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KINEMATICS-2D

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MOTION IN TWO AND THREE DIMENSIONS : (Using 3rd dimension z is optional)

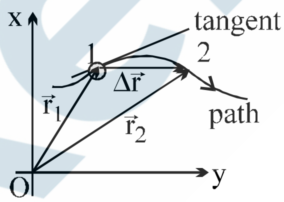
When a particle is moving in space then its motion can be broken up in three co-ordinate axes (x, y& z). The motion in these three directions is governed only by velocity & acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions. Lets say a particle is moving in space

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Gives position of particle in space.

VELOCITY

Using the language of calculus, we may write $\vec{v}$ as the derivative



$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \left(\frac{dz}{dt}\right)\hat{k}$$

where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt},\ v_z = \frac{dz}{dt}$$

Differentiating $\vec{r}$ w.r.t. time gives us velocity vector of particle at that time.

ACCELRATION

Similarly, if we differentiate $\vec{V}$ w.r.t. time we get acceleration of particle $\vec{a} = \frac{d\vec{V}}{dt}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

where the scalar components of $\vec{a}$ are

$$a_x = \frac{dv_x}{dt},\ a_y = \frac{dv_y}{dt},\ a_z = \frac{dv_z}{dt}$$

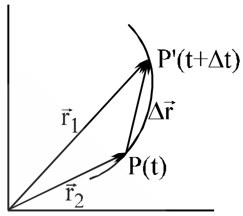
Now, collecting equations of motion relating to x & y axes separately

x-axis	y-axis
$V_x = \frac{dx}{dt}$	$V_y = \frac{dy}{dt}$
$a_x = \frac{dV_x}{dt}$	$a_y = \frac{dV_y}{dt}$

Thus we can see that motion in plane is composed of two straight line motions. **These motions are completely independent of each other**. Only thing connecting them is fact that they are occuring simultaneously.

Velocity is along tangent of path

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle position. $\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$



$\Delta\vec{r}$ will be along tangent

The result is the same in three dimensions:

Ex. A particle with velocity $\vec{v}_0 = -2\hat{i} + 4\hat{j}$ (in meters per second) at t = 0 undergoes a constant acceleration $\vec{a}$ of magnitude a = 3 m/s^2 at an angle θ = 127° from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at t = 5 sec, in unit vector notation ?

Sol. We know that $v = v_0 + at$

now $v_x = v_{0x} + a_x t$ and $v_y = v_{0y} + a_y t$

$a_x = a \cos\theta = (3\ m/s^2)(\cos 127°) = -1.80\ m/s^2$

$a_y = a \sin\theta = (3\ m/s^2)(\sin 127°) = +2.40\ m/s^2$

at time t = 5 sec

$v_x = -2\ m/s + (-1.80\ m/s^2)(5\ sec) = -11\ m/s$

$v_y = 4\ m/s + (2.40\ m/s^2)(5\ sec) = 16\ m/s$

Thus, at t = 5 sec,

$\vec{v} = (-11 m/s)\hat{i} + (16 m/s)\hat{j}$ **Ans.**

Ex. A particle moves in the x–y plane according to the law x = at; y = at(1–αt) where a and α are positive constants and t is time. Find the velocity and acceleration vector. The moment t_0 at which the velocity vector forms angle of 90° with acceleration vector.

Sol. $V_x = a$; $V_y = a - 2a\alpha t \Rightarrow \vec{V} = a\hat{i} + (a - 2a\alpha t)\hat{j}$

$a_x = 0$; $a_y = -2a\alpha \Rightarrow \vec{a} = -2a\alpha\hat{j}$

for 90°, $\vec{V} \cdot \vec{a} = 0$

$-2a\alpha(a - 2a\alpha t) = 0$

$1 - 2\alpha t = 0 \Rightarrow t = 1/(2\alpha)$ sec.

PROJECTILE MOTION

We next consider a special case of two-dimensional motion: A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a projectile (meaning that it is projected or launched) and its motion is called **projectile motion**.

Assumptions:–

Particle remains close to earth's surface, so acceleration due to gravity remains constant.

Air resistance is neglected.

Distance that projectile travels is small so that earth can be treated as plane surface.

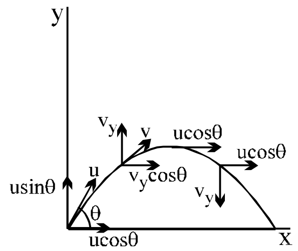
Two straight line motions:–

Our goal here is to analyse projectile motion using the tools for two dimensional motion. This feature allows us to break up a problem involving two dimensional motion into two separate and easier one-dimensional problems,

(a) The horizontal motion is motion with uniform velocity (no effect of gravity)

(b) The vertical motion is motion of uniform acceleration, or freely falling bodies.

Note: In projectile motion, the horizontal motion and the vertical motion are independent of each other, that is either motion does not affects the other.



Treating as two straight line motions:–

The horizontal Motion(x axis):

Because there is no accelration in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion,

The vertical motion(y axis):

The vertical motion is the motion we discussed for a particle in free fall.

As is illustrated in figure and equation (1.3), the vertical component behaves just as for a ball thrown vertically upward. It is directed upward initially and its magnitude steadily decreasing to zero, which marks the maximum height of the path. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

x-axis	**y-axis**
Initial velocity(u_x) = $u\cos\theta$	Initial velocity(u_y) = $u\sin\theta$
acceleration(a_x) = 0	acceleration(a_y) = $-g$
Thus, velocity after time t	Thus, velocity after time t
$v_x = u\cos\theta$	$v_y = u\sin\theta - gt$
Displacement after time t	Displacement after time t
$x = u\cos\theta t$	$y = u\sin\theta t - gt^2/2$

Resultant velocity

$$(\vec{V}_R) = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|\vec{V}_R| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

$$\& \quad \tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$$

where α is angle that velocity vector makes with horizontal. Also known as direction or angle of motion

Time of flight(T)

$$T = \frac{2u\sin\theta}{g}$$

Considering vertical motion

$s_y = 0$; $u_y = v\sin\theta$; $a_y = -g$

$$0 = u\sin\theta T - gT^2/2 \Rightarrow T = \frac{2u\sin\theta}{g}$$

Maximum Height(H)

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

Vertical velocity at maximum height $v_y = 0$

$$0 = u^2\sin^2\theta - 2gH \Rightarrow H = \frac{u^2 \sin^2 \theta}{2g}$$

Horizontal Range(R)

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

Total time $T = \dfrac{2u \sin \theta}{g}$

Velocity in horizontal direction $u_x = u\cos\theta$
Total displacement in horizontal direction $R = u\cos\theta T$

$$R = \frac{u^2 \sin 2\theta}{g}$$

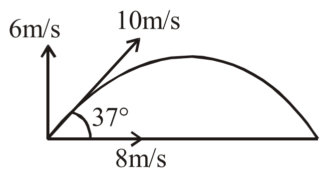
Note:– *For complementry angles i.e. $\theta + \alpha = 90°$, the range is same for same projection speed but maximum height and time of flight are different.*

Ex. A body is thrown with initial velocity 10m/sec. at an angle 37° from horizontal. Find

(i) Time of flight (ii) Maximum height.
(iii) Range (iv) Position vector after t = 1 sec.

Ans. (i) 1.2 sec, (ii) 1.8 m, (iii) 9.6 m, (iv) $(16\hat{i} - 8\hat{j}) - (8\hat{i} + \hat{j})$

Sol. (i) Time of flight $T = \dfrac{2u \sin \theta}{g} = \dfrac{2 \times 10 \times \frac{3}{5}}{10} = \dfrac{6}{5} = 1.2$ sec



(ii) Maximum height $H = \dfrac{u^2 \sin^2 \theta}{2g} = \dfrac{100\left(\frac{9}{25}\right)}{2 \times 10} = \dfrac{9}{5} = 1.8$ m

(iii) $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{100 \times 2 \times \frac{3}{5} \times \frac{4}{5}}{10} = \dfrac{240}{25} = 9.6\text{m}$

(iv) $x = 8 \times 1 = 8$ m

$$y = 6 \times 1 - \frac{1}{2} \times 10 \times (1)^2 = 1\text{m}$$

$$\vec{r} = 8\hat{i} + \hat{j}$$

Caution: This equation does not given the horizontal distance travelled by a projectile when the final height is not the launch height.

Maximum Range

$$R = \frac{u^2 \sin 2\theta}{g}$$

for $\theta = 45°$, R is maximum

$$R_{max} = \frac{u^2}{g}$$

Ex. A person can throw a ball vertically upto maximum height of 20 mt. How far can he throw the ball.

Sol. $H = \frac{u^2}{2g}$

$\therefore$ u = 20 m/s

$$R_{max} = \frac{u^2}{g} = 40 \text{ m}$$

Ex. A particle is projected with a speed u at an angle θ with horizontal. Find the average velocity of projectile for the period during which it crosses half of maximum height.

Sol. $u\cos\theta$ along horizontal

avg. velocity is a vector

First we will find vertical component

$$\vec{V}_y = \frac{\text{Total Displacement}}{\text{Total time}} = 0$$

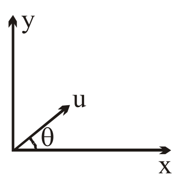
Horizontal

$$V_{1x} = V_{2x} = u\cos\theta$$

$$\overline{V}_x = u\cos\theta$$

EQUATION OF TRAJECTORY

Lets say point of projection is our origin and horizontal direction is x-axis and vertically upwards is positive y-axis.



We know $x = u\cos\theta t$

$$\therefore \quad t = \frac{x}{u\cos\theta} \qquad .....(1)$$

$$\text{also } y = u\sin\theta t - \frac{1}{2}gt^2 \qquad .....(2)$$

Putting value of 't' from eq. (1) in eq. (2), we get

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Ex. A particle is projected with a velocity 10 m/s at an angle 37° to the horizontal. Find the location at which the particle is at a height 1m from point of projection.

Ans. 1.6 m, 8 m.

Sol. $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

for y = 1; θ = 37°; u = 10 m/s

$$1 = \frac{3}{4}x - \frac{10x^2}{2\times 100\times\left(\frac{16}{25}\right)}$$

$$1 = \frac{3}{4}x - \frac{5}{64}x^2$$

$5x^2 - 48x + 64 = 0$

$5x^2 - 40x - 8x + 64 = 0$

x = 8m, 1.6m

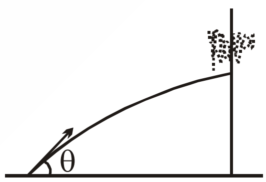
Ex. We have a hose pipe which disposes water at the speed of 10 ms^{-1}. The safe distance from a building on fire, on ground is 5 m. How high can this water go? (take : g = 10 ms^{-2})

Sol. Here we must understand that taking range of projectile as 10m and making projectile hit the building when it is at maximum height is wrong. By doing this we are not acheiving maximum y for given x = 5m. This just makes highest pt. of path to like on x = 5, But there may be other path for which y will be maximum for given x. This problem will be solved by using equation of trajectory by putting x = 5m and maximising y by varying θ.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Putting we get x = 5m

$$y = 5\tan\theta - \frac{10\times 25\sec^2\theta}{2\times 100}$$



$5\tan^2\theta - 20\tan\theta + (4y + 5) = 0$

for real roots discriminant must be positive.

$400 - 4 \times 5\,(4y + 5) > 0$

Solving $3.75 \geq y$

hence maximum y = 3.75 m

If we have taken range as 10 m then angle of projection will be θ = 45° corsponding maximum hight H = 2.5m which is smaller than our answer.

Ex. A projectile is fired horizontaly with a velocity of 98 m/s from the top of a hill 490 m high. Find

(i) Time to reach ground

(ii) The horizontal distance from foot of hill to ground.

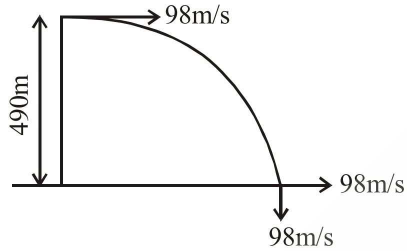
(iii) The speed with which it hits the ground.

Ans. (i) 10 sec, (ii) 980 m, (iii) $98\sqrt{2}$ m/se

Sol. $u_y = 0$; $u_x = 98$ m/s

on y-axis

$$s = ut + \frac{1}{2}at^2$$



$$-490 = 0 - \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{490 \times 2}{9.8}} = 10 \text{ sec}$$

on x-axis for horizontal distance,

$R = u_x t = 98 \times 10 = 980$ m

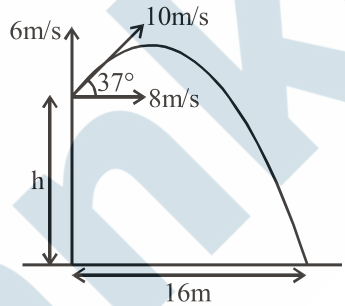
$v_x = 98$ m/s

$v_y = 0 - 9.8 \times 10 = -98$ m/s

So speed = $98\sqrt{2}$ m/s

Ex. A ball is thrown from the top of a tower with an initial velocity of 10m/s at an angle 37° above the horizontal, hits the ground at a distance 16m from the base of tower. Calculate height of tower. [$g = 10$ m/s^2]

Ans. 8 m



Sol. So time to reach ground is = $\frac{16}{8}$ = 2 sec

on y-axis (for height of tower)

$$-h = 6 \times 2 - \frac{1}{2} \times 10 \times (2)^2$$

$-h = 12 - 22$

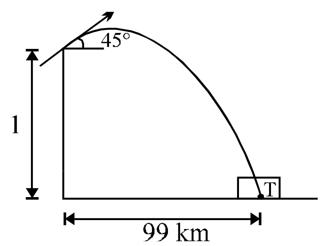
$h = 8$ m

Ex. Prithvi missile is fired to destroy an enemy military base situated on same horizontal level, situated 99 km away. The missile rises vertically for 1 km & then for remainder of flight, it followes parbolic path like a free body under earth's gravity, at an angle of 45°. Calculate its velocity at beginning of parabolic path. ($g = 10$ ms^{-2})

Sol. for horizontal motion time t

$$t = \frac{99 \times 10^3}{u \cos 45°}$$

for vertical



$$-1 \times 10^3 = u \sin 45°\, t - \frac{1}{2} \times 10 \times t^2$$

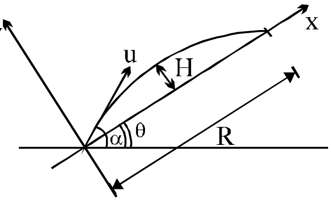
$$1 \times 10^3 + \frac{u \sin 45°}{u \sin 45°} \times 99 \times 10^3 = \frac{10}{2} \times \frac{(99 \times 10^3)^2 \times 2}{u^2}$$

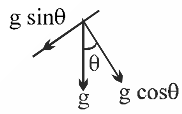
$$u^2 = \frac{(99 \times 10^3)^2 \times 10}{100 \times 10^3}$$

$$u = 99 \times 10^3 \sqrt{\frac{1}{10^4}} = 990 \text{ ms}^{-1}$$

PROJECTION ON INCLINED PLANE

There is an inclined plane making an angle θ with horizontal. A particle is projected at an angle α from horizontal.





x-axis

$u_x = u \cos(\alpha - \theta)$

$a_x = -g \sin\theta$

vel. at any time t

$v_x = u\cos(\alpha - \theta) - g\sin\theta\, t$

y-axis

$u_y = u\sin(\alpha - \theta)$

$a_y = -g\cos\theta$

vel. at any time t

$v_y = u\sin(\alpha - \theta) - g\cos\theta\, t$

Time of flight

Displacement in y direction $s_y = 0$

$$0 = u\sin(a - \theta)\,T - \frac{1}{2}g\cos\theta\,T^2$$

$$T = \frac{2u\sin(\alpha - \theta)}{g\cos\theta}$$

Maximum distance of particle from inclined plane

Pt. where $v_y = 0$ is max. height

$(0)^2 = u^2\sin^2(\alpha - \theta) - 2g\cos\theta\,H$

$$H = \frac{u^2\sin^2(\alpha - \theta)}{2g\cos\theta}$$

Range along the inclined plane

$$s_x = u_x T + \frac{1}{2}a_x T^2$$

$$R = \frac{u\cos(\alpha - \theta)\times 2u\sin(\alpha - \theta)}{g\cos\theta} - \frac{2\sin\theta\times 2\times 2u^2\sin^2(\alpha - \theta)}{2g^2\cos^2\theta}$$

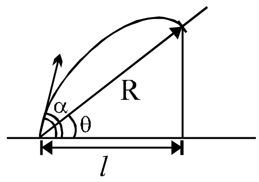
$$= \frac{2u^2\sin(\alpha - \theta)[\cos(\alpha - \theta)\cos\theta - \sin\theta\sin(\alpha - \theta)]}{g\cos^2\theta}$$

$$R = \frac{2u^2\sin(\alpha - \theta)\cos\alpha}{g\cos^2\theta}$$

$$R = \frac{u^2[\sin(2\alpha - \theta) - \sin\theta]}{g\cos^2\theta}$$

Alternate Method

$l = u \cos\alpha \, T$



$$R = \frac{l}{\cos\theta}$$

$$R = \frac{u\cos\alpha}{\cos\theta} \times \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$$R = \frac{2u^2\sin(\alpha-\theta)\cos\alpha}{g\cos^2\theta}$$

Note : Presence of incline plane does not affect the path of projectile in any way.

Maximum Range :

$$R = \frac{u^2[\sin(2\alpha-\theta)-\sin\theta]}{g\cos^2\theta}$$

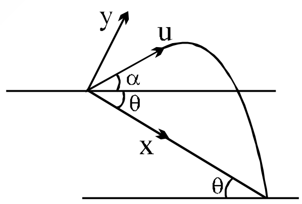
For max. range $2\alpha - \theta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4} + \frac{\theta}{2}$

so $R_{max} = \frac{u^2}{g(1+\sin\theta)}$

Projection from top of incline plane:

Incline plane is at an angle θ with horizontal and a particle is projected at an angle α from horizontal.

In all formulae replace θ with –θ



$$H = \frac{u^2\sin^2(\alpha+\theta)}{2g\cos\theta}$$

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$

$$R = \frac{2u^2\sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta}$$

$R_{max} = \frac{u^2}{g(1-\sin\theta)}$ and $\alpha = \frac{\pi}{4} - \frac{\theta}{2}$

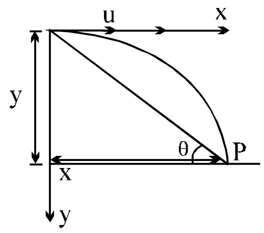
Note : If a particle strikes the incline plane ⊥ then its comp. of velocity along incline must be zero.

Ex. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far along the plane, from the point of projection will particle strike the plane?

Sol.

x-axis	**y-axis**
$u_x = u$	$u_x = 0$
$a_x = 0$	$a_x = g$
$x = ut$	$y = \frac{gt^2}{2}$

$$\Rightarrow \quad y = \frac{g x^2}{2u^2}$$



also $\frac{y}{x} = \tan\theta \Rightarrow x\tan\theta = \frac{g x^2}{2u^2}$

$x = 0,\ \frac{2u^2 \tan\theta}{g}$

$$x = \frac{2u^2 \tan\theta}{g}$$

$$\Rightarrow y = \frac{2u^2 \tan^2\theta}{g}$$

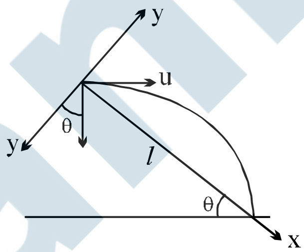
dist. $l = \sqrt{x^2 + y^2}$

$$l = \frac{2u^2 \tan\theta \sec\theta}{g}$$

<u>Alternate Method.</u>

$$R = \frac{2u^2 \sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta}$$

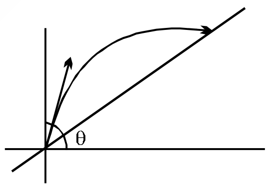
$R = 2\,u^2 \tan\theta \sec\theta$



Ex. A particle is projected up an inclined plane. Plane is inclined at an angle θ with horizontal and particle is projected at an angle α with horizontal. If particle strikes the plane horizontally prove that $\tan\alpha = 2\tan\theta$

Sol. We know time of flight

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$



considering vertical motion

$u = v\sin\alpha$

$a = -g$

$v = 0$

$$\therefore\ T = \frac{u\sin\alpha}{g} = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$\sin\alpha\cos\theta = 2\sin\alpha\ \cos\theta - 2\cos\alpha\sin\theta$

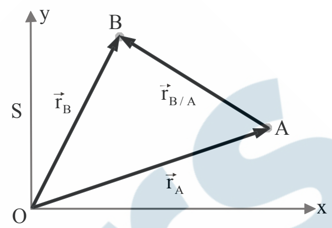
$2\cos\alpha\sin\theta = \sin\alpha\cos\theta$

$2\tan\theta = \tan\alpha$

Relative Motion

Motion of a body can only be observed, when it changes its position with respect to some other body. In this sense, motion is a relative concept. To analyze motion of a body say A, therefore we have to fix our reference frame to some other body say B. The result obtained is motion of body A relative to body B.

Relative position, Relative Velocity and Relative Acceleration



Let two bodies represented by particles A and B at positions defined by position vectors $\vec{r}_A$ and $\vec{r}_B$, moving with velocities $\vec{v}_A$ and $\vec{v}_B$ and accelerations $\vec{a}_A$ and $\vec{a}_B$ with respect to a reference frame S. For analyzing motion of terrestrial bodies the reference frame S is fixed with the ground.

The vectors $\vec{r}_{B/A}$ denotes position vector of B relative to A. Following triangle law of vector addition, we have

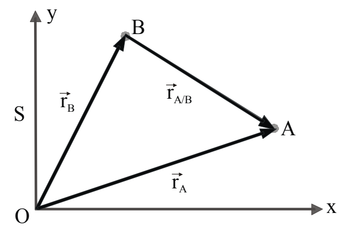
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad \text{...(i)}$$

First derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to velocity of particle A and velocity of particle B relative to frame S and first derivative of $\vec{r}_{B/A}$ with respect to time defines velocity of B relative to A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \text{...(ii)}$$

Second derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to acceleration of particle A and acceleration of particle B relative to frame S and second derivative of $\vec{r}_{B/A}$ with respect to time defines acceleration of B relative to A.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad \text{...(iii)}$$



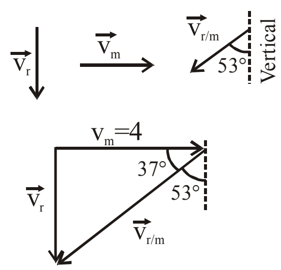
In similar fashion motion of particle A relative to particle B can be analyzed with the help of adjoining figure. You can observe in the figure that position vector of A relative to B is directed from B to A and therefore

$$\vec{r}_{B/A} = -\vec{r}_{A/B},\ \vec{v}_{B/A} = -\vec{v}_{A/B} \text{ and } \vec{a}_{B/A} = -\vec{a}_{A/B}.$$

The above equations elucidate that how a body A appears moving to another body B is opposite to how body B appears moving to body A.

Ex. A man when standstill observes the rain falling vertically and when he walks at 4 km/h he has to hold his umbrella at an angle of 53° from the vertical. Find velocity of the raindrops.

Sol. Assigning usual symbols $\vec{v}_m$, $\vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following eq. $\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$



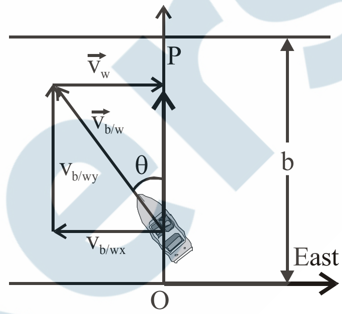
The above equation suggests that a standstill man observes velocity $\vec{v}_r$ of rain relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.

The addition of velocity vectors is represented according to the above equation is also represented. From the figure we have

$v_r = v_m \tan 37° = 3$ km/h **Ans.**

Ex. A boat can be rowed at 5 m/s on still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.
(a) In which direction must it be steered to cross the river perpendicular to current?
(b) How long will it take to cross the river in a direction perpendicular to the river flow?
(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Sol. (a) Velocity of a boat on still water is its capacity to move on water surface and equals to its velocity relative to water.

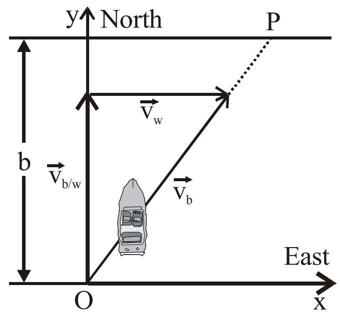


$\vec{v}_{b/w}$ = Velocity of boat relative to water = Velocity of boat on still water

On flowing water, the water carries the boat along with it. Thus velocity $\vec{v}_b$ of the boat relative to the ground equals to vector sum of $\vec{v}_{b/w}$ and $\vec{v}_w$. The boat crosses the river with the velocity $\vec{v}_b$.

$$\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w$$

(b) To cross the river perpendicular to current the boat must be steered in a direction so that one of the components of its velocity ($\vec{v}_{b/w}$) relative to water becomes equal and opposite to water flow velocity $\vec{v}_w$ to neutralize its effect. It is possible only when velocity of boat relative to water is grater than water flow velocity. In the adjoining figure it is shown that the boat starts from the point O and moves along the line OP (y-axis) due north relative to ground with velocity $\vec{v}_b$. To achieve this it is steered at an angle θ with the y-axis.



$v_{b/w} \sin\theta = v_w \rightarrow 5\sin\theta = 3 \Rightarrow \theta = 37°$ **Ans.**

(c) The boat will cover river width b with velocity

$v_b = v_{b/wy} = v_{b/w} \sin 37° = 4$ m/s in time t, which is given by

$t = b/v_b \rightarrow t = 50s$ **Ans.**

(d) To cross the river in minimum time, the component perpendicular to current of its velocity relative to ground must be kept to maximum value. It is achieved by steering the boat always perpendicular to current as shown in the adjoining figure. The boat starts from O at the south bank and reaches point P on the north bank. Time t taken by the boat is given by

$t = b/v_{b/w} \rightarrow t = 40s$ **Ans.**

Drift is the displacement along the river current measured from the starting point. Thus, it is given by the following equation. We denote it by x_d.

$$x_d = v_{bx} t$$

Substituting $v_{bx} = v_w = 3$ m/s, from the figure, we have

$x_d = 120$ m **Ans.**

EXERCISE (S-1)

General 2-D motion

1. The vertical height y and horizontal distance x of a projectile on a certain planet are given by $x = (3t)m$, $y = (4t - 6t^2)$ m where t is in seconds. Find the speed of projection (in m/s).

2. The position of a particle is given by

$$r = 3.0t\ \hat{i} - 2.0t^2\ \hat{j} + 4.0\ k\ m$$

where t is in seconds and the coefficients have the proper units for r to be in metres.

(a) Find the v and a of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s ?

3. A particle moves in xy plane such that $v_x = 50 - 16\ t$ and $y = 100 - 4t^2$ where v_x is in m/s and y is in m. It is also known that x = 0 when t = 0. Determine (i) Acceleration of particle (ii) Velocity of particle when y = 0.

4. The position of a particle is given by $x = 7 + 3t^3$ m and $y = 13 + 5t - 9t^2$ m, where x and y are the position coordinates, and t is the time in s. Find the speed (magnitude of the velocity) when the x component of the acceleration is 36 m/s^2.

Projectile motion

5. A particle is projected with a speed of 10 m/s at an angle 37° with the vertical. Find (i) time of flight (ii) maximum height above ground (iii) horizontal range.

6. A particle is thrown with a speed 60 ms^{-1} at an angle 60° to the horizontal. When the particle makes an angle 30° with the horizontal in downward direction, it's speed at that instant is v. What is the value of v^2 in SI units ?

7. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

8. A particle is projected upwards with a velocity of 100 m/s at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking g =10 m/s^2.

9. A particle is projected in x-y plane with y-axis along vertical, the point of projection is origin. The equation of a path is $y = \sqrt{3}\,x - \dfrac{gx^2}{2}$. Find angle of projection and speed of projection.

10. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{ox}}\right)$$

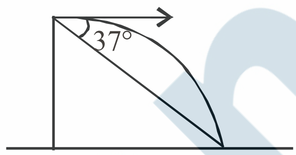
(b) Show that the projection angle θ_0 of a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

11. A particle is projected in the *x*-*y* plane with *y*-axis along vertical. Two second after projection the velocity of the particle makes an angle 45° with the *X*-axis. Four second after projection, it moves horizontally. Find the velocity of projection.

12. A ball is thrown horizontally from a cliff such that it strikes ground after 5 s. The line of sight from the point of projection to the point of landing makes an angle of 37° with the horizontal. What is the initial velocity of projection?



13. A Bomber flying upward at an angle of 53° with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 s after its release. Find: [Given sin 53° = 0.8; g = 10 m/s^2]

(i) The velocity of the bomber at the time of release of the bomb .

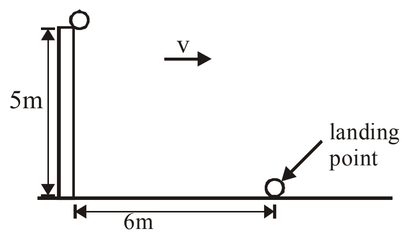
(ii) The maximum height attained by the bomb .

(iii) The horizontal distance travelled by the bomb before it strikes the ground

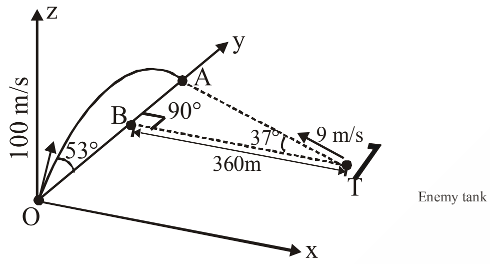
(iv) The velocity (magnitude & direction) of the bomb just when it strikes the ground.

14. A ball is projected at an angle of 30° above the horizontal from the top of a tower and strikes the ground in 5 s at an angle of 45° with the horizontal. Find the height of the tower and the speed with which it was projected.

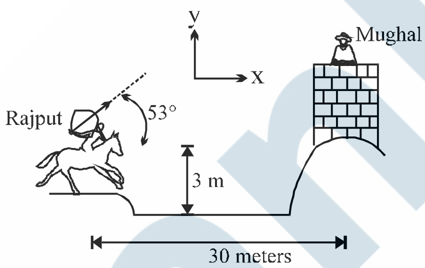
15. A ball is dropped from rest from a tower of height 5m. As a result of the wind it lands at a distance 6m from the bottom of the tower as shown. Assuming no air resistance but that the wind gives the ball a constant horizontal velocity v. Find value of v in m/s.



16. A tank is initially at a perpendicular distance BT = 360 m from the plane of firing as shown. The enemy tank is moving with a speed of 9 m/s in direction TA as shown in figure. A gun can fire shell in y-z plane only with a speed 100 m/s at an angle of 53° such that the shell lands at points A. If tank started at t = 0 then time interval (in sec) after which shell is to be fired to hit the tank is



17. A Rajput soldier, sits on a horse next to a river. Across the river there is a hill and atop the hill is a fortress. He sees a Mughal, sitting on the fortress's top wall. There is a full moon, so he angrily shoots an arrow at an angle 53° relative to the horizontal. The arrow hits Mughal after a 2 second flight. The horizontal distance from Rajput to Mughal is 30 meters. The arrow is 3 meters above the river when Rajput shoots it.

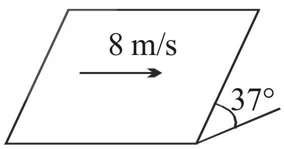


(a) What is the original velocity, $\vec{V}$, of the arrow when Rajput shoots it?

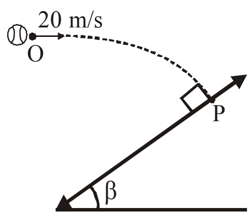
(b) What is Mughal elevation above the river?

(c) What is the flight direction of the arrow the instant before it strikes Mughal, i.e. what is the angle, θ, between its direction and the horizontal when it skins into Mughal's tender flesh?

18. A ball is projected on smooth inclined plane in direction perpendicular to line of greatest slope with velocity of 8 m/s. Find it's speed after 1 s.



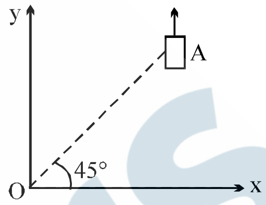
19. A ball is thrown horizontally from a point O with speed 20 m/s as shown. Ball strikes the incline plane along the normal to it after two seconds. Find value of x, if $\beta = \pi/x$ (where β is the angle of incline in degree).



Relative motion

20. A person decided to walk on an escalator which is moving at constant rate (speed). When he moves at the rate 1 step/sec, then he reaches top in 20 steps. Next day he goes 2 steps / sec. and reaches top in 32 steps. If speed of escalator is n steps / sec. Find the value of n.

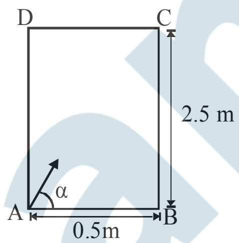
21. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley. A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.



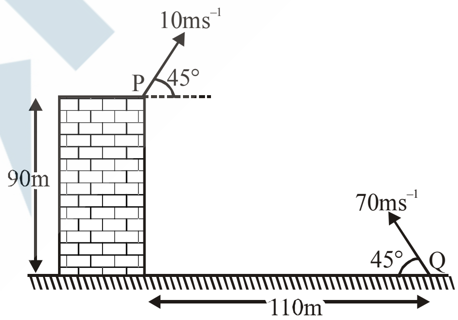
(a) The motion of the ball is observed from the frame of trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = \frac{4\theta}{3}$.

22. A cuboidal elevator cabin is shown in the figure. A ball is thrown from point A on the floor of cabin when the elevator is falling under gravity. The plane of motion is ABCD and the angle of projection of the ball with AB, relative to elevator, if the ball collides with point C, is α. Then find the value of $\tan\alpha$.



23. Two particles P and Q are launched simultaneously as shown in figure. Find the minimum distance between particles in meters.

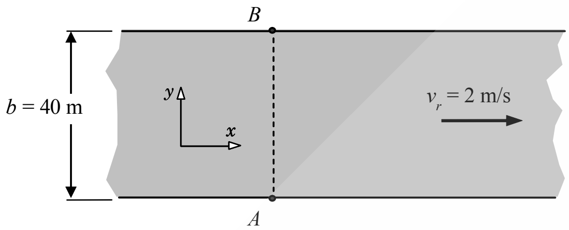


24. A man crosses a river by a boat. If he crosses the river in minimum time he takes 10 minutes with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 minutes. Assuming $v_{b/r} > v_r$, find (i) width of the river, (ii) velocity of the boat with respect to water ($v_{b/r}$) (iii) speed of the current (v_r)

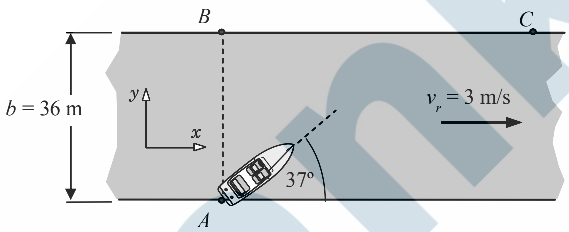
25. Rain is falling vertically with a speed of 20 m/s relative to air. A person is running in the rain with a velocity of 5 m/s and a wind is also blowing with a speed of 15 m/s (both towards east). Find the angle with the vertical at which the person should hold his umbrella for best protection from rain.

26. A glass wind-screen of adjustable inclination is mounted on a car. The car moves horizontally with a speed of 6 m/s. At what angle α with the vertical should the wind screen be adjusted so that the rain drops falling vertically with 2 m/s strike the wind screen perpendicularly?

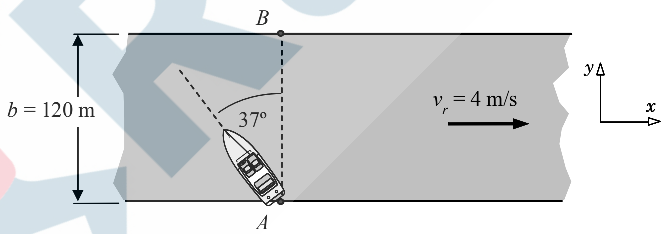
27. Boat moves with velocity 5m/s on still water. It is steered perpendicular to the river current.
(a) Will it reach point B or somewhere else on the other bank ?
(b) How long will it take to cross the river ?
(c) How far down stream, will it reach the other bank ?
(d) Does it take minimum time in this way ?



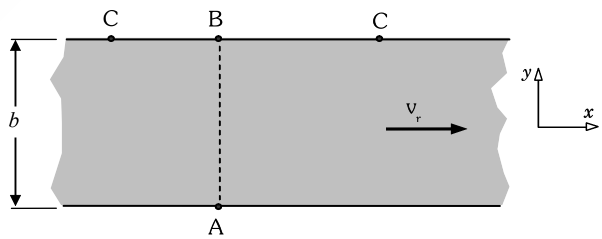
28. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown to reach point C. Find the time of the trip and distance between B and C.



29. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown. Where will it reach on the opposite bank?



30. Drift is distance along a river a boat covers in crossing the river. If the boat reaches point C, distance BC is called downstream drift and if the boat reaches point D, distance BD is called upstream drift. To cross a river without drift, what should be relation between v_{br} and v_r. If a boat crosses a river without drift, in which direction must it be steered.



EXERCISE (S-2)

1. A particle travels so that its acceleration is given by $\vec{a} = 5\cos t\,\hat{i} - 3\sin t\,\hat{j}$. If the particle is located at (–3, 2) at time t = 0 and is moving with a velocity given by $(-3\hat{i} + 2\hat{j})$. Find

(a) the velocity at time t and

(b) the position vector of the particle at time (t > 0).

2. A man can throw a stone with initial speed of 10 m/s. Find the maximum horizontal distance to which he can throw the stone in a room of height h for :

(i) $h = 2$ m &

(ii) $h = 4$ m

3. Two inclined planes OA and OB having inclinations 30° and 60° respectively, intersect each other at O as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}$ m/s along a direction perpendicular to plane OA. If the particle strikes the plane OB perpendicularly at Q. Calculate

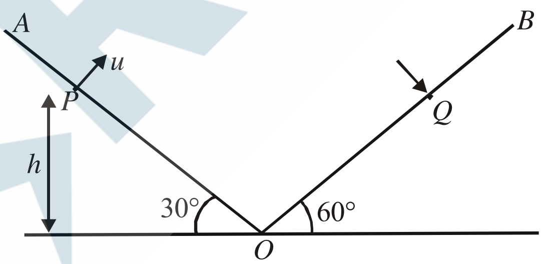
(i) Time of flight.

(ii) Velocity with which particle strikes the plane OB.

(iii) Vertical height of P from O.

(iv) Maximum height from O attained by the particle.

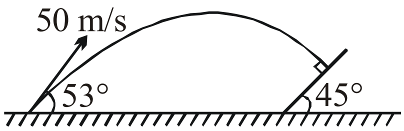
(v) Distance PQ.



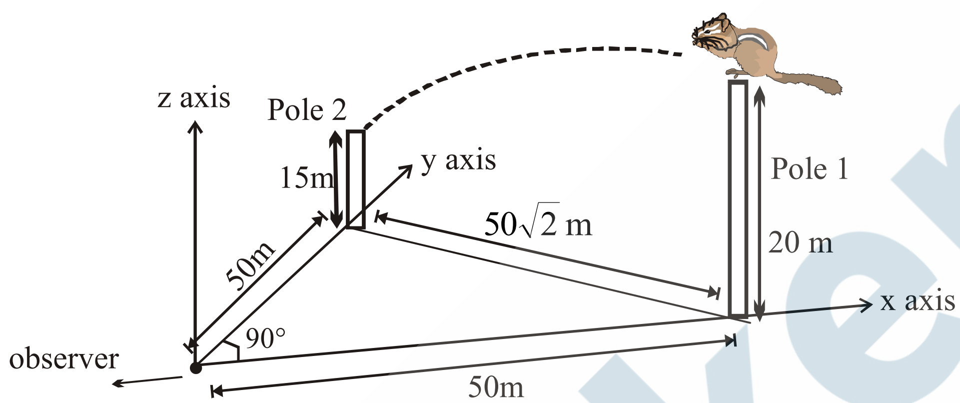
4. A projectile is thrown with velocity of 50 m/s towards an inclined plane from ground such that it strikes the inclined plane perpendicularly. The angle of projection of the projectile is 53° with the horizontal and the inclined plane is inclined at an angle of 45° to the horizontal.

(i) Find the time of flight.

(ii) Find the distance between the point of projection and the foot of inclined plane.



5. A small squirrel jumps from pole 1 to pole 2 in horizontal direction. Squirrels is observed by a very small observer at origin. What is average velocity vector of squirrel ? If average velocity vector is expressed as $v_x\hat{i}+v_y\hat{j}+v_z\hat{k}$, express your answer as sum of magnitudes of its components $|v_x|+|v_y|+|v_z|$ in unit m/s.

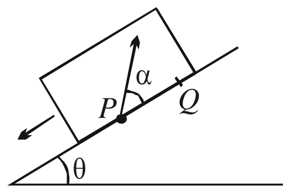


6. Two particles start simultaneously from a point and move along line *OP* and *OQ*, one with a uniform velocity 10 m/s and other from rest with a constant acceleration 5 m/s^2 respectively. Line *OP* makes an angle 37° with the line *OQ*. Find the time at which they appear to each other moving at minimum speed?

7. Two guns, situated at the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires at an angle of 60° up the horizontal. The shots collide in air at a point *P*. Find
(i) the time interval between the firings, and (ii) the coordinates of the point *P*.
(Take origin of the coordinates system at the foot of the hill vertically below the muzzle and trajectories in *X*-*Y* plane)

8. A hunter is riding an elephant of height 4 m moving in straight line with uniform velocity of 2 m/s. A deer starts running with uniform velocity from a point $4\sqrt{5}$ m away in front of the elephant along a line perpendicular to velocity of the elephant. If hunter can throw his spear with a speed of 10 m/s, relative to the elephant, at what angle θ to it's direction of motion must he throw his spear horizontally for a successful hit. Find also the speed of the deer.

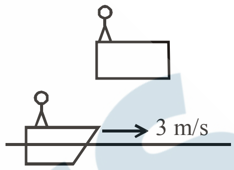
9. A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point *P* on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to box is *u* and the direction of projection makes an angle α with the bottom as shown in figure. **[JEE]**



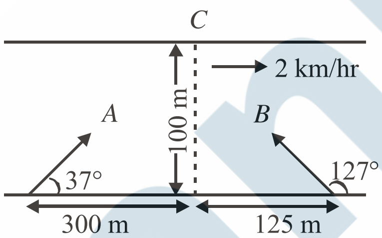
(i) Find the distance along the bottom of the box between the point of projection *P* and the point *Q* where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).
(ii) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

10. A particle is projected from ground towards a vertical wall 80 m away at an angle of 37° with horizontal with initial velocity of 50 m/s. After its collision with wall & then once with ground find at what distance in meter from wall will it strike the ground. The component of velocity normal to the surface becomes half after collision with each surface.

11. You are standing on the chambal Bridge watching the boats in the river. You see a motorboat pass directly below you, traveling perpendicular to the bridge at a speed of 3 m/s. A person on the boat throws a baseball at an initial speed of v_0 and at an angle of 37° from the vertical (Note: both v_0 and the angle are with respect to the boat). Find the value of v_0 (in m/s) necessary for the ball to travel straight up towards you.



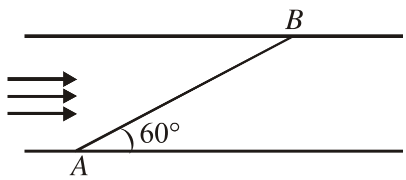
12. Two swimmers start a race. One who reaches the point *C* first on the other bank wins the race. Boy *A* makes his strokes in a direction of 37° to the river flow with velocity 5 km/hr relative to water. Boy *B* makes his strokes in a direction of 127° to the river flow with same relative velocity. River is flowing with speed of 2 km/hr and is 100 m wide. Who will win the race? Compute the time taken by *A* and *B* to reach the point *C* if the speeds of *A* and *B* on the ground are 8 km/hr and 6 km/hr respectively.



13. A swimmer starts to swim from point *A* to cross a river. He wants to reach point *B* on the opposite side of the river. The line *AB* makes an angle 60° with the river flow as shown. The velocity of the swimmer in still water is same as that of the water

(i) In what direction should he try to direct his velocity? Calculate angle between his velocity and river velocity.

(ii) Find the ratio of the time taken to cross the river in this situation to the minimum time in which he can cross this river.



14. Hailstones falling vertically with speed of 10 m/s hit with respect to himself the windscreen of a moving car and rebound elastically. Find the velocity of the car if the driver find the hailstones rebound vertically after striking. Windscreen makes an angle 30° with the horizontal.

15. A motor boat set out at 11 a.m. from a position –6i – 2j and travels at a steady speed of magnitude $\sqrt{53}$ on a direct course to intercept a ship. The ship maintains a steady velocity vector 3i + 4j and at 12 noon is at a position 3i – j. Find

(a) the velocity vector of the motor boat,

(b) the time of interception and

(c) the position vector of point of interception. If distances are measured in kilometres and speeds in kilometres per hour.

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

General 2-D motion

1. A particle is projected from a horizontal plane (x-z plane) such that its velocity vector at time t is given by $\vec{V} = a\hat{i} + (b - ct)\hat{j}$. Its range on the horizontal plane is given by :-

(A) $\frac{ba}{c}$ (B) $\frac{2ba}{c}$ (C) $\frac{3ba}{c}$ (D) None

2. A body of mass 5 kg starts from the origin with an initial velocity $\vec{u} = (30\hat{i} + 40\hat{j})\,\text{ms}^{-1}$. If a constant force $(-6\hat{i} - 5\hat{j})\,\text{N}$ acts on the body, the time in which the y component of the velocity becomes zero, is :-

(A) 5s (B) 20 s (C) 40 s (D) 80 s

Projectile motion

3. A boy throws a ball from shoulder height at an initial velocity of 30 m/s. Spending 4.8 s in air, the ball is caught by another boy as the same shoulder-height level. What is the angle of projection?

(A) 37° (B) 30° (C) 53° (D) 60°

4. A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of flight are R, H and T respectively. They are given by

$$R = \frac{u^2 \sin 2\theta}{g} \qquad H = \frac{u^2 \sin^2 \theta}{2g} \qquad \text{and} \qquad T = \frac{2u \sin\theta}{g}$$

Now keeping u fixed, θ is varied from 30° to 60°, then

(A) R will first increase then decrease, H will increase and T will decrease

(B) R will first increase then decrease while H and T both will increase

(C) R will decrease while H and T both will increase

(D) R will increase while H and T both will also increase

5. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?

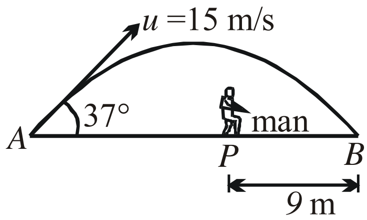
(A) The one with the farthest range.

(B) The one which reaches maximum height.

(C) The one with the greatest initial velocity.

(D) The one leaving the bat at 45° with respect to the ground.

6. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.

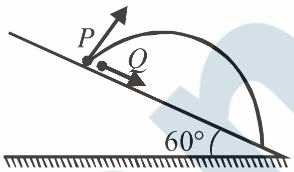


(A) 3 m/s (B) 5 m/s (C) 9 m/s (D) 12 m/s

7. A ball is projected horizontally. After 3 s from projection its velocity becomes 1.25 times of the velocity of projection. Its velocity of projection is :-

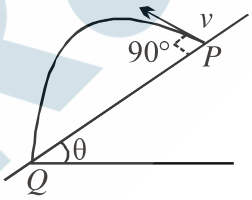
(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

8. A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide after t = 4 s. The speed of projection of P is :-



(A) 5 m/s (B) 10 m/s (C) 15 m/s (D) 20 m/s

9. In the given figure, if time taken by the projectile to reach Q is T, than PQ =



(A) $Tv \sin\theta$ (B) $Tv \cos\theta$ (C) $Tv \sec\theta$ (D) $Tv \tan\theta$

10. A particle is projected up the incline such that its component of velocity along the incline is 10m/s. Time of flight is 2 second and maximum perpendicular distance during the motion from the incline is 5 m. Then velocity of projection will be :-

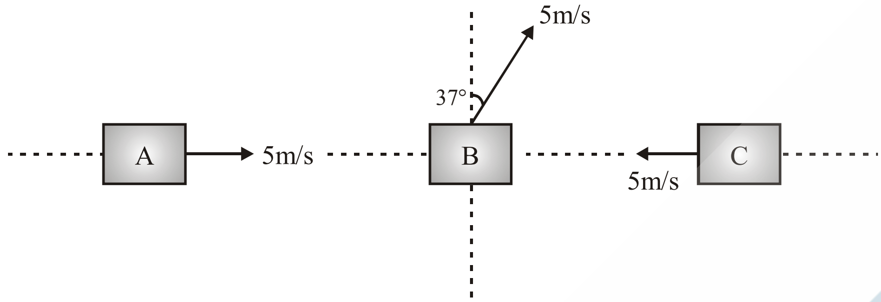
(A) 10 m/s (B) $10\sqrt{2}$ m/s (C) $5\sqrt{5}$ m/s (D) none of these

11. A particle A is projected with speed v_A from a point making an angle 60° with the horizontal. At the same instant, a second particle B is thrown vertically upward from a point directly below the maximum height point of parabolic path of A with velocity v_B. If the two particles collide then the ratio of v_A/v_B should be :-

(A) 1 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

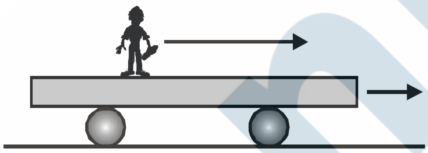
Relative motion

12. Consider the motion of three bodies as shown for an observer on B, what is the magnitude of relative velocity of A with respect to C ?



(A) 10 m/s (B) 0 (C) $\sqrt{20}$ m/s (D) 8 m/s

13. A trolley is moving horizontally with a constant velocity of v with respect to earth. A man starts running from one end of the trolley with a velocity 1.5 v with respect to trolley. After reaching the opposite end, the man returns back and continues running with a velocity of 1.5 v w.r.t. the trolley in the backward direction. If the length of the trolley is L then the displacement of the man with respect to earth during the process will be :-



(A) 2.5 L (B) 1.5 L (C) $\frac{5L}{3}$ (D) $\frac{4L}{3}$

14. An elevator car (lift) is moving upward with uniform acceleration of 2 m/s^2. At the instant, when its velocity is 2 m/s upwards a ball is thrown upward from its floor. The ball strikes back the floor 2 s after its projection. Find the velocity of projection of the ball relative to the lift.

(A) 10 m/s ↑ (B) 10 m/s ↓ (C) 12 m/s ↑ (D) 12 m/s ↓

15. A flag is mounted on a car moving due North with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20 km/hr. The flag will point in direction :-

(A) East (B) North-East (C) South-East (D) South-West

16. Three ships A, B & C are in motion. Ship A moves relative to B is with speed v towards North-East. Ship B moves relative to C with speed v towards the North-West. Then relative to A, C will be moving towards :-

(A) North (B) South (C) East (D) West

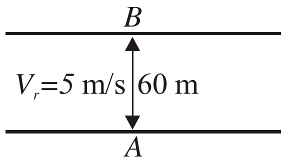
17. Wind is blowing in the north direction at speed of 2 m/s which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him :

(A) 2 m/s south (B) 2 m/s north (C) 4 m/s west (D) 4 m/s south

18. A boat having a speed of 5 km/hr in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river in Km/hr.

(A) 1 (B) 3 (C) 4 (D) √41

19. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across the river at a distance of 60 m in 5 sec. His velocity in still water should be :-



(A) 12 m/s (B) 13 m/s (C) 5 m/s (D) 10 m/s

20. A motor boat is to reach at a point 30° upstream (w.r.t. normal) on other side of a river flowing with velocity 5m/s. The angle 30° is measured from a direction perpendicular to river flow. Velocity of motorboat with respect to water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle

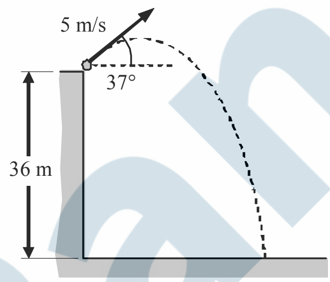
(A) 120° with respect to stream direction.

(B) 30° with respect to the perpendicular to the bank.

(C) 30° with respect to the line of destination from starting point.

(D) None of these.

21. A ball is thrown from the top of 36 m high tower with velocity 5 m/s at an angle 37° above the horizontal as shown. Its horizontal range on the ground is closest to [g = 10 m/s^2]



(A) 12 m (B) 18 m (C) 24 m (D) 30 m

MULTIPLE CORRECT TYPE QUESTIONS

22. A particle moves in the *xy*-plane and at time *t* is at the point $(t^2,\ t^3 - 2t)$. Then

(A) At $t = 2/3$ s, directions of velocity and acceleration are perpendicular

(B) At $t = 0$, directions of velocity and acceleration are perpendicular

(C) At $t = \sqrt{2/3}$ s, particle is moving parallel to *x*-axis

(D) Acceleration of the particle when it is at point (4, 4) is $\left(2\hat{i} + 12\hat{j}\right) m/s$

23. A particle moves in the *x*-*y* plane with a constant acceleration *g* in the negative *y*-direction. Its equation of motion is $y = ax - bx^2$, where *a* and *b* are constants. Which of the following is/are correct?

(A) The *x*-component of its velocity is constant.

(B) At the origin, the *y*-component of its velocity is $a\sqrt{\dfrac{g}{2b}}$.

(C) At the origin, its velocity makes an angle $\tan^{-1}(a)$ with the *x*-axis.

(D) The particle moves exactly like a projectile.

24. Two particles A and B projected along different directions from the same point P on the ground with the same velocity of 70 m/s in the same vertical plane . They hit the ground at the same point Q such that $PQ = 480$ m. Then [$g = 9.8$ m/s^2]

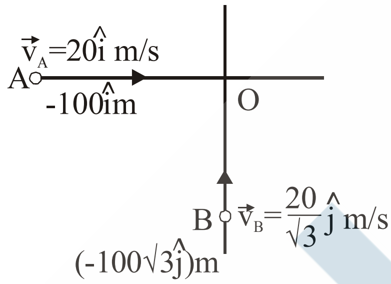
(A) Ratio of their times of flight is 4 : 5

(B) Ratio of their maximum heights is 9 : 16

(C) Ratio of their minimum speeds during flights is 4 : 3

(D) The bisector of the angle between their directions of projection makes 45° with horizontal

25. Positions of two vehicles A and B with reference to origin O and their velocities are as shown.



(A) they will collide

(B) distance of closest approach is 100 m.

(C) their relative speed is $\frac{40}{\sqrt{3}}$ m/s

(D) their relative velocity is $\frac{20}{\sqrt{3}}$ m/s

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 26 to 28

Two projectiles are thrown simultaneously in the same vertical plane from the same point. If their velocities of projection are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal, then answer the following questions

26. The trajectory of particle 1 with respect to particle 2 will be

(A) a parabola

(B) a straight line

(C) a vertical straight line

(D) a horizontal straight line

27. If $v_1\cos\theta_1 = v_2\cos\theta_2$, then choose the ***incorrect*** statement

(A) One particle will remain exactly below or above the other particle

(B) The trajectory of one with respect to other during the flight will be a vertical straight line

(C) Both will have the same range

(D) Both will attain same maximum height

28. If $v_1\sin\theta_1 = v_2\sin\theta_2$, then choose the ***correct*** statement

(A) The time of flight of both the particles will be same

(B) The maximum height attained by the particles will be same

(C) The trajectory of one with respect to another during the flight will be a horizontal straight line

(D) None of these

Paragraph for Question Nos. 29 to 31

A particle leaves the origin with initial velocity $\vec{v}_0 = 11\hat{i} + 14\hat{j}$ m/s. It undergoes a constant acceleration given by $\vec{a} = -\frac{22}{5}\hat{i} + \frac{2}{15}\hat{j}$ m/s^2.

29. When does the particle cross the y axis ?

(A) 2 sec (B) 4 sec (C) 5 sec (D) 7 sec

30. At the instant when particle crosses y-axis, direction in which particle is moving is :-

(A) At angle 37° from +x-axis towards +y-axis
(B) At angle 37° from –x-axis towards +y-axis
(C) At angle 53° from +x-axis towards +y-axis
(D) At angle 53° from –x-axis towards +y-axis

31. How far is it from the origin, at that time ?

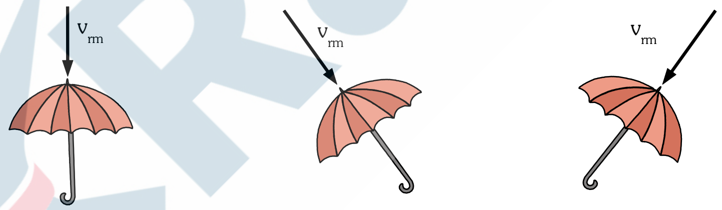
(A) 70 m (B) 71.67 m (C) 125 m (D) 15 m

Paragraph for Question no. 32 to 35: Rain and man

By the term velocity of rain, we mean velocity with which raindrops fall relative to the ground. In absence of wind, raindrops fall vertically and in presence of wind raindrops fall obliquely. Moreover raindrops acquire a constant terminal velocity due air resistance very quickly as they fall toward the earth. A moving man relative to himself observes an altered velocity of raindrops, which is known as velocity of rain relative to the man. It is given by the following equation.

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

A standstill man relative to himself observes rain falling with velocity, which is equal to velocity of the raindrops relative to the ground. To protect himself a man should hold his umbrella against velocity of raindrops relative to himself as shown in the following figure.



32. Rain is falling vertically with velocity 80 cm/s.

(a) How should you hold your umbrella?
(b) You start walking towards the east with velocity 60 cm/s. How should you hold your umbrella?
(c) You are walking towards the west with velocity 60 cm/s. How should you hold your umbrella?
(d) You are walking towards the north with velocity 60 cm/s. How should you hold your umbrella?
(e) You are walking towards the south with velocity 80 cm/s. How should you hold your umbrella?

33. When you are standstill in rain, you have to hold your umbrella vertically to protect yourself.

(a) When you walk with velocity 90 cm/s, you have to hold your umbrella at 53° above the horizontal. What is velocity of the raindrops relative to the ground and relative to you?
(b) If you walk with speed 160 cm/s, how should you hold your umbrella?

34. A man walks in rain at 72 cm/s due east and observes the rain falling vertically. When he stops, rain appears to strike his back at 37° from the vertical. Find velocity of raindrops relative to the ground.

35. When you walk in rain at 75 cm/s, you have to hold your umbrella vertically and when you double your speed in the same direction, you have to hold your umbrella at 53° above the horizontal. What is the rain velocity?

Paragraph for Question no. 36 to 38 : Flag in wind

When you are standstill holding a flag, the flag flutters in the direction of wind. When you start running the direction of fluttering of the flag changes in to the direction of the wind relative to you. In all case a flag flutters in the direction of the wind relative to the flag.

36. When you are standstill holding a flag the flag flutters in the north and when you run at 8 m/s due east, the flag flutters in direction 37° north of west. Find the wind velocity.

37. Wind is blowing uniformly due north everywhere with velocity 12 m/s. A car mounted with a flag starts running towards east. After 9 s from start the flag flutters in 53° north of west and after 16 s from the start the flag flutters in 37° north of west.

(a) Find velocity of the car 9 s after it starts.

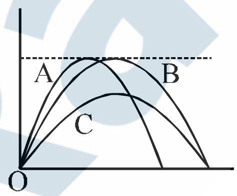
(b) Find velocity of the car 16 s after it starts.

(c) If the car maintains uniform acceleration, find acceleration of the car.

38. Holding a flag, when you run at 8 m/s due east, the flag flutters in the north and when you run at 2 m/s due south, the flag flutters in the northeast. If the wind velocity is uniform and remain constant, find the wind velocity.

MATRIX MATCH TYPE QUESTION

39. Trajectories are shown in figure for three kicked footballs. Initial vertical & horizontal velocity components are u_y and u_x respectively. Ignoring air resistance, choose the correct statement from **Column-II** for the value of variable in **Column-I**.



	Column-I		**Column-II**
(A)	time of flight	(P)	greatest for A only
(B)	u_y/u_x	(Q)	greatest for C only
(C)	u_x	(R)	equal for A and B
(D)	$u_x u_y$	(S)	equal for B and C

40. Trajectory of particle in a projectile motion is given as $y = x - \frac{x^2}{80}$. Here, x and y are in meters. For this projectile motion, match the following with $g = 10\ m/s^2$.

	Column–I		**Column–II**
(A)	Angle of projection (in degrees)	(P)	20
(B)	Angle of velocity with horizontal after 4s (in degrees)	(Q)	80
(C)	Maximum height (in metres)	(R)	45
(D)	Horizontal range (in metres)	(S)	30
		(T)	60

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A particle is moving in x-y plane. At certain instant, the components of its velocity and acceleration are as follows ; $V_x = 3$m/s, $V_y = 4$m/s, $a_x = 2$ m/s^2 and $a_y = 1$ m/s^2. The rate of change of speed at this moment is :-

(A) $\sqrt{10}$ m/s^2 (B) 4 m/s^2 (C) 10 m/s^2 (D) 2 m/s^2

2. A particle leaves the origin with an initial velocity $\vec{v} = \left(3\hat{i} + 4\hat{j}\right) ms^{-1}$ and a constant acceleration $\vec{a} = \left(-\hat{i} - 0.5\hat{j}\right) ms^{-2}$. When the particle reaches its maximum x–coordinate, what is the y–coordinate?

(A) $\frac{27}{4}$ m (B) $\frac{37}{4}$ m (C) $\frac{29}{4}$ m (D) $\frac{39}{4}$ m

3. The position vector of a particle is determined by $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$. The distance travelled in first 10 sec is :-

(A) 100 m (B) 150 m (C) 500 m (D) 300 m

4. A point moves in x–y plane according to the law x = 4sin6t and y = 4(1 – cos6t). The distance traversed by the particle in 4 seconds is (x and y are in metres)

(A) 96 m (B) 48 m (C) 24 m (D) 108 m

5. A particle moves in the x-y plane. It x and y coordinates vary with time t according to equations $x = t^2 + 2t$ and $y = 2t$. Possible shape of path followed by the particle is

(A) Straight line (B) Circle

(C) Parabola (D) More information is required to decide.

6. Particle is dropped from the height of 20 m from horizontal ground. A constant force acts on the particle in horizontal direction due to which horizontal acceleration of the particle becomes 6 m/s^2. Find the horizontal displacement of the particle till it reaches ground.

(A) 6 m (B) 10 m (C) 12 m (D) 24 m

7. A projectile is fired with a velocity u making an angle θ with the horizontal. What is the magnitude of change in velocity when it is at the highest point ?

(A) u cos θ (B) u (C) u sin θ (D) u cos θ – u

8. A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4m from it. The height of the wall is

(A) 3/4 m (B) 2/3 m (C) 4/3m (D) 1/3 m

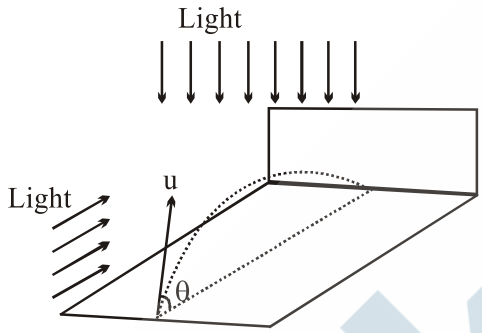
9. A ball was thrown by a boy A at angle 60° with horizontal at height 1m from ground. Boy B is running in the plane of motion of ball and catches the ball at height 1m from ground. He finds the ball falling vertically. If the boy is running at a speed 20 km/hr. Then the velocity of projection of ball is-

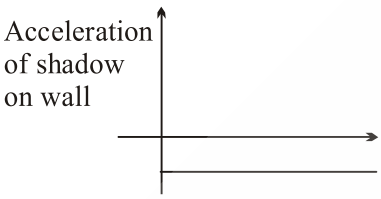
(A) 20 km/hr (B) 30 km/hr (C) 40 km/hr (D) 50 km/hr

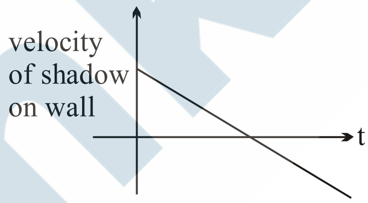
10. A light body is projected with a velocity $(10\hat{i}+20\hat{j}+20\hat{k})$ ms^{-1}. Wind blows along X-axis with an acceleration of 2.5 ms^{-2}. If Y-axis is vertical then the speed of particle after 2 second will be ($g = 10$ ms^{-2})

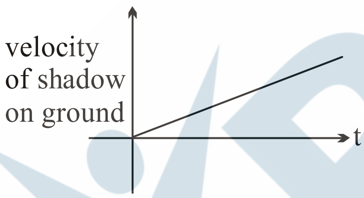
(A) 25 ms^{-1} (B) $10\sqrt{5}$ ms^{-1} (C) 30 ms^{-1} (D) None of these

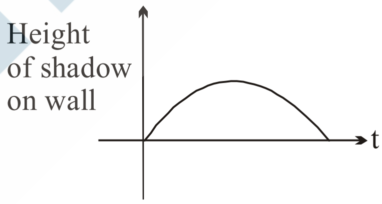
11. A projectile is projected as shown in figure. A proper light arrangement makes a shadow on the wall as well as on the floor ? Which of the following graphs is incorrect.



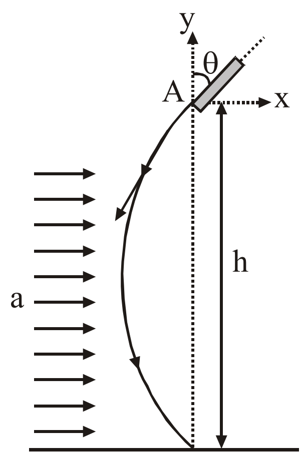
(A) 

(B) 

(C) 

(D) 

12. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y–axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x–directions. If the particle strikes the ground at a point directly under its released position and the downward y–acceleration is taken as g then



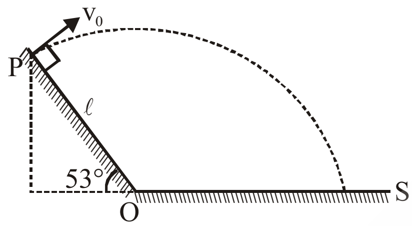
(A) $h = \dfrac{2v^2 \sin\theta \cos\theta}{a}$

(B) $h = \dfrac{2v^2 \sin\theta \cos\theta}{g}$

(C) $h = \dfrac{2v^2}{g}\sin\theta\left(\cos\theta + \dfrac{a}{g}\sin\theta\right)$

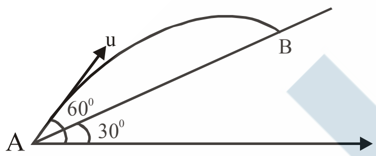
(D) $h = \dfrac{2v^2}{a}\sin\theta\left(\cos\theta + \dfrac{g}{a}\sin\theta\right)$

13. A stone is projected from point P on the inclined plane with velocity $v_0 = 10$ m/s directed perpendicular to the plane. The time taken by the stone to strike the horizontal ground S is
(Given PO = ℓ = 10 meter)



(A) 1.5 sec (B) 1.4 sec (C) 2 sec (D) 2.3 sec

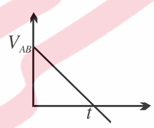
14. Time taken by the projectile to reach from A to B is t. Then the distance AB is equal to :-

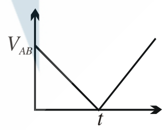


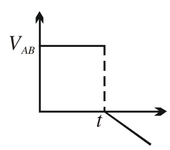
(A) $\frac{ut}{\sqrt{3}}$ (B) $\frac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}$ ut (D) 2 ut

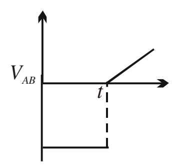
15. A particle is projected from a point *P* (2 m, 0 m, 0 m) with a velocity 10 m/s making an angle 45° with the horizontal. The plane of projectile motion passes through a horizontal line *PQ* which makes an angle of 37° with positive x–axis and *xy* plane is horizontal. The coordinates of the point where the particle will strike the line *PQ* is ($g = 10$ m/s^2)
(A) (10 m, 6 m, 0 m) (B) (8 m, 6 m, 0 m) (C) (10 m, 8 m, 0 m) (D) (6 m, 10 m, 0 m)

16. A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of *h*. Simultaneously another body *B* is dropped from height *h*. It strikes the ground and does not rebound. The velocity of *A* relative to *B* v/s time graph is best represented by (upward direction is positive)

(A) 

(B) 

(C) 

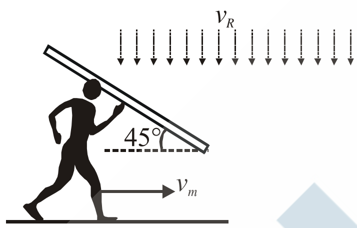
(D) 

17. An object moves to the East across a frictionless surface with constant speed. A person then applies a constant force to the North on the object. What is the resulting path that the object takes?
(A) A straight line path partly Eastward, partly Northward
(B) A straight line path totally to the North
(C) A parabolic path opening toward the North
(D) A parabolic path opening toward the East

18. A particle is thrown from a stationary platform with velocity v at an angle of 60° with the horizontal. The range obtained is R. If the platform moves horizontally in the direction of target with velocity v, the range will increase to :

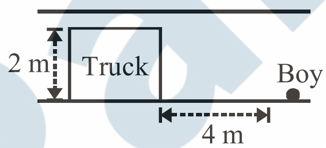
(A) $\frac{3R}{2}$ (B) $\frac{5R}{2}$ (C) 2 R (D) 3 R

19. On a particular day rain drops are falling vertically at a speed of 5 m/s. A man holding a plastic board is running to escape from rain as shown. The lower end of board is at a height half that of man and the board makes 45° with horizontal. The maximum speed of man so that his feet does not get wet, is



(A) 5 m/s (B) $5\sqrt{2}$ m/s (C) $5/\sqrt{2}$ m/s (D) zero

20. A 2 m wide truck is moving with a uniform speed of 8 m/s along a straight horizontal road. A pedestrian starts crossing the road at an instant when the truck is 4 m away from him. The minimum constant velocity with which he should run to avoid an accident is :-



(A) $1.6\sqrt{5}$ m/s (B) $1.2\sqrt{5}$ m/s (C) $1.2\sqrt{7}$ m/s (D) $1.6\sqrt{7}$ m/s

21. Two trucks are moving on parallel tracks. A person on one truck projects a ball vertically upward then path of the ball as seen by four observers: from the ground, from the second truck moving with same velocity as that first truck, from the second truck moving with speed greater than first one in same direction and from the second truck moving with speed less than the first truck in same direction are:

(A) Parabola, Parabola, Parabola and Parabola
(B) Straight line, Straight line, Parabola and Parabola
(C) Parabola, Straight line, Parabola and Parabola
(D) None of these

22. Man A sitting in a car moving at 54 km/hr observes a man B in front of the car crossing perpendicularly the road of width 15 m in three seconds. Then the velocity of man B will be

(A) $5\sqrt{10}$ towards the car (B) $5\sqrt{10}$ away from the car
(C) 5 m/s perpendicular to the road (D) None

23. A swimmer swims in still water at a speed = 5 km/hr. He enters a 200 m wide river, having river flow speed = 4 km/hr at point *A* and proceeds to swim at an angle of 127° with the river flow direction. Another point *B* is located directly across *A* on the other side. The swimmer lands on the other bank at a point *C*, from which he walks the distance *CB* with a speed = 3 km/hr. The total time in which he reaches from *A* to *B* is

(A) 5 minutes (B) 4 minutes (C) 3 minutes (D) None

24. A man wishes to swim across a river 400 m wide flowing with a speed of 3m/s, such that he reaches the point just infront on the other bank in time not greater than 100s. The angle made by the direction he swims and river flow direction is :-

(A) 90° (B) 127° (C) 150° (D) 143°

25. An observer on ground sees a boat cross a river of width 800 m perpendicular to its stream in 200seconds. He also finds a man on a raft floating at speed of 3 m/s with river. The distance travelled by boat as seen by man on the raft in crossing the river is-

(A) 800 m (B) 1000m (C) 1200m (D) 1600m

26. A boatman moves his boat with a velocity 'v' (relative to water) in river and finds to his surprise that velocity of river 'u' (with respect to ground) is more than 'v'. He has to reach a point directly opposite to the starting point on another bank by travelling minimum possible distance. Then

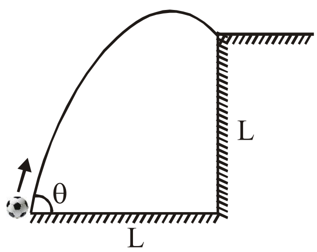
(A) he must steer the boat (with velocity v) at certain angle with river flow so that he can reach the opposite point on other bank directly.

(B) his velocity 'v' must be towards directly opposite point, So, that he can travel rest of distance by walking on other bank to reach the directly opposite point.

(C) boatman should maintain velocity v of boat at certain angle greater than 90° with direction of river flow to minimize drifting and then walk rest of distance on other bank.

(D) boat velocity 'v' should be at an angle less than 90° with direction of river flow to minimize the drift and then walk to the point.

27. A ball is thrown at an angle θ up to the top of a cliff of height L, from a point at a distance L from the base, as shown in figure. Assuming that one of the following quantities is the initial speed required to make the ball hit right at the edge of the cliff, which one is it :-



(A) $\sqrt{\dfrac{gL}{2(\tan\theta-1)}}$ (B) $\dfrac{1}{\cos\theta}\sqrt{\dfrac{gL}{2(\tan\theta-1)}}$ (C) $\dfrac{1}{\cos\theta}\sqrt{\dfrac{gL}{2(\tan\theta+1)}}$ (D) $\sqrt{\dfrac{gL\tan\theta}{2(\tan\theta+1)}}$

28. A particle is projected with a velocity of $\sqrt{20}$ m/s such that it strikes on the same level as the point of projection at a distance of $\sqrt{3}$ m. Which of the following options is/are incorrect (mass = 1kg) :

(A) The maximum height reached by the projectile can be 0.25 m.

(B) The minimum velocity during its motion can be $\sqrt{5}$ m/s

(C) The time taken for the flight can be $\sqrt{3/5}$ s.

(D) Minimum kinetic energy during its motion can be 6 J.

MULTIPLE CORRECT TYPE QUESTION

29. A particle is moving with a position vector, $\vec{r} = \left[a_0 \sin(2\pi t)\hat{i} + a_0 \cos(2\pi t)\hat{j}\right]$. Then-

(A) Magnitude of displacement of the particle between time t = 4 sec and t = 6 sec is zero

(B) Distance travelled by the particle in 1 sec is $2\pi a_0$

(C) The speed of particle in the whole motion is constant and equal to $2\pi a_0$.

(D) None of these

30. A point mass is moving in the x-y plane. Its acceleration is a constant vector perpendicular to the x-axis. Which of the following do/does not change with time?

(A) only y-component of its velocity vector

(B) only x-component of its velocity vector

(C) only y-component of its acceleration vector

(D) only x-component of its acceleration vector

31. A ball is thrown from ground such that it just crosses two poles of equal height kept 80 m apart. The maximum height attained by the ball is 80 m. When the ball passes the first pole, its velocity makes 45° with horizontal. The correct alternatives is/are :- (g = 10 m/s^2)

(A) Time interval between the two poles is 4 s.

(B) Height of the pole is 60 m.

(C) Range of the ball is 160 m.

(D) Angle of projection is $\tan^{-1}(2)$ with horizontal.

32. Position vector of a particle is expressed as function of time by equation $\vec{r} = 2t^2 + (3t-1)\hat{j} + 5\hat{k}$, where r is in meters and t is in seconds.

(A) It always moves in a plane that is parallel to the x-y plane.

(B) At the instant $t = 0$ s, it is observed at point (0 m, −1 m, 5 m), moving with velocity 3 m/s in the positive y-direction.

(C) Its acceleration vector is uniform.

(D) It is an example of three dimensional motion.

33. A projectile is thrown with speed u into air from a point on the horizontal ground at an angle θ with horizontal. If the air exerts a constant horizontal resistive force on the projectile then select correct alternative(s).

(A) At the farthest point, the velocity is horizontal.

(B) The time for ascent equals the time for descent.

(C) The path of the projectile may be parabolic.

(D) The path of the projectile may be a straight line.

34. A block is thrown horizontally with a velocity of 2 m/s (relative to ground) on a belt, which is moving with velocity 4 m/s in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 s it was dropped then choose the correct statement(s) :-

(A) Displacement with respect to ground is zero after 2.66 s and magnitude of displacement with respect to ground is 12 m after 4 s.

(B) Magnitude of displacement with respect to ground in 4 s is 4 m.

(C) Magnitude of displacement with respect to belt in 4 s is 12 m.

(D) Displacement with respect to ground is zero in 8/3 s.

35. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself

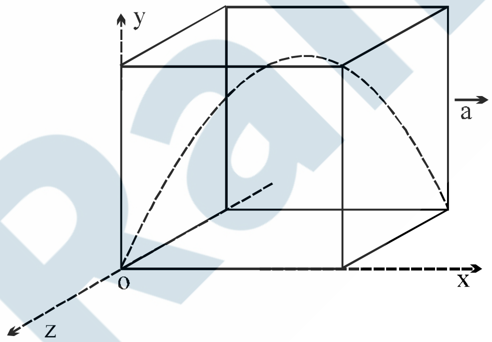
(A) The ball will always return to him.

(B) The ball will never return to him.

(C) The ball will return to him if the cart moves with constant velocity.

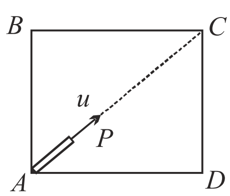
(D) The ball will fall behind him if the cart moves with some positive acceleration.

36. A cubical box dimension L = 5/4 metre starts moving with an acceleration $\vec{a} = 0.5\ m/s^2\,\hat{i}$ from the state of rest. At the same time, a stone is thrown from the origin with velocity $\vec{V} = v_1\hat{i} + v_2\hat{j} - v_3\hat{k}$ with respect to earth. Acceleration due to gravity $\vec{g} = 10 m/s^2(-\hat{j})$. The stone just touches the roof of box and finally falls at the diagonally opposite point then :



(A) $v_1 = \dfrac{3}{2}$ (B) $v_2 = 5$ (C) $v_3 = \dfrac{5}{4}$ (D) $v_3 = \dfrac{5}{2}$

37. A large rectangular box moves vertically downward with an acceleration a. A toy gun fixed at *A* and aimed towards *C* fires a particle *P*.



(A) *P* will hit *C* if $a = g$

(B) *P* will hit the roof *BC*, if $a > g$

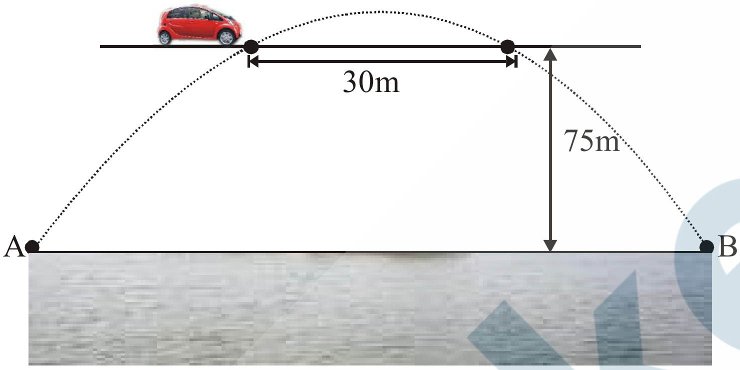
(C) *P* will hit the wall *CD* if $a < g$

(D) May be either (A), (B) or (C), depending on the speed of projection of *P*

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 38 to 40

In the figure shown there is a long horizontal bridge over a river 75 m high from water surface. A strong man throws a stone in the parallel plane of the bridge. A observer in a car travelling on the bridge finds the stone going pass by the car while ascending and also while descending between two points on the road 30 m away. The car is travelling at a speed of 15 m/s. The stone is thrown from the bank of river just at the same level of water.



38. What is the angle the velocity makes with the bridge when it goes past the car while ascending ?

(A) 30° (B) 45° (C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\tan^{-1}\left(\frac{3}{2}\right)$

39. Horizontal distance AB travelled by stone is :-

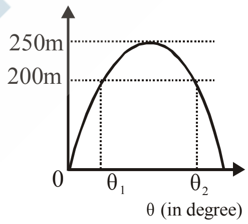
(A) 0 m (B) 75 m (C) 120 m (D) 240 m

40. What is the distance between car and stone at the instant when particle reaches at point B ?

(A) 0 m (B) 75 m (C) 120 m (D) 240 m

Paragraph for Question 41 & 42

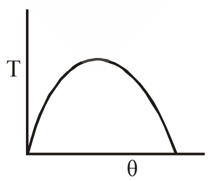
From the ground level, a ball is to be shot with a certain speed. Graph shows the range (R) of the particle versus the angle of projection from horizontal (θ).

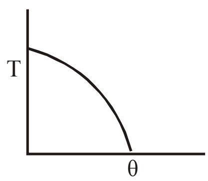


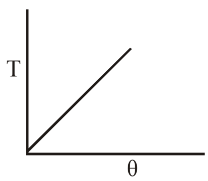
41. Values of θ_1 and θ_2 are

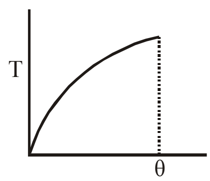
(A) 53° and 37° (B) 26.5° and 63.5° (C) 18.5° and 71.5 ° (D) 15° and 75°

42. The corresponding time of flight vs θ graph is :-

(A) 

(B) 

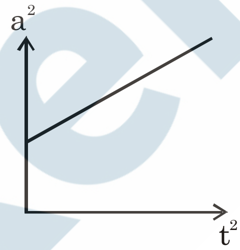
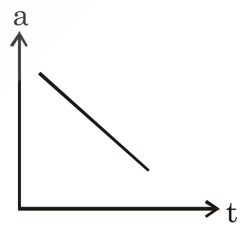
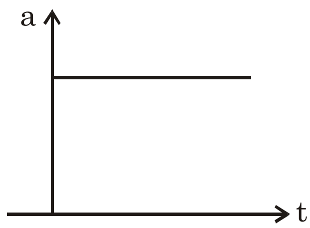
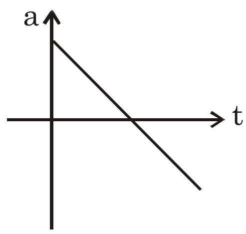
(C) 

(D) 

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT
(THREE COLUMNS AND FOUR ROWS)

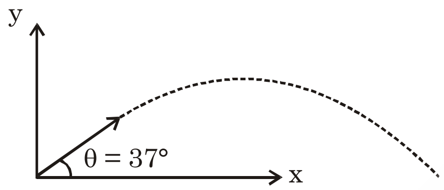
Answer Q.43, Q.44 and Q.45 by appropriately matching the information given in the three columns of the following table.

Match the following

Column–I	**Column-II**	**Column-III**
Time of flight (in sec)	**Range (in m) Along the ground/ along the inclined plane**	**Graph**
(I) 2	(i) 3840	(P) 
(II) 1	(ii) $20\sqrt{2}$	(Q) 
(III) 10	(iii) 7	(R) 
(IV) 32	(iv) 3600	(S) 

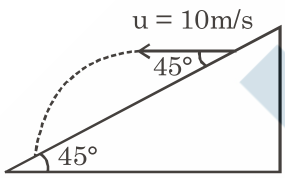
43. A particle is projected with initial speed $u = \frac{25}{3}$ m/s as shown, here acceleration vector is given as

$a_x = 2t\hat{i}\ \text{m/s}^2;\ a_y = -10\hat{j}\ \text{m/s}^2$



(A) (II) (iv) (P) (B) (II) (iii) (P) (C) (III) (iii) (S) (D) (II) (iii) (S)

44. A particle is projected from a large–fixed incline plane as shown. Here $\vec{a} = g$ (Vertically downward) take g = 10 m/s^2.



(A) (II) (iv) (P) (B) (IV) (ii) (S) (C) (I) (ii) (R) (D) (IV) (ii) (P)

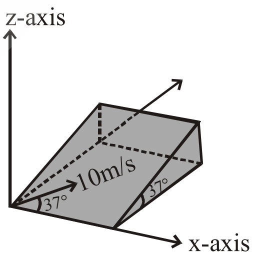
45. In ground to ground projection a particle is projected at 53° from horizontal. At t = 25 sec after projection, its velocity vecotor becomes perpendicular to its initial veocity vector.

(Given $\vec{a} = g \downarrow = 10\text{m/s}^2$)

(A) (IV) (i) (R) (B) (IV) (ii) (S) (C) (II) (ii) (P) (D) (I) (ii) (Q)

MATRIX MATCH TYPE QUESTION

46. A small ball is projected along the surface of a smooth inclined plane with speed 10m/s along the direction shown at t = 0. The point of projection is origin, z-axis is along vertical. The acceleration due to gravity is 10 m/s^2. Column-I lists values of certain parameters related to motion of ball and column-II lists different time instants. Match appropriately.

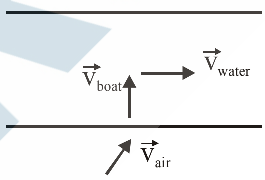


	Column-I		**Column-II**
(A)	Distance from x-axis is 2.25m	(P)	0.5 s
(B)	Speed is minimum	(Q)	1.0 s
(C)	Velocity makes angle 37° with x-axis	(R)	1.5 s
		(S)	2.0 s

47.

Column-I	Column-II
(A) Time for a boat to cross a river of width ℓ by the shortest distance ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water)	(P) $\dfrac{\ell}{\lvert\vec{v}+\vec{u}\rvert}$
(B) Time for two particles moving with velocities $\vec{v}$ and $\vec{u}$ in opposite directions to meet each other. (initial separation of particles is ℓ)	(Q) $\dfrac{\ell}{\sqrt{v^2-u^2}}$
(C) Time for a boat to cross a river of width ℓ in the shortest time ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water)	(R) $\dfrac{\ell}{\lvert\vec{v}\rvert+\lvert\vec{u}\rvert}$
(D) Time for a boat to travel a distance ℓ downstream ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water)	(S) $\dfrac{\ell}{\lvert\vec{v}\rvert}$
	(T) $\dfrac{\ell}{\sqrt{u^2+v^2}}$

48. A boat is being rowed in a river. Air is also blowing. Direction of velocity vectors of boat, water and air in ground frame are as shown in diagram.



Column-I	Column-II (possible directions)
(A) Direction in which boat is being steered	(P) $\nwarrow$
(B) Direction in which a flag on the boat may flutter	(Q) $\rightarrow$
(C) Direction of velocity of water relative to boat	(R) $\uparrow$
(D) Direction of velocity of air relative to a piece of wood floating on river.	(S) $\searrow$

EXERCISE (JM)

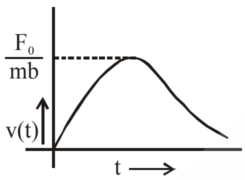
1. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is : **[AIEEE - 2010]**

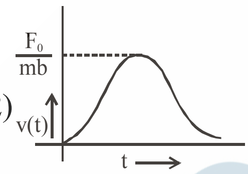
(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant (3) $y^2 = x$ + constant (4) xy = constant

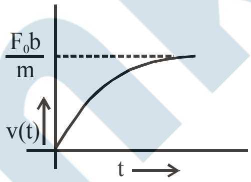
2. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :- **[AIEEE - 2011]**

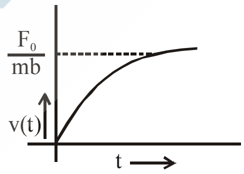
(1) $\frac{\pi}{2}\frac{v^4}{g^2}$ (2) $\pi\frac{v^2}{g^2}$ (3) $\pi\frac{v^2}{g}$ (4) $\pi\frac{v^4}{g^2}$

3. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves ? **[AIEEE - 2012]**

(1) 

(2) 

(3) 

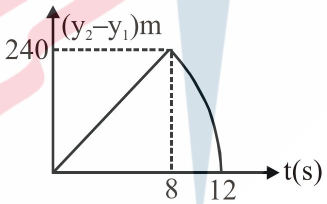
(4) 

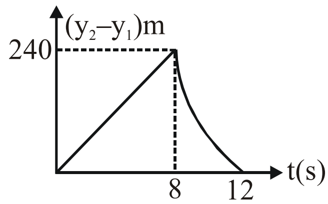
4. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s², the equation of its trajectory is : **[AIEEE - 2013]**

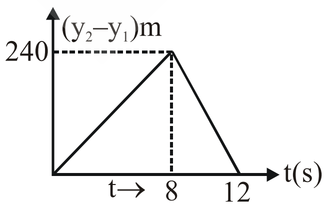
(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

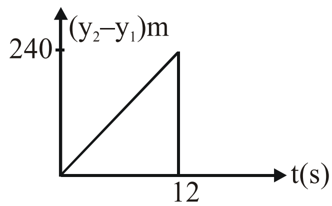
5. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s²) *(The figure are schematic and not drawn to scale)* **[JEE Main-2015]**

(1) 

(2) 

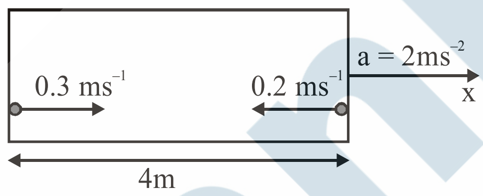
(3) 

(4) 

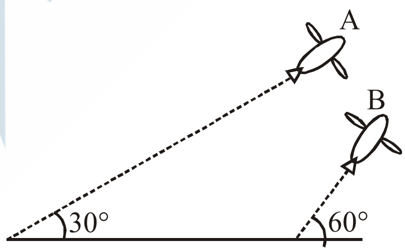
EXERCISE (JA)

1. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is **[IIT-JEE 2011]**

2. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[JEE Advanced 2014]**



3. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is **[JEE Advanced-2014]**



ANSWER KEY

EXERCISE (S-1)

1. Ans. 5

2. Ans. (a) $v(t) = \left(3.0\hat{i} - 4.0t\,\hat{j}\right)\,\hat{a}\left(t\right) = -4.0\,\hat{j}$ (b) 8.54 ms^{-1}, 70° with x-axis

3. Ans. (i) $\vec{a} = \left(-16\hat{i} - 8\hat{j}\right) m/s^2$ (ii) $\vec{v} = \left(-30\hat{i} - 40\hat{j}\right) m/s$

4. Ans. $\sqrt{2257}$ m/s

5. Ans. (i) 1.6 sec (ii) 3.2 m (iii) 9.6 m

6. Ans. 1200

7. Ans. 50 m

8. Ans. 20 s

9. Ans. 60°, 2 m/s.

11. Ans. $20\sqrt{5}$ m/s

12. Ans. $\frac{100}{3}$ m/s

13. Ans. (i) 100 m/s (ii) 980 m (iii) 1600 m (iv) $\left(80\hat{i} - 140\hat{j}\right)$

14. Ans. $u = 50(\sqrt{3} - 1)$ m/s, $H = 2.5(\sqrt{3} - 1)$ m

15. Ans. 6

16. Ans. 34

17. Ans. (a) With $\hat{i}$ to right and $\hat{j}$ up $\vec{V} = (15\hat{i} + 20\hat{j})$ m/s; (b) 23 meters; (c) It is horizontal. $\theta = 0$]

18. Ans. 10 m/s

19. Ans. 4

20. Ans. 3

21. Ans. (a) 45°, (b) 2 m/sec

22. Ans. 5

23. Ans. 6

24. Ans. 200 m, 20 m/min, 12 m/min

25. Ans. $\tan^{-1}(1/2)$

26. Ans. $\tan^{-1}(1/3)$

27. Ans. (a) Somewhere down stream (b) 8 s (c) 16 m (d) Yes

28. Ans. 6 s, 66 m

29. Ans. 30 m upstream

30. Ans. $v_{br} > v_r$, $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream of line AB

EXERCISE (S-2)

1. Ans. (a) $\vec{v} = (5\sin t - 3)\hat{i} + (3\cos t - 1)\hat{j}$, (b) $(2 - 5\cos t - 3t)\hat{i} + (2 + 3\sin t - t)\hat{j}$

2. Ans. (i) $4\sqrt{6}$ m, (ii) 10 m

3. Ans. (i) 2 s (ii) 10 m/s (iii) 5m (iv) 16.25 m (v) 20 m

4. Ans. (i) 7 s, (ii) 175 m

5. Ans. 0105

6. Ans. t = 1.6 s

7. Ans. (i) 1 s, (ii) ($5\sqrt{3}$ m, 5 m)

8. Ans. $\theta = 37°$, v = 6 m/s

9. Ans. (i) $\frac{u^2 \sin 2\alpha}{g\cos\theta}$ (ii) $v = \frac{u\cos(\alpha + \theta)}{\cos\theta}$

10. Ans. 140

11. Ans. 5

12. Ans. B, $t_A = 165$ s, $t_B = 150$ s

13. Ans. (i) 120° (ii) $2/\sqrt{3}$

14. Ans. $10/\sqrt{3}$ m/s

15. Ans. (a) 7i + 2j; (b) 12.30p.m, (c) 9/2i + j)

EXERCISE (O-1)

1. Ans. (B) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. (B)** **5. Ans. (B)** **6. Ans. (B)**
7. Ans. (D) **8. Ans. (B)** **9. Ans. (D)** **10. Ans. (B)** **11. Ans. (B)** **12. Ans. (A)**
13. Ans. (D) **14. Ans. (C)** **15. Ans. (C)** **16. Ans. (B)** **17. Ans. (B)** **18. Ans. (B)**
19. Ans. (B) **20. Ans. (C)** **21. Ans. (A)** **22. Ans. (A,B,C,D)**
23. Ans. (A,B,C,D) **24. Ans. (B,C,D)** **25. Ans. (B,C)** **26. Ans. (B)** **27. Ans. (C,D)**
28. Ans. (A,B,C) **29. Ans. (C)** **30. Ans. (D)** **31. Ans. (B)**

32. Ans. (a) Vertically (b) 53° above east (c) 53° above west (d) 53° above north (e) 45° above south

33. Ans. (a) 120 cm/s vertically150 cm/s 53° above horizontal (b) 37° above the horizontal.

34. Ans. 120 cm/s **35. Ans.** 125 cm/s at 37° from the vertical **36. Ans.** 6 m/s due north

37. Ans. (a) 9 m/s (b) 16 m/s (c) 1 m/s^2 **38. Ans.** 10 m/s, 37° north of east

39. Ans. (A) R; (B) P; (C) Q; (D) S **40. Ans.** (A) $\rightarrow$ (R); (B) $\rightarrow$ (R); (C) $\rightarrow$ (P); (D) $\rightarrow$ (Q)

EXERCISE (O-2)

1. Ans. (D) **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (A)** **5. Ans. (C)** **6. Ans. (C)**
7. Ans. (C) **8. Ans. (C)** **9. Ans. (C)** **10. Ans. (A)** **11. Ans. (C)** **12. Ans. (D)**
13. Ans. (C) **14. Ans. (A)** **15. Ans. (A)** **16. Ans. (C)** **17. Ans. (C)** **18. Ans. (D)**
19. Ans. (A) **20. Ans. (A)** **21. Ans. (C)** **22. Ans. (B)** **23. Ans. (B)** **24. Ans. (B)**
25. Ans. (B) **26. Ans. (C)** **27. Ans. (B)** **28. Ans. (D)** **29. Ans. (A,B,C)**
30. Ans. (B,C,D) **31. Ans. (A, B,C,D)** **32. Ans. (A,B,C)**
33. Ans. (B,C,D) **34. Ans. (B,C,D)** **35. Ans. (C,D)** **36. Ans. (A,B,C)**
37. Ans. (A,B) **38. Ans. (C)** **39. Ans. (C)** **40. Ans. (B)** **41. Ans. (B)**
42. Ans. (D) **43. Ans. (B)** **44. Ans. (C)** **45. Ans. (A)**
46. Ans. (A) - (P,R) ; (B) - (Q) ; (C) - (S)
47. Ans. (A) $\rightarrow$(P,Q); (B) $\rightarrow$(R); (C) $\rightarrow$(S); (D) $\rightarrow$(P,R)
48. Ans. (A)-P; (B)-Q, S; (C)-S; (D)-P,R

EXERCISE (JM)

1. Ans. (1) **2. Ans. (4)** **3. Ans. (4)** **4. Ans. (2)** **5. Ans. (1)**

EXERCISE (JA)

1. Ans. 5 **2. Ans. 8 or 2** **3. Ans. 5**