CONTENTS

SEQUENCE & PROGRESSION

JEE-Advanced Syllabus :

Arithmetic, geometric and harmonic progressions, arithmetic, geometric and harmonic means, sums of finite arithmetic and geometric progressions, infinite geometric series, sums of squares and cubes of the first n natural numbers.

JEE-Mains Syllabus :

Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers. Relation between A.M. and G.M. Sum upto n terms of special series: Sn, Sn^2, Sn^3. Arithmetico – Geometric progression.

SEQUENCE & SERIES

1. DEFINITION :

Sequence :

A succession of terms a_1, a_2, a_3, a_4........ formed according to some rule or law.

Examples are : 1, 4, 9, 16, 25

$-1, 1, -1, 1,$........

$\frac{x}{1!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, ...$

A finite sequence has a finite (i.e. limited) number of terms, as in the first example above. An infinite sequence has an unlimited number of terms, i.e. there is no last term, as in the second and third examples.

Series :

The indicated sum of the terms of a sequence. In the case of a finite sequence a_1, a_2, a_3................, a_n the corresponding series is $a_1 + a_2 + a_3 + ... + a_n = \sum_{k=1}^{n} a_k$. This series has a finite or limited number of terms and is called a finite series.

2. ARITHMETIC PROGRESSION (A.P.) :

A.P. is a sequence whose terms differ by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then A.P. can be written as

$a, a + d, a + 2d, ..., a + (n - 1) d, ...$

(a) n^{th} term of AP $T_n = a + (n-1)d$, where $d = t_n - t_{n-1}$

(b) The sum of the first n terms : $S_n = \frac{n}{2}[a + \ell] = \frac{n}{2}[2a + (n-1)d]$

where ℓ is n^{th} term.

Note :

(i) n^{th} term of an A.P. is of the form An + B i.e. a linear expression in 'n', in such a case the coefficient of n is the common difference of the A.P. i.e. A.

(ii) Sum of first 'n' terms of an A.P. is of the form $An^2 + Bn$ i.e. a quadratic expression in 'n', in such case the common difference is twice the coefficient of n^2. i.e. 2A

(iii) Also n^{th} term $T_n = S_n - S_{n-1}$

Illustration 1 :

If $(x + 1)$, $3x$ and $(4x + 2)$ are first three terms of an A.P. then its 5^{th} term is -

(A) 14 (B) 19 (C) 24 (D) 28

Solution :

(x + 1), 3x, (4x + 2) are in AP

$\Rightarrow \quad 3x - (x + 1) = (4x + 2) - 3x \quad \Rightarrow \quad x = 3$

$\therefore \quad a = 4, d = 9 - 4 = 5 \quad \Rightarrow \quad T_5 = 4 + (4)5 = 24$ **Ans. (C)**

Illustration 2 :

The sum of first four terms of an A.P. is 56 and the sum of it's last four terms is 112. If its first term is 11 then find the number of terms in the A.P.

Solution :

$a + a + d + a + 2d + a + 3d = 56$

$4a + 6d = 56$

$44 + 6d = 56$ (as $a = 11$)

$6d = 12$ hence $d = 2$

Let total number of terms = n

Now sum of last four terms.

$a + (n - 1)d + a + (n - 2)d + a + (n - 3)d + a + (n - 4)d = 112$

$\Rightarrow \quad 4a + (4n - 10)d = 112 \quad \Rightarrow \quad 44 + (4n - 10)2 = 112$

$\Rightarrow \quad 4n - 10 = 34$

$\Rightarrow \quad n = 11$ **Ans.**

Illustration 3 :

The sum of first n terms of two A.Ps. are in ratio $\frac{7n+1}{4n+27}$. Find the ratio of their 11^{th} terms.

Solution :

Let a_1 and a_2 be the first terms and d_1 and d_2 be the common differences of two A.P.s respectively then

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27} \Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

For ratio of 11^{th} terms

$$\frac{n-1}{2} = 10 \quad \Rightarrow \quad n = 21$$

so ratio of 11^{th} terms is $\frac{7(21)+1}{4(21)+27} = \frac{148}{111} = \frac{4}{3}$ **Ans.**

Do yourself - 1 :

(i) Write down the sequence whose n^{th} terms is : (a) $\frac{2^n}{n}$ (b) $\frac{3+(-1)^n}{3^n}$

(ii) For an A.P, show that $t_m + t_{2n+m} = 2t_{m+n}$

(iii) If the sum of p terms of an A.P. is q and the sum of its q terms is p, then find the sum of its (p + q) term.

3. PROPERTIES OF A.P. :

(a) If each term of an A.P. is increased, decreased, multiplied or divided by the some nonzero number, then the resulting sequence is also an A.P.

(b) Three numbers in A.P. : $a - d, a, a + d$

Four numbers in A.P. : $a - 3d, a - d, a + d, a + 3d$

Five numbers in A.P. : $a - 2d, a - d, a, a + d, a + 2d$

Six numbers in A.P. : $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.

(c) The common difference can be zero, positive or negative.

(d) k^{th} term from the last = $(n - k + 1)^{th}$ term from the beginning (If total number of terms = n).

(e) The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms. $\Rightarrow T_k + T_{n-k+1} = \text{constant} = a + \ell$.

(f) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it. $a_n = (1/2)(a_{n-k} + a_{n+k}),\ k < n$

For $k = 1,\ a_n = (1/2)(a_{n-1} + a_{n+1})$; For $k = 2,\ a_n = (1/2)(a_{n-2} + a_{n+2})$ and so on.

(g) If a, b, c are in AP, then $2b = a + c$.

Illustration 4 :

Four numbers are in A.P. If their sum is 20 and the sum of their squares is 120, then the middle terms are -

(A) 2, 4 (B) 4, 6 (C) 6, 8 (D) 8, 10

Solution :

Let the numbers are $a - 3d, a - d, a + d, a + 3d$

given, $a - 3d + a - d + a + d + a + 3d = 20 \Rightarrow 4a = 20 \Rightarrow a = 5$

and $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120 \Rightarrow 4a^2 + 20d^2 = 120$

$\Rightarrow 4 \times 5^2 + 20d^2 = 120 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

Hence numbers are 2, 4, 6, 8 or 8, 6, 4, 2 **Ans. (B)**

Illustration 5 :

If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ are in A.P. where $a_i > 0$ for all i, show that :

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{(n-1)}{\sqrt{a_1}+\sqrt{a_n}}$$

Solution :

$$\text{L.H.S.} = \frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

$$= \frac{1}{\sqrt{a_2}+\sqrt{a_1}}+\frac{1}{\sqrt{a_3}+\sqrt{a_2}}+\ldots\ldots+\frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{(a_2-a_1)}+\frac{\sqrt{a_3}-\sqrt{a_2}}{(a_3-a_2)}+\ldots\ldots+\frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}$$

Let 'd' is the common difference of this A.P.

then $a_2 - a_1 = a_3 - a_2 = ... = a_n - a_{n-1} = d$

Now L.H.S.

$$= \frac{1}{d}\left\{\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + ...+ \sqrt{a_{n-1}} - \sqrt{a_{n-2}} + \sqrt{a_n} - \sqrt{a_{n-1}}\right\} = \frac{1}{d}\left\{\sqrt{a_n} - \sqrt{a_1}\right\}$$

$$= \frac{a_n - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{a_1 + (n-1)d - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{1}{d}\frac{(n-1)d}{(\sqrt{a_n} + \sqrt{a_1})} = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}} = \text{R.H.S.}$$

Do yourself - 2 :

(i) Find the sum of first 24 terms of the A.P. a_1, a_2, a_3......, if it is know that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$.

(ii) Find the number of terms common to the two A.P.'s 3, 7, 11, ... 407 and 2, 9, 16, ..., 709

4. GEOMETRIC PROGRESSION (G.P.) :

G.P. is a sequence of non zero numbers each of the succeeding term is equal to the preceeding term multiplied by a constant. Thus in a GP the ratio of successive terms is constant. This constant factor is called the COMMON RATIO of the sequence & is obtained by dividing any term by the immediately previous term. Therefore a, ar, ar^2, ar^3, ar^4 , ... is a GP with 'a' as the first term & 'r' as common ratio.

(a) n^{th} term ; $T_n = a\, r^{n-1}$

(b) Sum of the first n terms; $S_n = \dfrac{a(r^n - 1)}{r-1}$, if $r \neq 1$

(c) Sum of infinite G.P. , $S_\infty = \dfrac{a}{1-r}$; $0 < |r| < 1$

5. PROPERTIES OF GP :

(a) If each term of a G.P. be multiplied or divided by the some non-zero quantity, then the resulting sequence is also a G.P.

(b) Three consecutive terms of a GP : a/r, a, ar ;
Four consecutive terms of a GP : a/r^3, a/r, ar, ar^3 & so on.

(c) If a, b, c are in G.P. then $b^2 = ac$.

(d) If in a G.P, the product of two terms which are equidistant from the first and the last term, is constant and is equal to the product of first and last term. $\Rightarrow T_k . T_{n-k+1}$ = constant = $a.\ell$

(e) If each term of a G.P. be raised to the same power, then resulting sequence is also a G.P.

(f) In a G.P., $T_r^2 = T_{r-k} . T_{r+k}$, $k < r$, $r \neq 1$

(g) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(h) If a_1, a_2, a_3.....a_n is a G.P. of positive terms, then log a_1, log a_2,.....log a_n is an A.P. and vice-versa.

(i) If a_1, a_2, a_3..... and b_1, b_2, b_3..... are two G.P.'s then a_1b_1, a_2b_2, a_3b_3..... & $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$.......... is also in G.P.

Illustration 6 :

If a, b, c, d and p are distinct real numbers such that

$\left(a^2+b^2+c^2\right)p^2-2p\left(ab+bc+cd\right)+\left(b^2+c^2+d^2\right)\le 0$ then a, b, c, d are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

Solution :

Here, the given condition $\left(a^2+b^2+c^2\right)p^2-2p\left(ab+bc+ca\right)+b^2+c^2+d^2\le 0$

$\Rightarrow \left(ap-b\right)^2+\left(bp-c\right)^2+\left(cp-d\right)^2\le 0$

$\because$ a square can not be negative

$\therefore \quad ap-b=0, bp-c=0, cp-d=0 \Rightarrow p=\frac{b}{a}=\frac{c}{b}=\frac{d}{c}\Rightarrow$ a, b, c, d are in G.P.

Ans. (B)

Illustration 7 :

If a, b, c are in G.P., then the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in -

(A) A.P. (B) G.P. (C) H.P. (D) none of these

Solution :

a, b, c are in G.P $\Rightarrow b^2=ac$

Now the equation $ax^2+2bx+c=0$ can be rewritten as $ax^2+2\sqrt{ac}x+c=0$

$\Rightarrow \left(\sqrt{a}x+\sqrt{c}\right)^2=0\Rightarrow x=-\sqrt{\frac{c}{a}}, -\sqrt{\frac{c}{a}}$

If the two given equations have a common root, then this root must be $-\sqrt{\frac{c}{a}}$.

Thus $d\frac{c}{a}-2e\sqrt{\frac{c}{a}}+f=0 \quad \Rightarrow \quad \frac{d}{a}+\frac{f}{c}=\frac{2e}{c}\sqrt{\frac{c}{a}}=\frac{2e}{\sqrt{ac}}=\frac{2e}{b} \quad \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. **Ans. (A)**

Illustration 8 :

A number consists of three digits which are in G.P. the sum of the right hand and left hand digits exceeds twice the middle digit by 1 and the sum of the left hand and middle digits is two third of the sum of the middle and right hand digits. Find the numbers.

Solution :

Let the three digits be a, ar and ar^2 then number is

$$100a + 10ar + ar^2 \quad ....(i)$$

Given, $\quad a + ar^2 = 2ar + 1$

or $\quad a(r^2 - 2r + 1) = 1$

or $\quad a(r-1)^2 = 1 \quad ....(ii)$

Also given $a + ar = \frac{2}{3}(ar + ar^2)$

$\Rightarrow \quad 3 + 3r = 2r + 2r^2 \quad \Rightarrow \quad 2r^2 - r - 3 = 0 \quad \Rightarrow \quad (r+1)(2r-3) = 0$

$\therefore \quad r = -1, 3/2$

for $\quad r = -1, \quad a = \frac{1}{(r-1)^2} = \frac{1}{4} \notin I \qquad \therefore r \neq -1$

for $\quad r = 3/2, \quad a = \frac{1}{\left(\frac{3}{2}-1\right)^2} = 4 \qquad$ {from (ii)}

From (i), number is $400 + 10.4.\frac{3}{2} + 4.\frac{9}{4} = 469$ **Ans.**

Illustration 9 :

Find the value of $0.32\overline{58}$

Solution :

Let $R = 0.32\overline{58} \quad \Rightarrow R = 0.32585858.... \quad .... (i)$

Here number of figures which are not recurring is 2 and number of figures which are recurring is also 2.

then $\quad 100\,R = 32.585858...... \quad ....(ii)$

and $\quad 10000\,R = 3258.5858..... \quad ....(iii)$

Subtracting (ii) from (iii) , we get

$9900\,R = 3226 \quad \Rightarrow \quad R = \frac{1613}{4950}$

Aliter Method : $R = .32 + .0058 + .0058 + .000058 + ...$

$$= .32 + \frac{58}{10^4}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + ...\infty\right)$$

$$= .32 + \frac{58}{10^4}\left(\frac{1}{1 - \frac{1}{100}}\right)$$

$$= \frac{32}{100} + \frac{58}{9900} = \frac{3168 + 58}{9900} = \frac{3226}{9900} = \frac{1613}{4950}$$

Do yourself - 3 :

(i) Find a three digit number whose consecutive digits form a G.P. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, then the resulting digits will form an A.P.

(ii) If the third term of G.P. is 4, then find the product of first five terms.

(iii) If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of the given G.P., then show that $(q-r)\log a + (r-p)\log b + (p-q)\log c = 0$, where $a, b, c > 0$.

(iv) Find three numbers in G.P., whose sum is 52 and the sum of whose products in pairs is 624.

(v) The rational number which equals the number $2.\overline{357}$ with recurring decimal is -

(A) $\frac{2357}{999}$ (B) $\frac{2379}{997}$ (C) $\frac{785}{333}$ (D) $\frac{2355}{1001}$

6. HARMONIC PROGRESSION (H.P.) :

A sequence is said to be in H.P. if the reciprocal of its terms are in AP.

If the sequence $a_1, a_2, a_3, ..., a_n$ is an HP then $1/a_1, 1/a_2, ..., 1/a_n$ is an AP . Here we do not have the formula for the sum of the n terms of an HP. The general form of a harmonic progression is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, ... \frac{1}{a+(n-1)d}$

Note : No term of any H.P. can be zero.

(i) If a, b, c are in HP, then $b = \frac{2ac}{a+c}$ or $\frac{a}{c} = \frac{a-b}{b-c}$

Illustration 10 :

The sum of three numbers are in H.P. is 37 and the sum of their reciprocals is 1/4. Find the numbers.

Solution :

Three numbers are in H.P. can be taken as

$$\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$$

then $\frac{1}{a-d} + \frac{1}{a} + \frac{1}{a+d} = 37$ ....(i)

and $a - d + a + a + d = \frac{1}{4} \Rightarrow a = \frac{1}{12}$

from (i), $\frac{12}{1-12d} + 12 + \frac{12}{1+12d} = 37 \Rightarrow \frac{12}{1-12d} + \frac{12}{1+12d} = 25$

$\Rightarrow \frac{24}{1-144d^2} = 25 \Rightarrow 1 - 144d^2 = \frac{24}{25} \Rightarrow d^2 = \frac{1}{25 \times 144}$

$\therefore d = \pm\frac{1}{60}$

$\therefore$ a – d, a, a + d are $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ or $\frac{1}{10}, \frac{1}{12}, \frac{1}{15}$

Hence, three numbers in H.P. are 15, 12, 10 or 10, 12, 15 **Ans.**

Illustration 11 :

Suppose a is a fixed real number such that $\frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$

If p, q, r are in A.P., then prove that x, y, z are in H.P.

Solution :

$\because$ p, q, r are in A.P.

$\therefore$ $q-p=r-q$ .... (i)

$\Rightarrow$ $p-q=q-r=k$ (let)

given $\frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$ $\Rightarrow$ $\frac{\frac{a}{x}-1}{p}=\frac{\frac{a}{y}-1}{q}=\frac{\frac{a}{z}-1}{r}$

$\Rightarrow$ $\frac{\left(\frac{a}{x}-1\right)-\left(\frac{a}{y}-1\right)}{p-q}=\frac{\left(\frac{a}{y}-1\right)-\left(\frac{a}{z}-1\right)}{q-r}$ (by law of proportion)

$\Rightarrow$ $\frac{\frac{a}{x}-\frac{a}{y}}{k}=\frac{\frac{a}{y}-\frac{a}{z}}{k}$ {from (i)}

$\Rightarrow$ $a\left(\frac{1}{x}-\frac{1}{y}\right)=a\left(\frac{1}{y}-\frac{1}{z}\right)$ $\Rightarrow$ $\frac{1}{x}-\frac{1}{y}=\frac{1}{y}-\frac{1}{z}$

$\therefore$ $\frac{2}{y}=\frac{1}{x}+\frac{1}{z}$

$\therefore$ $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P.

Hence x, y, z are in H.P.

Do yourself - 4 :

(i) If the 7^{th} term of a H.P. is 8 and the 8^{th} term is 7. Then find the 28^{th} term.

(ii) In a H.P., if 5^{th} term is 6 and 3^{rd} term is 10. Find the 2^{nd} term.

(iii) If the p^{th}, q^{th} and r^{th} terms of a H.P. are a,b,c respectively, then prove that $\frac{q-r}{a}+\frac{r-p}{b}+\frac{p-q}{c}=0$

7. MEANS

(a) ARITHMETIC MEAN :

If three terms are in A.P. then the middle term is called the A.M. between the other two, so if a, b, c are in A.P., b is A.M. of a & c. So A.M. of a and c = $\frac{a+c}{2}=b$.

n-ARITHMETIC MEANS BETWEEN TWO NUMBERS :

If a,b be any two given numbers & a, A_1, A_2, ..., A_n, b are in AP, then $A_1, A_2, \ldots\ldots A_n$ are the 'n' A.M's between a & b then. $A_1 = a + d$, $A_2 = a + 2d$,......, $A_n = a + nd$ or $b - d$, where

$$d = \frac{b-a}{n+1}$$

$$\Rightarrow A_1 = a + \frac{b-a}{n+1},\ \ A_2 = a + \frac{2(b-a)}{n+1}, \ldots\ldots.$$

Note : Sum of n A.M's inserted between a & b is equal to n times the single A.M. between a & b i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single A.M. between a & b.

(b) GEOMETRIC MEAN :

If a, b, c are in G.P., then b is the G.M. between a & c, $b^2 = ac$. So G.M. of a and c = $\sqrt{ac} = b$

n-GEOMETRIC MEANS BETWEEN TWO NUMBERS :

If a, b are two given positive numbers & a, G_1, G_2, ..., G_n, b are in G.P. Then G_1, G_2, G_3 ,.......G_n are 'n' G.Ms between a & b. where $b = ar^{n+1} \Rightarrow r = (b/a)^{1/n+1}$

$G_1 = a(b/a)^{1/n+1}$, $G_2 = a(b/a)^{2/n+1}$ ...., $G_n = a(b/a)^{n/n+1}$

$= ar$, $= ar^2$, ... $= ar^n = b/r$

Note : The product of n G.Ms between a & b is equal to n^{th} power of the single G.M. between a & b i.e. $\prod_{r=1}^{n} G_r = (G)^n$ where G is the single G.M. between a & b

(c) HARMONIC MEAN :

If a, b, c are in H.P., then b is H.M. between a & c. So H.M. of a and c = $\frac{2ac}{a+c} = b$.

Insertion of 'n' HM's between a and b :

a, H_1, H_2, H_3 ,........, H_n, $b \rightarrow$ H.P

$$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \ldots\ldots\ldots \frac{1}{H_n}, \frac{1}{b} \rightarrow \text{A.P.}$$

$$\frac{1}{b} = \frac{1}{a} + (n+1)D \Rightarrow \quad D = \frac{\frac{1}{b} - \frac{1}{a}}{n+1}$$

$$\frac{1}{H_n} = \frac{1}{a} + n\left(\frac{\frac{1}{b} - \frac{1}{a}}{n+1}\right)$$

Important note :

(i) If A, G, H, are respectively A.M., G.M., H.M. between two positive number a & b then

(a) $G^2 = AH$ (A, G, H constitute a GP) (b) $A \geq G \geq H$ (c) $A = G = H \Leftrightarrow a = b$

(ii) Let $a_1, a_2, \ldots, a_n$ be n positive real numbers, then we define their arithmetic mean (A), geometric mean (G) and harmonic mean (H) as $A = \dfrac{a_1 + a_2 + \ldots + a_n}{n}$

$G = (a_1 a_2 \ldots\ldots a_n)^{1/n}$ and $H = \dfrac{n}{\left(\dfrac{1}{a_1} + \dfrac{1}{a_2} + \dfrac{1}{a_3} + \ldots \dfrac{1}{a_n}\right)}$

It can be shown that $A \geq G \geq H$. Moreover equality holds at either place if and only if $a_1 = a_2 = \ldots = a_n$

Illustration 12 :

If $2x^3 + ax^2 + bx + 4 = 0$ (a and b are positive real numbers) has 3 real roots, then prove that $a + b \geq 6(2^{1/3} + 4^{1/3})$.

Solution :

Let α, β, γ be the roots of $2x^3 + ax^2 + bx + 4 = 0$. Given that all the coefficients are positive, so all the roots will be negative.

Let $\alpha_1 = -\alpha$, $\alpha_2 = -\beta$, $\alpha_3 = -\gamma$ $\Rightarrow$ $\alpha_1 + \alpha_2 + \alpha_3 = \dfrac{a}{2}$

$\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = \dfrac{b}{2}$

$\alpha_1\alpha_2\alpha_3 = 2$

Applying AM $\geq$ GM, we have

$\dfrac{\alpha_1 + \alpha_2 + \alpha_3}{3} \geq (\alpha_1\alpha_2\alpha_3)^{1/3} \Rightarrow a \geq 6 \times 2^{1/3}$

Also $\dfrac{\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_3}{3} > (\alpha_1\alpha_2\alpha_3)^{2/3} \Rightarrow b \geq 6 \times 4^{1/3}$

Therefore $a + b \geq 6(2^{1/3} + 4^{1/3})$.

Illustration 13 :

If $a_i > 0 \ \forall\ i \in N$ such that $\prod_{i=1}^{n} a_i = 1$, then prove that $(1 + a_1)(1 + a_2)(1 + a_3)\ldots\ldots(1 + a_n) \geq 2^n$

Solution :

Using A.M. $\geq$ G.M.

$1 + a_1 \geq 2\sqrt{a_1}$

$1 + a_2 \geq 2\sqrt{a_2}$

$\vdots$

$1 + a_n \geq 2\sqrt{a_n} \Rightarrow (1 + a_1)(1 + a_2)\ldots\ldots(1 + a_n) \geq 2^n(a_1a_2a_3\ldots\ldots a_n)^{1/2}$

As $a_1a_2a_3\ldots\ldots a_n = 1$

Hence $(1 + a_1)(1 + a_2)\ldots\ldots(1 + a_n) \geq 2^n$.

Illustration 14 :

If a, b, x, y are positive natural numbers such that $\frac{1}{x}+\frac{1}{y}=1$ then prove that $\frac{a^x}{x}+\frac{b^y}{y}\geq ab$.

Solution :

Consider the positive numbers a^x, a^x,.......y times and b^y, b^y,......x times

For all these numbers,

$$AM = \frac{\{a^x + a^x + ...y \text{ time}\} + \{b^y + b^y + ...x \text{ times}\}}{x+y} = \frac{ya^x + xa^y}{(x+y)}$$

$$GM = \left\{\left(a^x.a^x......y \text{ times}\right)\left(b^y.b^y......x \text{ times}\right)\right\}^{\frac{1}{(x+y)}} = \left[\left(a^{xy}\right).\left(b^{xy}\right)\right]^{\frac{1}{(x+y)}} = (ab)^{\frac{xy}{(x+y)}}$$

As $\frac{1}{x}+\frac{1}{y}=1$, $\frac{x+y}{xy}=1$, i.e, x + y = xy

So using AM ≥ GM $\frac{ya^x + xa^y}{x+y} \geq (ab)^{\frac{xy}{x+y}}$

$\therefore \quad \frac{ya^x + xa^y}{xy} \geq ab$ or $\frac{a^x}{x}+\frac{a^y}{y}\geq ab.$

Do yourself - 5 :

(i) If $\frac{a^n + b^n}{a^{n-1}+b^{n-1}}$ is the G.M. between a & b then find the value of 'n'.

(ii) If b is the harmonic mean between a and c, then prove that $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$.

8. ARITHMETICO - GEOMETRIC SERIES :

A series, each term of which is formed by multiplying the corresponding term of an A.P. & G.P. is called the Arithmetico-Geometric Series , e.g. $1+ 3x + 5x^2 + 7x^3 + ...$

Here 1, 3, 5, ... are in A.P. & 1, x, x^2, x^3 .... are in G.P.

(a) SUM OF N TERMS OF AN ARITHMETICO-GEOMETRIC SERIES :

Let $S_n = a + (a+d)r + (a+2d)r^2 + ... + [a+(n-1)d]r^{n-1}$

then $S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]\ r^n}{1-r}$, $r \neq 1$

(b) SUM TO INFINITY :

If $0<|r|<1$ & $n\to\infty$, then $\lim_{n\to\infty} r^n = 0, S_\infty = \frac{a}{1-r}+\frac{dr}{(1-r)^2}$

Illustration 15 :

Find the sum of series $4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + ... \infty$.

Solution :

Let $S = 4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + ... \infty$

$- Sx = - 4x + 9x^2 - 16x^3 + 25x^4 - 36x^5 + ... \infty$

On subtraction, we get

$S(1 + x) = 4 - 5x + 7x^2 - 9x^3 + 11x^4 - 13x^5 + \dots\dots \infty$

$-S(1 + x)x = -4x + 5x^2 - 7x^3 + 9x^4 - 11x^5 + \dots\dots \infty$

On subtraction, we get

$S(1 + x)^2 = 4 - x + 2x^2 - 2x^3 + 2x^4 - 2x^5 + \dots\dots \infty$

$= 4 - x + 2x^2 (1 - x + x^2 - \dots\dots\infty) = 4 - x + \dfrac{2x^2}{1+x} = \dfrac{4+3x+x^2}{1+x}$

$S = \dfrac{4+3x+x^2}{(1+x)^3}$ **Ans.**

Illustration 16 :

Find the sum of series upto n terms $\left(\dfrac{2n+1}{2n-1}\right)+3\left(\dfrac{2n+1}{2n-1}\right)^2+5\left(\dfrac{2n+1}{2n-1}\right)^3+\dots\dots\dots$.

Solution :

For $x \neq 1$, let

$S = x + 3x^2 + 5x^3 + \dots\dots + (2n-3)x^{n-1} + (2n-1)x^n$ .... (i)

$\Rightarrow$ $xS = x^2 + 3x^3 + \dots\dots + (2n-5)x^{n-1} + (2n-3)x^n + (2n-1)x^{n+1}$ .... (ii)

Subtracting (ii) from (i), we get

$(1-x)S = x + 2x^2 + 2x^3 + \dots\dots\dots + 2x^{n-1} + 2x^n - (2n-1)x^{n+1} = x + \dfrac{2x^2(1-x^{n-1})}{1-x} - (2n-1)x^{n+1}$

$= \dfrac{x}{1-x}[1 - x + 2x - 2x^n - (2n-1)x^n + (2n-1)x^{n+1}]$

$\Rightarrow$ $S = \dfrac{x}{(1-x)^2}[(2n-1)x^{n+1} - (2n+1)x^n + 1 + x]$

Thus $\left(\dfrac{2n+1}{2n-1}\right)+3\left(\dfrac{2n+1}{2n-1}\right)^2+\dots\dots+(2n-1)\left(\dfrac{2n+1}{2n-1}\right)^n$

$= \left(\dfrac{2n+1}{2n-1}\right)\left(\dfrac{2n-1}{2}\right)^2\left[(2n-1)\left(\dfrac{2n+1}{2n-1}\right)^{n+1}-(2n+1)\left(\dfrac{2n+1}{2n-1}\right)^n+1+\dfrac{2n+1}{2n-1}\right]$

$= \dfrac{4n^2-1}{4}.\dfrac{4n}{2n-1} = n(2n+1)$ **Ans.**

Do yourself - 6 :

(i) Find sum to n terms of the series $3+5\times\dfrac{1}{4}+7\times\dfrac{1}{4^2}+\dots\dots$

(ii) If the sum to the infinity of the series $3 + 5r + 7r^2 + \dots\dots\dots$ is $\dfrac{44}{9}$, then find the value of r.

(iii) If the sum to infinity of the series $3+(3+d).\dfrac{1}{4}+(3+2d).\dfrac{1}{4^2}+\dots\dots\dots$ is $\dfrac{44}{9}$ then find d.

9. SIGMA NOTATIONS (Σ)

THEOREMS :

(a) $\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$ **(b)** $\sum_{r=1}^{n} k\,a_r = k\sum_{r=1}^{n} a_r$ **(c)** $\sum_{r=1}^{n} k = nk$; where k is a constant.

10. RESULTS

(a) $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ (sum of the first n natural numbers)

(b) $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(c) $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2$ (sum of the cubes of the first n natural numbers)

(d) $\sum_{r=1}^{n} r^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$

(e) $\sum_{r=1}^{n}(2r-1) = n^2$ (sum of first n odd natural numbers)

(f) $\sum_{r=1}^{n} 2r = n(n+1)$ (sum of first n even natural numbers)

Note :

If n^{th} term of a sequence is given by $T_n = an^3 + bn^2 + cn + d$ where a, b, c, d are constants,

then sum of n terms $S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d$

This can be evaluated using the above results.

Illustration 17 :

Sum up to 16 terms of the series $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + ...$ is

(A) 450 (B) 456 (C) 446 (D) none of these

Solution :

$$t_n = \frac{1^3+2^3+3^3+....+n^3}{1+3+5+....(2n-1)} = \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\frac{n}{2}\{2+2(n-1)\}} = \frac{\frac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4} == \frac{n^2}{4} + \frac{n}{2} + \frac{1}{4}$$

$$\therefore \quad S_n = \Sigma t_n = \frac{1}{4}\Sigma n^2 + \frac{1}{2}\Sigma n + \frac{1}{4}\Sigma 1 \quad = \frac{1}{4}.\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}.\frac{n(n+1)}{2} + \frac{1}{4}.n$$

$$\therefore \quad S_{16} = \frac{16.17.33}{24} + \frac{16.17}{4} + \frac{16}{4} = 446$$

Ans. (C)

11. METHOD OF DIFFERENCE :

Some times the n^{th} term of a sequence or a series can not be determined by the method, we have discussed earlier. So we compute the difference between the successive terms of given sequence for obtained the n^{th} terms.

If $T_1, T_2, T_3,........,T_n$ are the terms of a sequence then some times the terms $T_2 - T_1, T_3 - T_2,.........$ constitute an AP/GP. n^{th} term of the series is determined & the sum to n terms of the sequence can easily be obtained.

Case 1 :

(a) If difference series are in A.P., then

Let $T_n = an^2 + bn + c$, where a, b, c are constant

(b) If difference of difference series are in A.P.

Let $T_n = an^3 + bn^2 + cn + d$, where a, b, c, d are constant

Case 2 :

(a) If difference are in G.P., then

Let $T_n = ar^n + b$, where r is common ratio & a, b are constant

(b) If difference of difference are in G.P., then

Let $T_n = ar^n + bn + c$, where r is common ratio & a, b, c are constant

Determine constant by putting n = 1, 2, 3 .... n and putting the value of T_1, T_2, T_3 ......

and sum of series $(S_n) = \sum T_n$

Do yourself - 7 :

(i) Find the sum of the series upto n terms $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \frac{1+2+3+4}{4} + ...$

(ii) Find the sum of 'n' terms of the series whose n^{th} term is $t_n = 3n^2 + 2n$.

<u>Miscellaneous Illustration</u> :

Illustration 18 :

If $\sum_{r=1}^{n} T_r = \frac{n}{8}(n+1)(n+2)(n+3)$, then find $\sum_{r=1}^{n} \frac{1}{T_r}$.

Solution : $\because$ $T_n = S_n - S_{n-1}$

$$= \sum_{r=1}^{n} T_r - \sum_{r=1}^{n-1} T_r = \frac{n(n+1)(n+2)(n+3)}{8} - \frac{(n-1)n(n+1)(n+2)}{8} = \frac{n(n+1)(n+2)}{8}[(n+3)-(n-1)]$$

$$T_n = \frac{n(n+1)(n+2)}{8}(4) = \frac{n(n+1)(n+2)}{2}$$

$$\Rightarrow \quad \frac{1}{T_n} = \frac{2}{n(n+1)(n+2)} = \frac{(n+2)-n}{n(n+1)(n+2)} = \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \qquad .... (i)$$

Let $V_n = \dfrac{1}{n(n+1)}$

$\therefore \quad \dfrac{1}{T_n} = V_n - V_{n+1}$

Putting n = 1, 2, 3, ... n

$$\Rightarrow \quad \frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3} + ... + \frac{1}{T_n} = (V_1 - V_{n+1}) \Rightarrow \sum_{r=1}^{n} \frac{1}{T_r} = \frac{n^2 + 3n}{2(n+1)(n+2)}$$

Illustration 19 :

Find the sum of n terms of the series 1 . 3 . 5 + 3 . 5 . 7 + 5 . 7 . 9 + ...

Solution :

The n^{th} term is (2n – 1)(2n + 1)(2n + 3)

$T_n = (2n - 1)\ (2n + 1)\ (2n + 3)$

$T_n = \dfrac{1}{8}(2n-1)\ (2n + 1)\ (2n + 3)\ \{(2n + 5) - (2n - 3)\}$

$= \dfrac{1}{8}(V_n - V_{n-1})$ [Let $V_n = (2n - 1)\ (2n + 1)\ (2n + 3)\ (2n + 5)$]

$S_n = \sum T_n = \dfrac{1}{8}[V_n - V_0]$

$\therefore \quad S_n = \dfrac{(2n-1)(2n+1)(2n+3)(2n+5)}{8} + \dfrac{15}{8} = n\ (2n^3 + 8n^2 + 7n - 2)$ **Ans.**

Illustration 20 :

Find the sum of n terms of the series 3 + 7 + 14 + 24 + 37 + ...

Solution :

Clearly here the differences between the successive terms are

7 – 3, 14 – 7, 24 – 14, ... i.e. 4, 7, 10, 13,........., which are in A.P.

Let $S = 3 + 7 + 14 + 24 + ... + T_n$

$S = \quad 3 + 7 + 14 + ... + T_{n-1} + T_n$

Subtracting, we get

$0 = 3 + [4 + 7 + 10 + 13 + ... (n - 1) \text{ terms}] - T_n$

$\therefore \quad T_n = 3 + S_{n-1}$ of an A.P. whose a = 4 and d = 3.

$\therefore \quad T_n = 3 + \left(\dfrac{n-1}{2}\right)(2.4 + (n-2)3) = \dfrac{6 + (n-1)(3n+2)}{4}$ or, $T_n = \dfrac{1}{2}(3n^2 - n + 4)$

Now putting n = 1, 2, 3,........, n and adding

$$\therefore \quad S_n = \frac{1}{2}\left[3\sum n^2 - \sum n + 4n\right] = \frac{1}{2}\left[3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n\right] = \frac{n}{2}(n^2+n+4)$$ **Ans.**

Aliter Method :

Let $T_n = an^2 + bn + c$

Now, $T_1 = 3 = a + b + c$ .....(i)

$T_2 = 7 = 4a + 2b + c$ .....(ii)

$T_3 = 14 = 8a + 3\,b + c$ .....(iii)

Solving (i), (ii) & (iii) we get

$$a = \frac{3}{2}, b = -\frac{1}{2} \ \& \ c = 2$$

$$\therefore \quad T_n = \frac{1}{2}(3n^2 - n + 4)$$

$$\Rightarrow \quad s_n = \Sigma T_n = \frac{1}{2}\left[3\sum n^2 - \sum n + 4n\right] = \frac{1}{2}\left[3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n\right] = \frac{n}{2}(n^2+n+4)$$ **Ans.**

Illustration 21 :

Find the sum of n-terms of the series 1 + 4 + 10 + 22 + ...

Solution :

Let $S = 1 + 4 + 10 + 22 + \ldots\ldots. + T_n$ .... (i)

$S = \quad 1 + 4 + 10 + \ldots\ldots. + T_{n-1} + T_n$ .... (ii)

(i) – (ii) $\Rightarrow T_n = 1 + (3 + 6 + 12 + \ldots\ldots.. + T_n - T_{n-1})$

$$T_n = 1 + 3\left(\frac{2^{n-1}-1}{2-1}\right)$$

$$T_n = 3 \,.\, 2^{n-1} - 2$$

So $S_n = \Sigma T_n = 3\Sigma 2^{n-1} - \Sigma 2$

$$= 3\left(\frac{2^n - 1}{2-1}\right) - 2n = 3.2^n - 2n - 3$$ **Ans.**

Aliter Method :

Let $T_n = ar^n + b$, where r = 2

Now $T_1 = 1 = ar + b$ ....(i)

$T_2 = 4 = ar^2 + b$ ....(ii)

Solving (i) & (ii), we get

$$a = \frac{3}{2}, b = -2$$

$\therefore \quad T_n = 3.2^{n-1} - 2$

$\Rightarrow \quad S_n = \Sigma T_n = 3\Sigma 2^{n-1} - \Sigma 2$

$$= 3\left(\frac{2^n - 1}{2 - 1}\right) - 2n = 3.2^n - 2n - 3$$ **Ans.**

Illustration 22 :

The series of natural numbers is divided into groups (1), (2, 3, 4), (5, 6, 7, 8, 9) .... and so on. Show that the sum of the numbers in n^{th} group is $n^3 + (n-1)^3$

Solution :

The groups are (1), (2, 3, 4), (5, 6, 7, 8, 9) ....

The number of terms in the groups are 1, 3, 5......

$\therefore$ The number of terms in the n^{th} group = $(2n - 1)$

the last term of the n^{th} group is n^2

If we count from last term common difference should be -1

So the sum of numbers in the n^{th} group $= \left(\frac{2n-1}{2}\right)\left\{2n^2 + (2n-2)(-1)\right\}$

$= (2n - 1)(n^2 - n + 1) = 2n^3 - 3n^2 + 3n - 1 = n^3 + (n - 1)^3$

Illustration 23 :

Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function f satisfied $f(x+y) = f(x).\ f(y)$ for all natural number x,y and further $f(1) = 2$.

Solution :

It is given that

$f(x+y) = f(x)\,f(y)$ and $f(1) = 2$

$f(1+1) = f(1)\,f(1) \Rightarrow f(2) = 2^2$, $f(1+2) = f(1)\,f(2) \Rightarrow f(3) = 2^3$, $f(2+2) = f(2)\,f(2) \Rightarrow f(4) = 2^4$

Similarly $f(k) = 2^k$ and $f(a) = 2^a$

Hence, $\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)f(k) = f(a)\sum_{k=1}^{n} f(k) = 2^a \sum_{k=1}^{n} 2^k = 2^a\{2^1 + 2^2 + ...+2^n\}$

$$= 2^a\left\{\frac{2(2^n - 1)}{2 - 1}\right\} = 2^{a+1}(2^n - 1)$$

But $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$

$2^{a+1}(2^n-1) = 16\ (2^n-1)$

$\therefore$ $2^{a+1} = 2^4$

$\therefore$ $a+1 = 4 \Rightarrow a = 3$ **Ans.**

ANSWERS FOR DO YOURSELF

1 : **(i)** **(a)** $\frac{2}{1}, \frac{4}{2}, \frac{8}{3}, \frac{16}{4}, \ldots\ldots,$ **(b)** $\frac{2}{3}, \frac{4}{9}, \frac{2}{27}, \frac{4}{81}, \ldots\ldots;$ **(iii)** $-(p+q)$

2 : **(i)** 900 **(ii)** 14

3 : **(i)** 931 **(ii)** 4^5 **(iv)** 4, 12, 36 **(v)** C

4 : **(i)** 2 **(ii)** 15

5 : **(i)** $\frac{1}{2}$

6 : **(i)** $4+\frac{8}{9}\left(1-\frac{1}{4^{n-1}}\right)-\left(\frac{2n+1}{3\times4^{n-1}}\right)$ **(ii)** $\frac{1}{4}$ **(iii)** 2

7 : **(i)** $\frac{n(n+3)}{4}$ **(ii)** $\frac{n(n+1)(2n+3)}{2}$

EXERCISE (O-1)

1. If $a_1, a_2, a_3,\ldots, a_n,\ldots$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P., is : [JEE-MAINS Online 2013]

(A) 15 m (B) 10 m (C) 12 m (D) 13 m

2. Let $a_1, a_2, a_3, \ldots$ be an A.P. such that $\frac{a_1 + a_2 + \ldots + a_p}{a_1 + a_2 + a_3 + \ldots + a_q} = \frac{p^3}{q^3}$; $p \neq q$. Then $\frac{a_6}{a_{21}}$ is equal to :

[JEE-MAINS Online 2013]

(A) $\frac{121}{1861}$ (B) $\frac{11}{41}$ (C) $\frac{121}{1681}$ (D) $\frac{41}{11}$

3. Given sum of the first n terms of an A. P. is $2n + 3n^2$. Another A. P. is formed with the same first term and double of the common difference, the sum of n terms of the new A. P. is :- [JEE-MAINS Online 2013]

(A) $n + 4n^2$ (B) $n^2 + 4n$ (C) $3n + 2n^2$ (D) $6n^2 - n$

4. If a, b, c are in AP, then $(a - c)^2$ equals

(A) $4(b^2 - ac)$ (B) $4(b^2 + ac)$ (C) $4b^2 - ac$ (D) $b^2 - 4ac$

5. If the sum of n terms of an AP is $Pn + Qn^2$, where P, Q are constants, then its common difference is

(A) 2Q (B) P + Q (C) 2P (D) P – Q

6. Given a sequence of 4 numbers, first three of which are in G.P. and the last three are in A.P. with common difference six. If first and last terms of this sequence are equal, then the last term is :

[JEE-MAINS Online 2013]

(A) 8 (B) 16 (C) 2 (D) 4

7. The first term of an infinite G.P. is 1 and every term is equals to the sum of the successive terms, then its fourth term will be-

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{16}$

8. If G be the GM between x and y, then the value of $\frac{1}{G^2 - x^2} + \frac{1}{G^2 - y^2}$ is equal to

(A) G^2 (B) $\frac{2}{G^2}$ (C) $\frac{1}{G^2}$ (D) $3G^2$

9. $2 + 4 + 7 + 11 + 16 + \ldots\ldots\ldots\ldots$ to n terms =

(A) $\frac{1}{6}(n^2 + 3n + 8)$ (B) $\frac{n}{6}(n^2 + 3n + 8)$ (C) $\frac{1}{6}(n^2 - 3n + 8)$ (D) $\frac{n}{6}(n^2 - 3n + 8)$

10. If a, b, c are in HP, then $\frac{a-b}{b-c}$ is equal to

(A) $\frac{a}{b}$ (B) $\frac{b}{a}$ (C) $\frac{a}{c}$ (D) $\frac{c}{b}$

11. If a, b and c are positive real numbers then $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$ is greater than or equal to

(A) 3 (B) 6 (C) 27 (D) 5

12. If $a_1, a_2, a_3 \ldots a_n \in R^+$ and $a_1 . a_2 . a_3 \ldots a_n = 1$, then minimum value of $(1+a_1+a_1^2)(1+a_2+a_2^2)(1+a_3+a_3^2)\ldots(1+a_n+a_n^2)$ is equal to :-

(A) 3^{n+1} (B) 3^n (C) 3^{n-1} (D) none of these

13. The value of $1^2 + 3^2 + 5^2 + \ldots\ldots + 25^2$ is : [JEE-MAINS Online 2013]

(A) 1728 (B) 1456 (C) 2925 (D) 1469

14. The sum of the series : $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots$ up to 10 terms, is:

[JEE-MAINS Online 2013]

(A) $\frac{22}{13}$ (B) $\frac{18}{11}$ (C) $\frac{20}{11}$ (D) $\frac{16}{9}$

15. If $a=\sum_{n=0}^{\infty} x^n, b=\sum_{n=0}^{\infty} y^n, c=\sum_{n=0}^{\infty}(xy)^n$ where $|x|,|y|<1$; then-

(A) abc = a + b + c (B) ab + bc = ac + b (C) ac + bc = ab + c (D) ab + ac = bc + a

16. If r > 1 and $x=a+\frac{a}{r}+\frac{a}{r^2}+\ldots\text{to }\infty$, $y=b-\frac{b}{r}+\frac{b}{r^2}-\ldots\text{to }\infty$ and $z=c+\frac{c}{r^2}+\frac{c}{r^4}+\ldots\text{to }\infty$, then

$\frac{xy}{z}=$

(A) $\frac{ab}{c}$ (B) $\frac{ac}{b}$ (C) $\frac{bc}{a}$ (D) None of these

17. If a, b, c are positive real numbers such that $ab^2c^3 = 64$ then minimum value of $\left(\frac{1}{a}+\frac{2}{b}+\frac{3}{c}\right)$ is equal to:-

(A) 6 (B) 2 (C) 3 (D) None of these

18. In a GP, first term is 1. If $4T_2 + 5T_3$ is minimum, then its common ratio is

(A) $\frac{2}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $-\frac{3}{5}$

19. The sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+.....$ upto 11-terms is :- [JEE-MAINS Online 2013]

(A) $\frac{11}{4}$ (B) $\frac{60}{11}$ (C) $\frac{7}{2}$ (D) $\frac{11}{2}$

20. The sum of the series : $(2)^2 + 2(4)^2 + 3(6)^2 + ...$ upto 10 terms is : [JEE-MAINS Online 2013]

(A) 11300 (B) 12100 (C) 12300 (D) 11200

EXERCISE (O-2)

1. If $x=\sum_{n=0}^{\infty}a^n$, $y=\sum_{n=0}^{\infty}b^n$, $z=\sum_{n=0}^{\infty}C^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, then x, y, z are in- **[AIEEE 2005]**

(A) HP (B) Arithmetic - Geometric Progression

(C) AP (D) GP

2. If a, b, c are distinct positive real in H.P., then the value of the expression, $\frac{b+a}{b-a}+\frac{b+c}{b-c}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

3. Along a road lies an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(A) 15 (B) 29 (C) 31 (D) 35

4. If $S = 1^2 + 3^2 + 5^2 + ... + (99)^2$ then the value of the sum $2^2 + 4^2 + 6^2 + ... + (100)^2$ is

(A) S + 2550 (B) 2S (C) 4S (D) S + 5050

5. In an A.P. with first term 'a' and the common difference d ($a, d \neq 0$), the ratio 'ρ' of the sum of the first n terms to sum of n terms succeeding them does not depend on n. Then the ratio $\frac{a}{d}$ and the ratio 'ρ', respectively are

(A) $\frac{1}{2}, \frac{1}{4}$ (B) $2, \frac{1}{3}$ (C) $\frac{1}{2}, \frac{1}{3}$ (D) $\frac{1}{2}, 2$

6. The arithmetic mean of the nine numbers in the given set {9, 99, 999, ... 999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit

(A) 0 (B) 2 (C) 5 (D) 9

7. If for an A.P. $a_1, a_2, a_3, ..., a_n, ...$

$a_1 + a_3 + a_5 = -12$ and $a_1 a_2 a_3 = 8$, then the value of $a_2 + a_4 + a_6$ equals

(A) – 12 (B) – 16 (C) – 18 (D) – 21

8. An H.M. is inserted between the number 1/3 and an unknown number. If we diminish the reciprocal of the inserted number by 6, it is the G.M. of the reciprocal of 1/3 and that of the unknown number. If all the terms of the respective H.P. are distinct then

(A) the unknown number is 27
(B) the unknown number is 1/27
(C) the H.M. is 15
(D) the G.M. is 21

9. If $x \in R$, the numbers $(5^{1+x} + 5^{1-x})$, $a/2$, $(25^x + 25^{-x})$ form an A.P. then 'a' must lie in the interval

(A) $[1, 5]$ (B) $[2, 5]$ (C) $[5, 12]$ (D) $[12, \infty)$

10. If the sum of the first 11 terms of an arithmetical progression equals that of the first 19 terms, then the sum of its first 30 terms, is

(A) equal to 0 (B) equal to – 1 (C) equal to 1 (D) non unique

11. Let s_1, s_2, s_3 .... and t_1, t_2, t_3 .... are two arithmetic sequences such that $s_1 = t_1 \neq 0$; $s_2 = 2t_2$ and $\sum_{i=1}^{10} s_i = \sum_{i=1}^{15} t_i$. Then the value of $\frac{s_2 - s_1}{t_2 - t_1}$ is

(A) 8/3 (B) 3/2 (C) 19/8 (D) 2

12. If $\frac{1+3+5+.....\text{upto n terms}}{4+7+10+....\text{upto n terms}} = \frac{20}{7\log_{10} x}$ and

$n = \log_{10} x + \log_{10} x^{\frac{1}{2}} + \log_{10} x^{\frac{1}{4}} + \log_{10} x^{\frac{1}{8}} + ...+\infty$, then x is equal to

(A) 10^3 (B) 10^5 (C) 10^6 (D) 10^7

13. Let a_n, $n \in N$ is an A.P. with common difference 'd' and all whose terms are non-zero. If n approaches infinity, then the sum $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + ... + \frac{1}{a_na_{n+1}}$ will approach

(A) $\frac{1}{a_1d}$ (B) $\frac{2}{a_1d}$ (C) $\frac{1}{2a_1d}$ (D) a_1d

14. The sum of the first three terms of an increasing G.P. is 21 and the sum of their squares is 189. Then the sum of its first n terms is

(A) $3(2^n - 1)$ (B) $12\left(1-\frac{1}{2^n}\right)$ (C) $6\left(1-\frac{1}{2^n}\right)$ (D) $6(2^n - 1)$

15. If $a \neq 1$ and $\ln a^2 + (\ln a^2)^2 + (\ln a^2)^3 + ... = 3\left(\ln a + (\ln a)^2 + (\ln a)^3 + (\ln a)^4 + ...\right)$ then 'a' is equal to

(A) $e^{1/5}$ (B) $\sqrt{e}$ (C) $\sqrt[3]{e}$ (D) $\sqrt[4]{e}$

16. If abcd = 1 where a, b, c, d are positive reals then the minimum value of

$$a^2 + b^2 + c^2 + d^2 + ab + ac + ad + bc + bd + cd \text{ is}$$

(A) 6 (B) 10 (C) 12 (D) 20

17. For which positive integers n is the ratio, $\dfrac{\sum_{k=1}^{n} k^2}{\sum_{k=1}^{n} k}$ an integer?

(A) odd n only
(B) even n only
(C) $n = 1 + 6k$ only, where $k \geq 0$ and $k \in I$
(D) $n = 1 + 3k$, integer $k \geq 0$

18. **Statement-1:** If $27\,abc \geq (a + b + c)^3$ and $3a + 4b + 5c = 12$ then $\dfrac{1}{a^2} + \dfrac{1}{b^3} + \dfrac{1}{c^5} = 10$; where a, b, c are positive real numbers.

Statement-2: For positive real numbers A.M. $\geq$ G.M.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

MULTIPLE

19. Let a_1, a_2, a_3 .... and b_1, b_2, b_3 ...... be arithmetic progressions such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$. Then

(A) the difference between successive terms in progression 'a' is opposite of the difference in progression 'b'.
(B) $a_n + b_n = 100$ for any n.
(C) $(a_1 + b_1), (a_2 + b_2), (a_3 + b_3)$, ... are in A.P.
(D) $\sum_{r=1}^{100} (a_r + b_r) = 10000$

20. If $\sin(x - y)$, $\sin x$ and $\sin(x + y)$ are in H.P., then $\sin x . \sec\dfrac{y}{2}$ can be

(A) 2
(B) $\sqrt{2}$
(C) $-\sqrt{2}$
(D) -2

EXERCISE (S-1)

1. The sum of n terms of two arithmetic series are in the ratio of $(7n + 1) : (4n + 27)$. Find the ratio of their n^{th} term.

2. In an AP of which 'a' is the Ist term, if the sum of the Ist p terms is equal to zero, show that the sum of the next q terms is $-\left(\dfrac{aq(p+q)}{p-1}\right)$

3. The interior angles of a convex polygon form an arithmetic progression with a common difference of 4°. Determine the number of sides of the polygon if its largest interior angle is 172°.

4. There are nAM's between 1 & 31 such that 7th mean : $(n-1)^{th}$ mean = 5 : 9, then find the value of n.

5. Find the value of the sum $\sum_{k=0}^{359} k.\cos k°$.

6. The first term of an arithmetic progression is 1 and the sum of the first nine terms equal to 369. The first and the ninth term of a geometric progression coincide with the first and the ninth term of the arithmetic progression. Find the seventh term of the geometric progression.

7. The sequence $a_1, a_2, a_3, ... a_{98}$ satisfies the relation $a_{n+1} = a_n + 1$ for n = 1, 2, 3, ...97 and has the sum equal to 4949. Evaluate $\sum_{k=1}^{49} a_{2k}$.

8. For an increasing G.P. a_1, a_2, a_3........., a_n, if $a_6 = 4a_4$, $a_9 - a_7 = 192$, then the value of $\sum_{i=1}^{\infty} \frac{1}{a_i}$ is

9. In a set of four numbers, the first three are in GP & the last three are in A.P. with common difference 6. If the first number is the same as the fourth, find the four numbers.

10. Find three numbers a,b,c between 2 & 18 such that ;

(i) their sum is 25

(ii) the numbers 2,a,b are consecutive terms of an AP &

(iii) the numbers b,c,18 are consecutive terms of a G.P.

11. If the 10^{th} term of an HP is 21 and 21^{st} term of the same HP is 10, then find the 210^{th} term.

12. The p^{th} term T_p of H.P. is q(p + q) and q^{th} term T_q is p(p + q) when p > 2, q > 2. Prove that

(a) $T_{p+q} = pq$; (b) $T_{pq} = p + q$; (c) $T_{p+q} > T_{pq}$

13. **(a)** The harmonic mean of two numbers is 4. The arithmetic mean A & the geometric mean G satisfy the relation $2A + G^2 = 27$. Find the two numbers.

(b) The AM of two numbers exceeds their GM by 15 & HM by 27. Find the numbers.

14. If A_1, A_2, A_3,........A_{51} are arithmetic means inserted between the numbers a and b, then find the value of $\left(\frac{b + A_{51}}{b - A_{51}}\right) - \left(\frac{A_1 + a}{A_1 - a}\right)$.

15. If a > 0, then minimum value of $a + 2a^2 + a^3 + 15 + a^{-1} + a^{-3} + a^{-4}$ is

16. If a, b, c, d > 0 such that a + 2b + 3c + 4d = 50, then find the maximum value of $\left(\frac{a^2b^4c^3d}{16}\right)^{1/10}$

17. If number of coins earned in n^{th} game is $n2^{n+2} - 2^n$ and total number of coins earned in first 10 games is $10(B.2^{10} + 1)$, where $B \in N$, then the value of B is

18. Find the n^{th} term and the sum to n terms of the sequence :

(i) 1 + 5 + 13 + 29 + 61 +...... (ii) 6 + 13 + 22 + 33 +........

19. Sum the following series to n terms and to infinity :

(i) $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + ...$ (ii) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$ (iii) $\sum_{r=1}^{n} \frac{1}{4r^2 - 1}$

EXERCISE (S-2)

1. If the first 3 consecutive terms of a geometrical progression are the real roots of the equation $2x^3 - 19x^2 + 57x - 54 = 0$ find the sum to infinite number of terms of G.P.

2. Find the sum of the infinite series $\frac{1.3}{2}+\frac{3.5}{2^2}+\frac{5.7}{2^3}+\frac{7.9}{2^4}+.......\infty$.

3. Let $S=\sum_{n=1}^{99}\frac{5^{100}}{(25)^n+5^{100}}$. Find [S].

Where [y] denotes largest integer less than or equal to y.

4. If $3^{2\sin 2x-1}$, 14, $3^{4-2\sin 2x}$ form first three terms of an A.P., then find the sum $1 + \sin 2x + \sin^2 2x + ...\infty$.

5. Given that the cubic $ax^3 - ax^2 + 9bx - b = 0$ $(a \neq 0)$ has all three positive roots. Find the harmonic mean of the roots independent of a and b, hence deduce that the root are all equal. Find also the minimum value of $(a + b)$ if a and $b \in N$.

6. If $\tan\left(\frac{\pi}{12}-x\right)$, $\tan\frac{\pi}{12}$, $\tan\left(\frac{\pi}{12}+x\right)$ in order are three consecutive terms of a G.P. then sum of all the solutions in [0, 314] is $k\pi$. Find the value of k.

7. If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then find c and all the roots.

8. In a GP the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Find the number of terms in the GP.

9. In the quadratic equation $A\left(\sqrt{3}-\sqrt{2}\right)x^2+\frac{B}{\left(\sqrt{3}+\sqrt{2}\right)}x+C=0$ with α,β as its roots.

If $A=\left(49+20\sqrt{6}\right)^{1/4}$; B = sum of the infinite G.P. as $8\sqrt{3}+\frac{8\sqrt{6}}{\sqrt{3}}+\frac{16}{\sqrt{3}}+.....\infty$

and $|\alpha-\beta|=\left(6\sqrt{6}\right)^k$ where $k=\log_6 10-2\ \log_6\sqrt{5}+\log_6\sqrt{\left(\log_6 18+\log_6 72\right)}$,

then find the value of C.

10. If a,b,c,d,e be 5 numbers such that a,b,c are in AP ; b,c,d are in GP & c,d,e are in HP then :

(i) Prove that a,c,e are in GP.

(ii) Prove that $e = (2b - a)^2/a$

(iii) If a = 2 & e = 18, find all possible values of b,c,d.

11. Prove that the average of the numbers $n \sin n°$, n = 2,4,6,........., 180, is cot 1°.

12. If one AM 'a' and two GM's p and q be inserted between any two given numbers then show that $p^3 + q^3 = 2apq$.

13. Find the sum of the n terms of the sequence $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+.......$

EXERCISE (JM)

1. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+....$ is :- **[AIEEE-2009]**

(1) 4 (2) 6 (3) 2 (4) 3

2. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is :- **[AIEEE-2010]**

(1) 24 minutes (2) 34 minutes (3) 125 minutes (4) 135 minutes

3. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :- **[AIEEE-2011]**

(1) 20 months (2) 21 months (3) 18 months (4) 19 months

4. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r}=\alpha$ and $\sum_{r=1}^{100} a_{2r-1}=\beta$, then the common difference of the A.P. is : **[AIEEE-2011]**

(1) $\frac{\alpha-\beta}{200}$ (2) $\alpha - \beta$ (3) $\frac{\alpha-\beta}{100}$ (4) $\beta - \alpha$

5. **Statement–1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement–2 : $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$, for any natural number n. **[AIEEE-2012]**

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

6. If 100 times the 100^{th} term of an A.P. with non-zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this A.P. is : **[AIEEE-2012]**

(1) zero (2) –150

(3) 150 times its 50^{th} term (4) 150

7. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is : **[JEE(Main)-2013]**

(1) $\frac{7}{81}(179-10^{-20})$ (2) $\frac{7}{9}(99-10^{-20})$ (3) $\frac{7}{81}(179+10^{-20})$ (4) $\frac{7}{9}(99-10^{-20})$

8. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha - \beta|$ is: **[JEE(Main)-2014]**

(1) $\frac{\sqrt{61}}{9}$ (2) $\frac{2\sqrt{17}}{9}$ (3) $\frac{\sqrt{34}}{9}$ (4) $\frac{2\sqrt{13}}{9}$

9. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is : **[JEE(Main)-2014]**

(1) $\sqrt{2}+\sqrt{3}$ (2) $3+\sqrt{2}$ (3) $2-\sqrt{3}$ (4) $2+\sqrt{3}$

10. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + ...+ 10 (11)^9 = k (10)^9$, then k is equal to : **[JEE(Main)-2014]**

(1) $\frac{121}{10}$ (2) $\frac{441}{100}$ (3) 100 (4) 110

11. If m is the A.M. of two distinct real numbers l and $n(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals - **[JEE(Main)-2015]**

(1) $4\ lmn^2$ (2) $4\ l^2m^2n^2$ (3) $4\ l^2mn$ (4) $4\ lm^2n$

12. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:- **[JEE(Main)-2016]**

(1) $\frac{7}{4}$ (2) $\frac{8}{5}$ (3) $\frac{4}{3}$ (4) 1

13. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+...,$ is $\frac{16}{5}$m, then m is equal to :- **[JEE(Main)-2016]**

(1) 99 (2) 102 (3) 101 (4) 100

14. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then : **[JEE(Main)-2017]**

(1) a, b and c are in G.P. (2) b, c and a are in G.P.

(3) b, c and a are in A.P. (4) a, b and c are in A.P.

15. Let $a_1, a_2, a_3, ... , a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to- **[JEE(Main)-2018]**

(1) 68 (2) 34 (3) 33 (4) 66

16. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2\cdot2^2 + 3^2 + 2\cdot4^2 + 5^2 + 2\cdot6^2 + ...$. If $B - 2A = 100\lambda$, then λ is equal to : **[JEE(Main)-2018]**

(1) 248 (2) 464 (3) 496 (4) 232

EXERCISE (JA)

1. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

[JEE 2009, 3 (–1)]

2. Let $a_1, a_2, a_3 .... a_{11}$ be real numbers satisfying
$a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3,4.........11.

If $\frac{a_1^2 + a_2^2 + ... + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + ... + a_{11}}{11}$ is equal to **[JEE 2010, 3+3]**

3. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is **[JEE 2011, 4]**

4. Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[JEE 2011, 4]**

5. Let a_1, a_2, a_3, ... be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[JEE 2012, 3 (–1)]**

(A) 22 (B) 23 (C) 24 (D) 25

6. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s) **[JEE-Advanced 2013, 4, (–1)]**

(A) 1056 (B) 1088 (C) 1120 (D) 1332

7. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller to the numbers on the removed cards is k, then k – 20 = **[JEE-Advanced 2013, 4, (–1)]**

8. Let a,b,c be positive integers such that $\frac{b}{a}$ is an integer. If a,b,c are in geometric progression and the arithmetic mean of a,b,c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is **[JEE(Advanced)-2014, 3]**

9. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is **[JEE 2015, 4M, –0M]**

10. Let $b_i > 1$ for i = 1, 2, ..., 101. Suppose $\log_e b_1, \log_e b_2, \ldots\ldots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots\ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \ldots\ldots + b_{51}$ and $s = a_1 + a_2 + \ldots\ldots + a_{51}$ then **[JEE(Advanced)-2016, 3(–1)]**

(A) $s > t$ and $a_{101} > b_{101}$ (B) $s > t$ and $a_{101} < b_{101}$

(C) $s < t$ and $a_{101} > b_{101}$ (D) $s < t$ and $a_{101} < b_{101}$

11. The sides of the right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ? **[JEE(Advanced)-2017, 3]**

12. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ... , and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ______ **[JEE(Advanced)-2018, 3]**

ANSWER KEY

EXERCISE (O-1)

1. D	**2.** C	**3.** D	**4.** A	**5.** A	**6.** A	**7.** B
8. C	**9.** B	**10.** C	**11.** A	**12.** B	**13.** C	**14.** C
15. C	**16.** A	**17.** C	**18.** B	**19.** D	**20.** B	

EXERCISE (O-2)

1. A	**2.** B	**3.** C	**4.** D	**5.** C	**6.** A	**7.** D
8. B	**9.** D	**10.** A	**11.** C	**12.** B	**13.** A	**14.** A
15. D	**16.** B	**17.** D	**18.** D	**19.** A,B,C,D	**20.** B,C	

EXERCISE (S-1)

1. $(14n-6)/(8n+23)$ **3.** 12 **4.** $n = 14$ **5.** -180 **6.** 27 **7.** 2499

8. 2 **9.** $(8, -4, 2, 8)$ **10.** $a = 5$, $b = 8$, $c = 12$ **11.** 1

13. (a) 6, 3 (b) 120, 30 **14.** 102 **15.** 22 **16.** 5 **17.** 7

18. (i) $2^{n+1}-3$; $2^{n+2}-4-3n$ (ii) n^2+4n+1; $(1/6)\,n(n+1)(2n+13)+n$

19. **(i)** $s_n = (1/24) - [1/\{6(3n+1)(3n+4)\}]$; $s_\infty = 1/24$ **(ii)** $(1/5)\,n(n+1)(n+2)(n+3)(n+4)$
(iii) $n/(2n+1)$

EXERCISE (S-2)

1. $\frac{27}{2}$ **2.** 23 **3.** 49 **4.** 2 **5.** 28 **6.** 4950

7. $C = 9$; $(3, -3/2, -3/5)$ **8.** $n = 38$ **9.** 128

10. (iii) $b = 4, c = 6, d = 9$ **or** $b = -2, c = -6, d = -18$ **13.** $\frac{n(n+1)}{2(n^2+n+1)}$

EXERCISE (JM)

1. 4	**2.** 2	**3.** 2	**4.** 3	**5.** 3	**6.** 1	**7.** 3
8. 4	**9.** 4	**10.** 3	**11.** 4	**12.** 3	**13.** 3	**14.** 3
15. 2	**16.** 1					

EXERCISE (JA)

1. C	**2.** 0	**3.** 8	**4.** 9 or 3	**5.** D	**6.** A,D	**7.** 5
8. 4	**9.** 9	**10.** B	**11.** 6	**12.** 3748		