CONTENTS

STATISTICS

MATHEMATICAL REASONING

JEE (Main) Syllabus :

MATHEMATICAL REASONING: Statements, logical operations and, or, implies, implied by, if and only if. Understanding of tautology, contradiction, converse and contrapositive

STATISTICS : Measures of Dispersion: Calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance and mean deviation for grouped and ungrouped data.

Ex.2 Find the mean of the following freq. dist.

x_i	5	15	25	35	45	55
f_i	12	18	27	20	17	6

Sol. Let assumed mean a = 35, h = 10

here $N = \Sigma f_i = 100$, $u_i = \dfrac{(x_i - 35)}{10}$

$\therefore\ \Sigma f_i u_i = (12 \times -3) + (18 \times -2) + (27 \times -1) + (20 \times 0) + (17 \times 1) + (6 \times 2) = -70$

$\therefore\ \bar{x} = a + \left(\dfrac{\Sigma f_i u_i}{N}\right) h = 35 + \dfrac{(-70)}{100} \times 10 = 28$

(v) Weighted mean : If $w_1, w_2, \ldots\ w_n$ are the weights assigned to the values $x_1, x_2, \ldots\ x_n$ respectively then their weighted mean is defined as

$$\text{Weighted mean} = \frac{w_1x_1 + w_2x_2 + \ldots + w_nx_n}{w_1 + \ldots + w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Ex.3 Find the weighted mean of first n natural numbers when their weights are equal to their squares respectively

Sol. $$\text{Weighted Mean} = \frac{1.1^2 + 2.2^2 + \ldots + n.n^2}{1^2 + 2^2 + \ldots + n^2} = \frac{1^3 + 2^3 + \ldots + n^3}{1^2 + 2^2 + \ldots + n^2} = \frac{[n(n+1)/2]^2}{[n(n+1)(2n+1)/6]} = \frac{3n(n+1)}{2(2n+1)}$$

(vi) Combined mean : If $\bar{x}_1$ and $\bar{x}_2$ be the means of two groups having n_1 and n_2 terms respectively then the mean (combined mean) of their composite group is given by

$$\text{combined mean} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

If there are more than two groups then, $\quad \text{combined mean} = \dfrac{n_1\bar{x}_1 + n_1\bar{x}_2 + n_3\bar{x}_3 + \ldots}{n_1 + n_2 + n_3 + \ldots}$

Ex.4 The mean income of a group of persons is Rs. 400 and another group of persons is Rs. 480. If the mean income of all the persons of these two groups is Rs. 430 then find the ratio of the number of persons in the groups.

Sol. Here $\bar{x}_1 = 400$, $\bar{x}_2 = 480$, $\bar{x} = 430$

$\because\ \bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 430 = \dfrac{400n_1 + 480n_2}{n_1 + n_2}$

$\Rightarrow \dfrac{n_1}{n_2} = \dfrac{5}{3}$

(vii) Properties of Arithmetic mean :

- Sum of deviations of variate from their A.M. is always zero i.e. $\Sigma(x_i - \bar{x}) = 0$, $\Sigma f_i(x_i - \bar{x}) = 0$
- Sum of square of deviations of variate from their A.M. is minimum i.e. $\Sigma(x_i - \bar{x})^2$ is minimum
- If $\bar{x}$ is the mean of variate x_i then
  A.M. of $(x_i + \lambda) = \bar{x} + \lambda$
  A.M. of $(\lambda x_i) = \lambda \bar{x}$
  A.M. of $(ax_i + b) = a\bar{x} + b$ (where λ, a, b are constant)
- A.M. is independent of change of assumed mean i.e. it is not effected by any change in assumed mean.

2. MEDIAN :

The median of a series is the value of middle term of the series when the values are written in ascending order. Therefore median, divided an arranged series into two equal parts.

Formulae of median :

(i) For ungrouped distribution : Let n be the number of variate in a series then

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)^{th} \text{term}, & \text{(when n is odd)} \\ \text{Mean of } \left(\dfrac{n}{2}\right)^{th} \text{ and } \left(\dfrac{n}{2}+1\right)^{th} \text{ terms}, & \text{(when n is even)} \end{cases}$$

(ii) For ungrouped freq. dist. : First we prepare the cumulative frequency (c.f.) column and Find value of N then

$$\text{Median} = \begin{cases} \left(\dfrac{N+1}{2}\right)^{th} \text{term}, & \text{(when N is odd)} \\ \text{Mean of } \left(\dfrac{N}{2}\right)^{th} \text{ and } \left(\dfrac{N}{2}+1\right)^{th} \text{ terms}, & \text{(when N is even)} \end{cases}$$

(iii) For grouped freq. dist : Prepare c.f. column and find value of $\frac{N}{2}$ then find the class which contain value of c.f. is equal or just greater to N/2, this is median class

$$\therefore \text{ Median} = \ell + \frac{\left(\frac{N}{2} - F\right)}{f} \times h$$

where ℓ — lower limit of median class
f — freq. of median class
F — c.f. of the class preceeding median class
h — Class interval of median class

Ex.5 Find the median of following freq. dist.

class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
f	8	30	40	12	10

Sol.

class	f_i	c.f.
0 – 10	8	8
10 – 20	30	38
20 – 30	40	78
30 – 40	12	90
40 – 50	10	100

Here $\frac{N}{2} = \frac{100}{2} = 50$ which lies in the value 78 of c.f. hence corresponding class of this c.f. is 20-30 is the median class, so $\ell = 20,\ f = 40,\ F = 38,\ h = 10$

$$\therefore \text{ Median} = \ell + \frac{\left(\frac{N}{2} - F\right)}{f} \times h = 20 + \frac{(50-38)}{40} \times 10 = 23$$

3. MODE :

In a frequency distribution the mode is the value of that variate which have the maximum frequency

Method for determining mode :

(i) For ungrouped dist. : The value of that variate which is repeated maximum number of times

(ii) For ungrouped freq. dist. : The value of that variate which have maximum frequency.

(iii) For grouped freq. dist. : First we find the class which have maximum frequency, this is model calss

$$\therefore \text{ Mode} = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

where ℓ — lower limit of model class

f_0 — freq. of the model class

f_1 — freq. of the class preceeding model class

f_2 — freq. of the class succeeding model class

h — class interval of model class

Ex. 6 Find the mode of the following frequecy dist

class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
f_i	2	18	30	45	35	20	6	3

Sol. Here the class 30–40 has maximum freq. so this is the model class

$\ell = 30,\ f_0 = 45,\ f_1 = 30,\ f_2 = 35,\ h = 10$

$$\therefore \text{ Mode} = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h = 30 + \frac{45 - 30}{2 \times 45 - 30 - 35} \times 10 = 36$$

4. RELATION BETWEEN MEAN, MEDIAN AND MODE :

In a moderately asymmetric distribution following relation between mean, median and mode of a distribution. It is known as imprical formula.

Mode = 3 Median – 2 Mean

Note (i) Median always lies between mean and mode

(ii) For a symmetric distribution the mean, median and mode are coincide.

5. MEASURES OF DISPERSION :

The dispersion of a statistical distribution is the measure of deviation of its values about the their average (central) value.

It gives an idea of scatteredness of different values from the average value.

Generally the following measures of dispersion are commonly used.

(i) Range (ii) Mean deviation (iii) Variance and standard deviation

(i) Range : The difference between the greatest and least values of variate of a distribution, are called the range of that distribution.

If the distribution is grouped distribution, then its range is the difference between upper limit of the maximum class and lower limit of the minimum class.

$$\text{Also, coefficient of range} = \frac{\text{difference of extreme values}}{\text{sum of extreme values}}$$

Ex.7 Find the range of following numbers 10, 8, 12, 11, 14, 9, 6

Sol. Here greatest value and least value of the distribution are 14 and 6 resp. therefore

Range = 14 – 6 = 8

(ii) Mean deviation (M.D.) : The mean deviation of a distribution is, the mean of absolute value of deviations of variate from their statistical average (Mean, Median, Mode).

If A is any statistical average of a distribution then mean deviation about A is defined as

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n} |x_i - A|}{n} \qquad \text{(for ungrouped dist.)}$$

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n} f_i\,|x_i - A|}{N} \qquad \text{(for freq. dist.)}$$

Note :- is minimum when it taken about the median

$$\text{Coefficient of Mean deviation} = \frac{\text{Mean deviation}}{A}$$

(where A is the central tendency about which Mean deviation is taken)

Ex.8 Find the mean deviation of number 3, 4, 5, 6, 7

Sol. Here n = 5, $\bar{x} = 5$

$$\therefore \quad \text{Mean deviation} = \frac{\Sigma |x_i - \bar{x}|}{n}$$

$$= \frac{1}{5}[|3-5| + |4-5| + |5-5| + |6-5| + |7-5|]$$

$$= \frac{1}{5}[2 + 1 + 0 + 1 + 2] = \frac{6}{5} = 1.2$$

Ex.9 Find the mean deviation about mean from the following data

x_i	3	9	17	23	27
f_i	8	10	12	9	5

Sol.

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
3	8	24	12	96
9	10	90	6	60
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60
	N = 44	$\Sigma f_i x_i = 660$		$\Sigma f_i \lvert x_i - \bar{x}\rvert = 312$

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{N} = \frac{660}{44} = 15$$

$$\text{Mean deviation} = \frac{\Sigma f_i |x_i - \bar{x}|}{N} = \frac{312}{44} = 7.09$$

(iii) Variance and standard deviation : The variance of a distribution is, the mean of squares of deviation of variate from their mean. It is denoted by σ^2 or var(x).

The positive square root of the variance are called the standard deviation. It is denoted by σ or S.D.

Hence standard deviation = $+\sqrt{\text{variance}}$

Formulae for variance :

(i) for ungrouped dist. :

$$\sigma_x^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\sigma_x^2 = \frac{\Sigma x_i^2}{n} - \bar{x}^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2, \quad \text{where } d_i = x_i - a$$

(ii) For freq. dist. :

$$\sigma_x^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N}$$

$$\sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$$\sigma_u^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] \quad \text{where } u_i = \frac{d_i}{h}$$

(iii) Coefficient of S.D. = $\frac{\sigma}{\bar{x}}$

Coefficient of variation = $\frac{\sigma}{\bar{x}} \times 100$ (in percentage)

Note :- $\sigma^2 = \sigma_x^2 = \sigma_d^2 = h^2\sigma_u^2$

Ex.10 Find the variance of first n natural numbers

Sol. $$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{\Sigma n^2}{n} - \left(\frac{\Sigma n}{n}\right)^2 = \frac{n(n+1)(2n+1)}{6n} - \left\{\frac{n(n+1)}{2n}\right\}^2 = \frac{n^2-1}{12}$$

Ex.11 If $\sum_{i=1}^{18}(x_i - 8) = 9$ and $\sum_{i=1}^{18}(x_i - 8)^2 = 45$, then find the standard deviation of $x_1, x_2, \ldots\, x_{18}$

Sol. Let $(x_i - 8) = d_i$

$$\therefore \sigma_x = \sigma_d = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{\frac{45}{18} - \left(\frac{9}{18}\right)^2} = \sqrt{\frac{5}{2} - \frac{1}{4}} = \frac{3}{2}$$

Ex.12 Find the coefficient of variation of first n natural numbers

Sol. For first n natural numbers.

$$\text{Mean } (\bar{x}) = \frac{n+1}{2}, \text{ S.D.}(\sigma) = \sqrt{\frac{n^2-1}{12}}$$

$$\therefore \text{ coefficient of variance } = \frac{\sigma}{\bar{x}} \times 100 = \sqrt{\frac{n^2-1}{12}} \times \frac{1}{\left(\frac{n+1}{2}\right)} \times 100 = \sqrt{\frac{(n-1)}{3(n+1)}} \times 100$$

6. MEAN SQUARE DEVIATION :

The mean square deviation of a distrubution is the mean of the square of deviations of variate from assumed mean. It is denoted by S^2

Hence $$S^2 = \frac{\Sigma(x_i - a)^2}{n} = \frac{\Sigma d_i^2}{n} \quad \text{(for ungrouped dist.)}$$

$$S^2 = \frac{\Sigma f_i(x_i - a)^2}{N} = \frac{\Sigma f_i d_i^2}{N} \quad \text{(for freq. dist.), where } d_i = (x_i - a)$$

7. RELATION BETWEEN VARIANCE AND MEAN SQUARE DEVIATION :

$$\because \ \sigma^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$$\Rightarrow \sigma^2 = s^2 - d^2, \quad \text{where } d = \bar{x} - a = \frac{\Sigma f_i d_i}{N}$$

$$\Rightarrow s^2 = \sigma^2 + d^2 \Rightarrow s^2 \geq \sigma^2$$

Hence the variance is the minimum value of mean square deviation of a distribution

Ex.13 Determine the variance of the following frequency dist.

class	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
f_i	2	7	12	19	9	1

Sol. Let a = 7, h = 2

class	x_i	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	$f_i u_i^2$
0 – 2	1	2	–3	–6	18
2 – 4	3	7	–2	–14	28
4 – 6	5	12	–1	–12	12
6 – 8	7	19	0	0	0
8 – 10	9	9	1	9	9
10 – 12	11	1	2	2	4
		N = 50		$\Sigma f_i u_i = -21$	$\Sigma f_i u_i^2 = 71$

$$\therefore \ \sigma^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] = 4\left[\frac{71}{50} - \left(\frac{-21}{50}\right)^2\right] = 4[1.42 - 0.1764] = 4.97$$

8. MATHEMATICAL PROPERTIES OF VARIANCE :

- $\text{Var.}(x_i + \lambda) = \text{Var.}(x_i)$

  $\text{Var.}(\lambda x_i) = \lambda^2.\text{Var}(x_i)$

  $\text{Var}(ax_i + b) = a^2.\text{Var}(x_i)$

  where λ, a, b, are constant

- If means of two series containing n_1, n_2 terms are $\bar{x}_1, \bar{x}_2$ and their variance's are σ_1^2, σ_2^2 respectively and their combined mean is $\bar{x}$ then the variance σ^2 of their combined series is given by following formula

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{(n_1 + n_2)} \qquad \text{where } d_i = \bar{x}_1 - \bar{x},\ d_2 = \bar{x}_2 - \bar{x}$$

i.e.

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\bar{x}_1 - \bar{x}_2)^2$$

SOLVED EXAMPLES

Ex.1 If in an examination different weights are assigned to different subjects Physics (2), Chemistry (1), English (1), Mathematics (2) A student scores 60 in Physics, 70 in Chemistry, 70 in English and 80 in Mathematics, then weighted mean is-

(1) 60 (2) 70 (3) 80 (4) 85

Sol.(2) Weighted mean = $\dfrac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} = \dfrac{2\times 60 + 1\times 70 + 1\times 70 + 2\times 80}{6} = 70$

Ex.2 The mean of two groups of sizes 200 and 300 are 25 and 10 respectively.Their standard deviation are 3 and 4 respectively. The variance of combined sample of size 500 is-

(1) 64 (2) 65.2 (3) 67.2 (4) 64.2

Sol.(3) Combined mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} = \dfrac{200\times 25 + 300\times 10}{500} = 16$

Here $d_1 = \bar{x}_1 - \bar{x} = 25 - 16 = 9$ and $d_2 = \bar{x}_2 - \bar{x} = 10 - 16 = -6$

We know that $\sigma^2 = \dfrac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1+n_2} = \dfrac{200(9+81)+300(16+36)}{500} = \dfrac{33600}{500} = 67.2$

Ex.3 If the mean of the series $x_1, x_2, \ldots\ldots, x_n$ is $\bar{x}$, then the mean of the series $x_i + 2i$, i = 1, 2, ..., n will be-

(1) $\bar{x} + n$ (2) $\bar{x} + n + 1$ (3) $\bar{x} + 2$ (4) $\bar{x} + 2n$

Sol.(2) As given $\bar{x} = \dfrac{x_1 + x_2 + \ldots.. + x_n}{n}$ ....(1)

If the mean of the series $x_i + 2i$, i = 1, 2, ..., n be $\bar{X}$, then

$$\bar{X} = \frac{(x_1+2)+(x_2+2.2)+(x_3+2.3)+\ldots..+(x_n+2.n)}{n}$$

$$= \frac{x_1+x_2+\ldots..+x_n}{n} + \frac{2(1+2+3+\ldots..+n)}{n}$$

$$= \bar{x} + \frac{2n(n+1)}{2n} \qquad \text{from (1)}$$

$$= \bar{x} + n + 1$$

Ex.4 The variance of first 20-natural numbers is-

(1) $\dfrac{133}{4}$ (2) $\dfrac{379}{12}$ (3) $\dfrac{133}{2}$ (4) $\dfrac{399}{4}$

Sol.(1) $\because \quad \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$$= \frac{1}{20}[1^2 + 2^2 + ... + 20^2] - \left[\frac{1}{20}(1+2+....+20)\right]^2$$

$$= \frac{1}{20}\,\frac{20\times21(2\times20+1)}{6} - \left[\frac{1}{20}\,\frac{20\times21}{2}\right]^2 = \frac{7\times41}{2} - \frac{441}{4} = \frac{133}{4}.$$

In fact, the variance of first n-natural numbers is $\frac{n^2-1}{12}$

Ex.5 The mean of the following freq. table is 50 and $\Sigma f = 120$

class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
f	17	f_1	32	f_2	19

the missing frequencies are-

(1) 28, 24 (2) 24, 36 (3) 36, 28 (4) None of these

Sol.(1) $\Sigma f = 120 = 17 + f_1 + 32 + f_2 + 19$

$\Rightarrow f_1 + f_2 = 52$ ...... (1)

and $\Sigma fx = (10 \times 17) + (30 \times f_1) + (50 \times 32) + (70 \times f_2) + (90 \times 19) = 30f_1 + 70f_2 + 3480$

$$\therefore \quad \bar{x} = \frac{\Sigma fx}{\Sigma f} \Rightarrow 50 = \frac{30f_1 + 70f_2 + 3480}{120}$$

$\Rightarrow 30f_1 + 70f_2 = 2520 \Rightarrow 3f_1 + 7f_2 = 252$ ..... (2)

by (1) and (2) we get $f_1 = 28$, $f_2 = 24$

Ex.6 A student obtained 75%, 80%, 85% marks in three subjects. If the marks of another subject are added then his average marks can not be less than-

(1) 60% (2) 65% (3) 80% (4) 90%

Sol.(1) Total marks obtained from three subjects out of 300 = 75 + 80 + 85 = 240

if the marks of another subject is added then total marks obtained out of 400 is greater than 240

if marks obtained in fourth subject is 0 then

minimum average marks = $\frac{240}{400} \times 100 = 60\%$

Ex.7 The mean and variance of a series containing 5 terms are 8 and 24 respectively. The mean and variance of another series containing 3 terms are also 8 and 24 respectively. The variance of their combined series will be-

(1) 20 (2) 24 (3) 25 (4) 42

Sol.(2) Using $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}(\bar{x}_1 - \bar{x}_2)^2 \Rightarrow \sigma^2 = \frac{5(24)+3(24)}{5+3} + \frac{5(3)}{(5+3)^2}(8-8)^2 = 24$

Ex.8 The mean deviation about median from the following data 340, 150, 210, 240, 300, 310, 320, is-

(1) 52.4 (2) 52.5 (3) 52.8 (4) none of these

Sol.(3) Arranging the observations in ascending order of magnitude, we have 150, 210, 240, 300, 310, 320, 340. Clearly, the middle observation is 300. So, median = 300

Calculation of Mean deviation

x_i	$\lvert x_i - 300 \rvert$
340	40
150	150
210	90
240	60
300	0
310	10
320	20
Total	$\sum \lvert x_i - 300 \rvert = 370$

Mean deviation from median = $\frac{1}{7}\sum |x_i - 300| = \frac{370}{7} = 52.8$

Ex.9 Variance of the data given below is

Size of item	3.5	4.5	5.5	6.5	7.5	8.5	9.5
Frequency	3	7	22	60	85	32	8

(1) 1.29 (2) 2.19 (3) 1.32 (4) none of these

Sol.(3) Let the assumed mean be a = 6.5

Calculation of variance

x_i	f_i	$d_i = x_i - 6.5$	f_id_i	$f_id_i^2$
3.5	3	–3	–9	27
4.5	7	–2	–14	28
5.5	22	–1	–22	22
6.5	60	0	0	0
7.5	85	1	85	85
8.5	32	2	64	128
9.5	8	3	24	72
	$N = \sum f_i = 217$		$\sum f_id_i = 128$	$\sum f_id_i^2 = 362$

Here N = 217, $\sum f_id_i = 128$ and $\sum f_id_i^2 = 362$

$$\therefore \quad \text{Var}(X) = \left(\frac{1}{N}\sum f_id_i^2\right) - \left(\frac{1}{N}\sum f_id_i\right)^2 = \frac{362}{217} - \left(\frac{128}{217}\right)^2 = 1.668 - 0.347 = 1.321$$

Ex.10 If a variable takes the value 0, 1, 2......n with frequencies proportional to the bionomial coefficients ${}^nC_0$, ${}^nC_1$,.....,${}^nC_n$ then the mean of the distribution is-

(1) $\frac{n(n+1)}{4}$ (2) $\frac{n}{2}$ (3) $\frac{n(n-1)}{2}$ (4) $\frac{n(n+1)}{2}$

Sol.(2) $N = \sum f_i = k\,[{}^nC_0 + {}^nC_1 + ... + {}^nC_n] = k2^n$

$$\sum f_i x_i = k\,[1.{}^nC_1 + 2.\,{}^nC_2 + ... + n\,{}^nC_n] = k\sum_{r=1}^{n} r.{}^nC_r = kn\sum_{r=1}^{n} {}^{n-1}C_{r-1} = kn2^{n-1}$$

Thus $\bar{x} = \frac{1}{2^n}(n\ \ 2^{n-1}) = \frac{n}{2}$.

Ex.11 The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. If from these observations three are 1, 2 and 6, then the remaining will be-

(1) 2, 9 (2) 5, 6 (3) 4, 7 (4) 3, 8

Sol.(3) As given $\bar{x} = 4$, n = 5 and $\sigma^2 = 5.2$. If the remaining observations are x_1, x_2 then

$$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} = 5.2$$

$$\Rightarrow \frac{(x_1-4)^2 + (x_2-4)^2 + (1-4)^2 + (2-4)^2 + (6-4)^2}{5} = 5.2$$

$\Rightarrow (x_1 - 4)^2 + (x_2 - 4)^2 = 9$ ....(1)

Also $\bar{x} = 4 \Rightarrow \frac{x_1 + x_2 + 1 + 2 + 6}{5} = 4 \Rightarrow x_1 + x_2 = 11$ ....(2)

from eq.(1), (2) $x_1, x_2 = 4, 7$

Ex.12 The mean deviation of the series a, a + d, a + 2d, ..., a + 2nd from its mean is-

(1) $\frac{n+1}{2n+1}|d|$ (2) $\frac{n(n+1)}{2n+1}|d|$ (3) $\frac{n(n-1)}{2n+1}|d|$ (4) none of these

Sol.(2) Number of terms in the series = 2n + 1

$$\therefore \text{ mean } \bar{x} = \frac{a + (a+d) + (a+2d) + ... + (a+2nd)}{2n+1} = \frac{1}{2n+1}\left[\frac{2n+1}{2}(a + a + 2nd)\right] = a + nd$$

Also $\sum|x_i - \bar{x}| = |-nd| + |(1-n)d| + ... + |-d| + 0 + |d| + ... + |nd|$

$$= 2|d|[n + (n-1) + ... + 1] = 2|d|\frac{n(n+1)}{2} = n(n+1)|d|$$

$$\therefore \text{ mean deviation from mean } = \frac{\sum|x_i - \bar{x}|}{N} = \frac{n(n+1)}{2n+1}|d|$$

Ex.13 Let $x_1, x_2 \ldots, x_n$ be values taken by a variable X and $y_1, y_2, \ldots, y_n$ be the values taken by a variable Y such that $y_i = ax_i + b$, $i = 1, 2, \ldots, n$. Then-

(1) $Var(Y) = a^2 Var(X)$ (2) $Var(Y) = a^2 Var(X) + b$ (3) $Var(Y) = Var(X) + b$ (4) None of these

Sol.(1) We have,

$$Var(Y) = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

$[\because y_i = ax_i + b; i = 1, 2, \ldots, n \Rightarrow \bar{Y} = a\bar{X} + b]$

$$\Rightarrow Var(Y) = \frac{1}{n}\sum_{i=1}^{n}a^2(x_i - \bar{X})^2$$

$$\Rightarrow Var(Y) = a^2\left\{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{X})^2\right\} = a^2 Var(X)$$

Ex.14 The mean square deviation of a set of n observations $x_1, x_2, \ldots, x_n$ about a point c is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$ The mean square deviation about –2 and 2 are 18 and 10 respectively, then standard deviation of this set of observations is-

(1) 3 (2) 2 (3) 1 (4) None of these

Sol.(1) $\because \frac{1}{n}\Sigma(x_i + 2)^2 = 18$ and $\frac{1}{n}\Sigma(x_i - 2)^2 = 10$

$\Rightarrow \Sigma(x_i + 2)^2 = 18n$ and $\Sigma(x_i - 2)^2 = 10n$

$\Rightarrow \Sigma(x_i + 2)^2 + \Sigma(x_i - 2)^2 = 28n$ and $\Sigma(x_i + 2)^2 - \Sigma(x_i - 2)^2 = 8n$

$\Rightarrow 2\Sigma x_i^2 + 8n = 28n$ and $8\Sigma x_i = 8n$

$\Rightarrow \Sigma x_i^2 = 10n$ and $\Sigma x_i = n$

$\Rightarrow \frac{\Sigma x_i^2}{n} = 10$ and $\frac{\Sigma x_i}{n} = 1$

$$\therefore \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{10 - (1)^2} = 3$$

Arithmetic mean, weighted mean, Combined mean

1. Mean of the first n terms of the A.P. a, (a + d), (a + 2d), ... is-

(1) $a+\frac{nd}{2}$ (2) $a+\frac{(n-1)d}{2}$

(3) a + (n – 1) d (4) a + nd

2. The A.M. of first n even natural number is -

(1) n(n + 1) (2) $\frac{n+1}{2}$ (3) $\frac{n}{2}$ (4) n + 1

3. The A.M. of ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ...${}^nC_n$ is -

(1) $\frac{2^n}{n}$ (2) $\frac{2^{n+1}}{n}$ (3) $\frac{2^n}{n+1}$ (4) $\frac{2^{n+1}}{n+1}$

4. If the mean of numbers 27, 31, 89, 107, 156 is 82, then the mean of numbers 130, 126, 68, 50, 1 will be-

(1) 80 (2) 82 (3) 75 (4) 157

5. If the mean of n observations x_1, x_2, ...x_n is $\bar{x}$, then the sum of deviations of observations from mean is :-

(1) 0 (2) $n\bar{x}$

(3) $\frac{\bar{x}}{n}$ (4) None of these

6. The mean of 9 terms is 15. if one new term is added and mean become 16, then the value of new term is :-

(1) 23 (2) 25 (3) 27 (4) 30

7. If the mean of first n natural numbers is equal to $\frac{n+7}{3}$, then n is equal to-

(1) 10 (2) 11

(3) 12 (4) none of these

8. The mean of first three terms is 14 and mean of next two terms is 18. The mean of all the five terms is-

(1) 15.5 (2) 15.0 (3) 15.2 (4) 15.6

9. If the mean of five observations x, x + 2 , x + 4, x+ 6 and x + 8 is 11, then the mean of last three obsevations is-

(1) 11 (2) 13 (3) 15 (4) 17

10. The mean of a set of numbers is $\bar{x}$. If each number is decreased by λ, the mean of the new set is-

(1) $\bar{x}$ (2) $\bar{x}+\lambda$ (3) $\lambda-\bar{x}$ (4) $\bar{x}-\lambda$

11. The mean of 50 observations is 36. If its two observations 30 and 42 are deleted, then the mean of the remaining observations is-

(1) 48 (2) 36

(3) 38 (4) none of these

12. In a frequency dist. , if d_i is deviation of variates from a number ℓ and mean = $\ell + \frac{\Sigma f_i d_i}{\Sigma f_i}$, then ℓ is :-

(1) Lower limit

(2) Assumed mean

(3) Number of observation

(4) Class interval

13. The A.M. of n observation is $\bar{x}$. If the sum of n – 4 observations is K, then the mean of remaining observations is-

(1) $\frac{\bar{x}-K}{4}$ (2) $\frac{n\bar{x}-K}{n-4}$

(3) $\frac{n\bar{x}-K}{4}$ (4) $\frac{n\bar{x}-(n-4)K}{4}$

14. The mean of values $1,\frac{1}{2},\frac{1}{3},......\frac{1}{n}$ which have frequencies 1, 2, 3, ... n resp., is :-

(1) $\frac{2n+1}{3}$ (2) $\frac{2}{n}$ (3) $\frac{n+1}{2}$ (4) $\frac{2}{n+1}$

15. The sum of squares of deviation of variates from their A.M. is always :-

(1) Zero

(2) Minimum

(3) Maximum

(4) Nothing can be said

16. If the mean of following feq. dist. is 2.6, then the value of f is :-

x_i	1	2	3	4	5
f_i	5	4	f	2	3

(1) 1 (2) 3

(3) 8 (4) None of these

17. The weighted mean (W.M.) is computed by the formula ?

(1) W.M. = $\frac{\Sigma x_i}{\Sigma w_i}$ (2) W.M.=$\frac{\Sigma w_i}{\Sigma x_i}$

(3) W.M. = $\frac{\Sigma w_i x_i}{\Sigma x_i}$ (4) W.M.=$\frac{\Sigma w_i x_i}{\Sigma w_i}$

18. The weighted mean of first n natural numbers when their weights are equal to corresponding natural number, is :-

(1) $\frac{n+1}{2}$ (2) $\frac{2n+1}{3}$

(3) $\frac{(n+1)(2n+1)}{6}$ (4) None of these

19. The average income of a group of persons is $\overline{x}$ and that of another group is $\overline{y}$. If the number of persons of both group are in the ratio 4 : 3, then average income of combined group is :-

(1) $\frac{\overline{x}+\overline{y}}{7}$ (2) $\frac{3\overline{x}+4\overline{y}}{7}$

(3) $\frac{4\overline{x}+3\overline{y}}{7}$ (4) None of these

20. In a group of students, the mean weight of boys is 65 kg. and mean weight of girls is 55 kg. If the mean weight of all students of group is 61 kg, then the ratio of the number of boys and girls in the group is :-

(1) 2 : 3 (2) 3 : 1 (3) 3 : 2 (4) 4 : 3

Median, Mode

21. The median of an arranged series of n even observations, will be :-

(1) $\left(\frac{n+1}{2}\right)$ th term

(2) $\left(\frac{n}{2}\right)$ th term

(3) $\left(\frac{n}{2}+1\right)$ th term

(4) Mean of $\left(\frac{n}{2}\right)$ th and $\left(\frac{n}{2}+1\right)$ th terms

22. The median of the numbers 6, 14, 12, 8, 10, 9, 11, is :-

(1) 8 (2) 10 (3) 10.5 (4) 11

23. Median of the following freq. dist.

x_i	3	6	10	12	7	15
f_i	3	4	2	8	13	10

(1) 7 (2) 10

(3) 8.5 (4) None of these

24. Median is independent of change of :-

(1) only Origin

(2) only Scale

(3) Origin and scale both

(4) Neither origin nor scale

25. A series which have numbers three 4's, four 5's, five 6's, eight 7's, seven 8's and six 9's then the mode of numbers is :-

(1) 9 (2) 8 (3) 7 (4) 6

26. Mode of the following freqency distribution

x :	4	5	6	7	8
f :	6	7	10	8	3

(1) 5 (2) 6 (3) 8 (4) 10

27. The mode of the following freq. dist is :-

Class	1–10	11–20	21–30	31–40	41–50
f_i	5	7	8	6	4

(1) 24 (2) 23.83

(3) 27.16 (4) None of these

Symmetric and asymmetric distribution, Range

28. For a normal dist :-

(1) mean = median

(2) median = mode

(3) mean = mode

(4) mean = median = mode

29. The relationship between mean, median and mode for a moderately skewed distribution is-

(1) mode = median – 2 mean

(2) mode = 2 median – mean

(3) mode = 2 median – 3 mean

(4) mode = 3 median – 2 mean

30. The range of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3 is :-

(1) 6 (2) 7 (3) 5.5 (4) 11

Mean Deviation

31. The mean deviation of a frequency dist. is equal to :-

(1) $\frac{\Sigma d_i}{\Sigma f_i}$ (2) $\frac{\Sigma |d_i|}{\Sigma f_i}$

(3) $\frac{\Sigma f_i d_i}{\Sigma f_i}$ (4) $\frac{\Sigma f_i |d_i|}{\Sigma f_i}$

32. Mean deviation from the mean for the observation –1, 0, 4 is-

(1) $\sqrt{\frac{14}{3}}$ (2) $\frac{2}{3}$

(3) 2 (4) none of these

33. Mean deviation of the observations 70, 42, 63, 34, 44, 54, 55, 46, 38, 48 from median is :-

(1) 7.8 (2) 8.6

(3) 7.6 (4) 8.8

34. Mean deviation of 5 observations from their mean 3 is 1.2, then coefficient of mean deviation is :-

(1) 0.24 (2) 0.4

(3) 2.5 (4) None of these

35. The mean deviation from median is

(1) greater than the mean deviation from any other central value

(2) less than the mean deviation from any other central value

(3) equal to the mean deviation from any other central value

(4) maximum if all values are positive

Variance and Standard Deviation

36. The variate x and u are related by $u = \frac{x-a}{h}$ then correct relation between σ_x and σ_u is :-

(1) $\sigma_x = h\sigma_u$ (2) $\sigma_x = h + \sigma_u$

(3) $\sigma_u = h\sigma_x$ (4) $\sigma_u = h + \sigma_x$

37. The S.D. of the first n natural numbers is-

(1) $\sqrt{\frac{n^2-1}{2}}$ (2) $\sqrt{\frac{n^2-1}{3}}$

(3) $\sqrt{\frac{n^2-1}{4}}$ (4) $\sqrt{\frac{n^2-1}{12}}$

38. The variance of observations 112, 116, 120, 125, 132 is :-

(1) 58.8 (2) 48.8

(3) 61.8 (4) None of these

39. If $\sum_{i=1}^{10}(x_i - 15) = 12$ and $\sum_{i=1}^{10}(x_i - 15)^2 = 18$ then the S.D. of observations x_1, x_2 .... x_{10} is :-

(1) $\frac{2}{5}$ (2) $\frac{3}{5}$

(3) $\frac{4}{5}$ (4) None of these

40. The S.D. of 7 scored 1, 2, 3, 4, 5, 6, 7 is-

(1) 4 (2) 2

(3) $\sqrt{7}$ (4) none of these

41. The variance of series a, a + d, a + 2d, ..., a + 2nd is :-

(1) $\frac{n(n+1)}{2}d^2$ (2) $\frac{n(n+1)}{3}d^2$

(3) $\frac{n(n+1)}{6}d^2$ (4) $\frac{n(n+1)}{12}d^2$

42. Variance is independent of change of-

(1) only origin

(2) only scale

(3) origin and scale both

(4) none of these

43. If the coefficient of variation and standard deviation of a distribution are 50% and 20 respectively, then its mean is-

(1) 40 (2) 30

(3) 20 (4) None of these

44. If each observation of a dist. whose S.D. is σ, is increased by λ, then the variance of the new observations is -

(1) σ (2) $\sigma + \lambda$ (3) σ^2 (4) $\sigma^2 + \lambda$

45. The variance of 2, 4, 6, 8, 10 is-

(1) 8 (2) $\sqrt{8}$

(3) 6 (4) none of these

46. If each observation of a dist., whose variance is σ^2, is multiplied by λ, then the S.D. of the new new observations is-

(1) σ (2) $\lambda\sigma$

(3) $|\lambda|\sigma$ (4) $\lambda^2 \sigma$

47. The standard deviation of variate x_i is σ. Then standard deviation of the variate $\frac{ax_i + b}{c}$, where a , b, c are constants is-

(1) $\left(\frac{a}{c}\right)\sigma$ (2) $\left|\frac{a}{c}\right|\sigma$

(3) $\left(\frac{a^2}{c^2}\right)\sigma$ (4) None of these

CHECK YOUR GRASP **ANSWER-KEY** **EXERCISE-I**

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	2	4	3	3	1	2	2	4	2	4	2	2	3	4	2	1	4	2	3	3
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans.	4	2	3	4	3	2	2	4	4	2	4	3	2	2	2	1	4	2	2	2
Que.	**41**	**42**	**43**	**44**	**45**	**46**	**47**													
Ans.	2	1	1	3	1	3	2													

1. The A.M. of the series 1, 2, 4, 8, 16, ..., 2^n is-

(1) $\frac{2^n-1}{n}$ (2) $\frac{2^{n+1}-1}{n+1}$

(3) $\frac{2^n-1}{n+1}$ (4) $\frac{2^{n+1}-1}{n}$

2. If the mean of n observations $1^2, 2^2, 3^2$, ... n^2 is $\frac{46n}{11}$, then n is equal to-

(1) 11 (2) 12

(3) 23 (4) 22

3. The weighted mean of first n natural numbers whose weights are equal, is :-

(1) $\frac{n+1}{2}$ (2) $\frac{2n+1}{2}$

(3) $\frac{2n+1}{3}$ (4) $\frac{(2n+1)(n+1)}{6}$

4. The average age of a group of men and women is 30years. If average age of men is 32 and that of women is 27, then the percentage of women in the group is-

(1) 60 (2) 50

(3) 40 (4) 30

5. Mean and median of four numbers a, b, c and d (b < a < d < c) is 35 and 25 respectively then the value of b + c – a – d will be :-

(1) 90 (2) 115 (3) 40 (4) 10

6. Variance of the group α, α + 2, α + 4, α + 6, ... upto n terms (α ≠ 0) is :-

(1) $\frac{n^2-1}{12}+2n+\alpha$ (2) $\frac{n^2-1}{3}+\alpha$

(3) $\frac{n^2-1}{3}$ (4) None

7. Product of n positive numbers is unit. The sum of these numbers can not be less than-

(1) 1 (2) n

(3) n^2 (4) none of these

8. The A.M. of first n terms of the series 1.3.5, 3.5.7, 5.7.9,....., is-

(1) $3n^3 + 6n^2 + 7n – 1$ (2) $n^3 + 8n^2 + 7n – 1$

(3) $2n^3 + 8n^2 – 7n – 2$ (4) $2n^3 + 8n^2 + 7n – 2$

9. The observations 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95 are arranged in ascending order and their median is 63 then the value of x is :-

(1) 61 (2) 62 (3) 62.5 (4) 63

10. If the mode of a distribution is 18 and the mean is 24, then median is-

(1) 18 (2) 24 (3) 22 (4) 21

11. If the mean and S.D. of n observations x_1, x_2,.....x_n are $\bar{x}$ and σ resp, then the sum of squares of observations is :-

(1) $n(\sigma^2 + \bar{x}^2)$ (2) $n(\sigma^2 – \bar{x}^2)$

(3) $n(\bar{x}^2 – \sigma^2)$ (4) None of these

12. The variance of observations 8, 12, 13, 15, 22, is :-

(1) 21 (2) 21.2

(3) 21.4 (4) None of these

13. If the mean of a set of observations x_1, x_2,......., x_{10} is 20, then the mean of x_1+ 4, x_2+ 8, x_3+12,......,x_{10}+ 40 is-

(1) 34 (2) 42 (3) 38 (4) 40

14. The mean of values 0, 1, 2, ..., n when their weights are 1, $^nC_1$, $^nC_2$, ..., ^nC_n, resp., is

(1) $\frac{2^n}{n+1}$ (2) $\frac{n+1}{2}$

(3) $\frac{2^{n+1}}{n(n+1)}$ (4) $\frac{n}{2}$

15. For 15 observations of x, mean and median were found to be 12 and 20 respectively. Later an observation which was 25 found to be wrong then replaced by its correct value 55, then new mean and median will be :-

(1) 14 and 50 respectively

(2) 12 and 20 respectively

(3) 14 and 20 respectively

(4) Mean is 14 but median can't be determined.

16. If a variable takes the discrete values $\alpha + 4$, $\alpha - \frac{7}{2}$, $\alpha - \frac{5}{2}$, $\alpha - 3$, $\alpha - 2$, $\alpha + \frac{1}{2}$, $\alpha - \frac{1}{2}$, $\alpha + 5 (\alpha > 0)$, then the median of these values-

(1) $\alpha - \frac{5}{4}$ (2) $\alpha - \frac{1}{2}$

(3) $\alpha - 2$ (4) $\alpha + \frac{5}{4}$

17. The S.D. of first n odd natural numbers is :-

(1) $\sqrt{\frac{n^2-1}{2}}$ (2) $\sqrt{\frac{n^2-1}{3}}$

(3) $\sqrt{\frac{n^2-1}{6}}$ (4) $\sqrt{\frac{n^2-1}{12}}$

18. If the sum and sum of squares of 10 observations are 12 and 18 resp., then, The S.D. of observations is :-

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

19. The mean of n values of a distribution is $\bar{x}$. If its first value is increased by 1 , second by 2, ... then the mean of new values will be-

(1) $\bar{x} + n$ (2) $\bar{x} + n/2$

(3) $\bar{x} + \left(\frac{n+1}{2}\right)$ (4) None of these

20. The mean of the series $x_1, x_2, \ldots.., x_n$ is $\bar{X}$. If x_2 is replaced by λ, then the new mean is-

(1) $\frac{\bar{X} - x_2 + \lambda}{n}$ (2) $\frac{n\bar{X} + x_2 - \lambda}{n}$

(3) $\frac{(n-1)\bar{X} + \lambda}{n}$ (4) $\frac{n\bar{X} - x_2 + \lambda}{n}$

21. The mean square deviation about –1 and +1 of a set of observations are 7 and 3 respectively then standard deviation of the set is :-

(1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) 2 (4) None

22. The mean deviation of the numbers 1, 2, 3, 4, 5 is-

(1) 0 (2) 1.2

(3) 2 (4) 1.4

23. If mean = (3 median – mode) x, then the value of x is-

(1) 1 (2) 2 (3) 1/2 (4) 3/2

24. A man spends equal ammount on purchasing three kinds of pens at the rate 5 Rs/pen, 10 Rs/pen, 20 Rs/pen, then average cost of one pen is :-

(1) 10 Rs (2) $\frac{35}{3}$ Rs

(3) $\frac{60}{7}$ Rs (4) None of these

25. The median of 21 observation is 40. if each observations greater than the median are increased by 6, then the median of the observations will be-

(1) 40 (2) 46

(3) 46 + 40/21 (4) 46 – 40/21

26. The coefficient of range of the following distribution 10, 14, 11, 9, 8, 12, 6

(1) 0.4 (2) 2.5

(3) 8 (4) 0.9

27. The S.D. of the following freq. dist. :-

Class	0 – 10	10 – 20	20 – 30	30 – 40
f_i	1	3	4	2

(1) 7.8 (2) 9
(3) 8.1 (4) 0.9

28. The mean of a dist. is 4. if its coefficient of variation is 58%. Then the S.D. of the dist. is:-
(1) 2.23 (2) 3.23
(3) 2.32 (4) None of these

29. The mean of a set of observations is $\bar{x}$. If each observation is divided by α, (α ≠ 0) and then is increased by 10, then the mean of the new set is
(1) $\frac{\bar{x}}{\alpha}$ (2) $\frac{\bar{x}+10}{\alpha}$
(3) $\frac{\bar{x}+10\alpha}{\alpha}$ (4) $\frac{\alpha\bar{x}+10}{\alpha}$

30. The average age of a teacher and three students is 20 years. If all students are of equal age and the difference between the age of the teacher and that of a student is 20 years, then the age of the teacher is-
(1) 25 years (2) 30 years
(3) 35 years (4) 45 years

31. Median of 5 observations i.e.
$$3^{\log_9 4}, 5^{\log_{1/2} 8}, e^{2\ell n 3}, \ell n\left(\frac{1}{e}-2\right)+3, e^{2\ell n 3+\frac{1}{\log_4 e}} \text{ :-}$$
(1) 1 (2) 2 (3) 9 (4) 36

32. Median of ${}^{2n}C_0$, ${}^{2n}C_1$, ${}^{2n}C_2$,....,${}^{2n}C_n$ (when n is even) is-
(1) ${}^{2n}C_{\frac{n-1}{2}}$ (2) ${}^{2n}C_{\frac{n}{2}}$
(3) ${}^{2n}C_{\frac{n+1}{2}}$ (4) None of these

33. The mean deviation from mean of observations 5, 10, 15, 20, ...85 is :-
(1) 43.71 (2) 21.17
(3) 38.7 (4) None of these

34. If standard deviation of variate x_i is 10, then variance of the variate ($50 + 5x_i$) will be-
(1) 50 (2) 250
(3) 500 (4) 2500

35. The S.D. of the numbers 31, 32, 33, ... 47 is-
(1) $2\sqrt{6}$ (2) $4\sqrt{3}$
(3) $\sqrt{\frac{47^2-1}{12}}$ (4) None of these

36. The sum of the squares of deviation of 10 observations from their mean 50 is 250, then coefficient of variation is-
(1) 10% (2) 40%
(3) 50% (4) None of these

37. The median and standard deviation (S.D.) of a distribution will be, If each term is increased by 2 -
(1) median and S.D. will increased by 2
(2) median will increased by 2 but S.D. will remain same
(3) median will remain same but S.D. will increased by 2
(4) median and S.D. will remain same

38. If $\bar{X}_1$ and $\bar{X}_2$ are the means of two series such that $\bar{X}_1 < \bar{X}_2$ and $\bar{X}$ is the mean of the combined series, then-
(1) $\bar{X} < \bar{X}_1$ (2) $\bar{X} > \bar{X}_2$
(3) $\bar{X}_1 < \bar{X} < \bar{X}_2$ (4) $\bar{X} = \frac{\bar{X}_1+\bar{X}_2}{2}$

39. The median of 19 observations of a group is 30. If two observations with values 8 and 32 are further included, then the median of the new group of 21 observation will be

(1) 28 (2) 30

(3) 32 (4) 34

40. The coefficient of mean deviation from median of observations 40, 62, 54, 90, 68, 76 is :-

(1) 2.16 (2) 0.2

(3) 5 (4) None of these

41. A group of 10 observations has mean 5 and S.D. $2\sqrt{6}$. another group of 20 observations has mean 5 and S.D. $3\sqrt{2}$, then the S.D. of combined group of 30 observations is :-

(1) $\sqrt{5}$ (2) $2\sqrt{5}$

(3) $3\sqrt{5}$ (4) None of these

42. For the values $x_1, x_2 ...... x_{101}$ of a distribution $x_1 < x_2 < x_3 < ..... < x_{100} < x_{101}$. The mean deviation of this distribution with respect to a number k will be minimum when k is equal to -

(1) x_1

(2) x_{51}

(3) x_{50}

(4) $\dfrac{x_1 + x_2 + ... + x_{101}}{101}$

43. In any discrete series (when all the value are not same) the relationship between M.D. about mean and S.D. is-

(1) M.D. = S.D. (2) M.D. > S.D.

(3) M.D. < S.D. (4) M.D. ≤ S.D.

44. Median of observations x_i such that $(x_i^2 - 7x_i + 12)(x_i^3 - x_i^2 - 4x_i + 4) = 0$ will be :-

(1) 1 (2) 2 (3) 3 (4) None

BRAIN TEASERS **ANSWER-KEY** **EXERCISE-II**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	1	3	3	3	2	4	2	3	1	2	2	4	3	1	2	3	3	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	2	3	3	1	1	2	3	3	3	3	2	2	4	1	1	2	3	2	2
Que.	41	42	43	44																
Ans.	2	2	3	2																

1. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given? **[JEE(Main)-2013]**

(1) mean (2) median

(3) mode (4) variance

2. The variance of first 50 even natural numbers is :- **[JEE(Main)-2014]**

(1) $\frac{833}{4}$ (2) 833

(3) 437 (4) $\frac{437}{4}$

3. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is : **[JEE(Main)-2015]**

(1) 15.8 (2) 14.0

(3) 16.8 (4) 16.0

4. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true ? **[JEE(Main)-2016]**

(1) $3a^2 - 23a + 44 = 0$

(2) $3a^2 - 26a + 55 = 0$

(3) $3a^2 - 32a + 84 = 0$

(4) $3a^2 - 34a + 91 = 0$

5. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is - **[JEE(Main)-2018]**

(1) 4 (2) 2 (3) 3 (4) 9

6. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is : **[JEE(Main)-19]**

(1) 22 (2) 20 (3) 16 (4) 18

7. A data consists of n observations :

$x_1, x_2, \ldots\ldots, x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is : **[JEE(Main)-19]**

(1) 5 (2) $\sqrt{5}$

(3) $\sqrt{7}$ (4) 2

8. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is : **[JEE(Main)-19]**

(1) 4 : 9 (2) 6 : 7

(3) 5 : 8 (4) 10 : 3

9. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals :- **[JEE(Main)-19]**

(1) 2 (2) $\frac{\sqrt{5}}{2}$

(3) $\frac{2}{3}$ (4) $\sqrt{2}$

10. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then then absolute value of the difference of the other two observations, is : **[JEE(Main)-19]**

(1) 1 (2) 3

(3) 7 (4) 5

11. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :

[JEE(Main)-19]

(1) 40 (2) 49

(3) 48 (4) 45

12. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to

(1) $2\sqrt{\frac{10}{3}}$ (2) $2\sqrt{6}$

(3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

13. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

[JEE(Main)-19]

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(1) 2.8 (2) 3.2

(3) 3.0 (4) 2.5

14. If both the mean and the standard deviation of 50 observations $x_1, x_2, \ldots, x_{50}$ are equal to 16, then the mean of $(x_1-4)^2, (x_2-4)^2, \ldots (x_{50}-4)^2$ is :

[JEE(Main)-19]

(1) 525 (2) 380

(3) 480 (4) 400

PREVIOUS YEARS QUESTIONS **ANSWER-KEY** **EXERCISE-III**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	4	2	2	3	2	2	2	1	4	3	3	2	1	4

MATHEMATICAL REASONING

1. STATEMENT :

A sentence which is either true or false but cannot be both are called a statement. A sentence which is an exclamatory or a wish or an imperative or an interrogative can not be a statement.

If a statement is true then its truth value is T and if it is false then its truth value is F

For ex.

(i) "New Delhi is the capital of India", a true statement

(ii) "3 + 2 = 6", a false statement

(iii) "Where are you going ?" not a statement beasuse it connot be defined as true or false

Note : A statement cannot be both true and false at a time

2. SIMPLE STATEMENT :

Any statement whose truth value does not depend on other statement are called simple statement

For ex. (i) " $\sqrt{2}$ is an irrational number" (ii) "The set of real number is an infinite set"

3. COMPOUND STATEMENT :

A statement which is a combination of two or more simple statements are called compound statement Here the simple statements which form a compound statement are known as its sub statements

For ex.

(i) "If x is divisible by 2 then x is even number"

(ii) "ΔABC is equilatral if and only if its three sides are equal"

4. LOGICAL CONNECTIVES :

The words or phrases which combined simple statements to form a compound statement are called logical connectives.

In the following table some possible connectives, their symbols and the nature of the compound statement formed by them

S.N.	Connectives	symbol	use	operation
1.	and	$\wedge$	$p \wedge q$	conjunction
2.	or	$\vee$	$p \vee q$	disjunction
3.	not	$\sim$ or '	$\sim p$ or p'	negation
4.	If .... then	$\Rightarrow$ or $\rightarrow$	$p \Rightarrow q$ or $p \rightarrow q$	Implication or conditional
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	$p \Leftrightarrow q$ or $p \leftrightarrow q$	Equivalence or Bi-conditional

Explanation :

(i) $p \wedge q \equiv$ statement p and q

($p \wedge q$ is true only when p and q both are true otherwise it is false)

(ii) $p \vee q \equiv$ statement p or q

($p \vee q$ is true if at least one from p and q is true i.e. $p \vee q$ is false only when p and q both are false)

(iii) $\sim p \equiv$ not statement p

(~ p is true when p is false and ~ p is false when p is true)

(iv) $p \Rightarrow q \equiv$ statement p then statement q

($p \Rightarrow q$ is false only when p is true and q is false otherwise it is true for all other cases)

(v) $p \Leftrightarrow q \equiv$ statement p if and only if statement q

($p \Leftrightarrow q$ is true only when p and q both are true or false otherwise it is false)

5. TRUTH TABLE :

A table which shows the relationship between the truth value of compound statement S(p, q, r ....) and the truth values of its sub statements p, q, r, ... is said to be truth table of compound statement S

If p and q are two simple statements then truth table for basic logical connectives are given below

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	(~ p)
T	F
F	T

Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$ or $p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note : If the compound statement contain n sub statements then its truth table will contain 2^n rows.

6. LOGICAL EQUIVALENCE :

Two compound statements S_1(p, q, r...) and S_2(p, q, r ....) are said to be logically equivalent or simply equivalent if they have same truth values for all logically possibilities

Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of their truth table are same. If statements S_1 and S_2 are equivalent then we write $S_1 \equiv S_2$

For ex. The truth table for $(p \rightarrow q)$ and $(\sim p \vee q)$ given as below

p	q	(~ p)	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that last two columns of the above truth table are identical hence compound statements

$(p \rightarrow q)$ and $(\sim p \vee q)$ are equivalent

i.e. $\boxed{p \rightarrow q \equiv \sim p \vee q}$

7. **TAUTOLOGY AND CONTRADICTION :**

(i) Tautology : A statement is said to be a tautology if it is true for all logical possibilities i.e. its truth value always T. it is denoted by t.

For ex. the statement $p \vee \sim (p \wedge q)$ is a tautology

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Clearly, The truth value of $p \vee \sim (p \wedge q)$ is T for all values of p and q. so $p \wedge \sim (p \wedge q)$ is a tautology

(ii) Contradiction : A statement is a contradiction if it is false for all logical possibilities.

i.e. its truth value always F. It is denoted by c.

For ex. The statement $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction

p	q	$\sim p$	$\sim q$	$p \vee q$	$(\sim p \wedge \sim q)$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Clearly, then truth value of $(p \vee q) \wedge (\sim p \wedge \sim q)$ is F for all value of p and q. So $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

Note : The negation of a tautology is a contradiction and negation of a contradiction is a tautology

8. **DUALITY :**

Two compound statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$

If a compound statement contains the special variable t (tautology) and c (contradiction) then obtain its dual we replaced t by c and c by t in addition to replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

Note :

(i) the connectives $\wedge$ and $\vee$ are also called dual of each other.

(ii) If $S^*(p, q)$ is the dual of the compound statement $S(p, q)$ then

(a) $S^*(\sim p, \sim q) \equiv \sim S(p, q)$ (ii) $\sim S^*(p, q) \equiv S(\sim p, \sim q)$

For ex. The duals of the following statements

(i) $(p \wedge q) \vee (r \vee s)$ (ii) $(p \vee t) \wedge (p \vee c)$

(iii) $\sim(p \wedge q) \vee [p \wedge \sim(q \vee \sim s)]$

are as given below

(i) $(p \vee q) \wedge (r \wedge s)$

(ii) $(p \wedge c) \vee (p \wedge t)$

(iii) $\sim(p \vee q) \wedge [p \vee \sim(q \wedge \sim s)]$

9. CONVERSE, INVERSE AND CONTRAPOSITIVE OF THE CONDITIONAL STATEMENT ($p \to q$):

(i) Converse : The converse of the conditional statement $p \to q$ is defined as $q \to p$

(ii) Inverse : The inverse of the conditional statement $p \to q$ is defined as $\sim p \to \sim q$

(iii) Contrapositive : The contrapositive of conditional statement $p \to q$ is defined as $\sim q \to \sim p$

10. NEGATION OF COMPOUND STATEMENTS :

If p and q are two statements then

(i) **Negation of conjunction :** $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	~ p	~ q	$(p \wedge q)$	$\sim(p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(ii) Negation of disjunction : $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	~ p	~ q	$(p \vee q)$	$(\sim p \vee q)$	$(\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

(iii) Negation of conditional : $\sim(p \to q) \equiv p \wedge \sim q$

p	q	~ q	$(p \to q)$	$\sim(p \to q)$	$(p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

(iv) Negation of biconditional : $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

we know that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\therefore \sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$

$\equiv \sim(p \to q) \vee \sim(q \to p)$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Note : The above result also can be proved by preparing truth table for $\sim(p \leftrightarrow q)$ and $(p \wedge \sim q) \vee (q \wedge \sim p)$

11. ALGEBRA OF STATEMENTS :

If p, q, r are any three statements then the some low of algebra of statements are as follow

(i) Idempotent Laws :

(a) $p \wedge p \equiv p$ (b) $p \vee p \equiv p$

i.e. $p \wedge p \equiv p \equiv p \vee p$

p	$(p \wedge p)$	$(p \vee p)$
T	T	T
F	F	F

(ii) Comutative laws :

(a) $p \wedge q \equiv q \wedge p$ (b) $p \vee q \equiv q \vee p$

p	q	$(p \wedge q)$	$(q \wedge p)$	$(p \vee q)$	$(q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	F	T	T
F	F	F	F	F	F

(iii) Associative laws :

(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

p	q	r	$(p \wedge q)$	$(q \wedge r)$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Similarly we can proved result (b)

(iv) Distributive laws : (a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (c) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$

(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (d) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

p	q	r	$(q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Similarly we can prove result (b), (c), (d)

(v) De Morgan Laws : (a) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$(p \wedge q)$	$\sim (p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Similarly we can proved resulty (b)

(vi) Involution laws (or Double negation laws) : $\sim(\sim p) \equiv p$

p	$\sim p$	$\sim (\sim p)$
T	F	T
F	T	F

(vii) Identity Laws : If p is a statement and t and c are tautology and contradiction respectively then

(a) $p \wedge t \equiv p$ (b) $p \vee t \equiv t$ (c) $p \wedge c \equiv c$ (d) $p \vee c \equiv p$

p	t	c	$(p \wedge t)$	$(p \vee t)$	$(p \wedge c)$	$(p \vee c)$
T	T	F	T	T	F	T
F	T	F	F	T	F	F

(viii) Complement Laws :

(a) $p \wedge (\sim p) \equiv c$ (b) $p \vee (\sim p) \equiv t$ (c) $(\sim t) \equiv c$ (d) $(\sim c) \equiv t$

p	$\sim p$	$(p \wedge \sim p)$	$(p \vee \sim p)$
T	F	F	T
F	T	F	T

(ix) Contrapositive laws : $p \to q \equiv \sim q \to \sim p$

p	q	~ p	~ q	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

12. QUANTIFIED STATEMENTS AND QUANTIFIERS :

The words or phrases "All", "Some", "None", "There exists a" are examples of quantifiers.

A statement containing one or more of these words (or phrases) is a quantified statement.

E.g. (1) All dogs are poodles

(2) Some books have hard covers

(3) There exists an odd number which is prime.

Note : Phrases "There exists a" and "Atleast one" and the word "some" have the same meaning.

NEGATION OF QUANTIFIED STATEMENTS :

(1) '**None**' is the negation of '**at least one**' or '**some**' or '**few**'

Statement : Some dogs are poodles.

Negation : No dogs are poodles.

Similarly negation of '**some**' is '**none**'

(2) The negation of "**some A are B**" or "**There exist A which is B**" is "**No A are (is) B**" or "**There does not exist any A which is B".**

Statement-1 : Some boys in the class are smart

Statement-2 : There exists a boy in the class who is smart

Statement-3 :Alteast one boy in the class is smart

All the three above statements have same meaning as they all indicate "**existence**" of smart boy in the class.

Negation of these statements is

No boy in the class is smart.

or

There does not exist any boy in the class who is smart.

(3) Negation of "**All A are B**" is "**Some A are not B**".

Statement : All boys in the class are smart.

Negation : Some boys in the class are not smart.

or

There exists a boy in the class who is not smart.

SOLVED EXAMPLES

Ex.1 Which of the following is correct for the statements p and q ?

(1) $p \wedge q$ is true when at least one from p and q is true

(2) $p \to q$ is true when p is true and q is false

(3) $p \leftrightarrow q$ is true only when both p and q are true

(4) $\sim(p \vee q)$ is true only when both p and q are false

Sol.(4) We know that $p \wedge q$ is true only when both p and q are true so option (1) is not correct

we know that $p \to q$ is false only when p is true and q is false so option (2) is not correct

we know that $p \leftrightarrow q$ is true only when either p and q both are true or both are flase

so option (3) is not correct

we know that $\sim(p \vee q)$ is true only when $(p \vee q)$ is false

i.e. p and q both are false

So option (4) is correct

Ex.2 $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to-

(1) p (2) ~p (3) q (4) ~q

Sol.(2) $\because \sim(p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ (By Demorgan Law)

$\equiv \sim p \wedge (\sim q \vee q)$ (By distributive laws)

$\equiv \sim p \wedge t$ (By complement laws)

$\equiv \sim p$ (By Identity Laws)

Ex.3 Which of the following is logically equivalent to $(p \wedge q)$?

(1) $p \to \sim q$ (2) $\sim p \vee \sim q$ (3) $\sim(p \to \sim q)$ (4) $\sim(\sim p \wedge \sim q)$

Sol.(3) $\because p \to \sim q \equiv \sim p \vee \sim q \equiv \sim(p \wedge q)$ $(\because p \to q \equiv \sim p \vee q)$

i.e. $\sim(p \to \sim q) \equiv p \wedge q$

$\because \sim p \vee \sim q \equiv \sim(p \wedge q)$

and $\sim(\sim p \wedge \sim q) \equiv p \vee q$

Ex.4 If $p \to (q \vee r)$ is false, then the truth values of p, q, r respectively are-

(1) T, F, F (2) F, F, F (3) F, T, T (4) T, T, F

Sol.(1) We know $p \to (q \vee r)$ is false only when p is true and $(q \vee r)$ is false. but $(q \vee r)$ is false only when q and r both are false

Hence truth values of p, q, r are respectively T, F, F

Ex.5 Statement $(p \wedge \sim q) \wedge (\sim p \vee q)$ is

(1) a tautology (2) a contradiction

(3) neither a tautology not a contradiction (4) None of these

Sol.(2) $\because (p \wedge \sim q) \wedge (\sim p \vee q)$

$\equiv (p \wedge \sim q) \wedge \sim(p \wedge \sim q)$ (By Demargon Laws)

$\equiv c$, where c is contradiction (By complement laws)

Ex.6 Negation of the statement $p \to (q \wedge r)$ is-

(1) $\sim p \to \sim(q \wedge r)$ (2) $\sim p \vee (q \wedge r)$ (3) $(q \wedge r) \to p$ (4) $p \wedge (\sim q \vee \sim r)$

Sol.(4) $\sim(p \to (q \wedge r)) \equiv p \wedge \sim(q \wedge r)$ $(\because \sim(p \to q) \equiv p \wedge \sim q)$

$\equiv p \wedge (\sim q \vee \sim r)$

Ex.7 If $x = 5$ and $y = -2$ then $x - 2y = 9$. The contrapositive of this statement is-

(1) If $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$ (2) If $x - 2y \neq 9$ then $x \neq 5$ and $y \neq -2$

(3) If $x - 2y = 9$ then $x = 5$ and $y = -2$ (4) None of these

Sol.(1) Let p, q, r be the three statements such that

$p : x = 5,\quad q : y = -2\ \text{ and }\ r : x - 2y = 9$

Here given statement is $(p \wedge q) \to r$ and its contrapositive is $\sim r \to \sim(p \wedge q)$

i.e. $\sim r \to (\sim p \vee \sim q)$

i.e. if $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$

Ex.8 Which of the following is wrong ?

(1) $p \to q$ is logically equivalent to $\sim p \vee q$

(2) If the $(p \vee q) \wedge (q \vee r)$ is true then truth values of p, q, r are T, F, T respectively

(3) $\sim(p \wedge (q \vee r)) \equiv (\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$

(4) The truth value of $p \wedge \sim(p \vee q)$ is always T

Sol.(4) We know that $p \to q \equiv \sim p \vee q$

If $(p \vee q) \wedge (q \vee r)$ is true then

$(p \vee q)$ and $(q \vee r)$ both are true.

i.e. truth values of p, q, r may be T, F, T respectively

$\because\ \sim(p \wedge (q \vee r)) \equiv \sim((p \wedge q) \vee (p \wedge r)) \equiv \sim(p \wedge q) \wedge \sim(p \wedge r) \equiv (\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$

If p is true and q is false then $\sim(p \vee q)$ is false i.e. $p \wedge \sim(p \vee q)$ is false

Ex.9 If $S^*(p, q, r)$ is the dual of the compound statement $S(p, q, r)$ and $S(p, q, r) = \sim p \wedge [\sim(q \vee r)]$ then $S^*(\sim p, \sim q, \sim r)$ is equivalent to-

(1) $S(p, q, r)$ (2) $\sim S(\sim p, \sim q, \sim r)$ (3) $\sim S(p, q, r)$ (4) $S^*(p, q, r)$

Sol.(3) $\because\ S(p, q, r) = \sim p \wedge [\sim(q \vee r)]$

So $S(\sim p, \sim q, \sim r) \equiv \sim(\sim p) \wedge [\sim(\sim q \vee \sim r)] \equiv p \wedge (q \wedge r)$

$S^*(p, q, r) \equiv \sim p \vee [\sim(q \wedge r)]$

$S^*(\sim p, \sim q, \sim r) \equiv p \vee (q \vee r)$

Clearly $S^*(\sim p, \sim q, \sim r) \equiv \sim S(p, q, r)$

Ex.10 The negation of the statement "If a quadrilateral is a square then it is a rhombus"

(1) If a quadrilateral is not a square then is a rhombus it

(2) If a quadrilateral is a square then it is not a rhombus

(3) a quadrilateral is a square and it is not a rhombus

(4) a quadritateral is not a square and it is a rhombus

Sol.(3) Let p and q be the statements as given below

p : a quadrilateral is a square

q : a quadritateral is a rhombus

the given statement is $p \to q$

$\because\ \sim(p \to q) \equiv p \wedge \sim q$

Therefore the negation of the given statement is a quadrilateral is a square and it is not a rhombus

CHECK YOUR GRASP MATHEMATICAL REASONING EXERCISE-I

1. The inverse of the statement $(p \wedge \sim q) \rightarrow r$ is-
(1) $\sim(p \vee \sim q) \rightarrow \sim r$ (2) $(\sim p \wedge q) \rightarrow \sim r$
(3) $(\sim p \vee q) \rightarrow \sim r$ (4) None of these

2. $(\sim p \vee \sim q)$ is logically equivalent to-
(1) $p \wedge q$ (2) $\sim p \rightarrow q$
(3) $p \rightarrow \sim q$ (4) $\sim p \rightarrow \sim q$

3. The equivalent statement of $(p \leftrightarrow q)$ is-
(1) $(p \wedge q) \vee (p \vee q)$
(2) $(p \rightarrow q) \vee (q \rightarrow p)$
(3) $(\sim p \vee q) \vee (p \vee \sim q)$
(4) $(\sim p \vee q) \wedge (p \vee \sim q)$

4. If the compound statement $p \rightarrow (\sim p \vee q)$ is false then the truth value of p and q are respectively-
(1) T, T (2) T, F (3) F, T (4) F, F

5. The statement $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$ is-
(1) a tautology
(2) a contradiction
(3) neither a tautology nor a contradiction
(4) None of these

6. Negation of the statement $(p \wedge r) \rightarrow (r \vee q)$ is-
(1) $\sim(p \wedge r) \rightarrow \sim(r \vee q)$ (2) $(\sim p \vee \sim r) \vee (r \vee q)$
(3) $(p \wedge r) \wedge (r \wedge q)$ (4) $(p \wedge r) \wedge (\sim r \wedge \sim q)$

7. The dual of the statement $\sim p \wedge [\sim q \wedge (p \vee q) \wedge \sim r]$ is-
(1) $\sim p \vee [\sim q \vee (p \vee q) \vee \sim r]$
(2) $p \vee [q \vee (\sim p \wedge \sim q) \vee r]$
(3) $\sim p \vee [\sim q \vee (p \wedge q) \vee \sim r]$
(4) $\sim p \vee [\sim q \wedge (p \wedge q) \wedge \sim r]$

8. Which of the following is correct-
(1) $(\sim p \vee \sim q) \equiv (p \wedge q)$
(2) $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$
(3) $\sim(p \rightarrow \sim q) \equiv (p \wedge \sim q)$
(4) $\sim(p \leftrightarrow q) \equiv (p \rightarrow q) \vee (q \rightarrow p)$

9. The contrapositive of $p \rightarrow (\sim q \rightarrow \sim r)$ is-
(1) $(\sim q \wedge r) \rightarrow \sim p$ (2) $(q \rightarrow r) \rightarrow \sim p$
(3) $(q \vee \sim r) \rightarrow \sim p$ (4) None of these

10. The converse of $p \rightarrow (q \rightarrow r)$ is-
(1) $(q \wedge \sim r) \vee p$ (2) $(\sim q \vee r) \vee p$
(3) $(q \wedge \sim r) \wedge \sim p$ (4) $(q \wedge \sim r) \wedge p$

11. If p and q are two statement then $(p \leftrightarrow \sim q)$ is true when-
(1) p and q both are true
(2) p and q both are false
(3) p is false and q is true
(4) None of these

12. Statement $(p \wedge q) \rightarrow p$ is-
(1) a tautology (2) a contradiction
(3) neither (1) nor (2) (4) None of these

13 If statements p, q, r have truth values T, F, T respectively then which of the following statement is true-
(1) $(p \rightarrow q) \wedge r$ (2) $(p \rightarrow q) \vee \sim r$
(3) $(p \wedge q) \vee (q \wedge r)$ (4) $(p \rightarrow q) \rightarrow r$

14. If statement $p \rightarrow (q \vee r)$ is true then the truth values of statements p, q, r respectively-
(1) T, F, T (2) F, T, F
(3) F, F, F (4) All of these

15. Which of the following statement is a contradiction-
(1) $(p \wedge q) \wedge (\sim(p \vee q))$ (2) $p \vee (\sim p \wedge q)$
(3) $(p \rightarrow q) \rightarrow p$ (4) $\sim p \vee \sim q$

16. The negative of the statement "If a number is divisible by 15 then it is divisible by 5 or 3"
(1) If a number is divisible by 15 then it is not divisible by 5 and 3
(2) A number is divisible by 15 and it is not divisible by 5 or 3
(3) A number is divisible by 15 or it is not divisible by 5 and 3
(4) A number is divisible by 15 and it is not divisible by 5 and 3

17. If $x = 5$ and $y = -2$ then $x - 2y = 9$. The contrapositive of this statement is-
(1) If $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$
(2) If $x - 2y \neq 9$ then $x \neq 5$ and $y \neq -2$
(3) If $x - 2y = 9$ then $x = 5$ and $y = -2$
(4) None of these

18. The negation of the statement "2 + 3 = 5 and 8 < 10" is-

(1) $2 + 3 \neq 5$ and $8 \nless 10$
(2) $2 + 3 \neq 5$ or $8 > 10$
(3) $2 + 3 \neq 5$ or $8 \geq 10$
(4) None of these

19. For any three simple statement p, q, r the statement $(p \wedge q) \vee (q \wedge r)$ is true when-

(1) p and r true and q is false
(2) p and r false and q is true
(3) p, q, r all are false
(4) q and r true and p is false

20. Which of the following statement is a tautology-

(1) $(\sim p \vee \sim q) \vee (p \vee \sim q)$
(2) $(\sim p \vee \sim q) \wedge (p \vee \sim q)$
(3) $\sim p \wedge (\sim p \vee \sim q)$
(4) $\sim q \wedge (\sim p \vee \sim q)$

21. Which of the following statement is a contradiction-

(1) $(\sim p \vee \sim q) \vee (p \vee \sim q)$
(2) $(p \rightarrow q) \vee (p \wedge \sim q)$
(3) $(\sim p \wedge q) \wedge (\sim q)$
(4) $(\sim p \wedge q) \vee (\sim q)$

22. The negation of the statement $q \vee (p \wedge \sim r)$ is equivalent to-

(1) $\sim q \wedge (p \rightarrow r)$ (2) $\sim q \wedge \sim(p \rightarrow r)$
(3) $\sim q \wedge (\sim p \wedge r)$ (4) None of these

23. Let Q be a non empty subset of N. and q is a statement as given below :-

q : There exists an even number $a \in Q$.

Negation of the statement q will be :-

(1) There is no even number in the set Q.
(2) Every $a \in Q$ is an odd number.
(3) (1) and (2) both
(4) None of these

24. The statement $\sim(p \rightarrow q) \leftrightarrow (\sim p \vee \sim q)$ is-

(1) a tautology
(2) a contradiction
(3) neither a tautology nor a contradiction
(4) None of these

25. Which of the following is equivalent to $(p \wedge q)$

(1) $p \rightarrow \sim q$ (2) $\sim(\sim p \wedge \sim q)$
(3) $\sim(p \rightarrow \sim q)$ (4) None of these

26. The dual of the following statement "Reena is healthy and Meena is beautiful" is-

(1) Reena is beauful and Meena is healthy
(2) Reena is beautiful or Meena is healthy
(3) Reena is healthy or Meena is beutiful
(4) None of these

27. If p is any statement, t and c are a tautology and a contradiction respectively then which of the following is not correct-

(1) $p \wedge t \equiv p$ (2) $p \wedge c \equiv c$
(3) $p \vee t \equiv c$ (4) $p \vee c \equiv p$

28. If S*(p, q) is the dual of the compound statement S(p, q) then S*(~p, ~q) is equivalent to-

(1) S(~p, ~q) (2) ~S(p, q)
(3) ~S*(p, q) (4) None of these

29. If p is any statement, t is a tautology and c is a contradiction then which fo the following is not correct-

(1) $p \wedge (\sim c) \equiv p$
(2) $p \vee (\sim t) \equiv p$
(3) $t \vee c \equiv p \vee t$
(4) $(p \wedge t) \vee (p \vee c) \equiv (t \wedge c)$

30. If p, q, r are simple statement with truth values T, F, T respectively then the truth value of $((\sim p \vee q) \wedge \sim r) \rightarrow p$ is-

(1) True
(2) False
(3) True if r is false
(4) True if q is true

31. Which of the following is wrong-

(1) $p \vee \sim p$ is a tautology
(2) $\sim(\sim p) \leftrightarrow p$ is a tautology
(3) $p \wedge \sim p$ is a contradiction
(4) $((p \wedge p) \rightarrow q) \rightarrow p$ is a tautology

32. The statement "If $2^2 = 5$ then I get first class" is logically equivalent to-

(1) $2^2 = 5$ and I do not get first class

(2) $2^2 = 5$ or I do not get first class

(3) $2^2 \neq 5$ or I get first class

(4) None of these

33. If statement $(p \vee \sim r) \rightarrow (q \wedge r)$ is false and statement q is true then statement p is-

(1) true (2) false

(3) may be true or false (4) None of these

34. Which of the following statement are not logically equivalent-

(1) $\sim(p \vee \sim q)$ and $(\sim p \wedge q)$

(2) $\sim(p \rightarrow q)$ and $(p \wedge \sim q)$

(3) $(p \rightarrow q)$ and $(\sim q \rightarrow \sim p)$

(4) $(p \rightarrow q)$ and $(\sim p \wedge q)$

35. Consider the following statements

p : Virat kohli plays cricket.

q : Virat kohli is good at maths

r : Virat kohli is successful.

then negation of the statement "If virat kohli plays cricket and is not good at maths then he is successful" will be :-

(1) $\sim p \wedge (q \wedge r)$ (2) $(\sim p \vee q) \wedge r$

(3) $p \wedge (\sim q \wedge \sim r)$ (4) None of these

36. Let p statement "If 2 > 5 then earth will not rotate" and q be the statement "$2 \not> 5$ or earth will not rotate".

Statement–1 : p and q are equivalent.

Statement–2 : $m \rightarrow n$ and $\sim m \vee n$ are equivalent.

(1) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1.

37. Which of the following is a tautology :-

(1) $[(\sim p \wedge p) \rightarrow q] \longrightarrow (p \wedge p)$

(2) $[(\sim p \wedge p) \rightarrow q] \longrightarrow (\sim p \rightarrow p)$

(3) $[(\sim p \wedge p) \rightarrow q] \longrightarrow (p \rightarrow p)$

(4) None of these

38. Negation of the statement "No one in the class is fond of music" is :-

(1) everyone in the class is fond of music.

(2) Some of the students in the class are fond of music.

(3) There exists a student in the class who is fond of music.

(4) (2) and (3) both

CHECK YOUR GRASP **ANSWER-KEY** **EXERCISE-I**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	3	4	2	2	4	3	2	1	1	3	1	4	4	1	4	1	3	4	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38		
Ans.	3	1	3	3	3	3	3	2	4	1	4	3	3	4	3	4	3	4		

1. The negation of the statement **[JEE(Main)-2012]**

"If I become a teacher, then I will open a school", is :

(1) I will not become a teacher or I will open a school.

(2) I will become a teacher and I will not open a school.

(3) Either I will not become a teacher or I will not open a school.

(4) Neither I will become a teacher nor I will open a school.

2. Consider :

Statement-I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tuatology. **[JEE(Main)-2013]**

(1) Statement-I is true, Statement-II is true; statement-II is a **correct** explanation for Statement-I.

(2) Statement-I is true, Statement-II is true; statement-II is **not** a correct explanation for Statement-I.

(3) Statement-I is true, Statement-II is false.

(4) Statement-I is false, Statement-II is true.

3. The statement $\sim(p \leftrightarrow \sim q)$ is : **[JEE(Main)-2014]**

(1) equivalent to $p \leftrightarrow q$

(2) equivalent to $\sim p \leftrightarrow q$

(3) a tautology

(4) a fallacy

4. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[JEE(Main)-2015]**

(1) $s \vee (r \vee \sim s)$ (2) $s \wedge r$

(3) $s \wedge \sim r$ (4) $s \wedge (r \wedge \sim s)$

5. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to :- **[JEE(Main)-2016]**

(1) $p \vee \sim q$ (2) $\sim p \wedge q$

(3) $p \wedge q$ (4) $p \vee q$

6. The following statement

$(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is : **[JEE(Main)-2017]**

(1) a fallacy

(2) a tautology

(3) equivalent to $\sim p \rightarrow q$

(4) equivalent to $p \rightarrow \sim q$

7. The Boolean expression $\sim (p \vee q) \vee (\sim p \wedge q)$ is equivalent to : **[JEE(Main)-2018]**

(1) p (2) q

(3) ~q (4) ~p

8. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is: **[JEE(Main)-19]**

(1) $(\wedge, \vee)$ (2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$ (4) $(\vee, \wedge)$

9. The logical statement

$$[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)]$$

is equivalent to : **[JEE(Main)-19]**

(1) $(p \wedge r) \wedge \sim q$ (2) $(\sim p \wedge \sim q) \wedge r$

(3) $\sim p \vee r$ (4) $(p \wedge \sim q) \vee r$

10. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ? **[JEE(Main)-19]**

(1) $(P \wedge Q) \vee (\sim R)$

(2) $(\sim P) \wedge (\sim Q \wedge R)$

(3) $(\sim P) \vee (Q \wedge R)$

(4) $P \vee (\sim Q \wedge R)$

11. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology? **[JEE(Main)-19]**

(1) $(p \vee r) \rightarrow (p \wedge r)$

(2) $p \vee r$

(3) $p \wedge r$

(4) $(p \wedge r) \rightarrow (p \vee r)$

12. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is :- **[JEE(Main)-19]**

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal.

13. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is : **[JEE(Main)-19]**

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not a citizen of India, then you are not born in India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not born in India, then you are not a citizen of India.

14. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is **[JEE(Main)-19]**

(1) $p \wedge q$ (2) $p \leftrightarrow q$

(3) $\sim p \vee \sim q$ (4) $\sim p \wedge \sim q$

15. If the truth value of the statement $P \rightarrow (\sim p \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively : **[JEE(Main)-19]**

(1) F, T, T (2) T, F, F

(3) T, T, F (4) T, F, T

16. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to : **[JEE(Main)-19]**

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$

(3) $q \Rightarrow \sim p$ (4) $p \wedge q$

PREVIOUS YEARS QUESTIONS **ANSWER-KEY** **EXERCISE-II**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	2	2	1	2	4	2	4	1	1	4	4	1	2	4	3	4