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JEE (Main) Syllabus :

Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, distance of a point from a line, equations of internal and external bisectors of angles between two lines, coordinates of centroid, orthocentre and circumcentre of a triangle, equation of family of lines passing through the point of intersection of two lines.

JEE (Advanced) Syllabus :

Cartesian coordinates, distance between two points, section formulae, shift of origin. Equation of a straight line in various forms, angle between two lines, distance of a point from a line; Lines through the point of intersection of two given lines, equation of the bisector of the angle between two lines, concurrency of lines; Centroid, orthocentre, incentre and circumcentre of a triangle.

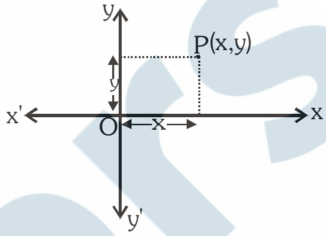
POINT & STRAIGHT LINE

1. INTRODUCTION OF COORDINATE GEOMETRY :

Coordinate geometry is the combination of algebra and geometry. A systematic study of geometry by the use of algebra was first carried out by celebrated French philosopher and mathematician René Descartes. The resulting combination of analysis and geometry is referred as ***analytical geometry***.

2. CARTESIAN CO-ORDINATES SYSTEM :

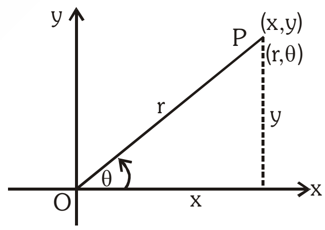
In two dimensional coordinate system, two lines are used; the lines are at right angles, forming a rectangular coordinate system. The horizontal axis is the x-axis and the vertical axis is y-axis. The point of intersection O is the origin of the coordinate system. Distances along the x-axis to the right of the origin are taken as positive, distances to the left as negative.



Distances along the y-axisabove the origin are positive; distances below are negative. The position of a point anywhere in the plane can be specified by two numbers, the coordinates of the point, written as (x, y). The x-coordinate (or abscissa) is the distance of the point from the y-axis in a direction parallel to the x-axis (i.e. horizontally). The y-coordinate (or ordinate) is the distance from the x-axis in a direction parallel to the y-axis (vertically). The origin O is the point (0, 0).

3. POLAR CO-ORDINATES SYSTEM :

A coordinate system in which the position of a point is determined by the length of a line segment from a fixed origin together with the angle that the line segment makes with a fixed line. The origin is called the pole and the line segment is the radius vector (r).



The angle θ between the polar axis and the radius vector is called the vectorial angle. By convention, positive values of θ are measured in an anticlockwise sense, negative values in clockwise sense. The coordinates of the point are then specified as (r, θ).

If (x,y) are cartesian co-ordinates of a point P, then : $x = r\cos\theta$, $y = r\sin\theta$

and $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$

4. DISTANCE FORMULA AND ITS APPLICATIONS :

If $A(x_1,y_1)$ and $B(x_2,y_2)$ are two points, then $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note :

(i) Three given points A,B and C are collinear, when sum of any two distances out of AB,BC, CA is equal to the remaining third otherwise the points will be the vertices of a triangle.

(ii) Let A,B,C & D be the four given points in a plane. Then the quadrilateral will be :

(a)	Square if AB = BC = CD = DA & AC = BD	;	$AC \perp BD$
(b)	Rhombus if AB = BC = CD = DA and $AC \neq BD$	;	$AC \perp BD$
(c)	Parallelogram if AB = DC, BC = AD; $AC \neq BD$	;	$AC \not\perp BD$
(d)	Rectangle if AB = CD, BC = DA, AC = BD	;	$AC \not\perp BD$

Illustration 1 : The number of points on x-axis which are at a distance $c(c < 3)$ from the point (2, 3) is

(A) 2 (B) 1 (C) infinite (D) no point

Solution : Let a point on x-axis is $(x_1, 0)$ then its distance from the point (2, 3)

$$=\sqrt{(x_1-2)^2+9}=c \quad \text{or} \quad (x_1-2)^2=c^2-9$$

$\therefore\ x_1-2=\pm\sqrt{c^2-9}$ since $c<3 \Rightarrow c^2-9<0$

$\therefore\ x_1$ will be imaginary. **Ans. (D)**

Illustration 2 : The distance between the point $P(a\cos\alpha, a\sin\alpha)$ and $Q(a\cos\beta, a\sin\beta)$, where $a > 0$ &
$\alpha > \beta$, is -

(A) $4a\sin\dfrac{\alpha-\beta}{2}$ (B) $2a\sin\dfrac{\alpha+\beta}{2}$ (C) $2a\sin\dfrac{\alpha-\beta}{2}$ (D) $2a\cos\dfrac{\alpha-\beta}{2}$

Solution : $d^2=(a\cos\alpha-a\cos\beta)^2+(a\sin\alpha-a\sin\beta)^2=a^2(\cos\alpha-\cos\beta)^2+a^2(\sin\alpha-\sin\beta)^2$

$$=a^2\left\{2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right\}^2+a^2\left\{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right\}^2$$

$$=4a^2\sin^2\frac{\alpha-\beta}{2}\left\{\sin^2\frac{\alpha+\beta}{2}+\cos^2\frac{\alpha+\beta}{2}\right\}=4a^2\sin^2\frac{\alpha-\beta}{2}\Rightarrow d=2a\sin\frac{\alpha-\beta}{2}$$

Ans. (C)

Do yourself - 1 :

(i) Find the distance between the points P(–3, 2) and Q(2, –1).

(ii) If the distance between the points P(–3, 5) and Q(–x, – 2) is $\sqrt{58}$, then find the value(s) of x.

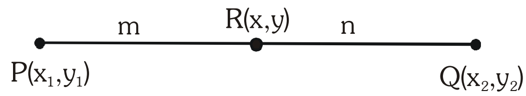
(iii) A line segment is of the length 15 units and one end is at the point (3, 2), if the abscissa of the other end is 15, then find possible ordinates.

5. SECTION FORMULA :

The co-ordinates of a point dividing a line joining the points $P(x_1,y_1)$ and $Q(x_2,y_2)$ in the ratio m:n is given by :

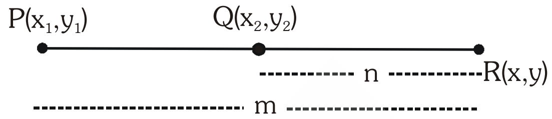
(a) For internal division : P - R - Q $\Rightarrow$ R divides line segment PQ, internally.

$$(x, y) \equiv \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$



(b) For external division : R - P - Q or P - Q - R $\Rightarrow$ R divides line segment PQ,externally.

$$(x, y) \equiv \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$



$\frac{(PR)}{(QR)} < 1 \Rightarrow$ R lies on the left of P & $\frac{(PR)}{(QR)} > 1 \Rightarrow$ R lies on the right of Q

(c) Harmonic conjugate : If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be harmonic conjugate of each other w.r.t. AB. Mathematically ; $\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$ i.e. AP, AB & AQ are in **H.P.**

Illustration 3 : Determine the ratio in which $y - x + 2 = 0$ divides the line joining (3, –1) and (8, 9).

Solution : Suppose the line $y - x + 2 = 0$ divides the line segment joining A(3, –1) and B(8, 9) in the ratio $\lambda : 1$ at a point P, then the co-ordinates of the point P are $\left(\frac{8\lambda+3}{\lambda+1}, \frac{9\lambda-1}{\lambda+1}\right)$

But P lies on $y - x + 2 = 0$ therefore $\left(\frac{9\lambda-1}{\lambda+1}\right) - \left(\frac{8\lambda+3}{\lambda+1}\right) + 2 = 0$

$\Rightarrow \quad 9\lambda - 1 - 8\lambda - 3 + 2\lambda + 2 = 0$

$\Rightarrow \quad 3\lambda - 2 = 0$ or $\lambda = \frac{2}{3}$

So, the required ratio is $\frac{2}{3} : 1$, i.e., 2 : 3 (internally) since here λ is positive.

Do yourself - 2 :

(i) Find the co-ordinates of the point dividing the join of A(1, – 2) and B(4, 7) :

(a) Internally in the ratio 1 : 2 (b) Externally in the ratio of 2 : 1

(ii) In what ratio is the line joining A(8, 9) and B(– 7, 4) divided by

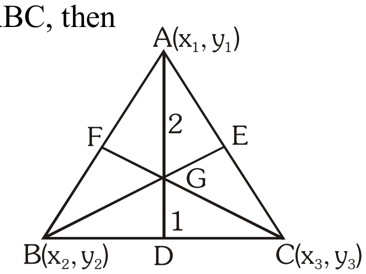
(a) the point (2, 7) (b) the x-axis (c) the y-axis.

6. CO-ORDINATES OF SOME PARTICULAR POINTS :

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of any triangle ABC, then

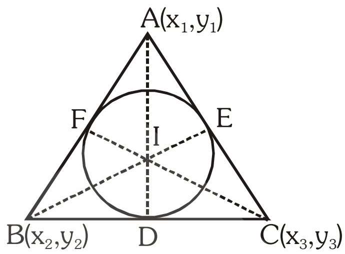
(a) Centroid :

The centroid is the point of intersection of the medians (line joining the mid point of sides and opposite vertices). Centroid divides each median in the ratio of 2 : 1.



Co-ordinates of centroid $G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

(b) Incenter :



The incenter is the point of intersection of internal bisectors of the angles of a triangle. Also it is a centre of the circle touching all the sides of a triangle.

Co-ordinates of incenter $I\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

where a, b, c are the sides of triangle ABC.

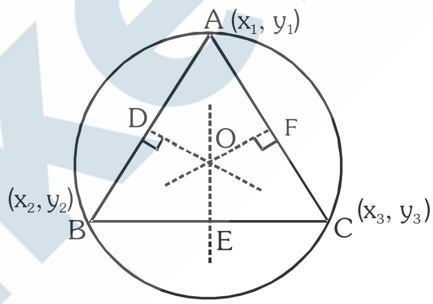
Note :

(i) Angle bisector divides the opposite sides in the ratio of remaining sides. e.g.

$$\frac{BD}{DC}=\frac{AB}{AC}=\frac{c}{b}$$

(ii) Incenter divides the angle bisectors in the ratio $(b+c):a,\ (c+a):b,\ (a+b):c$.

(c) Circumcenter :



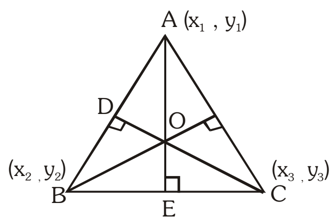
It is the point of intersection of perpendicular bisectors of the sides of a triangle. If O is the circumcenter of any triangle ABC, then $OA^2=OB^2=OC^2$. Also it is a centre of a circle touching all the vertices of a triangle.

Note :

(i) If the triangle is right angled, then its circumcenter is the mid point of hypotenuse.

(ii) Co-ordinates of circumcenter $\left(\frac{x_1\sin 2A+x_2\sin 2B+x_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}, \frac{y_1\sin 2A+y_2\sin 2B+y_3\sin 2C}{\sin 2A+\sin 2B+\sin 2C}\right)$

(d) Orthocenter :



It is the point of intersection of perpendiculars drawn from vertices on the opposite sides of a triangle and it can be obtained by solving the equation of any two altitudes.

Note :

(i) If a triangle is right angled, then orthocenter is the point where right angle is formed.

(ii) Co-ordinates of circumcenter $\left(\frac{x_1\tan A+x_2\tan B+x_3\tan C}{\tan A+\tan B+\tan C}, \frac{y_1\tan A+y_2\tan B+y_3\tan C}{\tan A+\tan B+\tan C}\right)$

Remarks :

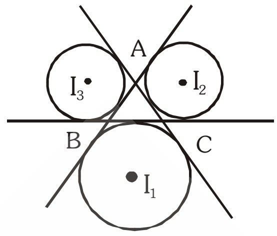
(i) If the triangle is equilateral, then centroid, incentre, orthocenter, circumcenter, coincide.

(ii) Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1

(iii) In an isosceles triangle centroid, orthocentre, incentre & circumcentre lie on the same line.

(e) Ex-centers :

The centre of a circle which touches side BC and the extended portions of sides AB and AC is called the ex-centre of ΔABC with respect to the vertex A. It is denoted by I_1 and its coordinates are $I_1\left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$



Similarly ex-centers of ΔABC with respect to vertices B and C are denoted by I_2 and I_3 respectively, and

$$I_2\left(\frac{ax_1-bx_2+cx_3}{a-b+c}, \frac{ay_1-by_2+cy_3}{a-b+c}\right), I_3\left(\frac{ax_1+bx_2-cx_3}{a+b-c}, \frac{ay_1+by_2-cy_3}{a+b-c}\right)$$

Illustration 4 : If $\left(\frac{5}{3}, 3\right)$ is the centroid of a triangle and its two vertices are (0, 1) and (2, 3), then find its third vertex, circumcentre, circumradius & orthocentre.

Solution : Let the third vertex of triangle be (x, y), then

$\frac{5}{3} = \frac{x+0+2}{3} \Rightarrow x = 3$ and $3 = \frac{y+1+3}{3} \Rightarrow y = 5$. So third vertex is (3, 5).

Now three vertices are A(0, 1), B(2, 3) and C(3, 5)

Let circumcentre be P(h, k),

then AP = BP = CP = R (circumradius) $\Rightarrow$ $AP^2 = BP^2 = CP^2 = R^2$

$h^2 + (k-1)^2 = (h-2)^2 + (k-3)^2 = (h-3)^2 + (k-5)^2 = R^2$ .... (i)

from the first two equations, we have

$h + k = 3$ .... (ii)

from the first and third equation, we obtain

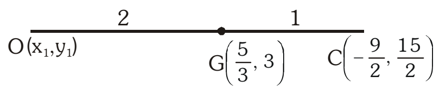
$6h + 8k = 33$ .... (iii)

On solving, (ii) & (iii), we get

$h = -\frac{9}{2}$, $k = \frac{15}{2}$

Substituting these values in (i), we have

$R = \frac{5}{2}\sqrt{10}$



Let $O(x_1, y_1)$ be the orthocentre, then $\frac{x_1+2\left(-\frac{9}{2}\right)}{3} = \frac{5}{3} \Rightarrow x_1 = 14, \frac{y_1+2\left(\frac{15}{2}\right)}{3} = 3$

$\Rightarrow$ $y_1 = -6$. Hence orthocentre of the triangle is (14, –6).

Illustration 5 : The vertices of a triangle are A(0, –6), B(–6, 0) and C(1,1) respectively, then coordinates of the ex-centre opposite to vertex A is :

(A) $\left(\frac{-3}{2}, \frac{-3}{2}\right)$ (B) $\left(-4, \frac{3}{2}\right)$ (C) $\left(\frac{-3}{2}, \frac{3}{2}\right)$ (D) (–4, 6)

Solution :

$$a = BC = \sqrt{(-6-1)^2 + (0-1)^2} = \sqrt{50} = 5\sqrt{2}$$

$$b = CA = \sqrt{(1-0)^2 + (1+6)^2} = \sqrt{50} = 5\sqrt{2}$$

$$c = AB = \sqrt{(0+6)^2 + (-6-0)^2} = \sqrt{72} = 6\sqrt{2}$$

coordinates of ex-centre opposite to vertex A will be :

$$x = \frac{-ax_1 + bx_2 + cx_3}{-a+b+c} = \frac{-5\sqrt{2}.0 + 5\sqrt{2}(-6) + 6\sqrt{2}(1)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{-24\sqrt{2}}{6\sqrt{2}} = -4$$

$$y = \frac{-ay_1 + by_2 + cy_3}{-a+b+c} = \frac{-5\sqrt{2}(-6) + 5\sqrt{2}.0 + 6\sqrt{2}(1)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{36\sqrt{2}}{6\sqrt{2}} = 6$$

Hence coordinates of ex-centre is (–4, 6) **Ans. (D)**

Do yourself - 3 :

(i) The coordinates of the vertices of a triangle are (0, 1), (2, 3) and (3, 5) :

(a) Find centroid of the triangle.

(b) Find circumcentre & the circumradius.

(c) Find orthocentre of the triangle.

7. AREA OF TRIANGLE :

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of a triangle, then

$$\text{Area of } \Delta ABC = \left|\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right| = \frac{1}{2}\left|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

To remember the above formula, take the help of the following method :

$$= \frac{1}{2}\begin{bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{bmatrix} = \left|\frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)]\right|$$

Remarks :

(i) If the area of triangle joining three points is zero, then the points are collinear.

(ii) **Area of Equilateral triangle :** If altitude of any equilateral triangle is P, then its area = $\frac{P^2}{\sqrt{3}}$. If 'a' be the side of equilateral triangle, then its area = $\left(\frac{a^2\sqrt{3}}{4}\right)$.

(iii) Area of quadrilateral with given vertices $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3), D(x_4, y_4)$

$$\text{Area of quad. ABCD} = \frac{1}{2}\left|\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}\right|$$

Note : Area of a polygon can be obtained by dividing the polygon into disjoined triangles and then adding their areas.

Illustration 6 : If the vertices of a triangle are (1, 2), (4, –6) and (3, 5) then its area is

(A) $\frac{25}{2}$ sq. units (B) 12 sq. units (C) 5 sq. units (D) 25 sq. units

Solution : $\Delta = \frac{1}{2}[1(-6-5)+4(5-2)+3(2+6)] = \frac{1}{2}[-11+12+24] = \frac{25}{2}$ square units **Ans. (A)**

Illustration 7 : The point A divides the join of the points (–5, 1) and (3, 5) in the ratio k:1 and coordinates of points B and C are (1, 5) and (7, –2) respectively. If the area of ΔABC be 2 units, then k equals -

(A) 7, 9 (B) 6, 7 (C) $7, \frac{31}{9}$ (D) $9, \frac{31}{9}$

Solution : $A \equiv \left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$

Area of $\Delta ABC = 2$ units $\Rightarrow \frac{1}{2}\left[\frac{3k-5}{k+1}(5+2)+1\left(-2-\frac{5k+1}{k+1}\right)+7\left(\frac{5k+1}{k+1}-5\right)\right] = \pm 2$

$\Rightarrow 14k-66 = \pm 4(k+1) \Rightarrow k = 7$ or $\frac{31}{9}$ **Ans. (C)**

Illustration 8 : Prove that the co-ordinates of the vertices of an equilateral triangle can not all be rational.

Solution : Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle ABC. If possible let x_1, y_1, x_2, y_2, x_3, y_3 be all rational.

Now area of $\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ = Rational .... (i)

Since ΔABC is equilateral

$\therefore$ Area of $\Delta ABC = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(AB)^2 = \frac{\sqrt{3}}{4}\{(x_1-x_2)^2+(y_1-y_2)^2\}$ = Irrational ...... (ii)

From (i) and (ii),

Rational = Irrational

which is contradiction.

Hence $x_1, y_1, x_2, y_2, x_3, y_3$ cannot all be rational.

8. CONDITIONS FOR COLLINEARITY OF THREE GIVEN POINTS :

Three given points A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) are collinear if any one of the following conditions are satisfied.

(a) Area of triangle ABC is zero i.e. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(b) Slope of AB = slope of BC = slope of AC. i.e. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_3 - y_1}{x_3 - x_1}$

(c) Find the equation of line passing through 2 given points, if the third point satisfies the given equation of the line, then three points are collinear.

Do yourself - 4 :

(i) Find the area of the triangle whose vertices are A(1,1), B(7, – 3) and C(12, 2)

(ii) Find the area of the quadrilateral whose vertices are A(1,1) B(7, – 3), C(12,2) and D(7, 21)

(iii) Prove that the points A(a, b + c), B(b, c + a) and C(c, a + b) are collinear (By determinant method)

9. LOCUS :

The locus of a moving point is the path traced out by that point under one or more geometrical conditions.

(a) Equation of Locus :

The equation to a locus is the relation which exists between the coordinates of any point on the path, and which holds for no other point except those lying on the path.

(b) Procedure for finding the equation of the locus of a point :

(i) If we are finding the equation of the locus of a point P, assign coordinates (h, k) to P.

(ii) Express the given condition as equations in terms of the known quantities to facilitate calculations. We sometimes include some unknown quantities known as parameters.

(iii) Eliminate the parameters, so that the eliminant contains only h, k and known quantities.

(iv) Replace h by x, and k by y, in the eliminant. The resulting equation would be the equation of the locus of P.

Illustration 9 : The ends of the rod of length ℓ moves on two mutually perpendicular lines, find the locus of the point on the rod which divides it in the ratio $m_1 : m_2$

(A) $m_1^2x^2 + m_2^2y^2 = \frac{\ell^2}{(m_1 + m_2)^2}$

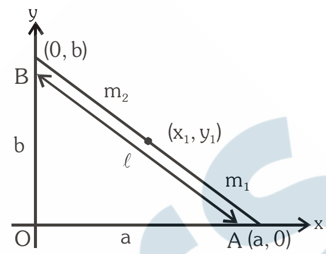
(B) $(m_2x)^2 + (m_1y)^2 = \left(\frac{m_1m_2\ell}{m_1 + m_2}\right)^2$

(C) $(m_1x)^2 + (m_2y)^2 = \left(\frac{m_1m_2\ell}{m_1 + m_2}\right)^2$

(D) none of these

Solution : Let (h,k) be the point that divide the rod $AB = \ell$, in the ratio $m_1 : m_2$, and $OA = a,\ OB = b$ say

$\therefore\quad a^2 + b^2 = \ell^2$ ..... (i)



$$\text{Now } h = \left(\frac{m_2 a}{m_1 + m_2}\right) \Rightarrow a = \left(\frac{m_1 + m_2}{m_2}\right)h$$

$$k = \left(\frac{m_1 b}{m_1 + m_2}\right) \Rightarrow b = \left(\frac{m_1 + m_2}{m_1}\right)k$$

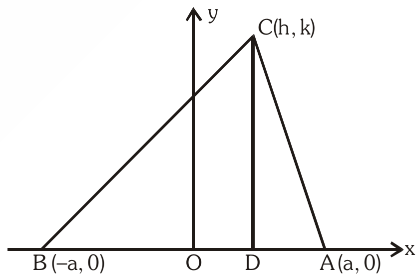
putting these values in (i) $\dfrac{(m_1 + m_2)^2}{m_2^2}h^2 + \dfrac{(m_1 + m_2)^2}{m_1^2}k^2 = \ell^2$

$\therefore$ Locus of (h,k) is $m_1^2 x^2 + m_2^2 y^2 = \left(\dfrac{m_1 m_2 \ell}{m_1 + m_2}\right)^2$ **Ans. (C)**

Illustration 10 : A(a, 0) and B(–a, 0) are two fixed points of ΔABC. If its vertex C moves in such a way that cotA + cotB = λ, where λ is a constant, then the locus of the point C is -

(A) $y\lambda = 2a$ (B) $y = \lambda a$ (C) $ya = 2\lambda$ (D) none of these

Solution : Given that coordinates of two fixed points A and B are (a, 0) and (–a, 0) respectively. Let variable point C is (h, k). From the adjoining figure



$$\cot A = \frac{DA}{CD} = \frac{a-h}{k}$$

$$\cot B = \frac{BD}{CD} = \frac{a+h}{k}$$

But cotA + cotB = λ, so we have

$$\frac{a-h}{k} + \frac{a+h}{k} = \lambda \Rightarrow \frac{2a}{k} = \lambda$$

Hence locus of C is $y\lambda = 2a$ **Ans. (A)**

Do yourself - 5 :

(i) Find the locus of a variable point which is at a distance of 2 units from the y-axis.

(ii) Find the locus of a point which is equidistant from both the axes.

10. STRAIGHT LINE :

Introduction : A relation between x and y which is satisfied by co-ordinates of every point lying on a line is called equation of the straight line. Here, remember that every one degree equation in variable x and y always represents a straight line i.e. **ax + by + c = 0 ; a & b ≠ 0** simultaneously.

(a) Equation of a line parallel to x-axis at a distance 'a' is **y = a** or **y = – a.**

(b) Equation of x-axis is **y = 0.**

(c) Equation of a line parallel to y-axis at a distance 'b' is **x = b** or **x = – b.**

(d) Equation of y-axis is **x = 0.**

Illustration 11 : Prove that every first degree equation in x, y represents a straight line.

Solution : Let $ax + by + c = 0$ be a first degree equation in x, y where a, b, c are constants.

Let $P(x_1, y_1)$ & $Q(x_2, y_2)$ be any two points on the curve represented by $ax + by + c = 0$.
Then $ax_1 + by_1 + c = 0$ and $ax_2 + by_2 + c = 0$

Let R be any point on the line segment joining P & Q

Suppose R divides PQ in the ratio $\lambda : 1$. Then, the coordinates of R are $\left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}\right)$

We have $a\left(\frac{\lambda x_2 + x_1}{\lambda+1}\right) + b\left(\frac{\lambda y_2 + y_1}{\lambda+1}\right) + c = \lambda\ 0 + 0 = 0$

$\therefore$ $R\left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}\right)$ lies on the curve represented by $ax + by + c = 0$. Thus every point on the line segment joining P & Q lies on $ax + by + c = 0$.

Hence $ax + by + c = 0$ represents a straight line.

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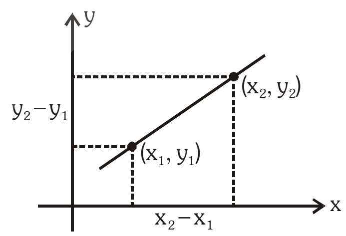
11. SLOPE OF LINE :

If a given line makes an angle θ(**0° ≤ θ < 180°, θ ≠ 90°**) with the positive direction of x-axis, then slope of this line will be $\tan\theta$ and is usually denoted by the letter **m** i.e. $\mathbf{m = \tan\theta}$. If $A(x_1, y_1)$ and $B(x_2, y_2)$ & $x_1 \neq x_2$ then slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$



Remark :

(i) If $\theta = 90°$, **m does not exist** and line is parallel to **y-axis.**

(ii) If $\theta = 0°$, **m = 0** and the line is parallel to **x-axis.**

(iii) Let m_1 and m_2 be slopes of two given lines (none of them is parallel to y-axis)

(a) If lines are parallel, $\mathbf{m_1 = m_2}$ and vice-versa.

(b) If lines are perpendicular, $\mathbf{m_1 m_2 = -1}$ and vice-versa

12. STANDARD FORMS OF EQUATIONS OF A STRAIGHT LINE :

(a) **Slope Intercept form :** Let m be the slope of a line and c its intercept on y-axis. Then the equation of this straight line is written as : $y = mx + c$

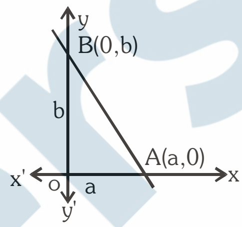
If the line passes through origin, its equation is written as $y = mx$

(b) **Point Slope form :** If m be the slope of a line and it passes through a point (x_1, y_1), then its equation is written as : $y - y_1 = m(x - x_1)$

(c) **Two point form :** Equation of a line passing through two points (x_1,y_1) and (x_2,y_2) is written as :

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1) \text{ or } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0$$

(d) **Intercept form :** If a and b are the intercepts made by a line on the axes of x and y, its equation is written as : $\frac{x}{a}+\frac{y}{b}=1$



(i) Length of intercept of line between the coordinate axes $=\sqrt{a^2+b^2}$

(ii) Area of triangle AOB $=\frac{1}{2}OA.OB=\left|\frac{1}{2}ab\right|$

Illustration 12 : The equation of the lines which passes through the point (3, 4) and the sum of its intercepts on the axes is 14 is -

(A) $4x-3y=24$, $x-y=7$ (B) $4x+3y=24$, $x+y=7$

(C) $4x+3y+24=0$, $x+y+7=0$ (D) $4x-3y+24=0$, $x-y+7=0$

Solution : Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ .....(i)

This passes through (3 , 4) , therefore $\frac{3}{a}+\frac{4}{b}=1$ .....(ii)

It is given that $a+b=14 \Rightarrow b=14-a$. Putting $b=14-a$ in (ii) , we get

$\frac{3}{a}+\frac{4}{14-a}=1 \Rightarrow a^2-13a+42=0 \Rightarrow (a-7)(a-6)=0 \Rightarrow a=7, 6$

For $a=7$, $b=14-7=7$ and for $a=6$, $b=14-6=8$

Putting the values of a and b in (i) , we get the equations of the lines

$\frac{x}{7}+\frac{y}{7}=1$ and $\frac{x}{6}+\frac{y}{8}=1$ or $x+y=7$ and $4x+3y=24$ **Ans. (B)**

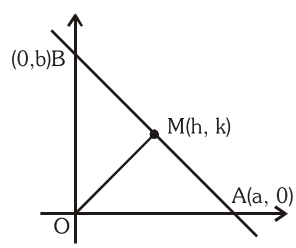
Illustration 13 : Two points A and B move on the positive direction of x-axis and y-axis respectively, such that OA + OB = K. Show that the locus of the foot of the perpendicular from the origin O on the line AB is $(x+y)(x^2+y^2)=Kxy$.

Solution : Let the equation of AB be $\frac{x}{a}+\frac{y}{b}=1$ .... (i)

given, $a+b=K$ .... (ii)

now, $m_{AB}\times m_{OM}=-1 \Rightarrow ah=bk$ .... (iii)

from (ii) and (iii),



$a = \dfrac{kK}{h+k}$ and $b = \dfrac{hK}{h+k}$

$\therefore$ from (i) $\dfrac{x(h+k)}{k.K} + \dfrac{y(h+k)}{h.K} = 1$

as it passes through (h, k)

$\dfrac{h(h+k)}{k.K} + \dfrac{k(h+k)}{h.K} = 1 \Rightarrow \quad (h+k)(h^2+k^2) = Khk$

$\therefore$ locus of (h, k) is $(x+y)(x^2+y^2) = Kxy$.

(e) Normal form : If p is the length of perpendicular on a line from the origin, and α the angle which this perpendicular makes with positive x-axis, then the equation of this line is written as : $\mathbf{x\cos\alpha + y\sin\alpha = p}$ (p is always positive) where $\mathbf{0 \le \alpha < 2\pi}$.

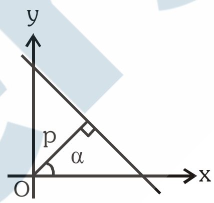
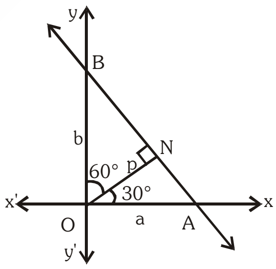


Illustration 14 : Find the equation of the straight line on which the perpendicular from origin makes an angle 30° with positive x-axis and which forms a triangle of area $\left(\dfrac{50}{\sqrt{3}}\right)$ sq. units with the co-ordinates axes.

Solution : $\angle NOA = 30°$

Let ON = p > 0, OA = a, OB = b



In ΔONA, $\cos 30° = \dfrac{ON}{OA} = \dfrac{p}{a} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{p}{a}$

or $\quad a = \dfrac{2p}{\sqrt{3}}$

and in ΔONB, $\cos 60° = \dfrac{ON}{OB} = \dfrac{p}{b} \Rightarrow \dfrac{1}{2} = \dfrac{p}{b}$

or $\quad b = 2p$

$\because$ Area of $\Delta OAB = \dfrac{1}{2}\,ab = \dfrac{1}{2}\left(\dfrac{2p}{\sqrt{3}}\right)(2p) = \dfrac{2p^2}{\sqrt{3}}$

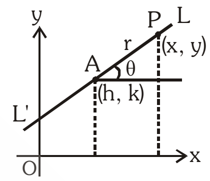
$\therefore \quad \dfrac{2p^2}{\sqrt{3}} = \dfrac{50}{\sqrt{3}} \quad \Rightarrow \quad p^2 = 25$

or $\quad p = 5$

$\therefore$ Using $x\cos\alpha + y\sin\alpha = p$, the equation of the line AB is $x\cos 30° + y\sin 30° = 5$

or $\quad x\sqrt{3} + y = 10$

(f) Parametric form : To find the equation of a straight line which passes through a given point A(h, k) and makes a given angle θ with the positive direction of the x-axis. P(x, y) is any point on the line LAL'.



Let AP = r, then **x – h = r cosθ, y – k = r sinθ** & $\frac{x-h}{\cos\theta}=\frac{y-k}{\sin\theta}$ **= r** is the equation of the straight line LAL'.

Any point P on the line will be of the form (h + r cosθ, k + r sinθ), where |r| gives the distance of the point P from the fixed point (h, k).

Illustration 15 : Equation of a line which passes through point A(2, 3) and makes an angle of 45° with x axis. If this line meet the line x + y + 1 = 0 at point P then distance AP is -

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $5\sqrt{2}$ (D) $2\sqrt{5}$

Solution : Here $x_1 = 2$, $y_1 = 3$ and θ = 45° hence $\frac{x-2}{\cos 45^\circ}=\frac{y-3}{\sin 45^\circ}=r$

from first two parts ⇒ x – 2 = y – 3 ⇒ x – y + 1 = 0

Co-ordinate of point P on this line is $\left(2+\frac{r}{\sqrt{2}}, 3+\frac{r}{\sqrt{2}}\right)$.

If this point is on line x + y + 1 = 0 then

$\left(2+\frac{r}{\sqrt{2}}\right)+\left(3+\frac{r}{\sqrt{2}}\right)+1=0$ ⇒ $r=-3\sqrt{2}$; $|r|=3\sqrt{2}$ **Ans. (B)**

Illustration 16 : A variable line is drawn through O, to cut two fixed straight lines L_1 and L_2 in A_1 and A_2, respectively. A point A is taken on the variable line such that $\frac{m+n}{OA}=\frac{m}{OA_1}+\frac{n}{OA_2}$.

Show that the locus of A is a straight line passing through the point of intersection of L_1 and L_2 where O is being the origin.

Solution : Let the variable line passing through the origin is $\frac{x}{\cos\theta}=\frac{y}{\sin\theta}=r_i$ ...... (i)

Let the equation of the line L_1 is $p_1x + q_1y = 1$ ...... (ii)

Equation of the line L_2 is $p_2x + q_2y = 1$ ...... (iii)

the variable line intersects the line (ii) at A_1 and (iii) at A_2.

Let $OA_1 = r_1$.

Then $A_1 = (r_1\cos\theta, r_1\sin\theta)$ ⇒ A_1 lies on L_1

⇒ $r_1 = OA_1 = \frac{1}{p_1\cos\theta+q_1\sin\theta}$

Similarly, $r_2 = OA_2 = \frac{1}{p_2\cos\theta+q_2\sin\theta}$

Let OA = r

Let co-ordinate of A are (h, k) ⇒ (h, k) ≡ (rcosθ, rsinθ)

Now $\frac{m+n}{r}=\frac{m}{OA_1}+\frac{n}{OA_2} \Rightarrow \frac{m+n}{r}=\frac{m}{r_1}+\frac{n}{r_2}$

$$\Rightarrow \quad m+n = m(p_1 r\cos\theta + q_1 r\sin\theta) + n(p_2 r\cos\theta + q_2 r\sin\theta)$$

$$\Rightarrow \quad (p_1 h + q_1 k - 1) + \frac{n}{m}(p_2 h + q_2 k - 1) = 0$$

Therefore, locus of A is $(p_1x+q_1y-1) + \frac{n}{m}(p_2x+q_2y-1) = 0$

$$\Rightarrow \quad L_1 + \lambda L_2 = 0 \text{ where } \lambda = \frac{n}{m}.$$

This is the equation of the line passing through the intersection of L_1 and L_2.

Illustration 17 : A straight line through P(–2, –3) cuts the pair of straight lines $x^2 + 3y^2 + 4xy - 8x - 6y - 9 = 0$ in Q and R. Find the equation of the line if PQ. PR = 20.

Solution : Let line be $\frac{x+2}{\cos\theta} = \frac{y+3}{\sin\theta} = r$

$\Rightarrow \quad x = r\cos\theta - 2,\ y = r\sin\theta - 3 \qquad$ .... (i)

Now, $x^2 + 3y^2 + 4xy - 8x - 6y - 9 = 0 \qquad$ .... (ii)

Taking intersection of (i) with (ii) and considering terms of r^2 and constant (as we need PQ . PR = $r_1 . r_2$ = product of the roots)

$r^2(\cos^2\theta + 3\sin^2\theta + 4\sin\theta\cos\theta) + (\text{some terms})r + 80 = 0$

$\therefore \quad r_1.r_2 = PQ.\ PR = \dfrac{80}{\cos^2\theta + 4\sin\theta\cos\theta + 3\sin^2\theta}$

$\therefore \quad \cos^2\theta + 4\sin\theta\cos\theta + 3\sin^2\theta = 4 \qquad (\because\ PQ\ .\ PR = 20)$

$\therefore \quad \sin^2\theta - 4\sin\theta\cos\theta + 3\cos^2\theta = 0$

$\Rightarrow \quad (\sin\theta - \cos\theta)(\sin\theta - 3\cos\theta) = 0$

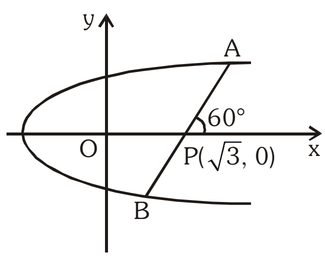
$\therefore \quad \tan\theta = 1,\ \tan\theta = 3$

hence equation of the line is $y + 3 = 1(x + 2) \Rightarrow x - y = 1$

and $y + 3 = 3(x + 2) \Rightarrow 3x - y + 3 = 0$.

Illustration 18 : If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then find the value of PA.PB {where $P \equiv (\sqrt{3}, 0)$}

Solution : Slope of line $y - \sqrt{3}x + 3 = 0$ is $\sqrt{3}$.



If line makes an angle θ with x-axis, then $\tan\theta = \sqrt{3}$

$\therefore \quad \theta = 60°$

$\frac{x-\sqrt{3}}{\cos 60°} = \frac{y-0}{\sin 60°} = r \Rightarrow \left(\sqrt{3} + \frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$ be a point on the parabola $y^2 = x + 2$

then $\frac{3}{4}r^2 = \sqrt{3} + \frac{r}{2} + 2 \Rightarrow 3r^2 - 2r - 4(2 + \sqrt{3}) = 0$

$$\therefore \quad PA.PB = r_1 r_2 = \left|\frac{-4(2+\sqrt{3})}{3}\right| = \frac{4(2+\sqrt{3})}{3}$$

Do yourself - 6 :

(i) Reduce the line $2x - 3y + 5 = 0$,

(a) In slope- intercept form and hence find slope & Y-intercept

(b) In intercept form and hence find intercepts on the axes.

(c) In normal form and hence find perpendicular distance from the origin and angle made by the perpendicular with the positive x-axis.

(ii) Find distance of point A (2, 3) measured parallel to the line $x - y = 5$ from the line $2x + y + 6 = 0$.

(g) **General form :** We know that a first degree equation in x and y, $ax + by + c = 0$ always represents a straight line. This form is known as general form of straight line.

(i) Slope of this line $= \frac{-a}{b} = -\frac{\text{coeff. of } x}{\text{coeff. of } y}$

(ii) Intercept by this line on x-axis $= -\frac{c}{a}$ and intercept by this line on y-axis $= -\frac{c}{b}$

(iii) To change the general form of a line to normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2 + b^2}$.

13. EQUATION OF LINES PARALLEL AND PERPENDICULAR TO A GIVEN LINE :

(a) Equation of line parallel to line $ax + by + c = 0$

$\mathbf{ax + by + \lambda = 0}$

(b) Equation of line perpendicular to line $ax + by + c = 0$

$\mathbf{bx - ay + k = 0}$

Here λ, k, are parameters and their values are obtained with the help of additional information given in the problem.

14. ANGLE BETWEEN TWO LINES :

(a) If θ be the angle between two lines : $y = m_1x + c_1$ and $y = m_2x + c_2$, then $\tan\theta = \pm\left(\frac{m_1 - m_2}{1 + m_1m_2}\right)$

Note :

(i) There are two angles formed between two lines but usually the acute angle is taken as the angle between the lines. So we shall find θ from the above formula only by taking positive value of $\tan\theta$.

(ii) Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0$; $L_2 = 0$; $L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the Δ ABC found by these formulas are given by,

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2} ; \tan B = \frac{m_2 - m_3}{1 + m_2 m_3} \;\&\; \tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$$

(b) If equation of lines are $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, then these lines are -

(i) Parallel $\Leftrightarrow$ $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

(ii) Perpendicular $\Leftrightarrow$ $a_1a_2+b_1b_2=0$

(iii) Coincident $\Leftrightarrow$ $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Illustration 19 : If $x + 4y - 5 = 0$ and $4x + ky + 7 = 0$ are two perpendicular lines then k is -

(A) 3 (B) 4 (C) –1 (D) –4

Solution : $m_1 = -\frac{1}{4} \quad m_2 = -\frac{4}{k}$

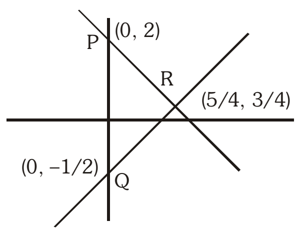
Two lines are perpendicular if $m_1 m_2 = -1$

$\Rightarrow \left(-\frac{1}{4}\right)\times\left(-\frac{4}{k}\right) = -1 \Rightarrow k = -1$ **Ans. (C)**

Illustration 20 : A line L passes through the points (1, 1) and (0, 2) and another line M which is perpendicular to L passes through the point (0, –1/2).The area of the triangle formed by these lines with y-axis is -

(A) 25/8 (B) 25/16 (C) 25/4 (D) 25/32

Solution : Equation of the line L is $y-1=\frac{-1}{1}(x-1) \Rightarrow y=-x+2$



Equation of the line M is $y = x - 1/2$.

If these lines meet y-axis at P and Q, then $PQ = 5/2$.

Also x-coordinate of their point of intersection $R = 5/4$

$\therefore$ area of the $\Delta PQR = \frac{1}{2}\left(\frac{5}{2}\times\frac{5}{4}\right) = 25/16.$ **Ans. (B)**

Illustration 21 : If the straight line $3x + 4y + 5 - k(x + y + 3) = 0$ is parallel to y-axis, then the value of k is -

(A) 1 (B) 2 (C) 3 (D) 4

Solution : A straight line is parallel to y-axis, if its y - coefficient is zero, i.e. $4 - k = 0$ i.e. $k = 4$ **Ans. (D)**

15. STRAIGHT LINE MAKING A GIVEN ANGLE WITH A LINE :

Equation of line passing through a point (x_1,y_1) and making an angle α, with the line $y=mx+c$ is written as :

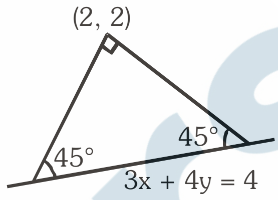
$$y-y_1=\frac{m\pm\tan\alpha}{1\mp m\tan\alpha}(x-x_1)$$

Illustration 22 : Find the equation to the sides of an isosceles right-angled triangle, the equation of whose hypotenuse is $3x + 4y = 4$ and the opposite vertex is the point (2, 2).

Solution : The problem can be restated as :

Find the equations of the straight lines passing through the given point (2, 2) and making equal angles of 45° with the given straight line $3x + 4y - 4 = 0$. Slope of the line $3x + 4y - 4 = 0$ is $m_1 = -3/4$.

$\Rightarrow \quad \tan 45° = \pm\dfrac{m - m_1}{1 + m_1 m}$, i.e., $1 = \pm\dfrac{m + 3/4}{1 - \dfrac{3}{4}m}$



$m_A = \dfrac{1}{7}$, and $m_B = -7$

Hence the required equations of the two lines are

$y - 2 = m_A(x - 2)$ and $y - 2 = m_B(x - 2)$

$\Rightarrow \quad 7y - x - 12 = 0$ and $7x + y = 16$ **Ans.**

Do yourself - 7 :

(i) Find the angle between the lines $3x + y - 7 = 0$ and $x + 2y - 9 = 0$.

(ii) Find the line passing through the point (2, 3) and perpendicular to the straight line $4x - 3y = 10$.

(iii) Find the equation of the line which has positive y-intercept 4 units and is parallel to the line $2x - 3y - 7 = 0$. Also find the point where it cuts the x-axis.

(iv) Classify the following pairs of lines as coincident, parallel or intersecting :

(a) $x + 2y - 3 = 0$ & $-3x - 6y + 9 = 0$

(b) $x + 2y + 1 = 0$ & $2x + 4y + 3 = 0$

(c) $3x - 2y + 5 = 0$ & $2x + y - 5 = 0$

16. LENGTH OF PERPENDICULAR FROM A POINT ON A LINE :

Length of perpendicular from a point (x_1, y_1) on the line $ax + by + c = 0$ is $\left|\dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

In particular, the length of the perpendicular from the origin on the line $ax + by + c = 0$ is $P = \dfrac{|c|}{\sqrt{a^2 + b^2}}$

Illustration 23 : If the algebraic sum of perpendiculars from n given points on a variable straight line is zero then prove that the variable straight line passes through a fixed point.

Solution : Let n given points be (x_i, y_i) where i = 1, 2.... n and the variable straight line is $ax + by + c = 0$.

Given that $\sum_{i=1}^{n}\left(\dfrac{ax_i + by_i + c}{\sqrt{a^2 + b^2}}\right) = 0 \quad \Rightarrow \quad a\Sigma x_i + b\Sigma y_i + cn = 0 \Rightarrow a\dfrac{\Sigma x_i}{n} + b\dfrac{\Sigma y_i}{n} + c = 0$.

Hence the variable straight line always passes through the fixed point $\left(\dfrac{\Sigma x_i}{n}, \dfrac{\Sigma y_i}{n}\right)$.

Ans.

Illustration 24 : Prove that no line can be drawn through the point (4, –5) so that its distance from (–2, 3) will be equal to 12.

Solution : Suppose, if possible.

Equation of line through (4, –5) with slope m is $y + 5 = m(x - 4)$

$\Rightarrow \quad mx - y - 4m - 5 = 0$

Then $\dfrac{|m(-2) - 3 - 4m - 5|}{\sqrt{m^2 + 1}} = 12$

$\Rightarrow \quad |-6m - 8| = 12\sqrt{(m^2 + 1)}$

On squaring, $\quad (6m + 8)^2 = 144(m^2 + 1)$

$\Rightarrow \quad 4(3m + 4)^2 = 144(m^2 + 1) \Rightarrow (3m + 4)^2 = 36(m^2 + 1)$

$\Rightarrow \quad 27m^2 - 24m + 20 = 0 \qquad \text{......... (i)}$

Since the discriminant of (i) is $(-24)^2 - 4.27.20 = -1584$ which is negative, there is no real value of m. Hence no such line is possible.

17. DISTANCE BETWEEN TWO PARALLEL LINES :

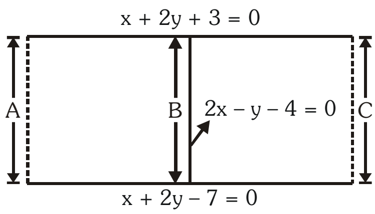
(a) The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $= \dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

(**Note :** The coefficients of x & y in both equations should be same)

(b) The area of the parallelogram $= \dfrac{p_1 p_2}{\sin\theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides . Note that area of the parallelogram bounded by the lines $y = m_1x + c_1$, $y = m_1x + c_2$ and $y = m_2x + d_1$, $y = m_2x + d_2$ is given by $\left|\dfrac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

Illustration 25 : Three lines $x + 2y + 3 = 0$, $x + 2y - 7 = 0$ and $2x - y - 4 = 0$ form 3 sides of two squares. Find the equation of remaining sides of these squares.

Solution : Distance between the two parallel lines is $\dfrac{|7 + 3|}{\sqrt{5}} = 2\sqrt{5}$.



The equations of sides A and C are of the form $2x - y + k = 0$.

Since distance between sides A and B = distance between sides B and C

$\Rightarrow \quad \dfrac{|k - (-4)|}{\sqrt{5}} = 2\sqrt{5} \Rightarrow \dfrac{k + 4}{\sqrt{5}} = \pm 2\sqrt{5} \Rightarrow k = 6, -14.$

Hence the fourth sides of the two squares are (i) $2x - y + 6 = 0$ (ii) $2x - y - 14 = 0$.

Ans.

Do yourself - 8 :

(i) Find the distances between the following pair of parallel lines :

(a) $3x + 4y = 13,\ 3x + 4y = 3$

(b) $3x - 4y + 9 = 0,\ 6x - 8y - 15 = 0$

(ii) Find the points on the x-axis such that their perpendicular distance from the line $\frac{x}{a}+\frac{y}{b}=1$ is 'a', a, $b > 0$.

(iii) Show that the area of the parallelogram formed by the lines

$2x - 3y + a = 0,\ 3x - 2y - a = 0,\ 2x - 3y + 3a = 0$ and $3x - 2y - 2a = 0$ is $\frac{2a^2}{5}$ square units.

18. POSITION OF TWO POINTS WITH RESPECT TO A GIVEN LINE :

Let the given line be $ax + by + c = 0$ and $P(x_1, y_1)$, $Q(x_2, y_2)$ be two points. If the expressions $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same signs, then both the points P and Q lie on the same side of the line $ax + by + c = 0$. If the quantities $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have opposite signs, then they lie on the opposite sides of the line.

19. CONCURRENCY OF LINES :

(a) Three lines $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent,

if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_2 & c_3 \end{vmatrix} = 0$

(b) To test the concurrency of three lines, first find out the point of intersection of any two of the three lines. If this point lies on the remaining line (i.e. coordinates of the point satisfy the equation of the line) then the three lines are concurrent otherwise not concurrent.

Illustration 27 : If the lines $ax + by + p = 0$, $x\cos\alpha + y\sin\alpha - p = 0$ $(p \neq 0)$ and $x\sin\alpha - y\cos\alpha = 0$ are concurrent and the first two lines include an angle $\frac{\pi}{4}$, then $a^2 + b^2$ is equal to -

(A) 1 (B) 2 (C) 4 (D) p^2

Solution : Since the given lines are concurrent,

$$\begin{vmatrix} a & b & p \\ \cos\alpha & \sin\alpha & -p \\ \sin\alpha & -\cos\alpha & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad a\cos\alpha + b\sin\alpha + 1 = 0$ .... (i)

As $ax + by + p = 0$ and $x\cos\alpha + y\sin\alpha - p = 0$ include an angle $\frac{\pi}{4}$.

$$\pm\tan\frac{\pi}{4} = \frac{-\frac{a}{b}+\frac{\cos\alpha}{\sin\alpha}}{1+\frac{a}{b}\frac{\cos\alpha}{\sin\alpha}}$$

$\Rightarrow \quad -a\sin\alpha + b\cos\alpha = \pm(b\sin\alpha + a\cos\alpha)$

$\Rightarrow \quad -a\sin\alpha + b\cos\alpha = \pm 1$ [from (i)] .... (ii)

Squaring and adding (i) & (ii), we get

$a^2 + b^2 = 2.$ **Ans. (B)**

Do yourself - 9 :

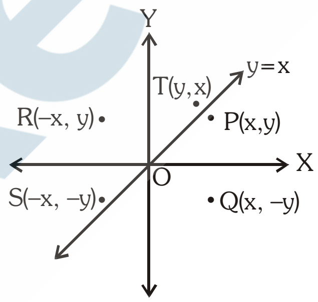
(i) Examine the positions of the points (3, 4) and (2, – 6) w.r.t. $3x - 4y = 8$

(ii) If (2, 9), (– 2, 1) and (1, –3) are the vertices of a triangle, then prove that the origin lies inside the triangle.

(iii) Find the equation of the line joining the point (2, – 9) and the point of intersection of lines $2x + 5y - 8 = 0$ and $3x - 4y - 35 = 0$.

(iv) Find the value of λ, if the lines $3x - 4y - 13 = 0$, $8x - 11y - 33 = 0$ and $2x - 3y + \lambda = 0$ are concurrent.

20. REFLECTION OF A POINT :



Let P(x, y) be any point, then its image with respect to

(a) x-axis is Q(x, –y)

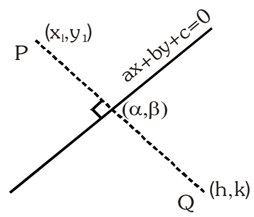
(b) y-axis is R(–x, y)

(c) origin is S(–x,–y)

(d) line y = x is T(y, x)

(e) Reflection of a point about any arbitrary line : The image (h,k) of a point $P(x_1, y_1)$ about the line $ax + by + c = 0$ is given by following formula.

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -2\frac{(ax_1 + by_1 + c)}{a^2 + b^2}$$

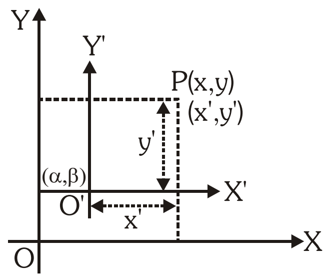


and the foot of perpendicular (α,β) from a point (x_1, y_1) on the line $ax + by + c = 0$ is given by following formula.

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

21. TRANSFORMATION OF AXES

(a) Shifting of origin without rotation of axes :



Let P (x, y) with respect to axes OX and OY.

Let O' (α, β) is new origin with respect to axes OX and OY and let P (x', y') with respect to axes O'X' and O'Y' , where OX and O'X' are parallel and OY and O'Y' are parallel.

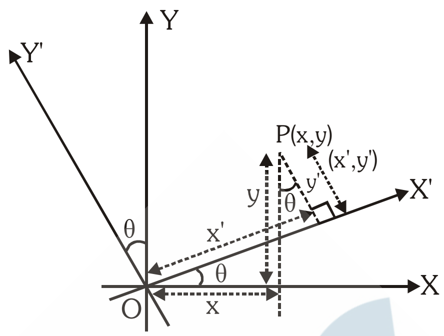
Then $\mathbf{x = x' + \alpha, \quad y = y' + \beta}$

or $\quad \mathbf{x' = x - \alpha, \quad y' = y - \beta}$

Thus if origin is shifted to point (α, β) without rotation of axes, then new equation of curve can be obtained by putting $x + \alpha$ in place of x and $y + \beta$ in place of y.

(b) Rotation of axes without shifting the origin :

Let O be the origin. Let P (x, y) with respect to axes OX and OY and let P (x', y') with respect to axes OX' and OY' where $\angle X'OX = \angle YOY' = \theta$, where θ is measured in anticlockwise direction.



then $x = x' \cos\theta - y' \sin\theta$

$y = x' \sin\theta + y' \cos\theta$

and $x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

The above relation between (x, y) and (x', y') can be easily obtained with the help of following table

New \ Old	$x \downarrow$	$y \downarrow$
$x' \rightarrow$	$\cos\theta$	$\sin\theta$
$y' \rightarrow$	$-\sin\theta$	$\cos\theta$

Illustration 28 : Through what angle should the axes be rotated so that the equation $9x^2 - 2\sqrt{3}\,xy + 7y^2 = 10$ may be changed to $3x^2 + 5y^2 = 5$?

Solution : Let angle be θ then replacing (x, y) by $(x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$

then $9x^2 - 2\sqrt{3}xy + 7y^2 = 10$ becomes

$$9(x\cos\theta - y\sin\theta)^2 - 2\sqrt{3}(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta) + 7(x\sin\theta + y\cos\theta)^2 = 10$$

$$\Rightarrow\quad x^2(9\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta + 7\sin^2\theta) + 2xy(-9\sin\theta\cos\theta - \sqrt{3}\cos 2\theta + 7\sin\theta\cos\theta)$$

$$+ y^2(9\cos^2\theta + 2\sqrt{3}\sin\theta\cos\theta + 7\cos^2\theta) = 10$$

On comparing with $3x^2 + 5y^2 = 5$ (coefficient of xy = 0)

We get $\quad -9\sin\theta\cos\theta - \sqrt{3}\cos 2\theta + 7\sin\theta\cos\theta = 0$

or $\quad \sin 2\theta = -\sqrt{3}\cos 2\theta \quad$ or $\quad \tan 2\theta = -\sqrt{3} = \tan(180° - 60°)$

or $\quad 2\theta = 120° \quad \therefore \quad \theta = 60°$

Do yourself - 10 :

(i) The point (4, 1) undergoes the following transformations, then the match the correct alternatives:

Column-I	**Column-II**
(A) Reflection about x-axis is	(p) (4, –1)
(B) Reflection about y-axis is	(q) (–4, –1)
(C) Reflection about origin is	(r) $\left(-\frac{12}{25}, -\frac{59}{25}\right)$
(D) Reflection about the line y = x is	(s) (–4, 1)
(E) Reflection about the line 4x + 3y – 5 = 0 is	(t) (1, 4)

(ii) To what point must the origin be shifted, so that the coordinates of the point (4, 5) become (– 3, 9).

(iii) If the axes be turned through an angle $\tan^{-1}2$ (in anticlockwise direction), what does the equation $4xy - 3x^2 = a^2$ become ?

22. EQUATION OF BISECTORS OF ANGLES BETWEEN TWO LINES :

If equation of two intersecting lines are $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, then equation of bisectors of the angles between these lines are written as :

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \qquad \text{..........(i)}$$

(a) Equation of bisector of angle containing origin :

If the equation of the lines are written with constant terms $\mathbf{c_1}$ and $\mathbf{c_2}$ positive, then the equation of the bisectors of the angle containing the origin is obtained by taking positive sign in (i)

(b) Equation of bisector of acute/obtuse angles :

To find the equation of the bisector of the acute or obtuse angle :

(i) let ϕ be the angle between one of the two bisectors and one of two given lines. Then if $\mathbf{\tan\phi < 1}$ i.e. $\phi < 45°$ i.e. $2\phi < 90°$, the angle bisector will be bisector of acute angle.

(ii) See whether the constant terms c_1 and c_2 in the two equation are +ve or not. If not then multiply both sides of given equation by –1 to make the constant terms positive.

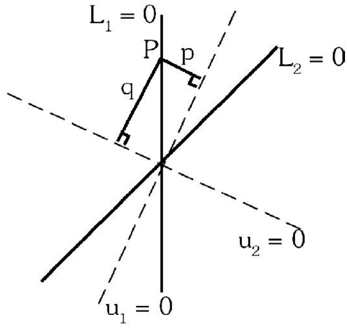
Determine the sign of $\mathbf{a_1a_2 + b_1b_2}$

If sign of $a_1a_2 + b_1b_2$	For obtuse angle bisector	For acute angle bisector
+	use + sign in eq. (1)	use – sign in eq. (1)
–	use – sign in eq. (1)	use + sign in eq. (1)

i.e. if $a_1a_2 + b_1b_2 > 0$, then the bisector corresponding to + sign gives obtuse angle bisector

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

(iii) Another way of identifying an acute and obtuse angle bisector is as follows :



Let $L_1 = 0$ & $L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ & $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ & $u_2 = 0$ as shown . If,

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector .

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector .

$|p| = |q| \Rightarrow$ the lines L_1 & L_2 are perpendicular .

Note : Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

Illustration 29 : For the straight lines $4x + 3y - 6 = 0$ and $5x + 12y + 9 = 0$, find the equation of the

(i) bisector of the obtuse angle between them.

(ii) bisector of the acute angle between them.

(iii) bisector of the angle which contains origin.

(iv) bisector of the angle which contains (1, 2).

Solution : Equations of bisectors of the angles between the given lines are

$$\frac{4x+3y-6}{\sqrt{4^2+3^2}} = \pm\frac{5x+12y+9}{\sqrt{5^2+12^2}} \Rightarrow 9x - 7y - 41 = 0 \text{ and } 7x + 9y - 3 = 0$$

If θ is the acute angle between the line $4x + 3y - 6 = 0$ and the bisector

$$9x - 7y - 41 = 0, \text{ then } \tan\theta = \left|\frac{-\frac{4}{3}-\frac{9}{7}}{1+\left(\frac{-4}{3}\right)\frac{9}{7}}\right| = \frac{11}{3} > 1$$

Hence

(i) bisector of the obtuse angle is $9x - 7y - 41 = 0$

(ii) bisector of the acute angle is $7x + 9y - 3 = 0$

(iii) bisector of the angle which contains origin

$$\frac{-4x-3y+6}{\sqrt{(-4)^2+(-3)^2}} = \frac{5x+12y+9}{\sqrt{5^2+12^2}} \Rightarrow 7x+9y-3=0$$

(iv) $L_1(1, 2) = 4 \times 1 + 3 \times 2 - 6 = 4 > 0$
$L_2(1, 2) = 5 \times 1 + 12 \times 2 + 9 = 38 > 0$

+ve sign will give the required bisector, $\dfrac{4x+3y-6}{5} = +\dfrac{5x+12y+9}{13}$

$\Rightarrow \quad 9x - 7y - 41 = 0.$

Alternative :

Making c_1 and c_2 positive in the given equation, we get $-4x - 3y + 6 = 0$ and $5x + 12y + 9 = 0$

Since $a_1a_2 + b_1b_2 = -20 - 36 = -56 < 0$, so the origin will lie in the acute angle.

Hence bisector of the acute angle is given by

$$\frac{-4x-3y+6}{\sqrt{4^2+3^2}} = \frac{5x+12y+9}{\sqrt{5^2+12^2}} \quad \Rightarrow \quad 7x+9y-3=0$$

Similarly bisector of obtuse angle is $9x - 7y - 41 = 0$.

Illustration 30 : A ray of light is sent along the line $x - 2y - 3 = 0$. Upon reaching the line mirror $3x - 2y - 5 = 0$, the ray is reflected from it. Find the equation of the line containing the reflected ray.

Solution : Let Q be the point of intersection of the incident ray and the line mirror, then

$$x_1 - 2y_1 - 3 = 0 \;\;\&\;\; 3x_1 - 2y_1 - 5 = 0$$

on solving these equations, we get

$$x_1 = 1 \;\;\&\;\; y_1 = -1$$

Since P(–1, –2) be a point lies on the incident ray, so we can find the image of the point P on the reflected ray about the line mirror (by property of reflection).

Let P'(h, k) be the image of point P about line mirror, then

$$\frac{h+1}{3} = \frac{k+2}{-2} = \frac{-2(-3+4-5)}{13} \Rightarrow h = \frac{11}{13} \text{ and } k = \frac{-42}{13}.$$

So $\quad P'\left(\frac{11}{13}, \frac{-42}{13}\right)$

Then equation of reflected ray will be

$$(y+1) = \frac{\left(\frac{-42}{13}+1\right)(x-1)}{\left(\frac{11}{13}-1\right)}$$

$\Rightarrow \quad 2y - 29x + 31 = 0.$

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23. FAMILY OF LINES :

If equation of two lines be $P \equiv a_1x + b_1y + c_1 = 0$ and $Q \equiv a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is : $P + \lambda Q = 0$ or $a_1x + b_1y + c_1 + \lambda(a_2x + b_2y + c_2) = 0$. The value of λ is obtained with the help of the additional informations given in the problem.

Illustration 31 : Prove that each member of the family of straight lines $(3\sin\theta + 4\cos\theta)x + (2\sin\theta - 7\cos\theta)y + (\sin\theta + 2\cos\theta) = 0$ (θ is a parameter) passes through a fixed point.

Solution : The given family of straight lines can be rewritten as

$(3x + 2y + 1)\sin\theta + (4x - 7y + 2)\cos\theta = 0$

or, $(4x - 7y + 2) + \tan\theta(3x + 2y + 1) = 0$ which is of the form $L_1 + \lambda L_2 = 0$

Hence each member of it will pass through a fixed point which is the intersection of

$4x - 7y + 2 = 0$ and $3x + 2y + 1 = 0$ i.e. $\left(\frac{-11}{29}, \frac{2}{29}\right)$.

Do yourself - 11 :

(i) Find the equations of bisectors of the angle between the lines $4x + 3y = 7$ and $24x + 7y - 31 = 0$. Also find which of them is (a) the bisector of the angle containing origin (b) the bisector of the acute angle.

(ii) Find the equations of the line which pass through the point of intersection of the lines $4x - 3y = 1$ and $2x - 5y + 3 = 0$ and is equally inclined to the coordinate axes.

(iii) Find the equation of the line through the point of intersection of the lines $3x - 4y + 1 = 0$ & $5x + y - 1 = 0$ and perpendicular to the line $2x - 3y = 10$.

24. PAIR OF STRAIGHT LINES :

(a) Homogeneous equation of second degree :

(i) Let us consider the homogeneous equation of 2nd degree as

$ax^2 + 2hxy + by^2 = 0$ ....(i)

which represents pair of straight lines passing through the origin.

Now, we divide by x^2, we get

$$a + 2h\left(\frac{y}{x}\right) + b\left(\frac{y}{x}\right)^2 = 0$$

$$\frac{y}{x} = m \quad \text{(say)}$$

then $a + 2hm + bm^2 = 0$ ....(ii)

if m_1 & m_2 are the roots of equation (ii), then $m_1 + m_2 = -\frac{2h}{b}$, $m_1m_2 = \frac{a}{b}$

and also, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{1 + m_1m_2}\right| = \left|\frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{1 + \frac{a}{b}}\right| = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

These line will be :

(1) **Real and different**, if $h^2 - ab > 0$ (2) **Real and coincident**, if $h^2 - ab = 0$

(3) **Imaginary**, if $h^2 - ab < 0$

(ii) The condition that these lines are :

(1) At **right angles** to each other is $\mathbf{a + b = 0}$. i.e. coefficient of $\mathbf{x^2}$ + coefficient of $\mathbf{y^2} = 0$.

(2) **Coincident** is $\mathbf{h^2 = ab}$.

(3) **Equally inclined** to the axes of x is $\mathbf{h = 0}$. i.e. coefficient of $\mathbf{xy = 0}$.

(iii) Homogeneous equation of 2nd degree $\mathbf{ax^2 + 2hxy + by^2 = 0}$ always represent a pair of straight lines whose equations are

$$y = \left(\frac{-h \pm \sqrt{h^2 - ab}}{b}\right)x \equiv y = m_1x \;\&\; y = m_2x \quad \text{and} \quad m_1 + m_2 = -\frac{2h}{b}\;;\; m_1m_2 = \frac{a}{b}$$

These straight lines passes through the origin.

(iv) Pair of straight lines perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$ and through origin are given by $bx^2 - 2hxy + ay^2 = 0$.

(v) The product of the perpendiculars drawn from the point (x_1, y_1) on the lines $ax^2 + 2hxy + by^2 = 0$ is $\left|\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}\right|$

Note : A homogeneous equation of degree n represents n straight lines passing through **origin**.

(b) The combined equation of angle bisectors :

The combined equation of angle bisectors between the lines represented by homogeneous equation of 2nd degree is given by $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$, $\mathbf{a \neq b, h \neq 0}$.

Note :

(i) If $\mathbf{a = b}$, the bisectors are $\mathbf{x^2 - y^2 = 0}$ i.e. $\mathbf{x - y = 0, x + y = 0}$

(ii) If $\mathbf{h = 0}$, the bisectors are $\mathbf{xy = 0}$ i.e. $\mathbf{x = 0, y = 0}$.

(iii) The two bisectors are always at **right angles**, since we have coefficient of $\mathbf{x^2}$ + coefficient of $\mathbf{y^2 = 0}$

(c) General Equation and Homogeneous Equation of Second Degree :

(i) The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ i.e. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

(ii) If θ be the angle between the lines, then $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$

Obviously these lines are

(1) Parallel, if $\Delta = 0$, $h^2 = ab$ or if $h^2 = ab$ and $bg^2 = af^2$

(2) Perpendicular, if $a + b = 0$ i.e. coeff. of x^2 + coeff. of $y^2 = 0$.

(iii) The product of the perpendiculars drawn from the origin to the lines

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\left|\frac{c}{\sqrt{(a - b)^2 + 4h^2}}\right|$

Illustration 32 : If $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, then λ is equal to -

(A) 4 (B) 3 (C) 2 (D) 1

Solution : Here $a = \lambda$, $b = 12$, $c = -3$, $f = -8$, $g = 5/2$, $h = -5$

Using condition $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, we have

$\lambda(12)(-3) + 2(-8)(5/2)(-5) - \lambda(64) - 12(25/4) + 3(25) = 0$

$\Rightarrow -36\lambda + 200 - 64\lambda - 75 + 75 = 0 \quad \Rightarrow \quad 100\lambda = 200$

$\therefore \quad \lambda = 2$ **Ans. (C)**

Do yourself - 12 :

(i) Prove that the equation $x^2 - 5xy + 4y^2 = 0$ represents two lines passing through the origin. Also find their equations.

(ii) If the equation $3x^2 + kxy - 10y^2 + 7x + 19y = 6$ represents a pair of lines, find the value of k.

(iii) If the equation $6x^2 - 11xy - 10y^2 - 19y - 6 = 0$ represents a pair of lines, find their equations. Also find the angle between the two lines.

(iv) Find the point of intersection and the angle between the lines given by the equation : $2x^2 - 3xy - 2y^2 + 10x + 5y = 0$.

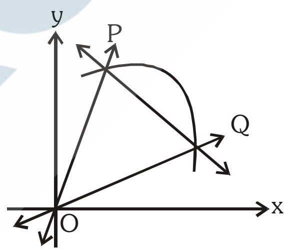
25. EQUATIONS OF LINES JOINING THE POINTS OF INTERSECTION OF A LINE AND A CURVE TO THE ORIGIN :

(a) Let the equation of curve be :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \qquad ...(i)$$

and straight line be

$$lx + my + n = 0 \qquad ...(ii)$$



Now joint equation of line OP and OQ joining the origin and points of intersection P and Q can be obtained by making the equation (i) homogenous with the help of equation of the line. Thus required equation is given by

$$ax^2+2hxy+by^2+2(gx+fy)\left(\frac{\ell x+my}{-n}\right)+c\left(\frac{\ell x+my}{-n}\right)^2 = 0$$

$\Rightarrow \quad (an^2 + 2gln + cl^2)x^2 + 2(hn^2 + gmn + fln + clm)xy + (bn^2 + 2fmn + cm^2)y^2 = 0 \quad ......(iii)$

All points which satisfy (i) and (ii) simultaneously, will satisfy (iii)

(b) Any second degree curve through the four points of intersection of $f(x, y) = 0$ & $xy = 0$ is given by $f(x, y) + \lambda\, xy = 0$ where $f(x, y) = 0$ is also a second degree curve.

Illustration 35 : The chord $\sqrt{6}y = \sqrt{8}px + \sqrt{2}$ of the curve $py^2 + 1 = 4x$ subtends a right angle at origin then find the value of p.

Solution : $\sqrt{3}y - 2px = 1$ is the given chord. Homogenizing the equation of the curve, we get,

$$py^2 - 4x(\sqrt{3}\,y - 2px)+(\sqrt{3}\,y - 2px)^2 = 0$$

$$\Rightarrow (4p^2 + 8p)x^2 + (p + 3)y^2 - 4\sqrt{3}\,xy - 4\sqrt{3}\,pxy = 0$$

Now, angle at origin is 90°

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$\therefore$ $4p^2 + 8p + p + 3 = 0 \Rightarrow 4p^2 + 9p + 3 = 0$

$\therefore$ $p = \dfrac{-9 \pm \sqrt{81-48}}{8} = \dfrac{-9 \pm \sqrt{33}}{8}$.

Do yourself - 13 :

(i) Find the angle subtended at the origin by the intercept made on the curve $x^2 - y^2 - xy + 3x - 6y + 18 = 0$ by the line $2x - y = 3$.

(ii) Find the equation of the lines joining the origin to the points of intersection of the curve $2x^2 + 3xy - 4x + 1 = 0$ and the line $3x + y = 1$.

ANSWERS FOR DO YOURSELF

1 : **(i)** $PQ = \sqrt{34}$ units ; **(ii)** $x = 6$ or $x = 0$ **(iii)** $11, -7$

2 : **(i) (a)** $(2,1)$; **(b)** $(7,16)$; **(ii) (a)** $2 : 3$ (internally) ; **(b)** $9 : 4$ (externally) ; **(c)** $8 : 7$ (internally)

3 : **(i)** **(a)** $\left(\dfrac{5}{3}, 3\right)$; **(b)** $\left(-\dfrac{9}{2}, \dfrac{15}{2}\right)$, $\dfrac{5\sqrt{10}}{2}$, **(c)** $(14, -6)$

4 : **(i)** 25 square units ; **(ii)** 132 square units ;

5 : **(i)** $x = \pm 2$; **(ii)** $y = \pm x$;

6 : **(i)** **(a)** $y = \dfrac{2x}{3} + \dfrac{5}{3}, \dfrac{2}{3}, \dfrac{5}{3}$; **(b)** $\dfrac{x}{(-5/2)} + \dfrac{y}{(5/3)} = 1, -\dfrac{5}{2}, \dfrac{5}{3}$;

(c) $-\dfrac{2x}{\sqrt{13}} + \dfrac{3y}{\sqrt{13}} = \dfrac{5}{\sqrt{13}}, \dfrac{5}{\sqrt{13}}$, $\alpha = \pi - \tan^{-1}\left(\dfrac{3}{2}\right)$; **(ii)** $13\sqrt{2}/3$ units

7 : **(i)** $\theta = 135°$ or $45°$; **(ii)** $3x + 4y = 18$; **(iii)** $2x - 3y + 12 = 0$, $(-6, 0)$

(iv) **(a)** Coincident, **(b)** Parallel, **(c)** Intersecting

8 : **(i)** **(a)** 2 (b) 33/10 ; **(ii)** $\left(\dfrac{a}{b}\left(b \pm \sqrt{a^2 + b^2}\right), 0\right)$

9 : **(i)** opposite sides of the line; **(iii)** $-y + x = 11$; **(iv)** $\lambda = -7$

10 : (i) (A) $\to$ (p), (B) $\to$ (s), (C) $\to$ (q), (D) $\to$ (t), (E) $\to$ (r),; **(ii)** $(7, -4)$; **(iv)** $x^2 - 4y^2 = a^2$

11 : (i) $x - 2y + 1 = 0$ & $2x + y - 3 = 0$; **(a)** $x - 2y + 1 = 0$; **(b)** $2x + y - 3 = 0$

(ii) $x + y = 2$, $x = y$; **(iii)** $69x + 46y - 25 = 0$

12 : (i) $x - y = 0$ & $x - 4y = 0$; **(ii)** $k = -1$, or $-\dfrac{127}{6}$;

(iii) $2x - 5y - 2 = 0$ & $3x + 2y + 3 = 0$; $\pm\tan^{-1}\left(\dfrac{19}{4}\right)$ **(iv)** $(-1, 2)$, $90°$

13 : (i) $\theta = \pm\tan^{-1}\dfrac{4}{7}$; **(ii)** $x^2 - y^2 - 5xy = 0$

EXERCISE (O-1)

1. Coordinates of the vertices of a triangle ABC are (12,8), (–2,6) and (6,0) then the **correct** statement is-

(A) triangle is right but not isosceles

(B) triangle is isosceles but not right

(C) triangle is obtuse

(D) the product of the abscissa of the centroid, orthocenter and circumcenter is 160.

2. Find the value of x_1 if the distance between the points $(x_1, 2)$ and (3, 4) be 8.

(A) $3 \pm 2\sqrt{15}$ (B) $3 \pm \sqrt{15}$ (C) $2 \pm 3\sqrt{15}$ (D) $2 \pm \sqrt{15}$

3. If P(1,2), Q(4,6), R(5,7) & S(a,b) are the vertices of a parallelogram PQRS, then :

(A) a = 2, b = 4 (B) a = 3, b = 4 (C) a = 2, b = 3 (D) a = 3, b = 5

4. The four points whose co-ordinates are (2,1),(1,4),(4,5),(5,2) form :

(A) a rectangle which is not a square (B) a trapezium which is not a parallelogram

(C) a square (D) a rhombus which is not a square

5. The length of a line segment AB is 10 units. If the coordinates of one extremity are (2,–3) and the abscissa of the other extremity is 10 then the sum of all possible values of the ordinate of the other extremity is -

(A) 3 (B) –4 (C) 12 (D) –6

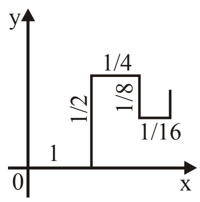
6. If A and B are the points (–3,4) and (2,1), then the co-ordinates of the point C on AB produced such that AC = 2BC are :

(A) (2,4) (B) (3,7) (C) (7,–2) (D) $\left(-\frac{1}{2}, \frac{5}{2}\right)$

7. The orthocenter of the triangle ABC is 'B' and the circumcenter is 'S' (a,b). If A is the origin then the co-ordinates of C are :

(A) (2a,2b) (B) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2}, 0\right)$ (D) none

8. A particle begins at the origin and moves successively in the following manner as shown,



1 unit to the right, 1/2 unit up, 1/4 unit to the right,

1/8 unit down, 1/16 unit to the right etc.

The length of each move is half the length of the previous move and movement continues in the 'zigzag' manner infinitely. The co-ordinates of the point to which the 'zigzag' converges is -

(A) (4/3, 2/3) (B) (4/3,2/5) (C) (3/2,2/3) (D) (2,2/5)

9. The medians of a triangle meet at (0,–3) and its two vertices are at (–1,4) and (5,2). Then the third vertex is at -

(A) (4,15) (B) (–4,–15) (C) (–4,15) (D) (4,–15)

10. If the two vertices of a triangle are (7,2) and (1,6) and its centroid is (4,6) then the coordinate of the third vertex are (a,b). The value of (a + b), is-

(A) 13 (B) 14 (C) 15 (D) 16

11. If in triangle ABC, A ≡ (1,10), circumcenter ≡ $\left(-\frac{1}{3},\frac{2}{3}\right)$ and orthocenter ≡ $\left(\frac{11}{3},\frac{4}{3}\right)$ then the co-ordinates of mid-point of side opposite to A is-

(A) (1,–11/3) (B) (1,5) (C) (1,–3) (D) (1,6)

12. Suppose ABC is a triangle with 3 acute angles A,B and C. The point whose coordinates are (cosB – sinA, sinB – cosA) can be in the -

(A) first and 2^{nd} quadrant (B) second the 3^{rd} quadrant

(C) third and 4^{th} quadrant (D) second quadrant only

13. Consider the points P(2,–4); Q(4,–2) and R(7,1). The points P,Q,R -

(A) form an equilateral triangle

(B) form a right angled triangle

(C) form an isosceles triangle which is not equilateral

(D) are collinear.

14. A triangle has two of its vertices at (0,1) and (2,2) in the cartesian plane. Its third vertex lies on the x-axis. If the area of the triangle is 2 square units then the sum of the possible abscissae of the third vertex, is-

(A) –4 (B) 0 (C) 5 (D) 6

15. A point P(x,y) moves so that the sum of the distance from P to the coordinate axes is equal to the distance from P to the point A(1,1). The equation of the locus of P in the first quadrant is -

(A) $(x + 1)(y + 1) = 1$ (B) $(x + 1)(y + 1) = 2$

(C) $(x – 1)(y – 1) = 1$ (D) $(x – 1)(y – 1) = 2$

16. Let A(2,–3) and B(–2,1) be vertices of a ΔABC. If the centroid of ΔABC moves on the line $2x + 3y = 1$, then the locus of the vertex C is-

(A) $2x + 3y = 9$ (B) $2x – 3y = 7$ (C) $3x + 2y = 5$ (D) $3x – 2y = 3$

17. A stick of length 10 units rests against the floor and a wall of a room. If the stick begins to slide on the floor then the locus of its middle point is :

(A) $x^2 + y^2 = 2.5$ (B) $x^2 + y^2 = 25$ (C) $x^2 + y^2 = 100$ (D) none

18. Given the points A(0,4) and B(0,–4), the equation of the locus of the point P such that |AP – BP| = 6 is-

(A) $9x^2 – 7y^2 + 63 = 0$ (B) $9x^2 – 7y^2 – 63 = 0$

(C) $7x^2 – 9y^2 + 63 = 0$ (D) $7x^2 – 9y^2 – 63 = 0$

19. A line passes through (2,2) and cuts a triangle of area 9 square units from the first quadrant. The sum of all possible values for the slope of such a line, is-

(A) –2.5 (B) –2 (C) –1.5 (D) –1

20. The diagonals of a parallelogram PQRS are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then PQRS must be a :

(A) rectangle (B) square

(C) cyclic quadrilateral (D) rhombus

21. A and B are any two points on the positive x and y axis respectively satisfying 2(OA) + 3(OB) = 10. If P is the middle point of AB then the locus of P is-

(A) $2x + 3y = 5$ (B) $2x + 3y = 10$ (C) $3x + 2y = 5$ (D) $3x + 2y = 10$

22. The co-ordinates of the orthocentre of the triangle bounded by the lines, $4x - 7y + 10 = 0$; $x + y = 5$ and $7x + 4y = 15$ is-

(A) (2,1) (B) (–1,2) (C) (1,2) (D) (1,–2)

23. If the x intercept of the line $y = mx + 2$ is greater than 1/2 then the gradient of the line lies in the interval-

(A) (–1,0) (B) (–1/4,0) (C) $(-\infty,-4)$ (D) (–4,0)

24. The greatest slope along the graph represented by the equation $4x^2 - y^2 + 2y - 1 = 0$, is-

(A) –3 (B) –2 (C) 2 (D) 3

25. The extremities of the base of an isosceles triangle ABC are the points A(2,0) and B(0,1). If the equation of the side AC is x = 2 then the slope of the side BC is -

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\sqrt{3}$

26. Number of lines that can be drawn through the point(4,–5) so that its distance from (–2,3) will be equal to 12 is equal to-

(A) 0 (B) 1 (C) 2 (D) 3

27. Two mutually perpendicular straight lines through the origin from an isosceles triangle with the line $2x + y = 5$. Then the area of the triangle is :

(A) 5 (B) 3 (C) 5/2 (D) 1

28. Point 'P' lies on the line l $\{(x,y) \mid 3x + 5y = 15\}$. If 'P' is also equidistant from the coordinate axes, then P can be located in which of the four quadrants -

(A) I only (B) II only (C) I or II only (D) IV only

29. The area of the parallelogram formed by the lines $3x + 4y = 7a$; $3x + 4y = 7b$; $4x + 3y = 7c$ and $4x + 3y = 7d$ is-

(A) $\frac{|(a-b)(c-d)|}{7}$ (B) $|(a-b)(c-d)|$ (C) $\frac{|(a-b)(c-d)|}{49}$ (D) $7|(a-b)(c-d)|$

30. If x_1,y_1 are the roots of $x^2 + 8x - 20 = 0$, x_2,y_2 are the roots of $4x^2 + 32x - 57 = 0$ and x_3,y_3 are the roots of $9x^2 + 72x - 112 = 0$, then the points, (x_1,y_1), (x_2,y_2) and (x_3,y_3) -

(A) are collinear (B) form an equilateral triangle

(C) form a right angled isosceles triangle (D) are concyclic

31. If the lines $\left.\begin{array}{l}\lambda x+(\sin\alpha)y+\cos\alpha=0\\ x+(\cos\alpha)y+\sin\alpha=0\\ x-(\sin\alpha)y+\cos\alpha=0\end{array}\right]$ pass through the same point where $\alpha \in R$ then λ lies in the interval -

(A) $[-1,1]$ (B) $\left[-\sqrt{2},\sqrt{2}\right]$ (C) $[-2,2]$ (D) $(-\infty,\infty)$

32. Family of lines represented by the equation $(\cos\theta + \sin\theta)x + (\cos\theta - \sin\theta)y - 3(3\cos\theta + \sin\theta) = 0$ passes through a fixed point M for all real values of θ. The reflection of M in the line $x - y = 0$, is-

(A) (6,3) (B) (3,6) (C) (–6,3) (D) (3,–6)

33. m,n are integer with $0 < n < m$. A is the point (m,n) on the cartesian plane. B is the reflection of A in the line $y = x$. C is the reflection of B in the y-axis, D is the reflection of C in the x-axis and E is the reflection of D in the y-axis. The area of the pentagon ABCDE is -

(A) $2m(m + n)$ (B) $m(m + 3n)$ (C) $m(2m + 3n)$ (D) $2m(m + 3n)$

34. Area of the quadrilateral formed by the lines $|x| + |y| = 2$ is :

(A) 8 (B) 6 (C) 4 (D) none

35. Consider a parallelogram whose sides are represented by the lines $2x + 3y = 0$; $2x + 3y - 5 = 0$; $3x - 4y = 0$ and $3x - 4y = 3$. The equation of the diagonal not passing through the origin, is-

(A) $21x - 11y + 15 = 0$ (B) $9x - 11y + 15 = 0$

(C) $21x - 29y - 15 = 0$ (D) $21x - 11y - 15 = 0$

36. If the straight lines, $ax + amy + 1 = 0$, $bx + (m + 1)\,by + 1 = 0$ and $cx + (m + 2)cy + 1 = 0$, $m \neq 0$ are concurrent then a,b,c are in :

(A) A.P. only for m = 1 (B) A.P. for all m

(C) G.P. for all m (D) H.P. for all m

37. A ray of light passing through the point A(1,2) is reflected at a point B on the x-axis and then passes through (5,3). Then the equation of AB is :

(A) $5x + 4y = 13$ (B) $5x - 4y = -3$

(C) $4x + 5y = 14$ (D) $4x - 5y = -6$

38. If the line $y = mx$ bisects the angle between the lines $ax^2 + 2h\,xy + by^2 = 0$ then m is a root of the quadratic equation :

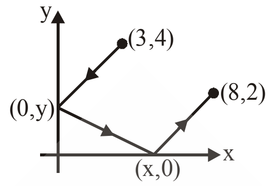
(A) $hx^2 + (a - b)x - h = 0$ (B) $x^2 + h(a - b)x - 1 = 0$

(C) $(a - b)x^2 + hx - (a - b) = 0$ (D) $(a - b)x^2 - hx - (a - b) = 0$

39. If the equation $ax^2 - 6xy + y^2 + 2gx + 2fy + c = 0$ represents a pair of lines whose slopes are m and m^2, then sum of all possible values of a is-

(A) 17 (B) –19 (C) 19 (D) –17

40. Suppose that a ray of light leaves the point (3,4), reflects off the y-axis towards the x-axis, reflects off the x-axis, and finally arrives at the point (8,2). The value of x, is-



(A) $x = 4\frac{1}{2}$ (B) $x = 4\frac{1}{3}$

(C) $x = 4\frac{2}{3}$ (D) $x = 5\frac{1}{3}$

41. Let S = {(x,y)| $x^2 + 2xy + y^2 - 3x - 3y + 2 = 0$}, then S -

(A) consists of two coincident lines.

(B) consists of two parallel lines which are not coincident.

(C) consists of two intersecting lines.

(D) is a parabola.

[MATRIX LIST TYPE]

42. Find the equation to the straight line :

	Column-I		**Column-II**
(P)	passing through the point (2, 3) and perpendicular to the straight line $4x - 3y = 10$.	(1)	$4y + 11x = 10$
(Q)	passing through the point (–6, 10) and perpendicular to the straight line $7x + 8y = 5$.	(2)	$4y + 3x = 18$
(R)	passing through the point (2, –3) and perpendicular to the straight line joining the points (5, 7) and (–6, 3).	(3)	$x + 4y + 16 = 0$
		(4)	$9y - 8x = 118$
(S)	passing through the point (–4, –3) and perpendicular to the straight line joining (1, 3) and (2, 7).		

Codes :

	P	Q	R	S
(A)	1	2	3	4
(B)	2	4	1	3
(C)	4	3	2	1
(D)	1	3	4	2

[MATRIX MATCH]

43.

	Column-I		**Column-II**
(A)	The ratio in which the point $\left(\frac{10}{7}, \frac{33}{7}\right)$ divides the line segment joining the points (1, 3) and (2, 7) is p : q, then (p + q) is (p & q are coprime)	(P)	$\frac{1}{5}$
(B)	The ratio in which the point (–2, –9) divides The line segment joining the points (1, 3) and (2, 7) is p : q, then (p + q) is (p & q are coprime)	(Q)	–1
(C)	If (p, q) divides internally the line joining (–1, 2) and (4, –5) in ratio 2 : 3 then (p + q) is	(R)	5
(D)	If (p, q) divides externally the line joining (–1, 2) (4, –5) in ratio 2 : 3 then (p + q) is	(S)	7

44. **Column-I** | **Column-II**

(A) The points (2,–2), (8,4), (5,7) and (–1,1) taken in order constitute the vertices of a

(B) The points (0,–1),(2,1),(0,3) and (–2,1) taken in order are the vertices of a

(C) The points (3,–5), (–5,–4),(7,10),(15,9) taken in order are the vertices of a

(D) The points (–3,4), (–1,0), (1,0) and (3,4) taken in order are the vertices of a

(P) square

(Q) rectangle

(R) trapezium

(S) parallelogram

(T) cyclic quadrilateral

45. **Column-I** | **Column-II**

(A) The points (2,–2), (–2,1) and (5,2)

(B) The points (1,–2),(–3,0) and (5,6)

(C) The points (3,7),(6,5) and (15,–1)

(D) The points (2,2), (–2,–2) and $\left(-2\sqrt{3}, 2\sqrt{3}\right)$

(P) are the vertices of a right angled triangle

(Q) are the vertices of a right angle isosceles triangle

(R) are the vertices of an equilateral triangle

(S) do not form a triangle

46. **Equation of Straight Line**

Column-I | **Column-II**

(A) Which cuts-off an intercept 4 on the x-axis and passes through the point (2,–3).

(B) Which cuts-off equal intercepts on the co-ordinate axes and passes through (2,5)

(C) Which makes an angle of 135° with the axis of x and which cuts the axis of y at a distance -8 from the origin and

(D) Through the point (4,1) and making with the axes in the first quadrant a triangle whose area is 8.

(P) $2x + y + 1 = 0$

(Q) $x + y = 7$

(R) $3x - 2y = 12$

(S) $x + 4y = 8$

(T) $x + y + 8 = 0$

47. **Column-I** | **Column-II**

(A) The four lines $3x - 4y + 11 = 0$; $3x - 4y - 9 = 0$; $4x + 3y + 3 = 0$ and $4x + 3y - 17 = 0$ enclose a figure which is

(B) The lines $2x + y = 1$, $x + 2y = 1$, $2x + y = 3$ and $x + 2y = 3$ form a figure which is

(C) If 'O' is the origin, P is the intersection of the lines $2x^2 - 7xy + 3y^2 + 5x + 10y - 25 = 0$, A and B are the points in which these lines are cut by the line $x + 2y - 5 = 0$, then the points O,A,P,B (in some order) are the vertices of

(P) a quadrilateral which is neither a parallelogram nor a trapezium nor a kite.

(Q) a parallelogram which is neither a rectangle nor a rhombus

(R) a rhombus which is not a square

(S) a square

EXERCISE (O-2)

1. The line x= c cuts the triangle with corners (0,0); (1,1) and (9,1) into two region. For the area of the two regions to be the same c must be equal to-

(A) 5/2 (B) 3 (C) 7/2 (D) 3 or 15

2. If m and b are real numbers and $mb > 0$, then the line whose equation is $y = mx + b$ cannot contain the point-

(A) (0,2009) (B) (2009,0) (C) (0,–2009) (D) (20,–100)

3. In a triangle ABC, side AB has the equation $2x + 3y = 29$ and the side AC has the equation, $x + 2y = 16$. If the mid- point of BC is (5,6) then the equation of BC is :

(A) $x - y = -1$ (B) $5x - 2y = 13$ (C) $x + y = 11$ (D) $3x - 4y = -9$

4. The vertex of the right angle of a right angled triangle lies on the straight line $2x - y - 10 = 0$ and the two other vertices, at points (2,–3) and (4,1) then the area of triangle in sq. units is-

(A) $\sqrt{10}$ (B) 3 (C) $\frac{33}{5}$ (D) 11

5. The line $(k + 1)^2x + ky - 2k^2 - 2 = 0$ passes through a point regardless of the value k. Which of the following is the line with slope 2 passing through the point ?

(A) $y = 2x - 8$ (B) $y = 2x - 5$ (C) $y = 2x - 4$ (D) $y = 2x + 8$

6. A triangle ABC is formed by the lines $2x - 3y - 6 = 0$; $3x - y + 3 = 0$ and $3x + 4y - 12 = 0$. If the points $P(\alpha,0)$ and $Q(0,\beta)$ always lie on or inside the ΔABC, then :

(A) $\alpha \in [-1,2]$ and $\beta \in [-2,3]$ (B) $\alpha \in [-1,3]$ and $\beta \in [-2,4]$

(C) $\alpha \in [-2,4]$ and $\beta \in [-3,4]$ (D) $\alpha \in [-1,3]$ and $\beta \in [-2,3]$

7. The co-ordinates of a point P on the line $2x - y + 5 = 0$ such that $|PA - PB|$ is maximum where A is (4,–2) and B is (2,–4) will be :

(A) (11,27) (B) (–11,–17) (C) (–11,17) (D) (0,5)

8. If the straight lines joining the origin and the points of intersection of the curve
$5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$
are equally inclined to the x- axis then the value of k :

(A) is equal to 1 (B) is equal to –1

(C) is equal to 2 (D) does not exist in the set of real numbers

9. Through a point A on the x-axis a straight line is drawn parallel to y-axis so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ in B and C. If AB = BC then-

(A) $h^2 = 4ab$ (B) $8h^2 = 9ab$ (C) $9h^2 = 8ab$ (D) $4h^2 = ab$

[MULTIPLE CHOICE]

10. The area of triangle ABC is 20 square units. The co-ordinates of vertex A are (–5,0) and B are (3,0). The vertex C lies on the line, $x - y = 2$. The co-ordinates of C are -

(A) (5,3) (B) (–3,–5) (C) (–5,–7) (D) (7,5)

11. Three vertices of a triangle are A(4,3); B(1,–1) and C(7,k). Value(s) of k for which centroid, orthocentre, incentre and circumcentre of the ΔABC lie on the same straight line is/are-

(A) 7 (B) –1 (C) –19/8 (D) none

12. Line $\frac{x}{a}+\frac{y}{b}=1$ cuts the co-ordinate axes at A(a,0) and B(0,b) and the line $\frac{x}{a'}+\frac{y}{b'}=-1$ at A'(–a',0) and B'(0,–b'). If the points A,B,A',B' are concyclic then the orthocentre of the triangle ABA' is -

(A) (0,0) (B) (0,b') (C) $\left(0,\frac{aa'}{b}\right)$ (D) $\left(0,\frac{bb'}{a}\right)$

13. A line passes through the origin and makes an angle of $\pi/4$ with the line $x - y + 1 = 0$. Then :

(A) equation of the line is x = 0

(B) the equation of the line is y = 0

(C) the point of intersection of the line with the given line is (–1,0)

(D) the point of intersection of the line with the given line is (0,1)

14. Equation of a straight line passing through the point (2,3) and inclined at an angle of arc tan $\frac{1}{2}$ with the line $y + 2x = 5$ is-

(A) y = 3 (B) x = 2 (C) $3x + 4y - 18 = 0$ (D) $4x + 3y - 17 = 0$

15. Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then :

(A) the lines will pass through a fixed point (B) there will be a set of parallel lines

(C) all the lines intersect the line $x = x_1$ (D) all the lines will be parallel to the line $y = x_1$.

16. If one vertex of an equilateral triangle of side 'a' lies at the origin and the other lies on the line $x-\sqrt{3}y=0$ then the co-ordinates of the third vertex are :

(A) (0,a) (B) $\left(\frac{\sqrt{3}a}{2},-\frac{a}{2}\right)$ (C) (0,–a) (D) $\left(-\frac{\sqrt{3}a}{2},\frac{a}{2}\right)$

17. The sides of a triangle are the straight lines $x + y = 1$; $7y = x$ and $\sqrt{3}y+x=0$. Then which of the following is an interior point of the triangle ?

(A) circumcentre (B) centroid

(C) incentre (D) orthocentre

18. If $a^2 + 9b^2 - 4c^2 = 6ab$ then the family of lines $ax + by + c = 0$ are concurrent at :

(A) (1/2, 3/2) (B) (–1/2, –3/2) (C) (–1/2, 3/2) (D) (1/2, –3/2)

19. The x-coordinates of the vertices of a square of unit area are the roots of the equation $x^2-3|x|+2=0$ and the y-coordinates of the vertices are the roots of the equation $y^2-3y+2=0$ then the possible vertices of the square is/are-

(A) (1,1), (2,1), (2,2), (1,2) (B) (–1,1), (–2,1), (–2,2), (–1,2)

(C) (2,1), (1,–1),(1,2),(2,2) (D) (–2,1), (–1,–1),(–1,2), (–2,2)

20. If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of the triangle PQR is/are always rational point(s) ?

(A) centroid (B) incentre (C) circumcentre (D) orthocentre

21. The lines L_1 and L_2 denoted by $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$ intersect at the point P and have gradients m_1 and m_2 respectively. The acute angles between them is θ. Which of the following relations hold good ?

(A) $m_1 + m_2 = 5/4$
(B) $m_1m_2 = 3/8$
(C) acute angle between L_1 and L_2 is $\sin^{-1}\left(\frac{2}{5\sqrt{5}}\right)$.
(D) sum of the abscissa and ordinate of the point P is –1.

22. Let B(1,–3) and D(0,4) represent two vertices of rhombus ABCD in (x,y) plane, then coordinates of vertex A if $\angle BAD = 60°$ can be equal to-

(A) $\left(\frac{1-7\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}\right)$ (B) $\left(\frac{1+7\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$

(C) $\left(\frac{1-14\sqrt{3}}{2}, \frac{1-2\sqrt{3}}{2}\right)$ (D) $\left(\frac{1+14\sqrt{3}}{2}, \frac{1+2\sqrt{3}}{2}\right)$

Paragraph for Question Nos. 23 to 25

Let ABCD is a square with sides of unit length. Points E and F are taken on sides AB and AD respectively so that AE= AF. Let P be a point inside the square ABCD.

23. The maximum possible area of quadrilateral CDFE is -

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{5}{8}$ (D) $\frac{3}{8}$

24. The value of $(PA)^2 – (PB)^2 + (PC)^2 – (PD)^2$ is equal to-

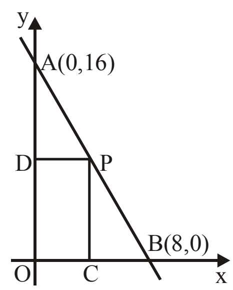
(A) 3 (B) 2 (C) 1 (D) 0

25. Let a line passing through point A divides the square ABCD into two parts so that area of one portion is double the other, then the length of portion of line inside the square is -

(A) $\frac{\sqrt{10}}{3}$ (B) $\frac{\sqrt{13}}{3}$ (C) $\frac{\sqrt{11}}{3}$ (D) $\frac{2}{\sqrt{3}}$

Paragraph for Question Nos. 26 to 28

In the diagram, a line is drawn through the points A(0,16) and B(8,0). Point P is chosen in the first quadrant on the line through A and B. Points C and D are chosen on the x and y axis respectively, so that PDOC is a rectangle.



26. Perpendicular distance of the line AB from the point (2, 2) is -

(A) $\sqrt{4}$ (B) $\sqrt{10}$

(C) $\sqrt{20}$ (D) $\sqrt{50}$

27. Sum of the coordinates of the point P if PDOC is a square is -

(A) $\frac{32}{3}$ (B) $\frac{16}{3}$ (C) 16 (D) 11

28. Number of possible ordered pair(s) of all positions of the point P on AB so that the area of the rectangle PDOC is 30 sq. units, is-

(A) three (B) two (C) one (D) zero

Paragraph for question nos. 29 and 30

An equilateral triangle ABC has its centroid at the origin and the base BC lies along the line $x + y = 1$.

29. Area of the equilateral ΔABC is -

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{3\sqrt{3}}{4}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $\frac{2\sqrt{3}}{4}$

30. Gradient of the other two lines are -

(A) $\sqrt{3}, \sqrt{2}$ (B) $\sqrt{3}, \frac{1}{\sqrt{3}}$ (C) $\sqrt{2}+1, \sqrt{2}-1$ (D) $2+\sqrt{3}, 2-\sqrt{3}$

Paragraph for Question Nos. 31 to 33

Consider two points $A \equiv (1,2)$ and $B \equiv (3,-1)$. Let M be a point on the straight line $L \equiv x + y = 0$.

31. If M be a point on the line $L = 0$ such that AM + BM is minimum, then the reflection of M in the line $x = y$ is -

(A) $(1,-1)$ (B) $(-1,1)$ (C) $(2,-2)$ (D) $(-2,2)$

32. If M be a point on the line $L = 0$ such that $|AM - BM|$ is maximum, then the distance of M from $N \equiv (1,1)$ is-

(A) $5\sqrt{2}$ (B) 7 (C) $3\sqrt{5}$ (D) 10

33. If M be a point on the line $L = 0$ such that $|AM - BM|$ is minimum, then the area of ΔAMB equals-

(A) $\frac{13}{4}$ (B) $\frac{13}{2}$ (C) $\frac{13}{6}$ (D) $\frac{13}{8}$

EXERCISE (S-1)

1. Line $\frac{x}{6} + \frac{y}{8} = 1$ intersects the x and y axes at M and N respectively. If the coordinates of the point P lying inside the triangle OMN (where 'O' is origin) are (a, b) such that the areas of the triangle POM, PON and PMN are equal. Find

(a) the coordinates of the point P and

(b) the radius of the circle escribed opposite to the angle N.

2. Two vertices of a triangle are $(4, -3)$ & $(-2, 5)$. If the orthocentre of the triangle is at $(1, 2)$, find the coordinates of the third vertex.

3. The point A divides the join of $P(-5, 1)$ & $Q(3, 5)$ in the ratio $K : 1$. Find the two values of K for which the area of triangle ABC, where B is $(1, 5)$ & C is $(7, -2)$, is equal to 2 units in magnitude.

4. Determine the ratio in which the point $P(3, 5)$ divides the join of $A(1, 3)$ & $B(7, 9)$. Find the harmonic conjugate of P w.r.t. A & B.

5. A line is such that its segment between the straight lines $5x - y - 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$. Obtain the equation.

6. The area of a triangle is 5. Two of its vertices are (2, 1) & (3, –2). The third vertex lies on $y = x + 3$. Find the third vertex.

7. A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ & $\frac{x}{b}+\frac{y}{a}=1$, meets the coordinate axes in A & B. Find the locus of the mid point of AB.

8. Consider the family of lines $(x - y - 6) + \lambda(2x + y + 3) = 0$ and $(x + 2y - 4) + \lambda(3x - 2y - 4) = 0$. If the lines of these 2 families are at right angle to each other then find the locus of their point of intersection.

9. Two consecutive sides of a parallelogram are $4x + 5y = 0$ & $7x + 2y = 0$. If the equation to one diagonal is $11x + 7y = 9$, find the equation to the other diagonal.

10. The line $3x + 2y = 24$ meets the y-axis at A & the x-axis at B. The perpendicular bisector of AB meets the line through (0, –1) parallel to x-axis at C. Find the area of the triangle ABC.

11. If the straight line drawn through the point P $(\sqrt{3}, 2)$ & inclined at an angle $\frac{\pi}{6}$ with the x-axis, meets the line $\sqrt{3}x - 4y + 8 = 0$ at Q. Find the length PQ.

12. A triangle has side lengths 18, 24 and 30. Find the area of the triangle whose vertices are the incentre, circumcentre and centroid of the triangle.

13. The points (1, 3) & (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$. Find c & the remaining vertices.

14. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line.

15. Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ & its third side passes through the point (1, –10). Determine the equation of the third side.

16. The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are $x - y + 5 = 0$ & $x + 2y = 0$, respectively. If the point A is (1, –2) find the equation of the line BC.

17. Given vertices A (1, 1), B (4, –2) & C (5, 5) of a triangle, find the equation of the perpendicular dropped from C to the interior bisector of the angle A.

18. Consider a triangle ABC with sides AB and AC having the equations $L_1 = 0$ and $L_2 = 0$. Let the centroid, orthocentre and circumcentre of the ΔABC are G, H and S respectively. L = 0 denotes the equation of sides BC.

(a) If $L_1 : 2x - y = 0$ and $L_2 : x + y = 3$ and G(2, 3) then find the slope of the line L = 0.

(b) If $L_1 : 2x + y = 0$ and $L_2 : x - y + 2 = 0$ and H(2, 3) then find the y-intercept of L = 0.

(c) If $L_1 : x + y - 1 = 0$ and $L_2 : 2x - y + 4 = 0$ and S(2, 1) then find the x-intercept of the line L = 0.

19. Let P be the point (3, 2). Let Q be the reflection of P about the x-axis. Let R be the reflection of Q about the line $y = -x$ and let S be the reflection of R through the origin. PQRS is a convex quadrilateral. Find the area of PQRS.

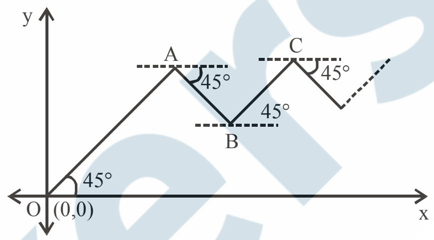
20. Let O(0, 0), A(6, 0) and B(3, $\sqrt{3}$) be the vertices of ΔOAB. Let R be the region consisting of all those points P inside ΔOAB which satisfy $d(P, OA) \le \text{minimum}(d(P,OB), d(P,AB))$ where d(P,OA), d(P,OB) and d(P,AB) represent the distance of P from the sides OA,OB and AB respectively. If the area of region R is $9(a - \sqrt{b})$ where a and b are coprime, then find the value of (a + b)

21. Consider a line pair $2x^2 + 3xy - 2y^2 - 10x + 15y - 28 = 0$ and another line L passing through origin with gradient 3. The line pair and line L form a triangle whose vertices are A, B and C.
(a) Find the sum of the cotangents of the interior angles of the triangle ABC.
(b) Find the area of triangle ABC
(c) Find the radius of the circle touching all the 3 sides of the triangle.

22. A straight line is drawn from the point (1, 0) to the curve $x^2 + y^2 + 6x - 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line.

EXERCISE (S-2)

1. Point O, A, B, C .... are shown in figure where OA = 2AB = 4BC = ... so on. Let A is the centroid of a triangle whose orthocentre and circumcenter are (2, 4) and $\left(\frac{7}{2}, \frac{5}{2}\right)$ respectively. If an insect starts moving from the point O(0,0) along the straight line in zig-zag fashion and terminates ultimately at point $P(\alpha, \beta)$ then find the value of $(\alpha + \beta)$



2. Let ABC be a triangle such that the coordinates of A are (–3, 1). Equation of the median through B is $2x + y - 3 = 0$ and equation of the angular bisector of C is $7x - 4y - 1 = 0$. Then match the entries of column-I with their corresponding correct entries of column-II.

	Column-I		**Column-II**
(A)	Equation of the line AB is	(P)	$2x + y - 3 = 0$
(B)	Equation of the line BC is	(Q)	$2x - 3y + 9 = 0$
(C)	Equation of CA is	(R)	$4x + 7y + 5 = 0$
		(S)	$18x - y - 49 = 0$

3. The equations of perpendiculars of the sides AB & AC of triangle ABC are $x - y - 4 = 0$ and $2x - y - 5 = 0$ respectively. If the vertex A is (– 2, 3) and point of intersection of perpendiculars bisectors is $\left(\frac{3}{2}, \frac{5}{2}\right)$, find the equation of medians to the sides AB & AC respectively.

4. The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are A(–8, 5); B(–15, –19) and C(1, – 7) has the equation $ax + 2y + c = 0$. Find 'a' and 'c'.

5. Find the equation of the straight lines passing through (–2, –7) & having an intercept of length 3 between the straight lines $4x + 3y = 12$, $4x + 3y = 3$.

6. Two sides of a rhombus ABCD are parallel to the lines $y = x + 2$ & $y = 7x + 3$. If the diagonals of the rhombus intersect at the point (1, 2) & the vertex A is on the y-axis, find the possible coordinates of A.

7. Find the equations of the sides of a triangle having (4, –1) as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles.

8. P is the point (–1, 2), a variable line through P cuts the x & y axes at A & B respectively Q is the point on AB such that PA, PQ, PB are H.P. Find the locus of Q.

9. Consider a ΔABC whose sides AB, BC and CA are represented by the straight lines $2x + y = 0$, $x + py = q$ and $x - y = 3$ respectively. The point P is (2, 3).

(a) If P is the centroid, then find the value of $(p + q)$.

(b) If P is the orthocentre, then find the value of $(p + q)$.

(c) If P is the circumcentre, then find the value of $(p + q)$.

10. The sides of a triangle have the combined equation $x^2 - 3y^2 - 2xy + 8y - 4 = 0$. The third side, which is variable always passes through the point $(-5, -1)$. If the range of values of the slope of the third line so that the origin is an interior point of the triangle, lies in the interval (a, b), then find $\left(a + \frac{1}{b^2}\right)$.

11. Show that all the chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 - 2x + 4y = 0$? If yes, what is the point of concurrency & if not, give reasons.

12. The two line pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon find **(i)** area of the polygon; **(ii)** length of its diagonals.

EXERCISE (JM)

1. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are Perpendicular to a common line for : **[AIEEE 2009]**

(1) Exactly two values of p
(2) More than two values of p
(3) No value of p
(4) Exactly one value of p

2. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is : **[AIEEE-2010]**

(1) $\frac{23}{\sqrt{15}}$
(2) $\sqrt{17}$
(3) $\frac{17}{\sqrt{15}}$
(4) $\frac{23}{\sqrt{17}}$

3. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[AIEEE 2011]**

Statement - 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement - 2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is false.
(2) Statement-1 is false, Statement-2 is true
(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

4. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval : **[AIEEE 2011]**

(1) $(-1, 1]$
(2) $(0, \infty)$
(3) $[1, \infty)$
(4) $(-1, \infty)$

5. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[AIEEE 2012]**

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) –4 (4) –2

6. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : **[AIEEE 2012]**

(1) $\frac{11}{5}$ (2) $\frac{29}{5}$ (3) 5 (4) 6

7. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is : **[JEE-MAIN 2013]**

(1) $y=x+\sqrt{3}$ (2) $\sqrt{3}y=x-\sqrt{3}$ (3) $y=\sqrt{3}x-\sqrt{3}$ (4) $\sqrt{3}y=x-1$

8. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1)(1, 1) and (1, 0) is : **[JEE-MAIN 2013]**

(1) $2+\sqrt{2}$ (2) $2-\sqrt{2}$ (3) $1+\sqrt{2}$ (4) $1-\sqrt{2}$

9. A light ray emerging from the point source placed at P(1, 3) is reflected at a point Q in the axis of *x*. If the reflected ray passes through the point R(6, 7), then the abscissa of Q is : **[JEE-MAIN Online 2013]**

(1) 3 (2) $\frac{7}{2}$ (3) 1 (4) $\frac{5}{2}$

10. If the three lines x–3y = p, ax + 2y = q and ax + y = r from a right – angled triangle then: **[JEE-MAIN Online 2013]**

(1) $a^2-6a-12=0$ (2) $a^2-9a+12=0$

(3) $a^2-9a+18=0$ (4) $a^2-6a-18=0$

11. If the x-intercept of some line L is double as that of the line, 3x + 4y = 12 and the y-intercept of L is half as that of the same line, then the slope of L is :- **[JEE-MAIN Online 2013]**

(1) –3 (2) –3/2 (3) –3/8 (4) –3/16

12. If the extremities of the base of an isoscelestriangle are the points (2a, 0) and (0, a) and the equation of one of the sides isx = 2a, then the area of the triangle, in square units, is : **[JEE-MAIN Online 2013]**

(1) $\frac{5}{2}a^2$ (2) $\frac{5}{4}a^2$ (3) $\frac{25a^2}{4}$ (4) $5a^2$

13. Let θ_1 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_2 = 0$, and θ_2 be the angle between two lines $2x + 3y + c_1 = 0$ and $-x + 5y + c_3 = 0$, where c_1, c_2, c_3 are any real numbers : **[JEE-MAIN Online 2013]**

Statement–1 : If c_2 and c_3 are proportional, then $\theta_1 = \theta_2$.

Statement–2 : $\theta_1 = \theta_2$ for all c_2 and c_3.

(1) Statement-1 is true and Statement - 2 is true, Statement-2 is not a correct explanation for Statement-1.

(2) Statement-1 is false and Statement-2 is true.

(3) Statement-1 is true and Statement-2 is false.

(4) Statement-1 is true and Statement - 2 is true, Statement-2 is a correct explanation for Statement-1.

14. Let A (-3, 2) and B (-2,1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line $3x + 4y + 2 = 0$, then the vertex C lies on the line : **[JEE-MAIN Online 2013]**

(1) $4x + 3y + 5 = 0$ (2) $3x + 4y + 5 = 0$ (3) $3x + 4y + 3 = 0$ (4) $4x + 3y + 3 = 0$

15. If the image of point P(2, 3) in a line L is Q (4, 5) then, the image of point R (0, 0) in the same line is : **[JEE-MAIN Online 2013]**

(1) (4, 5) (2) (2, 2) (3) (3, 4) (4) (7, 7)

16. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then : **[JEE(Main)-2014]**

(1) $2bc - 3ad = 0$ (2) $2bc + 3ad = 0$ (3) $3bc - 2ad = 0$ (4) $3bc + 2ad = 0$

17. Let PS be the median of the triangle with vertices P (2, 2), Q (6, –1) and R (7, 3). The equation of the line passing through (1, –1) and parallel to PS is : **[JEE(Main)-2014]**

(1) $4x - 7y - 11 = 0$ (2) $2x + 9y + 7 = 0$ (3) $4x + 7y + 3 = 0$ (4) $2x - 9y - 11 = 0$

18. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a

(1) circle of radius $\sqrt{2}$ (2) circle of radius $\sqrt{3}$

(3) straight line parallel to x-axis (4) straight line parallel to y-axis

[JEE(Main)-2015]

19. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus ? **[JEE(Main)-2016]**

(1) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (2) (–3, –9) (3) (–3, –8) (4) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

20. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[JEE(Main)-2017]**

(1) $\left(2, \frac{1}{2}\right)$ (2) $\left(2, -\frac{1}{2}\right)$ (3) $\left(1, \frac{3}{4}\right)$ (4) $\left(1, -\frac{3}{4}\right)$

21. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is: **[JEE(Main)-2018]**

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$ (3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

22. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is : **[JEE(Main)-2018]**

(1) $2x + 3y = xy$ (2) $3x + 2y = xy$ (3) $3x + 2y = 6xy$ (4) $3x + 2y = 6$

EXERCISE (JA)

1. Consider the lines given by

$L_1 = x + 3y - 5 = 0$

$L_2 = 3x - ky - 1 = 0$

$L_3 = 5x + 2y - 12 = 0$

Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in OMR.

	Column-I		**Column-II**
(A)	L_1, L_2, L_3 are concurrent, if	(P)	$k = -9$
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(Q)	$k = -\frac{6}{5}$
(C)	L_1, L_2, L_3 form a triangle, if	(R)	$k = \frac{5}{6}$
(D)	L_1, L_2, L_3 do not form a triangle, if	(S)	$k = 5$

[JEE 2008, 6]

2. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square **[JEE 2010, 3]**

3. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersect the x-axis, then the equation of L is **[JEE 2011, 3 (–1)]**

(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

4. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then **[JEE-Advanced 2013, 2]**

(A) $a + b - c > 0$ (B) $a - b + c < 0$

(C) $a - b + c > 0$ (D) $a + b - c < 0$

5. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

[JEE(Advanced)-2014, 3]

ANSWER KEY

EXERCISE (O-1)

1. D 2. A 3. C 4. C 5. D 6. C 7. A 8. B
9. B 10. B 11. A 12. D 13. D 14. A 15. B 16. A
17. B 18. A 19. A 20. D 21. A 22. C 23. D 24. C
25. A 26. A 27. A 28. C 29. D 30. A 31. B 32. B
33. B 34. A 35. D 36. D 37. A 38. A 39. B 40. B
41. B 42. B 43. (A) S; (B) S; (C) P; (D) R 44. (A) Q,S,T; (B) P,Q,S,T; (C) S; (D) R,T
45. (A) P,Q; (B) P; (C) S; (D) R 46. (A) R, (B) Q (C) T (D) S 47. (A) S; (B) R ; (C) Q

EXERCISE (O-2)

1. B 2. B 3. C 4. B 5. A 6. D 7. B 8. B
9. B 10. B,D 11. B,C 12. B,C 13. A,B,C,D 14. B,C 15. B,C
16. A,B,C,D 17. B,C 18. C,D 19. A,B 20. A,C,D 21. B,C,D 22. A,B
23. C 24. D 25. B 26. C 27. A 28. B 29. A 30. D
31. B 32. D 33. A

EXERCISE (S-1)

1. (a) $\left(2, \frac{8}{3}\right)$; (b) 4 2. (33, 26) 3. $K = 7$ or $\frac{31}{9}$ 4. 1 : 2 ; Q(–5, –3)

5. $83x - 35y + 92 = 0$ 6. $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$ 7. $2xy(a + b) = ab(x + y)$

8. $x^2 + y^2 - 3x + 4y - 3 = 0$ 9. $x - y = 0$ 10. 91 sq.units 11. 6 units

12. 3 units 13. $c = -4$; B(2, 0); D(4, 4) 14. $x + 5y + 5\sqrt{2} = 0$ or $x + 5y - 5\sqrt{2} = 0$

15. $x - 3y - 31 = 0$ or $3x + y + 7 = 0$ 16. $14x + 23y = 40$ 17. $x - 5 = 0$

18. (a) 5; (b) 2; (c) 3/2 19. 15 20. 5 21. (a) $\frac{50}{7}$; (b) $\frac{63}{10}$; (c) $\frac{3}{10}\left(8\sqrt{5} - 5\sqrt{10}\right)$

22. $x + y = 1$; $x + 9y = 1$

EXERCISE (S-2)

1. 8 2. (A) R; (B) S; (C) Q 3. $x + 4y = 4$; $5x + 2y = 8$ 4. $a = 11$, $c = 78$

5. $7x + 24y + 182 = 0$ or $x = -2$ 6. (0,0) or $\left(0, \frac{5}{2}\right)$

7. $2x - y + 3 = 0$, $2x + y - 7 = 0$, $x - 2y - 6 = 0$ 8. $y = 2x$. 9. (a) 74; (b) 50; (c) 47

10. 24 11. (1,–2), yes $\left(\frac{1}{3}, -\frac{2}{3}\right)$ 12. (i) area = 6 sq. units, (ii) diagonals are $\sqrt{5}$ & $\sqrt{53}$

EXERCISE (JM)

1. 4 2. 4 3. 1 4. 3 5. 4 6. 4 7. 2 8. 2 9. 4 10. 3 11. 4 12. 1
13. 4 14. 3 15. 4 16. 3 17. 2 18. 1 19. 4 20. 1 21. 2 22. 2

EXERCISE (JA)

1. (A) S; (B) P,Q; (C) R ; (D) P,Q,S 2. A 3. B 4. A or C or A,C 5. 6