Solved Paper 2018*

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

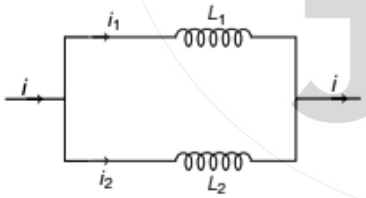
	No. of Questions
Part I Physics	1–40
Part II Chemistry	41–80
Part III a. English Proficiency	81–95
b. Logical Reasoning	96–105
Part IV Mathematics	106–150

- All questions are Multiple Choice Questions having four options out of which **only one** is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.
- Time allotted to complete this paper is 3 hrs.

PART I

Physics

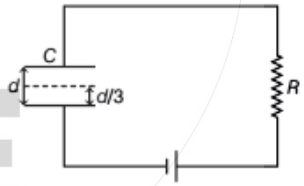
1. Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. The ratio of current $\frac{i_1}{i_2}$ at any time t is



a. $\frac{L_2}{L_1}$ *b.* $\frac{L_1}{L_2}$

c. $\frac{L_2^2}{(L_1+L_2)^2}$ *d.* $\frac{L_1^2}{(L_1+L_2)^2}$

2. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$, the level of liquid is $d/3$, initially. Suppose, the liquid level decreases at a constant speed v, the time constant as a function of time is



a. $\frac{6\varepsilon_0 R}{5d+3vt}$ *b.* $\frac{(15d+9vt)\varepsilon_0 R}{2d^3-3dvt-gv^2t^2}$

c. $\frac{6\varepsilon_0 R}{5d-3vt}$ *d.* $\frac{(15d-9vt)\varepsilon_0 R}{2d^3+3dvt-9v^2t^2}$

3. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent to 0.2 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of steam condensed (in kg) is

(Given, latent heat of steam = 540 cal/g)

a. 0.130 *b.* 0.065 *c.* 0.260 *d.* 0.135

4. Dimension of which base quantity corresponds to that of $\sqrt{Gh/c^3}$ =?

a. Time *b.* Length *c.* Mass *d.* Temperature

5. A reservoir is at 827°C and Carnot's engine takes a thousand kilocalories of heat from it and exhausts it to a sink at 27°C. What is the amount of work and the efficiency of the engine?

a. 2.7×10^5 cal,70.70% *b.* 2.72×10^5 cal, 72.72%

c. 2.70×10^5 cal, 80.70% *d.* 3.70×10^5 cal, 70.70%

6. A train moves towards stationary observer with speed 34 m/s. The train blows whistle and its frequency is registered by the observer as f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s, then the ratio f_1/f_2 is

a. 19/18 *b.* 18/19 *c.* 2 *d.* 1/2

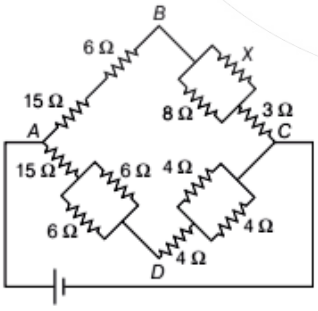
7. An object of mass 5 kg is projected with a velocity 20 ms^{-1} at an angle 60°, to the horizontal. At the highest point of its path, the projectile explodes and breaks up into two fragments of masses 1 kg and 4 kg. The fragments separate horizontally after the explosion, which releases internal energy such that the KE of the system at the highest point is doubled. The separation between the two fragments when they reach the ground is

a. 52.25 m *b.* 44.25 m *c.* 65.32 m *d.* 78.76 m

8. An automobile moving with a speed of 36 km/h reaches an upward inclined road of angle 30°, its engine becomes switch off. If the coefficient of friction is 0.1, then how much distance will automobile move before coming to rest?

a. 12.53 m *b.* 21.42 m *c.* 15.43 m *d.* 8.53 m

9. In the circuit given below, the value of resistance X, when the potential difference between the points B and D is zero, will be



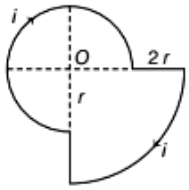
a. 9 Ω *b.* 8 Ω *c.* 4 Ω *d.* 6 Ω

10. A block of wood floats in water with (4/5) th of its volume submerged. If the same block just floats in a liquid, the density of liquid (in kg m^{-3}) is

a. 1250 *b.* 600

c. 400 *d.* 800

11. As current i flowing through the loop as shown in figure. The magnetic field at the centre O is



a. $\frac{7\mu_0 i}{12r}$ (acting downwards)

b. $\frac{5\mu_0 i}{12r}$ (acting upwards)

c. $\frac{7\mu_0 i}{12r}$ (acting upwards)

d. $\frac{5\mu_0 i}{12r}$ (acting downwards)

12. The ratio of angular momentum L to the atomic dipole moment μ_l for hydrogen like atoms and ions is

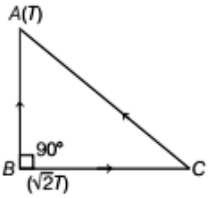
a. always constant and is equal to the ratio of mass to the charge of electron

b. always constant and is equal to twice the ratio of mass to the charge of electron

c. proportional to the principal quantum number n

d. proportional to $\frac{1}{n^2}$

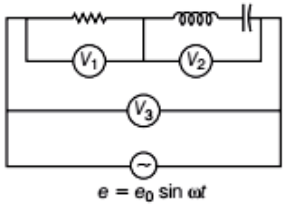
13. Three rods of identical cross-sectional area and made from the same metal, form the sides of an isosceles triangle ABC right angled at B as shown in figure. The point A and B are maintained at temperature T and $\sqrt{2}\,T$ respectively, in the steady state. Now, assuming that only heat conduction takes place. The temperature of point C will be



a. $\frac{T}{\sqrt{2}+1}$ *b.* $\frac{T}{\sqrt{2}-1}$

c. $\frac{3T}{\sqrt{2}+1}$ *d.* $\frac{\sqrt{3}T}{(\sqrt{2}+1)}$

14. A resistor R, an inductor L, a capacitor C and voltmeters V_1, V_2 and V_3 are connected to an oscillator in the circuit as shown in the adjoining diagram. When the frequency of the oscillation is increased, then at the resonant frequency, the voltmeter reading is zero in the case of



a. voltmeter V_1 only ***b.*** voltmeter V_2 only
c. voltmeter V_3 only ***d.*** All the three voltmeters

15. In Young's double slit experiment, intensity at a point is $\left(\frac{1}{4}\right)$ of the maximum intensity. Angular position of this point is

a. $\sin^{-1}\left(\frac{\lambda}{d}\right)$ ***b.*** $\sin^{-1}\left(\frac{\lambda}{2d}\right)$
c. $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ ***d.*** $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

16. The bob of simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob get suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

a. first increase and then decrease to the original value
b. first decrease and then increase to the original value
c. remain unchanged
d. increase towards a saturation value

17. At a certain temperature, the number density of charge carriers in a semiconductor is n. When an electric field is applied to it, the charge carriers drift with an average speed v. If the temperature of the semiconductor is raised, then

a. n will increase but v will decrease
b. n will decrease but v will increase
c. Both n and v will increase
d. Both n and v will decrease

18. Consider the acceleration, velocity and displacement of a tennis ball as its falls to the ground and bounces back. Directions of which of these change in the process?

a. Velocity only
b. Displacement and velocity
c. Acceleration, velocity and displacement
d. Displacement and acceleration

19. A convex lens shown in the figure is made up of two types of transparent materials. A point sources of light is placed on its principal axis. If reflections from the boundaries between layers are ignored, the lens will form

a. only one image ***b.*** two images
c. infinite images ***d.*** no image at all

20. If the time period is doubled, then the angular momentum of the body will (provided the moment of inertia of the body is constant)

a. remain constant ***b.*** quadruple
c. become half ***d.*** double

21. Breaking stress of a steel wire is p and the density of steel is ρ. The greatest length of steel wire that can hang vertically without breaking is

a. $\frac{p}{\rho g}$ ***b.*** $\frac{p}{2\rho g}$
c. $\frac{2p}{\rho g}$ ***d.*** None of these

22. A stone is projected with velocity $2\sqrt{gh}$, so that it just clears two walls of equal height h, at distance of $2h$ from each other. The time interval of passing between the two walls is

a. $\sqrt{h/g}$ ***b.*** $\sqrt{2h/g}$ ***c.*** $2\sqrt{h/g}$ ***d.*** $2h/g$

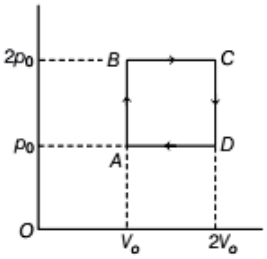
23. An object takes n times as much time to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the rough plane and the object is

a. $n^2 - 1$ ***b.*** $1 - \frac{1}{n^2}$ ***c.*** $n^2 + 1$ ***d.*** $1 + \frac{1}{n^2}$

24. Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days will the room be safe for occupation?

a. 20 days ***b.*** 34.8 days
c. 56.4 days ***d.*** 62.9 days

25. Helium gas goes through a cycle *ABCDA* (consisting of two isochoric and two isobaric lines) as shown in figure. The efficiency of this cycle is approximately



a. 15.4% ***b.*** 9.1% ***c.*** 10.5% ***d.*** 12.5%

26. The frequency and the intensity of incident beam of light falling on the surface of a photoelectric material is increased by a factor of two. This will

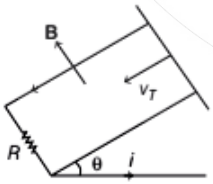
a. increase the maximum kinetic energy of the ejected photoelectrons by a factor of more than two and would increase the photoelectric current by a factor of two

b. increase the maximum kinetic energy of the photo electrons and would increase the photoelectric current both by a factor of two

c. increase the maximum kinetic energy of the photoelectrons by a factor of two and will have no effect on the magnitude of the photoelectric current produced

d. not produce any effect on the kinetic energy of the emitted photoelectrons but will increase the photoelectric current by a factor of two

27. A copper rod of mass m slides under gravity on two smooth parallel rails l distance apart and set an angle θ to the horizontal. At the bottom, the rails are joined by a resistance R in figure. There is a uniform magnetic field B prependicular to the plane of the rails. The terminal velocity of rod is



a. $\frac{mgR\tan\theta}{B^2l^2}$ ***b.*** $\frac{mgR\cot\theta}{B^2l^2}$

c. $\frac{mgR\sin\theta}{B^2l^2}$ ***d.*** $\frac{mgR\cos\theta}{B^2l^2}$

28. An asteroid of mass m is approaching earth, initially at a distance 10 R_e with speed v_i. It hits earth with a speed v_f (R_e and M_e are radius and mass of earth), then

a. $v_f^2 = v_i^2 + \frac{2Gm}{R_e}\left(1+\frac{1}{10}\right)$

b. $v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1+\frac{1}{10}\right)$

c. $v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1-\frac{1}{10}\right)$

d. $v_f^2 = v_i^2 + \frac{2Gm}{R_e}\left(1-\frac{1}{10}\right)$

29. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then choose the correct statement(s).

a. The work function of A is 1.50 eV.

b. The work function of B is 4.0 eV.

c. $T_A = 3.2$ eV

d. All of the above

30. A pulley of radius 2 m is rotated about its axis by a force $=(20t - 5t^2)$ N (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, then the number of rotation made by the pulley before its direction of motion is reversed is

a. more than 3 but less than 6

b. more than 6 but less than 9

c. more than 9

d. less than 3

31. A planoconvex lens has thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. The focal length of the lens is (assume thickness of lens to be negligible)

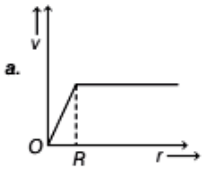
a. 85 cm ***b.*** 59 cm ***c.*** 75 cm ***d.*** 7.5 cm

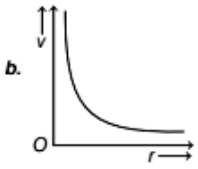
32. A spherically symmetric gravitational system of particles has mass density

$$\rho = \begin{cases} \rho_0 \text{ for } r \le R \\ 0 \text{ for } r > R \end{cases}$$

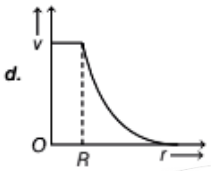
where, ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a

function of distance $r\,(0 < r < \infty)$ from the centre of the system is represented by

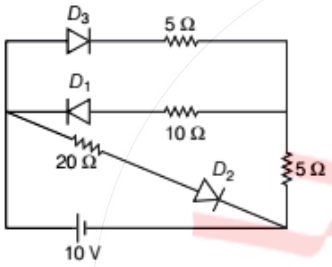
a.


b.


c.


d.


33. In the given circuit,



The current through battery is

a. 0.5 A ***b.*** 1 A ***c.*** 1.5 A ***d.*** 2.5 A

34. A trolley having mass of 200 kg moves with uniform speed of 36 kmh^{-1} on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to its motion and ultimately jumps out of the trolley. With how much velocity has the trolley moved from the time the child begins to run?

a. 10.36 ms^{-1} ***b.*** 11.36 ms^{-1}

c. 12.36 ms^{-1} ***d.*** 14.40 ms^{-1}

35. A gas has molar heat capacity $C = 37.55$ J $mol^{-1}K^{-1}$, in this process pT = constant. The number of degree of freedom of the molecule of gas is

a. 2 ***b.*** 3 ***c.*** 5 ***d.*** 7

36. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing SHM of time period T. Then, which of the following does not change with time?

a. $\frac{aT}{x}$ ***b.*** $aT + 2\pi v$

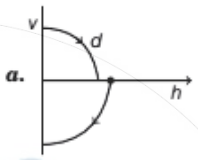
c. $\frac{aT}{v}$ ***d.*** $a^2T^2 + 4\pi^2v^2$

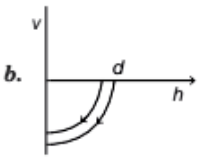
37. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased, so that new resistance becomes twice its initial resistance, by what factor does its voltage sensitivity change?

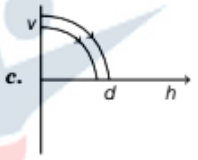
a. Increases by 15% ***b.*** Decreases by 15%

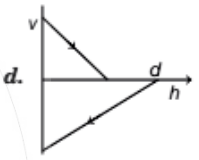
c. Increases by 25% ***d.*** Decreases by 25%

38. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with height h above the ground as

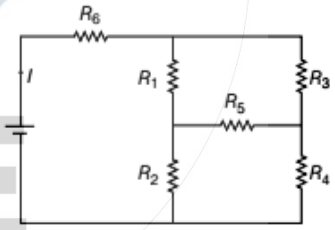
a.


b.


c.


d.


39. In the given circuit, it is observe that the current I is independent of the value of resistance R_5. Then, the resistance value must satisfy



a. $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} \cdot \frac{1}{R_3 + R_4}$

b. $R_1R_4 = R_2R_3$

c. $R_1R_2R_5 = R_3R_4R_6$

d. $R_1R_3 = R_2R_4 = R_5R_6$

40. If a drop of liquid breaks into smaller droplets, it results in lowering of temperature of the droplets. Let a drop of radius R, break into N small droplets each of radius r, then decrease (drop) in temperature Q' (given, specific heat of liquid drop = S and surface tension = T)

a. $\frac{3T}{\rho S}\left[\frac{1}{r} - \frac{1}{R}\right]$ ***b.*** $-\frac{2T}{\rho S}\left[\frac{1}{r} - \frac{1}{R}\right]$

c. $\frac{2T}{\rho S}\left[\frac{1}{R} - \frac{1}{r}\right]$ ***d.*** $\frac{3T}{\rho S}\left[\frac{1}{R} - \frac{1}{r}\right]$

PART II

Chemistry

41. A metallic element has a cubic lattice. Each edge of the unit cell is 2Å and the density of metal is 2.5 g cm^{-3}. The unit cell in 200 g of metal are

a. 1×10^{24} *b.* 1×10^{22} *c.* 1×10^{20} *d.* 1×10^{25}

42. Four gases *P*, *Q*, *R* and *S* have almost same values of '*b*' but their '*a*' values (*a* and *b* are van der Waals' constant) are in the order $Q<R<S<P$. At a particular temperature, among the four gases, the most easily liquefiable one is

a. *P* *b.* *Q* *c.* *R* *d.* *S*

43. If $\left[\frac{0.51\times10^{-10}}{4}\right]$ metre is the radius of smallest electron orbit in hydrogen like atom, then the atom is

a. hydrogen atom *b.* He^{+}

c. Li^{2+} *d.* Be^{3+}

44. Which of the following is the correct order for the wavelength, of absorption in the visible region?

a. $[Ni(NO_2)_6]^{4-} < [Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+}$

b. $[Ni(NO_2)_6]^{4-} < [Ni(H_2O)_6]^{2+} < [Ni(NH_3)_6]^{2+}$

c. $[Ni(H_2O)_6]^{2+} < [Ni(NH_3)_6]^{2+} < [Ni(NO_2)_6]^{4-}$

d. $[Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+} < [Ni(NO_2)_6]^{4-}$

45. The empirical formula and molecular mass of a compound are CH_2O and 180g respectively. The molecular formula of the compound will be

a. $C_9H_{18}O_9$ *b.* CH_2O *c.* $C_6H_{12}O_6$ *d.* $C_2H_4O_2$

46. In the context of the Hall-Heroult process for the extraction of Al, which of this following statement is incorrect?

a. CO and CO_2 are produced in this process.

b. Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity.

c. Al^{3+} is reduced at the cathode to Al.

d. Na_3AlF_6 serves as the electrolyte.

47. Which test among the following is not used for the distinction among 1°, 2° and 3° aliphatic amine?

a. Hinsberg's reagent test

b. Carbylamine reaction

c. Azo dye test

d. Action with nitrous acid

48. The incorrect statement about carbonate (CO_3^{2-}) ion is,

a. It has planar structure

b. It has one coordinate bond

c. It has three resonating structure

d. Hydrolysis of CO_3^{2-} ion gives basic solution

49. Under the same reaction conditions, initial concentration of 1.386 mol dm^{-3} of a substance becomes half in 40 s and 20 s through first order and zero order kinetics, respectively. Ratio $\left(\frac{k_1}{k_0}\right)$ of the rate constants for first order (k_1) and zero order (k_0) of the reaction is

a. 0.5 $mol^{-1}dm^3$ *b.* 1.0 mol dm^{-3}

c. 1.5 mol dm^{-3} *d.* 2.0 $mol^{-1}dm^3$

50. Which substance has a dipole moment?

a. CCl_4 *b.* CH_2Cl_2

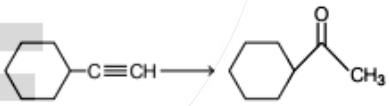
c. C_2Cl_2 *d.* C_2Cl_4

51. Enthalpy of combustion of methane and ethane are –210 kcal/mol and –368 kcal/mol respectively. The enthalpy of combustion of decane is

a. –1582 kcal *b.* –1632 kcal

c. –1700 kcal *d.* –1480 kcal

52. The correct sequence of reagents for the following conversion will be



a. O_3 / Red P, $AlCl_3$, MeCOOH

b. $H_2SO_4 + HgSO_4$, H_2O / Heat

c. O_3 / Zn - AcOH, $H_2SO_4 + HgSO_4$ / H_2O / Heat

d. CH_3COOH, $H_2O_2 + \bar{O}H$ / H_2O

53. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%, certainty with which the position of the electron can be located is ($h = 6.6\times10^{-34}$ kg m^2s^{-1}, mass of electron, $e_m = 9.1\times10^{-31}$ kg)

a. 1.52×10^{-4} m *b.* 5.10×10^{-3} m

c. 1.92×10^{-3} m *d.* 3.84×10^{-3} m

54. Which of the following diatomic molecules would be stabilised by the removal of an electron?

a. C_2 ***b.*** CN

c. N_2 ***d.*** O_2

55. 0.5F of electricity is passed through 500 mL of copper sulphate solution. The amount of copper which can be deposited will be

a. 63.5g

b. 31.75g

c. 15.80g

d. unpredictable

56. Consider the following sequence of reactions.

$$Z \xrightarrow{PCl_5} X \xrightarrow{\text{Alc. KOH}} Y \xrightarrow[2H_2O/\Delta]{\text{Conc. } H_2SO_4} Z$$

'Z' is

a. $CH_3CH_2CH_2OH$

b. $(CH)_2COH$

c. $CH_3-\underset{\underset{OH}{|}}{CH}-CH_3$

d. None of the above

57. An equilibrium mixture at 300K contains N_2O_4 and NO_2 at 0.28 and 1.1 atm pressure, respectively. If the volume of the container is doubled, the new equilibrium pressure of these two gases are respectively.

a. 0.064 atm and 0.095 atm

b. 0.64 atm and 0.095 atm

c. 0.095 atm and 0.632 atm

d. 0.095 atm and 0.64 atm

58. Which among the following actinoids does not have stable electronic configuration?

a. Protactinium

b. Nobelium

c. Americium

d. Lawrencium

59. Which of the following statement is incorrect regarding the equation?

$$MeCHO + [Ag(NH_3)_2]^+ + \bar{O}H \longrightarrow MeCOO^- + Ag$$

a. The equivalent weight of MeCHO is 22.

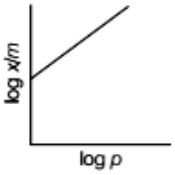
b. Three moles of $\bar{O}H$ are required in the reaction.

c. MeCHO acts as an oxidising agent.

d. $[Ag(NH_3)_2]^+$ gets reduced.

60. Which of the following graph is correctly represented according to Freundlich isotherm?

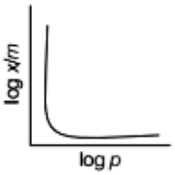
a.



b.

c.

d.



61. pH of a saturated solution of $Ba(OH)_2$ is 12. The value of its K_{sp} is

a. 3.3×10^{-7} M ***b.*** 5.0×10^{-7}M

c. 4.0×10^{-6} M ***d.*** 5.0×10^{-6}M

62. An unsaturated hydrocarbon '*X*' gives white precipitate with Tollen's reagent. If *X* is gaseous in nature, the molecular formula of *X* is

a. C_3H_6 ***b.*** C_2H_4

c. C_2H_2 ***d.*** C_4H_8

63. The rate of a reaction triples when temperature changes from 20°C to 50°C. The energy of activation for the reaction is ($R = 8.314\ JK^{-1}\ mol^{-1}$)

a. 181.327 J mol^{-1} ***b.*** 428.141 J mol^{-1}

c. 32.4321 kJ mol^{-1} ***d.*** 28.8118 kJ mol^{-1}

64. Which of the following compound will give blood red colour while doing the Lassaigne's test for N?

a. $(NH_2)C{=}O$ ***b.*** $H_2N(C_6H_4)SO_3H$

c. $C_6H_5SO_3H$ ***d.*** $CHCl_3$

65. For a reaction, $A + B^{2+} \longrightarrow B + A^{2+}$, at 25°C $E° = 0.2955$ V. The value of K_{eq} is

a. 10 ***b.*** 10^{10}

c. -10 ***d.*** 10^{-10}

66. Which of the following is the correct order of stability of conformations for $NH_2-CH_2-CH_2-CH_2-OH$?

a. Gauche > Eclipsed > Anti

b. Gauche > Anti > Eclipsed

c. Eclipsed > Gauche > Anti

d. Anti > Eclipsed > Gauche

67. When H_2S gas is passed into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ion in an acidified aqueous solution, the precipitates formed are

a. CuS and HgS
b. MnS and CuS
c. MnS and NiS
d. NiS and HgS

68. A mixture of bromo trichloride and hydrogen is subjected to silent electric discharge to form x and HCl. x is mixed with NH_3 and heated to 200°C to form y. The formula of y is

a. B_2O_3 ***b.*** $B_3N_3H_6$
c. H_3BO_3 ***d.*** B_2H_6

69. Which of the following reactions increase the production of dihydrogen from synthesis gas?

a. $CH_4(g) + H_2O(g) \xrightarrow[Ni]{1270K} CO(g) + 3H_2(g)$

b. $C(s) + H_2O(g) \xrightarrow{1270\ K} CO(g) + H_2(g)$

c. $CO(g) + H_2O(g) \xrightarrow[Catalyst]{673\ K} CO_2(g) + H_2(g)$

d. $C_2H_6 + 2H_2O \xrightarrow[Ni]{1270K} 2CO + 5H_2$

70. When the heat of a reaction at constant pressure is -2.5×10^3 cal and entropy change for the reaction is 7.4 cal deg^{-1}, it is predicted that the reaction at 25°C is

a. reversible ***b.*** spontaneous
c. non-spontaneous ***d.*** irreversible

71. The addition of HBr to 1-butene gives a mixture of products x, y and z.

The mixture consists of

a. x and y as major and z as minor products
b. y as major, x and z as minor products
c. y as minor, x and z as major products
d. x and y as minor and z as major products

72. The correct statement about silicone is

a. they are ketones with silyl group (SiH_3) similar to alkyl, $(SiH_3)_2CO$.
b. they are synthetic polymer containing repeated R_2SiO_2 units.
c. they are formed by hydrolysis of R_2SiCl_2.
d. All of the above

73. When dil. sulphuric acid reacts with aqueous solution of potassium chromate, the colour changes from yellow to orange. This shows that

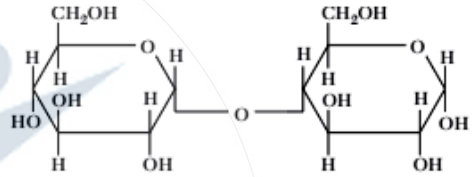
a. chromate ions reduced
b. chromate ions are oxidised
c. monocentric complex is converted into dicentric complex
d. oxygen gets removed from chromate ions

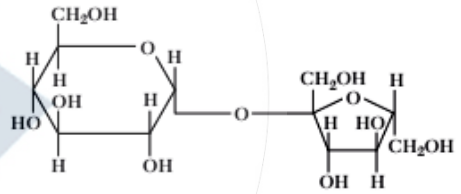
74. Valence electrons in the element A are 3 and that in element B are 6. Most probable compound formed from A and B is

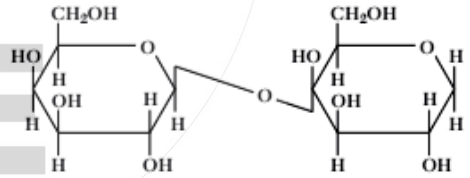
a. A_2B ***b.*** AB_2
c. A_6B_3 ***d.*** A_2B_3

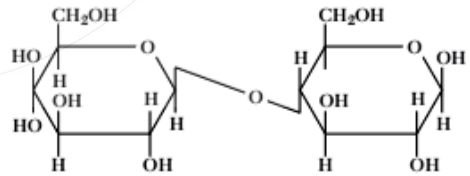
75. In non-reducing disaccharide, the reducing group of monosaccharides i.e. aldehydic or ketonic group are bounded.

Which of the following disaccharide is a non-reducing sugar?

a.


b.


c.


d.


76. Which of the following 0.1 M aqueous solution will have lowest freezing point?

a. Potassium sulphate
b. Sodium chloride
c. Urea
d. Glucose

77. A penicillin is a member of a family of drugs that have a
- ***a.*** four membered cyclic amide fused to a five membered thiazole ring.
- ***b.*** three membered cyclic amide fused to a five-membered thiazole ring
- ***c.*** four-membered cyclic amide fused to have a four membered thiazole ring.
- ***d.*** five-membered cyclic amide fused to have a five membered thiazole ring

78. Which of the following property of alkaline earth metal increases with their atomic number?
- ***a.*** electronegativity
- ***b.*** solubility of their hydroxides in water
- ***c.*** solubility of their sulphate in water
- ***d.*** ionisation energy

79. Which of the following expression is correct for the rate of reaction given below ?

$$5Br^-(aq) + BrO_3^-(aq) + 6H^+(aq) \longrightarrow 3Br_2(aq) + 3H_2O(l)$$

- ***a.*** $\frac{\Delta[Br^-]}{\Delta t} = \frac{5}{6}\frac{\Delta[H^+]}{\Delta t}$
- ***b.*** $\frac{\Delta[Br^-]}{\Delta t} = \frac{6}{5}\frac{\Delta[H^+]}{\Delta t}$
- ***c.*** $\frac{\Delta[Br^-]}{\Delta t} = \frac{5\Delta[H^+]}{\Delta t}$
- ***d.*** $\frac{\Delta[Br^-]}{\Delta t} = \frac{6\Delta[H^+]}{\Delta t}$

80. Match the polymer given in column I with correct monomer of column II and choose the correct option.

	Column I		Column II
A.	Neoprene	I.	Isoprene
B.	Natural rubber	II.	Tetrafluoro ethane
C.	Teflon	III.	Chloroprene
D.	Acrilan	IV.	Acryl nitrite

Codes

	A	B	C	D
a.	IV	III	II	I
b.	I	II	III	IV
c.	III	I	II	IV
d.	II	IV	I	III

PART III

a. English Proficiency

Directions (Q. Nos. 81-83) *In the following questions, the sentences may or may not be grammatically correct. Find out which part of a sentence has an error and mark that part. If there is no error, mark part 'd' as your answer.*

81. The captain alongwith his team ***(a)***/are practising very hard ***(b)***/ for the forthcoming match. ***(c)***/ No error ***(d)***

82. I am going ***(a)***/to have this certificate ***(b)***/attest by the director. ***(c)***/No error ***(d)***

83. He is ***(a)***/ having many ***(b)***/friends here.***(c)***/No error ***(d)***

Directions (Q. Nos. 84-85) *Fill in the blanks with suitable preposition from the alternatives given under each sentence.*

84. Is not learning superior ..... wealth?
- ***a.*** than
- ***b.*** from
- ***c.*** by
- ***d.*** to

85. He could not cope ..... the heavy workload.
- ***a.*** in with
- ***b.*** up with
- ***c.*** up
- ***d.*** with

Directions (Q. Nos. 86-88) *Select the word or the phrase which is closest to the opposite in meaning of the italicized word or phrase.*

86. He was in a *dejected* mood.
- ***a.*** jubilant
- ***b.*** rejected
- ***c.*** irritable
- ***d.*** romantic

87. The attack on the freedom of the press is a *retrograde* step.
- ***a.*** progressive
- ***b.*** stubborn
- ***c.*** punitive
- ***d.*** aggressive

88. We should not *belittle* the value of small things.
- ***a.*** exalt
- ***b.*** praise
- ***c.*** inflate
- ***d.*** expand

Directions (Q. Nos. 89-90) *Choose the word nearest in meaning to the underlined word.*

89. Before I could <u>make out anything</u> he had spoken again.
- ***a.*** find out
- ***b.*** apprehend
- ***c.*** explain
- ***d.*** reveal

90. He wrote a <u>scathing</u> review of the prize winning novel.
- ***a.*** biased
- ***b.*** scornful
- ***c.*** unbalanced
- ***d.*** subjective

Directions (Q. Nos. 91-95) *Read the passage given below and answer the questions that follow.*

The megalomaniac differs from the narcissist by the fact that he wishes to be powerful rather than charming and seeks to be feared rather than loved. To this type belong many lunatics and most of the great men in history. Love of power, like vanity, is a strong element in normal human nature and as such is to be accepted; it becomes deplorable only when it is excessive or associated with an insufficient sense of reality. Where this occurs, it makes a man unhappy or foolish, if not both. The lunatic who thinks he is crowned head may be, in a sense, happy, but his happiness is not of a kind that any sane person would envy. Alexander the Great was psychologically of the same type as the lunatic, though he possessed the talent to achieve the lunatic's dream.

He could not, however, achieve his own dream, which enlarged his scope as his achievement grew. When it became clear that he was the greatest conqueror known to fame, he decided that he was a God. Was he a happy man? His drunkenness, his furious rages, his indifference to women and his claim to divinity, suggest that he was not. There is no ultimate satisfaction in the cultivation of one element of human nature at the expense of all the others, nor in viewing all the world as raw material for the magnificence of one's own ego.

91. What is the difference between an ordinary megalomaniac and a megalomaniac like Alexander the Great?

a. The ordinary megalomaniac does not have excessive desire for power which Alexander the Great had.

b. The ordinary megalomaniac does not have the talent to realise his wish which Alexander the Great had.

c. The ordinary megalomaniac is a lunatic while Alexander the Great was not a lunatic.

d. The ordinary megalomaniac is not great while Alexander the Great was great.

92. How does a megalomaniac differ from a narcissist?

a. By wishing to be charming and feared

b. By wishing to be loved and not feared

c. By wishing to be powerful and not feared

d. By wishing to be powerful and feared

93. In "Where this occurs it makes a man..... if not both," 'this' refers to

a. vanity ***b.*** lunacy

c. love of power ***d.*** excessive lover of power

94. Which among the following is the reason for unhappiness?

a. Dealing with the raw material of the world

b. Realising one's dream as a megalomaniac

c. The nurturing of only one element in human nature

d. Being indifferent towards women

95. Why has love of power to be accepted?

a. Because it can become unreal

b. Because it is an excess in human nature

c. Because it is a part of human nature

d. Because it is vanity in human nature

b. Logical Reasoning

96. 'Umpire' is related to 'Match' in the same way as 'Judge' is related to

a. Bar council ***b.*** Lawyer

c. Judgement ***d.*** Lawsuit

97. Find the odd one from the following options.

a. 9, 49 ***b.*** 13, 121

c. 10, 61 ***d.*** 7, 25

98. Complete the series by replacing question mark '?'.

10, 9, 16, 45, 176, ?

a. 815 ***b.*** 222 ***c.*** 555 ***d.*** 875

99. In a cricket team, Dhoni is taller than Virat but not as tall as Raina, Rohit is shorter than Dhoni but taller than Shikhar. Who among them is the shortest?

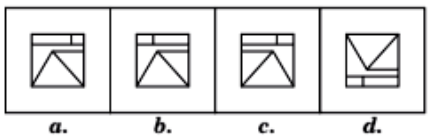
a. Dhoni ***b.*** Virat

c. Shikhar ***d.*** Cannot be determined

100. Identify the missing part of the question figure and select it from given answer figures.

Question figure

Answer figures

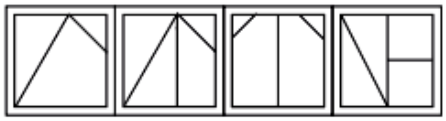


a. ***b.*** ***c.*** ***d.***

101. Find out which of the figure (a), (b), (c) and (d) can be formed from the pieces given in question figures.

Question figure

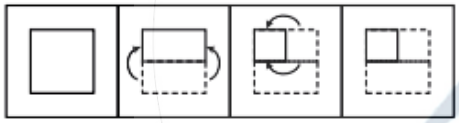
Answer figures



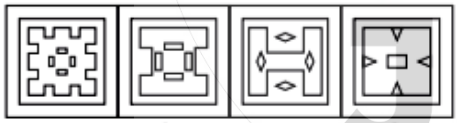
a. *b.* *c.* *d.*

102. In this question, a piece of paper is folded and then cut as shown below. The dotted lines shown are the portion which have been folded. The curve arrow shows the directions of folding. And the number of scissors beneath the figure show the number of portions cut. From the given responses, indicate how it will appear when opened. The opening is in the same order as folding.

Question figures



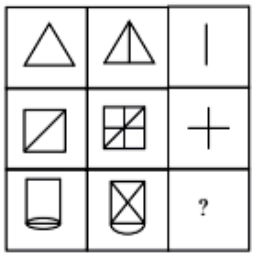
Answer figures



a. *b.* *c.* *d.*

103. Which of the answer figures (a), (b), (c) or (d) completes the figure matrix ?

Questions figures



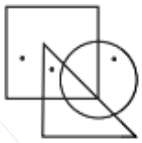
Answer figures

a. *b.* *c.* *d.*

104. In the following question, one or more dots are place in the question figure.

This figure is followed by four alternatives marked as (a), (b), (c) and (d). One out of these four options contains region(s) common to circle, square and triangles, similar to that marked by the dot in question figure. Find that figure.

Question figure



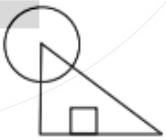
Answer figures

a.

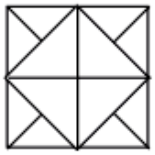
b.

c.

d.



105. How many triangles are there in the following figure?



a. 16 *b.* 20 *c.* 12 *d.* 22

PART IV

Mathematics

106. The coefficient of x^{-n} in $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is

a. 0 *b.* 1 *c.* 2^n *d.* $2n$

107. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is

a. $\frac{26840}{9}$ *b.* $\frac{24840}{9}$

c. $\frac{25840}{9}$ *d.* None of these

108. The nth roots of unity are in

a. AP *b.* GP

c. HP *d.* None of these

109. If $P=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, $abc = P^T P = I$, then the value of $a^3+b^3+c^3$ is

a. 2 *b.* 1 *c.* 0 *d.* 5

110. If $x^2 = \begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 1 \\ \sin\theta & \cos\theta & 2 \end{vmatrix}$, then the value of $4x^2 + x\sin\frac{3\pi}{2} + 5$ is

a. $13-\sqrt{2}$ *b.* $13+\sqrt{2}$

c. $\sqrt{2}-13$ *d.* Both (a) and (b)

111. If a, b, c are in GP and $\log a - \log 2b$, $\log 2b - \log 3c$ and $\log 3c - \log a$ are in AP, then a, b and c are the lengths of the sides of a triangle, which is

a. equilateral *b.* right angled

c. acute-angled *d.* obtuse angled

112. If $\sum_{r=1}^{n} t_r = \frac{n(n+1)(n+2)(n+3)}{8}$, where t_r denotes the rth term of a series, then $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{t_r}$ is

a. $\frac{1}{8}$ *b.* $\frac{1}{4}$ *c.* $\frac{1}{2}$ *d.* 1

113. Which of the following statement is a tautology?

a. $(p \vee q) \vee (\sim p)$ *b.* $(\sim q \wedge p) \vee (p \vee \sim p)$

c. Both (a) and (b) *d.* None of these

114. If a parallelogram is cut by two sets of m lines parallel to its sides, then the number of parallelogram thus formed, is

a. ${}^{m}C_2 \times {}^{m}C_2$ *b.* $2({}^{m+2}C_2)$

c. $({}^{m+2}C_2)^2$ *d.* None of these

115. The inverse of the function $f(x) = \log_a(x+\sqrt{x^2+1})$ (where, $a<0, a\neq 1$) is

a. $\frac{1}{2}(a^x - a^{-x})$ *b.* not defined for all x

c. defined for $x>0$ *d.* None of these

116. The value of $S = \sum_{n=1}^{\infty} \tan^{-1}\frac{2n}{n^4+n^2+2}$ is equal to

a. $\frac{\pi}{2}$ *b.* π

c. $\frac{\pi}{4}$ *d.* None of these

117. Equation $\sin x + \cos(t+x) + \cos(t-x) = 2$ has real solution, then $\sin t$ can be

a. 1/2 *b.* 1/5 *c.* 3/4 *d.* –3/4

118. A line makes angles α, β, γ with the coordinate axes. If $\alpha+\beta=\frac{\pi}{2}$, then $(\cos\alpha+\cos\beta+\cos\gamma)^2$ is equal to

a. $1+\cos 2\alpha$ *b.* $1-\sin 2\alpha$

c. $1+\sin 2\alpha$ *d.* None of these

119. Straight lines $3x+4y=5$ and $4x-3y=15$ intersect at the point A. If point B and C are chosen on these two lines such that $AB = AC$, then the possible equation of the line BC passing through the point (1, 2) is

a. $x+7y+13=0$ or $7x+y+9=0$

b. $x+7y+13=0$ or $7x+2y+7=0$

c. $x-7y+13=0$ or $7x+y-9=0$

d. None of the above

120. Normals drawn to $y^2=4ax$ at the points where it is intersected by the line $y=mx+c$ intersected at P. Coordinates of foot of the another normal drawn to the parabola from the point 'P' is

a. $\left(\frac{a}{m^2}, -\frac{2a}{m}\right)$ *b.* $\frac{9}{m}, -\frac{6a}{m}$

c. $(am^2, -2am)$ *d.* $\left(\frac{4a}{m^2}, -\frac{4a}{m}\right)$

121. The area of the triangle formed by joining the origin to the point of intersection of the line $x\sqrt{5}+2y=3\sqrt{5}$ and circle $x^2+y^2=10$ is

a. 3 *b.* 4 *c.* 5 *d.* 6

122. Radius of the largest circle which passes through the focus of the parabola $y^2=4x$ and contained in it, is

a. 8 *b.* 4 *c.* 2 *d.* 5

123. A tangent drawn to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ sq. units, with coordinate axes. If the eccentricity of hyperbola is e, then the value of e^2-9 is

a. 9 *b.* 10 *c.* 11 *d.* 8

124. If the sum of squares of distances of a point from the planes $x+y+z=0$, $x-z=0$ and $x-2y+z=0$ is p^2, then locus of the point is

a. $x^2+z^2=p^2$

b. $x^2+2xy+y^2+z^2=p^2$

c. $x+y+z=p^2$

d. $x^2+y^2+z^2=p^2$

125. Line $\frac{(x+1)}{\lambda}=y-1=\frac{(z+2)}{-4}$ is perpendicular to $2x+2y-8z+5=0$, then λ is

a. 1 *b.* -4

c. -5 *d.* -3

126. $OPQR$ is a square and M, N are the middle points of the sides PQ and QR respectively, then the ratio of the areas of the square and ΔOMN is

a. 4 : 1 *b.* 2 : 1

c. 8 : 3 *d.* 4 : 3

127. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2+9y^2=9$ meets its auxiliary circle at the point M. Then, the area of the triangle with vertices at A, M and the origin O is

a. $\frac{31}{10}$ *b.* $\frac{29}{10}$

c. $\frac{21}{10}$ *d.* $\frac{27}{10}$

128. If e_1 and e_2 are the eccentricities of a hyperbola $3x^2-3y^2=25$ and its conjugate, then

a. $e_1^2+e_2^2=2$ *b.* $e_1^2+e_2^2=4$

c. $e_1+e_2=4$ *d.* $e_1+e_2=\sqrt{2}$

129. Let $f:R\to R$ be a function satisfying $f(x+y)=f(x)+2y^2+kxy$ for all $x, y\in R$. If $f(1)=2$ and $f(2)=8$, then $f(x)$ is equal to

a. $2x^2$ *b.* $6x-4$

c. x^2+3x-2 *d.* $-x^2+9x-6$

130. If the planes $\mathbf{r}\cdot(2\hat{\mathbf{i}}-\lambda\hat{\mathbf{j}}+3\hat{\mathbf{k}})=0$ and $\mathbf{r}\cdot(\lambda\hat{\mathbf{i}}+5\hat{\mathbf{j}}-\hat{\mathbf{k}})=5$ are perpendicular to each other, then the value of $\lambda^2+\lambda$ is

a. 0 *b.* 2

c. 1 *d.* 3

131. Solution of the differential equation $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$ is equal to

a. $\log\left(2+\sec\frac{x+y}{2}\right)=x+C$

b. $\log(1+\tan(x+y))=x+C$

c. $\log\left(1+\tan\frac{x+y}{2}\right)=y+C$

d. $\log\left(1+\tan\frac{x+y}{2}\right)=x+C$

132. The value of α, so that $\lim_{x\to 0}\frac{1}{x^2}(e^{\alpha x}-e^x-x)=\frac{3}{2}$, is

a. 1 *b.* 0

c. 4 *d.* 2

133. An inverted conical flask is being filled with water at the rate of 3 cm^3/s. The height of the flask is 10 cm and the radius of the base is 5 cm. How fast is the water level rising when the level is 4 cm?

a. $\frac{4}{3}\pi$ cm/s *b.* $\frac{3}{4\pi}$ cm/s

c. $\frac{3\pi}{4}$ cm/s *d.* $\frac{4}{3\pi}$ cm/s

134. The equation of the curve whose slope at any point is equal to $y+2x$ and which passes through the origin is

a. $y=2(x-1)$ *b.* $y=2(e^x-x-1)$

c. $y=2(e^x-1)$ *d.* $y=2(e^x x-1)$

135. Let $f(x)=\begin{cases}x^p\sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$, then $f(x)$ is continuous but not differentiable at $x=0$, if

a. $p<0$ *b.* $p=0$

c. $0<p\le 1$ *d.* $p\ge 1$

136. The solution of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$, is

a. $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$

b. $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$

c. $-\frac{\cos 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$

d. None of the above

137. For which interval the given function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is decreasing?

a. $(-2, \infty)$,

b. $(-2, -1)$

c. $(-\infty, -1)$

d. $(-\infty, -2)$ or $(-1, \infty)$

138. If θ is the angle between the vectors $4(\hat{i} - \hat{k})$ and $\hat{i} + \hat{j} + \hat{k}$, then $(\sin\theta + \cos\theta)$ equals to

a. 0 *b.* $\frac{1}{2}$

c. 1 *d.* 2

139. In a ΔABC, D, E, F are the mid-points of the sides BC, CA and AB respectively, the vector AD is equal to

a. BE + CF *b.* BE − CF

c. CF − BE *d.* − BE − CF

140. The arithmetic mean of a set of observation is $\overline{X}$. If each observation is divided by α and increased by 10, then the mean of the new series is

a. $\frac{\overline{X}}{\alpha}$ *b.* $\frac{\overline{X} + 10}{\alpha}$

c. $\frac{\overline{X} + 10\alpha}{\alpha}$ *d.* $\alpha\overline{X} + 10$

141. If h is the altitude of a parallelopiped determined by the vectors a, b, c and the base is taken to be the parallelogram determined by a and b where $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} + \hat{j} + 3\hat{k}$, then the value of $19h^2$ is

a. 19 *b.* 16

c. 8 *d.* None of these

142. The mean and variance of a binomial distribution **BD** for 3 trials is 2.7, then the **BD** is given by

a. $(0.2 + 0.8)^5$ *b.* $(0.3 + 0.7)^5$

c. $(0.4 + 0.6)^5$ *d.* None of these

143. Let $P(x) = \int \frac{dx}{e^x + 8e^{-x} + 4e^{-3x}}$,

$Q(x) = \int \frac{dx}{e^{3x} + 8e^x + 4e^{-x}}$ and

$R(x) = P(x) - 2Q(x)$.

If $R(x) = \frac{1}{2}A\left(\frac{B + 2e^{-x}}{C}\right) + K$, then the value of (A, B, C) is

a. $(\tan^{-1}, 2, e^x)$ *b.* $(\tan^{-1}, e^x, 2)$

c. $\left(\tan^{-1}, \frac{1}{2}, \frac{1}{e^x}\right)$ *d.* $\left(\tan^{-1}, \frac{1}{e^x}, \frac{1}{2}\right)$

144. The value of $\int_0^1 \cot^{-1}(1 - x + x^2)dx$ is

a. $\log 2$ *b.* $\frac{\pi}{2} - \log 2$

c. $\frac{\pi}{2} + \log 2$ *d.* $-\log 2$

145. The area of the region included between the curves $x^2 + y^2 = a^2$ and $\sqrt{|x|} + \sqrt{|y|} = \sqrt{a}$ $(a > 0)$, is

a. $\left(\pi - \frac{2}{3}\right)a^2$ sq units

b. $\left(\frac{2}{3} - \pi\right)a^2$ sq units

c. $\frac{2}{3}\pi a^2$ sq units

d. $\left(\pi + \frac{2}{3}\right)a^2$ sq units

146. Let A and B are two independent events. If the probability that both A and B occur together is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$, then the probability of occurrence of A is

a. 0 or 1 *b.* $\frac{1}{2}$ or $\frac{1}{3}$

c. $\frac{1}{2}$ or $\frac{1}{4}$ *d.* $\frac{1}{3}$ or $\frac{1}{4}$

147. In a test an examiner either guesses or copies or knows the answer to a multiple choice question with 4 choices. The probability that he/she makes a guess is $\frac{1}{3}$. The probability that he/she copies the answer is $\frac{1}{6}$. If the probability that the answer is correct, given that he/she copied,

it is $\frac{1}{8}$, then the probability that he/she knows the answer to a question given that he/she correctly answered it, is

a. $\frac{27}{29}$ *b.* $\frac{26}{29}$

c. $\frac{25}{29}$ *d.* $\frac{24}{29}$

148. If p : 4 is an even prime number, q : 6 is a divisor of 12 and r : the HCF of 4 and 6 is 2, then which of the following is correct?

a. $(p \wedge q)$ *b.* $(p \vee q) \wedge \sim r$

c. $\sim(q \wedge r) \vee p$ *d.* $\sim p \vee (q \wedge r)$

149. The maximum value of $Z = 4x + 2y$ subject to constraints $2x + 3y \leq 18$, $x + y \geq 10$ and $x, y \geq 0$ is

a. 20 *b.* 36

c. 40 *d.* None of these

150. The coordinates of the point at which minimum value of $Z = 7x - 8y$ subject to constraints $x + y - 20 \leq 0$, $y \geq 5$, $x \geq 0$, $y \geq 0$ is attained, is

a. (20, 0) *b.* (15, 5)

c. (0, 5) *d.* (0, 20)

Answers

Physics

1. (a)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (d)
11. (d)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (c)
21. (a)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (a)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (a)
31. (c)	**32.** (c)	**33.** (c)	**34.** (a)	**35.** (c)	**36.** (a)	**37.** (d)	**38.** (a)	**39.** (b)	**40.** (d)

Chemistry

41. (d)	**42.** (a)	**43.** (d)	**44.** (a)	**45.** (c)	**46.** (d)	**47.** (c)	**48.** (b)	**49.** (a)	**50.** (b)
51. (b)	**52.** (b)	**53.** (c)	**54.** (d)	**55.** (c)	**56.** (c)	**57.** (d)	**58.** (a)	**59.** (c)	**60.** (a)
61. (b)	**62.** (c)	**63.** (d)	**64.** (b)	**65.** (b)	**66.** (b)	**67.** (a)	**68.** (b)	**69.** (c)	**70.** (b)
71. (a)	**72.** (d)	**73.** (c)	**74.** (d)	**75.** (b)	**76.** (a)	**77.** (a)	**78.** (b)	**79.** (a)	**80.** (c)

English Proficiency

81. (a)	**82.** (c)	**83.** (b)	**84.** (d)	**85.** (d)	**86.** (a)	**87.** (a)	**88.** (a)	**89.** (b)	**90.** (b)
91. (b)	**92.** (d)	**93.** (d)	**94.** (c)	**95.** (c)					

Logical Reasoning

96. (d)	**97.** (c)	**98.** (d)	**99.** (d)	**100.** (c)	**101.** (b)	**102.** (a)	**103.** (d)	**104.** (b)	**105.** (b)

Mathematics

106. (b)	**107.** (c)	**108.** (b)	**109.** (a)	**110.** (d)	**111.** (d)	**112.** (c)	**113.** (c)	**114.** (c)	**115.** (a)
116. (c)	**117.** (a)	**118.** (c)	**119.** (c)	**120.** (d)	**121.** (c)	**122.** (b)	**123.** (d)	**124.** (d)	**125.** (a)
126. (c)	**127.** (d)	**128.** (b)	**129.** (a)	**130.** (a)	**131.** (d)	**132.** (d)	**133.** (b)	**134.** (b)	**135.** (c)
136. (a)	**137.** (d)	**138.** (c)	**139.** (d)	**140.** (c)	**141.** (c)	**142.** (b)	**143.** (b)	**144.** (b)	**145.** (a)
146. (b)	**147.** (d)	**148.** (d)	**149.** (d)	**150.** (d)					

Hints & Solutions

Physics

1. ***(a)*** As the inductors are in parallel, induced emf across the two inductors is the same, i.e.

$$e_1 = e_2$$

$$L_1\left(\frac{di_1}{dt}\right) = L_2\left(\frac{di_2}{dt}\right)$$

On integrating both sides, we get

$$L_1\int\frac{di_1}{dt}dt = L_2\int\frac{di_2}{dt}dt$$

$$\Rightarrow \quad L_1\int di_1 = L_2\int di_2$$

$$L_1 i_i = L_2 i_2$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{L_2}{L_1}$$

2. ***(a)*** We have, the time constant, $\tau = RC'$ ...(i)

Now, $C' = \dfrac{C_1C_2}{C_1 + C_2}$

$$= \frac{\left(\frac{A\varepsilon_0}{d-x}\right)\left(\frac{KA\,\varepsilon_0}{x}\right)}{\frac{A\varepsilon_0}{d-x} + \frac{KA\varepsilon_0}{x}} \quad \left[\because C_1 = \frac{A\varepsilon_0}{d-x} \text{ and } C_2 = \frac{kA\varepsilon_0}{x}\right]$$

$$\therefore \quad C' = \frac{KA\varepsilon_0}{x + K(d-x)}$$

Putting the value of C' in Eq. (i), we get

$$\tau = R\left(\frac{KA\varepsilon_0}{x + K(d-x)}\right) \quad \text{...(ii)}$$

At time t, the level of liquid is $\dfrac{d}{3} - vt$.

$\therefore$ Putting $x = \dfrac{d}{3} - vt$ in Eq. (ii), we get

$$\tau = \frac{R\,K\,A\varepsilon_0}{\frac{d}{3} - vt + k\left(d - \frac{d}{3} + vt\right)}$$

Given, $A = 1$ and $K = 2$

$$\therefore \quad \tau = \frac{3 \times 2\,\varepsilon_0 \times R}{d - 3vt + 6d - 2d + 6vt} = \frac{6\,R\,\varepsilon_0}{5d + 3vt}$$

3. ***(a)*** According to principle of calorimetry,

Heat gained = Heat lost

Heat is lost by steam in two stages

(i) Change of state from steam to water at 100°C.

(ii) To change water at 100°C to water at 80°C.

Now we have,

The mass of water and the calorimeter

$$= (1.1 + 0.02)\text{kg} = 1.12\,\text{kg}$$

Specific heat capacity of water $= 4.184 \times 10^3\ \text{Jkg}^{-1}\text{K}^{-1}$

Now, by using, $Q = mc\Delta T$

Heat gained by the calorimeter and water is

$$Q = 1.12 \times 4.18 \times 10^3 \times (80 - 15)$$

$$= 1.12 \times 4.18 \times 10^3 \times 65\ \text{Jkg}^{-1}\text{K}^{-1} \quad \text{...(i)}$$

Let the mass of the steam be m kg.

So, latent heat of the vaporisation of water at 100°C is 540 cal/g $= 540 \times 4.184 \times 10^3$ J/kg

$\therefore$ Heat lost by the steam,

$$Q' = m(540 \times 4.184 \times 10^3) + m(4.184 \times 10^3 \times (100 - 80))\ \text{J/kgK} \quad \text{...(ii)}$$

From Eq. (i) and (ii), we get

$$m(540 + 20) = 1.12 \times 65$$

$$\Rightarrow \quad m = \frac{1.12 \times 65}{560} = 0.13\ \text{kg}$$

4. ***(b)*** As, gravitational constant,

$$[G] = [\text{M}^{-1}\text{L}^3\text{T}^{-2}]$$

Planck's constant, $[h] = [\text{ML}^2\text{T}^{-1}]$

Hence, $[G][h] = [\text{M}^{-1}\text{L}^3\text{T}^{-2}][\text{ML}^2\text{T}^{-1}]$

$$= [\text{M}^0\text{L}^5\text{T}^{-3}]$$

Velocity of light, $[c] = [\text{LT}^{-1}]$

Now, $\left[\dfrac{Gh}{c^3}\right]^{1/2} = \dfrac{[\text{L}^5\text{T}^{-3}]^{1/2}}{[\text{L}^3\text{T}^{-3}]^{1/2}}$

$$= [\text{L}^2]^{1/2} = [\text{L}]$$

Hence, $[\text{L}] =$ length

5. ***(b)*** Given, $Q = 10^6$ cal

$T_1 = 827°\text{C} = (827 + 273) = 1100$ K

$T_2 = 27°\text{C} = (27 + 273) = 300$ K

As, $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$

$$\therefore \quad Q_2 = \frac{T_2}{T_1}Q_1 = \frac{300}{1100} \times 10^6$$

$$= 2.72 \times 10^5\ \text{cal}$$

Efficiency of the engine,

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$\eta = \left(1 - \frac{300}{1100}\right) \times 100$$

$$= 72.72\%$$

6. ***(a)*** According to Doppler's effect, the approximate frequency heard by the stationary observer,

$$\nu = \frac{v}{v - v_s}\nu_0$$

Case (i) $v_s = 34$ m/s

where, v = speed of sound in air,

v_s = speed of source

and v_0 = frequency of the source.

$$\therefore \quad \nu_1 = \frac{340}{340 - 34}\nu_0$$

$$= \frac{340}{306}\nu_0 \qquad \ldots (i)$$

Case (ii) $v_s = 17$ m/s

$$\therefore \quad \nu_2 = \frac{340}{340 - 17}\nu_0 = \frac{340}{323}\nu_0 \qquad \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{340/306}{340/323} \Rightarrow \frac{323}{306} = \frac{19}{18}$$

7. ***(b)*** Given, $m = 5$ kg, $v = 20\ \text{ms}^{-1}$, $\theta = 60°$

Vertical component of velocity, $v_y = v \sin 60°$

$$= 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}\ \text{ms}^{-1}$$

Time taken to reach the highest point = Time taken to reach the ground from highest point.

$$t = \frac{v \sin\theta}{g} = \frac{v_y}{g} = \frac{10\sqrt{3}}{9.8} = 1.77\ \text{s}$$

If the highest point, m splits up into two parts of masses $m_1 = 1$ kg and $m_2 = 4$ kg.

If their velocities v_1 and v_2 respectively, then applying the principle of conservation of linear momentum, we get

$$m_1 v_1 + m_2 v_2 = mv\cos\theta$$

$$v_1 + 4v_2 = 5 \times 20 \times \frac{1}{2} \qquad [\because \theta = 60°]$$

$$\Rightarrow \quad v_1 + 4v_2 = 5 \times 10 = 50 \qquad \ldots (i)$$

$$\text{Initial KE} = \frac{1}{2}m(v\cos\theta)^2$$

$$= \frac{1}{2} \times 5 \times (10)^2 = 250\ \text{J}$$

Final KE = 2 (initial KE) = $2 \times 250 = 500$ J

$$\therefore \quad \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = 500$$

$$\text{or} \quad \frac{1}{2} \times 1 \times v_1^2 + \frac{1}{2} \times 4 \times v_2^2 = 500$$

$$\text{or} \quad v_1^2 + 4v_2^2 = 1000 \qquad \ldots (ii)$$

Solving Eqs. (i) and (ii), we get

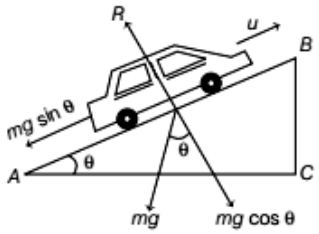
$$v_1 = 30\ \text{m/s}, \quad v_2 = 5\ \text{m/s}$$

Hence, the separation between the two fragments

$$= (v_1 - v_2) \times t = (30 - 5) \times 1.77\ \text{m} = 44.25\ \text{m}$$

8. ***(d)*** Given, initial speed, $u = 36$ km/h

$$= \frac{36 \times 1000}{60 \times 60} = 10\ \text{ms}^{-1}$$



$\theta = 30°, \mu = 0.1, s = ?$

Here, work done in moving up the inclined road

= KE of the vehicle

$$(mg\sin\theta + F)s = \frac{1}{2}mu^2$$

$$(mg\sin\theta + \mu R) \times s = \frac{1}{2}mu^2$$

$$(mg\sin\theta + \mu\, mg\cos\theta) \times s = \frac{1}{2}mu^2$$

$$s = \frac{\frac{1}{2}mu^2}{mg(\sin\theta + \mu\cos\theta)} = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$$

$$= \frac{10 \times 10}{2 \times 10 \times (\sin 30° + 0.1\cos 30°)} = 8.53\ \text{m}$$

9. ***(b)*** $P = 15 + 6 = 21\ \Omega$,

$$Q = \frac{8X}{8 + X} + 3$$

$$R = 15 + \frac{6 \times 6}{6 + 6} = 18\ \Omega$$

$$S = 4 + \frac{4 \times 4}{4 + 4} = 6\ \Omega$$

As, potential difference between B and D is zero, only when the bridge is balanced i.e.,

$$\frac{P}{Q} = \frac{R}{S}, \text{ so } Q = \frac{PS}{R} = \frac{21 \times 6}{18} = 7\ \Omega$$

$$\Rightarrow \quad 3 + \frac{8X}{8 + X} = 7 \Rightarrow X = 8\Omega$$

10. ***(d)*** Let V be the volume of the block. When block floats in water, then $V\rho_{\text{block}}\, g = \left(\frac{4}{5}V\right)\rho_{\text{water}}\, g$

$$\text{or} \quad \rho_{\text{block}} = \frac{4}{5}\rho_{\text{water}} \qquad \ldots (i)$$

When block floats in liquid.

$$V\rho_{\text{block}}\, g = V\rho_{\text{liquid}}\, g$$

$$\rho_{\text{block}} = \rho_{\text{liquid}}$$

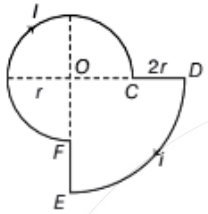
$$\rho_{liquid} = \frac{4}{5}\rho_{water} \qquad \text{[from Eq. (i)]}$$

$$= \frac{4}{5} \times 10^3 \text{ kg m}^{-3}$$

$$\Rightarrow \qquad \rho_{liquid} = 800 \text{ kg m}^{-3}$$

11. ***(d)*** The angle, subtended by arc DE at O is $\frac{\pi}{2}$ and FC at $O = \frac{3\pi}{2}$ the effective magnetic field at O is

$$B = B_{DE} + B_{FC}$$

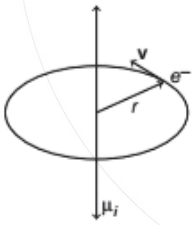


$$= \frac{\mu_0 i}{4\pi 3r} \times \frac{\pi}{2} + \frac{\mu_0 i}{4\pi r} \times \left(\frac{3\pi}{2}\right) \quad [\because OD = 3r \text{ and } OC = r]$$

$$= \frac{\mu_0 i}{8r}\left[\frac{1}{3} + 3\right]$$

$$= \frac{5\mu_0 i}{12r} \text{ (acting downwards)}$$

12. ***(b)*** The electron moving with a speed v in the circular Bohr orbit of radius r constitutes a current of magnitude.



$i = \frac{e}{T}$, where T is the orbital period of the electron.

$$T = 2\pi r / v \Rightarrow i = ev/2\pi r$$

By definition of atomic dipole moment μ_l, its magnitude is given by

$$\mu_l = iA = \frac{ev}{2\pi r} \cdot \pi r^2 = \frac{evr}{2}$$

$\therefore$ Angular momentum, $L = mvr$

$\therefore$ $$\frac{L}{\mu_l} = \frac{2mvr}{evr} = \frac{2m}{e} = \text{constant}$$

13. ***(c)*** Let T_0 be the temperature of point C and x be the length of rod AB or BC.

Then, $CA = \sqrt{x^2 + x^2} = \sqrt{2}\,x$

At steady state, the rate of heat flowing from B to C = rate of heat flowing from C to A.

So, $$\frac{KA(\sqrt{2}T - T_0)}{x} = \frac{KA(T_0 - T)}{\sqrt{2}\,x}$$

$$\Rightarrow \qquad \sqrt{2}(\sqrt{2}\,T - T_0) = T_0 - T$$

By solving, we get

$$T_0 = \frac{3T}{(\sqrt{2}+1)}$$

14. ***(b)*** At resonance, $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$

Voltage across the series L-C combination,

$$V_2 = i(X_L - X_C) = 0$$

15. ***(c)*** $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$$\therefore \qquad \frac{I_{max}}{4} = I_{max}\cos^2\frac{\phi}{2} \qquad \left[\text{Given } I, = \frac{I_{max}}{4}\right]$$

$$\Rightarrow \qquad \cos\frac{\phi}{2} = \frac{1}{2}$$

$$\Rightarrow \qquad \frac{\phi}{2} = \frac{\pi}{3}$$

$$\Rightarrow \qquad \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right) \times \Delta x \qquad \ldots \text{(i)}$$

where, $\Delta x = d\sin\theta$

Putting the value of Δx in Eq. (i), we get

$$\sin\theta = \frac{\lambda}{3d}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$$

16. ***(a)*** The bob filled completely with water, has its centre of mass at its centre. The time period of oscillation is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

As the water starts coming out of the bob, its centre of mass shifts vertically downward as a result effective length of the pendulum increases and hence its period also increases. When the bob is empty, again its centre of mass appears at its centre and as a result, the period of oscillation again reaches to its original value.

17. ***(a)*** On raising the temperature of the semiconductor, covalent bonds start breaking up and thus more charge carriers release. This will increase the value of n and as a result the rate of collision of charge carriers will increase. This will decrease the drift speed v.

18. ***(b)*** When a tennis ball falls on the ground and bounces back, its velocity and displacement changes in reverse direction while acceleration remains unchanged.

19. ***(b)*** Since, the lens is made up of two kinds of transparent material, it has two refractive indices for the incident beam of light. Hence, there will be two focal lengths of the lens and therefore two images will be observed.

20. ***(c)*** As we know,

Angular momentum, $L = I\omega$

$$\Rightarrow \quad L = I \times \frac{2\pi}{T}$$

$$L \propto \frac{1}{T}$$

Given, $T_2 = 2T$

Hence, $$\frac{L_1}{L_2} = \frac{2T}{T}$$

$$\Rightarrow \quad L_2 = \frac{L_1}{2}$$

21. ***(a)*** Maximum stress $= \dfrac{\text{Maximum weight}}{\text{Cross - sectional area}}$

Now, maximum weight of steel wire

$$= \text{Volume} \times \text{Density} \times g$$

$$= Al\rho g$$

where, l is the maximum length of steel wire that can hang vertically without breaking, ρ is the density of steel and A is the cross-sectional area of steel wire.

$\therefore$ Maximum stress, $p = \dfrac{Al\rho g}{A} = l\rho g$

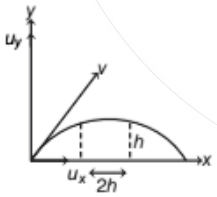
$$\Rightarrow \quad l = \frac{p}{\rho g}$$

22. ***(c)*** $2h = ut$

$$u_x = \frac{2h}{\Delta t} \qquad [\because u = u_x,\ t = \Delta t] \ \ldots (i)$$

By equation of motion,

$$h = u_y t - \frac{1}{2} g t^2$$



$$\Rightarrow \quad gt^2 - 2u_y t + 2h = 0$$

$$\Rightarrow \quad t_1 = \frac{2u_y + \sqrt{4u_y^2 - 8gh}}{2g}$$

$$\Rightarrow \quad t_2 = \frac{2u_y - \sqrt{4u_y^2 - 8gh}}{2g}$$

$$\Rightarrow \quad \Delta t = t_1 - t_2 = \frac{\sqrt{4\,u_y^2 - 8gh}}{g}$$

$$\Rightarrow \quad u_y^2 = \frac{g^2(\Delta t)^2}{4} + 2gh \qquad \ldots (ii)$$

As, $u_x^2 + u_y^2 = u^2 = (2\sqrt{gh})^2$

From Eqs. (i) and (ii), we get

$$\frac{4h^2}{(\Delta t)^2} + \frac{g^2(\Delta t)^2}{4} + 2gh = 4gh$$

$$\frac{g^2}{4}(\Delta t)^4 - 2gh(\Delta t)^2 + 4h^2 = 0$$

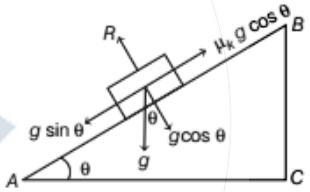
$$(\Delta t)^2 = \frac{2gh \pm \sqrt{4g^2h^2 - 4g^2h^2}}{g^2/2} = \frac{4h}{g}$$

$$\Rightarrow \quad \Delta t = 2\sqrt{\frac{h}{g}}$$

23. ***(b)*** $s = ut + \dfrac{1}{2}at^2,\ a = g\sin\theta,\ u = 0$

Then, $$s = 0 + \frac{1}{2}at^2$$

or $$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}}$$ [for smooth plane]



For the rough plane, the effective value of acceleration along the incline is

$$a' = g\sin\theta - \mu_k g\cos\theta$$

and $$t' = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta - \mu_k g\cos\theta}}$$

Now, $$\frac{t'}{t} = n = \sqrt{\frac{g\sin\theta}{g\sin\theta - \mu_k g\cos\theta}}$$

$$\Rightarrow \quad n = \sqrt{\frac{\sin 45^\circ}{\sin 45^\circ - \mu_k \cos 45^\circ}}$$

$$\Rightarrow \quad n = \sqrt{\frac{1}{1 - \mu_k}}$$

$$\Rightarrow \quad \frac{1}{n^2} = 1 - \mu_k$$

$$\Rightarrow \quad \mu_k = 1 - \frac{1}{n^2}$$

24. ***(c)*** Since, the initial activity is 50 times the activity for safe occupancy, therefore $R_0 = 50R$, where $R = \lambda N$.

Since, $R \propto N$

$$\frac{R}{R_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

or $$\left(\frac{1}{2}\right)^{t/10} = \frac{1}{50} \qquad [\because T_{1/2} = 10]$$

$$(2)^{t/10} = 50$$

Taking log both sides, we get

$$\Rightarrow \quad \frac{t}{10}\log_{10} 2 = \log_{10} 50$$

$$\Rightarrow \quad t = \frac{10\log_{10} 50}{\log_{10} 2} = \frac{10 \times 1.699}{0.301} = 56.4 \text{ days}$$

25. ***(a)*** Helium is monoatomic gas, for which

$$C_V = \frac{3}{2}R, C_p = \frac{5}{2}R$$

Work done by the gas in one complete cycle
W = area $ABCDA = p_0V_0$

From A to B,

Heat given to gas = $nC_V\Delta T$

$$= 1 \times \left(\frac{3}{2}R\right) \times \Delta T = \frac{3}{2}V_0(\Delta p) = \frac{3}{2}V_0p_0$$

From B to C, heat given to gas = $nC_p\Delta T$

$$= 1 \times \left(\frac{5}{2}R\right) \times \Delta T = \frac{5}{2}(2p_0)\Delta V = 5p_0V_0$$

$$\text{Efficiency of cycle} = \frac{\text{Work done by the gas / cycle}}{\text{Total heat given to gas / cycle}}$$

$$= \frac{p_0V_0}{\frac{3}{2}p_0V_0 + 5p_0V_0} = \frac{2}{13}$$

$$\text{Efficiency (\%)} = \frac{2}{13} \times 100 = 15.4\%$$

26. ***(a)*** Using Einstein's photoelectric equation

$$h\nu = \phi + eV_s$$

$$eV_s = h\nu - \phi \qquad \ldots (i)$$

Thus, when the frequency of incident light radiation (photons) ν increases to 2ν, the stopping potential V_s changes to V_s'.

$$eV_s' = h(2\nu) - \phi \qquad \ldots (ii)$$

On dividing Eq. (ii) by Eq. (i), we get

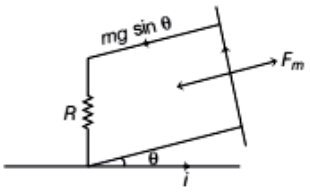
$$\frac{V_s'}{V_s} = \frac{2h\nu - \phi}{h\nu - \phi} = \frac{2h\nu - 2\phi + \phi}{h\nu - \phi}$$

$$\Rightarrow \quad = \frac{2(h\nu - \phi) + \phi}{h\nu - \phi} = 2 + \frac{\phi}{h\nu - \phi}$$

$$\therefore \quad \frac{V_s'}{V_s} > 2$$

$$\Rightarrow \quad V_s' > 2V_s$$

27. ***(c)*** Terminal velocity of the rod is attained when magnetic force on the rod (Bil) balances the component of weight of the rod ($mg\sin\theta$), as in figure.



So, $Bil = mg\sin\theta$

$$B\left(\frac{e}{R}\right)l = mg\sin\theta \qquad \left[\because i = \frac{e}{R}\right]$$

$$\Rightarrow \quad \frac{Ble}{R} = mg\sin\theta$$

$$\Rightarrow \quad \frac{Bl(Blv_T)}{R} = mg\sin\theta \qquad [\because e = Blv]$$

$$\Rightarrow \quad v_T = \frac{mg\ R\sin\theta}{B^2l^2}$$

28. ***(c)*** Initial energy of the asteroid is

$$E_i = K_i + U_i = \frac{1}{2}mv_i^2 - \frac{GM_em}{10R_e}$$

Final energy of the asteroid,

$$E_f = \frac{1}{2}mv_f^2 - \frac{GM_em}{R_e}$$

According to law of conservation of energy, $E_i = E_f$

$$\frac{1}{2}mv_i^2 - \frac{GM_em}{10R_e} = \frac{1}{2}mv_f^2 - \frac{GM_em}{R_e}$$

$$v_f^2 - \frac{2GM_e}{R_e} = v_i^2 - \frac{2GM_e}{10R_e}$$

$$\Rightarrow \quad v_f^2 = v_i^2 + \frac{2GM_e}{R_e}\left(1 - \frac{1}{10}\right)$$

29. ***(b)*** From Einstein photoelectric equation,

$$E = \phi_0 + \text{KE}_{\max}$$

For metal A, $4 = \phi_A + T_A$ … (i)

For metal B, $4.5 = \phi_B + (T_A - 1.5)$ …(ii)

From Eqs. (i) and (ii), we get

$$\phi_B - \phi_A = 2$$

Now, according to de-Broglie hypothesis,

$$\lambda_A = \frac{h}{mv} = \frac{h}{\sqrt{2mT_A}}$$

Similarly, $\lambda_B = \dfrac{h}{\sqrt{2mT_B}}$

$$\therefore \quad \frac{\lambda_A}{\lambda_B} = \sqrt{\frac{T_B}{T_A}} = \sqrt{\frac{T_A - 1.5}{T_A}} = \left(1 - \frac{1.5}{T_A}\right)^{1/2}$$

$$\left(\frac{1}{2}\right)^2 = 1 - \frac{1.5}{T_A}$$

On solving, we get $T_A = 2.0$ eV

So, $\phi_A = 4 - T_A = 4 - 2 = 2.0$ eV

$\phi_B = 6 - T_A = 6 - 2 = 4.0$ eV

30. ***(a)*** We have,

$$\text{Torque} = r \times F = I\alpha$$

So, $2(20t - 5t^2) = 10\alpha$

$$\alpha = 4t - t^2$$

But $\alpha = \frac{d\omega}{dt}$, so

$$\frac{d\omega}{dt} = 4t - t^2$$

$\Rightarrow$ $d\omega = (4t - t^2)\,dt$

Integrating, both sides, we get

$$\omega = 2t^2 - \frac{t^3}{3}$$

$\Rightarrow \omega$ will be zero at $t = 6$ s.

So, $\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

or $d\theta = \left(2t^2 - \frac{t^3}{3}\right)dt$

Again, integrating both sides, we get

$$\theta = \frac{2t^3}{3} - \frac{t^4}{12}$$

Since, $t = 6$ s

so, $\theta = \frac{2 \times 6^3}{3} - \frac{6^4}{12} = 36$

Number of turns, $n = \frac{\theta}{2\pi} = \frac{36}{2\pi} = 5.73$

So, option (a) is the correct

31. ***(c)***

Fig. (a)

Fig. (b)

In first case, as from Fig. (a), refraction of the rays, takes place from a plane surface, so we can use

$$d_{app} = \frac{d_{actual}}{\mu}$$

$$3 = 4/\mu \quad \Rightarrow \quad \mu = 4/3$$

where, d_{app} and d_{actual} = apparent and real depth, respectively.

Now, in second case, as from Fig. (b), refraction takes place from a spherical surface, so

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or $\frac{1}{(-25/8)} - \frac{4/3}{(-4)} = \frac{1 - 4/3}{-R}$

or $\frac{1}{3R} = \frac{1}{3} - \frac{8}{25} = \frac{1}{75}$

$\Rightarrow$ $R = 25$ cm

Now, using lens Maker's formula, to calculate focal length,

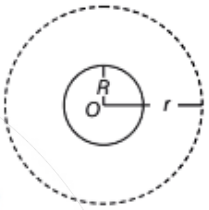
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\Rightarrow$ $\frac{1}{f} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{(-25)}\right) = \frac{1}{75}$

$\Rightarrow$ $f = 75$ cm

32. ***(c)*** When $r \le R$, then force on the test mass m at the surface of the sphere = mg

Force on the test mass at distance r from the centre of sphere is



If $r < R$, then $F = \frac{GMm}{R^2} \cdot \frac{r}{R} = \frac{GMm}{R^3} \cdot r$

$\therefore$ $\frac{mv^2}{r} = \frac{GMm}{R^3} \cdot r$

$\therefore$ $v \propto r$

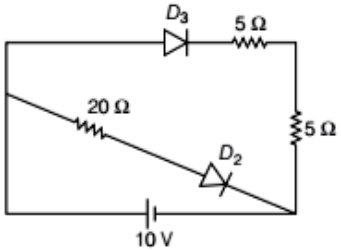
If $r > R$, then $F = \frac{GMm}{r^2}$

$\therefore$ $\frac{Mv^2}{r} = \frac{GMm}{r^2}$

$\therefore$ $v \propto \frac{1}{\sqrt{r}}$

Hence, option (*c*) is correct.

33. ***(c)*** In the given circuit, diode D_1 is reverse biased, so it will not conduct but D_2, D_3 are forward biased, so they will conduct, hence corresponding equivalent circuit. For the given circuit is



Now, the equivalent resistance of the circuit is

$$R_{eq} = \frac{(5+5)\times 20}{(5+5)+20} = \frac{20}{3}\ \Omega$$

Current through battery, $i = \dfrac{10}{\frac{20}{3}} = 1.5$ A

34. ***(a)*** Since, no external force is acting on the system, we can apply conservation of linear momentum.

Speed of 200 kg trolley $= \dfrac{36\times 1000}{60\times 60} = 10\ \text{ms}^{-1}$

If u be the initial velocity of trolley, v_b be the absolute velocity of the boy after the beginning of journey of the boy, their relative velocity is 4.

So, $\quad v' - v_b = 4 \quad \Rightarrow \quad v_b = (v' - 4)$

Now, applying law of conservation of momentum,

Momentum before the boy begins to run

= Momentum after the beginning of boy's running

$$\Rightarrow \quad 220\times 10 = 200\,v' + 20(v' - 4)$$

$$2200 = 220v' - 80$$

$$\Rightarrow \quad 220v' = 2280$$

$$v' = \frac{2280}{220} = 10.36\ \text{ms}^{-1}$$

35. ***(c)*** Given, $C = 37.55\ \text{J mol}^{-1}\ \text{K}^{-1}$

Also, $pT = \text{constant}\ (k)$ … (i)

According to ideal gas equation, $pV = RT$

$$\Rightarrow \quad p = \frac{RT}{V} \quad \text{… (ii)}$$

Putting the value of p in Eq. (i), we get

$$\frac{RT}{V}\times T = k \ \Rightarrow\ V = \frac{RT^2}{k}$$

On differentiating above equation both sides, we get

$$\frac{dV}{dT} = \frac{2RT}{k} \quad \text{… (iii)}$$

But $\quad \dfrac{T}{k} = \dfrac{1}{p}$ [from Eq. (i)]

Hence, Eq. (iii) becomes

$$\frac{dV}{dT} = \frac{2R}{p}$$

So, $\quad C = C_V + \dfrac{p\,dV}{dT}$

or $\quad C = C_V + \dfrac{p\times 2R}{p} = C_V + 2R$

or $\quad C_V = C - 2R \quad \text{… (iv)}$

As $C_V = \dfrac{nR}{2}$, where n = number of degrees of freedom.

Putting the value of C_V in Eq. (iv), we get

$$\frac{nR}{2} = C - 2R$$

$$n = \frac{2(C-2R)}{R} = \frac{2\,(37.55 - 2\times 8.3)}{8.3}$$

$$= 5.048 = \simeq 5$$

36. ***(a)*** We know, instantaneous displacement, $x = r\sin\omega t$

$\because$ Instantaneous velocity, $v = \dfrac{dx}{dt} = r\omega\cos\omega t$

$\because$ Instantaneous acceleration, $a = \dfrac{dv}{dt}$

$$= -r\omega^2\sin\omega t = -\omega^2 x$$

So, $\quad \dfrac{aT}{x} = \dfrac{-\omega^2 x\times T}{x} = -\omega^2 T = \dfrac{-4\pi^2}{T^2}\times T$

$$= \frac{-4\pi^2}{T} = \text{constant}$$

$$\Rightarrow \quad \frac{aT}{v} = \frac{-\omega^2 r\sin\omega t\times T}{\omega r\cos\omega t} = -\omega T\tan\omega T$$

$$= -\frac{2\pi}{T}\times T\tan\omega t = \text{not constant}$$

Similarly, $aT + 2\pi v$ and $a^2T^2 + 4\pi^2v^2$ is also not constant, i.e. both are function of t.

37. ***(d)*** New, current sensitivity, $I_s' = I_s + \dfrac{50}{100}I_s$

$$= \frac{150}{100}I_s = \frac{3}{2}I_s$$

New resistance, $R' = 2R$

Initial voltage sensitivity, $V_s = \dfrac{I_s}{R}$

Now, new voltage sensitivity,

$$V_s' = \frac{I_s'}{R'} = \frac{\frac{3}{2}I_s}{2R} = \frac{3}{4}V_s$$

% decrease in voltage sensitivity

$$= \frac{V_s - V_s'}{V_s}\times 100$$

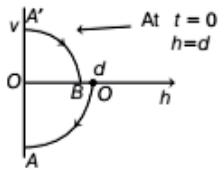
$$\Rightarrow \quad = \left(1 - \frac{V_s'}{V_s}\right)\times 100$$

$$\therefore \quad = \left(1 - \frac{3}{4}\right)\times 100 = 25\%$$

38. ***(a)*** For the uniformly accelerated/decelerated motion,

$$v^2 = u^2 \pm 2gh$$

So, from this equation, we can say v-h graph is parabola.



Initially, velocity is downwards (– ve), after collision, it reverses the direction with smaller magnitude and velocity is upwards (+ ve). So, graph (a) satisfies these conditions.

Also,

When $t = 0$, $h = d$

Velocity increases downwards $(0 \rightarrow A)$

When $t = 1$, velocity reverses its direction $(A' \rightarrow B)$.

39. ***(b)*** From the given circuit, we can say that I is independent of resistance R_5, so no current flows through R_5. This required that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4 ($\therefore$ Wheatstone bridge condition)

So, $$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{or} \quad R_1 R_4 = R_3 R_2$$

40. ***(d)*** Since, volume remains unchanged, during this phenomenon, so

$$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$$

$$N = \frac{R^3}{r^3}$$

Now, change in surface area $= 4\pi R^2 - N 4\pi r^2$

$$= 4\pi (R^2 - Nr^2)$$

Energy released $(\Delta U) = T \times$ change in surface area

$$= T \times 4\pi [R^2 - Nr^2]$$

Here, all this energy released is at the cost of lowering the temperature and mass of the big drop of liquid $= \frac{4}{3}\pi R^2 \rho$.

Now, change in temperature,

$$\Delta\theta = \frac{\Delta U}{MS} = \frac{T \times 4\pi(R^2 - Nr^2)}{\left(\frac{4}{3}\pi R^3 \rho\right) S}$$

$$\Rightarrow \quad = \frac{3T}{\rho S}\left(\frac{1}{R} - \frac{Nr^2}{R^3}\right)$$

$$\Rightarrow \quad = \frac{3T}{\rho S}\left(\frac{1}{R} - \frac{R^3 \times r^2}{r^3 \times R^3}\right)$$

$$\Rightarrow \quad \Delta\theta = \frac{3T}{\rho S}\left(\frac{1}{R} - \frac{1}{r}\right)$$

Chemistry

41. ***(d)*** Edge length of the unit cell $= 2\text{Å} = 2 \times 10^{-8}$ cm

Volume of unit cell $= a^3 = (2 \times 10^{-8})^3 = 8 \times 10^{-24}$ cm^3

Mass of one unit cell = volume × density

$$= 8 \times 10^{-24} \times 2.5$$

Number of unit cell in 200 g of metal

$$= \frac{\text{Mass of the metal}}{\text{Mass of one unit cell}} = \frac{200}{8 \times 10^{-24} \times 2.5} = 1 \times 10^{25}$$

42. ***(a)*** Higher the value of 'a', more will be the tendency to get liquefy. Since, value of a is highest for gas P. Thus, it is the most liquefiable gas among the given gases.

43. ***(d)*** Radius of hydrogen like atom,

$$r_n = \frac{n^2}{Z} r_o$$

where, $r_o = 0.51 \times 10^{-10}$ m

and $$r_n = \frac{0.51 \times 10^{-10}\text{ m}}{4}$$

In the ground state, $n = 1$

$$\therefore \quad \frac{0.51 \times 10^{-10}}{4} = \frac{(1)^2}{Z} \times 0.51 \times 10^{-10}$$

$$\therefore \quad Z = 4$$

So, the atom is triply ionised beryllium (Be^{3+}).

44. ***(a)*** When central metal ion is same (here Ni^{2+}), the absorption of colour depends on the ligand.

According to spectrochemical series, CFSE values of the ligands increases is as follows

$$I^- < Br^- < Cl^- < NO_3^- < F^- < H_2O$$
$$< NH_3 < NO_2^- < CN^- < CO$$

Thus, H_2O is the weakest ligand among these, therefore the absorbed energy will be lowest in $[Ni(H_2O)_6]^{2+}$, so it will absorb highest wavelength (red light).

As, $$E \propto \frac{1}{\lambda}$$

$\therefore$ The order of increasing wavelength is

$[Ni(NO_2)_6]^{4-} < [Ni(NH_3)_6]^{2+} < [Ni(H_2O)_6]^{2+}$.

45. ***(c)*** Empirical formula mass $= CH_2O = 12 + 2 + 1 \times 16$

$$= 30 \text{ g}$$

Molecular mass = 180 g

$$n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}} = \frac{180}{30} = 6$$

$\because$ Molecular formula = $n \times$ empirical formula

$\therefore$ Molecular formula $= 6 \times CH_2O = C_6H_{12}O_6$

46. ***(d)*** Statements (d) is incorrect. Whereas all other statements are correct.

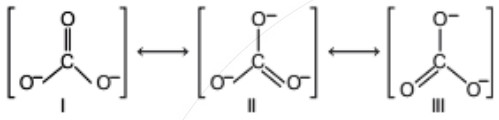
Corrected statement

Al_2O_3 serves as the electrolyte, undergoing the redox process. Na_3AlF_6 although is an electrolyte but serves as a solvent, not electrolyte.

47. ***(c)*** Azo dye test is not used for the distinction of 1°, 2° and 3° aliphatic amines. The other tests can be easily used to distinguish between 1°, 2° and 3° amines.

	Test	Primary amine	Secondary amine	Tertiary amine
(a)	Hinsberg's reagent test	Forms a sulphonamide soluble in alkali	Forms a sulphonamide insoluble in alkali	No reaction
(b)	Carbylamine reaction	Forms a carbylamine with unpleasant smell	No reaction	No reaction
(d)	Reaction with nitrous acid	Forms primary alcohol and evolves nitrogen gas with effervescence	Forms nitrosamine which gives Libermann's nitroso amine reaction	Forms nitrite salt

48. ***(b)*** Resonating structures of carbonate CO_3^{2-} ion are



I II III

Thus, there is no coordinate bond present in CO_3^{2-}

Hence, the second statement is incorrect.

49. ***(a)*** For first order reaction,

$$t_{1/2}=\frac{\ln 2}{k_1}=40\,s \qquad \ldots(i)$$

For zero order reaction,

$$t_{1/2}=\frac{[A]_0}{2k_0}=20\,s \qquad \ldots(ii)$$

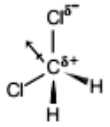
Divide Eq. (i) by Eq. (ii), we get

$$\frac{1}{2}=\frac{[A]_0}{2k_0}\times\frac{k_1}{\ln 2}$$

$$\frac{k_1}{k_0}=\frac{\ln 2}{[A]_0}=\frac{0.693}{1.386}=0.5\ mol^{-1}\ dm^3$$

50. ***(b)*** CH_2Cl_2 is similar to CH_4 (tetrahedral) but it has three different bond angles.

H—C—H, H—C—Cl, Cl—C—Cl, which are close to but not equal to 109.5°. Since, electronegativity of Cl > C > H the bond dipole moments do not cancel and the molecule has a dipole moment.



51. ***(b)*** Given, $CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$;

$$\Delta H=-210\ kcal/mol \qquad \ldots(i)$$

$$C_2H_6 + \frac{7}{2}O_2 \longrightarrow 2CO_2 + 3H_2O\ ;$$

$$\Delta H=-368\ kcal\ /mol \qquad \ldots(ii)$$

On subtracting Eq. (i) from Eq (ii), we get

$$CH_2 + \frac{3}{2}O_2 \longrightarrow CO_2 + H_2O\ ;$$

$$\Delta H=-158\ kcal/mol$$

$\therefore$ Enthalpy of combustion of one CH_2 unit

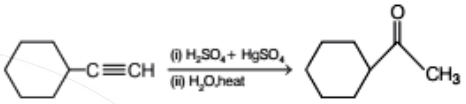
$$=-158\ kcal/mol$$

$$\Delta H_{comb}(C_{10}H_{22})=\Delta H_{comb}(CH_4)+9\times\Delta H_{comb}(CH_2)$$

$$=-210+(9\times-158)$$

$$=-1632\ kcal$$

52. ***(b)*** $R-C\equiv CH$ is converted to ketone by catalytic hydration with (i) $H_2SO_4 + HgSO_4$, (ii) H_2O, heat



53. ***(c)*** By Heisenberg's uncertainty principle,

$$\Delta x\times\Delta P\geq h/4\pi$$

$$\Delta x\times m\Delta v\geq h/4\pi$$

$$\text{So, } \Delta x\geq\frac{h}{4\pi m\Delta v} \qquad \ldots(i)$$

Given, Δv = 0.005% or 600 m/s

$$=\frac{600\times0.005}{100}=0.03$$

On putting all values we get,

$$\therefore\ \Delta x\times9.1\times10^{-31}\times0.03=\frac{6.6\times10^{-34}}{4\times3.14}$$

$$\Rightarrow\Delta x=\frac{6.6\times10^{-34}}{4\times3.14\times0.03\times9.1\times10^{-31}}=1.92\times10^{-3}\ m$$

54. ***(d)*** In O_2 molecule, two electrons are present in antibonding orbitals.

$$O_2(8+8=16)=\sigma1s^2,\sigma^*1s^2,\sigma2s^2\sigma^*2s^2,$$

$$\sigma(2p_z)^2,\pi2p_x^2=\pi2p_y^2,\pi^*2p_x^1=\pi^*2p_y^1$$

Removal of one electron from the O_2 molecule gives O_2^+ in which the number of antibonding electrons is one less and hence, BO (Bond order) increases. Thus, removal of one electron from O_2 stabilises the molecule.

55. ***(c)*** According to Faraday's first law, when an electric current is passed through an electrolyte, amount of substance deposited is directly proportional to the quantity of electric charge passed through the electrolyte.

For the reaction,

$$Cu^{2+} + 2e^- \longrightarrow Cu \qquad \ldots(i)$$

1 mol of 63.5g of Cu is obtained by passing 2F of electricity.

Mass of Cu obtained by passing 1F of electricity = $\frac{63.5}{2}$ g

∴ Mass of Cu obtained by passing 0.5F of electricity

$$= \frac{63.5}{2} \times 0.5 = 15.80 \text{ g}$$

56. ***(c)*** The sequence of reactions are as follows :

$$CH_3-\underset{\underset{\text{2-propanol}}{\underset{(Z)}{OH}}}{CH}-CH_3 \xrightarrow{PCl_5} CH_3-\underset{\underset{\text{2-chloropropane}}{\underset{(X)}{Cl}}}{CH}-CH_3 \xrightarrow{\text{Alc. KOH}}$$

$$CH_3-\underset{\underset{\text{2-propanol}}{\underset{(Z)}{OH}}}{CH}-CH_3 \xleftarrow[\text{conc. } H_2SO_4]{2H_2O/\Delta} CH_3-CH=CH_2 \quad \underset{(Y)}{\text{Propene}}$$

57. ***(d)*** $\underset{(0.28)}{N_2O_4(g)} \rightleftharpoons \underset{(1.1)}{2NO_2(g)}$

Pressure at equilibrium

$$K_p = \frac{p^2_{NO_2}}{p_{N_2O_4}} = \frac{(1.1)^2}{0.28} = 4.32 \text{ atm}$$

If volume of the container is doubled, the pressure will reduced to half

$$N_2O_4 \rightleftharpoons 2N_2O$$

$$p_{N_2O_4} = \frac{0.28}{2} = 0.14 - x \text{ atm}$$

$$p_{NO_2} = \frac{1.1}{2} = 0.55 + 2x \text{ atm}$$

$$K_p = \frac{[p_{NO_2}]}{[p_{N_2O_4}]}$$

$$K_p = \frac{(0.55+2x)^2}{(0.14-x)} = 4.32$$

$$0.30 + 1.1x + 4x^2 = 0.605 - 4.32x$$

$$4x^2 + 5.42x - 0.302 = 0$$

This is quadratic equation with solution

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{(5.42) \pm \sqrt{(5.42)^2 - 4(4)(-0.302)}}{2(4)}$$

$$x = -\frac{5.42 \pm 5.85}{8}$$

$$x = 0.045$$

or $x = -1.41$

(Negative value is discarded as it will lead to negative value of pressure.)

So, $x = 0.045$

$$p_{N_2O_4} = 0.14 - x = 0.14 - 0.045 = 0.095 \text{ atm}$$

$$p_{NO_2} = 0.55 + 2x = (0.55 + 2 \times 0.045) = 0.64 \text{ atm}$$

The new equilibrium pressure of the two gases are 0.095 and 0.64 atm respectively.

58. ***(a)*** Protactinium (Pa) is the element that does not have stable electronic configuration.

Protactinium (Pa) = $[Rn]5f^2 6d^1 7s^2$

(Atomic number = 91)

Nobelium (No) = $[Rn]5f^{14} 6d^0 7s^2$

(Atomic number = 102)

Lawrencium (Lr) = $[Rn]5f^{14} 6d^1 7s^2$

(Atomic number = 103)

Amercium (Am) = $[Rn]5f^7 6d^0 7s^2$

(Atomic number = 95)

59. ***(c)*** **Oxidation**

$$H_3C-CHO + 3\bar{O}H \longrightarrow CH_3COO^- + 2e^- + 2H_2O \quad \ldots(i)$$

Reduction $[Ag(NH_3)_2]^+ + e^- \longrightarrow Ag + 2NH_3] \times 2$

$$\therefore CH_3CHO + 2[Ag(NH_3)_2]^+ + 3\bar{O}H \longrightarrow CH_3C\bar{O}O + 2H_2O + 2Ag + 4NH_3$$

Molecular mass (M) of $CH_3CHO = 44 \text{ g mol}^{-1}$

$$\text{Equivalent weight} = \frac{M}{n\text{-factor}}$$

$$\omega = 44/2 = 22 \text{ g mol}^{-1}$$

∴ Acetaldehyde is reducing agent.

∴ Statement (c) is incorrect.

60. ***(a)*** According to Freundlich adsorption isotherm,

$$x/m = kp^{1/n}$$

where, x = mass of gas adsorbed on mass 'm' of the absorbent at pressure p.

k and n = constants.

Taking logarithm on both sides, we get

$$\log x/m = \log k + \frac{1}{n}\log p$$

This is equation of a straight line with slope of $1/n$ and intercept of log k.

61. ***(b)*** Given, pH of $Ba(OH)_2 = 12$

$$\because \quad pH + pOH = 14$$

$$\therefore \quad pOH = 14 - pH = 14 - 12 = 2$$

Now, $pOH = -\log[OH^-]$

$$\Rightarrow \quad 2 = -\log[OH^-]$$

$$[OH^-] = 10^{-2}$$

As conc. of Ba^{2+} is half of OH^- i.e. $[Ba^{2+}] = 0.5 \times 10^{-2}$

$$\underset{S \text{ mol}^{-1}}{Ba(OH)_2(s)} \rightleftharpoons \underset{S}{Ba^{2+}} + \underset{2S}{2OH^-}$$

$$K_{sp} = [Ba^{2+}][OH^-]^2$$

$$\Rightarrow \quad [0.5 \times 10^{-2}][10^{-2}]^2 = 0.5 \times 10^{-6}$$

$$= 5 \times 10^{-7} \text{M}$$

62. ***(c)*** With Tollen's reagent (ammoniacal $AgNO_3$), a white precipitate of silver salt is obtained on reaction with C_2H_2 (alkyne).

$$\underset{\text{Alkyne}}{H-C\equiv C-H} + 2AgNO_3 + 2NH_4OH \longrightarrow \underset{\text{Silver acetylide (white ppt.)}}{Ag-C\equiv C-Ag} + 2NH_4NO_3 + H_2O$$

63. ***(d)*** Arrhenius equation is given by,

$$\log_{10}\frac{K_2}{K_1}=\frac{E_a}{2.303\times R}\left[\frac{T_2-T_1}{T_1T_2}\right]$$

Given, $\frac{K_2}{K_1}=3;\ R=8.314\ \text{JK}^{-1}\text{mol}^{-1}$

$T_1 = 20+273 = 293\ \text{K}$

and $T_2=50+273=323\ \text{K}$

Substituting the given values in Arrhenius equation,

$$\log_{10}3=\frac{E_a}{8.314\times2.303}\left[\frac{323-293}{323\times293}\right]$$

$$E_a=\frac{2.303\times8.314\times323\times293\times0.477}{30}$$

$$= 28811.8\ \text{J mol}^{-1}$$

$$= 28.8118\ \text{kJ mol}^{-1}$$

64. ***(b)*** Compounds containing both N and S give blood red colour in Lassaigne's test due to the formation of $Fe(SCN)_2$. Thus, $H_2N(C_6H_4)SO_3H$ gives blood red colour in Lassaigne's test of nitrogen.

65. ***(b)*** Given that,

$$A + B^{2+} \longrightarrow B + A^{2+},\ E^\circ = 0.2955\text{V}$$

$n = 2,\ T = 298\ \text{K},\ E^\circ = 0.2955\ \text{V}$

Using Nernst equation,

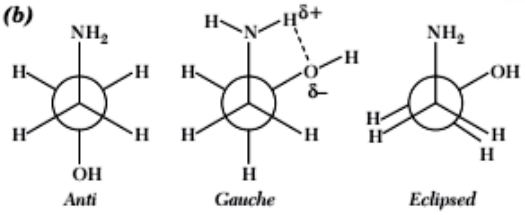
$$E^\circ = \frac{2.303nRT}{nF}\log K_{eq}$$

$$E^\circ = \frac{0.0591}{n}\log K_{eq}\ (\text{at } 25^\circ\text{C})$$

$$\Rightarrow \quad 0.2955 = \frac{0.0591}{2}\log K_{eq}$$

$$K_{eq} = 10^{10}$$

66. ***(b)***



Since, gauche form is stabilised by intermolecular hydrogen bonding, hence it is more stable than anti form while eclipsed form is least stable due to high angular strain.

∴ The correct order is, *gauche* > *anti* > *eclipsed*.

67. ***(a)*** In acidic medium, H_2S gas is very feebly ionised giving very small concentration of sulphide ion for precipitation. Therefore, the most insoluble salts CuS and HgS are precipitated. Since the solubility product of Mn^{2+}, Ni^{2+} are higher thus they remain unaffected.

68. ***(b)*** When a mixture of BCl_3 and H_2 is subjected to silent electric discharge, diborane and HCl are formed.

$$\underset{\text{Boron trichloride}}{2BCl_3} + 6H_2 \xrightarrow[\text{discharge}]{\text{Electric}} \underset{(x)}{B_2H_6} + 6HCl$$

Diborane reacts with NH_3 at 200°C to give borazine (inorganic benzene).

$$\underset{(x)}{3B_2H_6} + \underset{\text{Ammonia}}{6NH_3} \xrightarrow{200^\circ\text{C}} \underset{\text{Borazine } (y)}{2B_3N_3H_6} + 12H_2$$

69. ***(c)*** The process of producing *syn* gas or synthesis gas from coal is called 'coal gasification'.

$$\underset{\text{Coal}}{C(s)} + \underset{\text{Steam}}{H_2O(g)} \xrightarrow[\text{Ni}]{1270\ \text{K}} \underbrace{CO(g) + H_2(g)}_{\text{Synthesis gas}}$$

The production of hydrogen can be increased by reaching carbon monoxide of the *syn* gas with steam in the presence of iron chromate as a catalyst at 673 K.

$$CO(g) + H_2O(g) \xrightarrow{FeCrO_4,\ 673\ \text{K}} CO_2(g) + H_2(g)$$

CO_2 is removed by scrubbing with a solution of sodium arsenite.

70. ***(b)*** Heat at constant pressure means enthalpy. i.e.

$$\Delta H = -2.5\times10^3\ \text{cal}$$

$$\Delta S = 7.4\ \text{cal deg}^{-1}$$

$$T = 298\ \text{K}$$

$$\Delta G = \Delta H - T\Delta S$$

$$= -2.5\times10^3 - 298\times7.4$$

$$= -4705\ \text{cal}$$

The value of change in Gibbs free energy (ΔG) is negative.

Hence, the process is spontaneous.

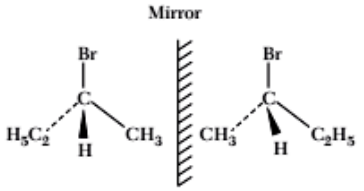
71. ***(a)*** The alkene is unsymmetrical, hence will follow Markownikoff's rule to give major product.

$$\underset{\text{But-1-ene}}{CH_3-CH_2-CH=CH_2} + H-Br \longrightarrow$$

$$\underset{\substack{(x)\\\text{Major}}}{CH_3-CH_2-CHBr-CH_3} + C_2H_5-\overset{\overset{Br}{|}}{\underset{\underset{H}{|}}{C^*}}-CH_3 \quad \underset{\substack{(y)\\\text{Major}}}{}$$

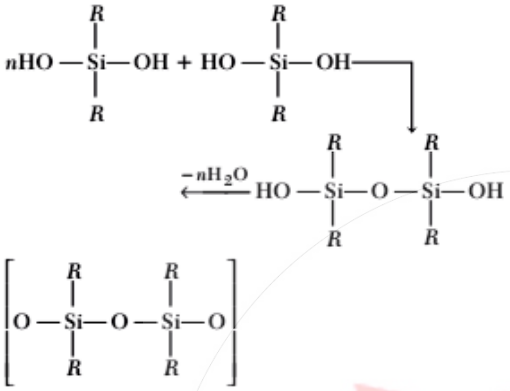
$$+ \underset{\substack{(z)\ \text{Minor}}}{CH_3-CH_2-CH_2-CH_2Br}$$

Since, *y* contains, a chiral carbon, it exists in two enantiomers (*x* and *y*) which are mirror images of each other.



72. ***(d)*** Silicones are synthetic organo-silicon polymers containing repeated R_2SiO units. Since the empirical formula is same as that of a ketone (R_2CO), the name silicon has been given to these materials. They can be formed by hydrolysis of dichlorosilanes (R_2SiCl_2).

$$R_2SiCl_2 + 2H_2O \longrightarrow R_2Si(OH)_2 + 2HCl$$



73. ***(c)*** On acidification of potassium chromate solution, yellow colour changes to orange colour due to formation of dichromate which suggests that monocentric complex is converted into dicentric complex.

$$\underset{\text{Potassium chromate}}{2K_2CrO_4} + \underset{\text{(Dil.)}}{H_2SO_4} \longrightarrow \underset{\text{Potassium dichromate}}{K_2Cr_2O_7} + K_2SO_4 + H_2O$$

74. ***(d)*** Valence electrons in $A = 3$.

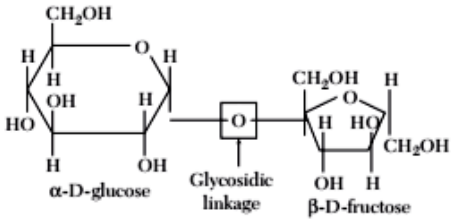
Valence electron in $B = 6$.

Thus, A is electropositive and B is electronegative, A can loose three electrons and B can gain two electrons to attain stable configuration

Hence, A exist as A^{3+} and B as B^{2-}

$\therefore$ Compound formed is A_2B_3.

75. ***(b)*** The non-reducing sugar is



This structure represents sucrose in which α-D-glucose and β-D fructose is attached to each other by $C_1 - C_2$ glycosidic linkage. Since reducing groups of glucose and fructose are involved in glycosidic bond formation, this is considered as non-reducing sugar.

76. ***(a)*** Potassium sulphate, $i = 3$

Sodium chloride, $i = 2$

Urea, $i = 1$

Glucose, $i = 1$

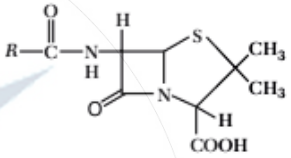
Depression is freezing point is given by

$$\Delta T_f = imK_f$$

$\therefore$ Greater the value of i, greater lowering in freezing point and hence, lower will be the freezing temperature.

Therefore, potassium sulphate (K_2SO_4) solution has the lowest freezing point.

77. ***(a)*** A penicillin is a member of a family of drugs that have a four membered cyclic amide fused to a five membered thiazole ring. It is narrow spectrum antibiotic



Penicillin

78. ***(b)*** The lattice energy of the hydroxides of alkaline earth metal decreases more rapidly than their hydration energy leading to more negative value of ΔH_{sol} down the group. More negative is ΔH_{sol}, more is the solubility of compounds. Hence, the solubility of hydroxides of alkaline earth metal increases with their atomic number.

79. ***(a)*** The given chemical reaction is

$$5Br^-(aq) + BrO_3^-(aq) + 6H^+(aq) \longrightarrow 3Br_2(aq) + 3H_2O\,(l)$$

Rate law expression for the given reaction can be written as

$$-\frac{1}{5}\frac{\Delta[Br]^-}{\Delta t} = -\frac{\Delta[BrO_3^-]}{\Delta t}$$

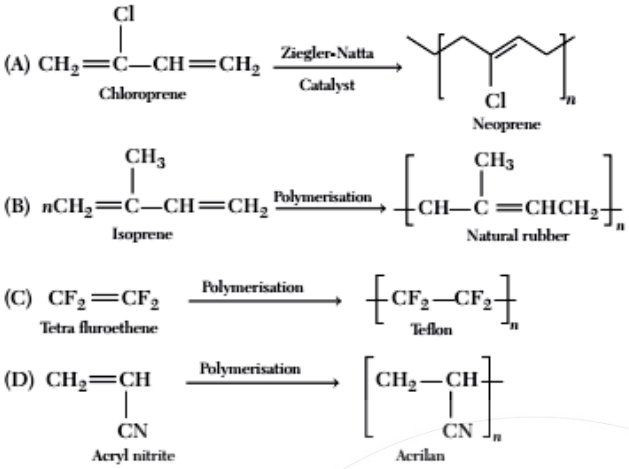
$$-\frac{1}{6}\frac{\Delta[H^+]}{\Delta t} = +\frac{1}{3}\frac{\Delta[Br_2]}{\Delta t}$$

$$\Rightarrow \quad -\frac{\Delta[Br^-]}{\Delta t} = -\frac{5\Delta[BrO_3^-]}{\Delta t}$$

$$= -\frac{5}{6}\frac{\Delta[H^+]}{\Delta t}$$

$$\Rightarrow \quad \frac{\Delta[Br^-]}{\Delta t} = \frac{5}{6}\frac{\Delta[H^+]}{\Delta t}$$

80. *(c)* The polymerisation reaction of the given monomers to form respective polymers are given below :
A-III, B-I, C-II, D-IV



a. English Proficiency

81. *(a)* Use 'is' in place of 'are' as when two subjects are joined 'along with', then helping verb is used according to first subject.

82. *(c)* Use 'attested' in place of 'attest' as sentence is in passive voice.

83. *(b)* Use 'making' in place of 'having' as 'have' is not used in progressive sense.

84. *(d)* 'Superior' is followed by 'to' preposition.

85. *(d)* 'Cope' always agree with preposition 'with' and use of any other auxiliary term is improper.

86. *(a)* 'Dejected' means very unhappy. 'Jubilant' is its opposite meaning word which means extremely happy.

87. *(a)* 'Retrograde' means making a situation worse. 'Progressive' is its opposite meaning word which means happening or developing steadily.

88. *(a)* 'Belittle' means to decrease the importance of something of somebody. 'Exalt' is its opposite meaning word which means to make somebody rise to a higher rank or position.

89. *(b)* 'Apprehend' means to grasp with the understanding. So, it is the nearest, meaning word of 'make out anything'.

90. *(b)* 'Scathing' means expressing a very strong negative opinion about somebody or something. 'Scornful' is also nearest meaning word of 'scathing' as its means feeling or showing disgust and anger to somebody or something.

91. *(b)* The ordinary megalomaniac does not have the talent to realise his wish which Alexander the Great had.

92. *(d)* Megalomaniac differ from a narcissist by wishing to be powerful and feared.

93. *(d)* In the given line, 'this' refers to excessive lover of power.

94. *(c)* The nurturing of only one element in human nature, is the reason for unhappiness.

95. *(c)* Love of power to be accepted because it is a part of human nature.

b. Logical Reasoning

96. *(d)* 'Umpire' is required to give decision in 'Match'. Likewise 'Judge' is required to give decision in a 'Lawsuit'.

97. *(c)* As, $9 - 2 = 7 \Rightarrow 7^2 = 49$

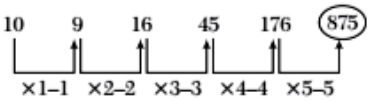
$13 - 2 = 11 \Rightarrow 11^2 = 121$

$10 - 2 = 8 \Rightarrow 8^2 = 64 \neq 61$

$7 - 2 = 5 \Rightarrow 5^2 = 25$

Thus, option (c) is odd.

98. *(d)* The pattern is as follows



99. *(d)* The data is inadequate because it is not given that who is taller between Virat and Shikhar. Hence, answer cannot be determined.

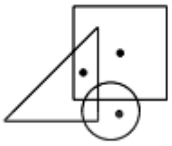
100. ***(c)*** Option (c) will complete the given pattern as follow.

101. ***(b)*** Options (b) can be formed from the pieces given in problem figures.

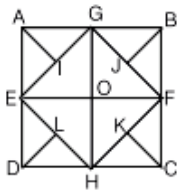
102. ***(a)*** After folding and cutting the paper, answer figure (a) will appear.

103. ***(d)*** The third figure is each row comprises of parts which are not common in the first two figures.

104. ***(b)*** Correct answer figure is (b).



105. ***(b)*** The figure in the question may be labelled as shown below



There are nine triangles in the upper half of the figure ABFE = ΔAEI, ΔAIG, ΔAEG, ΔGEO, ΔGBJ, ΔBFJ, ΔGBF, ΔGOF and ΔGEF

Similarly, there are nine triangles in the lower half figure, i.e. EFCD.

There are two more triangles ΔEGH and ΔFGH.

Hence, there are a total of 20 triangles.

Mathematics

106. ***(b)*** $(1+x)^n\left(1+\frac{1}{x}\right)^n=\left[(2+x)+\frac{1}{x}\right]^n$

$[\because a^n\cdot b^n=(a\cdot b)^n]$

$={}^nC_0(2+x)^n\cdot\left(\frac{1}{x}\right)^0+\ldots+{}^nC_n\cdot\frac{1}{x^n}$

$\therefore$ Coefficient of $x^{-n}={}^nC_n=1$

107. ***(c)*** $T_{r+1}=\sqrt{3}\cdot{}^{20}C_r\left(\frac{1}{\sqrt{3}}\right)^r$

$\Rightarrow \quad T_r=\sqrt{3}\cdot{}^{20}C_{r-1}\left(\frac{1}{\sqrt{3}}\right)^{r-1}$

Now, $\frac{T_{r+1}}{T_r}=\frac{20-r+1}{r}\left(\frac{1}{\sqrt{3}}\right)$

Since, $T_{r+1}\geq T_r$

$\Rightarrow \quad 20-r+1\geq\sqrt{3}r$

$\Rightarrow \quad r\leq\frac{21}{\sqrt{3}+1}=\frac{21}{2.73}$

$\Rightarrow \quad r\leq 7.692 \quad\Rightarrow\quad r=7$

$\therefore$ The greatest term is $T_8=\sqrt{3}\cdot{}^{20}C_7\left(\frac{1}{\sqrt{3}}\right)^7$

$=\frac{25840}{9}$

108. ***(b)*** Let $x=1$

$=\cos 0°+i\sin 0°$

$=\cos 2r\pi+i\sin 2r\pi=e^{i\,2r\pi}$

$\Rightarrow \quad x^{1/n}=e^{i(2r\pi)/n};\ r=0,1,2,\ldots$

Then, the roots are $1, e^{\frac{2\pi i}{n}}, e^{\frac{4\pi i}{n}}, \ldots\ldots$

Which are clearly in GP with common ratio $e^{2\pi i/n}$.

109. ***(a)*** Given, $P=\begin{bmatrix}a&b&c\\b&c&a\\c&a&b\end{bmatrix}$

$\therefore \quad P^T=\begin{bmatrix}a&b&c\\b&c&a\\c&a&b\end{bmatrix}$

Also given, $P^TP=I$

$\therefore \quad |P^TP|=|I|=1$

or $\begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}^2=1 \quad\Rightarrow\quad \begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}=\pm 1$

or $3abc-a^3-b^3-c^3=\pm 1$

or $3\mp 1=a^3+b^3+c^3$

$\therefore \quad a^3+b^3+c^3=2 \text{ or } 4$

110. ***(d)*** Given, $x^2=\begin{vmatrix}\sin\theta&\cos\theta&0\\-\cos\theta&\sin\theta&1\\\sin\theta&\cos\theta&2\end{vmatrix}$

On expanding $\begin{vmatrix}\sin\theta&\cos\theta&0\\-\cos\theta&\sin\theta&1\\\sin\theta&\cos\theta&2\end{vmatrix}$ along C_3,

we get

$x^2=0-1\begin{vmatrix}\sin\theta&\cos\theta\\\sin\theta&\cos\theta\end{vmatrix}+2\begin{vmatrix}\sin\theta&\cos\theta\\-\cos\theta&\sin\theta\end{vmatrix}$

$$= -1(\sin\theta\cos\theta - \sin\theta\cos\theta) + 2(\sin^2\theta + \cos^2\theta)$$

$$= -1 \times 0 + 2 \times 1$$

$\Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

If $x = \sqrt{2}$, then

$$4x^2 + x\sin\frac{3\pi}{2} + 5 = 4 \times (\sqrt{2})^2 - \sqrt{2} + 5$$

$$= 8 - \sqrt{2} + 5 = (13 - \sqrt{2})$$

If $x = -\sqrt{2}$, then $4x^2 + x\sin\frac{3\pi}{2} + 5$

$$= 4 \times (-\sqrt{2})^2 + \sqrt{2} + 5$$

$$= 8 + \sqrt{2} + 5 = (13 + \sqrt{2})$$

111. ***(d)*** Given, a, b, c are in GP and $\log a - \log 2b$, $\log 2b - \log 3c$ and $\log 3c - \log a$ are in AP.

$\therefore \quad b^2 = ac$

and $2(\log 2b - \log 3c) = \log a - \log 2b + \log 3c - \log a$

$\Rightarrow \quad b^2 = ac$ and $2b = 3c$

$\Rightarrow \quad c = \frac{4a}{9}$ and $b = \frac{2a}{3}$

$\because \quad a + b = \frac{5a}{3} > c, b + c = \frac{10a}{9} > a$

and $c + a = \frac{13a}{9} > b.$

$\therefore$ a, b, c are the sides of a triangle.

Also, a is the greatest side

$\therefore \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} = -\frac{29}{48} < 0$

$\therefore$ ΔABC is an obtuse angled triangle.

112. ***(c)*** Given, $\sum_{r=1}^{n} t_r = \frac{n(n+1)(n+2)(n+3)}{8} = S_n$ (say)

$\therefore \quad \sum_{r=1}^{n-1} t_r = \frac{(n-1)n(n+1)(n+2)}{8} = S_{n-1}$

Now, $t_n = S_n - S_{n-1} = \frac{n(n+1)(n+2)}{2}$

$$\therefore \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{t_r} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2}{n(n+1)(n+2)}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right)$$

$$= -\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{(n+1)(n+2)} - \frac{1}{n(n+1)}\right)$$

$$= -\lim_{n\to\infty} \left(\frac{1}{(n+1)(n+2)} - \frac{1}{2}\right) = -\left(0 - \frac{1}{2}\right) = \frac{1}{2}$$

113. ***(c)*** Truth table

p	q	~p	~q	p∨q	~q∧p	p∨~p	(p∨q) ∨(~p)	(~q∧p) ∨(p∨~p)
T	T	F	F	T	F	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	T	F	T	T	T
F	F	T	T	F	F	T	T	T

114. ***(c)*** The two sets of m parallel lines along with two sets of two parallel lines of the given parallelogram will form two sets of $(m + 2)$ parallel lines. Each parallelogram is formed by choosing two parallel lines from each of the above.

$\therefore$ Total number of parallelograms

$$= {}^{m+2}C_2 \times {}^{m+2}C_2 = ({}^{m+2}C_2)^2$$

115. ***(a)*** Let $f(x) = y$, then

$$a^y = x + \sqrt{x^2 + 1}$$

$$\Rightarrow \quad a^{-y} = \frac{1}{x + \sqrt{x^2 + 1}}$$

$$\Rightarrow \quad a^{-y} = \frac{x - \sqrt{x^2 + 1}}{-1} \quad \text{[rationalising]}$$

$$\therefore \quad a^y - a^{-y} = 2x$$

$$\Rightarrow \quad x = \frac{1}{2}(a^y - a^{-y})$$

$$\therefore \quad f^{-1}(y) = \frac{1}{2}(a^y - a^{-y})$$

$[\because f(x) = y \Rightarrow x = f^{-1}(y)]$

$$\Rightarrow \quad f^{-1}(x) = \frac{1}{2}(a^x - a^{-x}) \quad \text{[replacing } y \text{ by } x\text{]}$$

116. ***(c)*** Given,

$$S = \sum_{n=1}^{\infty} \tan^{-1} \frac{2n}{n^4 + n^2 + 2} \quad \ldots(i)$$

Let $n^4 + n^2 + 1 = [(n^2)^2 + 1^2 + 2(n^2)(1)] - n^2$

$$= (n^2 + 1)^2 - n^2$$

$$= (n^2 + n + 1)(n^2 - n + 1) \quad \ldots(ii)$$

Let $u_n = \tan^{-1}\left(\frac{2n}{n^4 + n^2 + 2}\right)$

$$= \tan^{-1} \frac{2n}{1 + (n^4 + n^2 + 1)}$$

$$= \tan^{-1} \frac{(n^2 + n + 1) - (n^2 - n + 1)}{1 + (n^2 + n + 1)(n^2 - n + 1)}$$

$$u_n = \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1) \quad \ldots(iii)$$

On putting $n = 1, 2, 3, \ldots\ldots$ successively in Eq. (iii), we get

$$u_1 = \tan^{-1} 3 - \tan^{-1} 1$$
$$u_2 = \tan^{-1} 7 - \tan^{-1} 3$$
$$u_3 = \tan^{-1} 13 - \tan^{-1} 7$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$u_n = \tan^{-1}(n^2 + n + 1) - \tan^{-1}(n^2 - n + 1)$$

On adding vertically, we get

$$\sum_{n=1}^{\infty} u_n = \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1$$

$$S = \lim_{n \to \infty} \sum_{n=1}^{\infty} u_n \qquad \text{[from Eq. (i)]}$$
$$= \lim_{n \to \infty} \tan^{-1}(n^2 + n + 1) - \tan^{-1} 1$$
$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

117. ***(a)*** Given, $\sin x + \cos(t + x) + \cos(t - x) = 2$

$\Rightarrow$ $\sin x + 2\cos t \cdot \cos x = 2$

For real solution

$$\sqrt{1 + 4\cos^2 t} \geq 2$$

$\Rightarrow$ $\cos^2 t \geq \frac{3}{4} \Rightarrow -\frac{1}{2} \leq \sin t \leq \frac{1}{2}$

118. ***(c)*** We have,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$\Rightarrow$ $\cos^2\alpha + \cos^2\left(\frac{\pi}{2} - \alpha\right) + \cos^2\gamma = 1$

$$\left[\text{given, } \alpha + \beta = \frac{\pi}{2}\right]$$

$\Rightarrow$ $\cos^2\alpha + \sin^2\alpha + \cos^2\gamma = 1$

$\Rightarrow$ $1 + \cos^2\gamma = 1$

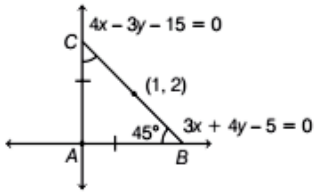
$\Rightarrow$ $\cos^2\gamma = 0 \Rightarrow \cos\gamma = 0$

$\therefore$ $(\cos\alpha + \cos\beta + \cos\gamma)^2 = (\cos\alpha + \sin\alpha)^2$

$$= 1 + 2\sin\alpha \cdot \cos\alpha = 1 + \sin 2\alpha$$

119. ***(c)*** The given straight lines are $3x + 4y = 5$ and $4x - 3y = 15$. Clearly, these straight lines are perpendicular to each other $(m_1 m_2 = -1)$ and intersect at A. Now, B and C are points on these lines such that $AB = AC$ and BC passes through $(1, 2)$.

From figure it is clear that $\angle B = \angle C = 45°$



Let slope of BC be m. Then,

$$\tan 45° = \left|\frac{m + \frac{3}{4}}{1 - \frac{3}{4}m}\right|$$

$\Rightarrow$ $\pm 1 = \frac{4m + 3}{4 - 3m}$

$\Rightarrow$ $4m + 3 = \pm(4 - 3m)$

$\Rightarrow$ $4m + 3 = 4 - 3m$

or $4m + 3 = -4 + 3m$

$\Rightarrow$ $m = \frac{1}{7}$ or $m = -7$

Hence, equation of BC is

$$y - 2 = \frac{1}{7}(x - 1)$$

or $y - 2 = -7(x - 1) \Rightarrow 7y - 14 = x - 1$

or $y - 2 = -7x + 7 \Rightarrow x - 7y + 13 = 0$

or $7x + y - 9 = 0$

120. ***(d)*** Let $y = mx + c$, intersect $y^2 = 4ax$ at $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

Then, $\frac{2}{t_1 + t_2} = m \Rightarrow t_1 + t_2 = \frac{2}{m}$

Let the foot of another normal be $C(at_3^2, 2at_3)$.

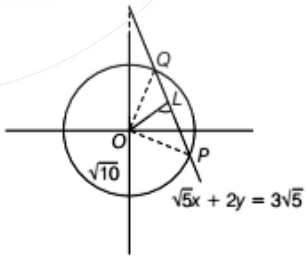
Then, $t_1 + t_2 + t_3 = 0$

$$t_3 = (t_1 + t_2) = -\frac{2}{m}$$

Thus, other foot is $\left(\frac{4a}{m^2}, \frac{-4a}{m}\right)$.

121. ***(c)*** Length of perpendicular from origin to the line $x\sqrt{5} + 2y = 3\sqrt{5}$ is

$$OL = \frac{3\sqrt{5}}{\sqrt{(\sqrt{5})^2 + 2^2}} = \frac{3\sqrt{5}}{\sqrt{9}} = \sqrt{5}$$



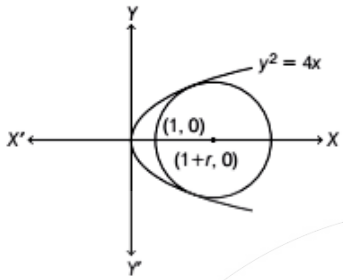
Radius of the given circle $= \sqrt{10} = OQ = OP$

$$PQ = 2QL$$
$$= 2\sqrt{OQ^2 - OL^2}$$
$$= 2\sqrt{10 - 5} = 2\sqrt{5}$$

Thus, area of $\Delta OPQ = \frac{1}{2} \times PQ \times OL$

$= \frac{1}{2} \times 2\sqrt{5} \times \sqrt{5} = 5$ sq units

122. ***(b)*** Let r be the radius of the largest circle passing through the focus $(1, 0)$ of $y^2 = 4x$



Clearly, centre of the circle will be on X-axis and its coordinates are $(1 + r, 0)$.

The equation of the circle, is $(x - 1 - r)^2 + y^2 = r^2$.

It touches $y^2 = 4x$. Therefore, the equation $(x - r - 1)^2 + 4x = r^2$ must have equal roots

$\therefore \quad 4(1 - r)^2 - 4(2r + 1) = 0$

$\Rightarrow \quad r = 4$

123. ***(d)*** The point $P\left(\frac{\pi}{6}\right)$ is $\left(a \sec\frac{\pi}{6}, b \tan\frac{\pi}{6}\right)$

or $P\left(\frac{2a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$

$\therefore$ Equation of tangent at P is $\frac{x}{\frac{\sqrt{3}a}{2}} - \frac{y}{\sqrt{3}\,b} = 1$

$\therefore$ Area of the triangle $= \frac{1}{2} \times \frac{\sqrt{3}a}{2} \times \sqrt{3}b = 3a^2$

$\therefore \quad \frac{b}{a} = 4$

$\therefore \quad e^2 = 1 + \frac{b^2}{a^2} = 17$

Now, $e^2 - 9 = 17 - 9 = 8$

124. ***(d)*** Let distances of a point $p(x, y, z)$ from the planes $x + y + z = 0$, $x - z = 0$ and $x - 2y + z = 0$ are $\frac{x + y + z}{\sqrt{3}}$, $\frac{x - z}{\sqrt{2}}$ and $\frac{x - 2y + z}{\sqrt{6}}$ respectively, then the sum of the squares of distances, is as

$\Rightarrow \left(\frac{x + y + z}{\sqrt{3}}\right)^2 + \left(\frac{x - z}{\sqrt{2}}\right)^2 + \left(\frac{x - 2y + z}{\sqrt{6}}\right)^2 = p^2$

$\Rightarrow 2(x + y + z)^2 + 3(x - z)^2 + (x - 2y + z)^2 = 6p^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 + 4xy + 4yz + 4zx + 3x^2 + 3z^2 - 6xz + x^2 + 4y^2 + z^2 - 4xy - 4yz + 2xz = 6p^2$

$\Rightarrow \quad 6x^2 + 6y + 6z^2 = 6p^2 \Rightarrow x^2 + y^2 + z^2 = p^2$

125. ***(a)*** When line

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

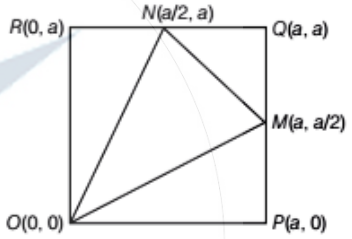
is perpendicular to plane $ax + by + cz + d = 0$, then

$$\frac{a_1}{a} = \frac{-b_1}{b} = \frac{c_1}{c}$$

$\therefore \quad \frac{\lambda}{2} = \frac{1}{2} = \frac{-4}{-8} \Rightarrow \lambda = 1$

126. ***(c)*** Taking the coordinates of vertices O, P, Q, R as $(0, 0)$, $(a, 0)$, (a, a), $(0, a)$, respectively.

$\therefore$ The coordinates of M is $\left(a, \frac{a}{2}\right)$ and N is $\left(\frac{a}{2}, a\right)$.



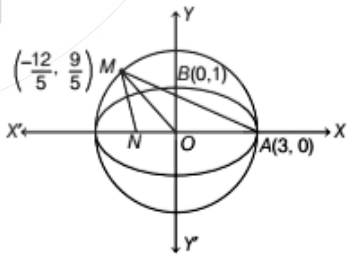
$\therefore$ Area of $\Delta OMN = \frac{1}{2}\begin{Vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{Vmatrix} = \frac{3a^2}{8}$

and area of the square $= a^2$

$\therefore$ The required ratio is $8 : 3$.

127. ***(d)*** Equation of auxiliary circle is

$$x^2 + 9y^2 = 9 \qquad \ldots(i)$$



$\therefore$ Equation of AM is $\frac{x}{3} + \frac{y}{1} = 1 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$$M\left(-\frac{12}{5}, \frac{9}{5}\right)$$

Now, area of $\Delta AOM = \frac{1}{2} OA \times MN = \frac{27}{10}$ sq units

128. ***(b)*** Given equation can be written as

$$x^2 - y^2 = \frac{25}{3}$$

$$\therefore \quad e_1 = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1+1} = \sqrt{2}$$

The equation of conjugate hyperbola is

$$-x^2 + y^2 = \frac{25}{3}$$

$$\therefore \quad e_2 = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1+1} = \sqrt{2}$$

$$\therefore \quad e_1^2 + e_1^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 4$$

129. ***(a)*** We have,

$$f(x+y) = f(x) + 2y^2 + kxy \text{ for all } x, y \in R$$

$$\Rightarrow \quad \frac{f(x+y) - f(x)}{y} = 2y + kx \text{ for all } x \in R$$

$$\Rightarrow \lim_{y \to 0} \frac{f(x+y) - f(x)}{y} = \lim_{y \to 0} (2y + kx)$$

$$\Rightarrow f'(x) = kx \text{ for all } x \in R$$

$$\Rightarrow f(x) = \frac{kx^2}{2} + C \text{ for all } x \in R \qquad \text{[by integration]}$$

But, $f(1) = 2$ and $f(2) = 8$.

$$\therefore \quad 2 = \frac{k}{2} + C \text{ and } 8 = 2k + C$$

$$\Rightarrow \quad k = 4 \text{ and } C = 0$$

Hence, $f(x) = 2x^2$ for all $x \in R$

130. ***(a)*** Since, given planes are perpendicular to each other, i.e. its normal are perpendicular.

$$\therefore \quad 2(\lambda) - \lambda(5) + 3(-1) = 0$$

$$\Rightarrow \quad -3\lambda - 3 = 0 \Rightarrow \lambda = -1$$

$$\therefore \quad \lambda^2 + \lambda = (-1)^2 - 1 = 0$$

131. ***(d)*** Put $x + y = v$ and $\frac{dv}{dx} = 1 + \frac{dy}{dx}$ in the given differential equation.

$$\therefore \frac{dv}{dx} = 1 + \sin v + \cos v$$

$$\Rightarrow \frac{dv}{2\cos^2\frac{v}{2} + 2\sin\frac{v}{2}\cos\frac{v}{2}} = dx \Rightarrow \frac{\frac{1}{2}\sec^2\frac{v}{2}}{1 + \tan\frac{v}{2}} dv = dx$$

$$\Rightarrow \quad \log\left(1 + \tan\frac{x+y}{2}\right) = x + C \text{ [by integration]}$$

132. ***(d)*** Since, the numerator tends to ∞ as $x \to 0$,

so $$\lim_{x \to 0} \frac{1}{x^2}(e^{\alpha x} - e^x - x)$$

$$= \frac{1}{2}\lim_{x \to 0} \frac{(\alpha e^{\alpha x} - e^x - 1)}{x}$$

For the last limit to exist we must have,

$$\lim_{x \to 0} (\alpha e^{\alpha x} - e^x - 1) = 0$$

$$\therefore \quad \alpha - 1 - 1 = 0$$

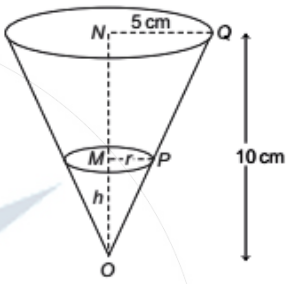
$$\Rightarrow \quad \alpha = 2$$

For $\alpha = 2$ the last limit and equal to

$$= \frac{1}{2}\lim_{x \to 0} \frac{(2e^{2x} - e^x - 1)}{x}$$

$$= \frac{1}{2}\lim_{x \to 0} (4e^{2x} - e^x) = \frac{3}{2}$$

133. ***(b)*** Let depth of water at time t be h and the radius of the base of water level be r.



From similar ΔOMP and ΔONQ, we have

$$\frac{OM}{ON} = \frac{PM}{QN}$$

$$\Rightarrow \quad \frac{h}{10} = \frac{r}{5}$$

$$\Rightarrow \quad r = \frac{5h}{10} \Rightarrow r = \frac{h}{2}$$

We have, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$$\Rightarrow \quad V = \frac{\pi h^3}{12}$$

On differentiating both sides, we get

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$$

Given, $\frac{dV}{dt} = 3 \text{ cm}^3/\text{s}$ when $h = 4$ cm, so we get

$$3 = \frac{\pi \times 4^2}{4}\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{3}{4\pi} \text{ cm/s}$$

Hence, the water level is rising at $\frac{3}{4\pi}$ cm/s.

134. ***(b)*** We have,

$$\frac{dy}{dx} = y + 2x$$

$$\Rightarrow \quad \frac{dy}{dx} - y = 2x$$

Now, $\text{IF} = e^{-\int 1\,dx} = e^{-x}$

$\therefore y \cdot e^{-x} = \int 2x\, e^{-x}\, dx + k$, k be the constant of integration

$= 2[x \int e^{-x} dx - \int 1 \cdot (-e^{-x})\, dx] + k$

[using integration by parts]

$\Rightarrow \quad y \cdot e^{-x} = -2xe^{-x} - 2e^{-x} + k \qquad \ldots(i)$

As curve (i) passes through (0, 0)

$\therefore \quad 0 = 0 - 2 + k$

$\Rightarrow \quad k = 2$

Thus, the curve is

$$ye^{-x} = -2xe^{-x} - 2e^{-x} + 2$$

$\therefore \quad y = 2(e^x - x - 1)$

135. ***(c)*** $f(0) = 0$

For $f(x)$ to be continuous at $x = 0$

$$\lim_{x \to 0} f(x) = 0$$

$\therefore \quad \lim_{x \to 0} x^p \sin\frac{1}{x} = 0$

This possible only when $p > 0 \qquad \ldots(i)$

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{h^p \sin\frac{1}{h} - 0}{h}$$

$$= \lim_{h \to 0} h^{p-1} \sin\frac{1}{h}$$

$\Rightarrow$ $f'(0)$ will exist only when $p > 0$

$\therefore$ $f(x)$ will not be differentiable if

$$p \le 1 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), for $f(x)$ to be not differentiable but continuous at $x = 0$, possible values of p are given by $0 < p \le 1$.

136. ***(a)*** Integrating the given differential equation, we have

$$\frac{dy}{dx} = \frac{-\cos 3x}{3} + e^x + \frac{x^3}{3} + C_1$$

But $y_1(0) = 1$

So, $\quad 1 = \left(-\frac{1}{3}\right) + 1 + C_1$

$\Rightarrow \quad C_1 = \frac{1}{3}$

$\therefore \quad \frac{dy}{dx} = \frac{-\cos 3x}{3} + e^x + \frac{x^3}{3} + \frac{1}{3}$

Again integrating, we get

$$y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + C_2$$

But $y(0) = 0$, so $0 = 0 + 1 + C_2$

$\Rightarrow \quad C_2 = -1$

Thus, $\quad y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$

137. ***(d)*** Given, $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\Rightarrow \quad f'(x) = -6x^2 - 18x - 12$

To be decreasing $f'(x) < 0$

$\Rightarrow \quad -6x^2 - 18x - 12 < 0$

$\Rightarrow \quad x^2 + 3x + 2 > 0$

$\Rightarrow \quad (x + 2)(x + 1) > 0$

Therefore, either $x < -2$ or $x > -1$

$\Rightarrow \quad x \in (-1, \infty)$ or $(-\infty, -2)$

138. ***(c)*** Let $\mathbf{a} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Since, θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

$\therefore \quad \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

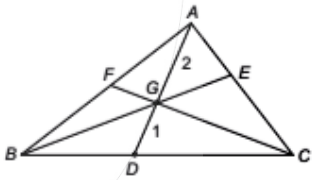
$$= \frac{(4\hat{\mathbf{i}} - 4\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})}{|\mathbf{a}||\mathbf{b}|}$$

$$= \frac{4 + 0 - 4}{|\mathbf{a}||\mathbf{b}|} = 0$$

$\cos\theta = \cos 90° \Rightarrow \theta = 90°$

$\therefore \quad \sin\theta + \cos\theta = \cos 90° + \sin 90° = 0 + 1 = 1$

139. ***(d)***



$\therefore \quad \mathbf{AD} = 3\mathbf{GD}$

$$= 3 \cdot \frac{1}{2}(\mathbf{GB} + \mathbf{GC})$$

$$= \frac{3}{2}\left(\frac{2}{3}\mathbf{EB} + \frac{2}{3}\mathbf{FC}\right)$$

$$= -\mathbf{BE} - \mathbf{CF}$$

140. ***(c)*** Let $x_1, x_2, \ldots, x_n$ be n observations.

Then, $\quad \bar{x} = \frac{1}{n}\Sigma x_i$

Let $\quad y_i = \frac{x_i}{\alpha} + 10$

Then, $\quad \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{\alpha}\left(\frac{1}{n}\Sigma x_i\right) + \frac{1}{n}(10n)$

$\Rightarrow \quad \bar{x}_{\text{new}} = \frac{1}{\alpha}\bar{X} + 10 = \frac{\bar{X} + 10\alpha}{\alpha}$

141. ***(c)*** Volume of the parallelopiped $= |[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]|$

$\Rightarrow$ (area of the base parallelogram) $\times h = |[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]|$

$\Rightarrow \quad |\mathbf{a} \times \mathbf{b}|\, h = |[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]|$

$$\Rightarrow \quad |-5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}|h = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 1 & 1 & 3 \end{vmatrix}$$

$$\Rightarrow \quad \sqrt{38}\, h = 4 \Rightarrow h = \frac{4}{\sqrt{38}}$$

$$\Rightarrow \quad 38h^2 = 4^2 \Rightarrow 19h^2 = 8$$

142. ***(b)*** Mean (m) for BD $= np$

and variance (σ^2) for BD $= npq$

Given, $np + npq = 2.7$ and $n = 3$

$$\therefore \quad np(1+q) = 2.7 \quad \Rightarrow \quad p(1+q) = \frac{2.7}{3}$$

$$\Rightarrow \quad p(1+q) = 0.9$$

$$\Rightarrow \quad (1-q)(1+q) = 0.9$$

[$\because$ for BD probability standard deviation (SD), $p + q = 1$]

$$\Rightarrow \quad 1 - q^2 = 0.9$$

$$\Rightarrow \quad q^2 = 0.1 \quad \Rightarrow \quad q = \pm 0.3$$

$$\therefore \quad q = 0.3$$

[$\because q = -0.3$ can't possible as $0 \le q \le 1$]

$$\Rightarrow \quad p = 0.7 \text{ and } q = 0.3$$

$$\therefore \quad \text{BD} = (0.3 + 0.7)^5$$

143. ***(b)*** We have,

$$R(x) = \int \frac{dx}{e^x + 8e^{-x} + 4e^{-3x}} - 2\int \frac{dx}{e^{3x} + 8e^x + 4e^{-x}}$$

$$\Rightarrow \quad R(x) = \int \frac{e^x (e^{2x} - 2)}{e^{4x} + 8e^{2x} + 4}\, dx$$

On substituting

$$e^x = t \quad \Rightarrow \quad e^x\, dx = dt,$$

we get

$$R(t) = \int \frac{(t^2 - 2)\, dt}{t^4 + 8t^2 + 4} = \int \frac{(1 - 2t^{-2})\, dt}{(t + 2t^{-1})^2 + 4}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{t + 2t^{-1}}{2}\right) + K$$

$$\Rightarrow \quad R(x) = \frac{1}{2}\tan^{-1}\left(\frac{e^x + 2e^{-x}}{2}\right) + K$$

Hence, $(A, B, C) = (\tan^{-1}, e^x, 2)$

144. ***(b)*** $\cot^{-1}(1 - x + x^2) = \tan^{-1}\left(\dfrac{1}{1 - x + x^2}\right)$

$$= \tan^{-1}\left(\frac{1}{1 - x(1-x)}\right)$$

$$= \tan^{-1}\left(\frac{x + (1-x)}{1 - x(1-x)}\right)$$

$$\Rightarrow \quad \cot^{-1}(1 - x + x^2) = \tan^{-1} x - \tan^{-1}(1 - x)$$

$$\therefore \int_0^1 \cot^{-1}(1 - x + x^2)\, dx$$

$$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(1 - x)\, dx$$

$$= \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1} x\, dx$$

$$\left[\because \int_0^a f(x)\, dx = -\int_0^a f(a - x)\, dx\right]$$

$$= 2\int_0^1 \tan^{-1} x\, dx$$

On evaluating by integration by parts, we have

$$= 2\left\{[\tan^{-1} x \cdot x]_0^1 - \int_0^1 \frac{x}{1 + x^2}\, dx\right\}$$

$$= 2\left\{\frac{\pi}{4} - \left[\frac{1}{2}\ln(1 + x^2)\right]_0^1\right\}$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right] = \frac{\pi}{2} - \log 2$$

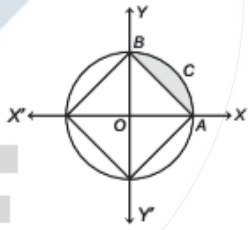
Hence, $\int_0^1 \cot^{-1}(1 - x + x^2)\, dx = \dfrac{\pi}{2} - \log 2$

145. ***(a)*** Given curves are

$$\sqrt{|x|} + \sqrt{|y|} = \sqrt{a} \qquad \ldots(\text{i})$$

and

$$x^2 + y^2 = a^2 \qquad \ldots(\text{ii})$$



Now, required areas $= 4$

[shaded area in the first quadrant]

$$= 4\left[\frac{\pi a^2}{4} - \int_0^a (\sqrt{a} - \sqrt{x})^2\, dx\right]$$

[from Eqs. (i) and (ii)]

$$= 4\left[\frac{\pi a^2}{4} - \int_0^a (a + x - 2\sqrt{a}\,\sqrt{x})\, dx\right]$$

$$= 4\frac{\pi a^2}{4} - 4\left[ax + \frac{x^2}{2} - \frac{4}{3}\sqrt{a}\, x^{3/2}\right]_0^a$$

$$= \left(\pi - \frac{2}{3}\right)a^2 \text{ sq units}$$

146. ***(b)*** $P(A \cap B) = \frac{1}{6}$ and $P(A^C \cap B^C) = \frac{1}{3}$

Now, $P(A \cup B)^C = P(A^C \cap B^C) = \frac{1}{3}$

$\Rightarrow \quad 1 - P(A \cup B) = \frac{1}{3}$

$\Rightarrow \quad P(A \cup B) = \frac{2}{3}$

But $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = \frac{5}{6}$ ...(i)

$\because$ A and B are independent events

$\therefore \quad P(A \cap B) = P(A)\,P(B)$

$\Rightarrow \quad P(A)\,P(B) = \frac{1}{6}$

$[P(A) - P(B)]^2 = [P(A) + P(B)]^2 - 4P(A)\,P(B)$

$= \frac{25}{36} - \frac{4}{6} = \frac{1}{36}$

$\Rightarrow \quad P(A) - P(B) = \pm\frac{1}{6}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$P(A) = \frac{1}{2}$ or $\frac{1}{3}$

147. ***(d)*** Let A be the event that the examinee gives the correct answer.

Let G, C, K stand for guessing, copying and knowing, respectively.

Given, $P(C) = \frac{1}{6}$, $P(G) = \frac{1}{3}$

and $P\left(\frac{A}{C}\right) = \frac{1}{8}$

$\Rightarrow \quad P(C) + P(G) + P(K) = 1$

$\Rightarrow \quad \frac{1}{6} + \frac{1}{3} + P(K) = 1$

$\therefore \quad P(K) = \frac{1}{2}$

Also, $P\left(\frac{A}{K}\right) = 1$, for if the examinee knows, he/she will correctly answer it and $P\left(\frac{A}{G}\right) = \frac{1}{4}$, since there are four choices.

Now, total probability

$P(A) = P(G)\,P\left(\frac{A}{G}\right) + P(C)\,P\left(\frac{A}{C}\right) + P(K)\,P\left(\frac{A}{K}\right)$

$= \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{6}\cdot\frac{1}{8} + \frac{1}{2}\cdot 1$

$= \frac{1}{12} + \frac{1}{48} + \frac{1}{2} = \frac{4+1+24}{48}$

$= \frac{29}{48}$

Now, $P\left(\frac{K}{A}\right) = \frac{P(K)\,P\left(\frac{A}{K}\right)}{P(A)}$

$= \frac{\frac{1}{2}\cdot 1}{\frac{29}{48}} = \frac{1}{2}\cdot\frac{48}{29}$

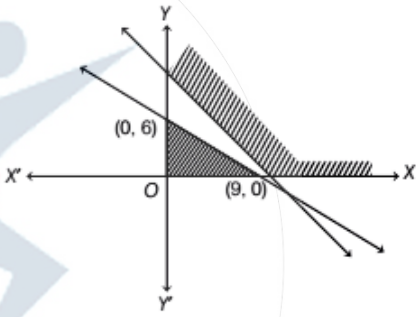
$= \frac{24}{29}$

148. ***(d)*** Since, p : 4 is an even prime number,

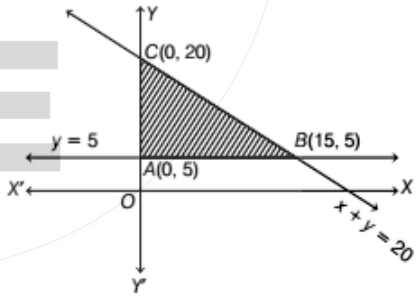
q : 6 is a divisor of 12 and r : the HCF of 4 and 6 is 2.

So, $\sim p \vee (q \wedge r)$ is correct.

149. ***(d)*** From the figure, it is clear that there is no common area. So, we cannot find maximum value of Z.



150. ***(d)*** Feasible region is $ABCA$ and $Z = 7x - 8y$



At $A(0, 5)$, $Z = 7 \times 0 - 8 \times 5 = -40$

At $B(15, 5)$, $Z = 7(15) - 8(5) = 65$

At $C(0, 20)$, $Z = 7(0) - 8(20) = -160$

Hence, the minimum value of Z is attained at point $(0, 20)$.