$ ***b.*** $L_1 = L_2 = L_3$

c. $L_1 = L_3, L_2 = 0$ ***d.*** $L_1 > L_2 > L_3$

2. Three capacitors C_1, C_2 and C_3 are connected as shown in the figure below. If capacitor C_3 breaks down electrically, then the change in total charge on the combination of capacitors, is



a. $(C_1 + C_2)V\left[1 - \left(\frac{C_3}{C_1 + C_2 + C_3}\right)\right]$

b. $(C_3 + C_2)V\left[1 - \left(\frac{C_1}{(C_1 + C_2 + C_3)}\right)\right]$

c. $(C_1 + C_2)V\left[1 - \left(\frac{C_2 + C_1}{C_1 + C_3}\right)\right]$

d. $(C_1 + C_2)V\left[1 - \left(\frac{C_2}{(C_1 + C_3)}\right)\right]$

3. A black body radiates energy at the rate $E\ Wm^{-2}$ at high temperature T K. When the temperature is reduced to $(T/4)$ K, then the new radiant energy is

a. $E/256$ ***b.*** $4E$ ***c.*** $E/4$ ***d.*** $E/16$

4. The length of the rectangle is $l = 15.2$ cm and breadth is $b = 2.9$ cm and the minimum possible measurement by scale = 0.1 cm. Then, the area of the rectangle is
(Taking, significant figures into consideration)

a. $44.08\ cm^2$ **b.** $24.8\ cm^2$
c. $44\ cm^2$ **d.** $94.008\ cm^2$

5. In an adiabatic process, where pressure is decreased by $\frac{3}{4}\%$, if $\frac{C_p}{C_V}=\frac{4}{3}$, then the volume increases by

a. $\frac{3}{4}\%$ **b.** $\frac{9}{16}\%$
c. $\frac{16}{9}\%$ **d.** $\frac{4}{3}\%$

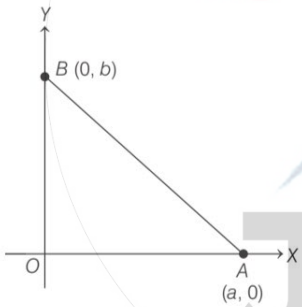
6. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y=4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

where, x and y are in cm and t is in second. Calculate the velocity of the particle at $x=7.5$ cm and $t=0.25$ s.

a. 4 m/s **b.** Zero
c. 16 m/s **d.** 9.8 m/s

7. A charge $+q$ is placed at the origin O of XY-axes as shown in the figure. The work done in taking a charge Q from A to B along the straight line AB is



a. $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{a-b}{ab}\right)$ **b.** $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$
c. $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{b}{a^2}-\frac{1}{b}\right)$ **d.** $\frac{qQ}{4\pi\varepsilon_0}\left(\frac{a}{b^2}-\frac{1}{b}\right)$

8. The hydrogen-like element that has a spectrum whose lines have wavelength four times shorter than those of atomic hydrogen is

a. lithium **b.** helium
c. berilliyum **d.** potassium

9. Two small conducting spheres of equal radius have charges $+20\,\mu C$ and $-40\,\mu C$ respectively and placed at a distance R from each other experience force F_1. If they are brought in contact and separated to the same distance, they experience force F_2. The ratio of F_1 to F_2 is

a. 1 : 4 **b.** 8 : 1
c. −8 : 1 **d.** 1 : 8

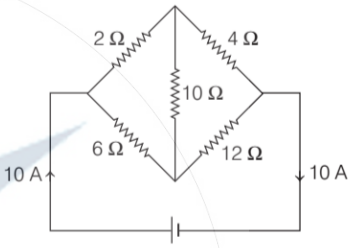
10. A Carnot engine has the same efficiency between 600 K to 300 K and 1600 K to x K, then the value of x is

a. 1600 K **b.** 800 K
c. 819 K **d.** 900 K

11. A ball of mass 0.5 kg is thrown up with initial speed 16 ms^{-1} and reaches maximum height of 9 m. How much energy is dissipated by air drag acting on the ball during the ascent?

a. 199 J **b.** 19.9 J
c. 20.9 J **d.** 9.9 J

12. In the circuit shown, if the 10 Ω resistor is replaced by a resistor of 15 Ω, then what is the amount of current drawn from the battery?



a. 100 A **b.** 10 A
c. 1 A **d.** 2.4 A

13. A time dependent force $F(=8t)$ acts on a particle of mass 2 kg. If the particle starts from rest, the work done by the force during the first 1s will be

a. 0.4 J **b.** 4 J
c. 19 J **d.** 4.5 J

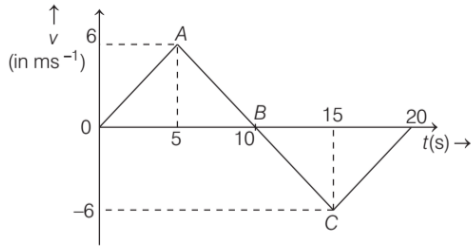
14. If L, R, C and V represent inductance, resistance, capacitance and potential difference respectively, then dimensions of $\frac{L}{RCV}$ are the same as those of

a. current **b.** $\frac{1}{\text{current}}$
c. charge **d.** $\frac{1}{\text{charge}}$

15. The magnetic field of a beam emerging from a fitter facing a flood light is given by $B=10\times10^{-8}\sin(1\times10^{7}z-3.6\times10^{15}t)$T. The average intensity of the beam is

a. $1.82\ W/m^2$ **b.** $1.19\ W/m^2$
c. $1.18\ W/m^2$ **d.** $1.17\ W/m^2$

16. From the velocity-time graph of a body moving in a straight line, the distance travelled and the average velocity in the time interval $t = 0$ to $t = 20$ s are, respectively,



a. 0, 0 ***b.*** 120 m, 60 m
c. 60 m, 0 ***d.*** 0, 60 m

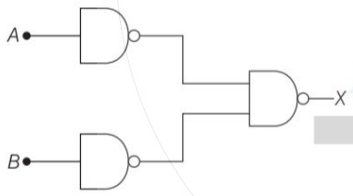
17. A thin equi-convex lens is made of glass of refractive index 1.5 and its focal length is 0.2 m. If it acts as a concave lens of focal length 0.5 m when dipped in a liquid, the refractive index of the liquid is

a. $\frac{17}{8}$ ***b.*** $\frac{15}{8}$ ***c.*** $\frac{13}{8}$ ***d.*** $\frac{9}{8}$

18. A moving coil galvanometer has a resistance of 60 Ω and it indicates full deflection on passing a current of 4.5 mA. A voltmeter is made using this galvanometer and a 4. 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to

a. 21 V ***b.*** 20.5 V ***c.*** 20 V ***d.*** 19.5 V

19. The combination of the gates shown in following figure yields



a. NAND gate ***b.*** OR gate
c. NOT gate ***d.*** XOR gate

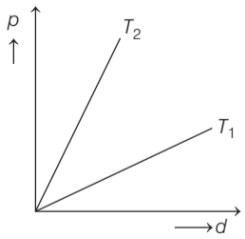
20. A vessel contains one mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is p. An identical vessel containing one mole of He gas (molar mass 4) at a temperature $2T$ has a pressure of

a. $\frac{p}{8}$ ***b.*** p ***c.*** $2p$ ***d.*** $8p$

21. The acceleration of a particle in m/s^2 is given by $a = (3t^2 - 2t + 1)$, where t is in second. If the particle starts with a velocity $v = 1$ m/s at $t = 1$ s, then velocity of the particle at the end of 4s is

a. 40 m/s ***b.*** 52 m/s ***c.*** 48 m/s ***d.*** 84 m/s

22. The figure shows graphs of pressure (p) *versus* density (d) for an ideal gas at two temperatures T_1 and T_2, then



a. $T_1 > T_2$ ***b.*** $T_1 < T_2$
c. $T_1 = T_2$ ***d.*** None of these

23. Two spheres of the same material and same radii r are touching each other. The gravitational force between the spheres is proportional to

a. $\frac{1}{r^2}$ ***b.*** r^2 ***c.*** $\frac{1}{r^4}$ ***d.*** r^4

24. A spherical lens of power –4 D is placed at a distance of 15 cm from another spherical lens of power 5 D. A beam of parallel light falls on the first spherical lens. The final image formed is

a. real and at a distance of 40 cm from the lens of power 5 D
b. real and at a distance of 10 cm from the lens of power – 4 D.
c. virtual and at a distance of 40 cm from the lens of power – 5 D
d. None of the above

25. The wheel of a car, accelerated uniformly from rest, rotates through 5 rad during the first second. The angle (in rad) rotated during the next second is

a. 15 ***b.*** 7.5 ***c.*** 12.5 ***d.*** 20

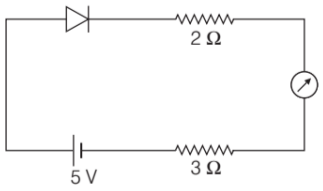
26. Lights of two different frequencies whose photons have energies 1.5 eV and 2.5 eV respectively illuminate a metallic surface whose work function is 0.5 eV successively. Ratio of maximum speeds of emitted electrons will be

a. 3 : 2 ***b.*** 2 : 3
c. $\sqrt{3} : \sqrt{2}$ ***d.*** $\sqrt{2} : \sqrt{3}$

27. When a body is dropped from a height h, then it hits the ground with a momentum p. If the same body is dropped from a height which is three times more than previous height, the percentage change in momentum when it hits the ground is

a. 25% ***b.*** 50%
c. 75% ***d.*** 100%

28. The reading of the ammeter for a germanium diode in the given circuit is



a. 0.94 A ***b.*** 0 ***c.*** 2.8 A ***d.*** 5 A

29. The decay constants of two radioactive substances X and Y are 4λ and λ respectively. At $t = 0$, a sample has the same number of two nuclei. The time taken for the ratio of number of nuclei to become $\frac{1}{e^3}$ will be

a. $\frac{1}{3\lambda}$ ***b.*** $\frac{1}{2\lambda}$ ***c.*** $\frac{2}{3\lambda}$ ***d.*** $\frac{3}{2\lambda}$

30. The magnitude of the force vector acting on a unit length of a thin wire carrying a current $I = 10$ A at a point O, if the wire is bent in the form of a semi-circle (shown below) with radius $R = 20\pi$ cm, is

a. 30 μN/m ***b.*** 40 μN/m
c. 50 μN/m ***d.*** 60 μN/m

31. A long cylindrical iron core of cross-sectional area 5 cm^2 is inserted into a long solenoid having 4000 turns/metre and carrying a current 5 A. The magnetic field inside the core is π T. Find the pole strength developed.

a. 1000 A-m ***b.*** 1240 A-m
c. 882 A-m ***d.*** 760 A-m

32. A plane requires for take off a speed of 72 kmh^{-1}, the run on the ground being 50 m. The mass of the plane is 10000 kg and the coefficient of friction between the plane and the ground is 0.2. Assume that the plane accelerates uniformly during take off. The minimum force required by the engine of the plane for take off is

a. 4.43×10^4 N ***b.*** 5.96×10^4 N
c. 2.25×10^4 N ***d.*** 3.45×10^4 N

33. In a fluorescent lamp choke, 120 V of reverse voltage is produced when the choke current changes uniformly 0.50 A to 0.20 A in a duration of 0.030 ms. The self inductance of the choke (in mH) is estimated to be

a. 12 H ***b.*** 12×10^{-3} mH
c. 12×10^{-3} H ***d.*** 0

34. A man grows into a giant such that his linear dimensions increase by a factor of 8. Assuming that his density remains same, the stress in his leg will change by a factor of

a. 1/8 ***b.*** 8 ***c.***36 ***d.*** 64

35. When 2 moles of a monoatomic gas are mixed with 3 moles of a diatomic gas, the value of adiabatic exponent for the mixture is

a. 15/16 ***b.*** 7/5 ***c.*** 31/21 ***d.*** 38/59

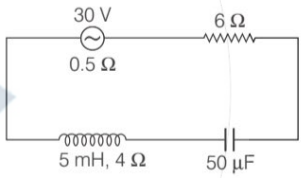
36. A simple pendulum is placed inside a lift, the lift is moving with a uniform acceleration. If the time periods of the pendulum, while the lift is moving upwards and downwards are in the ratio of 1 : 3, then the acceleration of the lift is [Take, acceleration due to gravity, $g = 10$ m/s^2]

a. 4 m/s^2 ***b.*** 6 m/s^2 ***c.*** 8 m/s^2 ***d.*** 10 m/s^2

37. A satellite is moving around the earth with speed v in a circular orbit of radius r. If the orbit radius is decreased by 2%, its speed will increase by

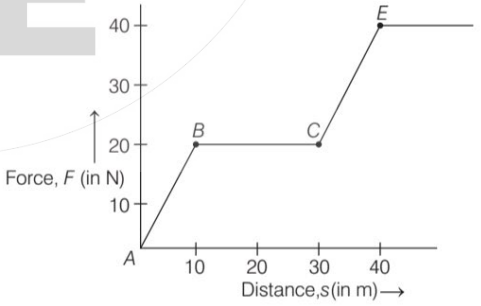
a. 1% ***b.*** 2% ***c.*** 1.5% ***d.*** 1.414%

38. In the circuit shown in figure, the AC source has angular frequency $\omega = 2000$ rad s^{-1}. The amplitude of the current will be nearest to



a. 2.85 A ***b.*** 3 A ***c.*** 0.5 A ***d.*** 3.625 A

39. The work done by a force acting on a body is as shown in the following graph. The total work done in covering an initial distance of 40 m is



a. 500 J ***b.*** 600 J ***c.***400 J ***d.*** 800 J

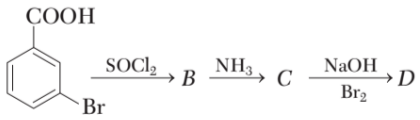
40. A potentiometer wire, 10 m long, has a resistance of 40 Ω. It is connected in series with a resistance box and a 4 V storage cell. If the potential gradient along the wire is 0.4 $mVcm^{-1}$, the resistance unplugged in the box is

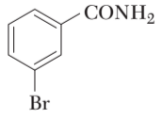
a. 220 Ω ***b.*** 360 Ω ***c.*** 760 Ω ***d.*** 848.3 Ω

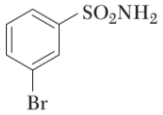
PART II

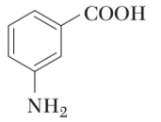
Chemistry

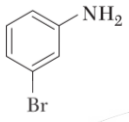
41. In a set of reactions, *m*-bromobenzoic acid gives a product *D*. Identify the product *D*.



a. 

b. 

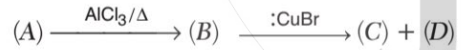
c. 

d. 

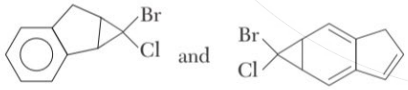
42. A volume of 50.00 mL of a weak acid of unknown concentration is titrated with 0.10 M solution of NaOH. The equivalence point is reached after 39.30 mL of NaOH solution has been added. At the half equivalence point (19.65 mL) the pH is 4.85. Thus, initial concentration of the acid and its pK_a values are

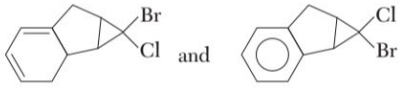
[HA]	pK_a
a. 0.1 M	4.85
b. 0.079 M	4.85
c. 0.1 M	3.70
d. 0.097 M	2.93

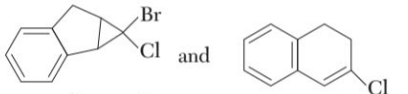
43. 1-phenyl-1, 3-dibromopropane on treatment with alc. KOH gives diastereomeric mixture in which compound (*A*) is major product. (*A*) gives the following reaction

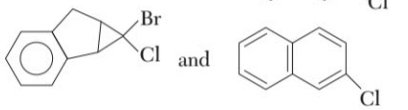


Compounds (*C*) and (*D*) are

a. 

b. 

c. 

d. 

44. Which of the following statement is incorrect?

a. α-D-fructose and β-D-fructose are enantiomers of each other.

b. D-glyceraldehyde and L-glyceraldehyde are enantiomers of each other.

c. The reserve carbohydrate of animals is glycogen.

d. Aldohexoses which react with phenyl hydrazine to give identical osazones are C-2 epimers.

45. Which of the following is not a mixed pair of oxides?

a. Mn_3O_4 and Co_3O_4

b. Co_3O_4 and Pb_3O_4

c. Pb_3O_4 and Mn_3O_4

d. Fe_3O_4 and Fe_2O_3

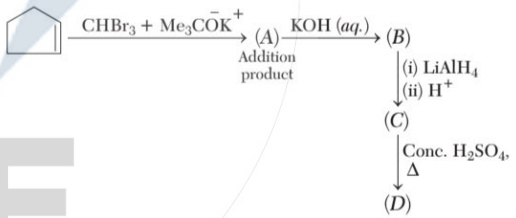
46. A solution of copper sulphate is electrolysed between copper electrodes by a current of 10.0A passing for one hour. Which of the following statements is correct regarding the changes that occur at the electrodes and in the solution?

a. 11.84 g of copper will deposit on the cathode

b. 11.84 g of copper will deposit on the anode

c. 11.84 g of copper will deposit on the anode as well as on the cathode

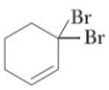
d. copper will not deposit on any of the electrode

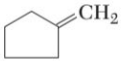
47. 

Identify structure of compound (*D*).

a. 

b. 

c. 

d. 

48. Which of the following statement is correct?

a. ΔS for $\frac{1}{2}Cl_2(g) \longrightarrow Cl(g)$ is positive.

b. $\Delta E < 0$ for combustion of $CH_4(g)$ in a sealed container with rigid adiabatic system.

c. ΔG is always zero for a reversible process in a closed system.

d. $\Delta G°$ for an ideal gas reaction is a function of pressure.

49. Which of the following statements on critical constants of gases are correct?

I. Larger the T_c / p_c value of a gas, larger would be the included volume.

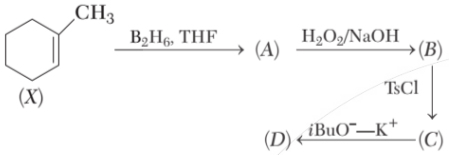
II. Critical temperature (T_c) of a gas is greater than its Boyle temperature (T_B).

III. At the critical point in the van der Waals' gas isotherm, $\left(\frac{\partial p}{\partial V_m}\right)_{T_c} = 0$

Select the correct answer using the codes given below.

a. Both I and II ***b.*** Both I and III
c. Both II and III ***d.*** I, II and III

50. In the given reaction,



Product (D) is

a. a positional isomer of X
b. identical to X
c. chain isomer of X
d. an oxidation product of X

51. $Na_2B_4O_7 + \text{conc.}H_2SO_4 + H_2O$

↓

(A)

↓ (i) C_2H_5OH (ii) Ignite

(B)

(B) is identified by the characteristic colour of the flame. (A) and (B) are respectively

a. H_3BO_3 and $Na_2B_4O_7$
b. $B(OC_2H_5)_3$ and H_3BO_3
c. $NaBO_2$ and H_3BO_3
d. H_3BO_3 and $B(OC_2H_5)_3$

52. How many mL of perhydrol is required to produce sufficient oxygen which can be used to completely convert 2 L of SO_2 gas?

a. 10 mL ***b.*** 5 mL
c. 20 mL ***d.*** 30 mL

53. At the top of a mountain the thermometer reads 0°C and the barometer reads 710 mm Hg. At the bottom of the mountain the temperature is 30°C and the pressure is 760 mm Hg. The ratio of the density of air at the top to that of the bottom is

a. 1 : 1.04 ***b.*** 0.4 : 1
c. 1.04 : 1 ***d.*** 1 : 04

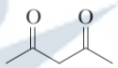
54. Match the following columns.

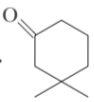
	Column I (Compound)		**Column II** (Structure)
A.	ClF_3	1.	Square planar
B.	PCl_5	2.	Tetrahedral
C.	IF_5	3.	Trigonal bipyramidal
D.	CCl_4	4.	Square pyramidal
E.	XeF_4	5.	T-shaped

Codes

	A	B	C	D	E
a.	5	4	3	2	1
b.	5	3	4	2	1
c.	5	3	4	1	2
d.	4	3	5	2	1

55. Most acidic hydrogen is present in

a. 

b. 

c. $(CH_3CO)_3CH$ ***d.*** $(CH_3)_3COH$

56. The process of 'eutrophication' is due to the

a. increase in concentration of insecticide in H_2O
b. increase in concentration of fluoride ion in H_2O
c. reduction in concentration of the dissolved oxygen in water due to phosphate pollution in water.
d. attack of younger leaves of a plant by peroxyacetyl nitrate

57. Ionisation energy of H-atom is 13.6 eV. The wavelengths of the spectral line emitted when an electron in Be^{3+} comes from 5th energy level to 2nd energy level is

a. 43.5 nm ***b.*** 4350 nm
c. 4.35 nm ***d.*** 435 nm

58. The enthalpies of combustion of carbon and carbon monoxide in excess of oxygen at 298 K and constant pressure are – 393.5 kJ/mol and –280.0 kJ/mol respectively. The heat of formation of carbon monoxide at constant volume is

a. + 111.7 kJ/mol ***b.*** –1111.7 kJ/mol
c. –111.7 kJ/mol ***d.*** –11.7 kJ/mol

59. If the quantum number l could have the value n also then, Sc(21) would have electronic configuration as (other rules strictly followed)

a. $1s^2\ 1p^6\ 2s^2\ 2p^6\ 2d^3\ 3s^2$

b. $1s^2\ 1p^6\ 2s^2\ 2p^6\ 3s^2\ 2d^3$

c. $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^1\ 4s^2$

d. $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^3$

60. 1.1 mole of A mixed with 2.2 moles of B and the mixture is kept in a 1 L flask and the equilibrium, $A + 2B \rightleftharpoons 2C + D$ is reached. If at equilibrium 0.2 mole of C is formed then the value of K_C will be

a. 0.1 ***b.*** 0.01 ***c.*** 0.001 ***d.*** 1

61. Choose the correct chemical reaction among the following:

a. $CaCN_2 + H_2O \longrightarrow Ca(OH)_2 + C_2H_2 + NH_3$

b. $2NH_3 + CaSO_4 + CO_2 + H_2O \longrightarrow CaCN_2 + (NH_4)_2SO_4$

c. $CaCl_2 + Na_2SO_4 \longrightarrow CaSO_4 + 2NaCl$

d. $CaC_2 + H_2O \longrightarrow Ca(OH)_2 + 2NaCl$

62. Which of the following reactions represents disproportionation?

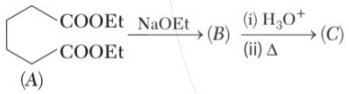
a. $CrO_5 \longrightarrow Cr^{3+} + O_2$

b. $IO_3^- + I^- + H^+ \longrightarrow I_2$

c. $CrO_2Cl_2 + NaOH \longrightarrow Na_2CrO_4 + NaCl + H_2O$

d. $Na_2S_2O_3 + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + S_8 + H_2O$

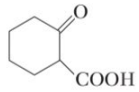
63.



(A)

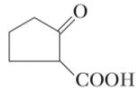
Compound C is

a.



b.

c.



d.

64. On the basis of Ellingham diagram, which of the following is not correct?

a. Entropy change for all metal oxides is roughly same.

b. Below the boiling point, $T\Delta S$ factor is same irrespective of metal.

c. Above $\Delta G = 0$ line, oxide decomposes into metal and oxygen.

d. If randomness increases the slope increases.

65. Which of the following is not explained by adsorption?

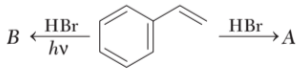
a. When acetic acid solution is shaken with charcoal the concentration of acid decreases.

b. The white ppt. of $Mg(OH)_2$ attains blue colour when precipitated in the presence of magneson reagent.

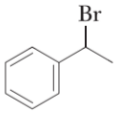
c. An aqueous solution of NaOH attains pink colour with a drop of phenolphthalein.

d. When animal charcoal is shaken with coloured methylene blue solution, the solution turns colourless.

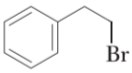
66. Identify 'A' and 'B' in the following reaction



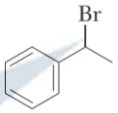
a. Both A and B are



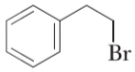
b. Both A and B are



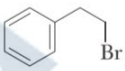
c. A is



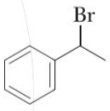
and B is



d. A is



and B is



67. An element occurs in two crystalline forms α and β. The α-form has an fcc with $a = 3.68$ Å and β form has a bcc with $a = 2.92$ Å. Calculate the ratio of their densities.

a. 1 : 1 ***b.*** 1 : 2

c. 2 : 1 ***d.*** 2 : 3

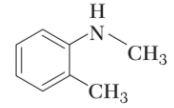
68. The molar heat of vaporisation of water at 100°C is 40.585 kJ/mol. The temperature at which a solution containing 5.60 g of $Al_2(SO_4)_3$ per 1000 g of water boil is ..... . (Assuming the degree of ionisation of salt to be 1).

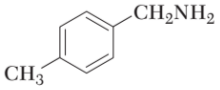
a. 1000.042° C ***b.*** 10.0042°C

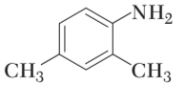
c. 100.042°C ***d.*** 105°C

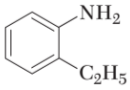
69. An organic base $C_8H_{11}N(X)$ reacts with nitrous acid at 0°C to give a clear solution. Heating the solution with KCN and cuprous cyanide followed by continued heating with conc. HCl gives a crystalline solid. Heating this solid with alkaline potassium permanganate gives a compound which dehydrates on heating to an anhydride ($C_8H_4O_3$).

Compound X is

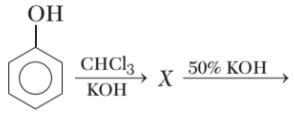
a. 

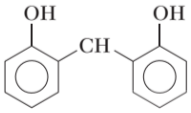
b. 

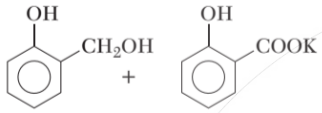
c. 

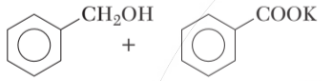
d. 

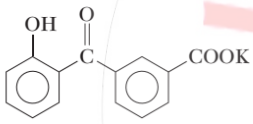
70. The final product of the following reaction is/are



a. 

b. 

c. 

d. 

71. The correct order of pseudohalide, polyhalide and interhalogen is

a. BrI_2^-, OCN^-, IF_5 *b.* IF_5, BrI_2^-, OCN^-

c. OCN^-, IF_2, BrI_2^- *d.* OCN^-, BrI_2^-, IF_5

72. An inorganic halide (A) reacts with water to form two acids (B) and (C). (A) also reacts with NaOH to form two salts (D) and (E) which remain in solution. The solution gives white precipitate with both $AgNO_3$ and $BaCl_2$ solutions respectively. (A) is a useful organic reagent. The compound (A) is

a. $SOCl_2$ *b.* SO_2Cl_2 *c.* S_2Cl_2 *d.* SF_4

73. Following statements regarding the periodic trends of chemical reactivity to the alkali metals and the halogens are given. Which of these statements gives the correct picture?

a. The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group.

b. In both the alkali metals and the halogens the chemical reactivity decreases with increase in atomic number down the group.

c. Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens.

d. In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group.

74. According to IUPAC nomenclature sodium nitroprusside is named as

a. sodiumpentacyanonitrosylferrate (II)

b. sodiumpentacyanonitrosylferrate (III)

c. sodiumnitroferricyanide

d. sodiumnitroferrocyanide

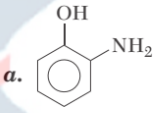
75. $A(g) \longrightarrow P(g) + Q(g) + R(g)$,

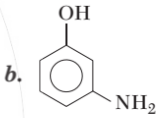
Follow first order kinetics with a half-life of 69.3 s at 500°C. Starting from the gas 'A' an container at 500°C and at a pressure of 0.4 atm, the total pressure of the system after 230 s will be

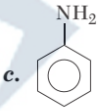
a. 1.15 atm *b.* 1.32 atm

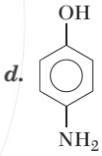
c. 1.22 atm *d.* 1.12 atm

76. Which of the following gives paracetamol on acetylation?

a. 

b. 

c. 

d. 

77. In qualitative analysis when H_2S is passed through an aqueous solution of salt acidified with dilute HCl, a black ppt. is obtained. On boiling the precipitate with dil. HNO_3, it forms a solution of blue colour. Addition of excess of aqueous solution of NH_3 to this solution gives

a. deep blue ppt. of $Cu(OH)_2$

b. deep blue solution of $[Cu(NH_3)_4]^{2+}$

c. deep blue solution of $Cu(NO_3)_2$

d. deep blue solution of $Cu(OH)_2 \cdot Cu(NO_3)_2$

78. Titration of 0.1467 g of primary standard $Na_2C_2O_4$ required 28.85 mL of $KMnO_4$ solution. Calculate the molar concentration of $KMnO_4$ solution.

a. 0.01518 M

b. 0.001518 M

c. 0.15180 M

d. 1.5180 M

79. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong?

a. The complex involves d^2sp^3 hybridisation and its octahedral in shape.

b. The complex is paramagnetic.

c. The complex is an outer orbital complex.

d. The complex gives white precipitate with silver nitrate solution.

80. Sulphuric acid is a dibasic acid. It ionises in two stages and hence, has two dissociation constants K_{a_1} and K_{a_2}. Which of the following is the correct observation regarding K_{a_1} and K_{a_2}?

a. $K_{a_1} > K_{a_2}$ ***b.*** $K_{a_1} < K_{a_2}$

c. $K_{a_1} = K_{a_2}$ ***d.*** $K_{a_1} = 1.2 \times 10^{-2}, K_{a_2} > 10$

PART III

a. English Proficiency

Directions (Q. Nos. 81-83) *In the following questions, find out which part of a sentence has an error. If there is no error, mark part (d) as your answer.*

81. The road **a.** / to famous monument **b.** / passes through a forest. **c.**/ No error **d**

82. The master did not know **a.**/ who of the servants **b.**/ broke the glass **c.**/ No error **d**

83. Had I come **a.**/ to know about his difficulties **b.**/ I would have certainly helped **c.**/ No error **d**

Directions (Q. Nos. 84 and 85) *Fill in the blanks with suitable preposition from the alternatives given under each sentence.*

84. Her trekking was met ..... obstacles.

a. with ***b.*** from ***c.*** by ***d.*** of

85. She has not got ..... the shock of losing her father.

a. over ***b.*** at ***c.*** from ***d.*** with

Directions (Q. Nos. 86-88) *Select the word or phrase which is closest to the opposite in meaning of the italicised word or phrase.*

86. Yuvraj Singh is suffering from a *BENIGN* cancer.

a. Unfriendly ***b.*** Friendly

c. Fatal ***d.*** Malignant

87. He is a *NOTED* figure of film industry.

a. Known ***b.*** Unknown

c. Famous ***d.*** Infamous

88. *SAGACIOUS* decisions taken at right time in one's career has long effects.

a. Foolish ***b.*** Intelligent

c. Thoughtful ***d.*** Intuitive

Directions (Q. Nos. 89 and 90) *Choose the word nearest in meaning to the italicised word.*

89. The actor got *PEEVISH* on asking personal questions.

a. Irritated ***b.*** Happy ***c.*** Shy ***d.*** Satisfied

90. The engineer *ROUGHED OUT* his ideas on a piece of paper while he talked.

a. Shaped soughly ***b.*** Rejected

c. Drew a quick plan ***d.*** Describe inaccurately

Directions (Q. Nos. 91-95) *Read the passage given below and answer the questions that follow.*

The Centre and the States must become partners in the planning process to determine national priorities together. The process of planning would undergo a change in view of the changes in domestic economic situation and momentous trends emerging in the world. The development of human resource and the building up of an institutional framework would have to receive priority attention. The role of the government would also have to be examined so as to fully involve the people in the process of nation-building. The main task would be to ensure that the real initiative is transferred to the people. The private sector which would register expansion hereafter should keep this objective firmly in view. The need for an effective population policy is an urgent necessity in the country's planning strategy. The family welfare programme should not be treated as the Centre's responsibility alone. The States should evolve a suitable mechanism for closer involvement of the Government agencies, Zilla Parishads and Panchayats for making the family welfare programme a success.

91. Which one of the following statements is correct?

a. Effective family welfare programme is Centre's responsibility alone.

b. Population policy and planning process are interlinked.

c. Family welfare programme should be left to the State Governments alone.

d. The State Government should use punitive measures to control population.

92. What should be given priority attention?

a. Role of the Government

b. Decentralisation of power

c. Involvement of people in labour welfare

d. Human resource and institutional framework

93. Which one of the following statements is not correct?

a. Role of the government in nation-building should be examined.

b. Real initiatives should be transferred to the people.

c. There should be no role for the government as far as planning is concerned.

d. The Centre and the States must become equal partners in the planning process.

94. What would force the planning process to undergo a change?

a. Free market forces

b. Domestic economic situation and world trends

c. Domestic compulsions

d. International pressures

95. Which one of the following is implied by the expression 'momentous trends'?

a. GDP growth of the country

b. Memorable historical events

c. Important changes in the international scene

d. Improvement of foreign exchange reserves

b. Logical Reasoning

96. Find out the wrong number.

2, 6, 12, 72, 865, 62208

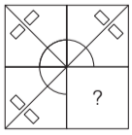
a. 72 ***b.*** 12 ***c.*** 62208 ***d.*** 865

97. Each of P, Q, T, A and B has different heights. T is taller than P and B but shorter than A and Q. P is not the shortest, who is the tallest?

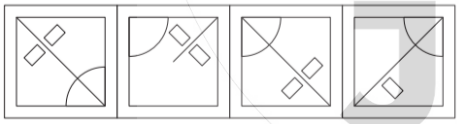
a. A ***b.*** Q ***c.*** A or Q ***d.*** P or B

98. Identify the missing part of the question figure and select it from given answer figures.

Question Figure



Answer Figures



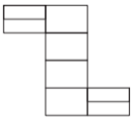
a. ***b.*** ***c.*** ***d.***

99. Select the related word from the given alternatives.

Mechanic : Spanner : : Carpenter :?

a. tree ***b.*** wood ***c.*** furniture ***d.*** saw

100. How many rectangles are there in the following figure?



a. 8 ***b.*** 18 ***c.*** 17 ***d.*** 20

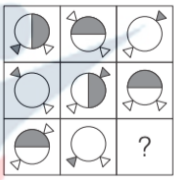
101. In the following question find the odd letters/group from the given alternatives.

a. BADC ***b.*** JILK

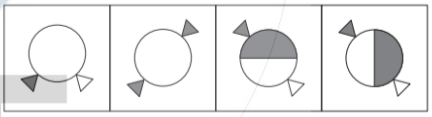
c. NMPO ***d.*** VUWX

102. Find out which of the answer figures (a), (b), (c) and (d) completes the figure matrix?

Question Figure



Answer Figures

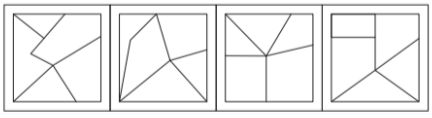


a. ***b.*** ***c.*** ***d.***

103. Among the four answer figures, which one can be formed from the cut out pieces given below in the question figures?

Question Figure

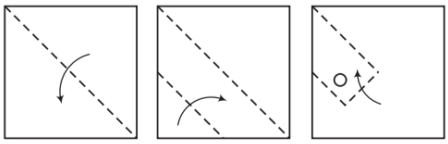
Answer Figures



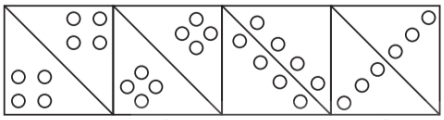
a. ***b.*** ***c.*** ***d.***

104. A piece of paper is folded and cut as shown below in the question figures. From the given answer figures, indicates how it will appear when opened.

Question Figures



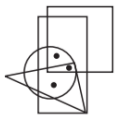
Answer Figures



a. *b.* *c.* *d.*

105. In the following question three dots are placed in the figure marked as (A). The figure is followed by four alternatives marked as (a), (b), (c) and (d). One out of these four options contains region(s) common to the circle, square, triangle, similar to that marked by the dot in figure (*A*).

Question Figure



(A)

Answer Figure

a. 

b. 

c. 

d. 

PART IV

Mathematics

106. If $A = \{x : x^2 = 1\}$ and $B = \{x : x^4 = 1\}$, then $A \Delta B$ is equal to

a. $\{-i, i\}$ *b.* $\{-1, i\}$

c. $\{-1, 1, -i, i\}$ *d.* None of these

107. If $2f(xy) = (f(x))^y + (f(y))^x$ for all $x, y \in R$ and $f(1) = a (\neq 1)$. Then $\sum_{k=1}^{n} f(k)$ is equal to

a. $(a^n - 1)/(a - 1)$

b. $a(a^{n-1} - 1)/(a - 1)$

c. $a(a^n - 1)/(a - 1)$

d. $(a^n - 1)/a + 1$

108. Let $f(x) = x - 3$ and $g(x) = 4 - x$. Then the set of values of x for which $|f(x) + g(x)| < |f(x)| + |g(x)|$ is true, is given by :

a. R *b.* $R - (3, 4)$

c. $R - [3, 4]$ *d.* None of these

109. If $a_1, a_2, a_3, \ldots, a_{20}$ are AM's between 13 and 67 then the maximum value of $a_1 \cdot a_2 \cdot a_3 \ldots, a_{20}$ is equal to

a. $(20)^{20}$ *b.* $(40)^{20}$

c. $(60)^{20}$ *d.* $(80)^{20}$

110. If p, q, r are in AP and are positive, the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for

a. $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ *b.* $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$

c. All p and r *d.* No p and r

111. The value of ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$ is equal to

a. ${}^{47}C_6$ *b.* ${}^{52}C_5$ *c.* ${}^{52}C_4$ *d.* None of these

112. The number of numbers divisible by 3 that can be formed by four different even digits is

a. 36 *b.* 18 *c.* 0 *d.* None of these

113. If $n(A) = 1000$, $n(B) = 500$, $n(A \cap B) \geq 1$ and $n(A \cup B) = P$, then

a. $500 \leq P \leq 1000$ *b.* $1001 \leq P \leq 1498$

c. $1000 \leq P \leq 1498$ *d.* $1000 \leq P \leq 1499$

114. $\left\{x \in R : \frac{2x - 1}{x^3 + 4x^2 + 3x} \in R\right\}$ is equal to

a. $R - \{0\}$ *b.* $R - \{0, 1, 3\}$

c. $R - \{0, -1, -3\}$ *d.* $R - \left\{0, -1, -3, \frac{1}{2}\right\}$

115. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in AP, then $f'(a), f'(b)$ and $f'(c)$ are in

a. AP
b. GP
c. Arithmetic-Geometric progression
d. None of the above

116. The value of $\lim_{x\to\infty} \frac{1}{n}\left\{\frac{1}{n+1}+\frac{2}{n+2}+....+\frac{3n}{4n}\right\}$ is

a. $5 - 2\log 2$ *b.* $4 - 2\log 2$
c. $3 - 2\log 2$ *d.* $2 - 2\log 2$

117. The coefficient of x^8 in the polynomial $(x-1)(x-2)....(x-10)$

a. 2640 *b.* 1320 *c.* 1370 *d.* 2740

118. If $z = \frac{7+i}{3+4i}$, then z^{14} is

a. 2^7 *b.* $2^7 i$ *c.* $(-2)^7$ *d.* $(-2)^7 i$

119. The solution of the equation

$$\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\tan y \sin y \text{ is}$$

a. $2y = \sin y(1 - 2cx^2)$ *b.* $2x = \cot y(1 + 2cx^2)$
c. $2x = \sin y\,(1 - 2cx^2)$ *d.* $2x\sin y = 1 - 2cx^2$

120. The value of the definite integral

$$\int_0^{\pi/2} \frac{dx}{\tan x + \cot x + \operatorname{cosec} x + \sec x}$$

a. $1 - \frac{\pi}{4}$ *b.* $1 + \frac{\pi}{4}$
c. $\pi + \frac{1}{4}$ *d.* None of these

121. If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $|\mathbf{a}| = 1, |\mathbf{b}| = 4$ and $\mathbf{a}\cdot\mathbf{b} = 2$, If $\mathbf{c} = (2\mathbf{a}\times\mathbf{b}) - 3\mathbf{b}$, then angle between $\mathbf{b}$ and $\mathbf{c}$

a. $\frac{\pi}{6}$ *b.* $\frac{\pi}{3}$
c. $\frac{2\pi}{3}$ *d.* $\frac{5\pi}{6}$

122. Let x_1 and x_2 be the real roots of the equation $x^2 - (k-2)x + (k^2 + 3k + 5) = 0$, then maximum value of $x_1^2 + x_2^2$ is

a. 19 *b.* 22
c. 18 *d.* 17

123. Circle centered at origin and having radius π units is divided by the curve $y = \sin x$ in two parts. Then area of upper parts equals to

a. $\frac{\pi^2}{2}$ *b.* $\frac{\pi^3}{4}$ *c.* $\frac{\pi^3}{2}$ *d.* $\frac{\pi^3}{8}$

124. The root of the equation $2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$, where $i = \sqrt{-1}$, which has greater modulus, is

a. $\frac{3-5i}{2}$ *b.* $\frac{5-3i}{2}$ *c.* $\frac{3+i}{2}$ *d.* $\frac{3i+1}{2}$

125. The equation $(\cos\beta - 1)x^2 + (\cos\beta)x + \sin\beta = 0$ in the variable x has real roots, then β lies in the interval

a. $(0, 2\pi)$ *b.* $(-\pi, 0)$ *c.* $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ *d.* $(0, \pi)$

126. An ordered pair (α, β) for which the system of linear $(1+\alpha)x + \beta y + z = 2, \alpha x + (1+\beta)y + z = 3$ and $\alpha x + \beta y + 2z = 2$ has a unique solution.

a. $(1, -3)$ *b.* $(-3, 1)$ *c.* $(2, 4)$ *d.* $(-4, 2)$

127. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45°. If flies off horizontally straight way from the point O. After one second, the elevation of the bird from O is reduced to 30°, then the speed (in m/s) of the bird is

a. $40(\sqrt{2} - 1)$ *b.* $40(\sqrt{3} - \sqrt{2})$
c. $20\sqrt{2}$ *d.* $20(\sqrt{3} - 1)$

128. If one GM, g and two AM's p and q are inserted between two numbers a and b, then

$(2p - q)(p - 2q)$ is equal to

a. g^2 *b.* $-g^2$ *c.* $2g$ *d.* $3g^2$

129. When x^{100} is divided by $x^2 - 3x + 2$, the remainder is $(2^{k+1} - 1)x - 2(2^k - 1)$, then k is

a. 97 *b.* 99 *c.* 100 *d.* 101

130. The mean of five observation is 5 and their variance is 9.20. If three of the given five observation are 1, 3 and 8, then a ratio of other two observations is

a. 4 : 9 *b.* 6 : 7
c. 5 : 8 *d.* 10 : 3

131. How many three digit number satisfy the property that the middle digit is arithmetic mean of the first and the last digit.

a. 41 *b.* 45 *c.* 43 *d.* 44

132. If $z = re^{i\theta}$, then $\arg(e^{iz})$ is

a. $-r\sin\theta$ *b.* $r\cos\theta$
c. $e^{-r\sin\theta}$ *d.* $-r\cos\theta$

133. If 4 dice are rolled, then the number of ways of getting the sum 10 is

a. 56 *b.* 64
c. 72 *d.* 80

134. Distance of point $A(1, 2)$ measured parallel to the line $3x - y = 10$ from the line $x + y + 5 = 0$, is
a. $2\sqrt{5}$ ***b.*** $2\sqrt{10}$ ***c.*** $4\sqrt{5}$ ***d.*** $4\sqrt{10}$

135. Let $f(x) = a_0 + a_1x^2 + a_2x^4 + a_3x^6 + \ldots + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_1 < a_2 < a_3 < \ldots < a_n$, the function $f(x)$ has
a. neither a maxima nor a minima
b. only one maxima
c. both maxima and minima
d. only one minima

136. If $f(x) = 2x^3 + x^4 + \log x$ and g is the inverse of f, then $g'(3)$ is equal to
a. 1/9 ***b.*** 1/7 ***c.*** 1/11 ***d.*** 1/8

137. A line passing through $P(3, 7, 1)$ and $R(2, 5, 7)$ meet the plane $3x + 2y + 11z - 9 = 0$ at Q. Then PQ is equal to
a. $\frac{5\sqrt{41}}{59}$ ***b.*** $\frac{\sqrt{41}}{59}$ ***c.*** $\frac{50\sqrt{41}}{59}$ ***d.*** $\frac{25\sqrt{41}}{59}$

138. $\int \frac{8x^{43} + 13x^{38}}{(x^{13} + x^5 + 1)^4}\,dx$ equals to
a. $\frac{x^{39}}{3(x^{13} + x^5 + 1)^3} + C$ ***b.*** $\frac{x^{39}}{(x^{13} + x^5 + 1)^3} + C$
c. $\frac{x^{39}}{5(x^{13} + x^5 + 1)^5} + C$ ***d.*** None of these

139. If $\mathbf{a} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}$, then find z component of a vector $\mathbf{r}$, which is coplanar with $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{r} \cdot \mathbf{b} = 0$ and $\mathbf{r} \cdot \mathbf{a} = 7$.
a. 0 ***b.*** 3 ***c.*** 6 ***d.*** 5/2

140. If x, y, z are three consecutive positive integers, $x - z + 2 = 0$, then
$\frac{1}{2}\log_e x + \frac{1}{2}\log_e z + \frac{1}{2xz+1} + \frac{1}{3}\left(\frac{1}{2xz+1}\right)^3 + \ldots$
is equal to
a. $\log_e x$ ***b.*** $\log_e y$
c. $\log_e z$ ***d.*** None of these

141. The solution of differential equation $(xy^5 + 2y)\,dx - x\,dy = 0$, is
a. $9x^8 + 4x^9y^4 = 9y^4C$ ***b.*** $9x^8 - 4x^9y^4 - 9y^4C = 0$
c. $x^8(9 + 4y^4) = 10y^4C$ ***d.*** None of these

142. The solution set of $\frac{|x-2|-1}{|x-2|-2} \le 0$ is
a. $[0, 1] \cup (3, 4)$ ***b.*** $[0, 1] \cup [3, 4]$
c. $[-1, 1] \cup (3, 4]$ ***d.*** None of these

143. Let $f(x) = \frac{x}{\sqrt{1+x^2}}$, the $\underbrace{fofofo\ldots\ldots of(x)}_{n \text{ times}}$ is
a. $\frac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} r\right)x^2}}$ ***b.*** $\frac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} 1\right)x^2}}$
c. $\left(\frac{x}{\sqrt{1+x^2}}\right)^x$ ***d.*** $\frac{nx}{\sqrt{1+nx^2}}$

144. If $\log_5 \frac{(a+b)}{3} = \frac{\log_5 a + \log_5 b}{2}$, then $\frac{a^4 + b^4}{a^2b^2}$ is equal to
a. 50 ***b.*** 47 ***c.*** 44 ***d.*** 53

145. The number of distinct solutions of the equation $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is
a. 8 ***b.*** 10 ***c.*** 6 ***d.*** 15

146. If the tangent at a point $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to $x^2 + y^2 - 2x = 15$, then ϕ equals
a. $\frac{\pi}{3}$ ***b.*** $\frac{\pi}{6}$ ***c.*** $-\frac{\pi}{6}$ ***d.*** $\frac{\pi}{4}$

147. The distance of point of intersection of the tangents to the parabola $x = 4y - y^2$ drawn at the points where it is meet by Y-axis, from its focus is
a. 11/4 ***b.*** 17/4 ***c.*** 13/4 ***d.*** 3

148. The value of the sum $\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}$ is
a. 5 ***b.*** 4 ***c.*** 3 ***d.*** 2

149. A curve passes through (2, 0) and the slope of the tangent at $P(x, y)$ is equal to $\frac{(x+1)^2 + y - 3}{x+1}$ then the equation of the curve is
a. $y = x^2 - 2x$ ***b.*** $y = x^3 - 8$
c. $y^2 = x^2 + 2x$ ***d.*** $y^2 = 5x^2 - 6$

150. Consider matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, if $A^{-1} = \alpha I + \beta A$, where $\alpha, \beta \notin R$, then $(\alpha + \beta)$ is equal to (where A^{-1} denotes the inverse of matrix A)
a. 1 ***b.*** $\frac{4}{3}$ ***c.*** $\frac{5}{3}$ ***d.*** $\frac{1}{3}$

Answers

Physics

1. (c)	2. (a)	3. (a)	4. (c)	5. (b)	6. (b)	7. (a)	8. (b)	9. (c)	10. (b)
11. (b)	12. (b)	13. (b)	14. (b)	15. (b)	16. (c)	17. (b)	18. (b)	19. (b)	20. (c)
21. (b)	22. (b)	23. (d)	24. (a)	25. (a)	26. (d)	27. (d)	28. (a)	29. (b)	30. (c)
31. (b)	32. (b)	33. (c)	34. (b)	35. (c)	36. (c)	37. (a)	38. (a)	39. (d)	40. (b)

Chemistry

41. (d)	42. (b)	43. (d)	44. (a)	45. (d)	46. (a)	47. (b)	48. (a)	49. (b)	50. (a)
51. (d)	52. (a)	53. (c)	54. (b)	55. (c)	56. (c)	57. (d)	58. (c)	59. (b)	60. (c)
61. (c)	62. (d)	63. (d)	64. (a)	65. (c)	66. (c)	67. (a)	68. (c)	69. (d)	70. (b)
71. (d)	72. (b)	73. (d)	74. (b)	75. (d)	76. (d)	77. (b)	78. (a)	79. (c)	80. (a)

English Proficiency

81. (b)	82. (b)	83. (c)	84. (a)	85. (b)	86. (c)	87. (b)	88. (a)	89. (a)	90. (c)
91. (b)	92. (d)	93. (c)	94. (b)	95. (c)					

Logical Reasoning

96. (d)	97. (c)	98. (c)	99. (d)	100. (b)	101. (d)	102. (d)	103. (c)	104. (b)	105. (b)

Mathematics

106. (a)	107. (c)	108. (c)	109. (b)	110. (a)	111. (c)	112. (a)	113. (d)	114. (c)	115. (a)
116. (c)	117. (b)	118. (b)	119. (c)	120. (a)	121. (d)	122. (a)	123. (c)	124. (a)	125. (d)
126. (c)	127. (d)	128. (b)	129. (b)	130. (a)	131. (b)	132. (b)	133. (d)	134. (b)	135. (d)
136. (c)	137. (d)	138. (a)	139. (b)	140. (b)	141. (a)	142. (b)	143. (b)	144. (b)	145. (a)
146. (a)	147. (b)	148. (d)	149. (a)	150. (a)					

Hints & Solutions

Physics

1. ***(c)*** As coil 2 with inductance L_2 has been wounded by a similar wire as coil 1 but the direction of windings is reversed in each layer, so flux through L_2 will be zero.

As, $L_2 \propto \phi \Rightarrow L_2 = 0$

Also, since the dimensions, number of turns and number of layers of windings are equal in coil 1 and coil 3.

So, $L_1 = L_3$

2. ***(a)*** Since, capacitors C_1 and C_2 are parallel, so their equivalent capacitance will be $(C_1 + C_2)$. Now, $(C_1 + C_2)$ and C_3 are in series, so the net equivalent capacitance of circuit will be

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} = \frac{C_1 + C_2 + C_3}{(C_1 + C_2)C_3}$$

or $$C = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3}$$

Since, V is the voltage of the battery, so charge on this system,

$$q = CV \Rightarrow q = \frac{(C_1 + C_2)C_3 V}{(C_1 + C_2 + C_3)}$$

If the capacitance C_3 breaks down, then total equivalent capacitance will become

$$C' = C_1 + C_2$$

$\therefore$ New charge stored,

$$q' = C'V$$

$\Rightarrow$ $$q' = (C_1 + C_2)V$$

Now, change in total charge,

$$\Delta q = q' - q \qquad [\because q' > q]$$

$$= (C_1 + C_2)V - \frac{(C_1 + C_2)C_3 V}{C_1 + C_2 + C_3}$$

or $$\Delta q = (C_1 + C_2)V\left[1 - \left(\frac{C_3}{C_1 + C_2 + C_3}\right)\right]$$

3. ***(a)*** Let E' be the new radiant energy.

According to Stefan's law,

$$E \propto T^4$$

$\Rightarrow$ $$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$$

$\Rightarrow$ $$\frac{E_2}{E} = \left\{\frac{(T/4)}{T}\right\}^4 = \left(\frac{1}{4}\right)^4$$

or $$E_2 = \frac{E}{256}$$

4. ***(c)*** Area of rectangle, $A = lb = 15.2 \times 2.9 = 44.08\ \text{cm}^2$

Minimum possible measurement of scale = 0.1 cm

As we know, the product should have significant figures that are present in the measurement with least number of significant figures which is 2 in this case. So, area measured by scale = 44 cm^2.

5. ***(b)*** In adiabatic process,

$$pV^{\gamma} = K$$

where, K is a constant.

Given that, $\gamma = \frac{4}{3} \Rightarrow pV^{4/3} = K$

Taking logarithm on both sides, we get

$$\log p + \frac{4}{3}\log V = \log K$$

$$\Rightarrow \frac{\Delta p}{p} + \frac{4}{3}\frac{\Delta V}{V} = 0 \Rightarrow \frac{\Delta V}{V} = -\frac{3}{4}\frac{\Delta p}{p}$$

Now, % change in volume,

$$\frac{\Delta V}{V} \times 100 = -\frac{3}{4}\left(\frac{\Delta p}{p} \times 100\right)$$

It is given that, pressure decreases by $\frac{3}{4}\%$,

i.e. $$\frac{\Delta p}{p} \times 100 = -\frac{3}{4}$$

$\therefore$ $$\frac{\Delta V}{V} \times 100 = -\frac{3}{4} \times \left(-\frac{3}{4}\right) = \frac{9}{16}\%$$

Hence, the volume increases by $\frac{9}{16}\%$.

6. ***(b)*** The given equation is

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t) \qquad \ldots(\text{i})$$

Now, the velocity of the string at a point x at time t is obtained by differentiating Eq. (i) w.r.t.

$$\therefore v = \frac{dy}{dt} = -(4 \times 96\pi) \times \sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$$

At $x = 7.5\ \text{cm} = 7.5 \times 10^{-2}$ m and $t = 0.25$ s,

$$v = -384\pi \sin\left(\pi \times \frac{7.5 \times 10^{-2}}{15}\right)\sin\left(96\pi \times \frac{25}{100}\right)$$

$$= -384\pi \sin\left(\frac{\pi}{200}\right)\sin(24\pi)$$

$$= 0 \qquad (\because \sin 24\pi = 0)$$

Hence, the velocity of the particle is zero.

7. ***(a)*** Potential at point A,

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}$$

Potential at point B,

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{b}$$

Work done in taking a charge Q from A to B,

$$W = Q(V_B - V_A) = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{b}-\frac{1}{a}\right] = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{a-b}{ab}\right]$$

8. ***(b)*** For hydrogen-like atom,

$$\frac{1}{\lambda'} = RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad \ldots(\text{i})$$

Similarly, for hydrogen atom,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \quad \ldots(\text{ii})$$

For the element to have hydrogen-like spectrum, the ratio of the wavelengths for transition of electron between any two levels n_1 and n_2 should be constant.

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\lambda'}{\lambda} = \frac{R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}{RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)} \quad \text{or} \quad \frac{\lambda'}{\lambda} = \frac{1}{Z^2}$$

It is given that, wavelength of the hydrogen atom is four times the wavelength of the hydrogen-like atom, i.e.

$$\lambda = 4\lambda'$$

Hence, $Z = 2$

Now, the element with atomic number 2 is helium.

9. ***(c)*** Given, two small conducting spheres of equal radius R.

Charge on first sphere, $q_1 = +20\,\mu C$

Charge on second sphere, $q_2 = -40\,\mu C$

$$\therefore \quad F_1 = \frac{k(+20)(-40)}{R^2} = \frac{-k(800)}{R^2} \quad \ldots(\text{i})$$

The spheres have equal radii, so their capacities will be same. Now, if they are brought in contact, then the net charge,

$$q_{net} = (+20-40)\mu C = -20\mu C$$

So, each sphere will have charge, $q = \frac{q_{net}}{2} = -10\mu C$

Now, $F_2 = \frac{k(q)(q)}{R^2} = \frac{k(-10)(-10)}{R^2} = \frac{k(100)}{R^2} \quad \ldots(\text{ii})$

Ratio, $\frac{F_1}{F_2} = -\frac{800}{100} = -8:1$

Thus, ratio of F_1 to $F_2 = -8:1$

10. ***(b)*** Efficiency of Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Given, $T_1 = 600$ K, $T_2 = 300$ K,

$T_1' = 1600$K and $T_2' = x$K

Carnot engine has same efficiency between 600 K to 300 K and 1600 K to xK, so

$$\left(1-\frac{T_2}{T_1}\right) = \left(1-\frac{T_2'}{T_1'}\right)$$

Substituting all the given values in above equation, we get

$$1-\frac{300}{600} = 1-\frac{x}{1600}$$

$$1-\frac{1}{2} = 1-\frac{x}{1600}$$

$$\frac{x}{1600} = 1-\frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \quad x = \frac{1600}{2} = 800 \text{ K}$$

11. ***(b)*** Given, initial speed of ball, $v = 16$ ms^{-1}

Maximum height, $h = 9$ m

Mass of ball, $m = 0.5$ kg

Energy supplied to ball while throwing up,

$$E_1 = \frac{1}{2}mv^2 = \frac{1}{2}\times 0.5\times 16\times 16 = 64 \text{ J}$$

Energy stored when ball reaches the maximum height,

$$E_2 = mgh = 0.5\times 9.8\times 9 = 44.1 \text{ J}$$

Energy dissipated by air drag acting on the ball during the ascent,

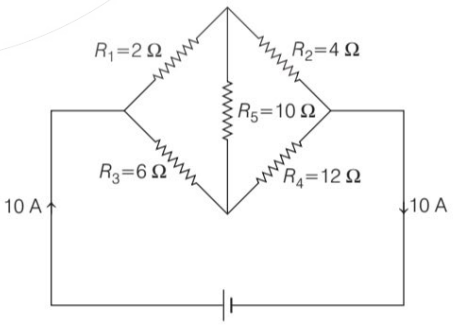
$$E = E_1 - E_2 = 64 - 44.1 = 19.9 \text{ J}$$

12. ***(b)*** From the following circuit diagram,

$\frac{R_1}{R_3} = \frac{R_2}{R_4}$, so the given Wheatstone bridge is balanced.

Hence, there is no current in the wire of 10 Ω resistance.

As,



Now, if the 10 Ω wire is replaced with 15 Ω wire, the bridge still remains balanced. Hence, current drawn from battery will be 10 A.

13. ***(b)*** Given, force, $F = 8t$ N

Mass, $m = 2$ kg

From Newton's second law,

Force = Rate of change of momentum

$$F = \frac{dp}{dt}$$

i.e. $dp = Fdt$

$\therefore$ Momentum, $p = \int dp = \int_0^1 F \cdot dt = \int_0^1 8t \cdot dt$

$$= 8\left[\frac{t^2}{2}\right]_0^1 = 8\left(\frac{1}{2} - 0\right)$$

$\Rightarrow$ $p = 4 \text{ kg ms}^{-1}$

Change in kinetic energy,

$$\Delta K = \frac{p^2}{2m} = \frac{(4)^2}{2 \times 2} = 4 \text{ J}$$

From work-energy theorem,

Work done = Change in kinetic energy

So, work done by the force during the first 1s,

$$W = \Delta K = 4 \text{J}$$

14. ***(b)*** As, RC is the time constant, so it has the dimension of time and V has the dimensions of $[L]\left[\frac{di}{dt}\right]$.

$\therefore$ Dimensions of $\frac{L}{RCV}, \left[\frac{\text{L}}{\text{RCV}}\right] = \frac{[\text{L}]}{[\text{RC}]\,[\text{V}]}$

$$\frac{[\text{L}]}{[\text{T}][\text{L}]\left[\frac{\text{A}}{\text{T}}\right]} = \frac{[\text{L}][\text{T}]}{[\text{T}][\text{L}][\text{A}]} = \frac{1}{[\text{A}]}$$

So, $\left(\frac{\text{L}}{\text{RCV}}\right)$ has the dimensions $[\text{A}^{-1}]$, i.e. same as the dimensions of $\frac{1}{\text{current}}$.

15. ***(b)*** Given, magnetic field,

$$B = 10 \times 10^{-8} \sin(1 \times 10^7 z - 3.6 \times 10^{15} t) \text{ T} \quad \ldots(\text{i})$$

Standard equation of magnetic field,

$$B = B_0 \sin(kz - \omega t) \quad \ldots(\text{ii})$$

On comparing Eqs. (i) and (ii), we get

$$B_0 = 10 \times 10^{-8} \text{ T}$$

The average intensity of the beam,

$$I_{\text{av}} = \frac{B_0^2 c}{2\mu_0} = \frac{1}{2} \times \frac{(10 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}}$$

$[\because c = 3 \times 10^8 \text{ ms}^{-1}]$

$$= 1.19 \text{ W/m}^2$$

16. ***(c)*** Distance = area under speed-time graph as given in question.

$\therefore$ Distance travelled in 20 s = area of ΔOAB + area of ΔBCD

$$= \frac{1}{2} \times 6 \times 10 + \frac{1}{2} \times 6 \times 10 = 60 \text{ m}$$

Displacement in 20 s = area of ΔOAB − area of ΔBCD

$$= \frac{1}{2} \times 6 \times 10 - \frac{1}{2} \times 6 \times 10 = 30 - 30 = 0$$

Hence, average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{0}{t} = 0$

17. ***(b)*** The focal length of a convex lens of refractive index μ_g in air,

$$\frac{1}{f_{\text{air}}} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(\text{i})$$

where, R_1, R_2 = radii of curvatures of its first and second surface.

When the lens is immersed in a liquid of refractive index μ_l, then refractive index of material of lens (glass) w.r.t. liquid is

$$_l\mu_g = \frac{\mu_g}{\mu_l} \quad \ldots(\text{ii})$$

Now, focal length of lens in liquid,

$$\frac{1}{f'} = (_l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots(\text{iii})$$

Dividing Eq. (i) by Eq (iii), we get

$$\frac{f'}{f_{\text{air}}} = \frac{\mu_g - 1}{(_l\mu_g - 1)} = \frac{\mu_g - 1}{\left(\frac{\mu_g}{\mu_l} - 1\right)}$$

Given, $f' = -0.5$ m, $f_{\text{air}} = 0.2$ m, $\mu_g = 1.5$

Substituting all these values in above equation, we get

$$-\frac{0.5}{0.2} = \frac{1.5 - 1}{\left(\frac{1.5}{\mu_l} - 1\right)} = \frac{0.5}{\left(\frac{1.5}{\mu_l} - 1\right)}$$

$\Rightarrow$ $\frac{1.5}{\mu_l} - 1 = -0.2$

$\Rightarrow$ $\frac{1.5}{\mu_l} = 1 - 0.2 = 0.8$

$\Rightarrow$ $\mu_l = \frac{15}{8}$

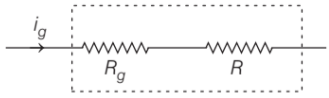
$\therefore$ Refractive index of liquid = 15/8

18. ***(b)*** Given, resistance of galvanometer, $R_g = 60\,\Omega$

Current, $I_g = 4.5 \text{ mA} = 4.5 \times 10^{-3}$ A

Resistance used in converting a galvanometer into voltmeter, $R = 4.5 \text{ k}\Omega = 4.5 \times 10^3\ \Omega$

To make the voltmeter, the galvanometer and the high resistance (R) have to be connected in series combination as shown below.

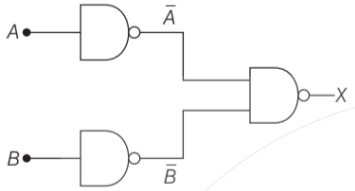


$\therefore$ Maximum current in galvanometer is

$$I_g = \frac{E}{R + R_g}$$

$$\Rightarrow \quad E = I_g(R + R_g)$$
$$= 4.5 \times 10^{-3} \times (4.5 \times 10^3 + 60)$$
$$= 20.5 \text{ V}$$

19. ***(b)*** The given combination of gates is shown below



The output of this combination can be determined by Boolean algebra.

$\therefore$ Output, $X = \overline{\bar{A} \cdot \bar{B}} = \bar{\bar{A}} + \bar{\bar{B}}$

[Using de-Morgan's theorem]

$$\Rightarrow \quad X = A + B$$

This is the output of OR gate. Hence, the given combination of gates yield OR gate.

20. ***(c)*** From gas equation,

$$pV = nRT \Rightarrow p = \frac{nRT}{V}$$

Pressure of one mole of O_2 gas,

$$(p)_{O_2} = \frac{1 \times RT}{V} \quad \ldots(i)$$

Pressure of one mole of He gas,

$$(p)_{He} = \frac{1 \times R(2T)}{V} \quad \ldots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\therefore \quad \frac{(p)_{He}}{(p)_{O_2}} = 2 \Rightarrow (p)_{He} = 2(p)_{O_2} = 2p$$

21. ***(b)*** Acceleration, $a = \frac{dv}{dt} \Rightarrow dv = adt$

Integrating both sides, $\int dv = \int adt$

Given, $a = (3t^2 - 2t + 1)\,\text{ms}^{-2}$

Substituting this value in above equation, we get

$$v = \int (3t^2 - 2t + 1)dt$$
$$= t^3 - t^2 + t + C \quad \ldots(i)$$

At $t = 1$ s, $v = 1$ m/s

Substituting these values in Eq. (i), we get

$$\because \quad 1 = 1 - 1 + 1 + C \quad \Rightarrow C = 0$$

Substituting the value of C in Eq. (i), we get

$$v = t^3 - t^2 + t$$

When $t = 4$ s, $v = (4)^3 - (4)^2 + 4 = 52$ m/s

22. ***(b)*** From ideal gas equation,

$$pV = nRT = \frac{mRT}{M}$$

where, $n = \dfrac{\text{Molar mass } (m)}{\text{Molecular weight } (M)}$

Here, n = number of moles,

R = gas constant

and V = volume.

$$\Rightarrow \quad \frac{pV}{m} = \frac{RT}{M}$$

$$\Rightarrow \quad \frac{p}{(m/V)} = \frac{RT}{M}$$

$$\frac{p}{d} = \frac{RT}{M} \qquad \left[\because \frac{m}{V} = d\right]$$

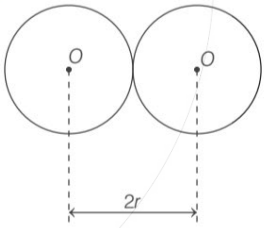
$$\Rightarrow \quad \frac{p}{d} \propto T$$

$\Rightarrow$ Slope of p *versus* d graph $\propto T$

Since, slope is less for the gas at temperature T_1.

Therefore, $T_1 < T_2$

23. ***(d)*** The two spheres are touching each other as shown below.



Let m be the mass of each sphere.

Gravitational force between the spheres,

$$F = G\frac{m \cdot m}{(2r)^2} = G.\frac{m^2}{4r^2}$$

$$= G.\frac{\left[\frac{4}{3}\pi r^3 \cdot \rho\right]^2}{4r^2} \quad [\text{Mass} = \text{volume} \times \text{density}]$$

$$= \frac{4G\pi^2 r^6 \rho^2}{9 \times 4r^2}$$

$$\Rightarrow \quad F = \left(\frac{4G\pi^2\rho^2}{9}\right) r^4 \Rightarrow F \propto r^4$$

24. ***(a)*** Power of first spherical lens, $P_1 = -4$ D

The power of this lens is negative, hence it is diverging lens.

Focal length of first spherical lens,

$$f_1 = \frac{1}{P_1} = \frac{1}{(-4\text{D})} = -0.25 \text{ m} = -25 \text{ cm}$$

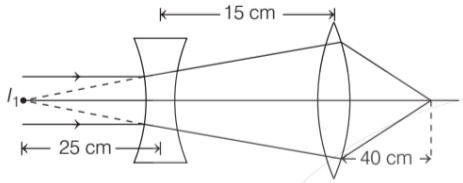
Power of second spherical lens, $P_2 = 5$D

The power of this lens is positive, hence it is converging lens.

Focal length of second spherical lens,

$$f_2 = \frac{1}{P_2} = \frac{1}{5\text{D}} = 0.2 \text{ m} = 20 \text{ cm}$$

The given situation is shown in the following figure.



Now, the image I_1 will be considered as object for converging lens.

For converging lens, distance of object,

$$u = -(25 + 15) = -40 \text{ cm}$$

and $f_2 = 20$ cm

$\therefore$ Using lens formula,

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{20} = \frac{1}{v} - \frac{1}{-40}$$

$$\Rightarrow \quad v = 40 \text{ cm}$$

Since v is positive, so image formed will be real and will be formed at a distance of 40 cm from the lens of power 5 D (converging lens).

25. ***(a)*** From second equation of rotational motion,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

Given, $\theta = 5$ rad, $t = 1$ s, $\omega_0 = 0$

Substituting all these values in above equation, we get

$$5 = 0 + \frac{1}{2}\alpha(1)^2$$

$$\Rightarrow \quad \alpha = 0 \text{ rad/s}^2$$

The angle rotated by wheel of car during first two seconds,

$$\theta_2 = 0 + \frac{1}{2} + \alpha t^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ rad}$$

Hence, angle rotated during the 2nd second

$$= \theta_2 - \theta$$
$$= 20 - 5 = 15 \text{ rad}$$

26. ***(d)*** Kinetic energy in photoelectric effect can be given as

$$\text{KE} = W - W_0 \qquad \begin{bmatrix} W \rightarrow \text{incident energy} \\ W_0 \rightarrow \text{work function} \end{bmatrix}$$

Given, $(\text{KE})_1 = 1.5 - 0.5 = 1$ eV

$(\text{KE})_2 = 2.5 - 0.5 = 1.5$ eV

$\because \quad \frac{(\text{KE})_1}{(\text{KE})_2} = \frac{1}{1.5} = \frac{2}{3}$

$$\Rightarrow \quad \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{2}{3}}$$

$\therefore \quad v_1 : v_2 = \sqrt{2} : \sqrt{3}$

So, the ratio of maximum speeds of emitted electrons is $\sqrt{2}:\sqrt{3}$.

27. ***(d)*** Initial momentum, $p_i = mv$

$$p_i = m\sqrt{2gh} \qquad [\because v = \sqrt{2gh}]$$

New height of body, $h_1 = h + 3h = 4h$

Now, final momentum,

$$p_f = m\sqrt{2g.h_1} = m\sqrt{2g(4h)}$$
$$= m \cdot 2\sqrt{2gh} = 2p_i$$

$\therefore$ % change in momentum $= \frac{p_f - p_i}{p_i} \times 100$

$$= \frac{2p_i - p_i}{p_i} \times 100 = 100\%$$

28. ***(a)*** In this case, the diode is forward biased. Potential drop in a germanium diode in forward bias is around 0.3 V.

Hence, current through the given circuit,

$$I = \frac{V_{\text{net}}}{R_{\text{eq}}} = \frac{5 - 0.3}{2 + 3} = \frac{4.7}{5}$$
$$= 0.94 \text{ A}$$

29. ***(b)*** Number of active nuclei after time t in a sample of radioactive substance is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = initial number of nuclei at $t = 0$

and λ = decay constant.

Here, at $t = 0$, number of nuclei in sample X and Y are equal,

i.e., $(N_0)_X = (N_0)_Y = N_0$

Also, $\lambda_X = 4\lambda$ and $\lambda_Y = \lambda$

So, after time t, number of active nuclei in X and Y are

$N_X = N_0 e^{-4\lambda t}$ and $N_Y = N_0 e^{-\lambda t}$

$$\therefore \quad \frac{N_X}{N_Y} = e^{-3\lambda t}$$

$$\Rightarrow \quad \frac{1}{e^3} = \frac{1}{e^{3\lambda t}} \qquad \left[\text{Given, } \frac{N_X}{N_Y} = \frac{1}{e^3}\right]$$

$$\Rightarrow \quad 3\lambda t = 3 \quad \Rightarrow t = \frac{1}{2\lambda}$$

30. ***(c)*** Given, current in the wire, $I = 10\text{A}$

Radius of the wire, $R = 20\pi \text{ cm} = 20\pi \times 10^{-2}$ m

Magnetic field produced by semi-circular current carrying thin wire at centre O,

$$B = \frac{\mu_0 I}{4R}$$

Substituting all the given values in above equation, we get

$$B = \frac{4\pi \times 10^{-7} \times 10}{4 \times 20\pi \times 10^{-2}}$$

$$= 5 \times 10^{-6} \text{ T}$$

Now, magnitude of force per unit length,

$$F = IB = 10 \times 5 \times 10^{-6}$$

$$= 50 \times 10^{-6} \text{ N/m}$$

$$= 50\,\mu\text{N/m}$$

31. ***(b)*** Given, area, $A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$

Number of turns per unit length, $n = 4000$ turns/m

Magnetic field, $B = \pi T$

Current, $i = 5$ A

Now, magnetic intensity,

$$H = ni = 4000 \times 5 = 2 \times 10^4 \text{ Am}^{-1}$$

As, $B = \mu_0 (H + I)$ [I = intensity of magnetisation]

Substituting all the values in above equation, we get

$$\pi = 4\pi \times 10^{-7} \{(2 \times 10^4) + I\}$$

$$\frac{1}{4} \times 10^7 = (2 \times 10^4) + I$$

$$250 \times 10^4 = 2 \times 10^4 + I$$

$$I = 250 \times 10^4 - 2 \times 10^4$$

$$\Rightarrow \quad I = 248 \times 10^4 \text{ A/m}$$

Now, pole strength, $m = I \times A = 248 \times 10^4 \times 5 \times 10^{-4}$

$$= 1240 \text{ A-m}$$

32. ***(b)*** Given, mass of plane, $M = 10000$ kg

Coefficient of friction, $\mu = 0.2$

Distance, $s = 50$ m

Initial speed, $u = 0$

Final speed, $v = 72 \text{ kmh}^{-1} = 20 \text{ ms}^{-1}$

Let a be the acceleration of the plane.

From third equation of motion,

$$v^2 - u^2 = 2as$$

$$(20)^2 - (0)^2 = 2 \times a \times 50$$

$$\therefore \quad a = \frac{20 \times 20}{100} = 4 \text{ ms}^{-2}$$

Force required to provide necessary acceleration to the plane,

$$F_1 = ma = 10000 \times 4 = 4 \times 10^4 \text{ N}$$

Force required to overcome friction by engine,

$$F_2 = \mu R = \mu mg = 0.2 \times 10000 \times 9.8$$

$$= 1.96 \times 10^4 \text{ N}$$

Therefore, the minimum force required by the engine,

$$F_1 + F_2 = 4 \times 10^4 + 1.96 \times 10^4$$

$$= 5.96 \times 10^4 \text{ N}$$

33. ***(c)*** Given, reverse voltage, $E = 120$ V

Change in current, $\Delta i = 0.50 - 0.20 = 0.30\text{A}$

Time, $\Delta t = 0.030 \text{ ms} = 0.030 \times 10^{-3}$ s

In an inductor (choke),

Emf induced, $E_{\text{induced}} = L\frac{di}{dt}$

$\therefore$ Self inductance of choke, $L = \frac{E \times \Delta t}{\Delta i}$

$$= \frac{120 \times 0.030 \times 10^{-3}}{0.30}$$

$$\Rightarrow \quad = 12 \times 10^{-3} \text{ H}$$

34. ***(b)*** Given, linear dimensions increase by a factor of 8.

So, volume will become $(8)^3$ times and area of cross-section will become $(8)^2$ times.

Now, stress $= \frac{\text{weight}}{\text{area}}$

Weight = (volume × density × g) will also become $(8)^3$ times.

$$\therefore \text{ New stress} = \frac{(8)^3 \times w_0}{(8)^2 \times A_0} = 8\left(\frac{w_0}{A_0}\right)$$

Hence, the stress increases by a factor of 8.

35. ***(c)*** Let, γ be the adiabatic exponent for the mixture.

For monoatomic gas, $n_1 = 2$ and $\gamma_1 = 5/3$

For diatomic gas, $n_2 = 3$ and $\gamma_2 = 7/5$

For the mixture, $\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

Substituting all the given values in above equation, we get

$$\frac{2+3}{\gamma - 1} = \frac{2}{\frac{5}{3} - 1} + \frac{3}{\frac{7}{5} - 1}$$

$$\Rightarrow \quad \frac{5}{\gamma - 1} = \frac{21}{2} \quad \Rightarrow \gamma = \frac{31}{21}$$

36. ***(c)*** Let a be the acceleration of the lift.
Given, the ratio of time periods of pendulum, while the lift is moving upwards and downwards,

$$T_1 : T_2 = 1 : 3$$

When lift is moving upwards, then total time period,

$$T_1 = 2\pi\sqrt{\frac{l}{g+a}} \quad \ldots(i)$$

When lift is moving downwards, then total time period,

$$T_2 = 2\pi\sqrt{\frac{l}{g-a}} \quad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{T_1}{T_2} = \sqrt{\frac{g-a}{g+a}}$$

$$\Rightarrow \quad \frac{1}{3} = \sqrt{\frac{g-a}{g+a}}$$

or $$\frac{g-a}{g+a} = \left(\frac{1}{3}\right)^2$$

or $$\frac{g-a}{g+a} = \frac{1}{9}$$

$$\Rightarrow \quad 9g - 9a = g + a$$

$$\Rightarrow \quad 9g - g = 9a + a \Rightarrow 10a = 8g$$

$$\Rightarrow \quad a = \frac{8g}{10} = \frac{8\times 10}{10} = 8\ \text{m/s}^2$$

37. ***(a)*** As, orbital velocity of a satellite, $v = \sqrt{\frac{GM}{r}}$

Hence, $$v \propto \frac{1}{\sqrt{r}}$$

% increase in speed $= \frac{1}{2}$ (% decrease in radius)

$$= \frac{1}{2}(2\%) = 1\%$$

Thus, the speed of satellite will increase by 1% .

38. ***(a)*** Given, $\omega = 2000\ \text{rad s}^{-1}$

$$L = 5\ \text{mH} = 5\times 10^{-3}\ \text{H}$$

$$C = 50\mu\text{F} = 50\times 10^{-6}\ \text{F}$$

$$V = 30\ \text{V}$$

Inductive reactance,

$$X_L = \omega L = 2000\times 5\times 10^{-3} = 10\ \Omega$$

and capactive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{50\times 10^{-6}\times 2000} = \frac{100}{10} = 10\ \Omega$$

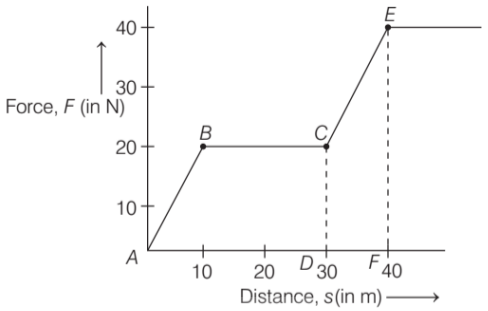
$$\Rightarrow \quad X_L = X_C$$

The given circuit is resonant circuit.
Hence, impedance of circuit, $Z = R$

$$= 4 + 6 + 0.5 = 10.5\ \Omega$$

Amplitude of current, $I_0 = \frac{V}{Z} = \frac{30}{10.5} = 2.85\ \text{A}$

39. ***(d)*** Work done, W = area under F-s graph



$\therefore$ Total work done is covering an initial distance of 40 m,

W = area of trapezium $ABCD$ + area of trapezium $CEFD$

$$= \frac{1}{2}\times 20(20+30) + \frac{1}{2}\times 10\times(20+40) = 800\ \text{J}$$

[$\because$ Area of trapezium $= \frac{1}{2}\times$ height $\times$ sum of parallel sides]

$$\Rightarrow \quad W = 800\ \text{J}$$

40. ***(b)*** Given, potential gradient $= 0.4\ \text{mV cm}^{-1}$

$$= 0.4\times 10^{-3}\ \text{V cm}^{-1}$$

Length of wire, $l = 10\ \text{m} = 1000\ \text{cm}$

Electromagnetic force of cell, $E = 4\ \text{V}$

Potential gradient along wire

$$= \frac{\text{Potential difference along wire}}{\text{Length of wire}}$$

$$\Rightarrow \quad 0.4\times 10^{-3} = \frac{i\times 40}{1000}$$

$\therefore$ Current in wire, $i = \frac{4}{400}\ \text{A} \quad \ldots(i)$

Let R be the required resistance unplugged in the box.

$\therefore$ $$i = \frac{E}{R+R'} = \frac{4}{40+R} \quad \ldots(ii)$$

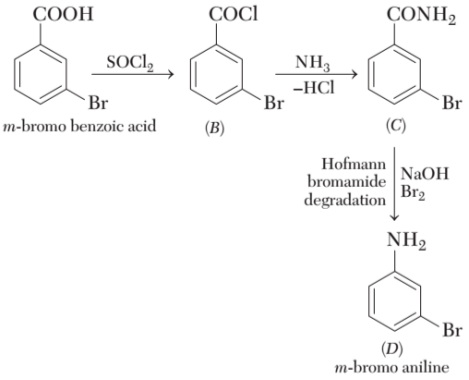
Equating Eqs. (i) and (ii), we get

$$\frac{4}{40+R} = \frac{4}{400}$$

$$\Rightarrow \quad R = 400 - 40 = 360\ \Omega$$

Chemistry

41. ***(d)*** The given reactions take place as follows



42. ***(b)*** From molarity equation, $M_1V_1 = M_2V_2$
(Acid, Base)

$$M_1 \times 50.0 = 0.10 \times 39.30$$

$$\therefore \quad M_1 = \frac{0.10 \times 39.30}{50}$$

$$= 0.0786 \text{ M}$$

$$\approx 0.079 \text{ M}$$

When half equivalence point is reached,

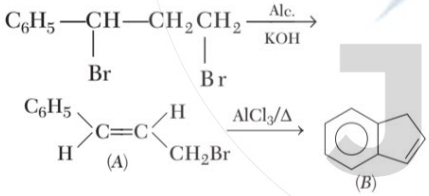
$$[HA] = [A^-]$$

Using Handerson-Hasselbalch equation

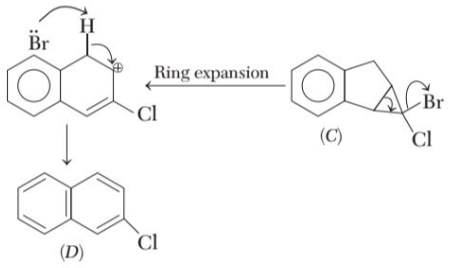
$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

$$\therefore \quad pH = pK_a = 4.85$$

43. ***(d)*** The complete equation is given as follows:



:CuBr compound gives :CClBr. This reaction is addition of carbene to double bond.



44. ***(a)*** Statement (a) is incorrect because α-D-fructose and β-D-fructose are anomers, diastereomers and geometrical isomers but not enantiomers.

Rest of the given statements are correct.

45. ***(d)*** Option (d) does not contain a pair of mixed oxides. Fe_2O_3 is not a mixed oxide.

While, Mn_3O_4 ($2MnO + MnO_2$); Co_3O_4 ($CoO + Co_2O_3$); Fe_3O_4 ($Fe_2O_3 + FeO$) and Pb_3O_4 ($2PbO + PbO_2$), all are mixed salts.

46. ***(a)*** According to Faraday's first law of electrolysis:

The reaction at cathode,

$$\underset{63.5\text{ g}}{Cu^{2+}} + 2e^- \longrightarrow \underset{2 \times 96500\text{ C}}{Cu}$$

The quantity of charge passed

$$= I \times t = (10 \text{ amp}) \times (60 \times 60\text{s}) = 36000 \text{ C}.$$

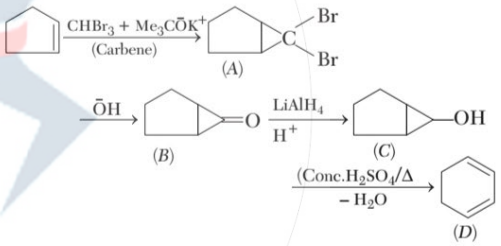
2×96500 C of charge deposit copper = 63.5 g

36000 C of charge deposit copper

$$= \frac{(63.5 \text{ g})}{(2 \times 96500 \text{ C})} \times (36000\text{C}) = 11.84 \text{ g}$$

Thus, 11.84 g of copper will dissolve from the anode and the same amount from the solution will get deposited on the cathode. The concentration of the solution will remain unchanged.

47. ***(b)*** The complete reaction is given as follows:



48. ***(a)*** Statement (a) is correct. Rest of the all statements are incorrect. These are explained as follows:

(a) $Cl_2(g) \longrightarrow 2Cl(g)$

As randomness is increasing. Therefore, ΔS is positive for this reaction.

∴ Statement is correct.

(b) In closed container, $\Delta V = 0$, hence work done is zero. There is no heat exchange as system is adiabatic. Hence, $\Delta E = q + W = 0$.

∴ Statement is incorrect.

(c) ΔG will be zero only when equilibrium is reached.

∴ Statement is incorrect.

(d) $\Delta G° = -RT \ln K_{eq}$, not a function of pressure. Thus, this statement is also incorrect.

49. ***(b)*** Statements I and III are correct and statement II is incorrect.

$$\text{(I)} \quad \frac{T_c}{p_c} = \frac{8a}{27Rb} \cdot \frac{27b^2}{a} = \frac{8b}{R}$$

Thus, $\frac{T_c}{p_c} \propto b$

Thus, larger the T_c / p_c value, larger would be b i.e. included volume.

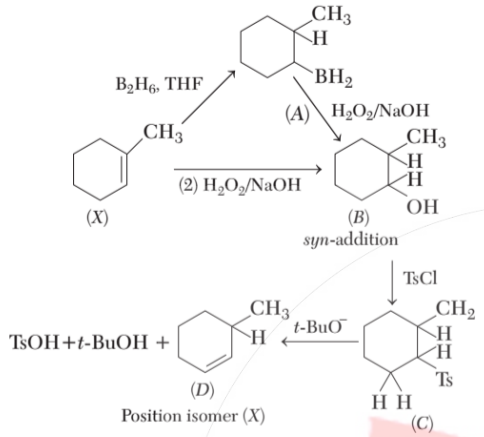
(II) $T_C = \frac{8a}{27Rb}$ and $T_B = \frac{a}{Rb}$

Thus, $T_B > T_C$

(III) When, T_C is attained.

$\therefore \quad (\partial p / \partial V)_{T_C} = 0$

50. ***(a)*** The reaction can be completed as follows:



51. ***(d)*** The reaction can be completed as follows

$$Na_2B_4O_7 + \text{conc.}H_2SO_4 + 5H_2O \longrightarrow Na_2SO_4 + \underset{(A)}{4H_3BO_3}$$

$$H_3BO_3 + 3C_2H_5OH \longrightarrow \underset{(B)}{B(OC_2H_5)_3}$$

Volatile

(Burn with green edged flame)

52. ***(a)*** Perhydrol means 30% solution of H_2O_2.

H_2O_2 decomposes as :

$$2H_2O_2 \longrightarrow 2H_2O + O_2$$

Volume strength of 30% H_2O_2 solution is 100 that means 1 mL of this solution on decomposition gives 100 mL oxygen.

$$SO_2 + \frac{1}{2}O_2 \longrightarrow SO_3$$

1 L 1/2 L 1 L

2 L 1 L 2 L

Since, 100 mL of oxygen is obtained by 1 mL of H_2O_2.

$\therefore$ 1000 mL of oxygen will be obtained by

$= \frac{1}{100} \times 1000$ mL of H_2O_2

$= 10$ mL of H_2O_2

53. ***(c)*** At top of a mountain,

$T_1 = 0 + 273\text{K} = 273\text{K}, p_1 = \frac{710}{760}$ atm and density $= \rho_1$

At bottom of mountain,

$T_2 = 30 + 273 = 303$ K , $p_2 = 1$ atm and density $= \rho_2$

From ideal gas equation,

$\because \quad pV = \frac{w}{M}RT \qquad \left[p = \frac{w}{MV}RT; \rho = \frac{w}{V}\right]$

or $\quad p = \rho\frac{RT}{M}$

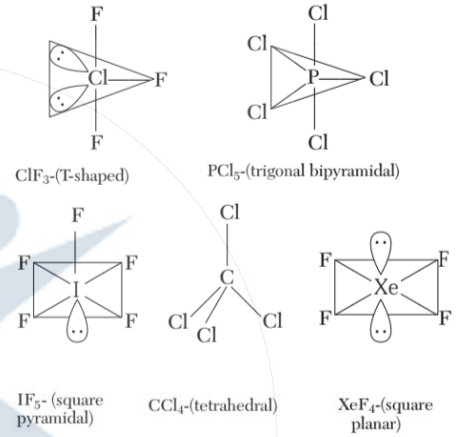
$\therefore \quad \frac{\rho_1}{\rho_2} = \frac{p_1}{T_1} \times \frac{T_2}{p_2}$

or $\quad \frac{\rho_1}{\rho_2} = \frac{710/760 \times 303}{273 \times 1} = \frac{1.04}{1}$

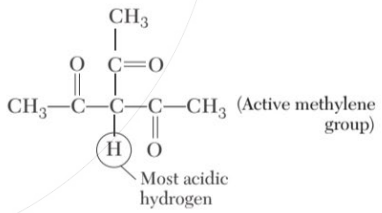
54. ***(b)*** The correct option is (b).

$A \to 5, B \to 3, C \to 4, D \to 2, E \to 1$.

The shapes of molecules are given below.



55. ***(c)*** In option (c), the hydrogen of —CH group is surrounded by three $-\overset{\overset{O}{\|}}{C}-CH_3$ groups as:



Here, O of CO attracts, the electron density towards itself.

Or

The anion formed after removal of acidic proton in $\left(CH_3-\overset{\overset{O}{\|}}{C}-\right)_3$ is stablised by $-R$-effect of three C = O groups.

Thus, (c) has most acidic hydrogen among given compounds.

56. ***(c)*** The process by which the aquatic life gets deprived oxygen and results in subsequent loss of biodiversity is known as eutrophication. It is because of reduction in concentration of the dissolved oxygen in water due to phosphate pollution in water.

57. ***(d)*** As we know,

$$\nu = \frac{1}{\lambda} = \frac{(IE)_H Z^2}{hc} x - \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Given, $n_1 = 2, n_2 = 5$

$$(IE)_H = 13.6 \text{ eV} = 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\nu = \frac{1}{\lambda} = \frac{13.6 \times 1.6 \times 10^{-19} \text{J}}{6.6 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}\left[\frac{1}{2^2} - \frac{1}{5^2}\right]$$

$$= 2.2988 \times 10^6 \text{ m}^{-1}$$

$$\therefore \quad \lambda = \frac{1}{\nu} \Rightarrow \lambda = 4.35 \times 10^{-7} \text{m} = 435 \times 10^{-9} \text{ m}$$

$$= 435 \text{ nm}$$

58. ***(c)*** Heat change at constant pressure means enthalpy change ($\Delta_r H = q_p$).

Heat of formation of CO is written as :

$$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g); \Delta_f H^\ominus = ?$$

Now we have,

$$\text{I. } C(g) + O_2(g) \longrightarrow CO_2(g); \quad \Delta_c H_1^\ominus = -393.5 \text{ kJ/mol}$$

$$\text{II. } CO(g) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g); \quad \Delta_c H_2^\ominus = -283 \text{ kJ/mol}$$

Substracting appropriately (i.e., I–II), we get,

$$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g); \ \Delta_f H^\ominus$$

From Hess's law, $\Delta_f H^\ominus = \Delta_c H_1^\ominus - \Delta_c H_2^\ominus$

$$\Rightarrow \Delta_f H^\ominus = -393.5 - (-283) = 110.5 \text{ kJ/mol}$$

Now, calculation of the heat of formation at constant volume means that we have to calculate change in internal energy (i.e. $\Delta_r U$).

Using, $\Delta_r H = \Delta_r U + p\Delta V$ [For a chemical reaction]

$= \Delta_r U + \Delta n_g RT$ [As $pV = nRT$]

[Δn_g = gaseous moles of products – gaseous moles of reactant]

$$\Rightarrow \quad \Delta_r U = \Delta_r H - \Delta n_g RT$$

Now putting the values : $\Delta n_g = 1 - \frac{1}{2} = \frac{1}{2}, T = 298 \text{ K},$

$R = 8.314$ J/K mol and $\Delta_f H^\ominus = -110.5$ kJ/mol

$$\Rightarrow \quad \Delta_f U^\ominus = -110.5 - \frac{1}{2} \times 8.314 \times 2.98 \times 10^{-3}$$

$$= -111.7 \text{ kJ/mol}$$

59. ***(b)*** If $n = n$, then $l = 0, 1, 2 \ldots n$.

n	l	Sub-orbit	(n + l)
1	0	1*s*	1
	1	1*p*	2
	0	2*s*	2
2	1	2*p*	3
	2	2*d*	4
	0	3*s*	3
3	1	3*p*	4
	2	3*d*	5
	3	3*f*	6

Thus, energy order is,

$1s < 1p < 2s < 2p < 3s < 2d < 3p$

Sc (21) will have electronic configuration as :

$$1s^2\, 1p^6\, 2s^2 2p^6\, 3s^2\, 2d^3$$

60. ***(c)***

	A	+	$2B$	$\rightleftharpoons$	$2C$	+	D	
Initial	1.1		2.2		0		0	
At equili.	1.1–x		2.2 – 2x		2x		x	[2x = 0.2]
	1.1 – 0.1		2.2–0.2		0.2		0.1	[x = 0.1]
	1		2		0.2		0.1	

$$K_C = \frac{[C^2][D]}{[A][B^2]} = \frac{(0.2)^2\,(0.1)}{(1)(2)^2} = 0.001$$

61. ***(c)*** Chemical reaction given in option 'c' is correct. Rest of the given chemical reactions are incorrect.

These can be corrected as:

(a) $CaCN_2 + 3H_2O \longrightarrow CaCO_3 + 2NH_3$

(b) $2NH_3 + CaSO_4 + CO_2 + H_2O \longrightarrow (NH_4)_2SO_4 + CaCO_3$

(d) $CaC_2 + H_2O \longrightarrow 2C_2H_2 + Ca(OH)_2$

62. ***(d)*** Disproportionation means a reaction in which a substance is oxidised as well as reduced simultaneously,

$Na_2S_2O_3 + H_2SO_4 \longrightarrow Na_2SO_4 + SO_2 + S_8 + H_2O$

Oxidation number of S in $Na_2S_2O_3 = +2$

Oxidation number of S in $H_2SO_4 = +6$

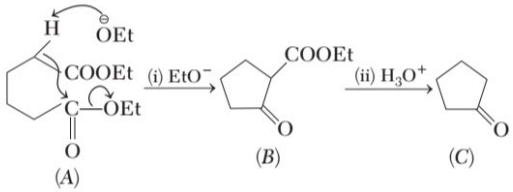
Oxidation number of S in $SO_2 = +4$

Oxidation number of S in $Na_2SO_4 = +6$

Oxidation number of S in $S_8 = 0$

So, sulphur can oxidise from + 2 to + 4 as well as reduce to 0 (S_8).

63. ***(d)*** Right answer is *d* because after heating (Δ), dcarboxylation occurs.



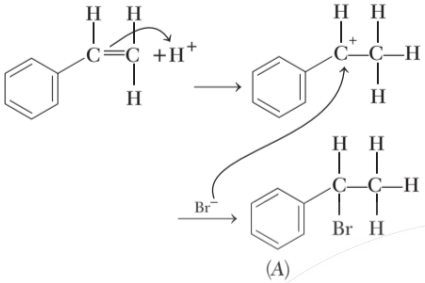
64. ***(a)*** $-\Delta S = \frac{\Delta G}{T}$, also $\frac{\Delta G}{T}$ is slope in Ellingham diagram. ΔS is different for different reactions as they have different slopes.

Rest of the statements (b), (c) and (d) are correct about Ellingham diagram.

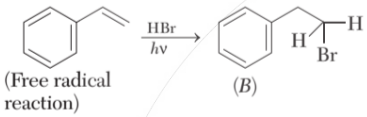
65. ***(c)*** Phenolphthalein indicator changes into pink colour due to change in pH and not due to adsorption.

66. ***(c)*** The reaction take place as follows

$$HBr \longrightarrow H^+ + Br^-$$



(A)



(Free radical reaction) (B)

67. ***(a)*** Z_{eff} for fcc = 4, Z_{eff} for bcc = 2

$$\text{Atomic volume of } \alpha\text{-form} = \frac{(3.68\times10^{-8})^3}{4}\times N_A$$

$$\text{Atomic volume of } \beta\text{-form} = \frac{(2.92\times10^{-8})^3}{2}\times N_A$$

As atomic weight is same, element is same, so the density ratio is

$$\rho_\alpha : \rho_\beta = V_\beta : V_\alpha = \frac{(2.92)^3}{2} : \frac{(3.68)^2}{4}$$

$$= \frac{24.9}{2} : \frac{4}{49.8} = 1:1$$

68. ***(c)*** $\therefore \quad K_b = \frac{RT_b^2 M_1}{\Delta H_{vap}}$

$$= \frac{8.314\times(373)^2\times0.018}{40585 \text{ J/mol}}$$

$$= 0.513 \text{ K kg/mol}$$

Now, molecular weight of $Al_2(SO_4)_3 = 342$

$$Al_2(SO_4)_3 \rightleftharpoons 2Al^{3+} + 3SO_4^{2-}$$

$$1-\alpha \qquad 2\alpha \qquad 3\alpha$$

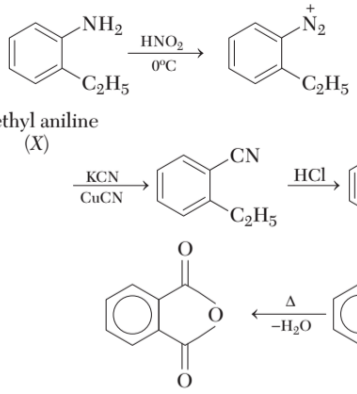
$$i = 1-\alpha+2\alpha+3\alpha = 1+4\alpha \Rightarrow i = 5 \qquad (\because \alpha = 1)$$

$$\Delta T_b = i\times K_b.m$$

$$= 5\times0.513\times\frac{5.6}{342\times1} = 0.042$$

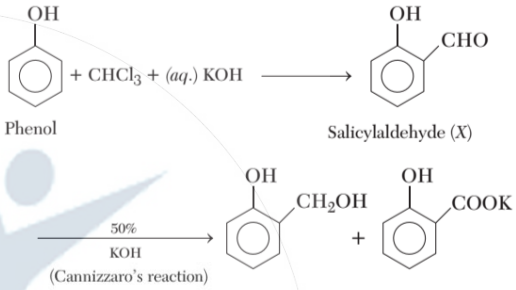
Boiling point of solution = 100 + 0.042 = 100.042°C

69. ***(d)*** The reaction take place as follows



o-ethyl aniline (X)

70. ***(b)*** Phenol on reaction with chloroform and KOH gives salicylaldehyde, which with 50% KOH solution undergoes Cannizzaro's reaction.



Phenol Salicylaldehyde (X)

(Cannizzaro's reaction)

71. ***(d)*** Some complex ions of nitrogen show similarities with halide ions. These are called pseudohalide ions e.g., OCN^-, NCO^-, CN^- etc.

Halogens, among themselves, form complex ions which are called polyhalide ions. e.g., I_3^-, BrI_2^- etc.

Similarly, interhalogens are the compound of halogens in which one halogen is cation and other halogen is anion. e.g., IF_7, ICl_5, BrF_5, IF_5 etc.

$\therefore$ The correct order of pseudohalide polyhalide and interhalogen is

$$OCN^-,\ BrI_2^-,\ IF_5$$

72. ***(b)*** The inorganic halide (A) is SO_2Cl_2.

$$\underset{(A)}{SO_2Cl_2} + 2H_2O \longrightarrow \underset{(B)}{H_2SO_4} + \underset{(C)}{2HCl}$$

$$\underset{(A)}{SO_2Cl_2} + 4NaOH \longrightarrow \underset{(D)}{Na_2SO_4} + \underset{(E)}{2NaCl} + 2H_2O$$

$$\underset{(D)}{Na_2SO_4} + BaCl_2 \longrightarrow \underset{\text{White ppt.}}{BaSO_4} + 2NaCl$$

$$\underset{(E)}{NaCl} + AgNO_3 \longrightarrow \underset{\text{White ppt.}}{AgCl} + NaNO_3$$

73. ***(d)*** In alkali metals, reactivity increases down the group as electropositivity increases but for halogens reactivity decreases down the group as molecular stability of halogens increases.

74. ***(b)*** IUPAC name of sodiumnitroprusside $Na_2[Fe(CN)_5NO]$ is sodium pentacyanonitrosyl ferrate (III) sodium pentacyanonitrosyl ferrate (III) because in it NO is neutral ligand and the oxidation number of Fe is III.

75. ***(d)***

	$A(g) \longrightarrow$	$P(g)$	$+ Q(g)$	$+ R(g)$
At $t = 0$,	0.4 atm	0	0	0
At time $= t$,	$0.4 - x$	x	x	x

$$P_t = 0.4 - x + x + x + x = 0.4 + 2x \text{ or } x = \frac{P_t - 0.4}{2}$$

$$P_A = P_0 - x = P_0 - \frac{P_t - 0.4}{2} = \frac{2 \times 0.4 - P_t + 0.4}{2}$$

$$= \frac{1.2 - P_t}{2}$$

For a first order reaction,

$$k = \frac{2.303}{t} \log \frac{P_0}{P_A}$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{230} \log\left(\frac{0.4 \times 2}{1.2 - P_t}\right)$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{230} \log\left(\frac{0.8}{1.2 - P_t}\right)$$

$$\log \frac{0.8}{1.2 - P_t} = \frac{0.693 \times 230}{69.3 \times 2.303} = 0.9987$$

$$\frac{0.8}{1.2 - P_t} = 10^{0.9987}$$

$$0.8 = (10)(1.2 - P_t)$$

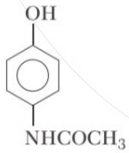
$$0.8 = 12 - 10P_t$$

$$10P_t = 12 - 0.8$$

$$10P_t = 11.2$$

$$P_t = \frac{11.2}{10} = 1.12 \text{ atm}$$

76. ***(d)*** The structure of an important antipyretic paracetamol is



So, *p*-amino phenol gives paracetamol on actylation.

77. ***(b)*** On passing H_2S gas in the aqueous solution of salt in the presence of dilute HCl, black ppt. of CuS is formed.

On boiling CuS(ppt.) with dil. HNO_3, $Cu(NO_3)_2$ (blue colour) is formed. Which gives deep blue solution of $[Cu(NH_3)_4]^{2+}$.

$$3CuS + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + 2NO + 3S + 4H_2O$$

$$Cu^{2+} + 4NH_4OH \longrightarrow \underset{\text{Deep blue complex}}{[Cu(NH_3)_4]^{2+}} + 4H_2O$$

78. ***(a)*** $5C_2O_4^{2-} + 2MnO_4^- + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

Note In a reaction (redox/neutralisation) number of equivalents of different reactants are same.

Equivalents of $C_2O_4^{2-}$ = Equivalents of MnO_4^-

$$\text{Equivalent weight of } C_2O_4^{2-} = \frac{\text{Molecular weight}}{2}$$

$\therefore$ Molarity $= 2 \times$ Normality

$$\text{Equivalent weight of } MnO_4^- = \frac{\text{Molecular weight}}{5}$$

Molarity $= 5 \times$ Normality

$$\text{Number of equivalents of } MnO_4^- = \frac{28.85 \times 5\ M_1}{1000}$$

$$\text{Number of equivalents of } C_2O_4^{2-} = \frac{0.1467}{67}$$

$$\therefore \quad \frac{28.85 \times 5\ M_1}{1000} = \frac{0.1467}{67}$$

$$M_1 = 0.01518 \text{ M}$$

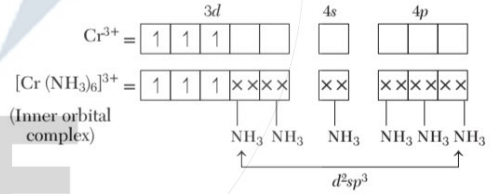
79. ***(c)*** $\underset{\text{Coordination sphere}}{[Cr(NH_3)_6]} \underset{\text{Ionisable}}{Cl_3} \rightleftharpoons [Cr(NH_3)_6]^{3+} + 3Cl^-$

$$[Cr(NH_3)_6]Cl_3 \xrightarrow{AgNO_3} \underset{\text{(White ppt.)}}{AgCl\downarrow}$$

Configuration of Cr (24) $= 3d^5\ 4s^1$

Configuration of $Cr^{3+} = 3d^3 4s^0$

$\therefore$ It is paramagnetic nature due to 3 unpaired electrons.



(a) d^2sp^3 hybridisation, octahedral. Thus, option (a) is correct.

(b) Here are three unpaired electrons, hence it is paramagnetic. Thus, option (b) is correct.

(c) d^2sp^3 inner orbital complex. Thus option (c) is incorrect.

(d) Due to ionisable Cl^- ions, white ppt. with $AgNO_3$. Thus, option (d) is correct.

So, the incorrect statement is (c).

80. ***(a)*** $H_2SO_4(aq) \longrightarrow H^+(aq) + HSO_4^-(aq)$;

$HSO_4^-(aq) \longrightarrow H^+(aq) + SO_4^-(aq)$

Here, $K_{a_2} < K_{a_1}$ because the negatively charged HSO_4^- ion has much less tendency to donate a proton to H_2O as compared to neutral H_2SO_4.

a. English Proficiency

81. ***(b)*** Article 'the' should be used before 'famous' as the sentence refers to a particular thing i.e. a famous monument.

82. ***(b)*** As the sentence refers to a choice among more than two persons (servants), 'which' will be used in place of 'who'.

83. ***(c)*** The sentence is incomplete as it does not answer the question 'helped whom?', So we add 'him' at the end of the sentence.

84. ***(a)*** Use of preposition 'with' is suitable to fill the given blank.

85. ***(b)*** Use of preposition 'at' is suitable to fill the given blank.

86. ***(c)*** 'Benign' means not likely to cause death. 'Fatal' would be its opposite.

87. ***(b)*** 'Noted' is known by many people because of particular qualities. Its opposite would be 'Unknown'.

88. ***(a)*** 'Sagacious' means having or showing understanding and the ability to make good judgements. So, 'Foolish' would be its correct antonym.

89. ***(a)*** 'Getting peevish' means to get irritated.

90. ***(c)*** 'Roughed out' is a phrasal verb which means to draw a 'rough draft'. So, 'drew a quick plan' is its correct synonym.

91. ***(b)*** The need for an effective population policy is an urgent necessity in the country's planning strategy. Hence, option (b) is the correct answer.

92. ***(d)*** The development of human resource and the building up of an institutional framework would have to receive priority attention.

93. ***(c)*** The Centre and the States government must become partners in the planning process to determine national priorities together. Hence, the statement given in option (c) is not correct.

94. ***(b)*** Domestic economic situation and world trends would force the planning process to undergo a change.

95. ***(c)*** Important changes in the international scene is implied by the expression 'momentous trends.'

b. Logical Reasoning

96. ***(d)*** The pattern is as follows

$2 \times 6 = 12$

$6 \times 12 = 72$

$12 \times 72 = 864 \neq \boxed{865}$

$72 \times 864 = 62208$

Hence, number 865 is wrong and should be replaced by 864.

97. ***(c)*** According to the question,

$$A / Q > T > P / B$$

But P is not the shortest

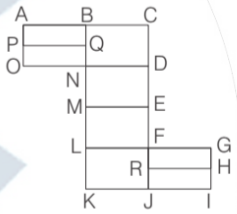
$$A / Q > T > P > B$$

So, tallest is either A or Q.

98. ***(c)*** Option figure (c) will complete the given pattern as

99. ***(d)*** 'Spanner' is used by 'mechanic' as a tool to loose or tight nut and bolt. In the same way, 'saw' is used by 'carpenter' as a tool to cut wood. Hence, option (d) is correct.

100. ***(b)***



Total 18 rectangles are as follow

ABQP,	PQNO,	BCDN,
NDEM,	MEFL,	LFJK,
FGHR,	RHIJ,	ABNO,
BCEM,	NDFL,	MEJK,
FGIJ,	ACDO,	BCFL,
NDJK,	LGIK and	BCJK

101. ***(d)*** As, $B \xrightarrow{-1} A \xrightarrow{+3} D \xrightarrow{-1} C$

$J \xrightarrow{-1} I \xrightarrow{+3} L \xrightarrow{-1} K$

$N \xrightarrow{-1} M \xrightarrow{+3} P \xrightarrow{-1} O$

But, $\boxed{V \xrightarrow{-1} U \xrightarrow{+2} W \xrightarrow{+1} X}$

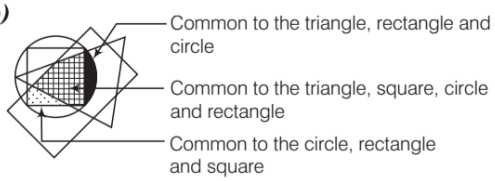
Except option (d), all others follow same pattern $(-1, +3, -1)$.

102. ***(d)*** Clearly, there are three types of shading of circles, one is unshaded another has its right half shaded and yet another has its upper half shaded. Also, two of the figures in each row have one triangle shaded.

103. ***(c)*** Option figure (c) can be formed from the cut out pieces given in question figure.

104. ***(b)*** After unfolding the folded and cut piece of paper it will look like as answer figure (b).

105. ***(b)***



Mathematics

106. ***(a)*** Given that, $A = \{x : x^2 = 1\} = \{-1, 1\}$ and $B = \{x : x^4 = 1\} = \{-1, 1, -i, i\}$

Now, $A - B = \phi$ and $B - A = \{-i, i\}$

$\therefore \quad A \Delta B = (A - B) \cup (B - A) = \{-i, i\}$

107. ***(c)*** We have, $2f(xy) = (f(x))^y + (f(y))^x$, $x, y \in R$ and $f(1) = a$.

Putting $x = 1, y = 2$, we get

$2f(2) = (f(1))^2 + f(2) \Rightarrow f(2) = a^2$

Putting $x = 1, y = 3$, we get

$2f(3) = (f(1))^3 + f(3) \Rightarrow f(3) = a^3$, and so on.

$\therefore \quad f(n) = a^n$

$$\sum_{i=1}^{n} f(i) = a + a^2 + .. + a^n = \frac{a(a^n - 1)}{a - 1}.$$

108. ***(c)*** The given inequality is equivalent to

$$1 < |x - 3| + |4 - x|$$

i.e., $$1 < |x - 3| + |x - 4| \qquad ...(i)$$

For $x < 3$, we have

$$3 - x + 4 - x = 7 - 2x > 1$$

i.e., $x < 3$, which is true in the domain.

For $3 \le x < 4$, Eq. (i) gives

$$x - 3 + 4 - x = 1 \not> 1 \qquad ...(ii)$$

For $4 \le x$, we have

$$2x - 7 > 1 \Rightarrow x > 4 \qquad ...(iii)$$

From Eqs. (i) and (iii), we have the solution of the inequality as $x < 3$ or $x > 4$.

i.e., $x \in]-\infty, 3) \cup (4, \infty[$ which is equal to $R - [3, 4]$.

109. ***(b)*** $\because$ $13, a_1, a_2, ..., a_{20}, 67$ are in AP.

$$\therefore \quad a_1 + a_2 + a_3 + \ldots + a_{20} = 20\left(\frac{13 + 67}{2}\right) = 800$$

Also, AM > GM

$$\Rightarrow \quad \frac{a_1 + a_2 + \ldots + a_{20}}{20} \ge (a_1 \cdot a_2 \cdot a_3 \cdot ..... \cdot a_{20})^{\frac{1}{20}}$$

$$\Rightarrow \quad 40 \ge (a_1 \cdot a_2 \cdot a_3 \ldots a_{20})^{\frac{1}{20}}$$

Hence, maximum value of $a_1 \cdot a_2 \cdot a_3 \ldots a_{20}$ is $(40)^{20}$.

110. ***(a)*** Since, p, q and r are positive and are in AP.

$$\therefore \quad q = \frac{p + r}{2} \qquad ...(i)$$

$\because$ The roots of the equation, $px^2 + qx + r = 0$ are real.

$$\Rightarrow \quad q^2 \ge 4pr \Rightarrow \left[\frac{p + r}{2}\right]^2 \ge 4pr \qquad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad p^2 + r^2 - 14pr \ge 0$$

$$\Rightarrow \quad \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \ge 0 \ (\because p > 0)$$

$$\Rightarrow \quad \left(\frac{r}{p} - 7\right)^2 - 48 \ge 0$$

$$\Rightarrow \quad \left(\frac{r}{p} - 7\right)^2 - (4\sqrt{3})^2 \ge 0 \Rightarrow \left|\frac{r}{p} - 7\right| \ge 4\sqrt{3}$$

111. ***(c)*** ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 = {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + ({}^{47}C_3 + {}^{47}C_4) = {}^{52}C_4$

112. ***(a)*** Possible even digits are 2,4,6,8,0.

Case **I** Number has digits 4,6,8,0.

(Here, sum of digits is 18, divisible by 3)

Number of arrangements $= 3 \times 3!$

[1st place can be filled using 4, 6, 8]

$= 3 \times 6 = 18$

Case **II** Number has digits 2, 4, 6, 0

(Here, sum of digits is 12, divisible by 3)

1st place cannot be filled by 0.

Number of arrangements $= 3 \times 3! = 18$

$\therefore$ Number of numbers $= 18 + 18 = 36$

113. ***(d)*** We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore \quad P = 1500 - n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 1500 - P$

Clearly, $1 \le n(A \cap B) \le 500$

$\Rightarrow \quad 1 \le 1500 - P \le 500$

$\Rightarrow \quad -1499 \le -P \le -1000$

$\Rightarrow \quad 1000 \le P \le 1499$

114. ***(c)*** Here, $\frac{2x-1}{x^3+4x^2+3x} \in R$ only when

$x^3 + 4x^2 + 3x \ne 0$

$\because \quad x(x^2 + 4x + 3) = 0$

$\Rightarrow \quad x(x+1)(x+3) = 0$

$\Rightarrow \quad x = 0, -1, -3$

$\therefore \quad \left\{x \in R : \frac{2x-1}{x^3+4x^2+3x} \in R\right\}$

$= R - \{0, -1, -3\}$

115. ***(a)*** Let $f(x) = Ax^2 + Bx + C$

$\therefore \quad f(1) = A + B + C$

and $\quad f(-1) = A - B + C$

$\because \quad f(1) = f(-1)$

$\Rightarrow \quad A + B + C = A - B + C$

$\Rightarrow \quad 2B = 0 \Rightarrow B = 0$

$\therefore \quad f(x) = Ax^2 + C \Rightarrow f'(x) = 2Ax$

$\Rightarrow \quad f'(a) = 2Aa$

$f'(b) = 2Ab$ and $f'(c) = 2Ac$

Also, a, b, c are in AP

So, $2Aa, 2Ab$ and $2Ac$ are in AP.

$\Rightarrow f'(a), f'(b)$ and $f'(c)$ are also in AP.

116. ***(c)*** $\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{n+1} + \frac{2}{n+2} + \ldots + \frac{3n}{n+3n}\right\}$

$= \lim_{n\to\infty} \sum_{r=1}^{3n} \frac{1}{n}\left(\frac{r}{n+r}\right) = \int_0^3 \frac{x}{1+x}\,dx$

$= \int_0^3 \left(1 - \frac{1}{1+x}\right) dx = [x - \log(1+x)]_0^3$

$= 3 - \log 4 = 3 - 2\log 2$

117. ***(b)*** We have,

$(x-1)(x-2)(x-3)\ldots(x-10)$

Coefficient of x^8 = Sum of the terms taken two at a time i.e., $\sum_{1 \le i < j \le 10}^{10} x_i \cdot x_j = \frac{1}{2}\left[\left(\sum_{i=1}^{10} x_i\right)^2 - \left(\sum_{i=1}^{10} x_i^2\right)\right]$

$= \frac{1}{2}[(1 + 2 + 3 + .. + 10)^2 - (1^2 + 2^2 + 3^2 + \ldots + 10^2)]$

$= \frac{1}{2}\left[\left(\frac{10 \times 11}{2}\right)^2 - \left(\frac{10 \times 11 \times 21}{6}\right)\right]$

$= \frac{1}{2}((55)^2 - 385) = \frac{1}{2}(3025 - 385) = \frac{2640}{2} = 1320$

118. ***(b)*** We have, $Z = \frac{7+i}{3+4i}$

$\Rightarrow \quad Z = \left(\frac{7+i}{3+4i}\right)\left(\frac{3-4i}{3-4i}\right)$

$= \frac{21 + 4 - 28i + 3i}{9 + 16} = 1 - i$

$\therefore \quad Z^{14} = (1-i)^{14} = [(1-i)^2]^7 = (1 - 1 - 2i)^7$

$= (-2)^7 (i)^7 = 2^7 i$

119. ***(c)*** We have, $\frac{dy}{dx} + \frac{\tan y}{x} = \frac{\tan y \sin y}{x^2}$

Now, divide by $\tan y \sin y$ both sides, we get

$$\cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{\operatorname{cosec} y}{x} = \frac{1}{x^2} \qquad \ldots(i)$$

Put $\operatorname{cosec} y = z$

$\therefore \quad -\operatorname{cosec} y \cot y \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore$ Eq. (i) reduces to

$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$

$\text{IF} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = 1/x$

Hence solution is

$\frac{z}{x} = \int -\frac{1}{x^3}\,dx = \frac{1}{2x^2} - c$

$\Rightarrow \quad \frac{1}{x \sin y} = \frac{1 - 2cx^2}{2x^2}$

$\Rightarrow \quad 2x = \sin y(1 - 2cx^2)$

120. ***(a)*** We have,

$I = \int_0^{\pi/2} \frac{dx}{\tan x + \cot x + \operatorname{cosec} x + \sec x}$

$\Rightarrow I = \int_0^{\pi/2} \frac{dx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\sin x} + \frac{1}{\cos x}}$

$\Rightarrow I = \int_0^{\pi/2} \frac{\sin x \cos x\,dx}{1 + \sin x + \cos x}$

$\Rightarrow I = \int_0^{\pi/2} \frac{2\sin\frac{x}{2}\cos\frac{x}{2}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)}{2\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}} dx$

$\Rightarrow I = \int_0^{\pi/2} \sin\frac{x}{2}\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) dx$

$\Rightarrow I = \frac{1}{2}\int_0^{\pi/2} (\sin x + \cos x - 1)\,dx$

$\Rightarrow I = \frac{1}{2}[-\cos x + \sin x - x]_0^{\pi/2}$

$$\Rightarrow I=\frac{1}{2}\left[\left(-\cos\frac{\pi}{2}+\sin\frac{\pi}{2}-\frac{\pi}{2}\right)-(-\cos 0+\sin 0-0)\right]$$

$$\Rightarrow I=\frac{1}{2}\left[0+1-\frac{\pi}{2}+1\right]=1-\frac{\pi}{4}$$

121. ***(d)*** We have given,

$$|\mathrm{a}|=1,|\mathrm{b}|=4 \text{ and } \mathrm{a}\cdot\mathrm{b}=2$$

Now, $\mathrm{c}=(2\mathrm{a}\times\mathrm{b})-3\mathrm{b}$

Multiplying by b both sides, we get

$$\mathrm{b}\cdot\mathrm{c}=(2\mathrm{a}\times\mathrm{b})\cdot\mathrm{b}-3|\mathrm{b}|^2$$

$$\Rightarrow \quad \mathrm{b}\cdot\mathrm{c}=0-3(4)^2=-48$$

and $|\mathrm{c}|^2=|(2\mathrm{a}\times\mathrm{b})-3\mathrm{b}|^2$

$$|\mathrm{c}|^2=(2\mathrm{a}\times\mathrm{b})^2+9|\mathrm{b}|^2=4(\mathrm{a}\times\mathrm{b})^2+9(4)^2$$

$$\Rightarrow \quad |\mathrm{c}|^2=4(|\mathrm{a}|^2\cdot|\mathrm{b}|^2-(\mathrm{a}\cdot\mathrm{b})^2)+144$$

$$=4(16-4)+144=48+144=192$$

$$\therefore \quad \mathrm{b}\cdot\mathrm{c}=|\mathrm{b}||\mathrm{c}|\cos\theta$$

$$\Rightarrow \quad \cos\theta=\frac{\mathrm{b}\cdot\mathrm{c}}{|\mathrm{b}||\mathrm{c}|}=\frac{-48}{4\times\sqrt{192}}=-\frac{\sqrt{3}}{2}$$

$$\theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

122. ***(a)*** We have,

$$x^2-(k-2)x+(k^2+3k+5)=0$$

x_1, x_2 are roots of equations

$$\therefore \quad x_1+x_2=k-2, x_1x_2=k^2+3k+5$$

$$x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2$$

$$=(k-2)^2-2(k^2+3k+5)$$

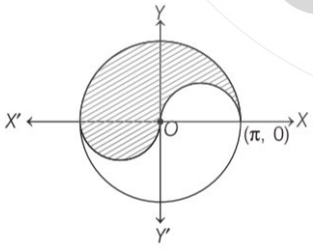
$$=k^2-4k+4-2k^2-6k-10$$

$$=-(k^2+10k+6)=-[(k+5)^2-19]$$

$$=19-(k+5)^2\le 19$$

$\therefore$ Maximum value of $x_1^2+x_2^2=19$

123. ***(c)*** Graph of circle whose centre is origin and radius is π units and $y=\sin x$ is



Area of shaded region $=\frac{1}{2}$ Area of circle

$$=\frac{1}{2}(\pi r^2)=\frac{1}{2}\pi(\pi^2) \quad [\because r=\pi]$$

$$=\pi^3/2$$

124. ***(a)*** The given equation is

$$2(1+i)x^2-4(2-i)x-5-3i=0$$

$$\Rightarrow x=\frac{4(2-i)\pm\sqrt{16(2-i)^2+8(1+i)(5+3i)}}{4(1+i)}$$

$$=-\frac{i}{1+i} \text{ or } \frac{4-i}{1+i}=\frac{-1-i}{2} \text{ or } \frac{3-5i}{2}$$

Now, $\left|\frac{-1-i}{2}\right|=\sqrt{\frac{1}{4}+\frac{1}{4}}=\sqrt{\frac{1}{2}}$

and $\left|\frac{3-5i}{2}\right|=\sqrt{\frac{9}{4}+\frac{25}{4}}=\sqrt{\frac{17}{2}}$

Also, $\sqrt{\frac{17}{2}}>\sqrt{\frac{1}{2}}$

Hence, required root is $\frac{3-5i}{2}$.

125. ***(d)*** For real roots, discriminant,

$$D=b^2-4ac\ge 0$$

$$=\cos^2\beta-4(\cos\beta-1)\sin\beta\ge 0$$

$$=\cos^2\beta+4(1-\cos\beta)\sin\beta\ge 0$$

So, $\sin\beta$ should be >0. $[\because \cos^2\beta\ge 0, 1-\cos\beta\ge 0]$

$\Rightarrow \beta\in(0,\pi)$

126. ***(c)*** We have,

$$(1+\alpha)x+\beta y+z=2$$

$$\alpha x+(1+\beta)y+z=3$$

$$\alpha x+\beta y+2z=2$$

For unique solution $\begin{vmatrix}1+\alpha & \beta & 1\\ \alpha & 1+\beta & 1\\ \alpha & \beta & 2\end{vmatrix}\neq 0$

Apply $R_1\to R_1-R_2$ and $R_2\to R_2-R_3$, we get

$$\begin{vmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ \alpha & \beta & 2\end{vmatrix}\neq 0$$

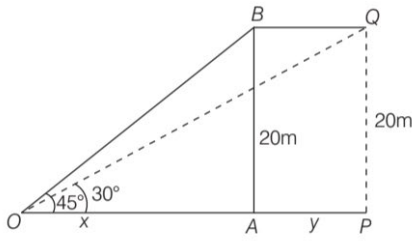
Expanding along R_1, we get

$$1(2+\beta)+1(0+\alpha)\neq 0=\alpha+\beta+2\neq 0$$

Only option (c) satisfied above equation.

127. ***(d)*** Let $AB=h=$ Height of vertical pole

$OA=x=$ Horizontal distance from O to pole



In ΔOAB

$$\tan 45° = \frac{AB}{OA} \Rightarrow 1 = \frac{20}{x}$$

$\Rightarrow \quad x = 20$ m

In ΔOPQ

$$\tan 30° = \frac{PQ}{OP} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{20}{x+y}$$

$\Rightarrow \quad x + y = 20\sqrt{3}$

$\Rightarrow \quad y = 20\sqrt{3} - x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$

$\therefore$ Speed of bird is $20(\sqrt{3} - 1)$ m/s.

128. ***(b)*** Since, $g = \sqrt{ab}$. Also, a, p, q and b are in AP.

So, common difference d is $\frac{b-a}{3}$

$$\therefore \quad p = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$q = b - d = b - \frac{b-a}{3} = \frac{a+2b}{3}$$

Now, $(2p - q)(p - 2q)$

$$= \frac{(4a + 2b - a - 2b)}{3} \cdot \frac{(2a + b - 2a - 4b)}{3}$$

$$= -ab = -g^2$$

129. ***(b)*** Let $f(x) = x^{100}$

$g(x) = x^2 - 3x + 2$

$q(x)$ = quotient

$r(x)$ = remainder $= (ax + b)$

$\therefore \quad f(x) = g(x)\, q(x) + r(x)$

$x^{100} = (x^2 - 3x + 2)\, q(x) + ax + b$

Put $\quad x = 1 \Rightarrow 1 = a + b$ ...(i)

Put $\quad x = 2 \Rightarrow 2^{100} = 2a + b$ ...(ii)

From Eqs. (i) and (ii), we get

$a = 2^{100} - 1, b = 2 - 2^{100}$

Remainder $= (2^{100} - 1)x + (2 - 2^{100})$

$= (2^{99+1} - 1)x - 2(2^{99} - 1)$

$\therefore \quad k = 99$

130. ***(a)*** Let two observations be x and y.

$$\therefore \text{ Mean} = \frac{1 + 3 + 8 + x + y}{5} = \frac{12 + x + y}{5}$$

$$\Rightarrow \quad 5 = \frac{x + y + 12}{5} \Rightarrow x + y = 13 \qquad \ldots(i)$$

$$\text{Now, variance} = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow \quad 9.20 = \frac{1^2 + 3^2 + 8^2 + x^2 + y^2}{5} - 25$$

$\Rightarrow \quad 46 = 74 + x^2 + y^2 - 125$

$\Rightarrow \quad x^2 + y^2 = 97$ ...(ii)

From Eqs. (i) and (ii), we get

$(x + y)^2 - 2xy = 97$

$\Rightarrow \quad (13)^2 - 2xy = 97$

$\Rightarrow \quad 2xy = 169 - 97 = 72$

$\Rightarrow \quad xy = 36$

$\therefore \quad x : y = 4 : 9$

131. ***(b)*** Let the three digit number be abc.

Given, middle digit is AM of first and last

$$\therefore \quad b = \frac{a + c}{2}$$

$\Rightarrow \quad 2b = a + c$

$2b$ is an even.

$\therefore$ a and c should be both even or both odd.

$\therefore$ Total number of ways $= {}^5C_1 \times {}^5C_1 + {}^4C_1 \times {}^5C_1$

$= 5 \times 5 + 4 \times 5 = 25 + 20 = 45$

132. ***(b)*** We have,

$Z = re^{i\theta}$

$Z = r(\cos\theta + i \sin\theta)$

$e^{iz} = e^{ir(\cos\theta + i \sin\theta)}$

$\Rightarrow \quad e^{iz} = e^{ir\cos\theta + i^2 r \sin\theta}$

$\Rightarrow \quad e^{iz} = e^{ir\cos\theta - r\sin\theta}$

$\Rightarrow \quad e^{iz} = e^{-r\sin\theta} \cdot e^{ir\cos\theta}$

$\therefore \quad \arg(e^{iz}) = r\cos\theta$

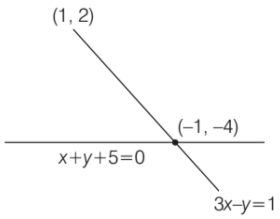
133. ***(d)*** With four 6-sided dics, there are possible combinations that can be rolled. Out of these, only 80 sum to 10 as the table shows,

$6 \times 6 \times 6 \times 6 = 6^4 = 1296$

Combinations	Count
1 – 1 – 2 – 6	$\frac{4!}{2!} = 12$
1 – 1 – 3 – 5	$\frac{4!}{2!} = 12$
1 – 1 – 4 – 4	$\frac{4!}{2!2!} = 6$
1 – 2 – 2 – 5	$\frac{4!}{2!} = 12$
1 – 2 – 3 – 4	$4! = 24$
1 – 3 – 3 – 3	$\frac{4!}{3!} = 4$
2 – 2 – 2 – 4	$\frac{4!}{3!} = 4$
2 – 2 – 3 – 3	$\frac{4!}{2!2!} = 6$
	Total Sum = 80

134. ***(b)*** A line passing through (1, 2) and parallel to the line $3x - y = 10$ is

$$3(x-1)-(y-2)=0$$



$\Rightarrow \quad 3x-3-y+2=0 \quad \Rightarrow \quad 3x-y-1=0$

The intersection point of line

$$x+y+5=0 \qquad \ldots(i)$$

and $\quad 3x-y-1=0 \qquad \ldots(ii)$

are $\quad (-1,-4)$

$\therefore$ Distance from the line $x+y+5$ to the point (1, 2) measured parallel to the line $3x-y=10$ is

$\Rightarrow \quad \sqrt{(-1-1)^2+(-4-2)^2}$

$\Rightarrow \quad \sqrt{4+36}=\sqrt{40}=2\sqrt{10}$

135. ***(d)*** We have,

$$f(x)=a_0+a_1x^2+a_2x^4+a_3x^6+\ldots+a_nx^{2n}$$

$$f'(x)=2a_1x+4a_2x^3+6a_3x^5+\ldots+2na_nx^{2n-1}$$

$$f'(x)=2x(a_1+2a_2x^2+3a_3x^4+\ldots+na_nx^{2n-2})$$

For maxima or minima

Put $f'(x)=0 \Rightarrow x=0$

$$f''(x)=2(a_1+6a_2x^2+15a_3x^4+\ldots+n(2n-1)a_nx^{2n-2})$$

$$(f''(x))_{x=0}=2a_1>0$$

$\therefore$ $f(x)$ has only one minima at $x=0$.

136. ***(c)*** We have,

$$f(x)=2x^3+x^4+\log x$$

Put $\quad x=1$, we get

$\therefore \quad f(1)=3 \Rightarrow 1=f^{-1}(3)$

Now, given g is the inverse of f

$\therefore \quad g(x)=f^{-1}(x) \Rightarrow f(g(x))=x$

Differentiating with respect to x, we get

$$f'(g(x))\cdot g'(x)=1$$

$$g'(x)=\frac{1}{f'(g(x))}$$

$$g'(x)=\frac{1}{6(g(x))^2+4(g(x))^3+\frac{1}{g(x)}}$$

$$\left[\because f'(x)=6x^2+4x^3+\frac{1}{x}\right]$$

$$g'(3)=\frac{1}{6(g(3))^2+4(g(3))^3+\frac{1}{g(3)}}$$

$$g'(3)=\frac{1}{6+4+1} \qquad [\because g(3)=f^{-1}(3)=1]$$

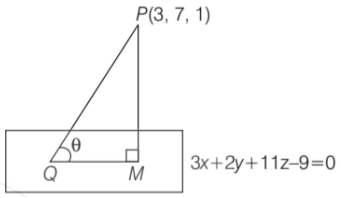
$$g'(3)=\frac{1}{11}$$

137. ***(d)*** We have $P(3, 7, 1)$ and $R(2, 5, 7)$

DR's of $PR=(-1, -2, 6)$

Angle between line PQ and plane is

$$\sin\theta=\frac{(-1)(3)+(-2)\times 2+6\times 11}{\sqrt{1+4+36}\sqrt{9+4+121}}$$



$$\sin\theta=\frac{59}{\sqrt{41}\sqrt{134}}$$

$$PM=\left|\frac{3(3)+2(7)+11(\times 1)-9}{\sqrt{9+4+121}}\right|$$

$$=\left|\frac{9+14+11-9}{\sqrt{134}}\right|=\frac{25}{\sqrt{134}}$$

$$\sin\theta=\frac{PM}{PQ}$$

$$\Rightarrow \quad PQ=\frac{PM}{\sin\theta}=\frac{25}{\sqrt{134}}\times\frac{\sqrt{41}\times\sqrt{134}}{59}=\frac{25\sqrt{41}}{59}$$

138. ***(a)*** We have,

$$I=\int\frac{8x^{43}+13x^{38}}{(x^{13}+x^5+1)^4}\,dx$$

$$=\int\frac{8x^{43}+13x^{38}}{x^{52}\left(1+\frac{1}{x^8}+\frac{1}{x^{13}}\right)^4}\,dx=\int\frac{8\left(\frac{1}{x^9}\right)+13\left(\frac{1}{x^{14}}\right)}{\left(1+\frac{1}{x^8}+\frac{1}{x^{13}}\right)^4}\,dx$$

Put, $\quad 1+\frac{1}{x^8}+\frac{1}{x^{13}}=t$

$$\Rightarrow \quad \left(\frac{-8}{x^9}-\frac{13}{x^{14}}\right)dx=dt$$

$$\therefore \quad I=-\int\frac{dt}{t^4}=\frac{1}{3t^3}+C$$

$$\Rightarrow I=\frac{1}{3\left(1+\frac{1}{x^8}+\frac{1}{x^{13}}\right)^3}+C=\frac{x^{39}}{3(x^{13}+x^5+1)^3}+C$$

139. ***(b)*** We have,

Vector r is coplanar with vector a and b

$\therefore \quad \mathbf{r} = \lambda(\mathbf{b} \times (\mathbf{a} \times \mathbf{b}))$

$\Rightarrow \quad \mathbf{r} = \lambda((\mathbf{b} \cdot \mathbf{b})\mathbf{a} - (\mathbf{b} \cdot \mathbf{a})\mathbf{b})$

$\Rightarrow \quad \mathbf{r} = \lambda(|\mathbf{b}|^2 \mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b})$

$\Rightarrow \quad \mathbf{r} = \lambda[5\mathbf{a} + \mathbf{b}] \qquad [\because |\mathbf{b}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

and $\mathbf{a} \cdot \mathbf{b} = (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{k}}) = -1]$

Given, $\mathbf{r} \cdot \mathbf{a} = 7$

$\therefore \quad \lambda(5\mathbf{a} + \mathbf{b}) \cdot \mathbf{a} = 7$

$\Rightarrow \quad \lambda[5|\mathbf{a}|^2 + \mathbf{a} \cdot \mathbf{b}] = 7$

$\Rightarrow \quad \lambda[15 - 1] = 7 \quad [\because |a|^2 = 1 + 1 + 1 = 3]$

$\Rightarrow \quad \lambda = 1/2$

$\therefore \quad \mathbf{r} = \frac{1}{2}(5\mathbf{a} + \mathbf{b})$

$$\mathbf{r} = \frac{1}{2}(-5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 5\hat{\mathbf{k}} + 2\hat{\mathbf{i}} + \hat{\mathbf{k}})$$

$$\mathrm{r} = \frac{-3}{2}\hat{\mathbf{i}} + \frac{5}{2}\hat{\mathbf{j}} + \frac{6}{2}\hat{\mathbf{k}}$$

z component of $\mathbf{r} = \frac{6}{2} = 3$

140. ***(b)*** Since x, y and z are three consecutive integers

$\therefore \quad 2y = x + z$

$\Rightarrow \quad (4y^2) = (x + z)^2$

$= (x - z)^2 + 4xz$

$= (-2)^2 + 4xz \qquad [\because x - z = -2]$

$\Rightarrow \quad 4y^2 = 4 + 4xz$

$\Rightarrow \quad y^2 = 1 + xz$

Now, $\frac{1}{2}\log_e x + \frac{1}{2}\log_e z + \frac{1}{2zx+1} + \frac{1}{3(2xz+1)^3} + \ldots$

$$= \frac{1}{2}\left[\log_e(xz) + 2\left(\frac{1}{2zx+1} + \frac{1}{3}\cdot\frac{1}{(2zx+1)^3} + \ldots\right)\right]$$

$$= \frac{1}{2}\left[\log_e(xz) + \log_e\left(\frac{1 + \frac{1}{2zx+1}}{1 - \frac{1}{2zx+1}}\right)\right]$$

$$= \frac{1}{2}\left[\log_e(xz) + \log_e\left(\frac{1+xz}{xz}\right)\right]$$

$$= \frac{1}{2}[\log_e(1+xz)] = \frac{1}{2}[\log_e(y)^2] = \log_e(y)$$

141. ***(a)*** We have, $(xy^5 + 2y)dx = x\,dy$

$$\Rightarrow \quad x\frac{dy}{dx} - 2y = xy^5$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{2y}{x} = y^5$$

$$\Rightarrow \quad y^{-5}\frac{dy}{dx} - \frac{2y^{-4}}{x} = 1 \qquad \ldots(i)$$

Put, $y^{-4} = t$

$$\Rightarrow \quad -4y^{-5}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \quad y^{-5}\frac{dy}{dx} = \frac{-1}{4}\frac{dt}{dx} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$-\frac{1}{4}\frac{dt}{dx} - \frac{2t}{x} = 1$$

$$\Rightarrow \quad \frac{dt}{dx} + \frac{8t}{x} = -4$$

Now, IF $= e^{\int \frac{8}{x}dx} = e^{8\log x} = x^8$

and the solution is $t \cdot x^8 = \int(-4)x^8 dx + C$

$$\Rightarrow \quad \frac{x^8}{y^4} = -\frac{4 \cdot x^9}{9} + C$$

$\Rightarrow \quad 9x^8 + 4x^9 \cdot y^4 = 9y^4 C$

142. ***(b)*** Given, $\frac{|x-2|-1}{|x-2|-2} \le 0$

Let $|x-2| = k$

Then, given equation,

$$\frac{k-1}{k-2} \le 0 \Rightarrow \frac{(k-1)(k-2)}{(k-2)^2} \le 0$$

$\Rightarrow \quad (k-1)(k-2) \le 0 \Rightarrow 1 \le k \le 2$

$\Rightarrow \quad 1 \le |x-2| \le 2$

Case I When $1 \le |x-2|$

$\Rightarrow \quad |x-2| \ge 1 \Rightarrow x - 2 \ge 1$ or $x - 2 \le -1$

$\Rightarrow \quad x \ge 3$ and $x \le 1 \qquad \ldots(i)$

Case II When $|x-2| \le 2$

$\Rightarrow \quad -2 \le x - 2 \le 2 \Rightarrow -2 + 2 \le x \le 2 + 2$

$\Rightarrow \quad 0 \le x \le 4 \qquad \ldots(ii)$

From Eqs. (i) and (ii), $x \in [0, 1] \cup [3, 4]$

143. ***(b)*** We have, $f(x) = \frac{x}{\sqrt{1+x^2}}$

$$\Rightarrow f(f(x)) = \frac{f(x)}{\sqrt{1 + (f(x))^2}} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1 + \frac{x^2}{1+x^2}}} = \frac{x}{\sqrt{1+2x^2}}$$

Similarly, $f(f(f(x))) = \frac{x}{\sqrt{1+3x^2}}$

$$\vdots \qquad \vdots$$

$$\underbrace{fofo\ldots of}_{n \text{ times}}(x) = \frac{x}{\sqrt{1+nx^2}} = \frac{x}{\sqrt{1 + \left(\sum_{r=1}^{n} 1\right)x^2}}$$

144. ***(b)*** We have,

$$\log_5\left(\frac{a+b}{3}\right)=\frac{\log_5 a+\log_5 b}{2}$$

$$\Rightarrow \quad 2\log_5\left(\frac{a+b}{3}\right)=\log_5 ab$$

$$\Rightarrow \quad \log_5\left(\frac{a+b}{3}\right)^2=\log_5 ab$$

$$\Rightarrow \quad \frac{(a+b)^2}{9}=ab \Rightarrow (a+b)^2=9ab$$

$$\Rightarrow \quad a^2+b^2=7ab$$

$$\Rightarrow \quad \frac{a}{b}+\frac{b}{a}=7$$

$$\Rightarrow \quad \left(\frac{a}{b}+\frac{b}{a}\right)^2=49$$

$$\Rightarrow \quad \frac{a^2}{b^2}+\frac{b^2}{a^2}+2=49$$

$$\Rightarrow \quad \frac{a^4+b^4}{a^2b^2}=49-2=47$$

145. ***(a)*** We have,

$$\frac{5}{4}\cos^2 2x+\cos^4 x+\sin^4 x+\cos^6 x+\sin^6 x=2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x+(\cos^2 x+\sin^2 x)^2-2\sin^2 x\cos^2 x$$
$$+(\cos^2 x+\sin^2 x)^3-3\sin^2 x\cos^2 x$$
$$(\sin^2 x+\cos^2 x)=2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x+1-2\sin^2 x\cos^2 x+1$$
$$-3\sin^2 x\cos^2 x=2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x-\frac{1}{2}\sin^2 2x-\frac{3}{4}\sin^2 2x=0$$

$$\Rightarrow \frac{5}{4}(\cos^2 2x-\sin^2 2x)=0$$

$$\Rightarrow \cos 4x=0$$

$$\Rightarrow 4x=2n\pi\pm\frac{\pi}{2}, n\in Z$$

$$\Rightarrow x=(4n\pm 1)\frac{\pi}{8}, n\in Z$$

$$\Rightarrow x=\frac{\pi}{8},\frac{3\pi}{8},\frac{5\pi}{8},\frac{7\pi}{8},\frac{9\pi}{8},\frac{11\pi}{8},\frac{13\pi}{8},\frac{15\pi}{8}$$

So, number of solution = 8

146. ***(a)*** Given equation of ellipse,

$$16x^2+11y^2=256 \quad \Rightarrow \frac{x^2}{16}+\frac{11y^2}{256}=1$$

Equation of tangent of ellipse at $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ is

$$\frac{4x\cos\phi}{16}+\frac{11\times 16y}{256\sqrt{11}}\sin\phi=1$$

$$\Rightarrow \quad \frac{x\cos\phi}{4}+\frac{\sqrt{11}}{16}\sin\phi\, y=1$$

is also a tangent of circle $x^2+y^2-2x=15$

$$\Rightarrow \quad (x-1)^2+y^2=16$$

radius of circle = 4 and centre = (1, 0)

Now, $\frac{x\cos\phi}{4}+\frac{\sqrt{11}}{16}\sin\phi\, y=1$ is tangent of circle

$$\therefore \ 4=\left|\frac{\frac{\cos\phi}{4}-1}{\sqrt{\frac{\cos^2\phi}{16}+\frac{11}{256}\sin^2\phi}}\right| \Rightarrow \phi=\pm\frac{\pi}{3}$$

$$\therefore \quad \phi=\frac{\pi}{3}$$

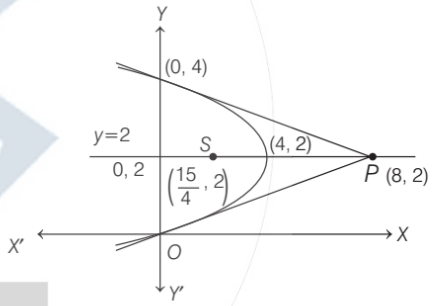
147. ***(b)*** Given equation of parabola is

$$x=4y-y^2$$

$$\Rightarrow \quad y^2-4y+4=-x+4$$

$$\Rightarrow \quad (y-2)^2=-(x-4)$$

$$a=\frac{1}{4}$$



Focus = $\left(\frac{15}{4}, 2\right)$

Parabola cuts Y-axis at (0, 0) and (0, 4).

Now, equation of tangent at (0, 0) is

$$x=4y \qquad \text{...(i)}$$

and equation of tangent at (0, 4) is

$$x+4y=16 \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$x=8, y=2$$

∴ Intersection point of tangent is $P(8, 2)$.

Distance from point P to focus S

$$\text{Distance}=\sqrt{\left(8-\frac{15}{4}\right)^2+(2-2)^2}$$

$$=8-\frac{15}{4}=\frac{17}{4}$$

148. ***(d)*** We have, $S_\infty = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\cdot\sum_{n=1}^{\infty}\frac{1}{2^n}$$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+..\right)$$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\left[\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{2^2}\cdots\right)\right]$$

$$S_\infty = \sum_{k=1}^{\infty}\frac{k}{2^k}\left(\frac{1}{2}\times\frac{1}{1-\frac{1}{2}}\right)=\sum_{k=1}^{\infty}\frac{k}{2^k}$$

$$S_\infty = \frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+\ldots \quad ...(i)$$

$$\frac{1}{2}S_\infty = \frac{1}{2^2}+\frac{2}{2^3}+\frac{3}{2^4}+\ldots \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}S_\infty = \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots$$

$$\frac{1}{2}S_\infty = \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right)=1$$

$$S_\infty = 2$$

149. ***(a)*** We have,

Slope of tangent of the curve is $\frac{(x+1)^2+y-3}{x+1}$

$$\therefore \quad \frac{dy}{dx}=\frac{(x+1)^2+y-3}{x+1}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{(x+1)^2-3}{x+1}+\frac{y}{x+1}$$

$$\Rightarrow \quad \frac{dy}{dx}-\frac{y}{x+1}=\frac{(x+1)^2-3}{x+1}$$

This is a form of linear differential equations

$$\therefore \quad \text{IF} = e^{\int\frac{-1}{x+1}dx} = e^{\log\frac{1}{1+x}} = \frac{1}{1+x}$$

Solution of given differential equation is

$$\frac{y}{x+1}=\int\frac{(x+1)^2-3}{(x+1)^2}dx$$

$$\frac{y}{x+1}=\int\left(1-\frac{3}{(x+1)^2}\right)dx$$

$$\frac{y}{x+1}=x+\frac{3}{x+1}+c$$

$\because$ Curve passes through $(2, 0)$

$$\therefore \quad 0=2+\frac{3}{3}+c$$

$$\Rightarrow \quad c=-3$$

Hence, equation of curve is

$$\frac{y}{x+1}=x+\frac{3}{x+1}-3$$

$$y=x^2+x+3-3x-3$$

$$y=x^2-2x$$

150. ***(a)*** We have,

$$A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

We know that, $|A-\lambda I|=0$

$$\left|\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\right|=0=\begin{vmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{vmatrix}=0$$

$$\Rightarrow \quad (2-\lambda)^2-1=0$$

$$\Rightarrow \quad 4-4\lambda+\lambda^2-1=0$$

$$\Rightarrow \quad \lambda^2-4\lambda+3=0$$

$$\therefore \quad A^2-4A+3I=0$$

Multiply by A^{-1}, we get

$$A-4I+3A^{-1}=0$$

$$\Rightarrow \quad A^{-1}=\frac{-1}{3}A+\frac{4}{3}I$$

$$\therefore \quad \alpha=-\frac{1}{3} \text{ and } \beta=\frac{4}{3}$$

$$\alpha+\beta=-\frac{1}{3}+\frac{4}{3}=\frac{3}{3}=1$$