CONTENTS

PROBABILITY

JEE (Main) Syllabus :

Addition and multiplication rules of probability, conditional probability, Bayes Theorem, independence of events, computation of probability of events using permutations and combinations.

JEE (Advanced) Syllabus :

Addition and multiplication rules of probability, conditional probability, Bayes Theorem, independence of events, computation of probability of events using permutations and combinations.

PROBABILITY

1 INTRODUCTION :

The theory of probability has been originated from the game of chance and gambling. In old days, gamblers used to gamble in a gambling house with a die to win the amount fixed among themselves. They were always desirous to get the prescribed number on the upper face of a die when it was thrown on a board. Shakuni of Mahabharat was perhaps one of them. People started to study the subject of probability from the middle of seventeenth century. The mathematicians Huygens, Pascal Fermat and Bernoulli contributed a lot to this branch of Mathematics. A.N. Kolmogorow proposed the set theoretic model to the theory of probability.

Probability gives us a measure of likelihood that something will happen. However probability can never predict the number of times that an occurrence actually happens. But being able to quantify the likely occurrence of an event is important because most of the decisions that affect our daily lives are based on likelihoods and not on absolute certainties.

2. DEFINITIONS :

(a) Experiment : An action or operation resulting in two or more well defined outcomes. e.g. tossing a coin, throwing a die, drawing a card from a pack of well shuffled playing cards etc.

(b) Sample space : A set S that consists of all possible outcomes of a random experiment is called a sample space and each outcome is called a sample point. Often, there will be more than one sample space that can describe outcomes of an experiment, but there is usually only one that will provide the most information.

e.g. in an experiment of "throwing a die", following sample spaces are possible :

(i) {even number, odd number}

(ii) {a number less than 3, a number equal to 3, a number greater than 3}

(iii) {1,2,3,4,5,6}

Here 3rd sample space is the one which provides most information.

If a sample space has a finite number of points it is called finite sample space and if it has an infinite number of points, it is called infinite sample space. e.g. (i) "in a toss of coin" either a head (H) or tail (T) comes up, therefore sample space of this experiment is S = {H,T} which is a finite sample space. (ii) "Selecting a number from the set of natural numbers", sample space of this experiment is S = {1,2,3,4,......} which is an infinite sample space.

(c) Event : An event is defined as an occurrence or situation, for example

(i) in a toss of a coin, it shows head,

(ii) scoring a six on the throw of a die,

(iii) winning the first prize in a raffle,

(iv) being dealt a hand of four cards which are all clubs.

In every case it is set of some or all possible outcomes of the experiment. Therefore event (A) is subset of sample space (S). If outcome of an experiment is an element of A we say that event A has occurred.

- An event consisting of a single point of S is called a simple or elementary event.
- ϕ is called impossible event and S (sample space) is called sure event.

Note : Probability of occurrence of an event A is denoted by P(A).

(d) **Compound Event :** If an event has more than one sample points it is called **Compound Event**. If A & B are two given events then $A \cap B$ is called compound event and is denoted by $A \cap B$ or AB or A & B.

(e) **Complement of an event :** The set of all outcomes which are in S but not in A is called the complement of the event A & denoted by $\bar{A}$, A^c, A' or 'not A'.

(f) **Mutually Exclusive Events :** Two events are said to be **Mutually Exclusive** (or disjoint or incompatible) if the occurrence of one precludes (rules out) the simultaneous occurrence of the other. If A & B are two mutually exclusive events then $P(A \cap B) = 0$.

Consider, for example, choosing numbers at random from the set {3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

If, Event A is the selection of a prime number,

Event B is the selection of an odd number,

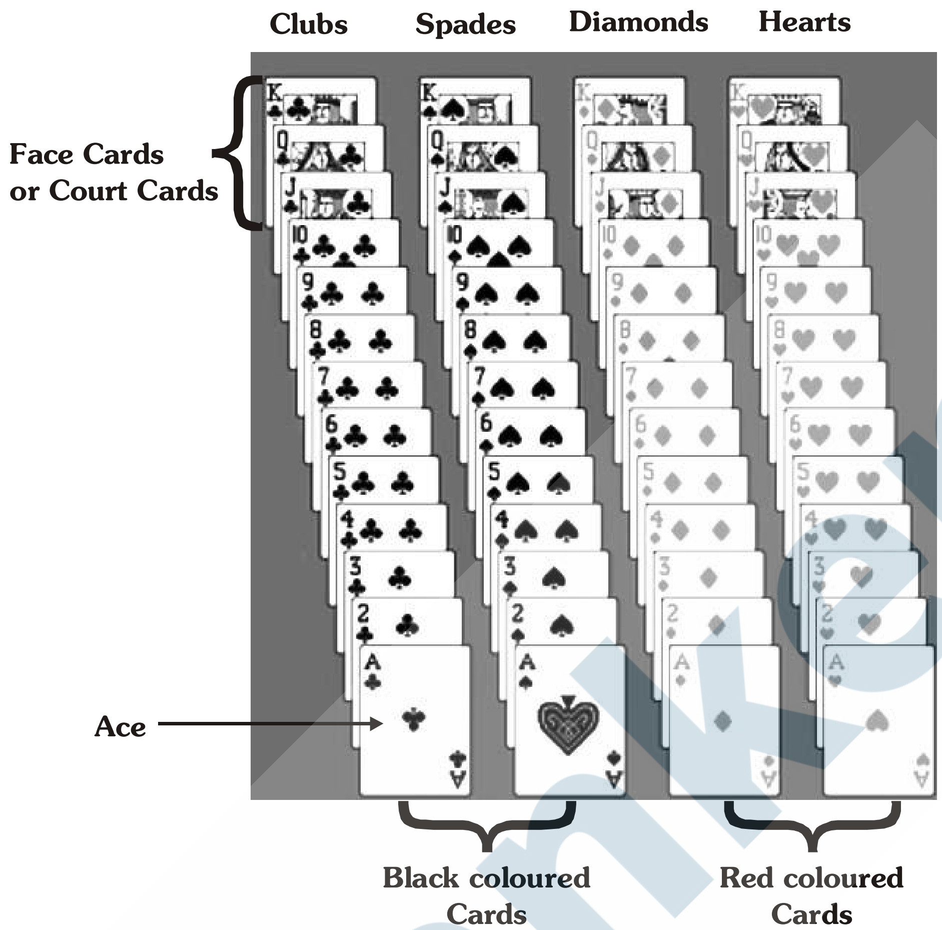
Event C is the selection of an even number,

then A and C are mutually exclusive as none of the numbers in this set is both prime and even. But A and B are not mutually exclusive as some numbers are both prime and odd (viz. 3, 5, 7, 11).

(g) **Equally Likely Events :** Events are said to be **Equally Likely** when each event is as likely to occur as any other event. Note that the term 'at random' or 'randomly' means that all possibilities are equally likely.

(h) **Exhaustive Events :** Events A,B,C........ N are said to be **Exhaustive Events** if no event outside this set can result as an outcome of an experiment. For example, if A & B are two events defined on a sample space S and A & B are exhaustive $\Rightarrow A \cup B = S \Rightarrow P(A \cup B) = 1$.

Note : Playing cards : A pack of playing cards consists of 52 cards of 4 suits, 13 in each, as shown in figure.



Comparative study of Equally likely, Mutually Exclusive and Exhaustive events :

	Experiment	Events	E/L	M/E	Exhaustive
1.	Throwing of a die	A: throwing an odd face {1, 3, 5} B : throwing a composite {4,6}	No	Yes	No
2.	A ball is drawn from an urn containing 2White, 3Red and 4Green balls	E_1 : getting a White ball E_2 : getting a Red ball E_3 : getting a Green ball	No	Yes	Yes
3.	Throwing a pair of dice	A : throwing a doublet {11, 22, 33, 44, 55, 66} B : throwing a total of 10 or more { 46, 64, 55, 56, 65, 66 }	Yes	No	No
4.	From a well shuffled pack of cards a card is drawn	E_1 : getting a heart E_2 : getting a spade E_3 : getting a diamond E_4 : getting a club	Yes	Yes	Yes
5.	From a well shuffled pack of cards a card is drawn	A = getting a heart B = getting a face card	No	No	No

Illustration 1 : A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 blue and 4 white balls; if it shows tail we throw a die. Describe the sample space of this experiment.

Solution : Let us denote blue balls by B_1, B_2, B_3 and the white balls by W_1, W_2, W_3, W_4. Then a sample space of the experiment is

$S = \{HB_1, HB_2, HB_3, HW_1, HW_2, HW_3, HW_4, T1, T2, T3, T4, T5, T6\}$.

Here HB_i means head on the coin and ball B_i is drawn, HW_i means head on the coin and ball W_i is drawn. Similarly, Ti means tail on the coin and the number i on the die.

Illustration 2 : Consider the experiment in which a coin is tossed repeatedly until a head comes up. Describe the sample space.

Solution : In the experiment head may come up on the first toss, or the 2nd toss, or the 3rd toss and so on. Hence, the desired sample space is S = {H, TH, TTH, TTTH, TTTTH,...}

Illustration 3 : A coin is tossed three times, consider the following events.

A : 'no head appears'

B : 'exactly one head appears'

C : 'at least two heads appear'

Do they form a set of mutually exclusive and exhaustive events ?

Solution : The sample space of the experiment is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Events A, B and C are given by

A = {TTT}

B = {HTT, THT, TTH}

C = {HHT, HTH, THH, HHH}

Now,

$A \cup B \cup C$ = {TTT, HTT,THT,TTH,HHT,HTH,THH,HHH} = S

Therefore A,B and C are exhaustive events. Also, $A \cap B = \phi$, $A \cap C = \phi$ and $B \cap C = \phi$. Therefore, the events are pair-wise disjoint, i.e., they are mutually exclusive. Hence, A,B and C form a set of mutually exclusive and exhaustive events.

Do yourself - 1 :

(i) Two balls are drawn from a bag containing 2 Red and 3 Black balls, write sample space of this experiment.

(ii) Out of 2 men and 3 women a team of two persons is to be formed such that there is exactly one man and one woman. Write the sample space of this experiment.

(iii) A coin in tossed and if head comes up, a die is thrown. But if tail comes up, the coin is tossed again. Write the sample space of this experiment.

(iv) In a toss of a die, consider following events :

A : An even number turns up. B : A prime number turns up.

These events are -

(A) Equally likely events (B) Mutually exclusive events

(C) Exhaustive events (D) None of these

3. CLASSICAL DEFINITION OF PROBABILITY :

If n represents the total number of equally likely, mutually exclusive and exhaustive outcomes of an experiment and m of them are favourable to the happening of the event A, then the probability of happening of the event A is given by P(A) = m/n. There are (n–m) outcomes which are favourable to the event that A does not happen. 'The event A does not happen' is denoted by $\bar{A}$ (and is read as 'not A')

Thus $P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n}$

i.e. $P(\bar{A}) = 1 - P(A)$

Note :

(i) $0 \le P(A) \le 1$

(ii) $P(A) + P(\bar{A}) = 1$,

(iii) If x cases are favourable to A & y cases are favourable to $\bar{A}$ then $P(A) = \frac{x}{(x+y)}$ and

$P(\bar{A}) = \frac{y}{(x+y)}$. We say that Odds In Favour Of A are x: y & Odds Against A are y : x

OTHER DEFINITIONS OF PROBABILITY :

(a) Axiomatic probability : Axiomatic approach is another way of describing probability of an event. In this approach some axioms or rules are depicted to assign probabilities.

Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0, 1] satisfying the following axioms :

(i) For any event E, $P(E) \ge 0$ (ii) $P(S) = 1$

(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

It follows from (iii) that $P(E \cap F) = P(\phi) = 0$.

Let S be a sample space containing outcomes $\omega_1, \omega_2, \ldots, \omega_n$, i.e., $S = \{\omega_1, \omega_2, \ldots \omega_n\}$

It follows from the axiomatic definition of probability that :

(i) $0 \le P(\omega_i) \le 1$ for each $\omega_i \in S$

(ii) $P(\omega_1) + P(\omega_2) + \ldots + P(\omega_n) = 1$

(iii) For any event A, $P(A) = \Sigma P(\omega_i)$, $\omega_i \in A$.

(b) Empirical probability : The probability that you would hit the bull's-eye on a dartboard with one throw of a dart would depend on how much you had practised, how much natural talent for playing darts you had, how tired you were, how good a dart you were using etc. all of which are impossible to quantify. A method which can be adopted in the example given above is to throw the dart several times (each throw is a trial) and count the number of times you hit the bull's-eye (a success) and the number of times you miss (a failure). Then an empirical value of the probability that you hit the bull's-eye with any one throw is

$$\frac{\text{number of successes}}{\text{number of successes + number of failures}}.$$

If the number of throws is small, this does not give a particular good estimate but for a large number of throws the result is more reliable.

When the probability of the occurrence of an event A cannot be worked out exactly, an empirical value can be found by adopting the approach described above, that is :

(i) making a large number of trials (i.e. set up an experiment in which the event may, or may not, occur and note the outcome),

(ii) counting the number of times the event does occur, i.e. the number of successes,

(iii) calculating the value of $\dfrac{\text{number of successes}}{\text{number of trials (i.e. successes + failures)}} = \dfrac{r}{n}$

The probability of event A occurring is defined as $P(A) = \lim\limits_{n \to \infty}\left(\dfrac{r}{n}\right)$

$n \to \infty$ means that the number of trials is large (but what should be taken as 'large' depends on the problem).

Illustration 4 : If the letters of INTERMEDIATE are arranged, then the odds in favour of the event that no two 'E's occur together, are-

(A) $\dfrac{6}{5}$ (B) $\dfrac{5}{6}$ (C) $\dfrac{2}{9}$ (D) none of these

Solution : $I \to 2,\ N \to 1,\ T \to 2,\ E \to 3,\ R \to 1,\ M \to 1,\ D \to 1, A \to 1$ (3'E's, Rest 9 letters)

First arrange rest of the letters $= \dfrac{9!}{2!\ 2!}$,

Now 3'E's can be placed by ${}^{10}C_3$ ways, so favourable cases $= \dfrac{9!}{2!\ 2!} \times {}^{10}C_3 = 3 \times 10!$

Total cases $= \dfrac{12!}{2!\,2!\,3!} = \dfrac{11}{2} \times 10!$; Non-favourable cases $= \left(\dfrac{11}{2} - 3\right) \times 10! = \dfrac{5}{2} \times 10!$

Odds in favour of the event $= \dfrac{3}{5/2} = \dfrac{6}{5}$ **Ans. (A)**

Illustration 5 : From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one of each category is-

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) none of these

Solution : $n(S) = {}^{10}C_4 = 210$.

$n(E) = {}^5C_2 \times {}^3C_1 \times {}^2C_1 + {}^5C_1 \times {}^3C_2 \times {}^2C_1 + {}^5C_1 \times {}^3C_1 \times {}^2C_2 = 105$

$\therefore \quad P(E) = \dfrac{105}{210} = \dfrac{1}{2}$. **Ans. (A)**

Illustration 6 : If four cards are drawn at random from a pack of fifty-two playing cards, find the probability that at least one of them is an ace.

Solution : If A is a combination of four cards containing at least one ace (i.e. either one ace, or two aces, or three aces or four aces) then $\bar{A}$ is a combination of four cards containing no aces.

$$\text{Now } P(\bar{A}) = \frac{\text{Number of combinations of four cards with no aces}}{\text{Total number of combinations of four cards}} = {}^{48}C_4 / {}^{52}C_4 = 0.72$$

Using $P(A) + P(\bar{A}) = 1$ we have $P(A) = 1 - P(\bar{A}) = 1 - 0.72 = 0.28$

Illustration 7 : A bag contains n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty. Show that the probability that each pair consists of one white and one red ball is $2^n/({}^{2n}C_n)$.

Solution : Let S be the sample space & E be the event that each of the n pairs of balls drawn consists of one white and one red ball.

$\therefore \quad n(S) = ({}^{2n}C_2)\,({}^{2n-2}C_2)\,({}^{2n-4}C_2)\ldots({}^{4}C_2)({}^{2}C_2)$

$$= \left\{\frac{(2n)(2n-1)}{1.2}\right\}\left\{\frac{(2n-2)(2n-3)}{1.2}\right\}\left\{\frac{(2n-4)(2n-5)}{1.2}\right\}\ldots\ldots\left\{\frac{4.3}{1.2}\right\}.\left\{\frac{2.1}{1.2}\right\}$$

$$= \frac{1.2.3.4\ldots.(2n-1)2n}{2^n} = \frac{2n!}{2^n}$$

and $n(E) = ({}^{n}C_1.{}^{n}C_1)\,({}^{n-1}C_1.{}^{n-1}C_1)\,({}^{n-2}C_1.{}^{n-2}C_1)\ldots({}^{2}C_1.{}^{2}C_1)\,({}^{1}C_1.{}^{1}C_1)$

$= n^2.(n-1)^2.(n-2)^2\ldots.2^2.1^2 = [1.2.3\ldots..(n-1)n]^2 = (n!)^2$

$$\therefore \quad \text{Required Probability, } P(E) = \frac{n(E)}{n(S)} = \frac{(n!)^2}{(2n)!/2^n} = \frac{2^n}{\frac{2n!}{(n!)^2}} = \frac{2^n}{{}^{2n}C_n}$$ **Ans.**

Do yourself - 2 :

(i) A coin is tossed successively three times. Find the probability of getting exactly one head or two heads.

(ii) A bag contains 5 red and 4 green balls. Four balls are drawn at random then find the probability that two balls are of red and two balls are of green colour.

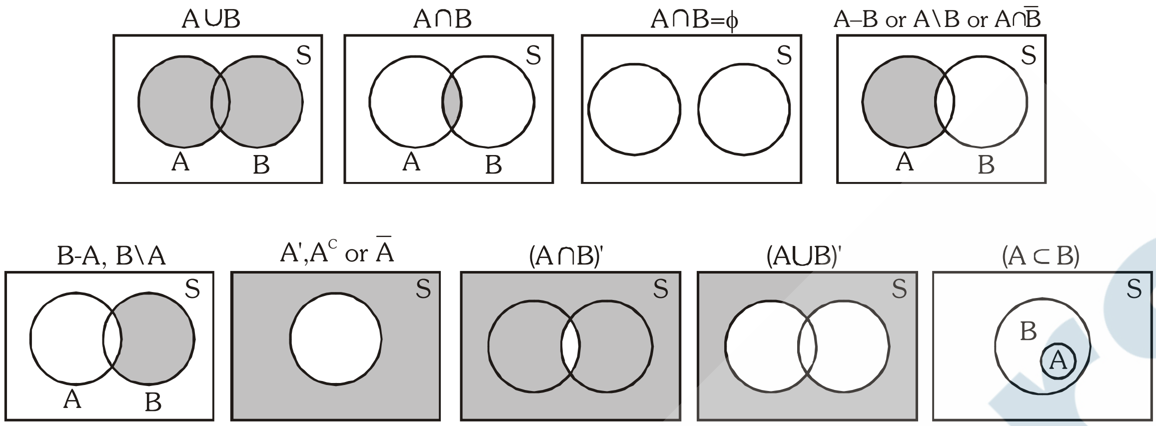
(iii) Two natural numbers are selected at random, find the probability that their sum is divisible by 10.

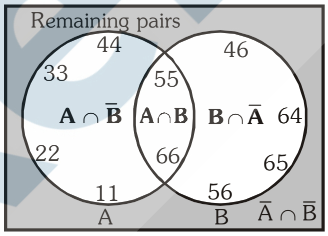
(iv) Five card are drawn successively from a pack of 52 cards with replacement. Find the probability that there is at least one Ace.

4. VENN DIAGRAMS :

A diagram used to illustrate relationships between sets. Commonly, a rectangle represents the universal set and a circle within it represents a given set (all members of the given set are represented by points within the circle). A subset is represented by a circle within a circle and intersection is indicated by overlapping circles.

Let S is the sample space of an experiment and A, B are two events corresponding to it :





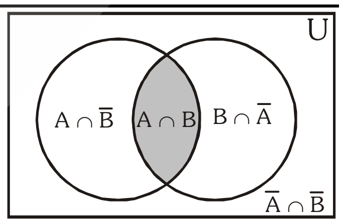
Example : Let us conduct an experiment of tossing a pair of dice.

Two events defined on the experiment are

A : getting a doublet $\{11, 22, 33, 44, 55, 66\}$

B : getting total score of 10 or more $\{64, 46, 55, 56, 65, 66\}$

5. ADDITION THEOREM :



$A \cup B = A + B = A$ or B denotes occurrence of at least A or B.

For 2 events A & B :

P(A∪B) = P(A) + P(B) – P(A∩B)

(a)

$$\left.\begin{array}{l} P(A\cup B) \\ P(A+B) \\ P(A \text{ or } B) \\ P(\text{occurence of atleast A or B}) \end{array}\right] = \begin{array}{l} P(A)+P(B)-P(A\cap B) \text{ (This is known as generallised addition theorem)} \\ P(A)+P(B\cap \bar{A}) \\ P(B)+P(A\cap \bar{B}) \\ P(A\cap \bar{B})+P(A\cap B)+P(B\cap \bar{A}) \\ 1-P(\bar{A}\cap \bar{B}) \\ 1-P(\overline{A\cup B}) \end{array}$$

Note :

(i) If A & B are mutually exclusive then $P(A\cup B) = P(A) + P(B)$.

(ii) If A & B are mutually exclusive and exhaustive, then $P(A\cup B) = P(A) + P(B) = 1$

(b) P(only A occurs) $= P(A\backslash B) = P(A-B) = P(A\cap B^C) = P(A) - P(A\cap B)$

(c) P(either A or B) = 1 – P(neither A nor B)

i.e. $P(A\cup B) = 1 - P(\bar{A}\cap \bar{B})$

(d) For any two events A & B

P(exactly one of A, B occurs) $= P\left(A\cap \bar{B}\right) + P\left(B\cap \bar{A}\right)$

$\Rightarrow$ **P(exactly one of A, B occurs) = $P(A) + P(B) - 2P(A \cap B)$**

$= P(A \cup B) - P(A \cap B) = P(A^c \cup B^c) - P(A^c \cap B^c)$

(e) $P(A \cap B) \le P(A), P(B) \le P(A \cup B) \le P(A) + P(B)$

6. DE MORGAN'S LAW :

If A & B are two subsets of a universal set U, then

(i) $(A \cup B)^c = A^c \cap B^c$ & (ii) $(A \cap B)^c = A^c \cup B^c$

Note :

(a) $(A \cup B \cup C)^C = A^C \cap B^C \cap C^C$ & $(A \cap B \cap C)^C = A^C \cup B^C \cup C^C$

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ & $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Illustration 8 : Given two events A and B. If odds against A are as 2 : 1 and those in favour of $A \cup B$ are as 3 : 1, then find the range of P(B).

Solution : Clearly $P(A) = 1/3$, $P(A \cup B) = 3/4$.

Now, $P(B) \le P(A \cup B)$

$\Rightarrow \quad P(B) \le 3/4$

Also, $P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$\Rightarrow \quad P(B) \ge P(A \cup B) - P(A) \quad (\because P(A \cap B) \ge 0)$

$\Rightarrow \quad P(B) \ge 3/4 - 1/3 \quad \Rightarrow \quad P(B) \ge \frac{5}{12}$

$\Rightarrow \quad \frac{5}{12} \le P(B) \le \frac{3}{4}$ **Ans.**

Illustration 9 : If A and B are two events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and $P(A^c) = \frac{2}{3}$. Then find -

(i) P(A) (ii) P(B) (iii) $P(A \cap B^c)$ (iv) $P(A^c \cap B)$

Solution : $P(A) = 1 - P(A^c) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(B) = P(A \cup B) + P(A \cap B) - P(A) = \frac{3}{4} + \frac{1}{4} - \frac{1}{3} = \frac{2}{3}$

$P(A \cap B^c) = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$P(A^c \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$ **Ans.**

Do yourself - 3 :

(i) Draw Venn diagram of (a) $(A^C \cap B^C) \cup (A \cap B)$ (b) $B^C \cup (A^C \cap B)$

(ii) If A and B are two mutually exclusive events, then-

(A) $P(A) \le P(\bar{B})$ (B) $P(\bar{A} \cap \bar{B}) = P(\bar{A}) - P(B)$ (C) $P(\bar{A} \cup \bar{B}) = 0$ (D) $P(\bar{A} \cap B) = P(B)$

(iii) A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting either a white or a black ball in a single draw.

(iv) In a class of 125 students, 70 passed in English, 55 in mathematics and 30 in both. Find the probability that a student selected at random from the class has passed in

(a) at least one subject (b) only one subject.

7. CONDITIONAL PROBABILITY AND MULTIPLICATION THEOREM :

(a) **Conditional Probability :** Let A and B be two events such that P (A) >0. Then P(B|A) denote the conditional probability of B given that A has occurred. Since A is known to have occurred, it becomes the new sample space replacing the original S. From this we led to the definition

$$\mathbf{P(B|A)} = \frac{\mathbf{P(A \cap B)}}{\mathbf{P(A)}}$$ = which is called conditional probability of B given A

(b) **Multiplication Theorem :** $P(A \cap B) = P(A)\ P(B|A)$ which is called compound probability or multiplication theorem. It says the probability that both A and B occur is equal to the probability that A occur times the probability that B occurs given that A has occurred.

Note : For any three events A_1, A_2, A_3 we have

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)\ P(A_2|A_1)\ P(A_3|(A_1 \cap A_2))$$

Illustration 10 : Two dice are thrown. Find the probability that the numbers appeared have a sum of 8 if it is known that the second die always exhibits 4

Solution : Let A be the event of occurrence of 4 always on the second die

$= \{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\}$; $\therefore\ n(A) = 6$

and B be the event of occurrence of such numbers on both dice whose sum is 8 = {(6,2), (5,3), (4,4), (3,5), (2,6)}.

Thus, $A \cap B = A \cap \{(4,4)\} = \{(4,4)\}$

$\therefore\ n(A \cap B) = 1$

$$\therefore\ P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{1}{6} \text{ or } \frac{P(A \cap B)}{P(A)} = \frac{1/36}{6/36} = \frac{1}{6}$$

Illustration 11 : A bag contains 3 red, 6 white and 7 blue balls. Two balls are drawn one by one. What is the probability that first ball is white and second ball is blue when first drawn ball is not replaced in the bag?

Solution : Let A be the event of drawing first ball white and B be the event of drawing second ball blue. Here A and B are dependent events.

$$P(A)=\frac{6}{16},\ P(B|A)=\frac{7}{15}$$

$$P(AB)=P(A).P(B|A)=\frac{6}{16}\times\frac{7}{15}=\frac{7}{40}$$

Illustration 12 : A bag contains 4 red and 4 blue balls. Four balls are drawn one by one from the bag, then find the probability that the drawn balls are in alternate colour.

Solution : E_1 : Event that first drawn ball is red, second is blue and so on.

E_2 : Event that first drawn ball is blue, second is red and so on.

$$\therefore\quad P(E_1)=\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}\text{ and }P(E_2)=\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$$

$$P(E)=P(E_1)+P(E_2)=2\times\frac{4}{8}.\frac{4}{7}.\frac{3}{6}.\frac{3}{5}=\frac{6}{35}$$ **Ans.**

Illustration 13 : If two events A and B are such that $P(\bar{A})=0.3$, P(B) = 0.4 and $P(A\bar{B})=0.5$ then $P(B|(A\cup\bar{B}))$ equals -

(A) 1/2 (B) 1/3 (C) 1/4 (D) 1/5

Solution : We have $P(B|(A\cup\bar{B}))=\dfrac{P[B\cap(A\cup\bar{B})]}{P(A\cup\bar{B})}=\dfrac{P[(B\cap A)\cup(B\cap\bar{B})]}{P(A)+P(\bar{B})-P(A\cap\bar{B})}$

$$=\frac{P(AB)}{P(A)+P(\bar{B})-P(A\bar{B})}=\frac{P(A)-P(A\bar{B})}{P(A)+P(\bar{B})-P(A\bar{B})}=\frac{0.7-0.5}{0.7+0.6-0.5}=\frac{0.2}{0.8}=\frac{1}{4}$$ **Ans.(C)**

Illustration 14 : Three coins are tossed. Two of them are fair and one is biased so that a head is three times as likely as a tail. Find the probability of getting two heads and a tail.

Solution : E_1 : Event that head occurs on fair coin

E_2 : Event that, head occurs on biased coin

$$\therefore\ P(E_1)=\frac{1}{2},\ P(E_2)=\frac{3}{4}$$

E : HHT or HTH or THH

$$\Rightarrow\ P(E)=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}\times\frac{3}{4}+\frac{1}{2}\times\frac{1}{2}\times\frac{3}{4}$$

$$=\frac{7}{16}$$

Illustration 15 : In a multiple choice test of three questions there are five alternative answers given to the first two questions each and four alternative answers given to the last question. If a candidate guesses answers at random, what is the probability that he will get-

(a) Exactly one right answer ?

(b) At least one right answer ?

Solution : E_1 : Event that, candidate guesses a correct answer for I question
E_2 : Event that, candidate guesses a correct answer for II question
E_3 : Event that, candidate guesses a correct answer for III question

$$\therefore\ P(E_1)=\frac{1}{5},\ P(E_2)=\frac{1}{5},\ P(E_3)=\frac{1}{4}$$

(a) E : Event that candidate get exactly one correct answer.

$$\therefore\ P(E)=P(E_1)P(\bar{E}_2)P(\bar{E}_3)+P(\bar{E}_1).P(E_2)P(\bar{E}_3)+P(\bar{E}_1)P(\bar{E}_2)P(E_3)$$

$$=\frac{1}{5}.\frac{4}{5}.\frac{3}{4}+\frac{4}{5}.\frac{1}{5}.\frac{3}{4}+\frac{4}{5}.\frac{4}{5}.\frac{1}{4}=\frac{2}{5}$$

(b) E : Event that candidate gets atleast one correct answer

$$\therefore\ P(E)=1-P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)=1-\frac{4}{5}.\frac{4}{5}.\frac{3}{4}=\frac{13}{25}$$

Illustration 16 : A speaks truth in 75% cases and B in 80% cases. What is the probability that they contradict each other in stating the same fact?

(A) 7/20 (B) 13/20 (C) 3/20 (D) 1/5

Solution : There are two mutually exclusive cases in which they contradict each other i.e. $\bar{A}B$ and $A\bar{B}$. Hence required probability = $P(A\bar{B}+\bar{A}B)=P(A\bar{B})+P(\bar{A}B)$

$$=P(A)P(\bar{B})+P(\bar{A})P(B)=\frac{3}{4}.\frac{1}{5}+\frac{1}{4}.\frac{4}{5}=\frac{7}{20}$$ **Ans. (A)**

Do yourself - 4 :

(i) A bag contains 2 black, 4 white and 3 red balls. One ball is drawn at random from the bag and kept aside. From the remaining balls another ball is drawn and kept aside the first. This process is repeated till all the balls are drawn. Then probability that the balls drawn are in sequence of 2 black, 4 white and 3 red is-

(A) $\frac{1}{1260}$ (B) $\frac{1}{7560}$ (C) $\frac{1}{210}$ (D) None of these

(ii) Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that the drawn cards are face cards of same suit ?

8. INDEPENDENT EVENTS :

Two events A & B are said to be independent if occurrence or non occurrence of one does not affect the probability of the occurrence or non occurrence of other.

(a) If the occurrence of one event affects the probability of the occurrence of the other event then the events are said to be **Dependent** or **Contingent.** For two independent events A and B : $P(A\cap B) = P(A)\ .\ P(B)$. Often this is taken as the definition of independent events.

Note : If A and B are independent events, then

(i) $P(\bar{A}\cap\bar{B})=P(\bar{A}).P(\bar{B})$ (ii) $P(A\cap\bar{B})=P(A).P(\bar{B})$

(b) Three events A, B & C are independent if & only if all the following conditions hold ;

$P(A\cap B) = P(A)\ .\ P(B)$; $P(B\cap C) = P(B).\ P(C)$

$P(C\cap A) = P(C)\ .\ P(A)$ and $P(A\cap B\cap C) = P(A)\ .\ P(B)\ .\ P(C)$

i.e. they must be pairwise as well as mutually independent.

(c) If three events A, B and C are pair wise mutually exclusive then they must be mutually exclusive.

i.e. $P(A \cap B) = P(B \cap C) = P(C \cap A) = 0 \Rightarrow P(A \cap B \cap C) = 0$. However the converse of this is not true.

Note :

Independent events are not in general mutually exclusive & vice versa. Mutually exclusiveness can be used when the events are taken from the same experiment & independence can be used when the events are taken from different experiments.

Illustration 17 : If A & B are independent events such that $P(A \cap \bar{B}) = \frac{1}{3}$ & $P(A \cup B) = \frac{11}{15}$, then $P(A \cap B)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{5}{11}$ (C) $\frac{2}{9}$ (D) $\frac{7}{9}$

Solution : $P(A) - P(A \cap B) = \frac{1}{3}$ & $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{11}{15}$

$\Rightarrow \quad P(B) = \frac{6}{15} = \frac{2}{5}$

$P(A) - P(A)\,P(B) = \frac{1}{3} \Rightarrow P(A) = \frac{5}{9}$

$\Rightarrow \quad P(A \cap B) = P(A)\,P(B) = \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$

Illustration 18 : A lot contains 50 defective and 50 non-defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as : **[IIT 1992]**

A = {The first bulb is defective}

B = {The second bulb is non-defective}

C = {The two bulbs are both defective or both non-defective}

Determine whether

(i) A, B, C are pairwise independent, (ii) A, B, C are independent.

Solution : We have $P(A) = \frac{50}{100}.1 = \frac{1}{2}$; $P(B) = 1.\frac{50}{100} = \frac{1}{2}$; $P(C) = \frac{50}{100}.\frac{50}{100} + \frac{50}{100}.\frac{50}{100} = \frac{1}{2}$

$A \cap B$ is the event that first bulb is defective and second is non-defective.

$\therefore \quad P(A \cap B) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

$A \cap C$ is the event that both bulbs are defective.

$\therefore \quad P(A \cap C) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

Similarly $P(B \cap C) = \frac{1}{4}$

Thus we have $P(A \cap B) = P(A) . P(B)$; $P(A \cap C) = P(A) . P(C)$; $P(B \cap C) = P(B).P(C)$

$\therefore$ A, B and C are pairwise independent.

There is no element in $A \cap B \cap C$

$\therefore$ $P(A \cap B \cap C) = 0$

$\therefore$ $P(A \cap B \cap C) \neq P(A) . P(B) . P(C)$

Hence A, B and C are not mutually independent.

Do yourself - 5 :

(i) For two independent events A and B, the probability that both A & B occur is 1/8 and the probability that neither of them occur is 3/8. The probability of occurrence of A may be -

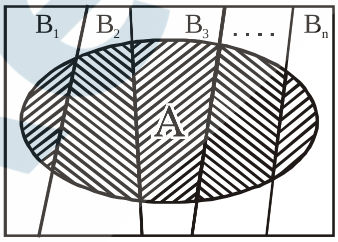
(A) 1/2 (B) 1/4 (C) 1/8 (D) 3/4

(ii) A die marked with numbers 1,2,2,3,3,3 is rolled three times. Find the probability of occurrence of 1,2 and 3 respectively.

9. TOTAL PROBABILITY THEOREM :

Let an event A of an experiment occurs with its n mutually exclusive & exhaustive events $B_1, B_2, B_3, \ldots\ldots\ldots B_n$

then total probability of occurrence of even A is



$$P(A) = P(AB_1) + P(AB_2) + \ldots\ldots. + P(AB_n) = \sum_{i=1}^{n} P(AB_i)$$

$$P(A) = P(B_1)\ P(A|B_1) + P(B_2)\ P(A|B_2) + \ldots\ldots\ldots.. + P(B_n)\ P(A|B_n)$$
$$= \Sigma P(B_i)\ P(A|B_i)$$

Illustration 19 : A purse contains 4 copper and 3 silver coins and another purse contains 6 copper and 2 silver coins. One coin is drawn from any one of these two purses. The probability that it is a copper coin is -

(A) $\frac{4}{7}$ (B) $\frac{3}{4}$ (C) $\frac{2}{7}$ (D) $\frac{37}{56}$

Solution : Let $A \equiv$ event of selecting first purse

$B \equiv$ event of selecting second purse

$C \equiv$ event of drawing a copper coin

Then given event has two disjoint cases: AC and BC

$$\therefore P(C) = P(AC + BC) = P(AC) + P(BC) = P(A)P(C|A) + P(B)P(C|B)$$

$$= \frac{1}{2}.\frac{4}{7} + \frac{1}{2}.\frac{6}{8} = \frac{37}{56}$$

Ans. (D)

Illustration 20 : Three groups A, B, C are contesting for positions on the Board of Directors of a Company. The probabilities of their winning are 0.5, 0.3, 0.2 respectively. If the group A wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for group B and C are 0.6 and 0.5 respectively. Find the probability that the new product will be introduced.

Solution : Given P(A) = 0.5, P(B) = 0.3 and P(C) = 0.2

$\therefore$ P(A) + P(B) + P(C) =1

then events A, B, C are exhaustive.

If P(E) = Probability of introducing a new product, then as given

P(E|A) = 0.7, P(E|B) =0.6 and P(E|C) = 0.5

$\therefore$ P(E) = P(A). P(E|A) + P(B). P(E|B) + P(C). P(E|C)

$= 0.5 \times 0.7 + 0.3 \times 0.6 + 0.2 \times 0.5 = 0.35 + 0.18 + 0.10 = 0.63$

Illustration21 : A pair of dice is rolled together till a sum of either 5 or 7 is obtained. Find the probability that 5 comes before 7.

Solution : Let E_1 = the event of getting 5 in a roll of two dice = {(1, 4), (2, 3), (3, 2), (4, 1)}

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{6\times 6} = \frac{1}{9}$$

Let E_2 = the event of getting either 5 or 7

= {(1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{6\times 6} = \frac{5}{18}$$

$\therefore$ the probability of getting neither 5 nor 7 $= P(\bar{E}_2) = 1 - P(E_2) = 1 - \frac{5}{18} = \frac{13}{18}$

The event of getting 5 before 7 = $E_1 \cup (\bar{E}_2 E_1) \cup (\bar{E}_2 \bar{E}_2 E_1) \cup ......$ to ∞

$\therefore$ the probability of getting 5 before 7

$$= P(E_1) + P(\bar{E}_2 E_1) + P(\bar{E}_2\bar{E}_2 E_1) + ... \text{ to } \infty = P(E_1) + P(\bar{E}_2)P(E_1) + P(\bar{E}_2)P(\bar{E}_2)P(E_1) + ... \text{ to } \infty$$

$$= \frac{1}{9} + \frac{13}{18}.\frac{1}{9} + \frac{13}{18}.\frac{13}{18}.\frac{1}{9} + ... \text{ to } \infty$$

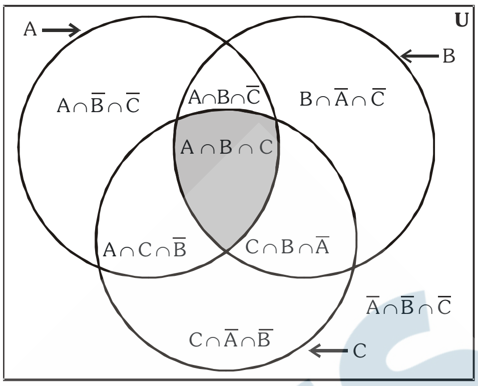
$$= \frac{1}{9}\left[1 + \frac{13}{18} + \left(\frac{13}{18}\right)^2 + ... \text{ to } \infty\right] = \frac{1}{9}.\frac{1}{1-\frac{13}{18}} = \frac{1}{9}.\frac{18}{5} = \frac{2}{5}$$

Do yourself - 6 :

(i) An urn contains 6 white & 4 black balls. A die is rolled and the number of balls equal to the number obtained on the die are drawn from the urn. Find the probability that the balls drawn are all black.

(ii) There are n bags such that i^{th} bag $(1 \le i \le n)$ contains i black and 2 white balls. Two balls are drawn from a randomly selected bag out of given n bags. Find the probability that the both drawn balls are white.

10. PROBABILITY OF THREE EVENTS :



For any three events A,B and C we have

(a) P(atleast one of A, B and C occurs)

$= P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A\cap B)$

$- P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

Note : $P(E_1 \cup E_2 \ldots. \cup E_n) = 1 - P(\bar{E}_1 \cap \bar{E}_2 \ldots. \cap \bar{E}_n)$

(b) P(at least two of A,B,C occur)

$= P(B\cap C) + P(C\cap A) + P(A\cap B) - 2P(A\cap B\cap C)$

(c) P(exactly two of A,B,C occur)

$= P(B\cap C) + P(C\cap A) + P(A\cap B) - 3P(A\cap B\cap C)$

(d) P(exactly one of A,B,C occurs)

$= P(A)+ P(B) + P(C) - 2P(B\cap C) - 2P(C\cap A) - 2P(A\cap B) + 3P(A\cap B\cap C)$

Illustration 22 : Let A,B,C be three events. If the probability of occurring exactly one event out of A and B is $1 - a$, out of B and C is $1 - 2a$, out of C and A is $1 - a$ and that of occurring three events simultaneously is a^2, then prove that the probability that at least one out of A,B,C will occur is greater than 1/2.

Solution : $P(A) + P(B) - 2P(A\cap B) = 1 - a$ .....(1)

and $P(B) + P(C) - 2P(B\cap C) = 1 - 2a$ .....(2)

and $P(C) + P(A) - 2P(C\cap A) = 1 - a$ .....(3)

and $P(A\cap B\cap C) = a^2$ .....(4)

$\because P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

$$= \frac{1}{2}\{P(A) + P(B) - 2P(A\cap B) + P(B) + P(C) - 2P(B\cap C) + P(C) + P(A) - 2P(C\cap A)\} + P(A\cap B\cap C)$$

$$= \frac{1}{2}\{1 - a + 1 - 2a + 1 - a\} + a^2 \qquad \{\text{from (1), (2), (3) \& (4)}\}$$

$$= \frac{3}{2} - 2a + a^2 = (a-1)^2 + \frac{1}{2} > \frac{1}{2}$$ **Ans.**

Do yourself - 7 :

(i) In a class, there are 100 students out of which 45 study mathematics, 48 study physics, 40 study chemistry, 12 study both mathematics & physics, 11 study both physics & chemistry, 15 study both mathematics & chemistry and 5 study all three subjects. A student is selected at random, then find the probability that the selected student studies

(a) only one subject (b) neither physics nor chemistry

11. BINOMIAL PROBABILITY DISTRIBUTION :

Suppose that we have an experiment such as tossing a coin or die repeatedly or choosing a marble from an urn repeatedly. Each toss or selection is called a trial. In any single trial there will be a probability associated with a particular event such as head on the coin, 4 on the die, or selection of a red marble. In some cases this probability will not change from one trial to the next (as in tossing a coin or die). Such trials are then said to be independent and are often called Bernoulli trials after James Bernoulli who investigated them at the end of the seventeenth century.

Let p be the probability that an event will happen in any single Bernoulli trial (called the probability of success). Then $q = 1 - p$ is the probability that the event will fail to happen in any single trial (called the probability of failure). The probability that the event will happen exactly x times in n trials (i.e., x successes and $n - x$ failures will occur) is given by the probability function.

$$f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x} \qquad \text{......... (i)}$$

where the random variable X denotes the number of successes in n trials and x = 0, 1,........., n.

Example : The probability of getting exactly 2 heads in 6 tosses of a fair coin is

$$P(X=2) = \binom{6}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2} = \frac{6!}{2!4!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2} = \frac{15}{64}$$

The discrete probability function (i) is often called the binomial distribution since for x = 0, 1, 2,...,n, it corresponds to successive terms in the binomial expansion

$$(q+p)^n = q^n + \binom{n}{1}q^{n-1}p + \binom{n}{2}q^{n-2}p^2 + \ldots.. + p^n = \sum_{x=0}^{n}\binom{n}{x}p^x q^{n-x}$$

Illustration 23 : If a fair coin is tossed 10 times, find the probability of getting

(i) exactly six heads (ii) atleast six heads (iii) atmost six heads

Solution : The repeated tosses of a coin are Bernoulli trials. Let X denotes the number of heads in an experiment of 10 trials.

Clearly, X has the binomial distribution with n = 10 and $p = \frac{1}{2}$

Therefore $P(X = x) = {}^nC_x q^{n-x} p^x,\ x = 0, 1, 2, \ldots., n$

Here $n = 10,\ p = \frac{1}{2},\ q = 1 - p = \frac{1}{2}$

Therefore $P(X = x) = {}^{10}C_x \left(\frac{1}{2}\right)^{10-x}\left(\frac{1}{2}\right)^x = {}^{10}C_x\left(\frac{1}{2}\right)^{10}$

Now (i) $P(X = 6) = {}^{10}C_6\left(\frac{1}{2}\right)^{10} = \frac{10!}{6! \times 4! 2^{10}} = \frac{105}{512}$

(ii) P(atleast six heads)

$= P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$

$$= {}^{10}C_6\left(\frac{1}{2}\right)^{10} + {}^{10}C_7\left(\frac{1}{2}\right)^{10} + {}^{10}C_8\left(\frac{1}{2}\right)^{10} + {}^{10}C_9\left(\frac{1}{2}\right)^{10} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}$$

$$= \left[\left(\frac{10!}{6!\times 4!}\right)+\left(\frac{10!}{7!\times 3!}\right)+\left(\frac{10!}{8!\times 2!}\right)+\left(\frac{10!}{9!\times 1!}\right)+\left(\frac{10!}{10!}\right)\right]\frac{1}{2^{10}} = \frac{193}{512}$$

(iii) P(at most six heads) $= P(X \leq 6)$

$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

$$= \left(\frac{1}{2}\right)^{10} + {}^{10}C_1\left(\frac{1}{2}\right)^{10} + {}^{10}C_2\left(\frac{1}{2}\right)^{10} + {}^{10}C_3\left(\frac{1}{2}\right)^{10} + {}^{10}C_4\left(\frac{1}{2}\right)^{10} + {}^{10}C_5\left(\frac{1}{2}\right)^{10} + {}^{10}C_6\left(\frac{1}{2}\right)^{10}$$

$$= \frac{848}{1024} = \frac{53}{64}$$

Illustration 24 : India and Pakistan play a 5 match test series of hockey, the probability that India wins at least three matches is -

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{5}{16}$

Solution : India win atleast three matches

$$= {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^5(16) = \frac{1}{2}$$

Ans. (A)

Illustration 25 : A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss on more than three occasions is -

(A) $\frac{1}{4}$ (B) $\frac{5}{8}$ (C) $\frac{1}{2}$ (D) none of these

Solution : The man has to win at least 4 times.

$\therefore$ the required probability

$$= {}^7C_4\left(\frac{1}{2}\right)^4\cdot\left(\frac{1}{2}\right)^3 + {}^7C_5\cdot\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6\cdot\frac{1}{2} + {}^7C_7\left(\frac{1}{2}\right)^7$$

$$= ({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7)\cdot\frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2}.$$

Ans. (C)

Do yourself - 8 :

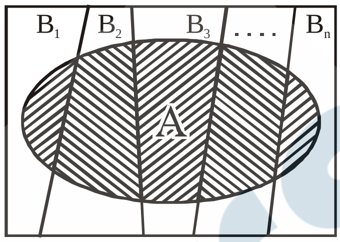
(i) An experiment succeeds twice as often as it fails. Find the probability that in next 6 trials, there will be more than 3 successes.

(ii) Find the probability of getting 4 exactly thrice in 7 throws of a die.

12. BAYE'S THEOREM :

Let an event A of an experiment occurs with its n mutually exclusive & exhaustive events $B_1, B_2, B_3, \ldots\ldots B_n$ & the probabilities $P(A/B_1)$, $P(A/B_2)$....... $P(A/B_n)$ are known, then

$$P(B_i/A) = \frac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$



Explanation :

$A \equiv$ event what we have ; $\quad B_i \equiv$ event what we want & remaining are alternative events.

Now, $P(AB_i) = P(A).\, P(B_i/A) = P(B_i).\, P(A/B_i)$

$$P(B_i/A) = \frac{P(B_i)\,.\,P(A/B_i)}{P(A)} = \frac{P(B_i)\,.\,P(A/B_i)}{\sum_{i=1}^{n} P(AB_i)}$$

$$P(B_i/A) = \frac{P(B_i)\,.\,P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\,.\,P(A/B_i)}$$

Illustration 26 : Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ?

Solution : Let E_1, E_2 and E_3 be the events that boxes I, II and III are chosen, respectively.

Then $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Also, let A be the event that 'the coin drawn is of gold'

Then $P(A|E_1)$ = P(a gold coin from box I) = $\frac{2}{2} = 1$

$P(A|E_2)$ = P(a gold coin from box II) = 0

$P(A|E_3)$ = P(a gold coin from box III) = $\frac{1}{2}$

Now, the probability that the other coin in the box is of gold
= the probability that gold coin is drawn from the box I.
= $P(E_1|A)$
By Baye's theorem, we know that

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)} = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times 0+\frac{1}{3}\times\frac{1}{2}} = \frac{2}{3}$$

Illustration 27 : A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the bag B.

Solution : Let E_1 = The event of ball being drawn from bag A

E_2 = The event of ball being drawn from bag B.

E = The event of ball being red.

Since, both the bags are equally likely to be selected, therefore

$$P(E_1) = P(E_2) = \frac{1}{2} \text{ and } P(E|E_1) = \frac{3}{5} \text{ and } P(E|E_2) = \frac{5}{9}$$

$\therefore$ Required probability

$$P(E_2|E) = \frac{P(E_2)P(E|E_2)}{P(E_1).P(E|E_1) + P(E_2)P(E|E_2)} = \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}} = \frac{25}{52}$$

Do yourself - 9 :

(i) A pack of cards was found to contain only 51 cards. If first 13 cards, which are examined, are all red, then find the probability that the missing card is black.

(ii) A man has 3 coins A, B & C. A is fair coin. B is biased such that the probability of occurring head on it is 2/3. C is also biased with the probability of occurring head as 1/3. If one coin is selected and tossed three times, giving two heads and one tail, find the probability that the chosen coin was A.

13. PROBABILITY THROUGH STATISTICAL (STOCHATIC) TREE DIAGRAM :

These tree diagrams are generally drawn by economist and give a simple approach to solve a problem.

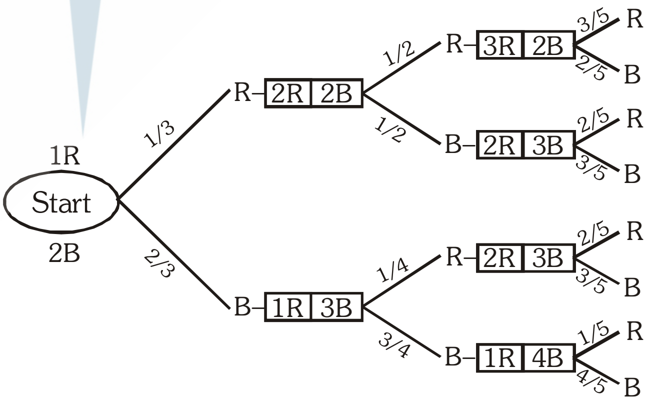
Illustration 28 : A bag initially contains 1 red ball and 2 blue balls. A trial consists of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. Given that three trials are made, draw a tree diagram illustrating the various probabilities. Hence, or otherwise, find the probability that

(a) atleast one blue ball is drawn

(b) exactly one blue ball is drawn

(c) Given that all three balls drawn are of the same colour find the probability that they are all red.

Solution :



Calculations :

$$P(A) = 1 - P(RRR) = 1 - \frac{1}{3}.\frac{1}{2}.\frac{3}{5} = 1 - \frac{1}{10} = \frac{9}{10}$$

$$P(\text{exactly one Blue}) = \frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{2}{3}.\frac{1}{4}.\frac{2}{5} = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5}$$

$$P(C) = P\left(\frac{RRR}{(RRR \cup BBB)}\right) = \frac{P(RRR)}{P(RRR) + P(BBB)} = \frac{\frac{1}{3}.\frac{1}{2}.\frac{3}{5}}{\frac{1}{3}.\frac{1}{2}.\frac{3}{5} + \frac{2}{3}.\frac{3}{4}.\frac{4}{5}} = \frac{\frac{1}{10}}{\frac{1}{10} + \frac{4}{10}} = \frac{1}{5}$$

14. PROBABILITY DISTRIBUTION (Not in JEE) :

(a) A Probability Distribution spells out how a total probability of 1 is distributed over several values of a random variable.

(b) Mean of any probability distribution of a random variable is given by :

$$\mu = \frac{\sum p_i\, x_i}{\sum p_i} = \sum p_i\, x_i \quad (\text{Since } \Sigma\, p_i = 1)$$

(c) Variance of a random variable is given by, $\sigma^2 = \sum (x_i - \mu)^2 . p_i$

$\sigma^2 = \sum p_i\, x_i^2 - \mu^2$ (Note that Standard Deviation (SD) = $+\sqrt{\sigma^2}$)

(d) The probability distribution for a binomial variate 'X' is given by ; P (X= r)= ${}^nC_r\, p^r\, q^{n-r}$ where: p = probability of success in a single trial, q = probability of failure in a single trial and p + q = 1.

(e) Mean of Binomial Probability Distribution (BPD) = np ; variance of BPD = npq.

(f) If p represents a person's chance of success in any venture and 'M' the sum of money which he will receive in case of success, then his expectations or probable value = pM

Illustration 29 : A random variable X has the probability distribution:

X :	1	2	3	4	5	6	7	8
p(X) :	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability $P(E \cup F)$ is -

(1) 0.35 (2) 0.77 (3) 0.87 (4) 0.50

Solution : E = x is a prime number

P(E) = P(2) + P(3) + P(5) + P(7) = 0.62

F = (x < 4), P(F) =P(1) + P(2) + P(3) = 0.50

$\therefore$ $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

= 0.62 + 0.50 – 0.35 = 0.77

Illustration 30 : The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is -

(1) $\frac{128}{256}$ (2) $\frac{219}{256}$ (3) $\frac{37}{256}$ (4) $\frac{28}{256}$

Solution : $\left.\begin{array}{r} np = 4 \\ npq = 2 \end{array}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$$P(X = 2) = {}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 = 28.\frac{1}{2^8} = \frac{28}{256}$$

<u>Miscellaneous Illustrations</u> **:**

Illustration 31 : Three persons A, B, C in order cut a pack of cards, replacing them after each cut, on the condition that the first who cuts a spade shall win a prize; find their respective chances.

[REE 1992]

Solution : Let p be the chance of cutting a spade and q the chance of not cutting a spade from a pack of 52 cards.

Then $p = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{1}{4}$ and $q = 1 - \frac{1}{4} = \frac{3}{4}$

Now A will win a prize if he cuts spade at 1st, 4th, 7th, 10th turns, etc. Note that A will get a second chance if A, B, C all fail to cut a spade once and then A cuts a spade at the 4th turn. Similarly he will cut a spade at the 7th turn when A, B, C fail to cut spade twice, etc.

Hence A's chance of winning the prize $= p + q^3p + q^6p + q^9p + \ldots\ldots = \frac{p}{1-q^3} = \frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^3} = \frac{16}{37}$

Similarly B's chance $= (qp + q^4p + q^7p + \ldots\ldots\ldots) = q(p + q^3p + q^6p + \ldots\ldots\ldots\ldots) = \frac{3}{4}.\frac{16}{37} = \frac{12}{37}$

and C's chance $= \frac{3}{4}$ of B's chance $= \frac{3}{4}.\frac{12}{37} = \frac{9}{37}$

Illustration 32 : (a) If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. **[IIT 1997]**

(b) Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have equal roots. **[REE 1998]**

Solution : (a) If roots of $x^2 + px + q = 0$ are real, then $p^2 - 4q \geq 0$ .... (i)

Both p, q belongs to set S = {1, 2, 3, ... 10} when p = 1, no value of q from S will satisfy (i)

p = 2	q = 1 will satisfy	1 value
p = 3	q = 1, 2	2 value
p = 4	q = 1, 2, 3, 4	4 value
p = 5	q = 1, 2, 3, 4, 5, 6	6 value
p = 6	q = 1, 2, 3, 4, 5, 6, 7, 8, 9	9 value

For p = 7, 8, 9, 10 all the ten values of q will satisfy.

Sum of these selections is 1 + 2 + 4 + 6 + 9 + 10 + 10 + 10 + 10 = 62

But the total number of selections of p and q without any order is 10 × 10 = 100

Hence the required probability is $= \frac{62}{100} = 0.62$

(b) Roots equal $\Rightarrow$ $b^2 - 4ac = 0$

$\therefore \quad \left(\frac{b}{2}\right)^2 = ac \qquad \ldots\ldots \text{(i)}$

Each coefficient is an integer, so we consider the following cases :

$b = 1 \quad \therefore \quad \frac{1}{4} = ac$

No integral values of a and c

$b = 2 \quad 1 = ac \quad \therefore \quad (1, 1)$

$b = 3 \quad 9/2 = ac$

No integral values of a and c

$b = 4 \quad 4 = ac \quad \therefore \quad (1, 4), (2, 2), (4, 1)$

$b = 5 \quad 25/2 = ac$

No integral values of a and c

$b = 6 \quad 9 = ac \quad \therefore \quad (3, 3)$

Thus we have 5 favourable ways for b = 2, 4, 6

Total number of equations is 6.6.6 = 216

$\therefore$ Required probability is $\frac{5}{216}$

Illustration 33 : A set A has n elements. A subset P of A is selected at random. Returning the element of P, the set Q is formed again and then a subset Q is selected from it. Find the probability that P and Q have no common elements. **[IIT 1990]**

Solution : Let P be the empty set, or one element set or two elements set .... or n elements set. Then the set Q will be chosen from amongst the remaining n elements or n – 1 elements or n – 2 elements .... or no elements. The probability of P being an empty set is ${}^nC_0/2^n$, the probability of P being one element set is ${}^nC_1/2^n$ and in general, the probability of P being an r element set is ${}^nC_r/2^n$.

When the set P consisting of r elements is chosen from A, then the probability of choosing the set Q from amongst the remaining n – r elements is $2^{n-r}/2^n$. Hence the probability that P and Q have no common elements is given by

$$\sum_{r=0}^{n} \frac{{}^nC_r}{2^n} \cdot \frac{2^{n-r}}{2^n} = \frac{1}{4^n} \sum_{r=0}^{n} {}^nC_r \cdot 2^{n-r} = \left(\frac{1}{4}\right)^n (1+2)^n = \left(\frac{3}{4}\right)^n \qquad \text{[By binomial theorem]}$$

Illustration 34 : The probabilities of three events A, B and C are P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$ and $P(A \cup B \cup C) \geq 0.85$, find $P(B \cap C)$. **[REE 1996]**

Solution : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + 0.4 - P(A \cap B)$

$\therefore \quad P(A \cap B) = 0.2$

Now $P(A \cup B \cup C) = S_1 - S_2 + S_3 = (0.6 + 0.4 + 0.5) - (0.2 + P(B \cap C) + 0.3) + 0.2$

$= 1.5 - 0.3 - P(B \cap C)$

We know $0.85 \leq P(A \cup B \cup C) \leq 1$

or $\quad 0.85 \leq 1.2 - P(B \cap C) \leq 1$

$\therefore \quad 0.2 \leq P(B \cap C) \leq 0.35$

ANSWERS FOR DO YOURSELF

1 : **(i)** $\{B_1R_1, B_2R_1, B_3R_1, B_1R_2, B_2R_2, B_3R_2, B_1B_2, B_2B_3, B_1B_3, R_1R_2\}$

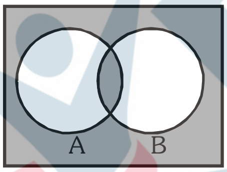
(ii) $\{M_1W_1, M_2W_1, M_1W_2, M_2W_2, M_1W_3, M_2W_3\}$

(iii) {H1, H2, H3, H4, H5, H6, TH,TT}

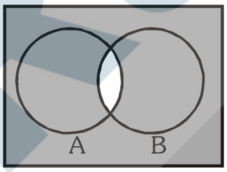
(iv) A

2 : **(i)** $\frac{3}{4}$ **(ii)** $\frac{10}{21}$ **(iii)** 1/10 **(iv)** $\frac{(13)^5 - (12)^5}{(13)^5}$

3 : **(i)** **(a)**



(b)



(ii) A,B,D **(iii)** 11/15

(iv) (a) $\frac{19}{25}$ (b) $\frac{13}{25}$

4 : **(i)** A **(ii)** 1/5525

5 : **(i)** A, B **(ii)** 1/36

6 : **(i)** 2/21 **(ii)** $\frac{1}{n+2}$

7 : **(i)** **(a)** 0.72 **(b)** 0.23

8 : **(i)** $\frac{496}{729}$ **(ii)** ${}^7C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^4$

9 : **(i)** 2/3 **(ii)** 9/25

EXERCISE (O-1)

PART # 1

1. 6 married couples are standing in a room. If 4 people are chosen at random, then the chance that exactly one married couple is among the 4 is-

(A) $\frac{16}{33}$ (B) $\frac{8}{33}$ (C) $\frac{17}{33}$ (D) $\frac{24}{33}$

2. The probability that a positive two digit number selected at random has its tens digit at least three more than its unit digit is -

(A) 14/45 (B) 7/45 (C) 36/45 (D) 1/6

3. A 5 digit number is formed by using the digits 0,1,2,3,4 & 5 without repetition. The probability that the number is divisible by 6 is :

(A) 8% (B) 17% (C) 18% (D) 36%

4. A cube with all six faces coloured is cut into 64 cubical blocks of the same size which are thoroughly mixed. Find the probability that the 2 randomly chosen blocks have 2 coloured faces each -

5. A card is drawn at random from a well shuffled deck of cards. Find the probability that the card is a-

(i) king or a red card (ii) club or a diamond (iii) king or a queen (iv) king or an ace

(v) spade or a club (vi) neither a heart nor a king

6. A bag contain 5 white, 7 black, and 4 red balls, find the chance that three balls drawn at random are all white.

7. If four coins are tossed, Two events A and B are defined as

A : No two consecutive heads occur

B : At least two consecutive heads occur.

Find P(A) and P(B). State whether the events are equally likely, mutually exclusive and exhaustive.

8. Thirteen persons take their places at a round table, Find the odds against two particular persons sitting together.

9. A has 3 shares in a lottery containing 3 prizes and 9 blanks, B has 2 shares in a lottery containing 2 prizes and 6 blanks. Compare their chances of success.

10. Mr. A forgot to write down a very important phone number. All he remembers is that it started with 713 and that the next set of 4 digit involved are 1,7 and 9 with one of these numbers appearing twice. He guesses a phone number and dials randomly. The odds in favour of dialing the correct telephone number, is -

(A) 1 : 35 (B) 1 : 71 (C) 1 : 23 (D) 1 : 36

11. Consider a function $f(x)$ that has zeroes 4 and 9. Given that Mr. A randomly selects a number from the set {–10, –9,–8,......8,9,10}, what is the probability that Mr. A chooses a zero of $f(x^2)$?

12. (a) A fair die is tossed. If the number is odd, find the probability that it is prime.

(b) Three fair coins are tossed. If both heads and tails appear, determine the probability that exactly one head appears.

13. Mr. A lives at origin on the cartesian plane and has his office at (4,5). His friend lives at (2,3) on the same plane. Mr. A can go to his office travelling one block at a time either in the +y or +x direction. If all possible paths are equally likely then the probability that Mr. A passed his friends house is -

(A) 1/2 (B) 10/21 (C) 1/4 (D) 11/21

14. I have 3 normal dice, one red, one blue and one green and I roll all three simultaneously. Let P be the probability that the sum of the numbers on the red and blue dice is equal to the number on the green die. If P is the written in lowest terms as a/b then the value of (a + b) equals -

(A) 79 (B) 77 (C) 61 (D) 57

15. There are three passengers on an airport shuttle bus that makes stops at four different hotels. The probability that all three passengers will be staying at different hotels, is -

(A) $\frac{1}{16}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{3}{4}$

PART # 2

1. In throwing 3 dice, the probability that atleast 2 of the three numbers obtained are same is -

(A) 1/2 (B) 1/3 (C) 4/9 (D) none

2. There are 4 defective items in a lot consisting of 10 items. From this lot we select 5 items at random. The probability that there will be 2 defective items among them is -

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{5}{21}$ (D) $\frac{10}{21}$

3. From a pack of 52 playing cards, face cards and tens are removed and kept aside then a card is drawn at random from the remaining cards. If

A: The event that the card drawn is an ace

H: The event that the card drawn is a heart

S : The event that the card drawn is a spade

then which of the following holds ?

(A) $9P(A) = 4P(H)$ (B) $P(S) = 4P(A\cap H)$

(C) $3P(H)= 4P(A\cup S)$ (D) $P(H)= 12P(A\cap S)$

4. If two of the 64 squares are chosen at random on a chess board, the probability that they have a side in common is -

(A) 1/9 (B) 1/18 (C) 2/7 (D) none

5. Two red counters, three green counters and 4 blue counters are placed in a row in random order. The probability that no two blue counters are adjacent is -

(A) $\frac{7}{99}$ (B) $\frac{7}{198}$ (C) $\frac{5}{42}$ (D) none

6. South African cricket captain lost the toss of a coin 13 times out of 14. The chance of this happening was

(A) $\frac{7}{2^{13}}$ (B) $\frac{1}{2^{13}}$ (C) $\frac{13}{2^{14}}$ (D) $\frac{13}{2^{13}}$

7. There are ten prizes, five A's, three B's and two C's, placed in identical sealed envelopes for the top ten contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8^{th} contestant goes to select the prize, the probability that the remaining three prizes are one A, one B and one C, is

(A) 1/4 (B) 1/3 (C) 1/12 (D) 1/10

8. A coin is tossed and a die is thrown. Find the probability that the outcome will be a head or a number greater than 4.

9. A coin is biased so that heads is three times as likely to appear as tails. Find P(H) and P(T). If such a coin is tossed twice find the probability that head occurs at least once.

10. Given two independent events A, B such that P(A) = 0.3, P(B) = 0.6. Determine

(i) P(A and B) (ii) P(A and not B) (iii) P(not A and B) (iv) P(neither A nor B)
(v) P(A or B)

11. The probabilities that a student will receive A, B, C or D grade are 0.40, 0.35, 0.15 and 0.10 respectively. Find the probability that a student will receive

(i) not an A grade (ii) B or C grade (iii) at most C grade

12. In a single throw of three dice, determine the probability of getting

(i) a total of 5 (ii) a total of atmost 5 (iii) a total of at least 5.

13. A natural number x is randomly selected from the set of first 100 natural numbers. Find the probability that it satisfies the inequality. $x + \frac{100}{x} > 50$

14. 3 students A, B and C are in a swimming race. A and B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins. Assume no two reach the winning point simultaneously.

15. A box contains 7 tickets, numbered from 1 to 7 inclusive. If 3 tickets are drawn from the box without replacement, one at a time, determine the probability that they are alternatively either odd-even-odd or even-odd-even.

16. Let a red die, a blue die, a green die and a white die are rolled once, the dice being fair. The outcomes on the red, blue, green and white die denote the numbers, a, b,c and d respectively. Let E denotes the event that absolute value of (a – 1) (b – 2) (c – 3) (d – 6) = 1, then P(E) is -

(A) $\frac{1}{324}$ (B) $\frac{1}{648}$ (C) $\frac{2}{324}$ (D) $\frac{1}{162}$

17. 5 different marbles are placed in 5 different boxes randomly. Find the probability that exactly two boxes remain empty. Given each box can hold any number of marbles.

18. Let A and B be events such that $P(\bar{A}) = 4/5$, P(B) = 1/3, P(A/B) = 1/6, then :

(a) $P(A \cap B)$
(b) $P(A \cup B)$
(c) P(B/A)
(d) Are A and B independent?

PART # 3

1. Let A & B be two events. Suppose P(A) = 0.4, P(B) = p & $P(A \cup B) = 0.7$. The value of p for which A & B are independent is :

(A) 1/3 (B) 1/4 (C) 1/2 (D) 1/5

2. A pair of numbers is picked up randomly (without replacement) from the set {1, 2, 3, 5, 7, 11, 12, 13, 17, 19}. The probability that the number 11 was picked given that the sum of the numbers was even, is nearly :

(A) 0.1 (B) 0.125 (C) 0.24 (D) 0.18

3. For a biased die the probabilities for the different faces to turn up are given below :

Faces :	1	2	3	4	5	6
Probabilities :	0.10	0.32	0.21	0.15	0.05	0.17

The die is tossed & you are told that either face one or face two has turned up. Then the probability that it is face one is :

(A) 1/6 (B) 1/10 (C) 5/49 (D) 5/21

4. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen has the value non negative is :

(A) 3/16 (B) 6/16 (C) 10/16 (D) 13/16

5. A card is drawn & replaced in an ordinary pack of 52 playing cards. Minimum number of times must a card be drawn so that there is atleast an even chance of drawing a heart, is

(A) 2 (B) 3 (C) 4 (D) more than four

6. A license plate is 3 capital letters (of English alphabets) followed by 3 digits. If all possible license plates are equally likely, the probability that a plate has either a letter palindrome or a digit palindrome (or both), is

(A) $\frac{7}{52}$ (B) $\frac{9}{65}$ (C) $\frac{8}{65}$ (D) none

7. Whenever horses a, b, c race together, their respective probabilities of winning the race are 0.3, 0.5 and 0.2 respectively. If they race three times the probability that "the same horse wins all the three races" and the probability that a, b, c each wins one race, are respectively (Assume no dead heat)

(A) $\frac{8}{50}$; $\frac{9}{50}$ (B) $\frac{16}{100}$, $\frac{3}{100}$ (C) $\frac{12}{50}$; $\frac{15}{50}$ (D) $\frac{10}{50}$; $\frac{8}{50}$

8. Two cubes have their faces painted either red or blue. The first cube has five red faces and one blue face. When the two cubes are rolled simultaneously, the probability that the two top faces show the same colour is 1/2. Number of red faces on the second cube, is

(A) 1 (B) 2 (C) 3 (D) 4

9. A committee of three persons is to be randomly selected from a group of three men and two women and the chair person will be randomly selected from the committee. The probability that the committee will have exactly two women and one man, and that the chair person will be a woman, is/are

(A) 1/5 (B) 8/15 (C) 2/3 (D) 3/10

10. An urn contains 3 red balls and n white balls.

Mr. A draws two balls together from the urn. The probability that they have the same colour is 1/2.

Mr. B draws one ball from the urn, notes its colour and replaces it. He then draws a second ball from the urn and finds that both balls have the same colour is, 5/8. The possible value of n is

(A) 9 (B) 6 (C) 5 (D) 1

11. The probability that an automobile will be stolen and found within one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is

(A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6

12. A box contains 100 tickets numbered 1, 2, 3,.... ,100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5, with probability

(A) $\frac{1}{9}$ (B) $\frac{2}{11}$ (C) $\frac{3}{19}$ (D) none

13. Two boys A and B find the jumble of n ropes lying on the floor. Each takes hold of one loose end randomly. If the probability that they are both holding the same rope is $\frac{1}{101}$ then the number of ropes is equal to

(A) 101 (B) 100 (C) 51 (D) 50

[REASONING TYPE]

14. For children A,B,C and D have 1,3,5 and 7 identical unbiased dice respectively and roll them with the condition that one who obtains an even score, wins. They keep playing till some one or the other wins.

Statement-1: All the four children are equally likely to win provided they roll their dice simultaneously.

Statement-2: The child A is most probable to win the game if they roll their dice in order of A,B,C and D respectively.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

15. In one day test match between India and Australia the umpire continues tossing a fair coin until the two consecutive throws either H T or T T are obtained for the first time. If it is H T, India wins and if it is T T, Australia wins.

Statement-1: Both India and Australia have equal probability of winning the toss.

Statement-2: If a coin is tossed twice then the events HT or TT are equiprobable.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

PART # 4

1. If E & F are events with $P(E) \le P(F)$ & $P(E \cap F) > 0$, then :

(A) occurrence of E $\Rightarrow$ occurrence of F

(B) occurrence of F $\Rightarrow$ occurrence of E

(C) non – occurrence of E $\Rightarrow$ non – occurrence of F

(D) none of the above implications holds.

2. Events A and C are independent. If the probabilities relating A, B and C are P (A) = 1/5; P (B) = 1/6 ; $P(A \cap C) = 1/20$; $P(B \cup C) = 3/8$ then

(A) events B and C are independent

(B) events B and C are mutually exclusive

(C) events B and C are neither independent nor mutually exclusive

(D) events A and C are equiprobable

3. An unbiased cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is

(A) $\frac{16}{216}$ (B) $\frac{50}{216}$ (C) $\frac{60}{216}$ (D) none

4. A bag contains 3 R & 3 G balls and a person draws out 3 at random. He then drops 3 blue balls into the bag & again draws out 3 at random. The chance that the 3 later balls being all of different colours is

(A) 15% (B) 20% (C) 27% (D) 40%

5. A biased coin with probability P, $0 < P < 1$, of heads, is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then the value of P is

(A) 1/4 (B) 1/6 (C) 1/3 (D) 1/2

6. Two numbers a and b are selected from the set of natural number then the probability that $a^2 + b^2$ is divisible by 5 is

(A) $\frac{9}{25}$ (B) $\frac{7}{18}$ (C) $\frac{11}{36}$ (D) $\frac{17}{81}$

7. When a missile is fired from a ship, the probability that it is intercepted is 1/3. The probability that the missile hits the target, given that it is not intercepted is 3/4. If three missiles are fired independently from the ship, the probability that all three hits the target, is

(A) 1/12 (B) 1/8 (C) 3/8 (D) 3/4

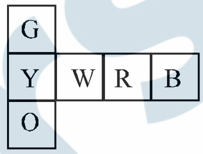
8. An urn contains 10 balls coloured either black or red. When selecting two balls from the urn at random, the probability that a ball of each colour is selected is $8/15$. Assuming that the urn contains more black balls than red balls, the probability that at least one black ball is selected, when selecting two balls, is

(A) $\frac{18}{45}$ (B) $\frac{30}{45}$ (C) $\frac{39}{45}$ (D) $\frac{41}{45}$

9. An unbiased die with numbers 1, 2, 3, 4, 6 and 8 on its six faces is rolled. After this roll if an odd number appears on the top face, all odd numbers on the die are doubled. If an even number appears on the top face, all the even numbers are halved. If the given die changes in this way then the probability that the face 2 will appear on the second roll is -

(A) 2/18 (B) 3/18 (C) 2/9 (D) 5/18

10. A butterfly randomly lands on one of the six squares of the T-shaped figure shown and then randomly moves to an adjacent square. The probability that the butterfly ends up on the R square is



(A) 1/4 (B) 1/3
(C) 2/3 (D) 1/6

11. A fair coin is tossed a large number of times. Assuming the tosses are independent which one of the following statement, is True?

(A) Once the number of flips is large enough, the number of heads will always be exactly half of the total number of tosses. For example, after 10,000 tosses one should have exactly 5,000 heads.

(B) The proportion of heads will be about 1/2 and this proportion will tend to get closer to 1/2 as the number of tosses increases

(C) As the number of tosses increases, any long run of heads will be balanced by a corresponding run of tails so that the overall proportion of heads is exactly 1/2

(D) All of the above

12. The number 'a' is randomly selected from the set {0, 1, 2, 3, ... 98, 99}. The number 'b' is selected from the same set. Probability that the number $3^a + 7^b$ has a digit equal to 8 at the units place, is

(A) $\frac{1}{16}$ (B) $\frac{2}{16}$ (C) $\frac{4}{16}$ (D) $\frac{3}{16}$

PART # 5

1. An examination consists of 8 questions in each of which one of the 5 alternatives is the correct one. On the assumption that a candidate who has done no preparatory work chooses for each question any one of the five alternatives with equal probability, the probability that he gets more than one correct answer is equal to

(A) $(0.8)^8$ (B) $3(0.8)^8$ (C) $1-(0.8)^8$ (D) $1-3(0.8)^8$

2. An ant is situated at the vertex A of the triangle ABC. Every movement of the ant consists of moving to one of other two adjacent vertices from the vertex where it is situated. The probability of going to any of the other two adjacent vertices of the triangle is equal. The probability that at the end of the fourth movement the ant will be back to the vertex A, is :

(A) 4/16 (B) 6/16 (C) 7/16 (D) 8/16

3. A key to room number C_3 is dropped into a jar with five other keys, and the jar is throughly mixed. If keys are randomly drawn from the jar without replacement until the key to room C_3 is chosen, then what are the odds in favour that the key to room C_3 will be obtained on the 2^{nd} try?

(A) 1 : 4 (B) 1 : 5 (C) 1 : 6 (D) 5 : 6

4. Lot A consists of 3G and 2D articles. Lot B consists of 4G and 1D article. A new lot C is formed by taking 3 articles from A and 2 from B. The probability that an article chosen at random from C is defective, is

(A) 1/3 (B) 2/5 (C) 8/25 (D) none

5. Mr. A and Mr. B each have a bag that contains one ball of each of the colours blue, green, orange, red and violet. 'A' randomly selects one ball from his bag and puts it into B's bag. 'B' then randomly selects one ball from his bag and puts it into A's bag. The probability that after this process the contents of the two bags are the same, is

(A) 1/6 (B) 1/5 (C) 1/3 (D) 1/2

6. An instrument consists of two units. Each unit must function for the instrument to operate. The reliability of the first unit is 0.9 & that of the second unit is 0.8. The instrument is tested & fails. The probability that "only the first unit failed & the second unit is sound" is :

(A) 1/7 (B) 2/7 (C) 3/7 (D) 4/7

7. A box contains 10 tickets numbered from 1 to 10. Two tickets are drawn one by one without replacement. The probability that the "absolute value of difference between the first drawn ticket number and the second is not less than 4" is

(A) $\frac{7}{30}$ (B) $\frac{7}{15}$ (C) $\frac{11}{30}$ (D) $\frac{10}{30}$

8. A purse contains 2 six sided dice. One is a normal fair die, while the other has 2 ones, 2 threes, and 2 fives. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3. The probability that a fair die was picked up, is

(A) 1/7 (B) 1/4 (C) 1/6 (D) 1/24

9. On a Saturday night 20% of all drivers is U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001. If a car on a Saturday night smashed into a tree, the probability that the driver was under the influence of alcohol, is -

(A) 3/7 (B) 4/7 (C) 5/7 (D) 6/7

10. A box has four dice in it. Three of them are fair dice but the fourth one has the number five on all of its faces. A die is chosen at random from the box and is rolled three times and shows up the face five on all the three occasions. The chance that the die chosen was a rigged die, is

(A) $\frac{216}{217}$ (B) $\frac{215}{219}$ (C) $\frac{216}{219}$ (D) none

11. On a normal standard die one of the 21 dots from any one of the six faces is removed at random with each dot equally likely to be chosen. The die is then rolled. The probability that the top face has an odd number of dots is

(A) $\frac{5}{11}$ (B) $\frac{5}{12}$ (C) $\frac{11}{21}$ (D) $\frac{6}{11}$

Paragraph for question nos. 12 to 14

A JEE aspirant estimates that she will be successful with an 80 percent chance if she studies 10 hours per day, with a 60 percent chance if she studies 7 hours per day and with a 40 percent chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively

12. The chance she will be successful, is

(A) 0.28 (B) 0.38 (C) 0.48 (D) 0.58

13. Given that she is successful, the chance she studied for 4 hours, is

(A) $\frac{6}{12}$ (B) $\frac{7}{12}$ (C) $\frac{8}{12}$ (D) $\frac{9}{12}$

14. Given that she does not achieve success, the chance she studied for 4 hour, is

(A) $\frac{18}{26}$ (B) $\frac{19}{26}$ (C) $\frac{20}{26}$ (D) $\frac{21}{26}$

[REASONING TYPE]

15. A fair coin is tossed 3 times consider the events

A : first toss is head

B : second toss is head

C : exactly two consecutive heads or exactly two consecutive tails.

Statement-1: A, B, C are independent events.

Statement-2: A, B, C are pairwise independent.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

PART # 6

1. A bowl has 6 red marbles and 3 green marbles. The probability that a blind folded person will draw a red marble on the second draw from the bowl without replacing the marble from the first draw, is

(A) 2/3 (B) 1/4 (C) 5/12 (D) 5/8

2. The probability that a radar will detect an object in one cycle is p. The probability that the object will be detected in n cycles is :

(A) $1 - p^n$ (B) $1 - (1 - p)^n$ (C) p^n (D) $p(1 - p)^{n-1}$

3. In a certain factory, machines A, B and C produce bolts. Of their production, machines A, B, and C produce 2%, 1% and 3% defective bolts respectively. Machine A produces 35% of the total output of bolts, machine B produces 25% and machine C produces 40%. A bolts is chosen at random from the factory's production and is found to be defective. The probability it was produced on machine C, is

(A) $\frac{6}{11}$ (B) $\frac{23}{45}$ (C) $\frac{24}{43}$ (D) $\frac{3}{11}$

4. Three numbers are chosen at random without replacement from {1, 2, 3,...... , 10}. The probability that the minimum of the chosen numbers is 3 or their maximum is 7 is

(A) 1/2 (B) 1/3 (C) 1/4 (D) 11/40

5. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then the probabilities

(i) neither bus will be late on a particular day and

(ii) bus A is late given that bus B is late, are respectively

(A) 2/25 and 12/28 (B) 18/25 and 22/28

(C) 7/10 and 18/28 (D) 12/25 and 2/28

6. If at least one child in a family with 3 children is a boy then the probability that exactly 2 of the children are boys, is

(A) 3/7 (B) 4/7 (C) 1/3 (D) 3/8

7. From an urn containing six balls, 3 white and 3 black ones, a person selects at random an even number of balls (all the different ways of drawing an even number of balls are considered equally probable, irrespective of their number). Then the probability that there will be the same number of black and white balls among them

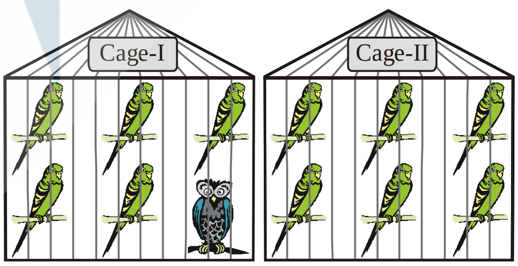
(A) 4/5 (B) 11/15 (C) 11/30 (D) 2/5

8. There are three main political parties namely 1, 2, 3. If in the adjoining table p_{ij} , (i, j=1, 2, 3) denote the probability that party j wins the general elections contested when party i is in the power. What is the probability that the party 2 will be in power after the next two elections, given that the party 1 is in the power?

$P_{11}=0.7$	$P_{12}=0.2$	$P_{13}=0.1$
$P_{21}=0.5$	$P_{22}=0.3$	$P_{23}=0.2$
$P_{31}=0.3$	$P_{32}=0.4$	$P_{33}=0.3$

(A) 0.27 (B) 0.24 (C) 0.14 (D) 0.06

9. Shalu bought two cages of birds : Cage-I contains 5 parrots and 1 owl, and Cage-II contains 6 parrots, as shown



Birds like to fly

One day Shalu forgot to lock both cages and two birds flew from Cage-I to Cage-II. Then two birds flew back from Cage-II to Cage-I. Assume that all birds have equal chance of flying, the probability that the Owl is still in Cage-I, is

(A) 1/6 (B) 1/3 (C) 2/3 (D) 3/4

10. Suppose families always have one, two or three children, with probabilities $\frac{1}{4}, \frac{1}{2}$ and $\frac{1}{4}$ respectively. Assume everyone eventually gets married and has children, the probability of a couple having exactly four grandchildren is

(A) $\frac{27}{128}$ (B) $\frac{37}{128}$ (C) $\frac{25}{128}$ (D) $\frac{20}{128}$

11. Miss C has either Tea or Coffee at morning break. If she has tea one morning, the probability she has tea the next morning is 0.4. If she has coffee one morning, the probability she has coffee next morning is 0.3. Suppose she has coffee on a Monday morning. The probability that she has tea on the following Wednesday morning is

(A) 0.46 (B) 0.49 (C) 0.51 (D) 0.61

12. In a maths paper there are 3 sections A, B & C. Section A is compulsory. Out of sections B & C a student has to attempt any one. Passing in the paper means passing in A & passing in B or C. The probability of the student passing in A, B & C are p, q & 1/2 respectively. If the probability that the student is successful is 1/2 then :

(A) p = q = 1 (B) p = q = 1/2 (C) p = 1, q = 0 (D) p = 1, q = 1/2

[REASONING TYPE]

13. From a well shuffled pack of 52 playing cards a card is drawn at random. Two events A and B are defined as

A : Red card is drawn.

B : Card drawn is either a Diamond or Heart

Statement-1: P(A + B) = P(AB)

Statement-2: $A \subseteq B$ and $B \subseteq A$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

EXERCISE (O-2)

[STRAIGHT OBJECTIVE TYPE]

1. n different books ($n \geq 3$) are put at random in a shelf. Among these books there is a particular book 'A' and a particular book B. The probability that there are exactly 'r' books between A and B is -

(A) $\frac{2}{n(n-1)}$ (B) $\frac{2(n-r-1)}{n(n-1)}$ (C) $\frac{2(n-r-2)}{n(n-1)}$ (D) $\frac{(n-r)}{n(n-1)}$

2. Of all the mappings that can be defined from the set A : {1, 2, 3, 4} → B(5, 6, 7, 8, 9}, a mapping is randomly selected. The chance that the selected mapping is strictly monotonic, is

(A) $\frac{1}{125}$ (B) $\frac{2}{125}$ (C) $\frac{5}{4096}$ (D) $\frac{5}{2048}$

3. A fair die is thrown 3 times. The chance that sum of three numbers appearing on the die is less than 11, is equal to -

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{5}{8}$

4. One bag contains 3 white & 2 black balls, and another contains 2 white & 3 black balls. A ball is drawn from the second bag & placed in the first, then a ball is drawn from the first bag & placed in the second. When the pair of the operations is repeated, the probability that the first bag will contain 5 white balls is:

(A) 1/25 (B) 1/125 (C) 1/225 (D) 2/15

5. If a, b and c are three numbers (not necessarily different) chosen randomly and with replacement from the set {1, 2, 3, 4, 5}, the probability that (ab + c) is even, is

(A) $\frac{35}{125}$ (B) $\frac{59}{125}$ (C) $\frac{64}{125}$ (D) $\frac{75}{125}$

6. A purse contains 100 coins of unknown value, a coin drawn at random is found to be a rupee. The chance that it is the only rupee in the purse, is (Assume all numbers of rupee coins in the purse to be equally likely.)

(A) $\frac{1}{5050}$ (B) $\frac{2}{5151}$ (C) $\frac{1}{4950}$ (D) $\frac{2}{4950}$

7. Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1. When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2. Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says : "French". The probability that the wine he tasted was Californian, is nearly equal to

(A) 0.14 (B) 0.24 (C) 0.34 (D) 0.44

8. Sixteen players $s_1, s_2, \ldots\ldots, s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 & s_2 is among the eight winners" is

(A) $\frac{4}{15}$ (B) $\frac{7}{15}$ (C) $\frac{8}{15}$ (D) $\frac{9}{15}$

9. A multiple choice test question has five alternative answers, of which only one is correct. If a student has done his home work, then he is sure to identify the correct answer; otherwise, he chooses an answer at random.

Let E : denotes the event that a student does his home work with P(E) = p and

F : denotes the event that he answer the question correctly.

(a) If p = 0.75 the value of P(E/F) equals

(A) $\frac{8}{16}$ (B) $\frac{10}{16}$ (C) $\frac{12}{16}$ (D) $\frac{15}{16}$

(b) The relation $P(E/F) \geq P(E)$ holds good for

(A) all values of p in [0, 1]
(B) all values of p in (0, 1) only
(C) all values of p in [0.5, 1] only
(D) no value of p.

(c) Suppose that each question has *n* alternative answers of which only one is correct, and p is fixed but not equal to 0 or 1 then P(E/F)

(A) decreases as *n* increases for all $p \in (0, 1)$
(B) increases as *n* increases for all $p \in (0, 1)$
(C) remains constant for all $p \in (0, 1)$
(D) decreases if $p \in (0, 0.5)$ and increases if $p \in (0.5, 1)$ as *n* increases

[MULTIPLE OBJECTIVE TYPE]

10. Which of the following statement(s) is/are correct?

(A) 3 coins are tossed once. Two of them atleast must land the same way. No matter whether they land heads or tails, the third coin is equally likely to land either the same way or oppositely. So, the chance that all the three coins land the same way is 1/2.

(B) Let $0 < P(B) < 1$ and $P(A/B) = P(A/B^c)$ then A and B are independent.

(C) Suppose an urn contains 'w' white and 'b' black balls and a ball is drawn from it and is replaced along with 'd' additional balls of the same colour. Now a second ball is drawn from it. The probability that the second drawn ball is white is independent of the value of 'd'.

(D) A, B, C simultaneously satisfy $P(ABC) = P(A)\cdot P(B)\cdot P(C)$ and $P(AB\overline{C}) = P(A)\cdot P(B)\cdot P(\overline{C})$ and $P(A\overline{B}C) = P(A)\cdot P(\overline{B})\cdot P(C)$ and $P(\overline{A}BC) = P(\overline{A})\cdot P(B)\cdot P(C)$ then A, B, C are independent.

11. Identify the correct statement :

(A) If the probability that a computer will fail during the first hour of operation is 0.01, then if we turn on 100 computers, exactly one will fail in the first hour of operation.

(B) A man has ten keys only one of which fits the lock. He tries them in a door one by one discarding the one he has tried. The probability that fifth key fits the lock is 1/10.

(C) Given the events A and B in a sample space. If P(A) = 1, then A and B are independent.

(D) When a fair six sided die is tossed on a table top, the bottom face can not be seen. The probability that the product of the numbers on the five faces that can be seen is divisible by 6 is one.

12. Two whole numbers are randomly selected and multiplied. Consider two events E_1 and E_2 defined as

E_1 : Their product is divisible by 5

E_2 : Unit's place in their product is 5.

Which of the following statement(s) is/are correct?

(A) E_1 is twice as likely to occur as E_2.
(B) E_1 and E_2 are disjoint
(C) $P(E_2/E_1) = 1/4$
(D) $P(E_1/E_2) = 1$

13. A boy has a collection of blue and green marbles. The number of blue marbles belong to the sets {2, 3, 4, ... 13}. If two marbles are chosen simultaneously and at random from his collection, then the probability that they have different colour is $1/2$. Possible number of blue marbles is :

(A) 2 (B) 3 (C) 6 (D) 10

14. If A & B are two events such that $P(B) \neq 1$, B^C denotes the event complementary to B, then

(A) $P\left(A/B^C\right) = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

(B) $P(A \cap B) \geq P(A) + P(B) - 1$

(C) $P(A) > < P(A/B)$ according as $P\left(A/B^C\right) > < P(A)$

(D) $P\left(A/B^C\right) + P\left(A^C/B^C\right) = 1$

15. For $P(A) = \dfrac{3}{8}$; $P(B) = \dfrac{1}{2}$; $P(A \cup B) = \dfrac{5}{8}$ which of the following do/does hold good?

(A) $P\left(A^c/B\right) = 2P\left(A/B^c\right)$ (B) $P(B) = P(A/B)$

(C) $15P\left(A^c/B^c\right) = 8P\left(B/A^c\right)$ (D) $P\left(A/B^c\right) = P(A \cap B)$

16. If E_1 and E_2 are two events such that $P(E_1) = 1/4$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2) = 1/4$

(A) then E_1 and E_2 are independent

(B) E_1 and E_2 are exhaustive

(C) E_2 is twice as likely to occur as E_1

(D) Probabilities of the events $E_1 \cap E_2$, E_1 and E_2 are in G.P.

17. Two events A and B are such that the probability that at least one of them occurs is 5/6 and both of them occurring simultaneously is 1/3. If the probability of not occurrence of B is 1/2 then

(A) A and B are equally likely (B) A and B are independent

(C) $P(A/B) = 2/3$ (D) $3P(A) = 4P(B)$

18. The probabilities of events, $A \cap B$, A, B & $A \cup B$ are respectively in A.P. with probability of second term equal to the common difference. Therefore the events A and B are

(A) mutually exclusive (B) independent

(C) such that one of them must occur (D) such that one is twice as likely as the other

19. A box contains 11 tickets numbered from 1 to 11. Six tickets are drawn simultaneously at random. Let E_1 denotes the event that the sum of the numbers on the tickets drawn is even and E_2 denotes the event that the sum of the numbers on the tickets drawn is odd. Which of the following hold good?

(A) E_1 and E_2 are equally likely (B) E_1 and E_2 are exhaustive

(C) $P(E_2) > P(E_1)$ (D) $P(E_1/E_2) = P(E_2/E_1)$

20. If $\overline{E}$ & $\overline{F}$ are the complementary events of events E & F respectively & if $0 < P(F) < 1$, then :

(A) $P(E|F) + P(\overline{E}|F) = 1$ (B) $P(E|F) + P(E|\overline{F}) = 1$

(C) $P(\overline{E}|F) + P(E|\overline{F}) = 1$ (D) $P(E|\overline{F}) + P(\overline{E}|\overline{F}) = 1$

21. Probability of **n** heads in **2n** tosses of a fair coin can be given by

(A) $\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right)$ (B) $\prod_{r=1}^{n}\left(\frac{n+r}{2r}\right)$ (C) $\sum_{r=0}^{n}\left(\frac{{}^nC_r}{2^n}\right)^2$ (D) $\dfrac{\sum_{r=0}^{n}\left({}^nC_r\right)^2}{\left(\sum_{r=0}^{n}{}^nC_r\right)^2}$

22. Which of the following statements is/are True?

(A) A fair coin is tossed *n* times where *n* is a positive integer. The probability that n^{th} toss results in head is 1/2.

(B) The conditional probability that the n^{th} toss results in head given that first $(n-1)$ tosses results in head is $1/2^n$

(C) Let E and F be the events such that F is neither impossible nor sure. If $P(E/F) > P(E)$ then $P(E/F^c) > P(E)$

(D) If A, B and C are independent then the events $(A \cup B)$ and C are independent.

[MATRIX MATCH TYPE]

23.

	Column-I		Column-II
(A)	Two different numbers are taken from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The probability that their sum and positive difference, are both multiple of 4, is $x/55$ then *x* equals	(P)	4
		(Q)	6
(B)	There are two red, two blue, two white and certain number (greater than 0) of green socks in a drawer. If two socks are taken at random from the drawer without replacement, the probability that they are of the same colour is 1/5 then the number of green socks are	(R)	8
(C)	A drawer contains a mixture of red socks and blue socks, atmost 17 in all. It so happens that when two socks are selected randomly without replacement, there is a probability of exactly $1/2$ that both are red or both are blue. The largest possible number of red socks in the drawer that is consistent with this data, is	(S)	10

EXERCISE (S-1)

1. In a box, there are 8 alphabets cards with the letters: S, S, A,A,A,H,H,H. Find the probability that the word 'ASH' will form if :

(i) the three cards are drawn one by one & placed on the table in the same other that they are drawn.

(ii) the three cards are drawn simultaneously.

2. There are 2 groups of subjects one of which consists of 5 science subjects & 3 engg. subjects & other consists of 3 science & 5 engg. subjects. An unbiased die is cast . If the number 3 or 5 turns up a subject is selected at random from first group, otherwise the subject is selected from 2^{nd} group. Find the probability that an engg. subject is selected.

3. A pair of fair dice is tossed. Find the probability that the maximum of the two numbers is greater than 4.

4. In a given race, the odds in favour of four horses A, B, C & D are 1 : 3, 1 : 4, 1 : 5 and 1 : 6 respectively. Assuming that a dead heat is impossible, find the chance that one of them wins the race.

5. A covered basket of flowers has some lilies and roses. In search of rose, Sweety and Shweta alternately pick up a flower from the basket but puts it back if it is not a rose. Sweety is 3 times more likely to be the first one to pick a rose. If sweety begin this 'rose hunt' and if there are 60 lilies in the basket, find the number of roses in the basket.

6. A certain drug, manufactured by a Company is tested chemically for its toxic nature. Let the event "THE DRUG IS TOXIC" be denoted by H and the event "THE CHEMICAL TEST REVEALS THAT THE DRUG IS TOXIC" be denoted by S. Let P(H) = a, P(S/H) = $P(\bar{S}/\bar{H})$ = 1 – a. Then show that the probability that the drug is not toxic given that the chemical test reveals that it is toxic' is free from 'a'.

7. Players A and B alternately toss a biased coin, with A going first. A wins if A tosses a Tail before B tosses a Head; otherwise B wins. If the probability of a head is p, find the value of p for which the game is fair to both players.

8. The entries in a two-by-two determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ are integers that are chosen randomly and independently, and , for each entry, the probability that the entry is odd is p. If the probability that the value of the determinant is even is 1/2, then find the value of p.

9. There are 4 urns. The first urn contains 1 white & 1 black ball, the second urn contains 2 white & 3 black balls, the third urn contains 3 white & 5 black balls & the fourth urn contains 4 white & 7 black balls. The selection of each urn is not equally likely. The probability of selecting i^{th} urn is $\frac{i^2+1}{34}$ (i = 1,2,3,4). If we randomly select one of the urns & draw a ball, then the probability of ball being white is p/q where p and q $\in$ N are in their lowest form. Find (p+q).

10. A room has three electric lamps. From a collection of 10 electric bulbs of which 6 are good 3 are selected at random & put in the lamps. Find the probability that the room is lighted.

11. Find the minimum number of tosses of a pair of dice so that the probability of getting the sum of the digits on the dice equal to 7 on at least one toss is greater than 0.95.
($\log_{10}2$ = 0.3010; $\log_{10}3$ = 0.4771)

12. The probability that a person will get an electric contract is $2/5$ and the probability that he will not get plumbing contract is $4/7$. If the probability of getting at least one contract is $2/3$, what is the probability that he will get both?

13. Five horses compete in a race. John picks two horses at random and bets on them. Find the probability that John picked the winner. Assume no dead heat.

14. There are 6 red balls and 6 green balls in a bag. Five balls are drawn out at random and placed in a red box. The remaining seven balls are put in a green box. If the probability that the number of red balls in the green box plus the number of green balls in the red box is not a prime number, is $\frac{p}{q}$ where p and q are relatively prime, then find the value of $(p + q)$

15. A lot contains 50 defective & 50 non defective bulbs . Two bulbs are drawn at random, one at a time, with replacement . The events A, B, C are defined as :

A = { the first bulb is defective}; B = { the second bulb is non defective}

C = { the two bulbs are both defective or both non defective}

Determine whether (i) A,B,C are pair wise independent (ii) A,B,C are independent

16. An unbiased normal coin is tossed 'n' times

Let E_1 : event that both Heads and Tails are present in 'n' tosses.

E_2 : event that the coin shows up Heads atmost once.

Find the value of 'n' for which E_1 and E_2 are independent.

17. A bomber wants to destroy a bridge . Two bombs are sufficient to destroy it . If four bombs are dropped, what is the probability that it is destroyed, if the chance of a bomb hitting the target is 0.4.

18. The chance of one event happening is the square of the chance of a 2^{nd} event, but odds against the first are the cubes of the odds against the 2^{nd} . Find the chances of each (assume that both events are neither sure nor impossible).

19. A bag contains N balls, some of which are white, the others are black, white being more in number than back. Two balls are drawn at random from the bag, without replacement. It is found that the probability that the two balls are of the same colour is the same as the probability that they are of different colour. It is given that 180 < N < 220. If K denotes the number of white balls, find the exact value of (K + N).

20. An aircraft gun can take a maximum of four shots at an enemy's plane moving away from it. The probability of hitting the plane at first, second, third & fourth shots are 0.4, 0.3, 0.2 & 0.1 respectively. What is the probability that the gun hits the plane.

21. In a batch of 10 articles, 4 articles are defective. 6 articles are taken from the batch for inspection. If more than 2 articles in this batch are defective, the whole batch is rejected Find the probability that the batch will be rejected.

22. An author writes a good book with a probability of 1/2. If it is good it is published with a probability of 2/3. If it is not, it is published with a probability of 1/4. Find the probability that he will get atleast one book published if he writes two.

23. A uniform unbiased die is constructed in the shape of a regular tetrahedron with faces numbered 2, 2, 3 and 4 and the score is taken from the face on which the die lands. If two such dice are thrown together, find the probability of scoring.

(i) exactly 6 on each of 3 successive throws.

(ii) more than 4 on at least one of the three successive throws.

24. A biased coin which comes up heads three times as often as tails is tossed. If it shows heads, a chip is drawn from urn-I which contains 2 white chips and 5 red chips. If the coin comes up tail, a chip is drawn from urn-II which contains 7 white and 4 red chips. Given that a red chip was drawn, what is the probability that the coin came up heads ?

25. Each of the 'n' passengers sitting in a bus may get down from it at the next stop with probability p. Moreover, at the next stop either no passenger or exactly one passenger boards the bus. The probability of no passenger boarding the bus at the next stop being $p_{o.}$ Find the probability that when the bus continuous on its way after the stop, there will again be 'n' passengers in the bus.

26. A normal coin is continued tossing unless a head is obtained for the first time. Find the probability that

(a) number of tosses needed are atmost 3.

(b) number of tosses are even.

27. Before a race the chance of three runners, A, B, C were estimated to be proportional to 5, 3, 2, but during the race A meets with an accident which reduces his chance to 1/3. What are the respective chance of B and C now?

28. A is one of the 6 horses entered for a race, and is to be ridden by one of two jockeys B or C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win; if C rides A, his chance is trebled, what are the odds against his winning?

29. A real estate man has eight master keys to open several new houses. Only one master key will open a given house. If 40% of these homes are usually left unlocked, find the probability that the real estate man can get into a specific home if he selects three master keys at random.

30. A, B are two inaccurate arithmeticians whose chance of solving a given question correctly are (1/8) and (1/12) respectively. They solve a problem and obtained the same result. If it is 1000 to 1 against their making the same mistake, find the chance that the result is correct.

31. During a power blackout, 100 persons are arrested on suspect of looting. Each is given a polygraph test. From past experience it is known that the polygraph is 90% reliable when administered to a guilty person and 98% reliable when given to some one who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrong doing. If the probability that a given suspect is innocent given that the polygraph says he is guilty is a/b where a and b are relatively prime, find the value of (a + b).

EXERCISE (S-2)

1. N fair coins are flipped once. The probability that at most 2 of the coins show up as heads is $\frac{1}{2}$. Find the value of N.

2. To pass a test a child has to perform successfully in two consecutive tasks, one easy and one hard task. The easy task he can perform successfully with probability 'e' and the hard task he can perform successfully with probability 'h', where h < e. He is allowed 3 attempts, either in the order (Easy, Hard, Easy) (option A) or in the order (Hard, Easy, Hard)(option B) whatever may be the order, he must be successful twice in a row. Assuming that his attempts are independent, in what order he choses to take the tasks, in order to maximise his probability of passing the test.

3. A box contains three coins two of them are fair and one two – headed. A coin is selected at random and tossed. If the head appears the coin is tossed again, if a tail appears, then another coin is selected from the remaining coins and tossed.

(i) Find the probability that head appears twice.

(ii) If the same coin is tossed twice, find the probability that it is two headed coin.

(iii) Find the probability that tail appears twice.

4. Eight players P_1 , P_2 , P_3 ,..........P_8 play a knock out tournament. It is known that whenever the players P_i and P_j play , the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the players P_4 reaches the final ?

5. A doctor is called to see a sick child. The doctor knows (prior to the visit) that 90% of the sick children in that neighbourhood are sick with the flu, denoted by F, while 10% are sick with the measles, denoted by M.

A well known symptom of measles is rash, denoted by R. The probability of having a rash for a child sick with the measles is 0.95. However, occasionally children with the flu also develop a rash, with conditional probability 0.08.

Upon examination the child, the doctor finds a rash. What is the probability that the child has the measles ?

If the probability can be expressed in the form of p/q where p,q $\in$ N and are in their lowest form, find (p + q).

6. A permutation of 5 digits from the set {1,2,3,4,5} where each digit is used exactly once, is chosen randomly. Let $\frac{p}{q}$ expressed as rational in lowest form be the probability that the chosen permutation changes from increasing to decreasing, or decreasing to increasing at most once e.g. the strings like 1 2 3 4 5, 5 4 3 2 1, 1 2 5 4 3 and 5 3 2 1 4 are acceptable but strings like 1 3 2 4 5 or 5 3 2 4 1 are not, find (p + q).

7. (a) Two natural numbers x and y are chosen at random. Find the probability that $x^2 + y^2$ is divisible by 10.

(b) Two numbers x & y are chosen at random from the set {1,2,3,4,....3n}. Find the probability that $x^2 - y^2$ is divisible by 3.

8. A hotel packed breakfast for each of the three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese and fruit rolls. The preparer wrapped each of the nine rolls and once wrapped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. If the probability that each guest got one roll of each type is m/n where m and n are relatively prime integers, find the value of (m + n).

9. A coin has probability 'p' of showing head when tossed. It is tossed 'n' times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that,

$$p_1 = 1,\ p_2 = 1 - p^2\ \&\ p_n = (1 - p)p_{n-1} + p(1 - p)p_{n-2},\ \text{for all } n \geq 3.$$

10. In a tournament, team X, plays with each of the 6 other teams once. For each match the probabilities of a win, drawn and loss are equal. Find the probability that the team X, finishes with more wins than losses.

11. A pair of students is selected at random from a probability class. The probability that the pair selected will consist of one male and one female student is $\frac{10}{19}$. Find the maximum number of students the class can contain.

12. 3 students {A, B, C} tackle a puzzle together and offers a solution upon which majority of the 3 agrees. Probability of A solving the puzzle correctly is p. Probability of B solving the puzzle correctly is also p. C is a dumb student who randomly supports the solution of either A or B. There is one more student D, whose probability of solving the puzzle correctly is once again, p. Out of the 3 member team {A, B, C} and one member team {D}, which one is more likely to solve the puzzle correctly.

13. In a knockout tournament 2^n equally skilled players; $S_1, S_2, \ldots\ldots\ S_{2^n}$ are participating. In each round players are divided in pair at random and winner from each pair moves in the next round. If S_2 reaches the semifinal then the probability that S_1 wins the tournament is 1/20. Find the value of 'n'.

14. All the face cards from a pack of 52 plying cards are removed. From the remaining pack half of the cards are randomly removed without looking at them and then randomly drawn two cards simultaneously from the remaining. If the probability that, two cards drawn are both aces, is $\frac{p({}^{38}C_{20})}{{}^{40}C_{20}.{}^{20}C_2}$, then find the value of p.

EXERCISE (JM)

1. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals **[AIEEE-2009]**

(1) 5/14 (2) 1/50 (3) 1/14 (4) 1/7

2. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[AIEEE-2009]**

(1) $\frac{9}{\log_{10}4 - \log_{10}3}$ (2) $\frac{4}{\log_{10}4 - \log_{10}3}$ (3) $\frac{1}{\log_{10}4 - \log_{10}3}$ (4) $\frac{1}{\log_{10}4 + \log_{10}3}$

3. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have difference colours is :- **[AIEEE-2010]**

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{21}$ (4) $\frac{2}{23}$

4. Four numbers are chosen at random (without replacement) from the set (1, 2, 3, ..., 20).

Statement–1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$

Statement–2 : In the four chosen numbers form an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}. **[AIEEE-2010]**

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(2) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is false, Statement–2 is true.

5. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :- **[AIEEE-2011]**

(1) $P(C|D) < P(C)$ (2) $P(C|D) = \frac{P(D)}{P(C)}$ (3) $P(C|D) = P(C)$ (4) $P(C|D) \geq P(C)$

6. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :- **[AIEEE-2011]**

(1) $\left[0, \frac{1}{2}\right]$ (2) $\left(\frac{11}{12}, 1\right]$ (3) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (4) $\left(\frac{3}{4}, \frac{11}{12}\right]$

7. Let A, B, C be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then $P(A^c \cap B^c | C)$ is equal to: **[AIEEE-2011]**

(1) $P(A^c) - P(B)$ (2) $P(A) - P(B^c)$ (3) $P(A^c) + P(B^c)$ (4) $P(A^c) - P(B^c)$

8. Three numbers are chosen at random without replacement from {1, 2, 3, ... , 8}. The probability that their minimum is 3, given that their maximum is 6, is : **[AIEEE-2012]**

(1) $\frac{2}{5}$ (2) $\frac{3}{8}$ (3) $\frac{1}{5}$ (4) $\frac{1}{4}$

9. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is : **[JEE-MAIN 2013]**

(1) $\frac{17}{3^5}$ (2) $\frac{13}{3^5}$ (3) $\frac{11}{3^5}$ (4) $\frac{10}{3^5}$

10. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are : **[JEE(Main)-2014]**

(1) mutually exclusive and independent. (2) equally likely but not independent.

(3) independent but not equally likely. (4) independent and equally likely.

11. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is **NOT true** ? **[JEE(Main)-2016]**

(1) E_1, E_2 and E_3 are independent. (2) E_1 and E_2 are independent.

(3) E_2 and E_3 are independent. (4) E_1 and E_3 are independent.

12. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one–by–one, with replacement, then the variance of the number of green balls drawn is :- **[JEE(Main)-2017]**

(1) $\frac{6}{25}$ (2) $\frac{12}{5}$ (3) 6 (4) 4

13. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is :- **[JEE(Main)-2017]**

(1) $\frac{7}{55}$ (2) $\frac{6}{55}$ (3) $\frac{12}{55}$ (4) $\frac{14}{45}$

14. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$. Then the probability that at least one of the events occurs, is :- **[JEE(Main)-2017]**

(1) $\frac{3}{16}$ (2) $\frac{7}{32}$ (3) $\frac{7}{16}$ (4) $\frac{7}{64}$

15. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

[JEE(Main)-2018]

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{4}$ (4) $\frac{3}{10}$

16. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is : **[JEE(Main)-2019]**

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

17. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is : **[JEE(Main)-2019]**

(1) $\frac{13}{36}$ (2) $\frac{19}{36}$ (3) $\frac{19}{72}$ (4) $\frac{15}{72}$

18. If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is : **[JEE(Main)-2019]**

(1) 6 (2) 5 (3) 4 (4) 3

19. Let S = {1, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is :- **[JEE(Main)-2019]**

(1) $\frac{6}{2^{20}}$ (2) $\frac{5}{2^{20}}$ (3) $\frac{4}{2^{20}}$ (4) $\frac{7}{2^{20}}$

20. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to : **[JEE(Main)-2019]**

(1) $\frac{150}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{200}{6^5}$ (4) $\frac{225}{6^5}$

21. In a game, a man wins Rs. 100 if he gets 5 of 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is : **[JEE(Main)-2019]**

(1) $\frac{400}{3}$ gain (2) $\frac{400}{3}$ loss (3) 0 (4) $\frac{400}{9}$ loss

22. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is : **[JEE(Main)-2019]**

(1) $\frac{1}{11}$ (2) $\frac{1}{17}$ (3) $\frac{1}{10}$ (4) $\frac{1}{12}$

23. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is : **[JEE(Main)-2019]**

(1) $\frac{3}{10}$ (2) $\frac{1}{10}$ (3) $\frac{3}{20}$ (4) $\frac{1}{5}$

EXERCISE (JA)

1. (a) Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is -

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

(b) A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is -

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

[JEE 2010, 3+5]

Paragraph for Question 2 and 3

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

2. The probability of the drawn ball from U_2 being white is -

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

3. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is -

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

[JEE 2011, 3+3]

4. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P(T) denotes the probability of occurrence of the event T, then -

(A) $P(E)=\frac{4}{5},\ P(F)=\frac{3}{5}$ (B) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(C) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ (D) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

[JEE 2011, 4M]

5. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and X_1, X_2, X_3 denotes respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

[JEE 2012, 4M]

(A) $P\left[X_1^c \mid X\right]=\frac{3}{16}$

(B) P[Exactly two engines of ship are functioning $\mid X]=\frac{7}{8}$

(C) $P[X \mid X_2]=\frac{5}{16}$

(D) $P[X \mid X_1]=\frac{7}{16}$

6. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is -

[JEE 2012, 4M]

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

7. Let X and Y be two events such that $P(X|Y)=\frac{1}{2}, P(Y|X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is(are) correct ? **[JEE 2012, 4M]**

(A) $P(X\cup Y)=\frac{2}{3}$ (B) X and Y are independent

(C) X and Y are not independent (D) $P(X^C\cap Y)=\frac{1}{3}$

8. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is **[JEE(Advanced) 2013, 2M]**

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

9. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$. All the given probabilities are assumed of lie in the interval (0,1).

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$ **[JEE-Advanced 2013, 4, (–1)]**

Paragraph for Question 10 and 11

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

10. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

[JEE(Advanced) 2013, 3, (–1)]

11. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

[JEE(Advanced) 2013, 3, (–1)]

12. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is - **[JEE(Advanced)-2014, 3(–1)]**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Paragraph For Questions 13 and 14

Box 1 contains three cards bearing numbers, 1,2,3 ; box 2 contains five cards bearing numbers 1,2,3,4,5; and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, i = 1,2,3.

13. The probability that $x_1 + x_2 + x_3$ is odd, is -

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

[JEE(Advanced)-2014, 3(–1)]

14. The probability that x_1,x_2,x_3 are in an arithmetic progression, is -

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

[JEE(Advanced)-2014, 3(–1)]

15. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is **[JEE 2015, 4M, –0M]**

Paragraph For Questions 16 and 17

Let n_1 and n_2 be the number of red and black balls respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

16. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)

[JEE 2015, 4M, –0M]

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$ (B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$ (D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

17. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are) **[JEE 2015, 4M, –0M]**

(A) $n_1 = 4$ and $n_2 = 6$ (B) $n_1 = 2$ and $n_2 = 3$
(C) $n_1 = 10$ and $n_2 = 20$ (D) $n_1 = 3$ and $n_2 = 6$

18. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P(computer turns out to be defective given that is produced in plant T_1)

= 10P(computer turns out to be defective given that it is produced in plant T_2)

where P(E) denotes the probability of an event E. A computer produces in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is **[JEE(Advanced)-2016, 3(–1)]**

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Paragraph For Questions 19 and 20

Football teams T_1 and T_2 have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of T_1 winning, drawing and losing a game against T_2 are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively. Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let X and Y denote the total points scored by teams T_1 and T_2, respectively, after two games

19. P(X > Y) is- **[JEE(Advanced)-2016, 3(0)]**

(A) $\frac{1}{4}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{7}{12}$

20. P(X = Y) is- **[JEE(Advanced)-2016, 3(0)]**

(A) $\frac{11}{36}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{1}{2}$

21. Let X and Y be two events such that $P(X)=\frac{1}{3}$, $P(X|Y)=\frac{1}{2}$ and $P(Y|X)=\frac{2}{5}$. Then

[JEE(Advanced)-2017, 4(–2)]

(A) $P(X'|Y)=\frac{1}{2}$ (B) $P(X\cap Y)=\frac{1}{5}$

(C) $P(X\cup Y)=\frac{2}{5}$ (D) $P(Y)=\frac{4}{15}$

22. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is **[JEE(Advanced)-2017, 3(–1)]**

(A) $\frac{36}{55}$ (B) $\frac{6}{11}$ (C) $\frac{5}{11}$ (D) $\frac{1}{2}$

Paragraph For Questions 23 and 24

There are five students S_1, S_2, S_4 and S_5 in a music class and for them there are five sets R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2, 3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on Paragraph "A". the question given below is one of them*)

23. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1 and **NONE** of the remaining students gets the seat previously allotted to him/her is -

[JEE(Advanced)-2018, 3(–1)]

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

24. For i = 1, 2, 3, 4, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is-

[JEE(Advanced)-2018, 3(–1)]

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

25. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls, Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}$, $\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct ?

[JEE(Advanced)-2019, 4(–1)]

(1) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(2) Probability that the chosen ball is green equals $\frac{39}{80}$

(3) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(4) Probability that the selected bag is B_3, given that the chosen balls is green, equals $\frac{5}{13}$

26. Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$$E_1 = \{A \in S : \det A = 0\} \text{ and}$$
$$E_2 = \{A \in S : \text{sum of entries of A is } 7\}.$$

If a matrix is chosen at random from S, then the conditional probability $P(E_1|E_2)$ equals ____

[JEE(Advanced)-2019, 3(0)]

27. Let |X| denote the number of elements in set X. Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1 \leq |B| < |A|$, equals **[JEE(Advanced)-2019, 3(0)]**

ANSWER KEY

EXERCISE (O-1)

PART # 1

1. A **2.** A **3.** C **4.** 23/168

5. (i) 7/13, (ii) 1/2, (iii) 2/13, (iv) 2/13, (v) 1/2, (vi) 9/13 **6.** 1/56 **7.** 1/2 ; 1/2

8. 5 : 1 **9.** 952 to 715 **10.** A **11.** 4/21 **12.** **(a)** 2/3, **(b)** 1/2

13. B **14.** B **15.** C

PART # 2

1. C **2.** D **3.** A **4.** B **5.** C **6.** A **7.** A

8. 2/3 **9.** 3/4, 1/4; 15/16 **10.** (i) 0.18, (ii) 0.12, (iii) 0.42, (iv) 0.28, (v) 0.72

11. (i) 0.6, (ii) 0.5, (iii) 0.25 **12.** (i) 1/36, (ii) 5/108, (iii) 53/54 **13.** 11/20

14. 3/5 **15.** 2/7 **16.** A **17.** 12/25 **18.** (a) 1/18, (b) 43/90, (c) 5/18, (d) NO

PART # 3

1. C **2.** C **3.** D **4.** D **5.** B **6.** A **7.** A

8. C **9.** A **10.** D **11.** B **12.** A **13.** C **14.** B

15. D

PART # 4

1. D **2.** A **3.** B **4.** C **5.** C **6.** A **7.** B

8. C **9.** C **10.** A **11.** B **12.** D

PART # 5

1. D **2.** B **3.** B **4.** C **5.** C **6.** B **7.** B

8. A **9.** C **10.** C **11.** C **12.** C **13.** B **14.** D

15. B

PART # 6

1. A **2.** B **3.** C **4.** D **5.** C **6.** A **7.** B

8. B **9.** D **10.** A **11.** B **12.** D **13.** A

EXERCISE (O-2)

1. B **2.** B **3.** A **4.** C **5.** B **6.** A **7.** C

8. C **9.** **(a)** D, **(b)** A, **(c)** B **10.** B,C,D **11.** B,C,D **12.** C, D **13.** B,C,D

14. A,B,C,D **15.** A,B,D **16.** A,C,D **17.** B,C,D **18.** A,D **19.** B,C,D **20.** A,D

21. A,C,D **22.** A,D **23.** (A) Q; (B) P; (C) S

EXERCISE (S-1)

1. **(i)** 3/56; **(ii)** 9/28 **2.** 13/24 **3.** 5/9 **4.** 319/420 **5.** 120

6. $P(\bar{H}/S) = 1/2$ **7.** $\frac{\sqrt{5}-1}{2}$ **8.** $\frac{\sqrt{2}}{2}$ **9.** 2065 **10.** $\frac{29}{30}$

11. 17 **12.** 17/105 **13.** 2/5 **14.** 37

15. **(i)** A,B,C are pairwise independent **(ii)** A,B,C are not independent **16.** 3

17. $\frac{328}{625}$ **18.** $\frac{1}{9},\frac{1}{3}$ **19.** 301 **20.** 0.6976 **21.** 19/42 **22.** 407/576

23. (i) $\frac{125}{16^3}$; (ii) $\frac{63}{64}$ **24.** 165/193 **25.** $(1-p)^{n-1}.\ [p_0\,(1-p) + np(1-p_0)]$

26. (a) 7/8, (b) 1/3 **27.** B = 2/5 ; C = 4/15 **28.** 13 to 5 **29.** 5/8

30. 13/14 **31.** 179

EXERCISE (S-2)

1. 5 **2.** Option B **3.** 1/2, 1/2, 1/12 **4.** 4/35 **5.** 262

6. 5 **7.** **(a)** $\frac{9}{50}$ **(b)** $\frac{(5n-3)}{(9n-3)}$ **8.** 79 **9.** 79 **10.** $\frac{98}{243}$ **11.** 20

12. Both are equally likely **13.** 4 **14.** 6

EXERCISE (JM)

1. 3	**2.** 3	**3.** 2	**4.** 3	**5.** 4	**6.** 1	**7.** 1	**8.** 3
9. 3	**10.** 3	**11.** 1	**12.** 2	**13.** 2	**14.** 3	**15.** 1	**16.** 2
17. 3	**18.** 2	**19.** 2	**20.** 2	**21.** 3	**22.** 1	**23.** 2	

EXERCISE (JA)

1. **(a)** C; **(b)** C **2.** B **3.** D **4.** A,D **5.** B,D **6.** A **7.** A,B

8. A	**9.** 6	**10.** D	**11.** A	**12.** A	**13.** B	**14.** C	**15.** 8
16. A,B	**17.** C,D	**18.** **C**	**19.** **B**	**20.** C	**21.** A,D	**22.** B	**23.** A
24. C	**25.** 2,3	**26.** 0.50	**27.** 422.00				