CONTENTS

Newton's laws of Motion & Friction

NEWTON'S LAWS OF MOTION & FRICTION

KEY CONCEPTS

FORCE

A push or pull that one object exerts on another.

FORCES IN NATURE

There are four fundamental forces in nature :

1. Gravitational force
2. Electromagnetic force
3. Strong nuclear force
4. Weak force

TYPES OF FORCES ON MACROSCOPIC OBJECTS

(a) Field Forces or Range Forces :

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies. (ii) Electrostatic force between two charges.

(b) Contact Forces :

Contact forces exist only as long as the objects are touching each another.

Ex. (i) Normal forces. (ii) Frictional force

(c) Attachment to Another Body :

Tension (T) in a string and spring force (F = kx) comes in this group.

NEWTON'S FIRST LAW OF MOTION (OR GALILEO'S LAW OF INERTIA)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion *"Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line".*

Inertia : Inertia is the property of the body due to which body oppose the change of its state. Inertia of a body is measured by mass of the body. $\boxed{\text{inertia} \propto \text{mass}}$

TYPES OF INERTIA

Inertia of rest : It is the inability of a body to change, its state of rest by itself.

Examples :

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.
- When a stone hits a window pane, the glass is broken into a number of pieces whereas if the high speed bullet strikes the pane, it leaves a clean hole.

Inertia of motion : It is the inability of a body to change its state of uniform motion by itself.

Examples :

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.
- A ball thrown upwards in a running train continues to move along with the train.

Inertia of direction : It is the inability of a body to change its direction of motion by itself.

Examples :

- When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud, mudguard over the wheels stop this mud.
- A body released from a balloon rising up, continues to move in the direction of balloon.

NEWTON'S SECOND LAW OF MOTION

Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or} \qquad \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

if m = constant then $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

- Newton's Second Law Provides the Definition of the Concept of Force.
- **Definition of the 1 Newton (N) :–** If an object of mass one kilogram has an acceleration of $1\,ms^{-2}$ relative to an inertial reference frame, then the net force exerted on the object is one newton.

CONSEQUENCES OF NEWTON'S II LAW OF MOTION

- **Concept of inertial mass :** From Newton's II law of motion $a = \frac{F}{M}$

  i.e., the magnitude of acceleration produced by a given body is inversely proportional to mass i.e. greater the mass, smaller is the acceleration produced in the body. Thus, mass is the measure of inertia of the body. The mass given by above equation is therefore called the inertial mass.

- **An accelerated motion is the result of application of the force :**

  There may be two types of accelerated motion :

(i) When only the magnitude of velocity of the body changes : In this types of motion the force is applied along the direction of motion or opposite to the direction of motion.

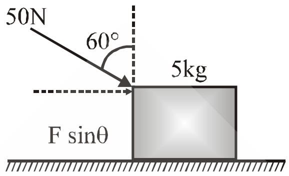
(ii) When only the direction of motion of the body changes : In this case the force is applied at right angles to the direction of motion of the body, e.g. uniform circular motion.

- **Acceleration produced in the body depends only on its mass and not on the final or initial velocity.**

Ex. A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})\,N$ produces acceleration of 1 ms^{-2} in a body. Calculate the mass of the body.

Sol. $\because$ Acceleration $a = \frac{|\vec{F}|}{m}$ $\qquad \therefore$ mass $m = \frac{|\vec{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = 10\sqrt{2}$ kg

Ex. A force of 50 N acts in the direction as shown in figure. The block of mass 5kg, resting on a smooth horizontal surface. Find out the acceleration of the block.



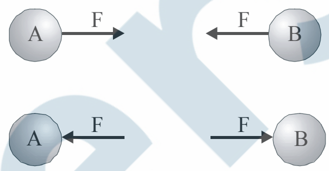
Sol. Horizontal component of the force = Fsinθ = 50sin60° = $\frac{50\sqrt{3}}{2}$N

acceleration of the block $a = \frac{F\sin\theta}{m} = \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}$ m/s^2

NEWTON'S THIRD LAW OF MOTION

The first and second laws are statements about a single object, whereas the third law is a statement about two objects.

- According to this law, every action has equal and opposite reaction. Action and reaction act on different bodies and they are simultaneous. There can be no reaction without action.
- If an object A exerts a force F on an object B, then B exerts an equal and opposite force (–F) on A.



The forces between two objects A and B are equal and opposite, whether they are attractive or repulsive.

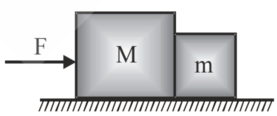
- Action and reaction never cancel each other, since they act on different bodies.

- **First law** : If no net force acts on a particle, then it is possible to select a set of reference frames, called inertial reference frames, observed from which the particle moves without any change in velocity.
- **Second law** : Observed from an inertial reference frame, the net force on a particle is proportional to the time rate of change of its linear momentum: $\frac{d(m\vec{v})}{dt}$
- **Third law** : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

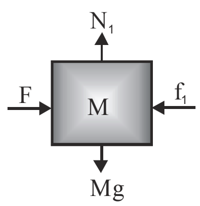
FREE BODY DIAGRAM

A diagram showing all external forces acting on an object is called "Free Body Diagram" (F.B.D.) In a specific problem, first we are required to choose a body and then we find the number of forces acting on it, and all the forces are drawn on the body, considering it as a point mass. The resulting diagram is known as free body diagram (FBD).

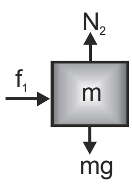
For example, if two bodies of masses m and M are in contact and a force F on M is applied from the left as shown in figure (a), the free body diagrams of M and m will be as shown in figure (b) and (c).



(a)



(b)



(c)

Important Point :

Two forces in Newton's third law never occur in the same free–body diagram. This is because a free–body diagram shows forces acting on a single object, and the action–reaction pair in Newton's third law always act on different objects.

MOTION OF BODIES IN CONTACT

Case I :

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called Force of Contact. These two bodies will move with same acceleration a.

(i) When the force F acts on the body with mass m_1 as shown in fig. (1) $F = (m_1 + m_2)\ a$.

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1 : $(F - f_1) = m_1\ a$

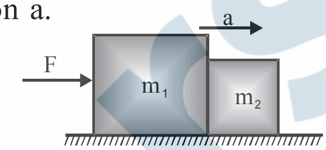


Fig.(1) : When the force F acts on mass m_1.

for body m_2 : $f_1 = m_2 a$

$\Rightarrow$ action of m_1 on m_2 : $f_1 = \dfrac{m_2 F}{m_1 + m_2}$

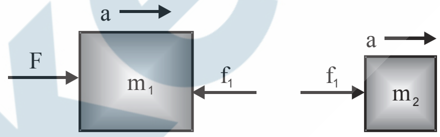


Fig. 1(a) : F.B.D. representation of action and reaction forces.

(ii) When the force F acts on the body with mass m_2 as shown in figure 2

$F = (m_1 + m_2)\ a$

for body with mass m_2

$F - f_2 = m_2\ a$

for body m_1, $f_2 = m_1 a$

$\Rightarrow$ action on m_1 , $f_2 = \dfrac{m_1 F}{m_1 + m_2}$

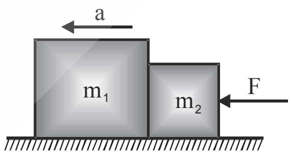


Fig. (2) : When the force F acts on mass m_2.

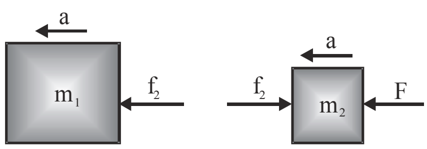
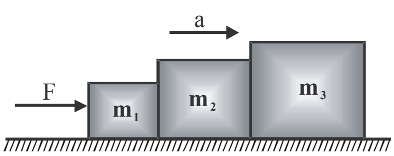


Fig. 2 (a) : F.B.D. representation of action and reaction forces.

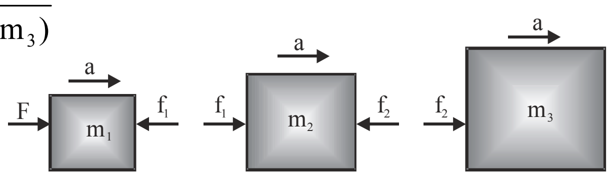
Case II :

Three bodies of masses m_1, m_2 and m_3 placed one after another and in contact with each other. Suppose a force F is applied horizontally on mass m_1



then $F = (m_1 + m_2 + m_3)\ a \Rightarrow a = \dfrac{F}{(m_1 + m_2 + m_3)}$

$f_1 = \dfrac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$ (action on both m_2 and m_3)

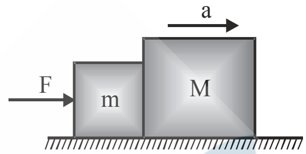


$F - f_1 = ma_1$ $f_1 - f_2 = ma_1$ $f_2 = ma_2$

and $f_2 = \dfrac{m_3 F}{(m_1 + m_2 + m_3)}$ (action on m_3 alone) when the force F is applied on m_3, then

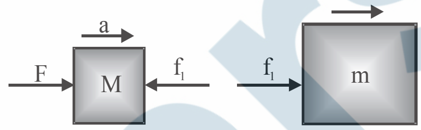
$$f_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)} \text{ (action on } m_1 \text{ alone) and } f_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)} \text{ (action on } m_1 \text{ and } m_2)$$

Ex. Two blocks of mass m = 2 kg and M = 5 kg are in contact on a frictionless table. A horizontal force F (= 35 N) is applied to m. Find the force of contact between the block, will the force of contact remain same if F is applied to M ?

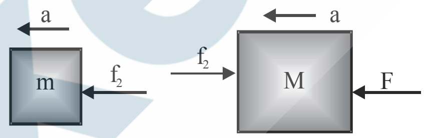


Sol. As the blocks are rigid under the action of a force F, both will move with same acceleration

$$a = \frac{F}{m + M} = \frac{35}{2 + 5} = 5 \text{ m/s}^2$$



force of contact $f_1 = Ma = 5 \times 5 = 25$ N

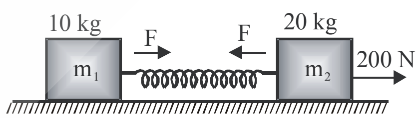


If the force is applied to M then its action on m will be

$f_2 = ma = 2 \times 5 = 10$ N.

From this problem it is clear that acceleration does not depends on the fact that whether the force is applied to m or M, but force of contact does.

Ex. Two masses 10 kg and 20 kg respectively are connected by a massless spring as shown in figure force of 200 N acts on the 20 kg mass. At the instant shown in figure the 10 kg mass has acceleration of 12 m/s², what is the acceleration of 20 kg mass?



Sol. Equation of motion for m_1 is $F = m_1 a_1 = 10 \times 12 = 120$ N.

Force on 10 kg–mass is 120 N to the right. As action and reaction are equal and opposite, the reaction force F on 20 kg mass F = 120 N to the left.

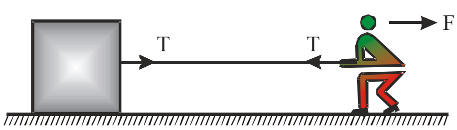
$\therefore$ Equation of motion for m_2 is $200 - F = 20\, a_2$

$$\Rightarrow 200 - 120 = 20\, a_2 \qquad \Rightarrow 20\, a_2 = 80 \qquad \Rightarrow a_2 = \frac{80}{20} = 4 \text{ ms}^{-2}$$

SYSTEM OF MASSES TIED BY STRINGS

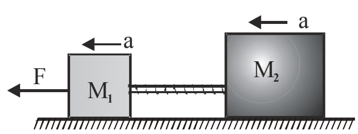
Tension in a String :

It is an intermolecular force between the atoms of a string, which acts or reacts when the string is streched.

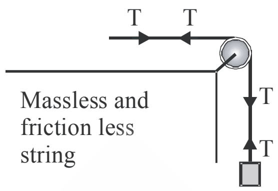


Important points about the tension in a string :

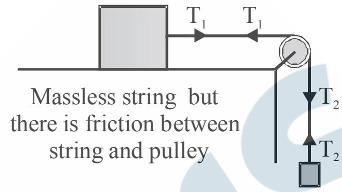
- Force of tension act on a body in the direction away from the point of contact or tied ends of the string.
- String is assumed to be inextensible so that the magnitude of accelerations of any number of masses connected through strings is always same.



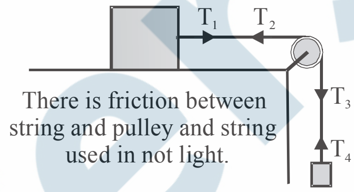
- If the string is extensible the acceleration of different masses connected through it will be different until the string can stretch.
- String is massless and frictionless so that tension throughout the string remains same.



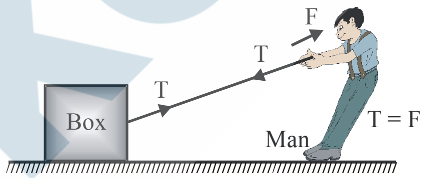
- If the string is massless but not frictionless, at every contact tension changes.



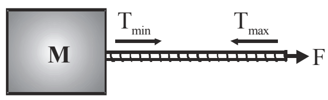
- If the string is not light, tension at each point will be different depending on the acceleration of the string.



- If a force is directly applied on a string as say man is pulling a tied string from the other end with some force the tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box will move or not, man will move or not.



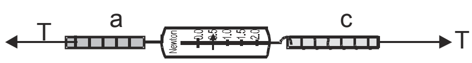
- String is assumed to be massless unless stated, hence tension in it everywhere remains the same and equal to applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.



- In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F. Here it must be noted that a string can never be compressed like a spring.

- If string is cut so that element b is replaced by a spring scale (the rest of the string being undisturbed), the scale reads the tension T.

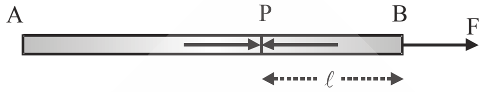
Three small elements
of a string, a, b, c



- Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break if the tension is increased beyond a certain limit. The maximum tension which a string can bear without breaking is called "breaking strength". It is finite for a string and depends on its material and dimensions.

Ex. A uniform rope of length L is pulled by a constant force F. What is the tension in the rope at a distance ℓ from the end where it is applied?

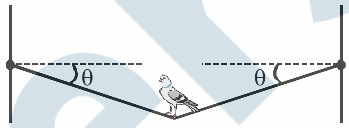
Sol. Let mass of rope is M and T be tension in the rope at point P, then. Acceleration of rope, $a = \dfrac{F}{M}$



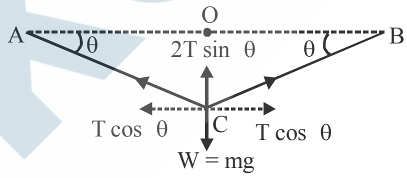
Equation of motion of part PB is F – T = $(m\ell)$ a

$$\Rightarrow T = F - (m\ell)\,a \;=\; F - \left(\frac{M}{L}\right)(\ell)\left(\frac{F}{M}\right) = \left[1 - \frac{\ell}{L}\right] F$$

Ex. A bird with mass m perches at the middle of a stretched string Show that the tension in the string is give by $T = \dfrac{mg}{2\sin\theta}$. Assume that each half of the string is straight.



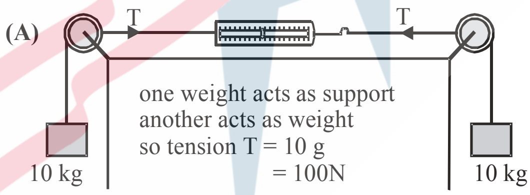
Sol. Initial position of wire = AOB. Final position of wire = ACB Let θ be the angle made by wire with horizontal, which is very small. Resolving tension T of string in horizontal and vertical directions, we note that the horizontal components cancel while vertical components add and balance the weight.



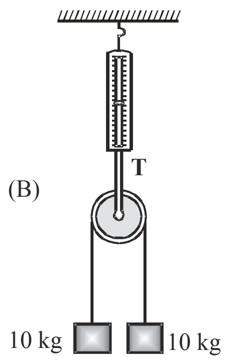
For equilibrium 2T sinθ = W = mg $\Rightarrow T = \dfrac{W}{2\sin\theta}$ = mg/2sinθ

Ex. The system shown in figure are in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? (g = 10 m/s²)

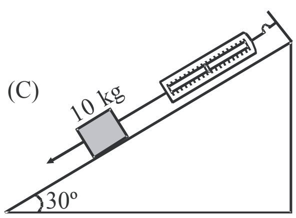
Sol.



one weight acts as support
another acts as weight
so tension T = 10 g
= 100N



T = 2×10×g = 2×10×10 = 200N



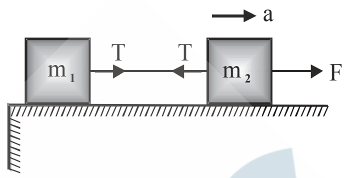
T = 10×10 sin30°
= 10 × 10 × ½ = 50 N

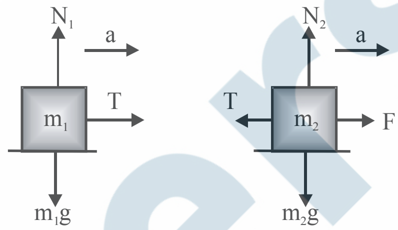
MOTION OF BODIES CONNECTED BY STRINGS

Case : I

Two bodies :

Let us consider the case of two bodies of masses m_1 and m_2 connected by a thread and placed on a smooth horizontal surface as shown in figure. A force F is applied on the body of mass m_2 in forward direction as shown. Our aim is to consider the acceleration of the system and the tension T in the thread. The forces acting separately on two bodies are also shown in the figure:





From figure $\quad T = m_1a$

and $\quad F - T = m_2a$

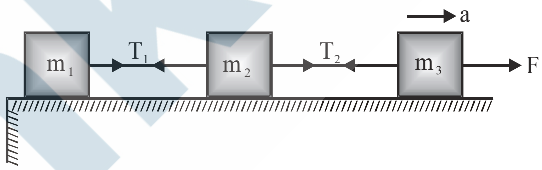
$\Rightarrow \quad F = (m_1 + m_2)a$

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2} \quad \& \quad T = \frac{m_1F}{m_1 + m_2}$$

Case II:

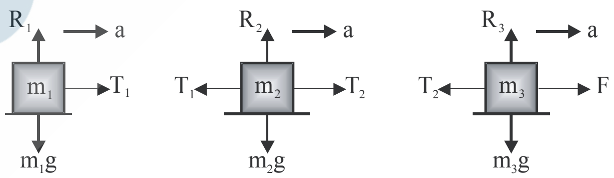
Three bodies

In case of three bodies, the situation is shown in figure



Acceleration $\quad a = \dfrac{F}{m_1 + m_2 + m_3}$,

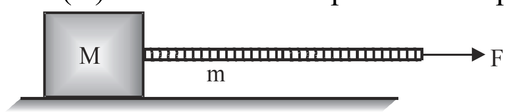
$$T_1 = m_1a = \frac{m_1F}{m_1 + m_2 + m_3}$$



$\because$ for block of mass $m_3 \quad F - T_2 = m_3a$

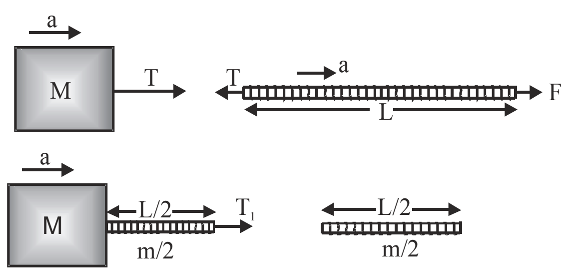
$$\therefore \quad T_2 = F - \frac{m_3F}{m_1 + m_2 + m_3} = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$$

Ex. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find (i) The acceleration of the rope and block (ii) The force that the rope exerts on the block. (iii) Tension in the rope at its mid point.



Sol. (i) Accelaration $\quad a = \dfrac{F}{(m + M)}$

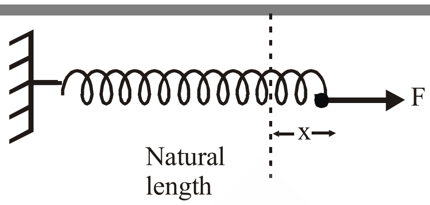
(ii) Force exerted by rope $\quad T = Ma = \dfrac{M.F}{(m + M)}$



(iii) $T_1 = \left(\dfrac{m}{2} + M\right) a = \left(\dfrac{m + 2M}{2}\right)\left(\dfrac{F}{m + M}\right)$

Tension in rope at midpiont $\quad T_1 = \dfrac{(m + 2M)F}{2(m + M)}$

Spring Force (According to Hooke's law) :

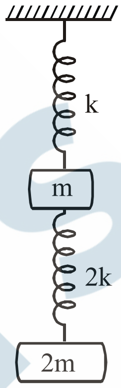


In equilibrium $F = kx$

k is spring constant

Note : Spring force is non impulsive in nature.

Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.

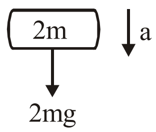


Sol. Intial stretches

$$x_{upper} = \frac{3mg}{k} \quad \& \quad x_{lower} = \frac{mg}{k}$$

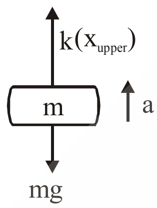
On cutting the lower spring, by virture of non-impulsive nature of spring the stretch in upper spring remains same. Thus,

Lower block :



$2mg = 2ma \Rightarrow a = g$

Upper block :



$k\left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2g$

FRAME OF REFERENCE

It is a conveniently chosen co–ordinate system which describes the position and motion of a body in space.

INERTIAL AND NON–INERTIAL FRAMES OF REFERENCE

Inertial frames of reference :

A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

- All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- All the fundamental laws of physics can be expressed as to have the same mathematical form in all the inertial frames of reference.
- The mechanical and optical experiments performed in an inertial frame in any direction will always yield the same results. It is called isotropic property of the inertial frame of reference.

Examples of inertial frames of reference :

- A frame of reference remaining fixed w.r.t. distant stars is an inertial frame of reference.
- A space–ship moving in outer space without spinning and with its engine cut–off is also inertial frame of reference.
- For practical purposes, a frame of reference fixed to the earth can be considered as an inertial frame. Strictly speaking, such a frame of reference is not an inertial frame of reference, because the motion of earth around the sun is accelerated motion due to its orbital and rotational motion. However, due to negligibly small effects of rotation and orbital motion, the motion of earth may be assumed to be uniform and hence a frame of reference fixed to it may be regarded as inertial frame of reference.

Non–inertial frame of reference :

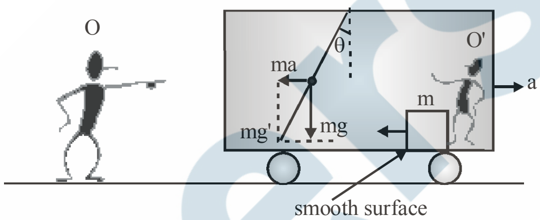
An accelerating frame of reference is called a non–inertial frame of reference.

Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Note : A rotating frame of references is a non–inertial frame of reference, because it is also an accelerating one due to its centripetal acceleration.

PSEUDO FORCE

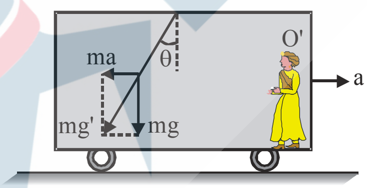
The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.

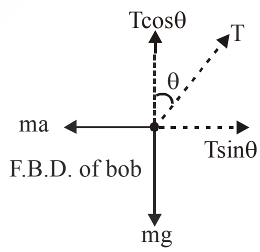


For observer O on ground train is moving with acceleration on "a" for observer O' in side the train there is pseudo force in opposite direction shown in figure.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass).
- But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

Ex. A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in equilibrium position.





Sol. Non–inertial frame of reference (Train)

F.B.D. of bob w.r.t. train. (real forces + pseudo force) : with respect to train, bob is in equilibrium

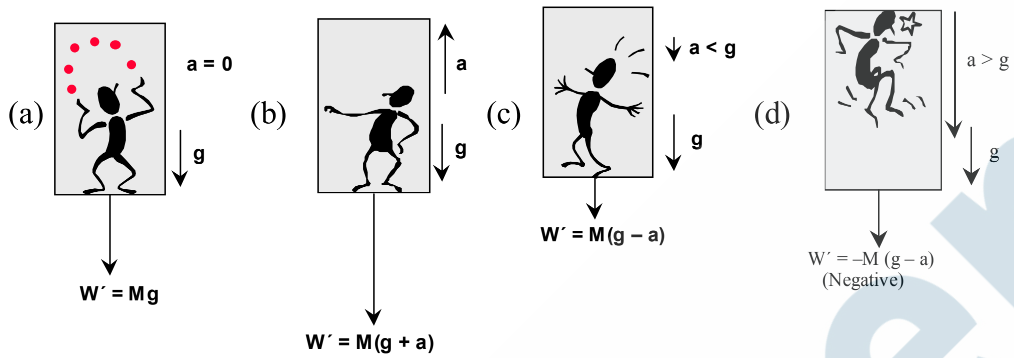
$\therefore\ \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ and

$\Sigma F_x = 0 \Rightarrow T\sin\theta = ma$

$$\Rightarrow \tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

MOTION IN A LIFT

The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg be standing in a lift. We consider the following cases :



Case (a) :

If the lift moving with constant velocity v upwards or downwards.

In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

So apparent weight W′ = Mg (Actual weight).

Case (b) :

If the lift is accelerated upward with constant acceleration a.

Then net forces acting on the man are (i) weight W = Mg downward (ii) fictitious force F_0 = Ma downward.

So apparent weight $W' = W + F_0 = Mg + Ma = M(g + a)$

Case (c) :

If the lift is accelerated downward with acceleration a < g

Then fictitious force F_0 = Ma acts upward while weight of man W = Mg always acts downward.

So apparent weight $W' = W + F_0 = Mg - Ma = M(g - a)$

Special Case : If a = g then W′ = 0 (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

Case (d) :

If lift accelerates downward with acceleration a > g :

Then as in Case c . Apparent weight W′ = M(g – a) is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

Ex. A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration?

Sol. (i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.

(ii) In this case the acceleration is upward, the fictitious force ma acts downward,therefore apparent weight is more than actual weight i.e. W′= m (g + a).

Hence scale shows a reading $= m(g + a) = \dfrac{mg\left(1+\dfrac{a}{g}\right)}{g} = \left(50 + \dfrac{50a}{g}\right)$ kgwt.

Ex. Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?

Sol. Yes, since both masses experience equal fictitious forces in magnitude as well as direction.

Ex. A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of 5° to the vertical?

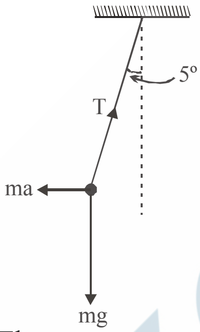
Sol. The ball is accelerated by the force T sin5°.

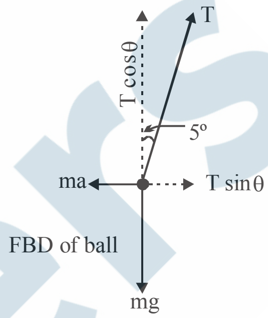
Therefore T sin5° = ma.

Vertical component $\Sigma F = 0$, so T cos5° = mg.

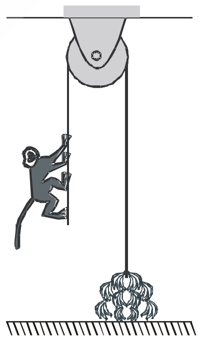
By solving $a = g \tan 5° = 0.0875\ g$

$= 0.86\ \text{ms}^{-2}$.





Ex. A 12 kg monkey climbs a light rope as shown in figure. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the pulley's only effect is to reverse the direction of the rope. What is the maximum acceleration the monkey can have without lifting the bananas?



Sol. Effective weight of monkey $W_m = M_m(g + a)$

As per given condition $W_m = W_b$

$\Rightarrow \quad M_m (g + a) = M_b\ g$

$$\Rightarrow \quad a = \frac{(M_b - M_m)\ g}{M_m} = \left(\frac{16-12}{12}\right) \times 9.8$$

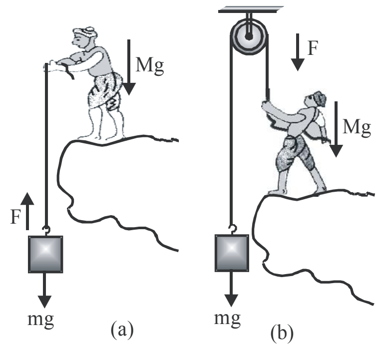
$$= \frac{9.8}{3} = 3.26\ \text{m/s}^2$$

PULLEY SYSTEM

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

It is clear from example given below.

Ex. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force of 700 N, which mode should be the man adopt to lift the block without the floor yielding ?



Sol. Mass of the block, m = 25 kg ; mass of the man, M = 50 kg

Force applied to lift the block F = mg = 25 × 9.8 = 245 N

Weight of the man, M g = 50 × 9.8 = 490 N

(a) When the block is raised by the man by applying force F in upward direction, reaction equal and opposite to F will act on the floor in addition to the weight of the man.

∴ action on the floor Mg + F = 490 + 245 = 735 N

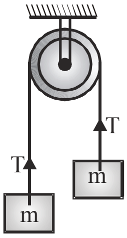
(b) When the block is raised by the mass applying force F over the rope (passed over the pulley) in downward direction, reaction equal and opposite to F will act on the floor,

$\therefore$ action on the floor $\quad Mg - F = 490 - 425 = 245$ N

floor yields to a normal force of 700 N, the mode (b) should be adopted by the man to lift block.

Some cases of pulley

I Case



$m_1 = m_2 = m$

Tension in the string $T = mg$

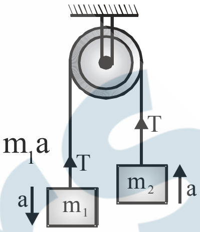
Acceleration 'a' = zero

Reaction at the suspension of the pulley

$R = 2T = 2\ mg.$

$$\text{Acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

II Case



$m_1 > m_2$

now for mass m_1, $m_1 g - T = m_1 a$

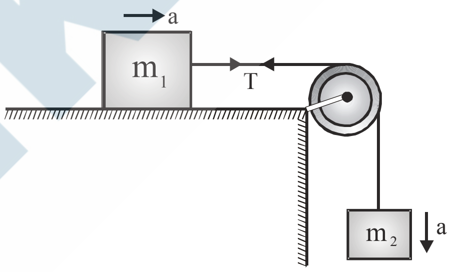
for mass m_2, $T - m_2 g = m_2 a$

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g \quad \text{and} \quad T = \frac{2m_1 m_2}{(m_1 + m_2)} g$$

$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

$$\text{Reaction at the suspension of pulley } R = 2T = \frac{4m_1 m_2 g}{(m_1 + m_2)}$$

III Case :

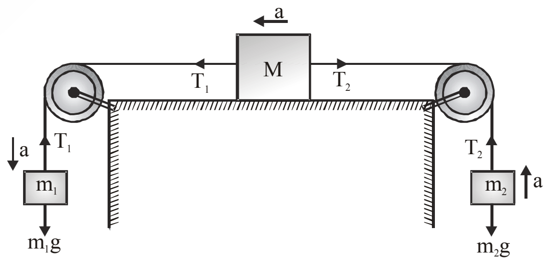


For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2 g - T = m_2 a$

$$\text{acceleration } a = \frac{m_2 g}{(m_1 + m_2)} \quad \text{and} \quad T = \frac{m_1 m_2}{(m_1 + m_2)} g$$

IV Case :



$(m_1 > m_2)$

For m_1, $m_1 g - T_1 = m_1 a$

For m_2, $T_2 - m_2 g = m_2 a$

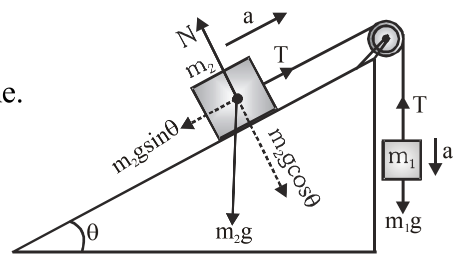
For M $T_1 - T_2 = Ma$

$$\Rightarrow a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)} g,$$

$$T_1 = \frac{(2m_2 + M) m_1 g}{m_1 + m_2 + M}, \quad T_2 = \frac{(2m_1 + M) m_2 g}{m_1 + m_2 + M}$$

V Case :

Mass suspended over a pulley from another on an inclined plane.



For mass m_1 : $m_1 g - T = m_1 a$

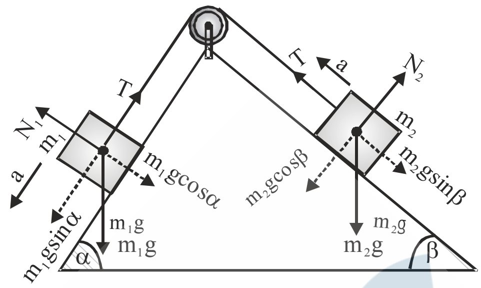
For mass m_2 : $T - m_2 g \sin\theta = m_2 a$

$$\text{acceleration } a = \frac{(m_1 - m_2 \sin\theta)}{(m_1 + m_2)} g$$

$$T = \frac{m_1 m_2 (1 + \sin\theta)}{(m_1 + m_2)} g$$

VI Case :

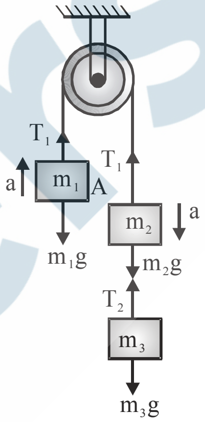
Masses m_1 and m_2 are connected by a string passing over a pulley ($m_1 > m_2$)



Acceleration $a = \dfrac{(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)} g$

Tension $T = \dfrac{m_1 m_2 (\sin\alpha + \sin\beta)}{(m_1 + m_2)} g$

VII Case :



For mass m_1 : $T_1 - m_1g = m_1a$

For mass m_2 : $m_2g + T_2 - T_1 = m_2a$

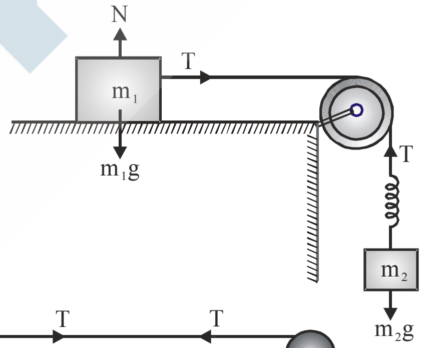
For mass m_3 : $m_3g - T_2 = m_3a$

Acceleration $a = \dfrac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} g$

we can calculate tensions T_1 and T_2 from above equations

VIII Case :

From case (iii)



tension $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$

If x is the extension in the spring,

then $\quad T = kx$

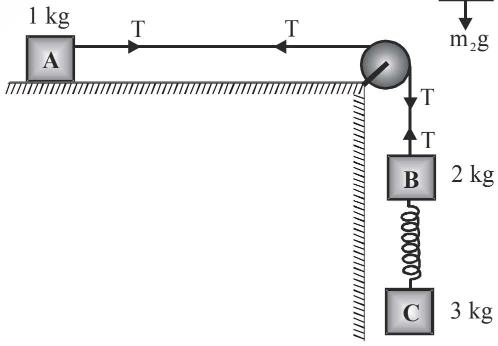
$x = \dfrac{T}{k} = \dfrac{m_1 m_2 g}{k(m_1 + m_2)}$

Ex. In the system shown in figure all surface are smooth, string is massless and inextensible. Find:

(a) acceleration of the system

(b) tension in the string and

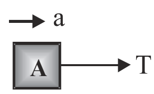
(c) extension in the spring if force constant of spring is $k = 50$ N/m (Take $g = 10$ m/s^2)



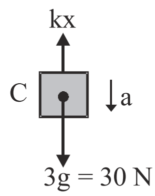
Sol. (a) In this case net pulling force $= m_cg + m_Bg = 50$ N and total mass to be pulled is $(1 + 2 + 3)$ kg $= 6$ kg.

$\therefore$ Acceleration of the system is $a = \dfrac{50}{6}$ ms^{-2}

(b) Free body diagram of 1 kg block gives $T = ma = (1)\left(\dfrac{50}{6}\right)N = \dfrac{50}{6}N$



(c) Free body diagram of 3 kg block gives



$30 - kx = ma \quad$ but $\quad ma = 3 \times \dfrac{50}{6} = 25N$

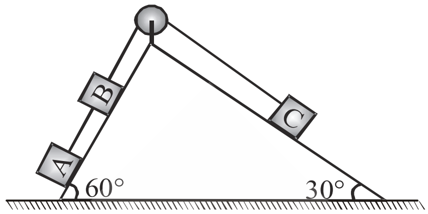
$x = \dfrac{30 - 25}{k} = \dfrac{5}{50} = 0.1 \; m = 10$ cm

Ex. In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively.

Find : (a) the acceleration of the system and

(b) tensions in the string

Neglect friction. ($g = 10\ ms^{-2}$)



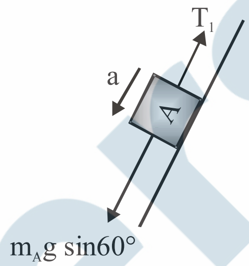
Sol. (a) In this case net pulling force

$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$

$= (m_A + m_B)\, g \sin 60° - m_C\, g \sin 30°$

$$= (1 + 3) \times 10 \times \frac{\sqrt{3}}{2} - 2 \times 10 \times \frac{1}{2}$$

$= 20\sqrt{3} - 10 = 20 \times 1.732 - 10 = 24.64$ N



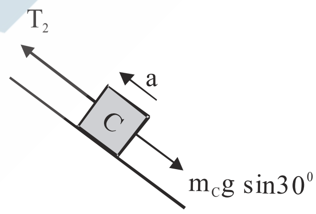
Total mass being pulled $= 1 + 3 + 2 = 6$ kg

$\therefore$ Acceleration of the system $a = \dfrac{24.64}{6} = 4.1\ m/s^2$

(b) For the tension T_1 in the string between A and B, $m_A g \sin 60° - T_1 = (m_A)(a)$

$\therefore\ T_1 = m_A\, g \sin 60° - m_A\, a = m_A\, (g \sin 60° - a)$

$$\Rightarrow\ T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56 \text{ N}$$

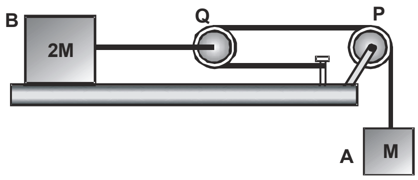


For the tension T_2 in the string between B and C.

$T_2 - m_C\, g \sin 30° = m_C a$

$$\Rightarrow T_2 = m_C\, (a + g \sin 30°) = 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2 \text{ N}$$

Ex.. Consider the situation shown in figure. Both the pulleys and the strings are light and all the surfaces are frictionless. Calculate



(a) the acceleration of mass M,

(b) tension in the string PQ and

(c) force exerted by the clamp on the pulley P.

Sol. As pulley Q is not fixed so if it moves a distance d the length of string between P and Q will changes by 2d (d from above and d from below), i.e., M will move 2d. This in turn implies that if a is the acceleration of M the acceleration of Q and of 2M will be $\dfrac{a}{2}$.

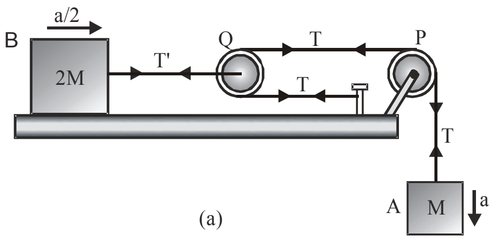
Now if we consider the motion of mass M, it is accelerated downward, so

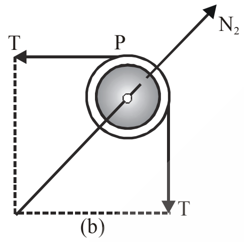
$T = M(g - a)$ ....(i)

And for the motion of Q

$$2T - T' = 0 \times \frac{a}{2} = 0 \quad \Rightarrow T' = 2T \qquad \text{........(ii)}$$

And for the motion of mass 2M $\quad T' = 2M\left(\dfrac{a}{2}\right), \quad \Rightarrow \quad T' = Ma \qquad$ ....(iii)





(a) From equation (ii) and (iii) as $T = \frac{1}{2}Ma$, so equation (i) reduces to

$$T = \frac{1}{2} Ma = M(g - a) \Rightarrow a = \frac{2}{3} g$$

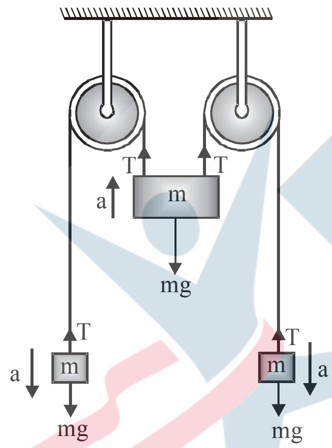
(b) So the acceleration of mass M is $\frac{2}{3}$ g while tension in the string PQ from equation (1) will be

$$T = M(g - \frac{2}{3} g) = \frac{1}{3} Mg$$

(c) Now from figure (b), it is clear that force on pulley by the clamp will be equal and opposite to the resultant of T and T at 90° to each other, i.e.,

$$(N_2) = \sqrt{T^2 + T^2} = \sqrt{2}\,T = \frac{\sqrt{2}}{3} Mg$$

Ex. Consider the double Atwood's machine as shown in the figure



(a) What is acceleration of the masses?

(b) What is the tension in each string?

Sol. (a) Here the system behaves as a rigid system, therefore every part of the system will move with same acceleration. Thus Applying newton's law

$mg - T = ma$ ...(1)

$2T - mg = ma$ ...(2)

Doubling the first equation and adding

$mg = 3\,ma \Rightarrow$ acceleration $a = \frac{1}{3}g$

(b) Tension in the string $T = m\,(g - a) = m\left(g - \frac{g}{3}\right) = \frac{2}{3}\,mg$

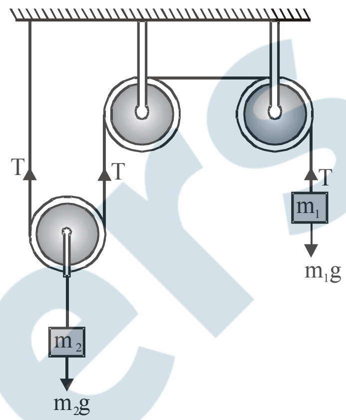
Ex. Consider the system of masses and pulleys shown in fig. with massless string and frictionless pulleys.

(a) Give the necessary relation between masses m_1 and m_2 such that system is in equilibrium and does not move.

(b) If m_1 = 6 kg and m_2 = 8 kg, calculate the magnitude and direction of the acceleration of m_1.

Sol. (a) Applying newton's law $m_2g - 2T = 0$ (because there is no acceleration) and $T - m_1g = 0$

$\Rightarrow (m_2 - 2m_1)\, g = 0 \Rightarrow m_2 = 2m_1$



(b) If the upwards acceleration of m_1 is a,

then acceleration of m_2 is $\frac{a}{2}$ downwards

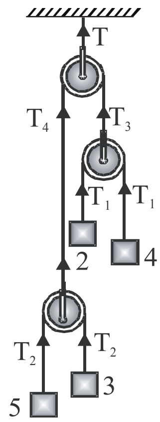
for mass m_2: $m_2g - 2T = m_2\left(\frac{a}{2}\right) \Rightarrow 2m_2g - 4T = m_2a$

for mass m_1 : $T - m_1g = m_1a$

$$\Rightarrow \quad a = \left(\frac{2m_2 - 4m_1}{m_2 + 4m_1}\right)g = \frac{2(8-12)}{8+24}g = -\frac{g}{4}$$

Negative sign shows that acceleration is opposite to considered direction i.e. it is downwards for m_1 and upwards for m_2.

Ex. In the given figure If $T_1 = 2T_2 = 50$ N then find the value of T.



Sol. As given in figure,

$$T_3 = 2T_1 = 2(2T_2) = 4T_2$$

and $\quad T_4 = 2T_2$

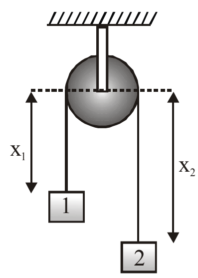
$$\therefore \quad T = T_3 + T_4 = 4T_2 + 2T_2 = 6T_2$$

$$= 6 \times \frac{50}{2} = 150\text{N}$$

CONSTRAINT RELATIONS

These equations establish the relation between accelerations (or velocities) of different masses attached by string(s). Normally number of constraint equations are equal to number of strings in the system under consideration.

Ex. Find the relation between acceleration of 1 and 2.



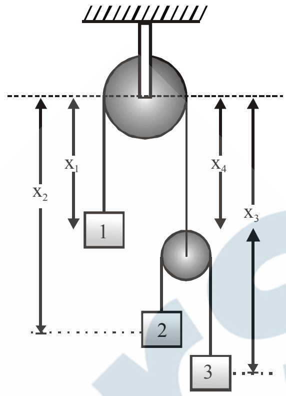
Sol. At any instant of time let x_1 and x_2 be the displacements of 1 and 2 from a fixed line. Then $x_1 + x_2$ = constant

Differentiating w.r.t. time, $v_1 + v_2 = 0$

Again differentiating w.r.t. time, $a_1 + a_2 = 0 \Rightarrow a_1 = -a_2$

So acceleration of 1 and 2 are equal but in opposite directions.

Ex. At certain moment of time, velocities of 1 and 2 both are 1 ms^{-1} upwards. Find the velocity of 3 at that moment.



Sol. $x_1 + x_4 = \ell_1$ (length of first string)

$x_2 - x_4 + x_3 - x_4 = \ell_2$ (length of second string)

$\Rightarrow v_1 + v_4 = 0$ & $v_2 + v_3 - 2v_4 = 0$

$\Rightarrow v_2 + v_3 + 2v_1 = 0$

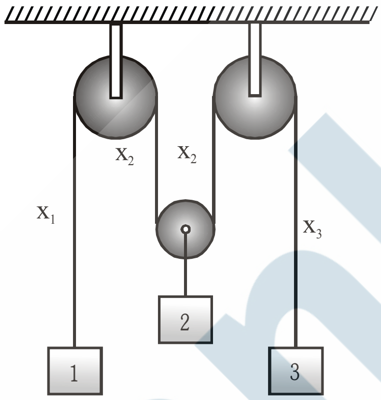
Taking upward direction as positive

$v_1 = v_2 = 1$

so $1 + v_3 + 2 \times 1 = 0 \Rightarrow v_3 = -3\ ms^{-1}$

i.e. velocity of block 3 is 3 ms^{-1} downwards.

Ex. Find the relation between acceleration of blocks a_1, a_2 and a_3.

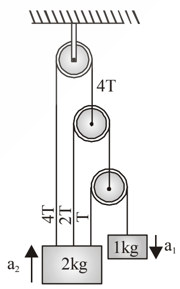


Sol. $x_1 + 2x_2 + x_3 = \ell$

$v_1 + 2v_2 + v_3 = 0$

$a_1 + 2a_2 + a_3 = 0$

Ex. Using contraint equation. Find the relation between a_1 and a_2.

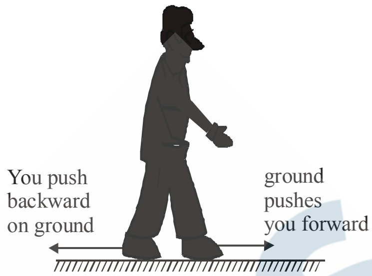


Sol. For this system $a_1T = a_2(4T+2T+T) \Rightarrow a_1=7a_2$

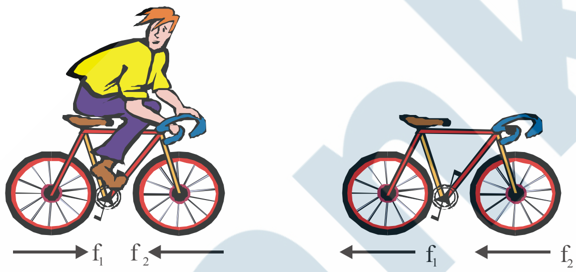
FRICTION

INTRODUCTION

Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.). Frictional forces may also exist between surfaces when there is no relative motion. Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.



- **Supporting :** Walking process can only take place because there is friction between the shoes and ground.
- **Opposing :** When a block slides over a surface the force of friction acts as an opposing force in the opposite direction of the motion
- **Both Supporting and Opposing :**
- **Pedaling :** When cyclist pedals the friction force on rear wheel acts as a supporting force and on front wheel as a opposing force.



Force of friction when pedaling

Force of friction when not pedaling

- **Non–Pedaling :** When cyclist not pedals the friction force on rear wheel & front wheel act as a opposing force.

CAUSE OF FRICTION

- **Old View :** When two bodies are in contact with each other, the irregularities in the surface of one body set interlocked in the irregularites of another surface. This locking opposes the tendency of motion.
- **Modern View :** Friction is arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.

TYPES OF FRICTION

Friction

- Static friction (No relative motion between objects)
- Dynamic friction (Kinetic friction) (There is relative motion between objects)

STATIC FRICTION

- It is the frictional force which is effective before motion starts between two planes in contact with each other.
- It's nature is self adjusting.
- Numerical value of static friction is equal to external force which creates the tendency of motion of body.
- Maximum value of static friction is called limiting friction. $0 \le f_s \le \mu_s N$, $\vec{f}_s = -\vec{F}_{applied}$

LAWS OF LIMITING FRICTION

- The magnitude of the force of limiting friction (F) between any two bodies in contact is directly proportional to the normal reaction (N) between them $F \propto N$
- The direction of the force of limiting friction is always opposite to the direction in which one body is on the verge of moving over the other.
- The force of limiting friction is independent of the apporent contact area, as long as normal reaction between the two bodies in contact remains the same.
- Limiting friction between any two bodies in contact depends on the nature of material of the surfaces in contact and their roughness and smoothness.
- Its value is more than to other types of friction force.

DYNAMIC FRICTION

The friction opposing the relative motion between two bodies is called dynamic or kinetic friction

$$\vec{f}_k = -(\mu_k N)$$

- This is always slightly less than the limitng friction

COEFFICIENT OF FRICTION

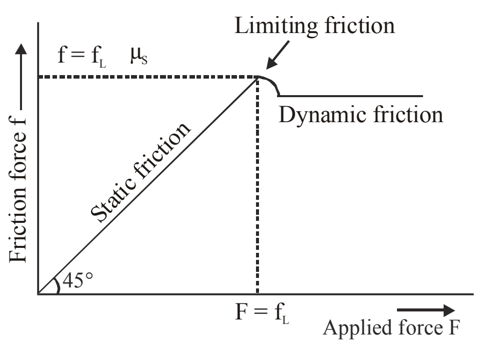
The frictional coefficient is a dimensionless scalar value which describes the ratio of the force of friction between two bodies and the force pressing them together.

- Static friction coefficient $\mu_s = \dfrac{F}{N}$
- Sliding friction coefficient $\mu_k = \dfrac{F_k}{N}$

The values of μ_s and μ_k depend on the nature of both the surfaces in contact.

GRAPH BETWEEN APPLIED FORCE AND FORCE OF FRICTION

If we slowly increase the force with which we are pulling the box, graph shows that the friction force increases with our force upto a certain critical value, f_L, the box suddenly begins to move, and as soon as it starts moving, a smaller force is required to maintain its motion as in motion friction is reduced. The friction value from 0 to f_L is known as static friction, which balances the external force on the body and prevent it from sliding. The value f_L is the maximum limit up to which the static friction acts is known as limiting friction, after which body starts sliding and friction reduces to kinetic friction.



- When two highly polished surfaces are pressed hard, then a situation similar to welding occurs. It is called **cold welding.**
- When two copper plates are highly polished and placed in contact with each other, then instead of decreasing, the force of friction increases. This arises due to the fact that for two highly polished surfaces in contact, the number of molecules coming in contact increases and as a result the cohesive/adhesive forces increases. This in turn, increases the force of friction.

  Net contact force is the resultant of normal reaction and frictional force.

APPROXIMATE COEFFICIENTS OF FRICTION

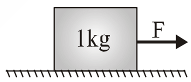
Materials	**Coefficient of static friction, μ_s**	**Coefficien of kinetic friction, μ_k**
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Copper on cast iron	1.05	0.29
Brass on steel	0.51	0.44
Teflon on teflon	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

Ex. A block of mass 1 kg is at rest on a rough horizontal surface having coefficient of static friction 0.2 and kinetic friction 0.15, find the frictional forces if a horizontal force,

(a) $F = 1N$

(b) $F = 1.96$ N

(c) $F = 2.5$ N , is applied on a block



Sol. Maximum force of friction $f_{max} = 0.2 \times 1 \times 9.8\ N = 1.96$ N

(a) for $F_{ext} = 1$ N, $F_{ext} < f_{max}$

So, body is in rest means static friction is present and hence $f_s = F_{ext} = 1$ N

(b) for $F_{ext} = 1.96$ N, $F_{ext} = f_{max} = 1.96$ N so $f = 1.96$ N

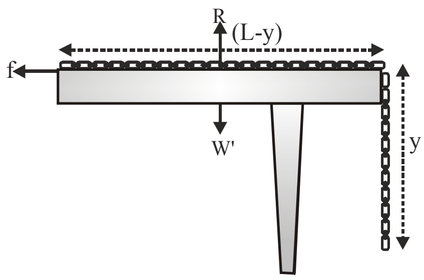
(c) for $F_{ext} = 2.5$ N, so $F_{ext} > f_{max.}$

now body is in moving condition

$\therefore\ f_{max.} = f_k = \mu_k N = \mu_k\ mg = 0.15 \times 1 \times 9.8 = 1.47$ N

Ex. Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can be hang from the table without sliding.

Sol. Let y be the maximum length of the chain can be hold out side the table without sliding.



Length of chain on the table = (L – y)

Weight of part of the chain on table $W' = \frac{M}{L}(L-y)g$

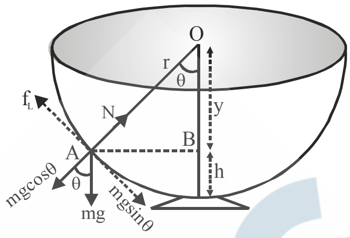
Weight of hanging part of the chain $W = \frac{M}{L}yg$

For equlibrium : limiting force of friction = weight of hanging part of the chain

$\mu R = W \Rightarrow \quad \mu W' = W \quad \Rightarrow \mu\frac{M}{L}(L-y)g = \frac{M}{L}yg \Rightarrow \mu L - \mu y = y \Rightarrow y = \frac{\mu L}{1+\mu}$

Ex. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bowl.

Sol. The insect will crawl up the bowl till the component of its weight along the bowl is balanced by limiting frictional force. So, resolving weight perpendicular to the bowl and along the bowl,



$$N = mg\cos\theta\ ,\ f_L = mg\sin\theta \Rightarrow \tan\theta = \frac{f_L}{N} \Rightarrow \tan\theta = \mu\ \ [\because f_L = \mu N]$$

$$\Rightarrow \sqrt{\frac{(r^2 - y^2)}{y}} = \mu \Rightarrow y = \frac{r}{\sqrt{1+\mu^2}} \quad \text{So } h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$$

Ex. A body of mass M is kept on a rough horizontal surface (friction coefficient = μ). A person is trying to pull the body by applying a horizontal force F, but the body is not moving. What is the force by the surface on A.

Sol. Let f is the force of friction and N is the normal reaction,

then the net force by the surface on the body is $F = \sqrt{N^2 + f^2}$

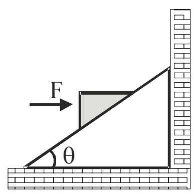
Let the applied force is F′ (varying), applied horizontally then $f \le \mu_s N$ (adjustable with f = F′). Now if F′ is zero, f = 0 and F_{min} = N = Mg

and when F′ is increased to maximum value permissible for no motion. $f = \mu_s N$,

giving $F_{max} = \sqrt{N^2 + \mu_s^2 N^2} = Mg\sqrt{1+\mu_s^2}$

therefore we can write $\quad Mg \le F \le Mg\sqrt{1+\mu_s^2}$

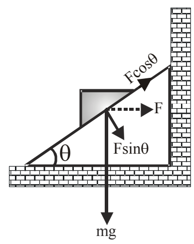
Ex. A block rest on a rough inclined plane as shown in fig. A horizontal force F is applied to it (a) Find out the force of reaction, (b) Can the force of friction be zero if yes when? and (c) Assuming that friction is not zero find its magnitude and direction of its limiting value.



Sol. (a) N = mg cos θ + F sin θ

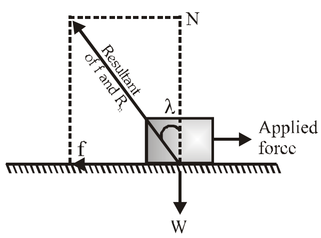
(b) Yes, if mg sin θ = F cos θ

(c) f = μR = μ (mg cos θ + F sin θ); up the plane if the body has tendency to slide down and down the plane if the body has tendency to move up.



ANGLE OF FRICTION

The angle of friction is the angle which the resultant of limiting friction f_S and normal reaction N makes with the normal reaction. It is represented by λ $\tan\lambda = \frac{f_S}{N} = \frac{\mu N}{N} = \mu$



- For smooth surface λ = 0

ANGLE OF REPOSE (θ)

If a body is placed on an inclined plane and if its angle of inclination is gradually increased, then at some angle of inclination θ the body will just on the point to slide down. The angle is called angle of repose (θ).

$F_S = mg\ \sin\theta$ and $N = mg\ \cos\theta$

so $\frac{F_S}{N} = \tan\theta \Rightarrow \mu = \tan\theta$

Relation between angle of friction (λ) and angle of repose (θ)

$\tan\lambda = \mu$ and $\mu = \tan\theta$, hence $\tan\lambda = \tan\theta \Rightarrow \theta = \lambda$

Thus, angle of repose = angle of friction

Ex. A block of mass 2 kg slides down an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.

(i) What force must be applied to the block so that the block moves down the plane without acceleration ?

(ii) What force should be applied to the block so that it can move up without any acceleration ?

Sol. Make a 'free–body' diagram of the block. Take the force of friction opposite to the direction of motion.

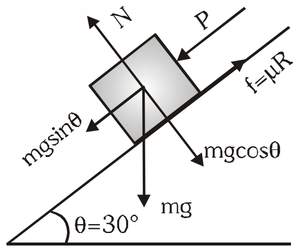
(i) Project forces along and perpendicular to the plane

perpendicular to plane $N = mg\cos\theta$

along the plane $F + mg\sin\theta - f = 0$

(∵ there is no acceleration along the plane)

$F + mg\sin\theta - \mu N = 0 \Rightarrow F + mg\sin\theta = \mu\, mg\cos\theta$



$F = mg(\mu\cos\theta - \sin\theta) = 2 \times 9.8\left(\frac{\sqrt{3}}{2}\cos 30° - \sin 30°\right)$

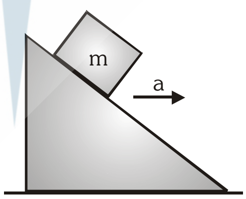
$= 19.6\left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 19.6\left(\frac{3}{4} - \frac{1}{2}\right) = 4.9\ \text{N}$

(ii) This time the direction of F is reversed and that of the frictional force is also reversed.

$\therefore\ N = mg\cos\theta\ ;\ F = mg\sin\theta + f$

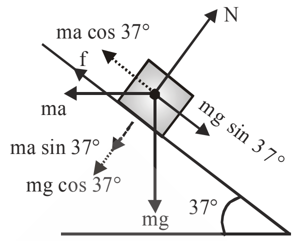
$\Rightarrow F = mg(\mu\cos\theta + \sin\theta) = 19.6\left(\frac{3}{4} + \frac{1}{2}\right) = 24.5\ \text{N}$

Ex. A block of mass 1 kg sits on an incline as shown in figure.



(a) What must be the frictional force between block and incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 m/s^2 ?

(b) What is the least value μ_s can have for this to happen ?



Sol. N = m (g cos37° + a sin37°) = 1(9.8 × 0.8 + 3 × 0.6) = 9.64 N

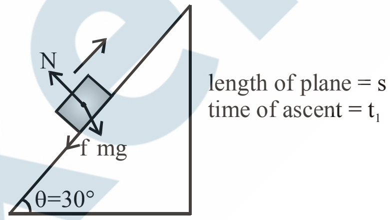
mg sin 37° = ma cos 37° + f

(a) f = 1(9.8 × 0.6 – 3 × 0.8) = 3.48

(b) $\because$ f = μN $\qquad \therefore \mu = \dfrac{f}{N} = \dfrac{3.48}{9.64} = 0.36$

Ex. A body of mass 5 × 10^{-3} kg is launched up on a rough inclined plane making an angle of 30° with the horizontal. Obtain the coefficient of friction between the body and the plane if the time of ascent is half of the time of descent.

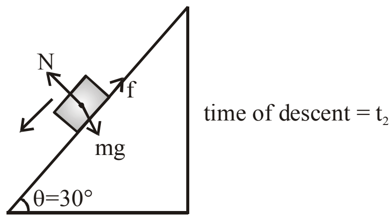
Sol. For upward motion: upward retardation $a_1 = \dfrac{\mu N + mg\sin\theta}{m}$



$a_1 = \mu g\cos 30° + g\sin 30° = (\sqrt{3}\,\mu + 1)\dfrac{g}{2}$

$\because s = \dfrac{1}{2}a_1 t_1^2 \quad \therefore t_1 = \sqrt{\dfrac{2s}{a_1}} = \sqrt{\dfrac{4s}{(\sqrt{3}\mu + 1)g}}$

For downward motion: downward acceleration $a_2 = \dfrac{mg\sin\theta - \mu N}{m}$



$a_2 = g\sin 30° - g\cos 30° = (1 - \sqrt{3}\,\mu)\dfrac{g}{2}$

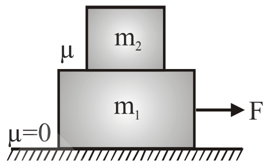
$\Rightarrow t_2 = \sqrt{\dfrac{2s}{a_2}} = \sqrt{\dfrac{4s}{(1-\sqrt{3}\mu)g}}$

Now according to question $2t_1 = t_2$

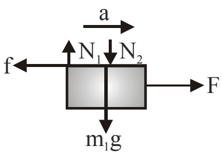
$$\Rightarrow 2\sqrt{\dfrac{4s}{(\sqrt{3}\mu+1)g}} = \sqrt{\dfrac{4s}{(1-\sqrt{3}\mu)g}}$$

$$\Rightarrow \dfrac{1-\sqrt{3}\mu}{1+\sqrt{3}\mu} = \dfrac{1}{4} \Rightarrow \mu = \dfrac{\sqrt{3}}{5}$$

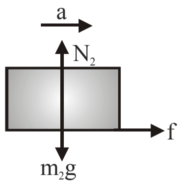
Ex. When force F applied on m_1 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ. What should be the minimum value of F so that there is no relative motion between m_1 and m_2



Sol. For m_1



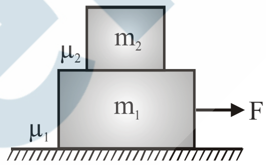
for m_2



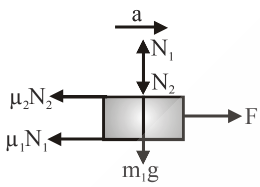
For system acceleration $a = \dfrac{F}{m_1 + m_2}$

For m_2 $f = m_2a \Rightarrow \mu m_2 g = m_2\left(\dfrac{F}{m_1 + m_2}\right) \Rightarrow F_{min} = \mu(m_1 + m_2)\ g$

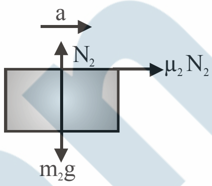
Ex. When force F applied on m_1 and the coefficient of friction between m_1 and surface. is μ_1 and the coefficient of friction between m_1 and m_2 is μ_2. What should be the minimum value of F so that there is no relative motion between m_1 and m_2 .



Sol. For m_1



for m_2

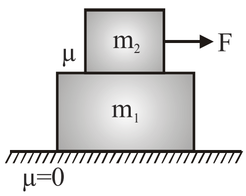


For system $a = \dfrac{F - \mu_1(m_1 + m_2)g}{m_1 + m_2}$

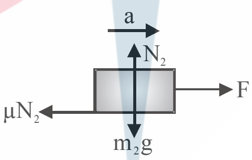
For m_2, $\mu_2\ (m_2g) = m_2a = m_2\left(\dfrac{F - \mu_1(m_1 + m_2)g}{m_1 + m_2}\right)$

$\Rightarrow \quad F_{min} = (m_1 + m_2)\ (\mu_1 + \mu_2)g$

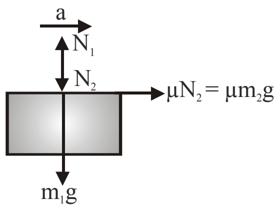
Ex. When force F applied on m_2 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ.. What should be the minimum value of F so that there is no relative motion between m_1 and m_2



Sol. For m_2



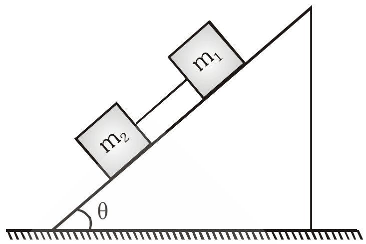
For m_1



for system : acceleration $= \dfrac{F}{m_1 + m_2}$

for m_1 : $\mu m_2 g = m_1 a = m_1\left(\dfrac{F}{m_1 m_2}\right)$, $F_{min} = (m_1 + m_2)\left(\dfrac{\mu m_2 g}{m_1}\right)$

Ex. Two blocks with masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a string and side down a plane inclined at an angle $\theta = 45^0$ with the horizontal. The coefficient of sliding friction between m_1 and plane is $\mu_1 = 0.4$, and that between m_2 and plane is $\mu_2 = 0.2$. Calculate the common acceleration of the two blocks and the tension in the string.



Sol. As $\mu_2 < \mu_1$, block m_2 has greater acceleration than m_1 if we separately consider the motion of blocks. But they are connected so they move together as a system with common acceleration. So acceleration of the blocks :

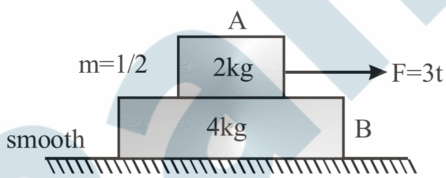
$$a = \frac{(m_1+m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1+m_2}$$

$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4\times1\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}}}{1+2} = \frac{22}{3\sqrt{2}}\ ms^{-2}$$

For block m_2 : $m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - T = m_2 a \Rightarrow T = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - m_2 a$

$$= 2\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}} - 2\times\frac{22}{3\sqrt{2}} = \frac{2}{3\sqrt{2}}\,N$$

Ex. For shown situation draw a graph showing accelerations of A and B on y–axis and time on x–axis. $(g=10ms^{-2})$



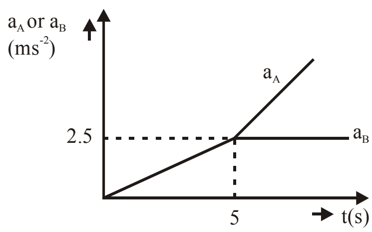
Sol. Limiting friction between A & B, $f_L = \mu m_A g = \left(\frac{1}{2}\right)(2)(10) = 10$ N

Block B moves due to friction only. So maximum acceleration of B,

$$a_{max} = \frac{f_L}{m_B} = \frac{10}{4} = 2.5\ ms^{-2}.$$

So both the blocks move together till the common acceleration becomes 2.5 ms^{-2}, after that acceleations of B will become constant while that of A will go on increasing. Slipping will starts between A & B at 2.5 ms^{-2}

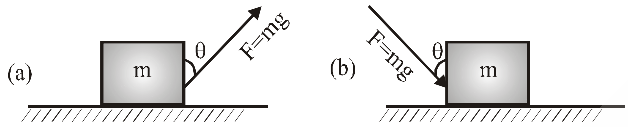
$$\Rightarrow 2.5 = \frac{F}{m_A+m_B} = \frac{3t}{6} \Rightarrow t = 5\ s$$



Hence for $t \le 5$s, $a_A = a_B = \frac{F}{m_A+m_B} = \frac{3t}{6} = \frac{t}{2}$

and for $t > 5$ s $a_B = 2.5\ ms^{-2}$, $a_A = \frac{F-f_L}{m_A} = \frac{3t-10}{2} = \frac{3}{2}t - 5$

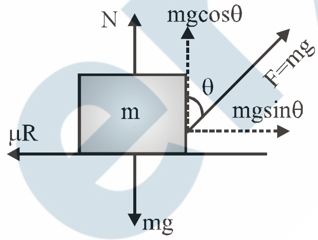
Ex. A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between block and surface is μ. A force $F = mg$ acting at an angle θ with the vertical side of the block. Find the condition for which block will move along the surface.



Sol. For (a) : normal reaction $N = mg - mg\cos\theta$, frictional force $= \mu N = \mu(mg - mg\cos\theta)$

Now block can be pulled when : Horizontal component of force $\geq$ frictional force

i.e. $mg\sin\theta \geq \mu(mg - mg\cos\theta)$



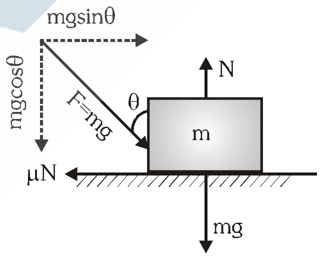
or $2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq \mu(1-\cos\theta)$

or $2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq 2\mu\sin^2\dfrac{\theta}{2}$ or $\cot\dfrac{\theta}{2} \geq \mu$

For (b) **:** Normal reaction $N = mg + mg\cos\theta = mg(1 + \cos\theta)$

Hence, block can be pushed along the horizontal surface when horizontal component of force $\geq$ frictional force

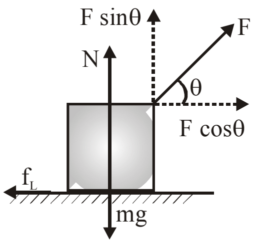
i.e. $mg\sin\theta \geq \mu\, mg(1 + \cos\theta)$



or $2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq \mu \times 2\cos^2\dfrac{\theta}{2} \Rightarrow \tan\dfrac{\theta}{2} \geq \mu$

Ex. A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

Sol. Let the force F be applied at an angle θ with the horizontal as shown in figure.



For vertical equilibrium,

$R + F\sin\theta = mg \Rightarrow N = mg - F\sin\theta$ (i)

for horizontal motion $F\cos\theta \geq f_L \Rightarrow F\cos\theta \geq \mu N$ [as $f_L = \mu N$] (ii)

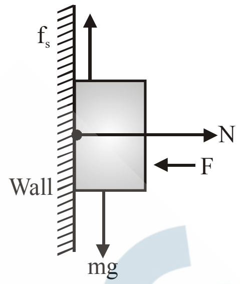
substituting value of R from equation (i) in (ii),

$$F\cos\theta \geq \mu(mg - F\sin\theta) \Rightarrow F \geq \frac{\mu mg}{(\cos\theta + \mu\sin\theta)} \quad \text{(iii)}$$

For the force F to be minimum $(\cos\theta + \mu\sin\theta)$ must be maximum,

maximum value of $\cos\theta + \mu\sin\theta$ is $\sqrt{1+\mu^2}$ so that $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$ with $\theta = \tan^{-1}(\mu)$

Ex. A book of 1 kg is held against a wall by applying a perpendicular force F. If $\mu_S = 0.2$ then what is the minimum value of F ?



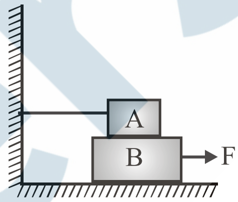
Sol. The situation is shown in fig. The forces acting on the book are–

For book to be at rest it is essential that $\quad Mg = f_s$

But $\quad f_{s\,max} = \mu_S N \quad$ and $N = F$

$\therefore\ Mg = \mu_S F \Rightarrow F = \dfrac{Mg}{\mu_s} = \dfrac{1\times 9.8}{0.2} = 49\ N$

Ex. A is a 100 kg block and B is a 200 kg block. As shown in fig., the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. (take g =10 m/s^2).

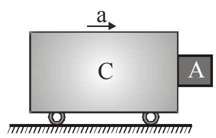


Sol. When B is tied to move, by applying a force F, then the frictional forces acting on the block B are f_1 and f_2 with limiting values, $f_1 = (\mu_S)_A m_A g$ and $f_2 = (\mu_S)_B\,(m_A + m_B)g$

then minimum value of F should be (for just tending to move),

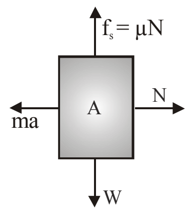
$F = f_1 + f_2 = 0.2 \times 100\ g + 0.3 \times 300\ g = 110\ g = 1100\ N$

Ex. Consider the figure shown here of a moving cart C. If the coefficient of friction between the block A and the cart is μ, then calculate the minimum acceleration a of the cart C so that the block A does not fall.



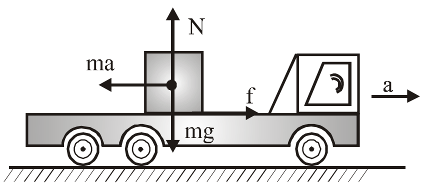
Sol. The forces acting on the block A (in block A's frame (i.e. non inertial frame) are :

For A to be at rest in block A's frame i.e. no fall,



we require $W = f_s \quad \Rightarrow mg = \mu(ma) \quad$ Thus $a = \dfrac{g}{\mu}$

Ex. A block of mass 1kg lies on a horizontal surface in a truck, the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block. If the acceleration of the truck is 5 m/s^2.



Sol. Fictitious force on the block $F = ma = 1 \times 5 = 5N$

While the limiting friction force

$F = \mu_s N = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88\ N$

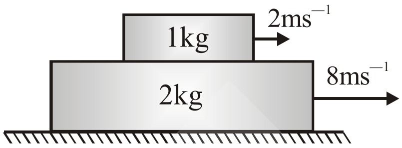
As required force F lesser than limiting friction force. The block will remain at rest in the truck and the force of friction will be equal to 5N and in the direction of acceleration of the truck.

Ex. Coefficient of friction between two blocks shown in figure is $\mu = 0.4$ The blocks are given velocities of $2ms^{-1}$ and $8\ ms^{-1}$ in the directions shown in figure. Find

(i) The time when relative motion between them will stop.

(ii) The common velocities of blocks upto that instant.

(iii) Displacement of blocks upto that instant ($g = 10\ ms^{-2}$)



Sol. (i) Frictional force between two blocks will oppose the relative motion. For 1 kg block friction support the motion & for 2 kg friction oppose the motion. Let common velocity be v then

for 1 kg $\quad v = 2 + a_1 t$ where $a_1 = \dfrac{\mu(1g)}{1} = \dfrac{0.4\times 10}{1} = 4\ ms^{-2}$

for 2 kg $v = 8 - a_2\, t$ where $a_2 = \dfrac{\mu(1g)}{2} = \dfrac{0.4\times 10}{2} = 2\ ms^{-2} \Rightarrow\ 2 + 4t = 8 - 2\,t \Rightarrow\ 6t = 6 \Rightarrow\ t = 1s$

(ii) $v = 2 + 4\,t = 2 + 4 \times 1 = 6\ ms^{-1}$

(iii) Displacement of 1 kg block from rest

$$s = ut + \frac{1}{2}at^2 \Rightarrow s_1 = 2 \times 1 + \frac{1}{2} \times 4 \times 1^2 = 2 + 2 = 4m$$

Displacement of 2 kg block from rest

$$s = ut + \frac{1}{2}at^2 \Rightarrow s_2 = 8 \times 1 - \frac{1}{2} \times 2 \times 1^2 = 8 - 1 = 7\ m$$

Friction is a Necessary Evil :

Friction is a necessary evil. It means it has advantage as well as disadvantages. In other words, friction is not desirable but without friction, we cannot think of survival.

Disadvantages :

(i) A significant amount of energy of a moving object is wasted in the form of heat energy to overcome the force of friction.

(ii) The force of friction restricts the speed of moving vehicles like buses, trains, aeroplanes, rockets etc.

(iii) The efficiency of machines decreases due to the presence of force of friction.

(iv) The force of friction causes lot of wear and tear in the moving parts of a machine.

(v) Sometimes, a machine gets burnt due to the friction force between different moving parts.

Advantages :

(i) The force of friction helps us to move on the surface of earth. In the absence of friction, we cannot think of walking on the surface. That is why, we fall down while moving on a smooth surface.

(ii) The force of friction between the tip of a pen and the surface of paper helps us to write on the paper. It is not possible to write on the glazed paper as there is not force of friction.

(iii) The force of friction between the tyres of a vehicle and the road helps the vehicle to stop when brake is applied. In the absence of friction, the vehicle skid off the road when brake is applied.

(iv) moving belts remain on the rim of a wheel because of friction.

(v) The force of friction between a chalk and the black board helps us to write on the board.

Thus, we observe that inspect of various disadvantages of the friction, it is very difficult to part with it. So, friction is a necessary evil.

METHODS OF REDUCING FRICTION

As friction causes the wastage of energy so it becomes necessary to reduce the friction. Friction can be reduced by the following methods.

(i) Polishing the surface. We know, friction between rough surface is much more than between the polished surfaces. So we polish the surface to reduce the friction. The irregularities on the surface are filled with polish and hence the friction decreases.

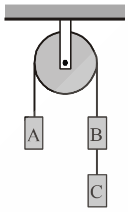
(ii) Lubrication. To reduce friction, lubricants like oil or greese are used. When the oil or greese is put in between the two surfaces, the irregularities remain apart and do not interlock tightly. Thus, the surface can move over each other with less friction between them.

(iii) By providing the streamlined shape. When a body (e.g. bus, train, aeroplane etc.) moves with high speed, air resistance (friction) opposes its motion. The effect of air resistance on the motion of the objects (stated above) is decreased by providing them a streamlined shape.

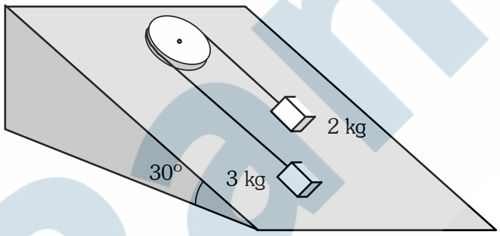
EXERCISE (S-1)

1. A force F applied to an object of mass m_1 produces an acceleration of 3.00 m/s^2. The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s^2.
(i) What is the value of the ratio m_1/m_2 ?
(ii) If m_1 and m_2 are combined, find their acceleration under the action of the force F.

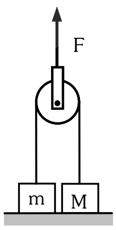
2. In the system shown, the blocks A, B and C are of weight 4W, W and W respectively. The system set free. The tension in the string connecting the blocks B and C is



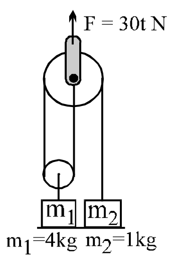
3. Two blocks of masses 2.0 kg and 3.0 kg are connected by light inextensible string. The string passes over an ideal pulley pivoted to a fixed axel on a smooth incline plane as shown in the figure. When the blocks are released, find magnitude of their accelerations.



4. In the system shown, pulley and strings are ideal. The vertically upward pull F is being increased gradually, find magnitude of F and acceleration of the 5 kg block at the moment the 10 kg block leaves the floor.

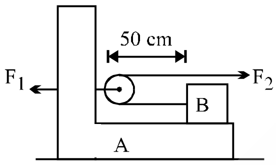


5. Force F is applied on upper pulley. If $F = 30t\ N$ where t is time in second. Find the time when m_1 loses contact with floor.

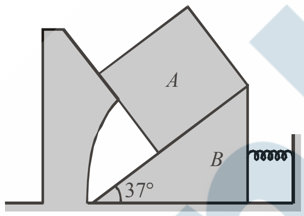


6. A 40 kg boy climbs a rope that passes over an ideal pulley. The other end of the rope is attached to a 60 kg weight placed on the ground. What is the maximum upward acceleration the boy can have without lifting the weight? If he climbs the rope with upward acceleration 2 g, with what acceleration the weight will rise up?

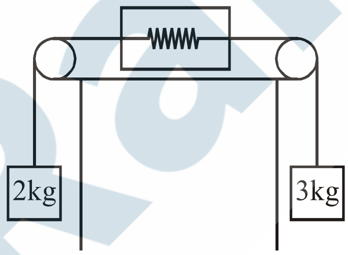
7. A 1kg block B rests as shown on a bracket A of same mass. Constant forces $F_1 = 20$N and $F_2 = 8$N start to act at time t = 0 when the distance of block B from pulley is 50cm. Time when block B reaches the pulley is _________.



8. In the figure shown, all surfaces are smooth and block A and wedge B have mass 10 kg and 20 kg respectively. Find normal reaction between block A & B, spring force and normal reaction of ground on block B. (g= 10 m/s^2).



9. Find the reading of the massless spring balance in the given condition



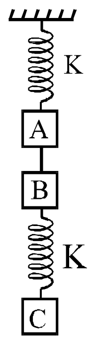
10. The system shown adjacent is in equilibrium. Find the acceleration of the blocks A, B & C all of equal masses m at the instant when (Assume springs to be ideal)

(i) The spring between ceiling & A is cut.

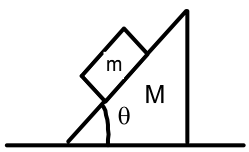
(ii) The string (inextensible) between A & B is cut.

(iii) The spring between B & C is cut.

Also find the tension in the string when the system is at rest and in the above 3 cases.

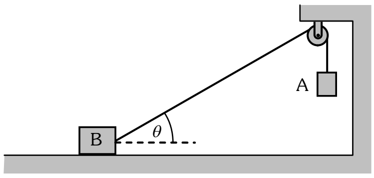


11. A block of mass m lies on wedge of mass M as shown in figure.

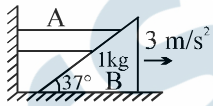


With what minimum acceleration must the wedge be moved towards right horizontally so that block m falls freely.

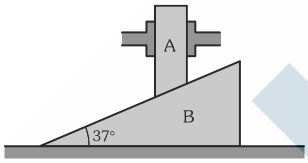
12. The block A is moving downward with constant velocity v_0. Find the velocity of the block B, when the string makes an angle θ with the horizontal



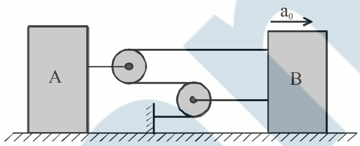
13. Find force in newton which mass A exerts on mass B if B is moving towards right with 3 m/s^2. Also find mass of A. (All surfaces are smooth)



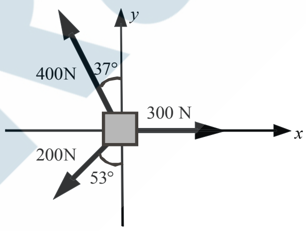
14. Rod A can slide in vertical direction pushing the triangular wedge B towards right. The wedge is moving toward right with uniform acceleration a_B. Find acceleration of the rod A.



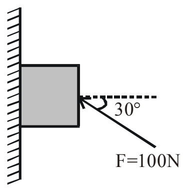
15. Calculate the relative acceleration of A w.r.t. B if B is moving with acceleration a_0 towards right.



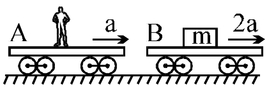
16. A block is placed on a rough horizontal plane. Three horizontal forces are applied on the block as shown in the figure. If the block is in equilibrium, find the friction force acting on the block.



17. A force of 100 N is applied on a block of mass 3 kg as shown in figure. The coefficient of friction between the wall and the surface of the block is 1/4. Calculate frictional force acting on the block.



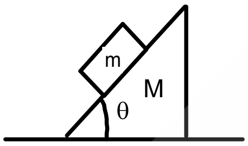
18. Two trolley A and B are moving with accelerations a and 2a respectively in the same direction. To an observer in trolley A, the magnitude of pseudo force acting on a block of mass m on the trolley B is



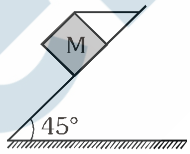
19. A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration 4 m/s^2. A bead can slide on the rod, and friction coefficient between them is 1/2. If the bead is released from rest at the top of the rod, find the time when it will reach at the bottom.[g = 10 m/s^2]

20. A block of mass 1 kg is horizontally thrown with a velocity of 10 m/s on a stationary long plank of mass 2 kg whose surface has μ = 0.5. Plank rests on frictionless surface. Find the time when block comes to rest w.r.t. plank.

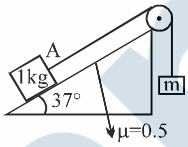
21. A block of mass *m* lies on wedge of mass *M* as shown in figure. Find the minimum friction coefficient required between wedge *M* and ground so that it does not move while block *m* slips down on it.



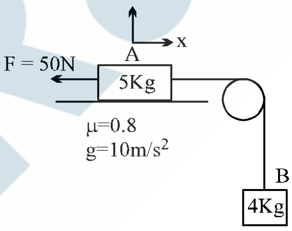
22. A block of mass 15 kg is resting on a rough inclined plane as shown in figure.The block is tied up by a horizontal string which has a tension of 50N. Calculate the minimum coefficient of friction between the block and inclined plane.



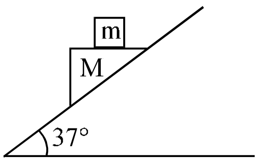
23. In the figure, what should be mass m so that block *A* slides up with a constant velocity?



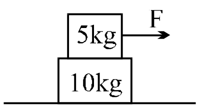
24. Find the acceleration of the blocks and magnitude & direction of frictional force between block *A* and table, if block *A* is pulled towards left with a force of 50 N.



25. Block *M* slides down on frictionless incline as shown. Find the minimum friction coefficient so that *m* does not slide with respect to *M*.

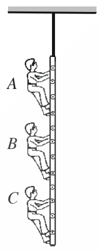


26. Coefficient of friction between 5 kg and 10 kg block is 0.5. If friction between them is 20 N. What is the value of force being applied on 5 kg. The floor is frictionless.

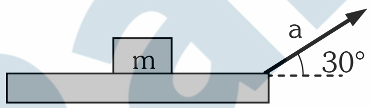


EXERCISE (S-2)

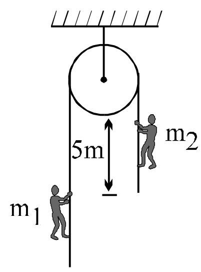
1. A ladder is hanging from ceiling as shown in figure. Three men *A*, *B* and *C* of masses 40 kg, 60 kg, and 50 kg are climbing the ladder. Man *A* is going up with retardation 2 m/s^2, *C* is going up with an acceleration of 1 m/s^2 and man *B* is going up with a constant speed of 0.5 m/s. Find the tension in the string supporting the ladder. [g = 9.8 m/s^2]



2. A box of mass *m* is placed on a smooth horizontal platform as shown in the figure. The platform is made to move in direction 30° above the horizontal with acceleration *a* so that the contact force between the box and the platform becomes $3mg/2$. Find the magnitude of the acceleration.



3. Two men of masses m_1 and m_2 hold on the opposite ends of a rope passing over a frictionless pulley. The man m_1 climbs up the rope with an acceleration of 1.2m/s^2 relative to the rope. The mann m_2 climbs up the rope with an acceleration of 2 m/s^2 relative to the rope. Find the tension in the rope if m_1 = 40 kg and m_2 = 60 kg. Also find the time after which they will be at same horizontal level if they start from rest and are initially separated by 5 m.



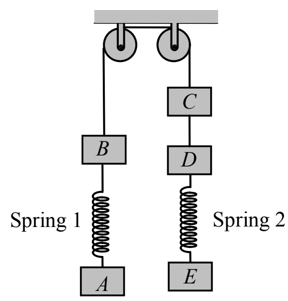
4. The system shown in the figure is initially in equilibrium. *A* is of mass 2*m* and *B*,*C*, *D* and *E* are of mass *m*. Certain actions are performed on the system. Every action has been taken individually when the system is intact. Find the direction and magnitude of acceleration of the blocks after each action of the following actions has been taken

(i) Spring 1 is cut

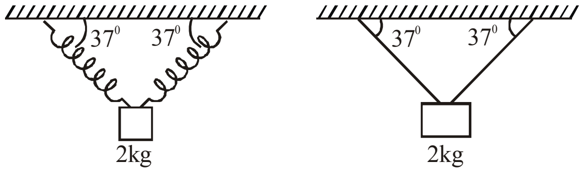
(ii) Spring 2 is cut

(iii) String between *C* and *D* is cut.

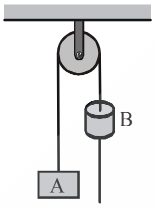
(iv) String between *B* and *C* is cut.



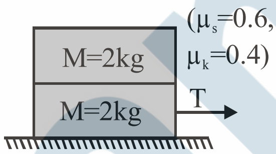
5. The blocks are of mass 2 kg shown is in equilibrium. At $t = 0$ right spring in figure (i) and right string in figure (ii) breaks. Find the ratio of instantaneous acceleration of blocks?



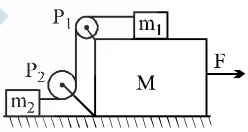
6. A 2 kg block A is attached to one end of a light string that passes over an an ideal pulley and a 1 kg sleeve B slides down the other part of the string with an acceleration of 5 m/s^2 with respect to the string. Find the acceleration of the block, acceleration of sleeve and tension in the string. [$g = 10$ m/s^2]



7. The coefficient of static and kinetic friction between the two blocks and also between the lower block and the ground are $\mu_S = 0.6$ and $\mu_K = 0.4$. Find the value of tension T applied on the lower block at which the upper block begins to slip relative to lower block.



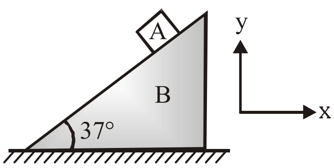
8. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg respectively. The co-efficient of friction between M and ground is zero. The co-efficient of friction between m_1 and M and that between m_2 and ground is 0.3. The pulleys and the string are massless. The string is perfectly horizontal between P_1 and m_1 and also between P_2 and m_2. The string is perfectly vertical between P_1 and P_2.An external horizontal force F is applied to the mass M. Take $g = 10$ m/s^2. **[IIT-JEE 2000]**



(i) Draw a free-body diagram for mass M, clearly showing all the forces.

(ii) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2\,f_2$. Find f_1 and f_2 . Write down equations of motion of all the masses . Find F, tension in the string and accelerations of the masses.

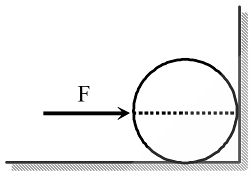
9. In the figure shown the acceleration of A is, $\vec{a}_A = (15\hat{i} + 15\hat{j})$ m/s^2. If A is sliding on B then the acceleration of B is.

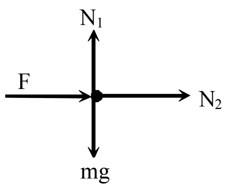


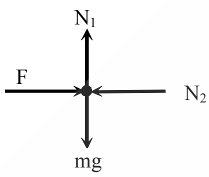
EXERCISE (O-1)

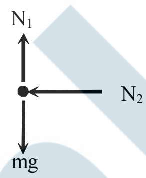
SINGLE CORRECT TYPE QUESTIONS

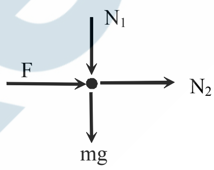
1. A ball of mass m kept at the corner as shown in the figure, is acted by a horizontal force F. The correct free body diagram of ball is



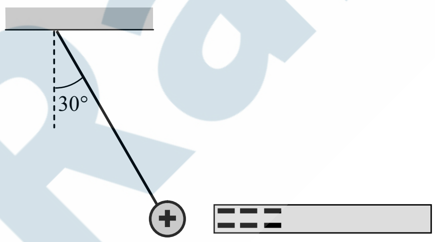
(A) 

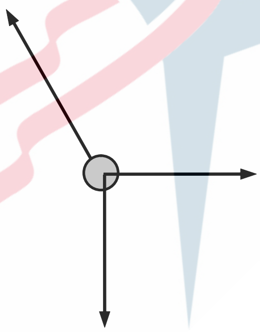
(B) 

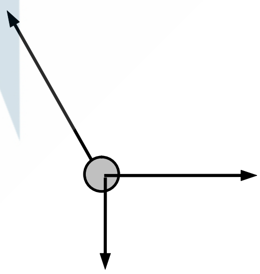
(C) 

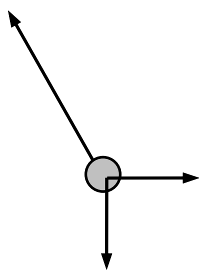
(D) 

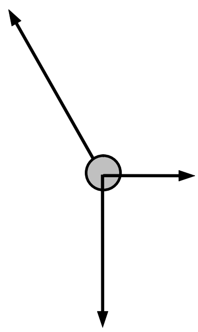
2. A small electrically charged sphere is suspended vertically from a thread. An oppositely charged rod is brought close to the sphere such that the sphere is in equilibrium displaced from the vertical by an angle of 30°. Which one of the following best represents the free body diagram for the sphere?



(A) 

(B) 

(C) 

(D) 

3. Under what condition(s) will an object be in equilibrium ?

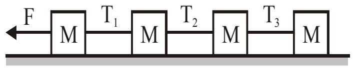
(A) Only if it is at rest

(B) Only if it is moving with constant velocity

(C) Only if it is moving with constant acceleration

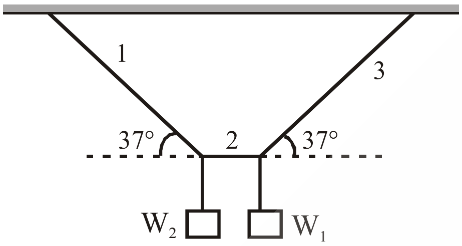
(D) If it is either at rest or moving with constant velocity

4. Four blocks of same mass connected by cords are pulled by force F on a smooth horizontal surface, as in figure. The tension T_1, T_2 and T_3 will be



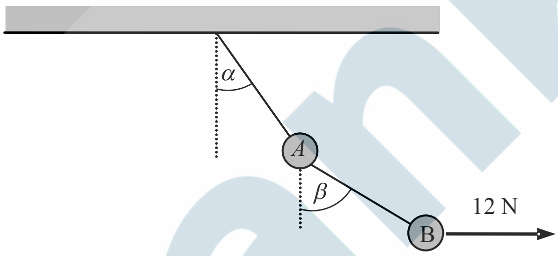
(A) $T_1 = F/4$, $T_2 = 3F/2$, $T_3 = F/4$
(B) $T_1 = F/4$, $T_2 = F/2$, $T_3 = F/2$
(C) $T_1 = 3F/4$, $T_2 = F/2$, $T_3 = F/4$
(D) $T_1 = 3F/4$, $T_2 = F/2$, $T_3 = F/2$

5. In a given figure system is in equilibrium. If $W_1 = 300$ N. Then W_2 is approximately equal to



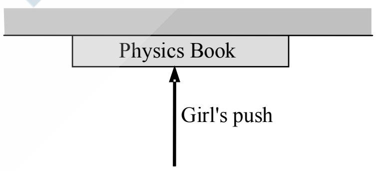
(A) 500 N
(B) 400 N
(C) 670 N
(D) 300 N

6. Two balls A and B weighing 7 N and 9 N are connected by a light cord. The system is suspended from a fixed support by connecting the ball A with another light cord. The ball B is pulled aside by a horizontal force 12 N and equilibrium is established. Angles α and β respectively are



(A) 30° and 60°
(B) 60° and 30°
(C) 37° and 53°
(D) 53° and 37°

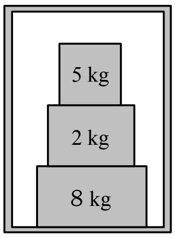
7. A girl pushes her physics book up against the horizontal ceiling of her room as shown in the figure. The book weighs 20 N and she pushes upwards with a force of 25 N. The choices below list the magnitudes of the contact force F_{CB} between the ceiling and the book, and F_{BH} between the book and her hand. Select the correct pair.



(A) $F_{CB} = 20$ N and $F_{BH} = 25$ N
(B) $F_{CB} = 25$ N and $F_{BH} = 45$ N
(C) $F_{CB} = 5$ N and $F_{BH} = 25$ N
(D) $F_{CB} = 5$ N and $F_{BH} = 45$ N

8. Two astronauts A and B connected with a rope stay stationary in free space relative to their spaceship. Mass of A is more than that of B and the rope is straight. Astronaut A starts pulling the rope but astronaut B does not. If you were the third astronaut in the spaceship, what do you observe?
(A) Astronaut B accelerates towards A and A remains stationery.
(B) Both accelerate towards each other with equal accelerations of equal modulus.
(C) Both accelerate towards each other but acceleration of B is greater than that of A.
(D) Both accelerate towards each other but acceleration of B is smaller than that of A.

9. Three boxes are placed in a lift. When acceleration of the lift is 4 m/s^2, the net force on the 8 kg box is closest to

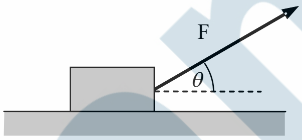


(A) 80 N (B) 48 N (C) 40 N (D) 32 N

10. A man is standing on a weighing machine with a block in his hand. The machine records w. When he takes the block upwards with some acceleration the machine records w_1. When he takes the block down with some acceleration, the machine records w_2.Then choose correct option

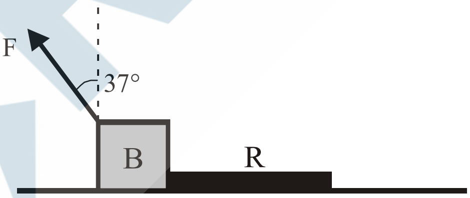
(A) $w_1 = w = w_2$ (B) $w_1 < w < w_2$ (C) $w_2 < w < w_1$ (D) $w_2 = w_1 > w$

11. A block is being pulled by a force F on a long frictionless level floor. Magnitude of the force is gradually increases from zero until the block lifts off the floor. Immediately before the block leaves the floor, its acceleration is



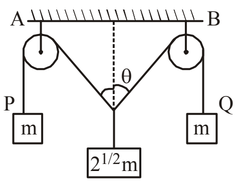
(A) gcosθ (B) gcotθ (C) gsinθ (D) g tanθ

12. A block B is tied to one end of a uniform rope R as shown. The mass of block is 2 kg and that of rope is 1 kg. A force F = 15 N is applied at angle 37° with vertical. The tension at the mid-point of rope is



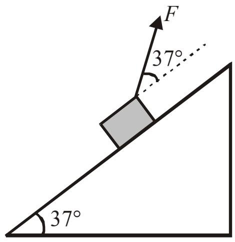
(A) 1.5 N (B) 2 N (C) 3N (D) 4.5 N

13. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be **[JEE (Scr) 2001]**



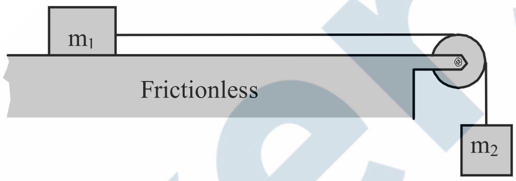
(A) 0° (B) 30° (C) 45° (D) 60°

14. A block resting on a smooth inclined plane is acted upon by a force F as shown. If mass of block is 2 kg and $F = 20$ N and $\sin 37° = 3/5$, the acceleration of block is



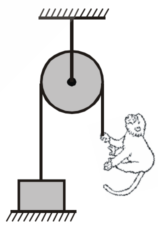
(A) 2 m/s^2 (B) 6 m/s^2 (C) 8 m/s^2 (D) zero

15. In the arrangement shown, the blocks of unequal masses are held at rest. When released, acceleration of the blocks is



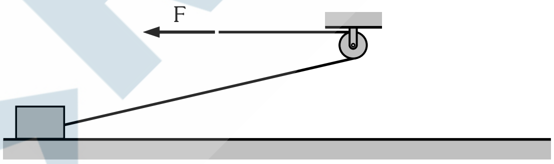
(A) g/2.
(B) g.
(C) a value between zero and g.
(D) a value that could be greater than g.

16. A monkey weighing 10 kg is climbing up a light rope and frictionless pulley attached to 15 kg mass at other end as in figure. In order to raise the 15 kg mass off the ground the monkey must climb-up



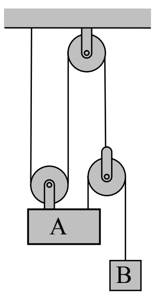
(A) with constant acceleration g/3.
(B) with an acceleration greater than g/2.
(C) with an acceleration greater than g/4.
(D) It is not possible because weight of monkey is lesser than the block.

17. A heavy cart is pulled by a constant force F along a horizontal track with the help of a rope that passes over a fixed pulley, as shown in the figure. Assume the tension in the rope and the frictional forces on the cart remain constant and consider motion of the cart until it reaches vertically below the pulley. As the cart moves to the right, its acceleration



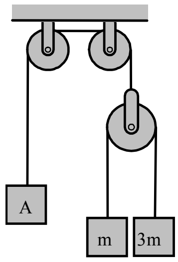
(A) decreases. (B) increases. (C) remains constant. (D) is zero

18. In arrangement shown the block A of mass 15 kg is supported in equilibrium by the block B. Mass of the block B is closest to



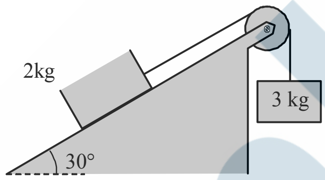
(A) 2 kg (B) 3 kg (C) 4 kg (D) 5 kg

19. In the given figure, find mass of the block A, if it remains at rest, when the system is released from rest. Pulleys and strings are massless. [g = 10 m/s^2]



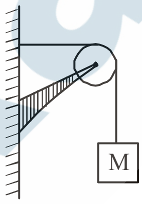
(A) m (B) 2m (C) 2.5m (D) 3m

20. In the arrangement shown, the 2 kg block is held to keep the system at rest. The string and pulley are ideal. When the 2 kg block is set free, by what amount the tension in the string changes? [g = 10 m/s^2]



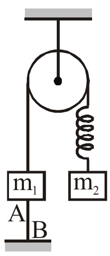
(A) Increase of 12 N (B) Decrease of 12 N (C) Increase of 18 N (D) Decrease of 18 N

21. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given **[JEE (Scr) 2001]**



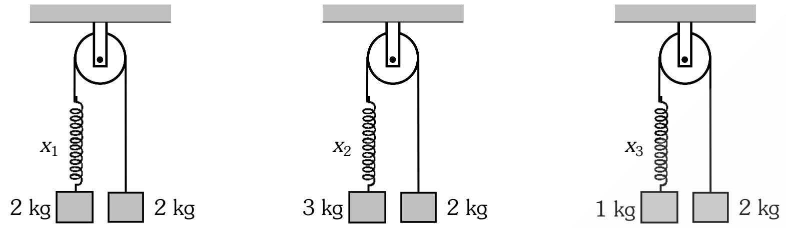
(A) $\sqrt{2}\,Mg$ (B) $\sqrt{2}\,mg$ (C) $\sqrt{(M+m)^2+m^2}$ g (D) $\sqrt{(M+m)^2+M^2}$ g

22. In a given figure two masses m_1 & m_2 ($m_2 > m_1$) are at rest in equilibrium position. Find the tension in string AB



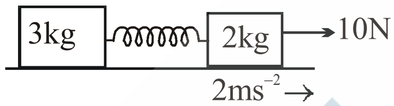
(A) m_1g (B) m_2g (C) $(m_1+m_2)g$ (D) $(m_2-m_1)g$

23. Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown. If x_1, x_2 and x_3 be the extensions in the spring in these three cases, when acceleration of both the blocks have same magnitude, then



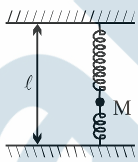
(A) $x_2 > x_3 > x_1$ (B) $x_2 > x_1 > x_3$ (C) $x_3 > x_1 > x_2$ (D) $x_1 > x_2 > x_3$

24. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms^{-2} as shown in figure.



(A) 3 ms^{-2} (B) 2 ms^{-2} (C) 0.5 ms^{-2} (D) zero

25. A small ball of mass M is held in equilibrium with two identical springs as shown in the figure. Force constant of each spring is k and relaxed length of each spring is $\ell/2$. What is distance between the ball and roof?

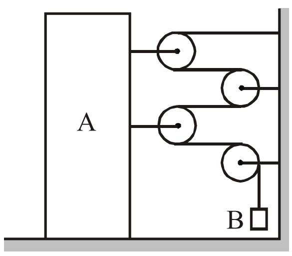


(A) $\frac{\ell}{2}+\frac{Mg}{k}$ (B) $\frac{\ell}{2}-\frac{Mg}{k}$ (C) $\frac{\ell}{2}+\frac{Mg}{2k}$ (D) $\frac{\ell}{2}-\frac{Mg}{2k}$

26. An elastic spring of relaxed length ℓ_o and force constant k is cut into two parts of lengths ℓ_1 and ℓ_2. The force constants of these parts are respectively

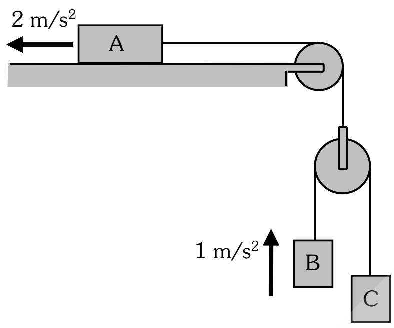
(A) $\frac{k\ell_o}{\ell_1}$ and $\frac{k\ell_o}{\ell_2}$ (B) $\frac{k\ell_1}{\ell_0}$ and $\frac{k\ell_2}{\ell_o}$ (C) $\frac{k\ell_0}{\ell_2}$ and $\frac{k\ell_0}{\ell_1}$ (D) $\frac{k\ell_2}{\ell_0}$ and $\frac{k\ell_1}{\ell_0}$

27. Block A is moving away from the wall at a speed v and acceleration a.



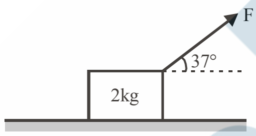
(A) Velocity of B is v with resect to A.
(B) Acceleration of B is a with respect to A.
(C) Acceleration of B is 4a with respect to A.
(D) Acceleration of B is $\sqrt{17}$a with respect to A.

28. In the setup shown, find acceleration of the block C.



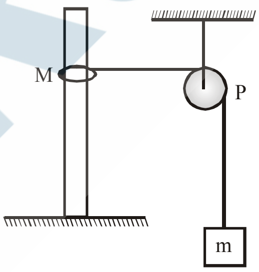
(A) 3 m/s² ↑ (B) 3 m/s² ↓ (C) 5 m/s² ↑ (D) 5 m/s² ↓

29. A block of mass 2 kg is kept on a rough horizontal floor and pulled with a force F. If the coefficient of friction is 0.5. then the minimum force required to move the block is :-



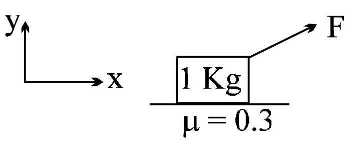
(A) 10 N (B) $\frac{100}{11}$ N (C) $\frac{100}{8}$ N (D) 20 N

30. In the figure shown a ring of mass M and a block of mass m are in equilibrium. The string is light and pulley P does not offer any friction and coefficient of friction between pole and M is μ. The frictional force offered by the pole on M is



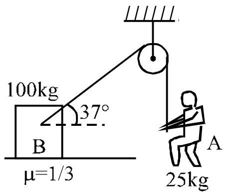
(A) Mg directed up (B) μmg directed up
(C) $(M - m)$g directed down (D) μmg directed down

31. A force $\vec{F} = \hat{i} + 4\hat{j}$ acts on block shown. The force of friction acting on the block is :



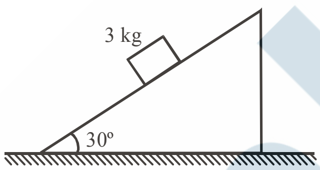
(A) $-\hat{i}$ (B) $-1.8\hat{i}$ (C) $-2.4\hat{i}$ (D) $-3\hat{i}$

32. Block *B* of mass 100 kg rests on a rough surface of friction coefficient $\mu = 1/3$. A rope is tied to block *B* as shown in figure. The maximum acceleration with which boy *A* of 25 kg can climbs on rope without making block move is :



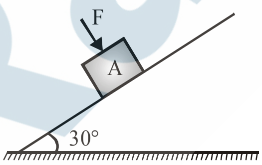
(A) $\frac{4g}{3}$ (B) $\frac{g}{3}$ (C) $\frac{g}{2}$ (D) $\frac{3g}{4}$

33. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be $(g = 10\ m/s^2)$



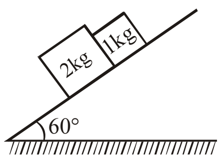
(A) 26 N (B) 19.5 N (C) 10 N (D) 30 N

34. A block of mass $m = 2$ kg is resting on a rough inclined plane of inclination 30° as shown in figure. The coefficient of friction between the block and the plane is $\mu = 0.5$. What minimum force F should be applied perpendicular to the plane on the block, so that block does not slip on the plane $(g=10m/s^2)$



(A) zero (B) 6.24 N (C) 2.68 N (D) 4.34 N

35. In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is $\mu_1=0.5$ and $\mu_2 = 0.4$ respectively, then

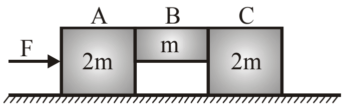


(A) both block will move together.
(B) both block will move separately.
(C) there is a non zero contact force between two blocks.
(D) none of these

36. A block is pushed with some velocity up a rough inclined plane. It stops after ascending few meters and then reverses its direction and returns back to point from where it started. If angle of inclination is 37° and the time to climb up is half of the time to return back then coefficient of friction is

(A) $\frac{9}{20}$ (B) $\frac{7}{5}$ (C) $\frac{7}{12}$ (D) $\frac{5}{7}$

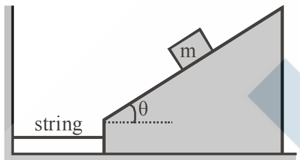
37. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from downward slipping is :-



(A) $\left(\frac{3}{2\mu}\right)mg$ (B) $\left(\frac{5}{2\mu}\right)mg$ (C) $\left(\frac{5}{2}\right)\mu mg$ (D) $\left(\frac{3}{2}\right)\mu mg$

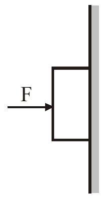
MULTIPLE CORRECT TYPE QUESTIONS

38. Refer the system shown in the figure. Block is sliding down the wedge. All surfaces are frictionless. Find correct statement(s)



(A) Acceleration of block is $g\sin\theta$
(B) Acceleration block is $g\cos\theta$
(C) Tension in the string is $mg\cos^2\theta$
(D) Tension in the string is $mg\sin\theta.\cos\theta$

39. A block of mass 1 kg is held at rest against a rough vertical surface by pushing by a force F horizontally. The coefficient of friction is 0.5. When

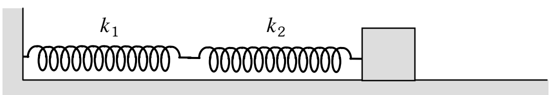


(A) F = 40 N, friction on the block is 20 N.
(B) F = 30 N, friction on the block is 10 N.
(C) F = 20 N, friction on the block is 10 N.
(D) Minimum value of force F to keep block at rest is 20 N.

40. A block is kept on a rough horizontal surface as shown. Its mass is 2 kg and coefficient of friction between block and surface (μ) = 0.5. A horizontal force F is acting on the block. When

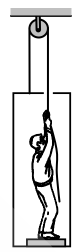
(A) F = 4 N, acceleration is zero.
(B) F = 4 N, friction is 10 N and acceleration is 3 m/s^2.
(C) F = 14 N, acceleration is 2 m/s^2.
(D) F = 14 N, friction is 14 N.

41. The mass in the figure can slide on a frictionless surface. When the mass is pulled out, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2. The spring constants are k_1 and k_2 respectively. Magnitude of spring force pulling back on the mass is



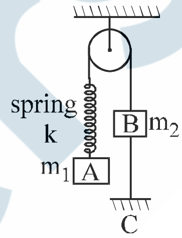
(A) k_1x_1 (B) k_2x_2 (C) $(k_1x_1 + k_2x_2)$ (D) $0.5\,(k_1 + k_2)\,(x_1 + x_2)$

42. A carpenter of mass 50 kg is standing on a weighing machine placed in a lift of mass 20 kg. A light string is attached to the lift. The string passes over a smooth pulley and the other end is held by the carpenter as shown. When carpenter keeps the lift moving upward with constant velocity :- ($g = 10$ m/s^2)



(A) the reading of weighing machine is 15 kg
(B) the man applies a force of 350 N on the string
(C) net force on the man is 150 N
(D) Net force on the weighing machine is 150 N

43. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt :

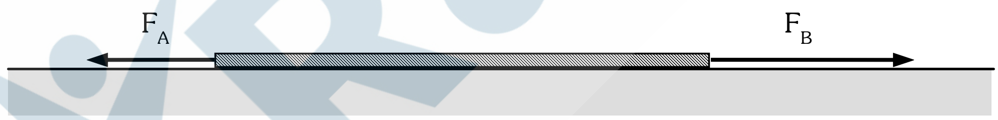


(A) initial acceleration of m_2 will be upwards
(B) magnitude of initial acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ g.
(C) initial acceleration of m_1 will be equal to zero
(D) magnitude of initial acceleration of two blocks will be non-zero and unequal.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 44 to 47

A uniform rope of mass (m) and length (L) placed on frictionless horizontal ground is being pulled by two forces F_A and F_B at its ends as shown in the figure. As a result, the rope accelerates toward the right.



44. Acceleration (A) of the rope is

(A) zero (B) $a = \frac{F_A + F_B}{m}$ (C) $a = \frac{F_A - F_B}{m}$ (D) $a = \frac{F_B - F_A}{m}$

45. Tension (T) at the mid point of the rope is

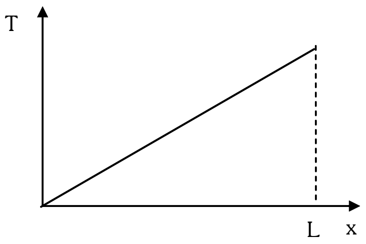
(A) $T = F_B - F_A$ (B) $T = F_A + F_B$ (C) $T = \frac{1}{2}(F_B - F_A)$ (D) $T = \frac{1}{2}(F_A + F_B)$

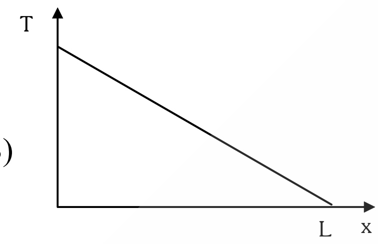
46. Expression (T_x) of tension at a point at distance x from the end A is

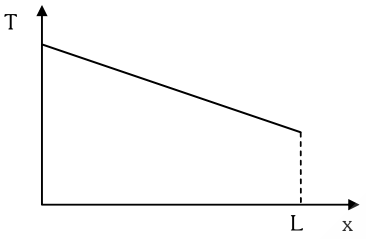
(A) $T_x = \left(\frac{F_B - F_A}{L}\right)x + F_A$ (B) $T_x = \left(\frac{F_B - F_A}{L}\right)x - F_A$

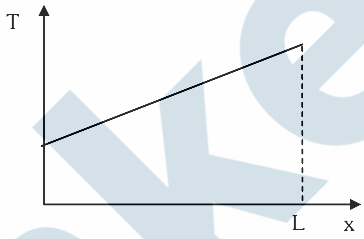
(C) $T_x = \left(\frac{F_B - F_A}{L}\right)x + F_B$ (D) $T_x = \left(\frac{F_B - F_A}{L}\right)x - F_B$

47. Which of the following graph best represents variation in tension at a point on the rope with distance x of the point from the end A?

(A)


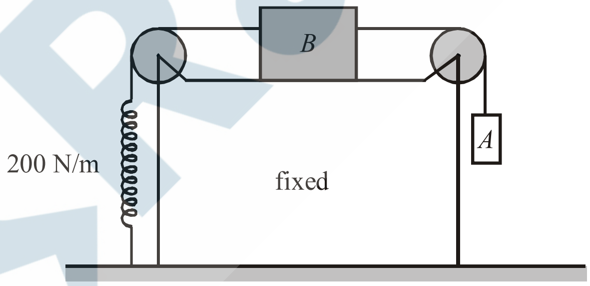
(B)


(B)


(D)


Paragraph for Question No. 48 to 50

The figure shown blocks A and B are of mass 2 kg and 8 kg and they are connected through strings to a spring connected to ground. The blocks are in equilibrium. (g = 10m/s^2)



48. The elongation of the spring is

(A) 1 cm (B) 10 cm (C) 0.1 cm (D) 1m

49. Now the block A is pulled downwards by a force gradually increasing to 20 N. The new elongation of spring is :-

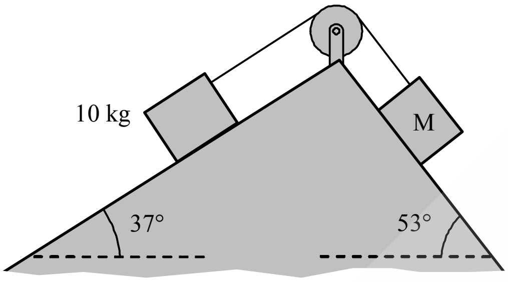
(A) 2 cm (B) 4 cm (C) 20 cm (D) 40 cm

50. Now the force on A is suddenly removed. The acceleration of block B becomes :-

(A) 1.0 m/s (B) 2.0 m/s^2 (C) 3.0 m/s^2 (D) 4.0 m/s^2

Paragraph for Question No. 51 to 53

The blocks are on frictionless inclined ramp and connected by a massless cord. The cord passes over an ideal pulley. [$g = 10\ m/s^2$]



51. When set free, the 10 kg block slides down the ramp with acceleration of $2\ m/s^2$. Mass M is closest to

(A) 13 kg (B) 8 kg (C) 5 kg (D) 4 kg

52. When the 10 kg block slides down the ramp with acceleration of $2\ m/s^2$, tension in the cord is closest to :-

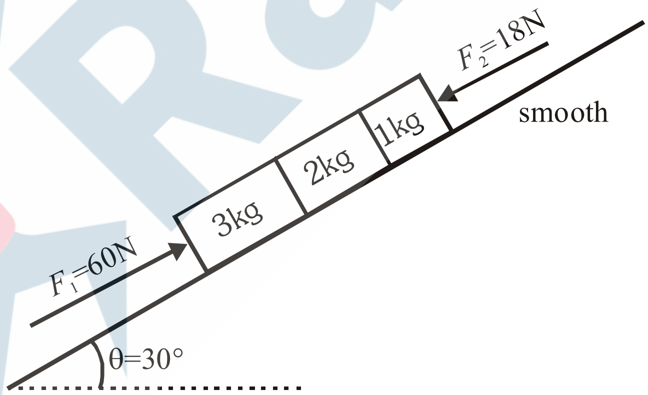
(A) 80 N (B) 60 N (C) 40 N (D) 30 N

53. What value of M would keep the system at rest.

(A) 10 kg (B) 8 kg (C) 7.5 kg (D) 6 kg

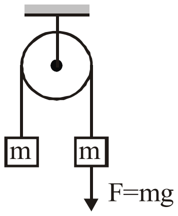
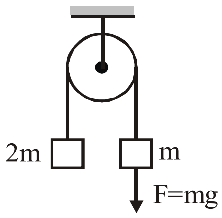
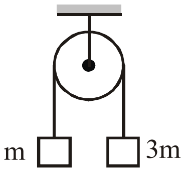
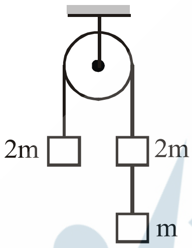
MATRIX MATCH TYPE QUESTION

54. In the diagram shown in figure ($g = 10\ m/s^2$)

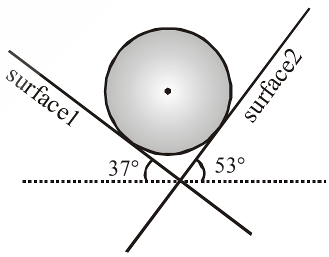


	Column I		Column II
(A)	Acceleration of 2 kg block in m/s^2	(P)	8
(B)	Net force on 3 kg block in newton	(Q)	25
(C)	Normal reaction between 2 kg and 1 kg in newton	(R)	2
(D)	Normal reaction between 3 kg and 2 kg in newton	(S)	45
		(T)	None

55. Match the situations in column I to the accelerations of blocks in the column II (acceleration due to gravity is g and F is an additional force applied to one of the blocks ?

Column I		**Column II**	
(A)		(P)	$\frac{g}{5}$
(B)		(Q)	$\frac{g}{3}$
(C)		(R)	$\frac{g}{2}$
(D)		(S)	$\frac{2g}{3}$
		(T)	zero

56. A sphere of mass 10 kg is placed in equilibrium in a V shaped groove plane made of two smooth surfaces 1 and 2 as shown in figure. (g = 10 ms^{-2})

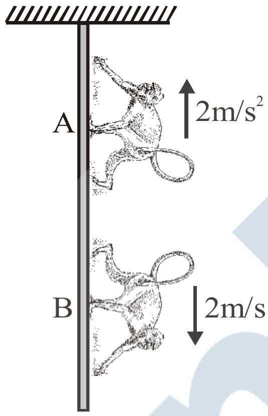


Column I		**Column II**	
(A)	Normal reaction by Surface 1	(P)	Zero
(B)	Normal reaction by surface 2	(Q)	60 N
(C)	Force on sphere by Earth	(R)	80 N
(D)	Net force on sphere	(S)	100 N
		(T)	120 N

EXERCISE (O-2)

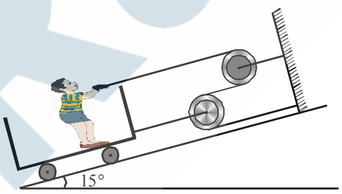
SINGLE CORRECT TYPE QUESTIONS

1. Two monkeys of masses 10 kg and 8 kg are moving along a vertical light rope, the former climbing up with an acceleration of 2 m/s^2, while the latter coming down with a uniform velocity of 2 m/s. Find tension in the rope at the fixed support.



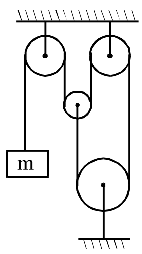
(A) 180 N (B) 200 N (C) 80 N (D) 216 N

2. A trolley is being pulled up an incline plane by a man sitting on it (as shown in figure). He applies a force of 250 N. If the combined mass of the man and trolley is 100 kg, the acceleration of the trolley will be [sin15° = 0.26]



(A) 2.4 m/s^2 (B) 9.4 m/s^2 (C) 6.9 m/s^2 (D) 4.9 m/s^2

3. If the string & all the pulleys are ideal, acceleration of mass *m* is :-

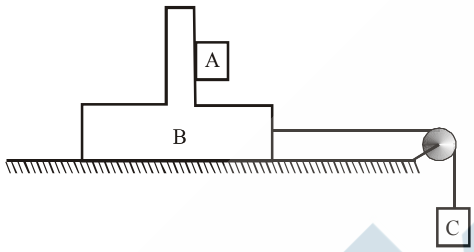


(A) $\frac{g}{2}$ (B) 0 (C) g (D) dependent on m

4. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end, $\mu = 0.15$ and $g = 10\ m/s^2$. The truck starts from rest with an acceleration of $2 m/s^2$ on a straight road. The distance moved by the truck when box starts fall down is :-

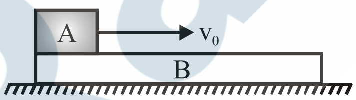
(A) 4 m (B) 8 m (C) 16 m (D) 32 m

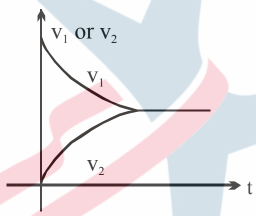
5. In the arrangement shown in the figure, mass of the block *B* and *A* is 2m and *m* respectively. Surface between *B* and floor is smooth. The block *B* is connected to the block *C* by means of a string-pulley system. If the whole system is released, then find the minimum value of mass of block *C* so that *A* remains stationary w.r.t. *B*. Coefficient of friction between *A* and *B* is μ.

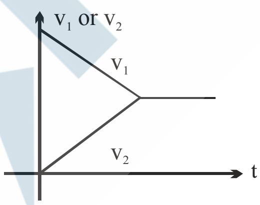


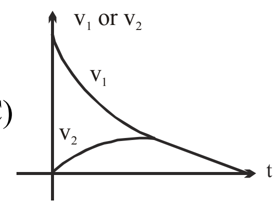
(A) $\dfrac{m}{\mu}$ (B) $\dfrac{2m+1}{\mu+1}$ (C) $\dfrac{3m}{\mu-1}$ (D) $\dfrac{6m}{\mu+1}$

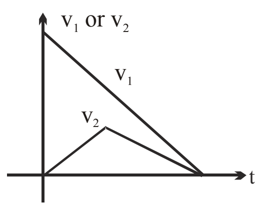
6. A block *A* is placed over a long rough plank *B* of same mass as shown in figure. The plank is placed over a smooth horizontal surface. At time t = 0, block *A* is given a velocity v_0 in horizontal direction. Let v_1 and v_2 be the velocities of *A* and *B* at time *t*. Then choose the correct graph between v_1 or v_2 and *t*.



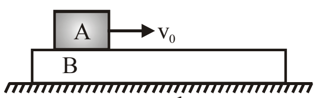
(A)


(B)


(C)


(D)


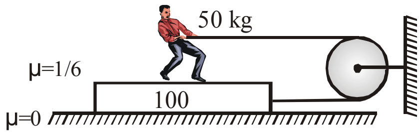
7. A block *A* of mass *m* is placed over a plank *B* of mass 2 m. Plank *B* is placed over a smooth horizontal surface. The coefficient of friction between *A* and *B* is 0.5. Block *A* is given a velocity v_0 towards right. Acceleration of *B* relative to *A* is :-



smooth

(A) $\dfrac{g}{2}$ (B) g (C) $\dfrac{3g}{4}$ (D) zero

8. A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man & plank move together, find force of friction acting on man.



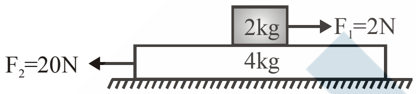
(A) $\frac{100}{3}$ N towards left

(B) $\frac{100}{3}$ N towards right

(C) $\frac{250}{3}$ N towards left

(D) $\frac{250}{3}$ N towards right

9. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = 1/2$. The force of friction acting between the two blocks is :-

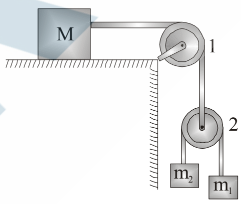


(A) 8 N (B) 10 N (C) 6 N (D) 4 N

10. A flexible chain of weight W hangs between two fixed points A & B which are at the same horizontal level. The inclination of the chain with the horizontal at both the points of support is θ. What is the tension of the chain at the mid point?

(A) $\frac{W}{2}.\,cosec\,\theta$ (B) $\frac{W}{2}.\,tan\,\theta$ (C) $\frac{W}{2}cot\,\theta$ (D) none

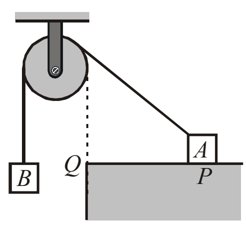
11. In the arrangement shown in figure m_1 = 1kg, m_2 = 2kg. Pulleys are massless and strings are light. For what value of M the mass m_1 moves with constant velocity (Neglect friction)



(A) 6 kg (B) 4 kg (C) 8 kg (D) 10 kg

MULTIPLE CORRECT TYPE QUESTIONS

12. Two small identical blocks are connected to the ends of a string passing over pulley as shown when the system is released from rest.



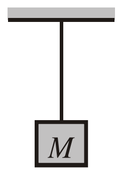
(A) block A and B do not move

(B) block A accelerates towards pulley along the string.

(C) block A does not leave contact with table till it reaches to the edge Q of the table

(D) Normal reaction of table on block A is less than weight of block A between P and Q and at Q it vanishes

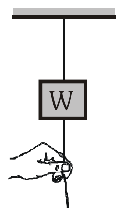
13. Consider a block suspended from a light string as shown in the figure. Which of the following pairs of forces constitute Newton's third law pair?



(A) Force with which string pulls on the ceiling and the force with which string pulls on block
(B) Force with which string pulls on the block and weight of the block
(C) Force acting on block due to the earth and force the block exerts on the earth
(D) Force with which block pulls on string and force with which the string pulls on the block

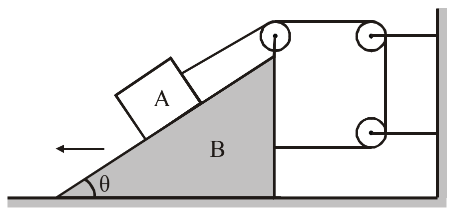
14. If a horizontal support exerts an upward force of 10 N on a block of weight 9.8 N placed on it, which of the following statements is/are correct. Assume acceleration due to gravity to be 9.8 m/s^2.
(A) The block exerts a force of 10 N on the support.
(B) The block exerts a force of 9.8 N on the support.
(C) The block has an upward acceleration.
(D) The block has a downward acceleration.

15. A block of mass m is suspended from a fixed support with the help of a cord. Another identical cord is attached to the bottom of the block. Which of the following statement is /are true?



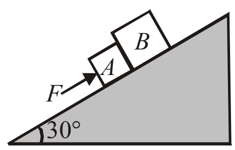
(A) If the lower cord is pulled suddenly, only the upper cord will break.
(B) If the lower cord is pulled suddenly, only the lower cord will break.
(C) If pull on the lower cord is increased gradually, only the lower cord will break.
(D) If pull on the lower cord is increased gradually, only the upper cord will break.

16. A block A and wedge B connected through a string as shown. The wedge B is moving away from the wall with acceleration 2 m/s^2 horizontally and acceleration of block A is vertical upwards. Then



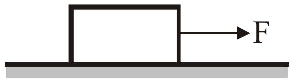
(A) Acceleration of A with respect to B is 4 m/s^2.
(B) Acceleration of A with respect to B is $2\sqrt{3}$ m/s^2.
(C) Angle θ is 60°.
(D) Acceleration of A is $2\sqrt{3}$ m/s^2.

17. Two blocks *A* and *B* of mass 2 kg and 4 kg respectively are placed on a smooth inclined plane and 2 kg block is pushed by a force F acting parallel to the plane as shown. If N be the magnitude of contact force applied on B by A, which of the following is/are correct?



(A) if $F = 0$ N, $N = 10$ N
(B) if F=15 N, N=10 N
(C) If $F = 30$ N, $N = 20$ N
(D) if F=45 N, N=30 N

18. A block is kept on a rough surface and applied with a horizontal force as shown which is gradually increasing from zero. The coefficient of static and kinetic friction are $1/\sqrt{3}$ then



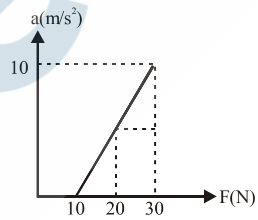
(A) When F is less than the limiting friction, angle made by net force on the block by the surface is less than 30° with vertical.

(B) When the block is just about to move, the angle made by net force by the surface on the block becomes equal to 30° with vertical.

(C) When the block starts to accelerate, the angle made by net force by the surface on the block becomes constant and equal to 30° vertical.

(D) The angle made by net force with vertical on the block by the surface, depends on the mass of the block.

19. A block placed on a rough horizontal surface is pushed with a force F acting horizontally on the block. The magnitude of F is increased and acceleration produced is plotted in the graph shown.



(A) Mass of the block is 2 kg.

(B) Coefficient of friction between block and surface is 0.5.

(C) Limiting friction between block and surface is 10 N.

(D) When F = 8 N, friction between block and surface is 10 N.

20. A block is placed over a plank. The coefficient of friction between the block and the plank is $\mu = 0.2$. Initially both are at rest, suddenly the plank starts moving with acceleration $a_0 = 4$ m/s^2. The displacement of the block in 1s is (g=10 m/s^2)

(A) 1 m relative to ground

(B) 1 m relative to plank

(C) zero relative to plank

(D) 2 m relative to ground

21. A block is released from rest from a point on a rough inclined place of inclination 37°. The coefficient of friction is 0.5.

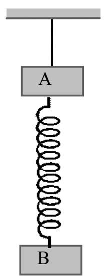
(A) The time taken to slide down 9 m on the plane is 3 s.

(B) The velocity of block after moving 4 m is 4 m/s.

(C) The block travels equal distances in equal intervals of time.

(D) The velocity of block increases linearly.

22. In the given figure both the blocks have equal mass. When the thread is cut, which of the following statements give correct description immediately after the thread is cut?



(A) Relative to the block A, acceleration of block B is 2g upwards.

(B) Relative to the block B, acceleration of block A is 2g downwards.

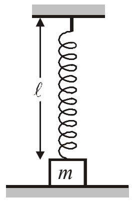
(C) Relative to the ground, accelerations of the blocks A and B are both g downwards.

(D) Relative to the ground, accelerations of the blocks A and B are 2g downwards and zero respectively.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 23 to 25

A block of mass m is placed on a smooth horizontal floor is attached to one end of spring. The other end of the spring is attached to fixed support. When spring is vertical it is relaxed. Now the block is pulled towards right by a force F, which is being increased gradually. When the spring makes angle 53° with the vertical, block leaves the floor.



23. When blocks leaves the table, the normal force on it from table is

(A) mg (B) zero (C) $\frac{4mg}{3}$ (D) $\frac{3mg}{4}$

24. Force constant of the spring is :-

(A) $\frac{5mg}{2\ell}$ (B) $\frac{15mg}{8\ell}$ (C) $\frac{5mg}{3\ell}$ (D) $\frac{5mg}{4\ell}$

25. When the block leaves the table, the force F is :-

(A) $\frac{3mg}{4}$ (B) $\frac{4mg}{3}$ (C) $\frac{3mg}{5}$ (D) $\frac{4mg}{5}$

Paragraph for Question No. 26 to 30

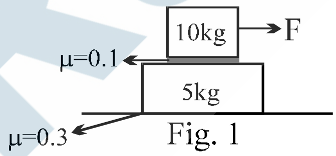


Fig. 1

26. When F = 2N, the frictional force between 5 kg block and ground is

(A) 2N (B) 0 (C) 8 N (D) 10 N

27. When F = 2N, the frictional force between 10 kg block and 5 kg block is

(A) 2N (B) 15 N (C) 10 N (D) None

28. The maximum F which will not cause motion of any of the blocks is

(A) 10 N (B) 15 N (C) data insufficient (D) None

29. The maximum acceleration of 5 kg block is :-

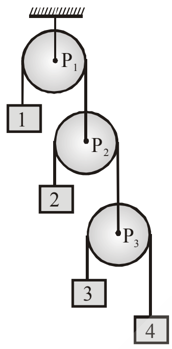
(A) 1 m/s^2 (B) 3 m/s^2 (C) 0 (D) None

30. The acceleration of 10 kg block when F = 30 N is

(A) 2 m/s^2 (B) 3 m/s^2 (C) 1 m/s^2 (D) None

MATRIX MATCH TYPE QUESTION

31. In the figure shown, acceleration of 1 is x (upwards). Acceleration of pulley P_3, w.r.t. pulley P_2 is y (downwards) and acceleration of 4 w.r.t. to pulley P_3 is z (upwards). Then



	Column I		Column II
(A)	Absolute acceleration of 2	(P)	(y–x) downwards
(B)	Absolute acceleration of 3	(Q)	(z–x–y) upwards
(C)	Absolute acceleration of 4	(R)	(x+y+z) downwards
		(S)	None

32. Velocity of three particles A, B and C varies with time t as, $\vec{v}_A = (2t\hat{i} + 6\hat{j})$ m/s; $\vec{v}_B = (3\hat{i} + 4\hat{j})$ m/s and $\vec{v}_C = (6\hat{i} - 4t\hat{j})$ m/s. Regarding the pseudo force match the following table

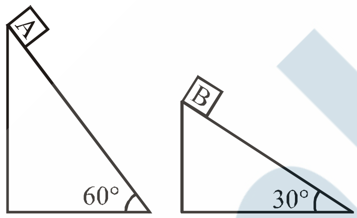
	Column I		Column II
(A)	On A as observed by B	(P)	Along positive x–direction
(B)	On B as observed by C	(Q)	Along negative x–direction
(C)	On A as observed by C	(R)	Along positive y–direction
(D)	On C as observed by A	(S)	Along negative y–direction
		(T)	Zero

EXERCISE (JM)

1. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m :- **[AIEEE - 2007]**

(1) $\frac{mF}{M}$ (2) $\frac{(M+m)F}{m}$ (3) $\frac{mF}{(m+M)}$ (4) $\frac{MF}{(m+M)}$

2. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ? **[AIEEE - 2010]**



(1) 4.9 ms^{-2} in vertical direction. (2) 4.9 ms^{-2} in horizontal direction
(3) 9.8 ms^{-2} in vertical direction (4) Zero

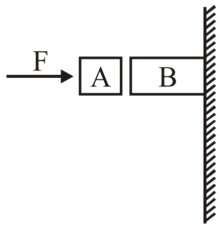
3. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$ then the ratio $\frac{F_1}{F_2}$ is :- **[AIEEE - 2011]**

(1) 4 (2) 1 (3) 2 (4) 3

4. A block of mass m is placed on a surface with a vertical cross section given by $y=\frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is :- **[JEE-Main-2014]**

(1) $\frac{1}{3}$m (2) $\frac{1}{2}$m (3) $\frac{1}{6}$m (4) $\frac{2}{3}$m

5. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :- **[JEE-Main-2015]**

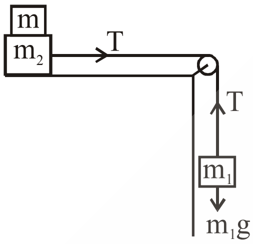


(1) 120 N (2) 150 N (3) 100 N (4) 80 N

6. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is: **[JEE-Main Online-2016]**

(1) 2 tan θ (2) 3 tan θ (3) tan θ (4) tan 2θ

7. Two masses m_1 = 5kg and m_2 = 10kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :- **[JEE-Main-2018]**



(1) 27.3 kg (2) 43.3 kg (3) 10.3 kg (4) 18.3 kg

EXERCISE (JA)

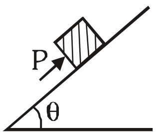
1. **Statement-1 :** A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table. **[IIT-JEE 2007]**
and
Statement-2 : For every action there is an equal and opposite reaction
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

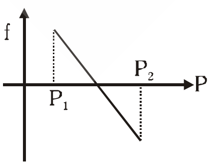
2. **Statement-1 :** It is easier to pull a heavy object that to push it on a level ground.
and **[IIT-JEE 2008]**
Statement-2 : The magnitude of frictional force depends on the nature of the two surfaces in contact.
(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

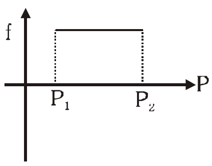
3. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is **[IIT-JEE-2009]**

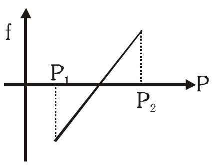
(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

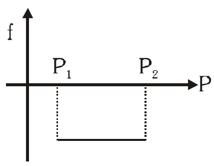
4. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tanθ >μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like **[IIT-JEE-2010]**



(A) 

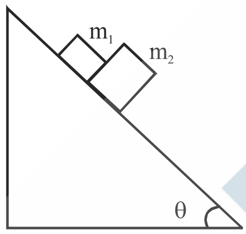
(B) 

(C) 

(D) 

5. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N =10μ, then N is **[IIT-JEE-2011]**

6. A block of mass m_1 = 1 kg another mass m_2 = 2kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.
[**useful information :** tan (5.5°) ≈ 0.1 ; tan(11.5°) ≈ 0.2 ; tan (16.5°) ≈ 0.3] **[IIT-JEE-2014]**



	List–I		**List–II**
(P)	$\theta = 5°$	(1)	$m_2 g \sin\theta$
(Q)	$\theta = 10°$	(2)	$(m_1 + m_2) g \sin\theta$
(R)	$\theta = 15°$	(3)	$\mu m_2 g \cos\theta$
(S)	$\theta = 20°$	(4)	$\mu (m_1 + m_2) g \cos\theta$

Code :

(A) P-1, Q-1, R-1, S-3
(B) P-2. Q-2, R-2, S-3
(C) P-2,. Q-2, R-2, S-4
(D) P-2, Q-2, R-3, S-3

ANSWER KEY

EXERCISE (S-1)

1. Ans. (i) $\frac{m_1}{m_2} = \frac{1}{3}$ (ii) $a = 3/4 \text{ m/s}^2$

2. Ans. $\frac{4}{3}W$

3. Ans. $\frac{g}{10} m/s^2$

4. Ans. 200 N, 10 m/s^2

5. Ans. 2 sec

6. Ans. $0.5g$, g

7. Ans. 0.5 s

8. Ans. 80 N, 48 N, 264 N

9. Ans. 24N

10. Ans. (i) $a_A = \frac{3g\downarrow}{2} = a_B$; $a_C = 0$; T = $mg/2$; (ii) $a_A = 2g\uparrow$, $a_B = 2g\downarrow$, $a_c = 0$, $T = 0$;

(iii) $a_A = a_B = g/2\uparrow$, $a_c = g\downarrow$, $T = \frac{3mg}{2}$;

11. Ans. $a = g \cot\theta$

12. Ans. $v_0/\cos\theta$

13. Ans. 5N, 16/31 kg

14. Ans. $3a_B/4$

15. Ans. $\frac{a_0}{2}$

16. Ans. $(100\hat{i} - 200\hat{j})$ N

17. Ans. 20 N vertically downward

18. Ans. (ma)

19. Ans. 1/2 s

20. Ans. 4/3 s

21. Ans. $\mu_{min} = \frac{m\sin\theta\cos\theta}{m\cos^2\theta + M}$

22. Ans. 0.5

23. Ans. 1 kg

24. Ans. $10\hat{i}$

25. Ans. 3/4

26. Ans. 30 N

EXERCISE (S-2)

1. Ans. 1440 N

2. Ans. g m/s^2

3. Ans. 556.8 N, 1.47 s

4. Ans. (i) $a_A = g\downarrow$, $a_B = \frac{2g}{3}\uparrow$, $a_c = \frac{2g}{3}\downarrow$, $a_D = \frac{2g}{3}\downarrow$, $a_E = 0$

(ii) $a_A = 0$, $a_B = \frac{g}{3}\downarrow$, $a_C = \frac{g}{3}\uparrow$, $a_D = \frac{g}{3}\uparrow$, $a_E = g\downarrow$

(iii) $a_A = 0, a_B = g\downarrow, a_C = g\uparrow, a_D = 2g\downarrow, a_E = 0$

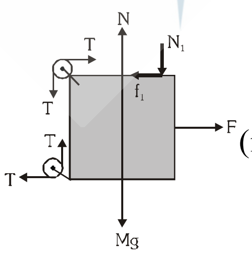
(iv) $a_A = 0, a_B = 3g\downarrow, a_C = \frac{3g}{2}\downarrow, a_D = \frac{3g}{2}\downarrow, a_E = 0$

5. Ans. $\frac{25}{24}$

6. Ans. 5 m/s^2 downwards, 0 m/s^2, 10 N

7. Ans. 40 N

8. Ans. (i)



(ii) a = 3/5 m/s^2 , T = 18 N, F = 60N

9. Ans. $-5\,\hat{i}$ m/s^2

EXERCISE (O-1)

1. Ans. (B) 2. Ans. (D) 3. Ans. (D) 4. Ans. (C) 5. Ans. (D) 6. Ans. (C)
7. Ans. (C) 8. Ans. (C) 9. Ans. (D) 10. Ans. (C) 11. Ans. (B) 12. Ans. (A)
13. Ans. (C) 14. Ans. (A) 15. Ans. (C) 16. Ans. (B) 17. Ans. (A) 18. Ans. (B)
19. Ans. (D) 20. Ans. (B) 21. Ans. (D) 22. Ans. (D) 23. Ans. (B) 24. Ans. (B)
25. Ans. (C) 26. Ans. (A) 27. Ans. (D) 28. Ans. (A) 29. Ans. (B) 30. Ans. (A)
31. Ans. (A) 32. Ans. (B) 33. Ans. (D) 34. Ans. (C) 35. Ans. (B) 36. Ans. (A)
37. Ans. (B) 38. Ans. (A,D) 39. Ans. (B,C,D) 40. Ans. (A,C) 41. Ans. (A,B) 42. Ans. (A,B)
43. Ans. (A,C) 44. Ans. (D) 45. Ans. (D) 46. Ans. (A) 47. Ans. (D)
48. Ans. (B) 49. Ans. (C) 50. Ans. (B) 51. Ans. (D) 52. Ans. (C) 53. Ans. (C)
54. Ans. (A) - (R); (B) - (T); (C) - (Q); (D) - (T) 55. Ans. (A) - (R); (B) - (T); (C) - (R); (D) - (P)
56. Ans. (A)-R; (B)-Q; (C)-S; (D)-P

EXERCISE (O-2)

1. Ans. (B) 2. Ans. (D) 3. Ans. (C) 4. Ans. (C) 5. Ans. (C) 6. Ans. (B)
7. Ans. (C) 8. Ans. (A) 9. Ans. (A) 10. Ans. (C) 11. Ans. (C) 12. Ans. (C, D)
13. Ans. (C,D) 14. Ans. (A,C) 15. Ans. (B,D) 16. Ans. (A,C,D)
17. Ans. (B,C,D) 18. Ans. (A,B,C,D) 19. Ans. (A,B,C) 20. Ans. (A,B)
21. Ans. (A,B,D) 22. Ans. (A,B,D) 23. Ans. (B) 24. Ans. (A) 25. Ans. (B)
26. Ans. (A) 27. Ans. (A) 28. Ans. (A) 29. Ans. (C) 30. Ans. (A)
31. Ans. (A) - (S) ; (B) - (R) ; (C) - (Q) 32. Ans. (A) - (T); (B) - (R); (C) - (R); (D) - (Q)

EXERCISE (JM)

1. Ans. (3) 2. Ans. (1) 3. Ans. (4) 4. Ans. (3) 5. Ans. (1) 6. Ans. (3)
7. Ans. (1)

EXERCISE (JA)

1. Ans. (B) 2. Ans. (B) 3. Ans. (B) 4. Ans. (A) 5. Ans. 5 6. Ans. (D)