CONTENTS

KINEMATICS-1D & CALCULUS

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Kinematics

Study of motion of objects without taking into account the factor which cause the motion (i.e. nature of force).

Motion

If a body changes its position with time, it is said to be moving else it is at rest. Motion is always relative to the observer.

Motion is a combined property of the object under study and the observer. There is no meaning of rest or motion without the viewer. In other words absolute motion or rest is meaningless.

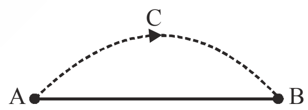
- To locate the position of a particle we need a reference frame. A commonly used reference frame is Cartesian coordinate system or simply coordinate system.

  The coordinates (x, y, z) of a particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest w.r.t. this frame.
- The reference frame is chosen according to the problems.
- If the frame is not mentioned, then ground is taken as the reference frame.

DISTANCE AND DISPLACEMENT

Total length of path covered by the particle, in definite time interval is called distance.

Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.



But overall, body is displaced from A to B. A vector from A to B, i.e. $\overrightarrow{AB}$ is its displacement vector or displacement that is the minimum distance and directed from initial position to final position.

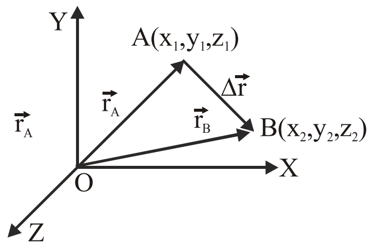
- Distance is a scalar while displacement is a vector.
- Distance depends on path while displacement is independent of path but depends only on final and initial position.
- For a moving body, distance can't have zero or negative values but displacement may be +ive, –ive or zero.
- For a moving/stationary object distance can't be decreasing.
- If motion is in straight line without change in direction then

  distance = |displacement| i.e. magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance.

  distance $\geq$ |displacement|

Displacement in terms of position vector

Let a body is displaced from A (x_1, y_1, z_1) to B (x_2, y_2, z_2) then its displacement is given by vector $\overrightarrow{AB}$.



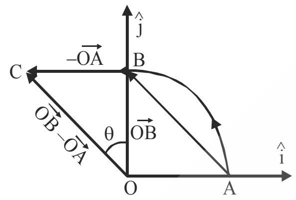
From ΔOAB $\vec{r}_A + \Delta\vec{r} = \vec{r}_B \Rightarrow \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \text{ and } \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_2\hat{k}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \Rightarrow \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Ex. A particle goes along a quadrant from A to B of a circle radius 10m as shown in fig. Find the direction and magnitude of displacement and distance along path AB.

Sol. Displacement $\vec{AB} = \vec{OB} - \vec{OA} = 10\hat{j} - 10\hat{i}$



$|\vec{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m

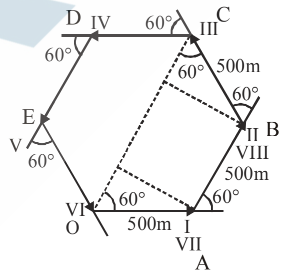
From ΔOBC $\tan\theta = \frac{OA}{OB} = \frac{10}{10} = 1 \Rightarrow \theta = 45°$

Angle between displacement vector $\vec{OC}$ and x–axis = 90° + 45° = 135°

Distance of path AB = $\frac{1}{4}$ (circumference) = $\frac{1}{4}(2\pi R)$ m = (5π) m

Ex. On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

Sol. **At III turn**



Displacement $= \overline{OA} + \overline{AB} + \overline{BC} = \overline{OC}$

= 500 cos 60° + 500 + 500 cos 60°

$= 500 \times \frac{1}{2} + 500 + 500 \times \frac{1}{2}$ = 1000 m from O to C

Distance = 500 + 500 + 500 = 1500 m. So $\frac{\text{Displacement}}{\text{Distance}} = \frac{1000}{1500} = \frac{2}{3}$

At VI turn

$\because$ initial and final positions are same so displacement = 0 and distance = 500 × 6 = 3000 m

$\therefore \frac{\text{Displacement}}{\text{Distance}} = \frac{0}{3000} = 0$

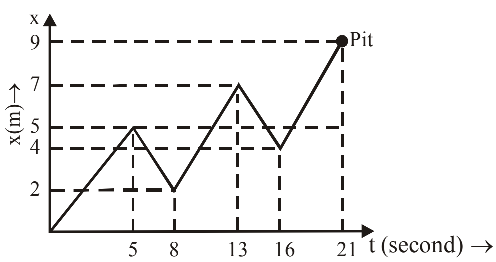
At VIII turn

Displacement $= 2(500)\cos\left(\frac{60°}{2}\right)$ = 1000 × cos 30° $= 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}$ m

Distance = 500 × 8 = 4000 m

$\therefore \frac{\text{Displacement}}{\text{Distance}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8}$

Ex. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Plot the x–t graph of his motion. Determine graphically or otherwise how long the drunkard takes to fall in a pit 9m away from the start.



Sol. from x–t graph time taken = 21 s

or (5m – 3m) + (5m – 3m) + 5m = 9m

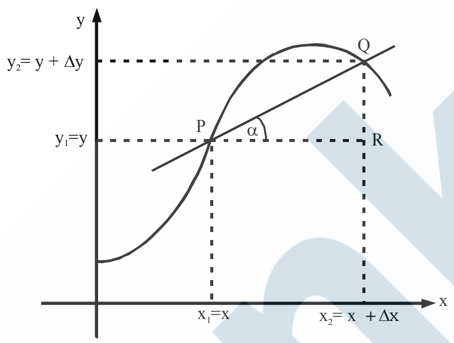
$\Rightarrow$ total steps = 21 $\Rightarrow$ time = 21 s

DERIVATIVE OF A FUNCTION

Average Rate of Change

Let a function y=f(x) be plotted by an arbitrary graph as shown in the figure. Average rate of change in y with respect to x in an interval $[x_1, x_2]$ is defined as

$$\text{Average rate of change} = \frac{\text{Change in y}}{\text{Change in x}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$



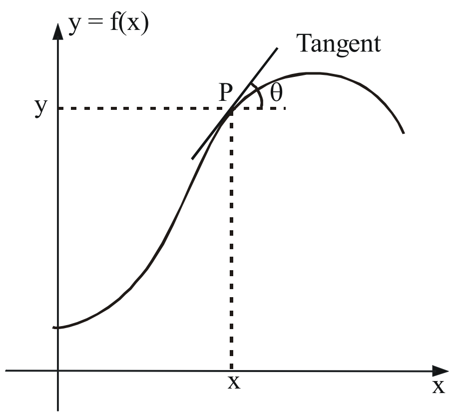
= slope of chord

If both the axes have equal scale

Average rate of change = slope of chord PQ = tanθ

Instantaneous Rate of Change : First derivative

It is defined as the rate of change in y with x at a particular value of x. It is measured graphically by the slope of the tangent drawn to the y-x graph at the point (x, y) and algebraically by the first derivative of the function y=f(x).



$$\text{Instantaneous rate of change} = \frac{dy}{dx} = \text{Slope of the tangent}$$

$$\text{If both the axes have equal scale then } \frac{dy}{dx} = \tan\theta$$

$$\text{Instantaneous rate of change} = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Derivatives of Commonly Used Functions.

- $y = \text{constant} \Rightarrow \frac{dy}{dx} = 0$
- $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$
- $y = e^x \Rightarrow \frac{dy}{dx} = e^x$
- $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$
- $y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x$
- $y = \tan x \Rightarrow \frac{dy}{dx} = \sec^2 x$
- $y = \cot x \Rightarrow \frac{dy}{dx} = -\text{cosec}^2 x$
- $y = \text{cosec}\, x \Rightarrow \frac{dy}{dx} = -\text{cosec}\, x \cot x$
- $y = \sec x \Rightarrow \frac{dy}{dx} = \sec x \tan x$

Method of Differentiation.

If $y = f(x)$, let us denote $\frac{dy}{dx} = f'(x)$

- Sum or Subtraction of two functions $y = f(x) \pm g(x) \Rightarrow \frac{dy}{dx} = f'(x) \pm g'(x)$
- Product of two functions

  $$y = f(x)\cdot g(x) \Rightarrow \frac{dy}{dx} = g(x)f'(x) + f(x)g'(x)$$
- Division of two functions.

  $$y = \frac{f(x)}{g(x)} \Rightarrow \frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{\{g(x)\}^2}$$
- Chain Rule

  $$y = f\{g(x)\} \Rightarrow \frac{dy}{dx} = g'(x)f'\{g(x)\}$$

Ex. Find $\frac{dy}{dx}$, when (i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$ (iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Sol. (i) $\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

(ii) $\frac{dy}{dx} = \frac{d}{dx}(x^5 + x^4 + 7) = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^4) + \frac{d}{dx}(7) = 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$

(iii) $\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2}) = \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$

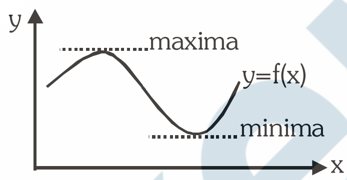
$$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2}) = 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3} = 2x - 2x^{-3/2} + 6x^{-3}$$

Second Derivative and it's meaning

Second derivative of a function y = f(x) is defined as $\frac{d}{dx}\left[\frac{dy}{dx}\right]$. It is obtained by differentiating the function with respect to x two times successively. Geometrically it expresses rate of change in slope of graph of the function.

Maxima & Minima

Maxima & minima of a function y = f(x)



for maximum value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} =$ negative

for minimum value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} =$ positive

Ex. Find minimum value of $y = 25x^2 - 10x + 5$.

Sol. For maximum/minimum value $\frac{dy}{dx} = 0 \Rightarrow 50x - 10 = 0 \Rightarrow x = \frac{1}{5}$

Now at $x = \frac{1}{5}$, $\frac{d^2y}{dx^2} = 50$ which is positive. So $y_{min} = 25\left(\frac{1}{5}\right)^2 - 10\left(\frac{1}{5}\right) + 5 = 1 - 2 + 5 = 4$

INTEGRATION

- This operation enables us to find sum of infinite number of infinitely small quantities.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{\infty} \Delta x_i = \int dx = x$$

- It is reverse operation of differentiation. If derivative, which is rate of change at a point, is given as a function $f(x) = \frac{dF(x)}{dx}$, operation of integration enables us to find original function F(x).

$$\int f(x)dx = F(x) + C$$

Here function f(x) is known as integrand, function F(x) as integral and C as constant of integration. Value of C is obtained by substituting initial, final or any other condition (known as boundary conditions) in the above equation.

This interpretation enables us to find integral of those functions whose derivative is known.

Integrand $f(x)=\frac{dF(x)}{dx}$	Integral $\int f(x)dx = F(x)+C$
k = Constant	kx+C
x^n	$\frac{x^{n+1}}{n+1}+C$ If $n\neq -1$
x^{-1}	$\ln x + C$
e^x	$e^x + C$
sin x	–cos x + C
cos x	sin x + C
f(ax+b)	$\frac{F(ax+b)}{a}+C$

Ex. Evaluate the following:

(i) $\int x^{-7}dx$ (ii) $\int x^{p/q}dx$

Sol. (i) $\int x^{-7}dx = \frac{x^{-7+1}}{-7+1}+c = -\frac{1}{6}x^{-6}+c$

(ii) $\int x^{\frac{p}{q}}dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1}+c = \frac{q}{p+q}x^{(p+q)/q}+c$

Ex. Evaluate $\int\left(x^2 - \cos x + \frac{1}{x}\right)dx$

Sol. $I = \int x^2dx - \int \cos x dx + \int \frac{1}{x}dx = \frac{x^{2+1}}{2+1} - \sin x + \log_e x + c = \frac{x^3}{3} - \sin x + \log_e x + c$

SPEED :

Speed is the rate of change of distance with respect to time.

Uniform speed :

An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, irrespective of duration of interval. The uniform speed is shown by straight line in distance–time graph.

For example, suppose a train travels 1000 m in 60 s. The train is said to be moving with uniform speed, if it travels 500 m. in 30s, 250 m in 15s, 125 m in 7.5s and so on.

Non Uniform speed :

An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, irrespective of duration of interval.

For example, suppose a train travels first 1000 m in 60s next 1000 m in 120s and next 1000m in 50s, then the train is moving with variable speed.

Average speed :

Speed is distance travelled per unit time. Average speed of a trip $v_{av} = \dfrac{\text{Total travelled distance}}{\text{Total time taken}}$

If a particle travels a distance s in time t_1 to t_2, the average speed is $v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{s}{t_2 - t_1}$

Instantaneous speed

The speed at a particular instant is defined as instantaneous speed (or speed) while average speed is defined for a time interval.

If Δt approaches zero, average speed becomes instantaneous speed. $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

i.e. instantaneous speed is the time derivative of distance.

If a particle travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 etc. respectively, then total travelled distance

$$s = s_1 + s_2 + s_3 + \ldots\ldots\ldots + s_n$$

Total time taken $t = \dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \dfrac{s_3}{v_3} + \ldots.. + \dfrac{s_n}{v_n}$

No sign is needed for distance or speed.
They are always positive quantities.

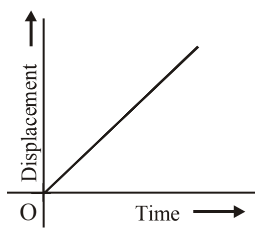
Average speed of the trip $= \dfrac{s_1 + s_2 + s_3 \ldots\ldots + s_n}{\left(\dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \ldots\ldots + \dfrac{s_n}{v_n}\right)}$

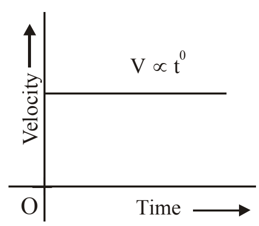
VELOCITY :

Velocity is the rate of change of displacement with respect to time.

Uniform Velocity :

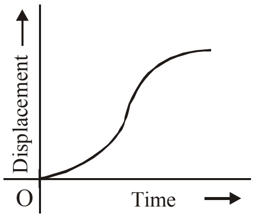
A body is said to move with uniform velocity, if it covers equal displacements in equal intervals of time, irrespective of duration of interval. When a body is moving with uniform velocity, then the magnitude and direction of the velocity of the body remains same at all points of its path.





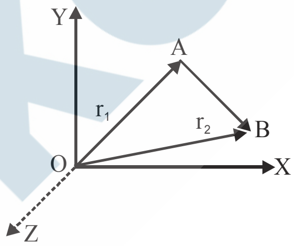
Non–uniform Velocity :

The particle is said to have non–uniform motion if it covers unequal displacements in equal intervals of time, irrespective of duration of interval. In this type of motion velocity does not remain constant.



Average Velocity

The average velocity of a particle in a time interval t_1 to t_2 is a defined as its displacement divided by the time interval. Let a particle is at a point A at time t_1 and B at time t_2. Position vectors of A and B are $\vec{r}_1$ and $\vec{r}_2$. The displacement in this time interval is the vector $\overrightarrow{AB} = (\vec{r}_2 - \vec{r}_1)$. The average velocity in this time interval is, $\vec{v}_{av} = \dfrac{\text{displacement vector}}{\text{time interval}}$

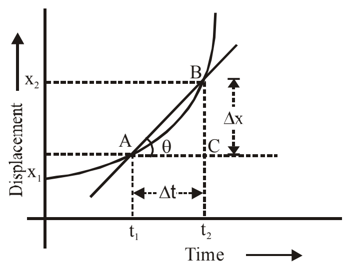


$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

here $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1$ = change in position vector.

Instantaneous velocity

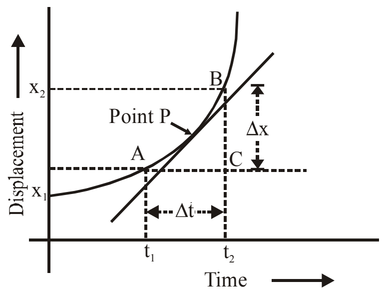
The velocity of the object at a given instant of time or at a given position during motion is called instantaneous velocity $\vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$



From fig., the average velocity between points A and B is

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \text{slope of secant AB} = \tan\theta$$

Average velocity is equal to slope of straight line joining two points on displacement time graph. If $\Delta t \to 0$, then average velocity becomes instantaneous velocity.



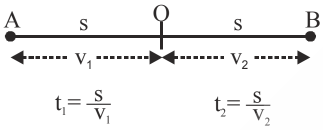
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \text{slope of tangent at P} = \tan\alpha$$

- Magnitude of instantaneous velocity is the instantaneous speed.

  Note : When a particle moves with constant velocity, its magnitude of average velocity, its magnitude of instantaneous velocity and its speed all are equal.

Ex. If a particle travels the first half distance with speed v_1 and second half distance with speed v_2. Find its average speed during journey.

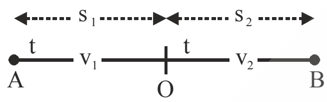
Sol. $$v_{avg.} = \frac{s+s}{t_1+t_2} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$



Note :– Here $v_{avg.}$ is the harmonic mean of two speeds.

Ex. If a particle travels with speed v_1 during first half time interval and with v_2 speed during second half time interval. Find its average speed during its journey.

Sol. Total distance = $s_1 + s_2 = v_1t + v_2t = (v_1 + v_2)t$



Total time $= t + t = 2t$ $\qquad v_{avg.} = \dfrac{s_1+s_2}{t+t} = \dfrac{(v_1+v_2)t}{2t} = \dfrac{v_1+v_2}{2}$

Note :– here v_{avg} is arithmatic mean of two speeds.

Ex. A car travels a distance A to B at a speed of 40 km/h and returns to A at a speed of 30 km/h.

(i) What is the average speed for the whole journey?

(ii) What is the average velocity?

Sol. (i) Let AB = s, time taken to go from A to B, $t_1 = \dfrac{s}{40}$ h

and time taken to go from B to A, $t_2 = \dfrac{s}{30}$ h

$\therefore$ total time taken $= t_1 + t_2 = \dfrac{s}{40} + \dfrac{s}{30} = \dfrac{(3+4)\ s}{120} = \dfrac{7s}{120}$ h

Total distance travelled = s + s = 2s

$$\therefore \text{ Average speed } = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{2s}{\frac{7s}{120}} = \frac{120\times 2}{7} = 34.3 \text{ km/h.}$$

(ii) Total displacement = zero, since the car returns to the original position.

$$\text{Therefore, average velocity} = \frac{\text{total displacement}}{\text{time taken}} = \frac{0}{2t} = 0$$

Ex. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. On reaching the market he instantly turns and walks back with a speed of 7.5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min (iii) 0 to 40 min.

Sol. Time taken by man to go from his home to market, $t_1 = \dfrac{\text{distance}}{\text{speed}} = \dfrac{2.5}{5} = \dfrac{1}{2}$ h

Time taken by man to go from market to his home, $t_2 = \dfrac{2.5}{7.5} = \dfrac{1}{3}$ h

$\therefore$ Total time taken $= t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}\text{h} = 50$ min.

(i) 0 to 30 min

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time interval}} = \frac{2.5}{\frac{30}{60}} = 5 \text{ km/h towards market}$$

$$\text{Average speed} = \frac{\text{distance}}{\text{time interval}} = \frac{2.5}{\frac{30}{60}} = 5 \text{ km/h}$$

(ii) 0 to 50 min

Total displacement = zero so average velocity = 0

$$\text{So, average speed} = \frac{5}{50/60} = 6 \text{ km/h}$$

Total distance travelled = 2.5 + 2.5 = 5 km.

(iii) 0 to 40 min

Distance moved in 30 min (from home to market) = 2.5 km.

Distance moved in 10 min (from market to home) with speed 7.5 km/h $= 7.5 \times \frac{10}{60} = 1.25$ km

So, displacement = 2.5 – 1.25 = 1.25 km (towards market)

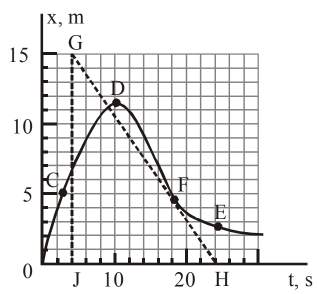
Distance travelled = 2.5 + 1.25 = 3.75 km

$$\text{Average velocity} = \frac{1.25}{\frac{40}{60}} = 1.875 \text{ km/h. (towards market)}$$

$$\text{Average speed} = \frac{3.75}{\frac{40}{60}} = 5.625 \text{ km/h.}$$

Note : Moving body with uniform speed may have variable velocity. e.g. in uniform circular motion speed is constant but velocity is non–uniform.

Ex. Refer to figure for the motion of an object along the x-axis.



What is the instantaneous velocity of the object at (a) F (b) D

Sol. **(a)** The tangent at F is the dashed line GH. Taking triangle GHJ,
$\Delta t = 24 - 4 = 20$ s $\quad \Delta x = 0 - 15 = -15$m

Hence slope at F is $v_F = \dfrac{\Delta x}{\Delta t} = \dfrac{-15\text{m}}{20\ \text{s}} = -0.75$ m/s

The negative sign tells us that the object is moving in the –x direction.

(b) At point D slope of curve is zero so $v = \dfrac{dx}{dt} = 0$.

ACCELERATION :

Acceleration

The acceleration is rate of change of velocity or change in velocity per unit time interval. Velocity is a vector quantity hence a change in its magnitude or in direction or in both, will change the velocity .

Uniform acceleration :

An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time, irrespective of duration of intervals.

Variable acceleration :

An object is said to be moving with a variable acceleration if its velocity changes by unequal amounts in equal intervals of time, irrespective of duration of intervals..

Average Acceleration :

When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken

$$\text{Average Acceleration} = \frac{\text{Total change in velocity}}{\text{total time taken}}$$

Suppose the velocity of a particle is $\vec{v}_1$ at time t_1 and $\vec{v}_2$ at time t_2. Then $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{\Delta \vec{v}}{\Delta t}$

Instantaneous Acceleration :

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration. Suppose the velocity of a particle at time $t_1 = t$ is $\vec{v}_1 = \vec{v}$ and becomes $\vec{v}_2 = \vec{v} + \Delta\vec{v}$ at time $t_2 = t + \Delta t$, Then, $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$

If Δt approaches to zero then the rate of change of velocity will be instantaneous acceleration.

Instantaneous acceleration $\vec{a} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}\left(\dfrac{d\vec{r}}{dt}\right) = \dfrac{d^2\vec{r}}{dt^2}$

Ex. An athlete takes 2 second to reach the maximum speed of 18 km/h from rest. What is the magnitude of his average acceleration.

Sol. Here, Initial velocity u = 0, $v = (v_{max}) = 18$ km/h $= 18 \times \dfrac{5}{18} = 5$ m/s, $t_1 = 0$s, $t_2 = 2$s.

$$a_{av} = \frac{v - u}{t_2 - t_1} = \frac{5.0}{2} = 2.5 \text{ m/s}^2$$

Ex. A car moving with a velocity of 20 ms^{-1} is brought to rest in 5 seconds by applying brakes. Calculate the retardation of the car.

Sol. Here, $u = 20\ ms^{-1}$, $v = 0$, $t = 5s$. acceleration $a = \frac{v-u}{t} = \frac{(0-20)}{5} = -4 m/s^2$

–ve acceleration is known as retardation. Thus, retardation of the car = 4 ms^{-2}.

Use of Mathematical Tools in Solving Problems of One–Dimensional Motion

If displacement–time equation is given, we can get velocity–time equation with the help of differentiation. Again, we can get acceleration–time equation with the help of differentiation.

If acceleration–time equation is given, we can get velocity–time equation by integration. From velocity equation, we can get displacement–time equation by integration.

$$\boxed{\textbf{Displacement}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\textbf{Velocity}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\textbf{Acceleration}}$$

Ex. The velocity of any particle is related with its displacement as; $x = \sqrt{v+1}$, Calculate acceleration at $x = 5$ m.

Sol. $\therefore x = \sqrt{v+1}$ $\therefore x^2 = v + 1 \Rightarrow v = (x^2-1)$

Therefore $a = \frac{dv}{dt} = \frac{d}{dt}\left(x^2-1\right) = 2x\frac{dx}{dt} = 2x\ \ v = 2x\left(x^2-1\right)$

At $x = 5$ m, $a = 2 \times 5\ (25-1) = 240\ m/s^2$

Ex. The velocity of a particle moving in the positive direction of x–axis varies as $v = \alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment $t = 0$, the particle was located at $x = 0$ find, (i) the time dependance of the velocity and the acceleration of the particle and (ii) the average velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

Sol. (i) Given that $v = \alpha\sqrt{x}$

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \quad \therefore \frac{dx}{\sqrt{x}} = \alpha\, dt \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt \quad 2\sqrt{x} = \alpha t \Rightarrow x = \left(\alpha^2 t^2/4\right)$$

Velocity $\frac{dx}{dt} = \frac{1}{2}\alpha^2 t$ and Acceleration $\frac{d^2x}{dt^2} = \frac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres $s = \frac{\alpha^2 t^2}{4} \Rightarrow t^2 = \frac{4s}{\alpha^2} \Rightarrow t = \frac{2\sqrt{s}}{\alpha}$;

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}} = \frac{s\alpha}{2\sqrt{s}} = \frac{1}{2}\sqrt{s\alpha}$$

Ex. A particle moves in the plane xy with constant acceleration a directed along the negative y–axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the partcle at the origin of coordinates.

Sol. Given that $y = px - qx^2$

$$\therefore \frac{dy}{dt} = p\frac{dx}{dt} - q.2x\frac{dx}{dt} \text{ and } \frac{d^2y}{dt^2} = p\frac{d^2x}{dt^2} - 2qx\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2 = (p-2qx)\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2$$

$$\because \frac{d^2x}{dt^2} = 0 \text{ (no acceleration along x–axis) and } \frac{d^2y}{dt^2} = -a$$

$$\therefore v_x^2 = \frac{a}{2q} \Rightarrow v_x = \sqrt{\frac{a}{2q}} \text{ Further, } \left(\frac{dy}{dt}\right)_{x=0} = p\frac{dx}{dt} \Rightarrow v_y = p\sqrt{\left(\frac{a}{2q}\right)}$$

$$\text{Now } v = \sqrt{\left(v_x^2 + v_y^2\right)} = \sqrt{\left(\frac{a}{2q} + \frac{ap^2}{2q}\right)} \Rightarrow v = \sqrt{\left[\frac{a\left(p^2+1\right)}{2q}\right]}$$

Ex. A particle moves along a straight line path such that its magnitude of velocity is given by $v = (3t^2 - 6t)\ ms^{-1}$, where t is the time in seconds. If it is initially located at the origin O then determine the magnitude of particle's average velocity and average speed in time interval from $t = 0$ to $t = 4s$.

Sol. $$\text{Average velocity} = \frac{\int v\,dt}{\int dt} = \frac{\int_0^4 \left(3t^2 - 6t\right)dt}{\int_0^4 dt} = \frac{\left(t^3 - 3t^2\right)_0^4}{(t)_0^4} = 4\ ms^{-1}$$

$$\text{Average speed} = \frac{\int |v|\,dt}{\int dt} = \frac{\int_0^4 \left|3t^2 - 6t\right|dt}{\int_0^4 dt} = \frac{\int_0^2 \left(6t - 3t^2\right)dt + \int_2^4 \left(3t^2 - 6t\right)dt}{\int_0^4 dt}$$

$$= \frac{\left(3t^2 - t^3\right)_0^2 + \left(t^3 - 3t^2\right)_2^4}{(t)_0^4} = \frac{24}{4} = 6\ ms^{-1}$$

Ex. The coordinates a particle moving in a plane are given by $x = 3\cos 2t$ and $y = 4\sin 2t$.

(i) Find the equation of the path of the particle.

(ii) Find the angle between $\vec{r}$ and $\vec{v}$ at $t = \frac{\pi}{4}$.

(iii) Prove that acceleration of the particle is always directed towards a fixed point.

Sol. (i) Eliminating t from $x = 3\cos 2t$ & $y = 4\sin 2t$. We get $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1 \Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$

(ii) $\vec{r} = x\hat{i} + y\hat{j} = 3\cos 2t\,\hat{i} + 4\sin 2t\,\hat{j} \Rightarrow \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = -6\sin 2t\,\hat{i} + 8\cos 2t\,\hat{j}$

At $t = \frac{\pi}{4}$, $\vec{r} = 4\hat{j}$, $\vec{v} = -6\hat{i}$

Angle between $\vec{r}$ and $\vec{v}$ $\cos\theta = \frac{\vec{r}.\vec{v}}{rv} = \frac{(4\hat{j}).(-6\hat{i})}{(4)(6)} = 0 \Rightarrow \theta = \frac{\pi}{2}$

(iii) $\vec{a} = \frac{d\vec{v}}{dt} = -12\cos 2t\,\hat{i} - 16\sin 2t\,\hat{j} = -4\,(3\cos 2t\,\hat{i} + 4\sin 2t\,\hat{j}) = -4\vec{r}$

So acceleation is always directed toward origin (a fixed point)

Ex. A particle moves in a straight line according to the relation $x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t$.

Find the displacement and distance travelled by the particle upto t = 4 sec.

Sol. $v = \frac{dx}{dt} = t^2 - 5t + 6$

The particle turns when $v = 0 = t^2 - 5t + 6 \Rightarrow (t-2)(t-3) = 0$ i.e. t = 2 sec, 3 secs

Displacement $= x(4) - x(0) = \frac{64}{3} - \frac{80}{2} + 24 = \frac{16}{3}$ m

Distance $= |x(2) - x(0)| + |x(3) - x(2)| + |x(4) - x(3)|$

$$= \left[\frac{8}{3} - 10 + 12\right] + \left|\left(9 - \frac{45}{2} + 18 - \frac{14}{3}\right)\right| + \left|\frac{16}{3} - \frac{9}{2}\right| = \frac{2}{3} + \frac{1}{6} + \frac{5}{6} = \frac{5}{3}\text{ m}$$

Alter : distance $= \int_0^4 |v|\,dt$

Equations of motion (motion with constant acceleration)

If a particle moves with acceleration $\vec{a}$, then by definition $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}dt$. Let at starting (t = 0)

initial velocity of the particle $\vec{u}$ and at time t its final velocity = $\vec{v}$ then $\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}dt$

If acceleration is constant

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \vec{a}\int_0^t dt \Rightarrow [\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{a}[t]_0^t \Rightarrow \vec{v} - \vec{u} = \vec{a}\,t \Rightarrow \vec{v} = \vec{u} + \vec{a}t \qquad \text{........(1)}$$

Now by definition of velocity, equation (1) reduces to

$$\vec{v} = \frac{d\vec{s}}{dt} = \vec{u} + \vec{a}t \Rightarrow \int_0^{\vec{s}} d\vec{s} = \int_0^t (\vec{u} + \vec{a}t)dt \Rightarrow \vec{s} = \left[\vec{u}t + \frac{1}{2}\vec{a}t^2\right]_0^t \Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \qquad \text{........(2)}$$

Now substituting the value of t from equation (1) to equation (2)

$$s = u\frac{(v-u)}{a}+\frac{1}{2}a\left[\frac{v-u}{a}\right]^2 \Rightarrow 2as = 2uv - 2u^2 + v^2 + u^2 - 2uv \Rightarrow v^2 = u^2 + 2as \text{........(iii)}$$

vector form of equation (iii) $\boxed{v^2 = u^2 + 2\vec{a}.\vec{s}}$ ....(3)

These three equation are called equations of motion and are applicable only and only when acceleration is constant.

Distance travalled by the body in n^{th} second

$$s_{n^{th}} = s_n - s_{n-1} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2 = un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{a}{2}$$

vector form of equation (iv)

$$\boxed{s_{n^{th}} = u + \frac{a}{2}(2n-1)} \qquad \text{........(4)}$$

Ex. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of $6.0 m/s^2$, find the distance travelled by the car after he sees the need to put the brakes on

Sol. Distance covered by the car during the application of brakes by driver –

$$s_1 = ut = \left(54\times\frac{5}{18}\right)(0.2) = 15 \times 0.2 = 3.0 \text{ meter}$$

After applying the brakes; $v = 0$ $u = 15$ m/s, $a = 6$ m/s^2 $s_2 = ?$

Using $v^2 = u^2 - 2as \Rightarrow 0 = (15)^2 - 2 \times 6 \times s_2 \Rightarrow 12\ s_2 = 225 \Rightarrow s_2 = \frac{225}{12} = 18.75$ metre

Distance travelled by the car after driver sees the need for it $s = s_1 + s_2 = 3 + 18.75 = 21.75$ metre.

Ex. A passenger is standing d distance away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed u towards the bus. What must be the minimum speed of the passenger so that he may catch the bus?

Sol. Let the passenger catch the bus after time t.

The distance travelled by the bus, $s_1 = 0 + \frac{1}{2}at^2$ ....(1)

and the distance travelled by the passenger $s_2 = ut + 0$ ....(2)

Now the passenger will catch the bus if $d + s_1 = s_2$ ....(3)

$$\Rightarrow d + \frac{1}{2}at^2 = ut \Rightarrow \frac{1}{2}at^2 - ut + d = 0 \Rightarrow t = \frac{[u \pm \sqrt{u^2 - 2ad}]}{a}$$

So the passenger will catch the bus if t is real, i.e., $u^2 \geq 2ad \Rightarrow u \geq \sqrt{2ad}$

So the minimum speed of passenger for catching the bus is $\sqrt{2ad}$.

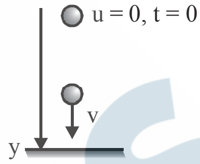
Vertical motion under gravity

If air resistance is neglected and a body is freely moving along vertical line near the earth surface then an acceleration downward which is $9.8 m/s^2$ or $980 cm/s^2$ or $32 ft/s^2$ is experienced by the body.

Freely falling bodies from a height h above the ground

Taking initial position as origin and direction of motion (i.e. downward direction) positive y axis, as body is just released/dropped $u = 0$

acceleration along +Y axis $a = g$

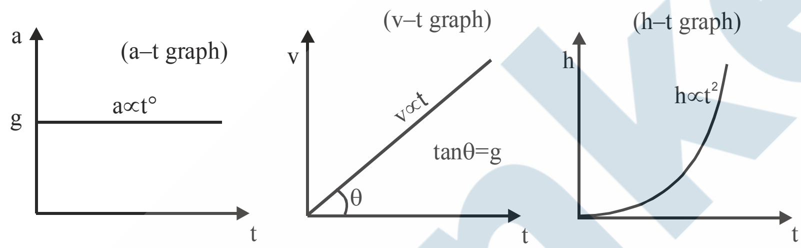


Use equations of motion to describe the motion, i.e.

$$v = u + at,\quad y = ut + \frac{1}{2}at^2,\quad v^2 = u^2 + 2ay$$

Let the body acquires velocity v(downward) after falling a distance h in time t, then

$$v = gt \Rightarrow t = v/g \because h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}},\ v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

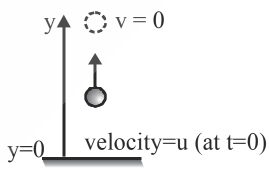


Body is projected vertically upward : With velocity u take initial position as origin and direction of motion

(i.e. vertically upward) as positive y–axis.

$v = 0$ at maximum height, at $t = T$,

$a = – g$ (because directed downward)



Put the values in equation of motion

$$v = u + at \Rightarrow 0 = u - gT \Rightarrow u = gT$$

$$s = ut + \frac{1}{2}at^2 \qquad \Rightarrow h = ut - \frac{1}{2}gt^2$$

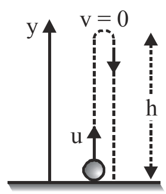
$$\Rightarrow h_{max} = uT - \frac{1}{2}gT^2 \qquad \Rightarrow h_{max} = (gT)T - \frac{1}{2}gT^2 = \frac{1}{2}gT^2$$

$$v^2 = u^2 + 2as \qquad \Rightarrow v^2 = u^2 - 2gh \Rightarrow 0 = u^2 - 2g\,h_{max} \Rightarrow u^2 = 2g\,h_{max} \Rightarrow u = \sqrt{2g\,h_{max}}$$

After attaining maximum height body turns and come back at ground. During complete flight acceleration is constant,

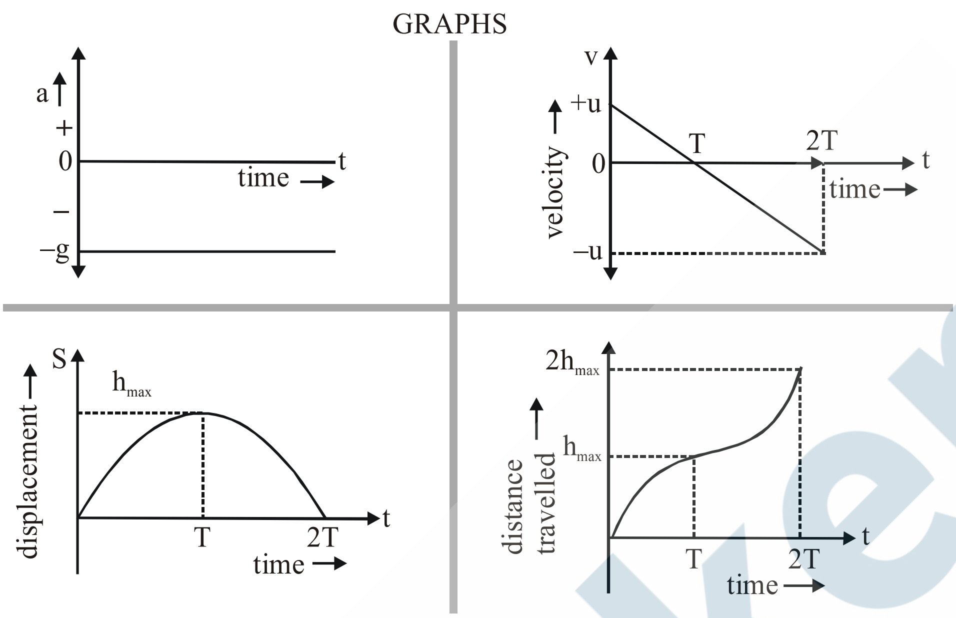
Time taken during up flight and down flight are equal

$$\text{Time for one side } T = \frac{u}{g} \text{ and total flight time} = 2T = \frac{2u}{g}$$



At each equal height from ground speed of body will be same either going up or coming down.

SOME RELATED GRAPHS FOR ABOVE MOTION'S



Ex. A body is freely dropped from a height h above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in 1^{st} second, in 2^{nd} second, in 3^{rd} second etc.

Sol. From second equation of motion, i.e. $h = \frac{1}{2}gt^2$ $(h = ut + \frac{1}{2}gt^2$ and $u = 0)$

$h_1 : h_2 : h_3 \ldots\ldots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 = 1^2 : 2^2 : 3^2 \ldots\ldots.. = 1 : 4 : 9 : \ldots\ldots .$

Now from the of distance travelled in n^{th} second

$$s_n = u + \frac{1}{2}a(2n - 1) \text{ here } u = 0, a = g \Rightarrow s_n = \frac{1}{2}g(2n - 1)$$

$$\Rightarrow s_1 : s_2 : s_3 \ldots\ldots.. = \frac{1}{2}g(2 \times 1 - 1) : \frac{1}{2}g(2 \times 2 - 1) : \frac{1}{2}g(2 \times 3 - 1) = 1 : 3 : 5 \ldots\ldots..$$

Ex. A rocket is fired vertically up from the ground with a resultant vertical acceleration of $10 m/s^2$. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After finishing fuel, calculate the time for which it continues its upwards motion. (Take $g = 10 m/s^2$)

Sol. (a) The distance travelled by the rocket during burning interval (1minute= 60s) in which resultant acceleration $10 m/s^2$ is vertically upwards will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000 m = 18 km$ and velocity acquired by it will be $v = 0 + 10 \times 60 = 600 m/s$

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to

gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero such that

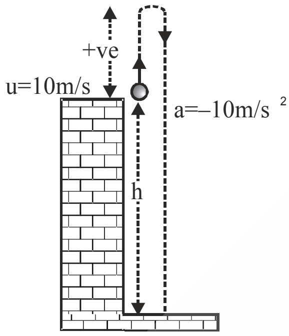
$$0 = (600)^2 - 2gh_2 \Rightarrow h_2 = 18000 m = 18 km \; [g = 10 ms^{-2}]$$

So the maximum height reached by the rocket from the ground, $H = h_1 + h_2 = 18 + 18 = 36 km$

(b) As after burning of fuel the initial velocity 600m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till it velocity v = 0

$0 = 600 - gt \Rightarrow t = 60$ s

Ex. A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10$ m/s^2)



Sol. In the problem $u = +10$ m/s, $a = -10$ m/s^2 and $s = -40$m (at the point where ball strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$

$-40 = 10t - 5t^2 \Rightarrow 5t^2 - 10t - 40 = 0 \Rightarrow t^2 - 2t - 8 = 0$

Solving this we have $t = 4$ s and -2s. Taking the positive value $t = 4$s.

Ex. The acceleration of a particle moving in a straight line varies with its displacement as, $a = 2s$ velocity of the particle is zero at zero displacement. Find the corresponding velocity displacement equation.

Sol. $a = 2s \Rightarrow \frac{dv}{dt} = 2s \Rightarrow \frac{dv}{ds}.\frac{ds}{dt} = 2s \Rightarrow \frac{dv}{ds}.v = 2s$

$\Rightarrow \int v\,dv = 2\int s\,ds \Rightarrow \left(\frac{v^2}{2}\right)_0^v = 2\left(\frac{s^2}{2}\right)_0^s$

$\Rightarrow \frac{v^2}{2} = s^2 \Rightarrow v = s\sqrt{2}$

Ex. If a body travels half its total path in the last second of its fall from rest, find :

(a) The time and

(b) height of its fall. Explain the physically unacceptable solution of the quadratic time equation. ($g = 9.8$ m/s^2)

Sol. If the body falls a height h in time t, then

$h = \frac{1}{2}gt^2$ [$u = 0$ as the body starts from rest] ... (1)

Now, as the distance covered in $(t-1)$ second is $h' = \frac{1}{2}g(t-1)^2$ ... (2)

So from Equations (1) and (2) distance travelled in the last second.

$h - h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$ i.e., $h - h' = \frac{1}{2}g(2t-1)$

But according to given problem as $(h - h') = \frac{h}{2}$

i.e., $\left(\frac{1}{2}\right)h = \left(\frac{1}{2}\right)g(2t-1)$ or $\left(\frac{1}{2}\right)gt^2 = g(2t-1)$ [as from equation (1) $h = \left(\frac{1}{2}\right)gt^2$]

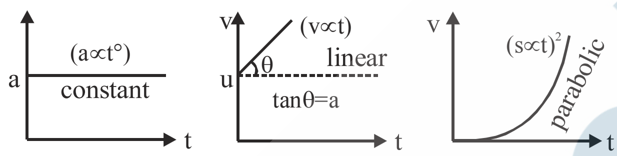
or $t^2 - 4t + 2 = 0$ or $t = [4 \pm \sqrt{(4^2 - 4\times 2)}]/2$ or $t = 2 \pm \sqrt{2}$ $\Rightarrow t = 0.59$ s or 3.41 s

0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground lesser than one sec while according to the given problem time of motion must be greater than 1s.

so $t = 3.41$s and $h = \frac{1}{2} \times (9.8) \times (3.41)^2 = 57$ m

Graphs based on 1–D

For constant acceleration, a/t, v/t and s/t curve from equations of motion are –



In case of constant acceleration motion in a straight line, scalar form of equations of motion can be applied and problem becomes fairly simple.

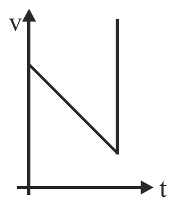
As $d\vec{v} = \vec{a}dt$ or $[\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{v} - \vec{u} = \int_{t_1}^{t_2} \vec{a}dt$ = Area between curve and time axis from t_1 to t_2.

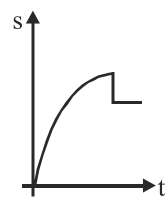
Area under the curve of a – t graph always gives the change in velocity.

Similarly $d\vec{s} = \int \vec{v}dt$ or $\vec{s} = \int_{t_1}^{t_2} \vec{v}dt$ = Area between curve and time axis from t_1 to t_2.

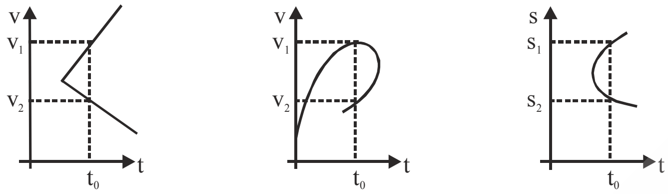
Here $\vec{s}$ is the displacement of particle in time interval t_1 to t_2, i.e. area under the curve of v/t graph always gives the displacement. If only magnitude of area is taken into account then sum of all area is the total distance travelled by the particle.

- Slopes of v–t or s–t graphs can never be infinite at any point, because infinite slope of v–t graph means infinite acceleration. Similarly, infinite slope of s–t graph means infinite velocity. Hence, the following graphs are not possible.





- At one time, two values of velocity or displacement are not possible Hence, the following graphs are not acceptable.



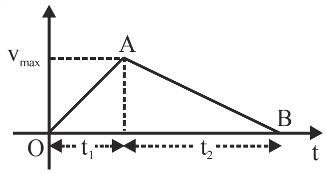
- The slope of velocity–time graph of uniform motion is zero.
- When a body is having uniform motion along a straight line in a given direction, the magnitude of the displacement of body is equal to the actual distance travelled by the body in the given time.
- The average and instantaneous velocity in a uniform motion are equal in magnitude.
- In a uniform motion along a straight line, the slope of position–time graph gives the velocity of the body.
- The position–time graph of a body moving along a straight line can never be a straight line parallel to position axis because it will indicate infinite velocity.
- The speed of a body can never be negative
- Medium effects the motion of a body falling freely under gravity due to thrust and viscous drag.

Ex. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t evaluate (a) the maximum velocity attained and (b) the total distance travelled.

Sol. (a) Let the car accelerates for time t_1 and decelerates for time t_2 then

$$t = t_1 + t_2 \qquad \text{......(i)}$$

and corresponding velocity–time graph will be as shown in. fig.



From the graph α = slope of line AB $= \frac{v_{max}}{t_1} \Rightarrow t_1 = \frac{v_{max}}{\alpha}$

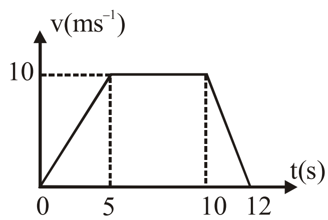
and β = – slope of line OB $= \frac{v_{max}}{t_2} \Rightarrow t_2 = \frac{v_{max}}{\beta}$

$$\Rightarrow \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t \Rightarrow v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t \Rightarrow v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$$

(b) Total distance = area under v–t graph $= \frac{1}{2} \times t \times v_{max} = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta} = \frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$

Note: This problem can also be solved by using equations of motion (v = u + at, etc.).

Ex. Draw displacement time and acceleration – time graph for the given velocity–time graph



Sol. For $0 \le t \le 5$ $v \propto t \Rightarrow s \propto t^2$ and a_1 = constant $\frac{10}{5} = 2\ ms^{-2}$

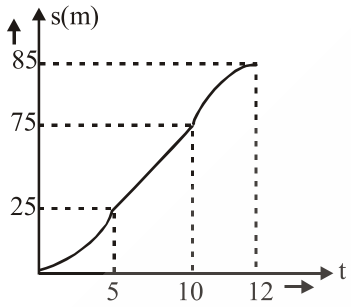
for whole interval s_1 = Area under the curve = $\frac{1}{2} \times 5 \times 10 = 25$ m

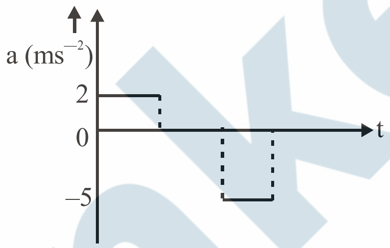
For $5 \le t \le 10$ $v = 10ms^{-1}$ $\Rightarrow a = 0$

for whole interval s_2 = Area under the curve = $\frac{1}{2} \times 5 \times 10 = 50$ m

For $10 \le t \le 12$ v linearly decreases with time $\Rightarrow a_3 = -\frac{10}{2} = -5\ ms^{-1}$

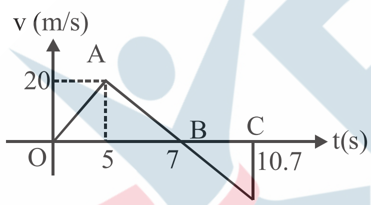
for whole interval s_3= Area under the curve = $\frac{1}{2} \times 2 \times 10 = 10$ m

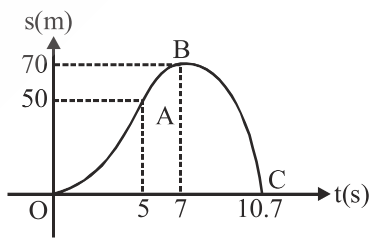




Ex. A rocket is fired upwards vertically with a net acceleration of 4 m/s^2 and initial velocity zero. After 5 seconds its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and return back to ground. Plot velocity–time and displacement–time graphs for the complete journey. Take g = 10 m/s^2.

Sol.





In the graphs, $v_A = at_{OA} = (4)(5) = 20$ m/s $v_B = 0 = v_A - gt_{AB}$

$\therefore t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2s$ $\therefore t_{OAB} = (5 + 2)s = 7s$

Now, s_{OAB} = area under v–t graph between 0 to 7 s = $\frac{1}{2}(7)(20) = 70$ m

Now, $s_{OAB} = s_{BC} = \frac{1}{2}gt^2_{BC}$ $\therefore 70 = \frac{1}{2}(10)t^2_{BC}$

$\therefore t_{BC} = \sqrt{14} = 3.7s$ $\therefore t_{OAB} = 7 + 3.7 = 10.7s$

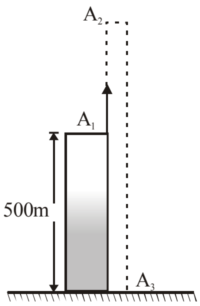
Also s_{OA} = area under v–t graph between OA = $\frac{1}{2}(5)(20) = 50$ m

Ex. At the height of 500m, a particle A is thrown up with $v = 75\ ms^{-1}$ and particle B is released from rest. Draw, accelearation –time, velocity–time, speed–time and displacement–time graph of each particle.

For particle A :

Time of flight

$$-500 = +75\,t - \frac{1}{2} \times 10t^2$$



$\Rightarrow t^2 - 15t - 100 = 0$

$\Rightarrow t = 20$ s

Time taken for A_1A_2

$= 75 - 10t \Rightarrow t = 7.5$ s

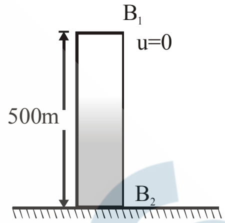
Velocity at A_3, $v = 75 - 10 \times 20 = -125\ ms^{-1}$

$$\text{Height } A_2A_1 = \frac{1}{2}(10)(7.5)^2 = 281.25 \text{ m}$$

For Particle B

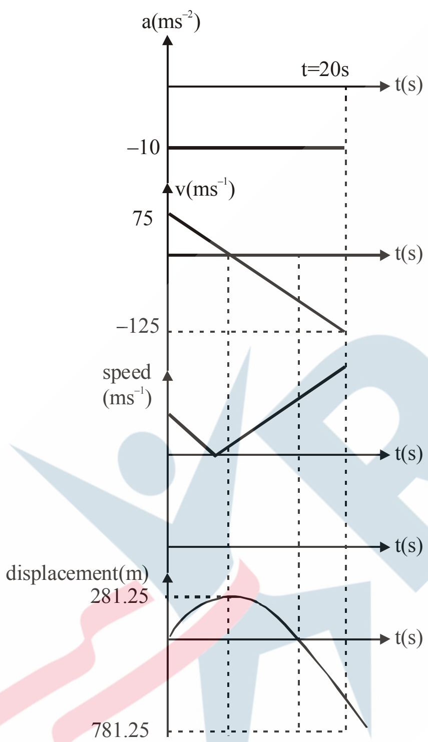
Time of flight

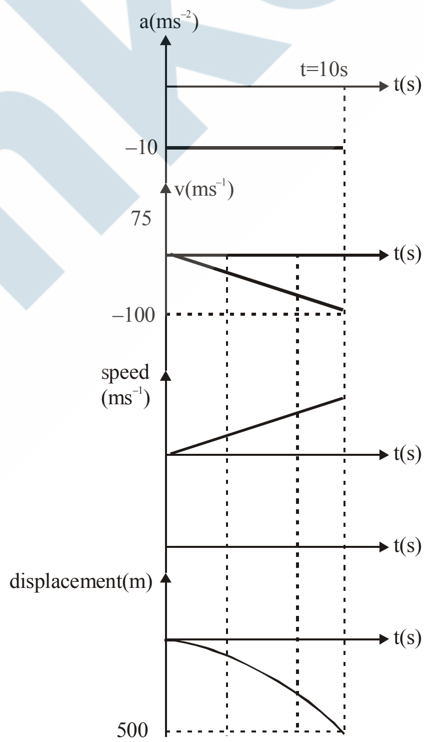
$$500 = \frac{1}{2}(10)t^2 \Rightarrow t = 10 \text{ s}$$

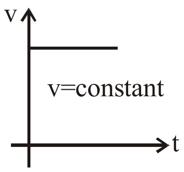
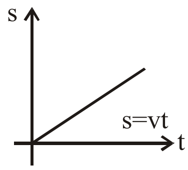
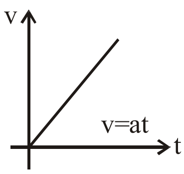
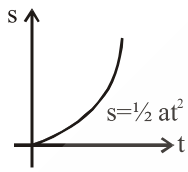
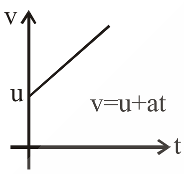
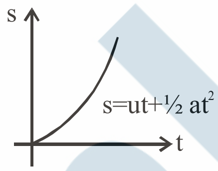
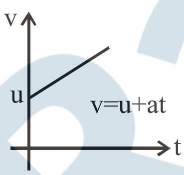
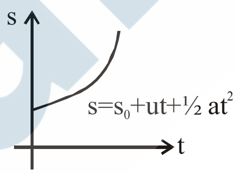
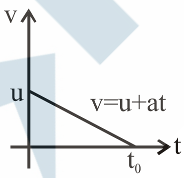
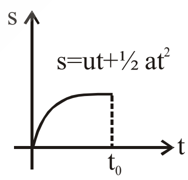
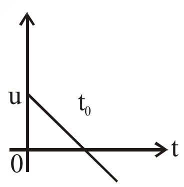
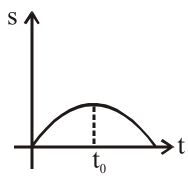


Velocity at B_2

$v = 0 - (10)(10) = -100\ ms^{-1}$



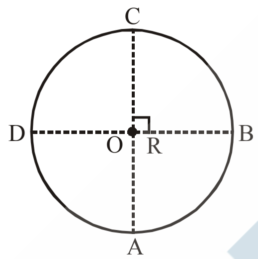


S.N.	Different Cases	v–t graph	s–t graph	Important Points
1.	Uniform motion			(i) Slope of s–t graph = v = constant (ii) In s–t graph s = 0 at t = 0
2.	Uniformly accelerated motion with u = 0 at t = 0			(i) u = 0, i.e. v = 0 at t = 0 (ii) u = 0, i.e., slope of s–t graph at t = u, should be zero (iii) a or slope of v – t graph is constant
3.	Uniformly accelerated with $u \neq 0$ at t = 0			(i) $u \neq 0$, i.e., v or slope motion of s – t graph at t = 0 is not zero (ii) v or slope of s – t graph gradually goes on increasing.
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at t = 0			(i) $s = s_0$ at t = 0
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of s – t graph at t = 0 gives u (ii) Slope of s – t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero.
6.	Uniformly retarded then accelerated in opposite direction			(i) At time $t = t_0$, v = 0 or slope of s – t graph is zero (ii) In s – t graph slope or velocity first decreases then increases with opposite sign.

EXERCISE (S-1)

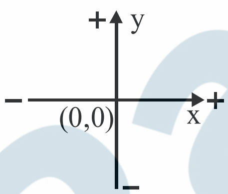
Definitions of kinematics variables

1. A particles starts from point A with constant speed v on a circle of radius R. Find magnitude of average velocity during its journey from :-



(a) A to B (b) A to C (c) A to D

2. A particle is moving along x-axis. Initially it is located 5 m left of origin and it is moving away from the origin and slowing down. In this coordinate system, what are the signs of the initial velocity and acceleration.



v_0	a

Motion with constant acceleration

3. A car accelerates with uniform rate from rest on a straight road. The distance travelled in the last second of a three second interval from the start is 15 m then find the distance travelled in first second in m.

4. A particle moving in one-dimension with constant acceleration of 10 m/s^2 is observed to cover a distance of 100 m during a 4s interval. How far will the particle move in the next 4s?

5. A particle starts from rest at t = 0 and x = 0 to move with a constant acceleration = +2 m/s^2, for 20 seconds. After that, it moves with –4 m/s^2 for the next 20 seconds. Finally, it moves with positive acceleration for 10 seconds until its velocity becomes zero.

(a) What is the value of the acceleration in the last phase of motion?

(b) What is the final x-coordinate of the particle?

(c) Find the total distance covered by the particle during the whole motion.

6. A body moving with uniform acceleration has a velocity of –11 cm/s when its x coordinate is 3.00 cm. If its x coordinate 2 s later is – 5 cm, what is the magnitude in cm/s^2 of its acceleration?

7. A driver travelling at speed 36 kmh^{-1} sees the light turn red at the intersection. If his reaction time is 0.6s, and then the car can deaccelerate at 4ms^{-2}. Find the stopping distance of the car.

8. The window of the fourth floor of SANKALP building is 5 m high. A man looking out of the window sees an object moving up and down the height of window for 2 sec. Find the height that the object reaches from the top end of the window.

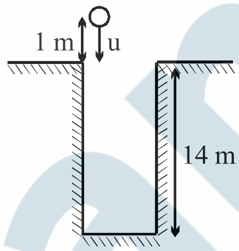
9. A body is dropped from a height of 300 m. Exactly at the same instant another body is projected from the ground level vertically up with a velocity of 150 ms^{-1}. Find when they will meet.

10. A stone is dropped from the top of a tall cliff, and 1s later a second stone is thrown vertically downward with a velocity of 20 ms^{-1}. How far below the top of the cliff will the second stone overtake the first?

11. Speed of train is increasing linearly with time. The train passes a hut with speed 2 m/s and acquires a speed of 12 m/s after 10 s. What is the speed of the train in m/s, 5 s after passing the hut?

12. Two particle A and B are moving in same direction on same straight line. A is ahead of B by 20m. A has constant speed 5 m/sec and B has initial speed 30 m/sec and retardation of 10 m/sec^2. Then if x (in m) is total distance travelled by B as it meets A for second time. Then value of x will be.

13. A boy throws a ball with speed u in a well of depth 14 m as shown. On bounce with bottom of the well the speed of the ball gets halved. What should be the minimum value of u (in m/s) such that the ball may be able to reach his hand again? It is given that his hands are at 1 m height from top of the well while throwing and catching.



14. From the top of a tower, a ball is thrown vertically upwards. When the ball reaches h below the tower, its speed is double of what it was at height h above the tower. Find the greatest height attained by the ball from the tower.

15. A rocket is fired vertically upwards with initial velocity 40 m/s at the ground level. Its engines then fired and it is accelerated at 2 m/s^2 until it reaches an altitude of 1000 m. At that point the engines shut off and the rocket goes into free-fall. If the velocity (in m/s) just before it collides with the ground is 40α. Then fill the value of α. Disregard air resistance ($g = 10 m/s^2$).

16. A balloon rises from rest on the ground with constant acceleration $\frac{g}{3}$. A stone is dropped when the balloon has rises to a height 60 metre. The time taken by the stone to reach the ground is.

Motion with variable acceleration & calculus

17. The position x of a particle w.r.t. time t along x–axis is given by $x = 9t^2 - t^3$ where x is in metre and t in second. Find

(a) Maximum speed along +x direction

(b) Position of turning point

(c) Displacement in first ten seconds

(d) Distance travelled in first ten seconds

18. The momentum of a particle moving in straight line is given by $p = \ln t + \frac{1}{t}$ (in kg m/s) find the time $t > 0$ at which the net force acting on particle is 0 and it's momentum at that time. [Hint : $F = \frac{dp}{dt}$]

19. The velocity of the particle is given as $v = 3t^3 + t - \frac{1}{t^2}$. Calculate the net force acting on the body at time t = 2 sec, if the mass of the body is 5 kg.

20. A wheel rotates so that the angle of rotation is proportional to the square of time. The first revolution was performed by the wheel for 8 sec. Find the angular velocity ω, 32 sec after the wheel started.
[**Hint:** Consider $\theta = kt^2$, find k]

21. The charge flowing through a conductor beginning with time t = 0 is given by the formula $q = 2t^2 + 3t + 1$ (coulombs). Find the current $i = \frac{dq}{dt}$ at the end of the 5^{th} second.

22. The angle θ through which a pulley turns with time t is specified by the function $\theta = t^2 + 3t - 5$. Find the angular velocity $\omega = \frac{d\theta}{dt}$ at $t = 5$ sec.

23. The motion of a particle in a straight line is defined by the relation $x = t^4 - 12t^2 - 40$ where x is in meters and t is in sec. Determine the position x, velocity v and acceleration a of the particle at t = 2sec.

24. A point moves in a straight line so that its distance from the start in time t is equal to

$$s = \frac{1}{4}t^4 - 4t^3 + 16t^2.$$

(a) At what times was the point at its starting position?
(b) At what times is its velocity equal to zero?

25. A body whose mass is 3 kg performs rectilinear motion according to the formula $s = 1 + t + t^2$, where s is measured in centimetres & t in seconds. Determine the kinetic energy $\frac{1}{2}mv^2$ of the body in 5 sec after its start.

26. A force of 40N is responsible for the motion of a body governed by the equation $s = 2t + 2t^2$ where s is in meters and t in sec. What is the momentum of the body at t = 2 sec?
[**Hint**: Find acc. then m = F/a & p = mv]

27. The angle rotated by a disc is given by $\theta = \frac{2}{3}t^3 - \frac{25}{2}t^2 + 77t + 5$, where θ is in rad and t in seconds.
(a) Find the times at which the angular velocity of the disc is zero.
(b) Its angular acceleration at these times.

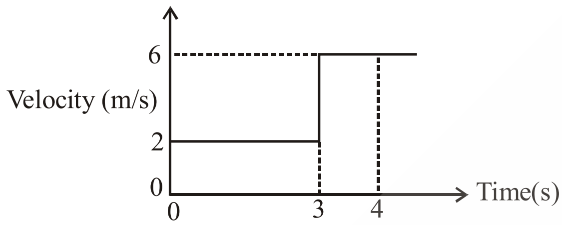
28. The acceleration of a particle starting from rest vary with respect to time is given by $a = (2t - 6)$, where t is in seconds. Find the time (in seconds) at which velocity of particle in negative direction is maximum.

29. Acceleration of a particle is defined as $a = \left(75V^2 - 30V + 3\right)$ (m/s²). If the constant speed achieved by the particle is given by V_C, then find the value of $10V_C$.

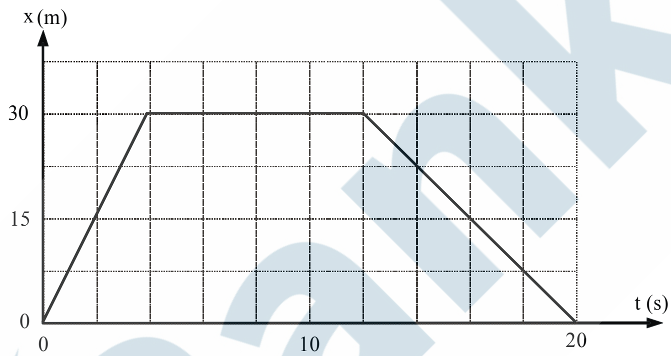
30. Position vector of a particle is given by $\vec{r} = 3t^3\hat{i} + 4t\hat{j} + t^2\hat{k}$ .find avg acceleration of particle from t = 1 to t = 2sec.

Question based on graph

31. In the following graph variation with time (t), in velocity (v) of a particle moving rectilinearly is shown. What is average velocity in m/s of the particle in time interval from 0 s to 4 s?



32. The graph illustrates motion of a bucket being lowered into a well from the top at the instant t = 0, down to the water level, filled with water and drawn up again. Here 'x' is the depth. Find the average speed of the bucket in m/s during whole operation.



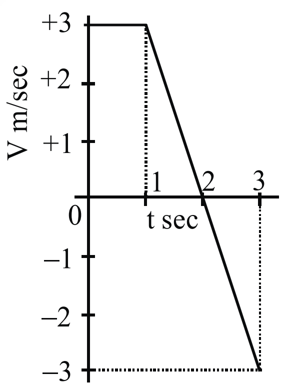
33. A particle moves along a straight line, x. At time t = 0, its position is at x = 0. The velocity, V, of the object changes as a function of time t, as indicated in the figure; t is in seconds, V in m/sec and x in meters.

(a) What is x at t = 3 sec?

(b) What is the instantaneous acceleration (in m/sec^2) at t = 2 sec?

(c) What is the average velocity (in m/sec) between t = 0 and t = 3 sec?

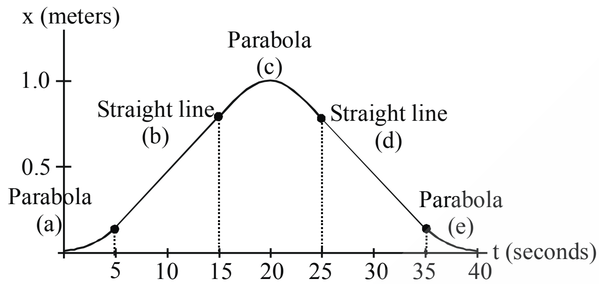
(d) What is the average speed (in m/sec) between t = 1 and t = 3 sec?



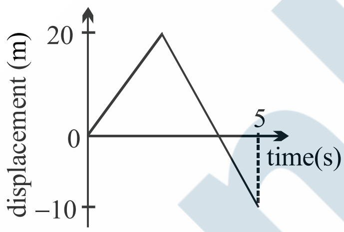
34. The figure below is a displacement vs time plot for the motion of an object, answer questions (i) & (ii) with the letter of appropriate section of the graph.

(i) Which section represents motion in the forward direction with positive acceleration?

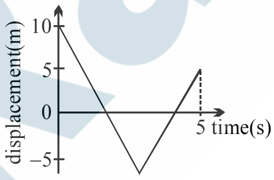
(ii) Which section represents uniform motion backwards (–x direction)?



35. **(a)** The diagram shows the displacement-time graph for a particle moving in a straight line. Find the average velocity for the interval from t = 0 to t = 5.

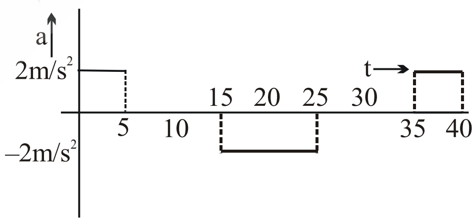


(b) The diagram shows the displacement-time graph for a particle moving in a straight line. Find the average speed for the interval from t = 0 to t = 5.



36. Figure shows a graph of acceleration of a particle moving on the x-axis. Plot the following graphs if the particle is at origin and at rest at t = 0.

(i) velocity-time graph (ii) displacement-time graph (iii) distance-time graph.



EXERCISE (S-2)

1. At a distance L = 400m from the traffic light, brakes are applied to a locomotive moving at a velocity v = 54 km/hr. Determine the position of the locomotive relative to the traffic light 1 minute after the application of the brakes if its acceleration is $-0.3 m/sec^2$.

2. A particle goes from A to B with a speed of 40km/h and B to C with a speed of 60km/h. If AB = 6BC, the average speed in km/h between A and C is.

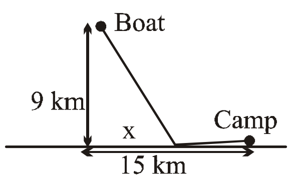
3. A flower pot falls off a window sill and falls past the window below. It takes 0.30 s to pass a window 3.45 m high. How far is the top of the window below upper window sill?

4. A juggler performs in a room whose ceiling is 3 m above the level of his hands. He throws a ball vertically upward so that it just reaches the ceiling.
(a) With what initial velocity does he throw the ball ?
(b) What time is required for the ball to reach the ceiling ?
He throws a second ball upward with the same initial velocity, at the instant that the first ball is at the ceiling.
(c) How long after the second ball is thrown do the two ball pass each other ?
(d) When the balls pass each other, how far are they above the juggler's hands ?

5. A train, travelling at 20 km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it to stop it at pole. At that instant the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance will the bird have flown before the train stops?

6. A helicopter takes off along the vertical with an acceleration of $3 m/sec^2$ & zero initial velocity. In a certain time, the pilot switches off the engine. At the point of takeoff, the sound dies away in 30 sec. Determine the velocity of the helicopter at the moment when its engine is switched off, assuming the velocity of sound is 320 m/sec.

7. A fishing boat is anchored 9 km away from the nearest point on shore. A messenger must be sent from the fishing boat to a camp, 15 km from the point on shore closest to the boat. If the messenger can walk at a speed of 5km per hour and can row at 4 km per hour.
(i) Form an expression relating time taken to reach the camp t with distance x on shore where he lands.
(ii) At what point on shore must he land in order to reach the camp in the shortest possible time?

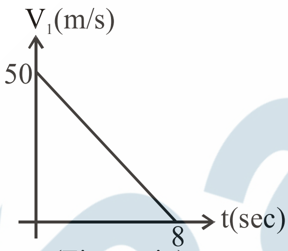


8. Two body move from the same point along a straight line. The first body moves with velocity $v = (3t^2 - 6t)$ m/s, the second with velocity $v = (10t + 20)$ m/s. At what instant and at what distance from the initial point will they meet.

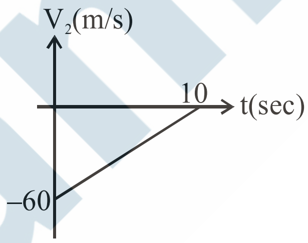
9. Velocity of a car depends on its distance ℓ from a fixed pole on a straight road as $v = 2\sqrt{\ell}$, where ℓ is in meters and v in m/s. Find acceleration (in m/s^2) when $\ell = 8$m.

10. A particle is moving with uniform acceleration along x-axis with initial velocity along positive x. At $t = \dfrac{3\sqrt{2}}{\sqrt{2}-1}$ s the magnitude of displacement becomes $\dfrac{1}{3}$ the total distance travelled. By this time the x coordinate of particle is still positive. The instant (in sec) at which displacement becomes zero is

11. Two trains are moving in opposite direction on same track. When their separation was 600 m their drivers notice the mistake and start slowing down to avoid collision. Graphs of their velocities as function of time is as shown. If separation between the drivers when first train stops is x then find the value of $\dfrac{x}{16}$.



(First train)



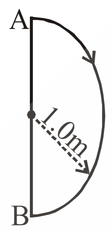
(Second train)

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Definitions of kinematics variables

1. In 1.0 sec. a particle goes from point A to point B moving in a semicircle of radius 1.0 m. The magnitude of average velocity is : **[JEE '99]**



(A) 3.14 m/sec (B) 2.0 m/sec (C) 1.0 m/sec (D) zero

2. An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention upwards as positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?

(A) On the way up $a_y > 0$, on the way down $a_y > 0$

(B) On the way up $a_y < 0$, on the way down $a_y > 0$

(C) On the way up $a_y > 0$, on the way down $a_y < 0$

(D) On the way up $a_y < 0$, on the way down $a_y < 0$

Motion with constant acceleration

3. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as :-

(A) 1 : 2 : 4 (B) 1 : 2 : 5 (C) 1 : 3 : 5 (D) 1 : 3 : 9

4. If a body starts from rest and travels 120 cm in the 6th second, with constant acceleration then what is the acceleration :

(A) 0.20 m/s^2 (B) 0.027 m/s^2 (C) 0.218 m/s^2 (D) 0.03 m/s^2

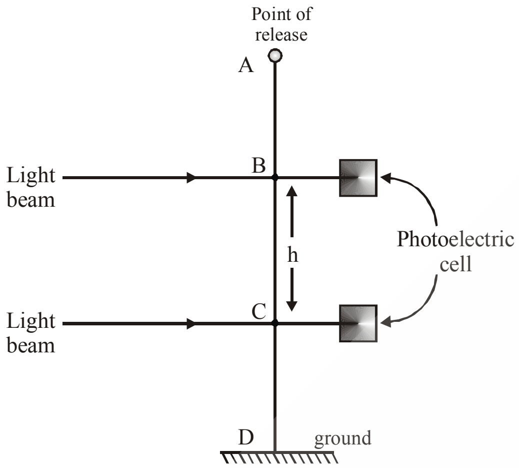
5. A particle travels 10m in first 5 sec and 10m in next 3 sec. Assuming constant acceleration what is the distance travelled in next 2 sec.

(A) 8.3 m (B) 9.3 m (C) 10.3 m (D) None of above

6. The engine of a motorcycle can produce a maximum acceleration 5 m/s^2. Its brakes can produce a maximum retardation 10 m/s^2. If motorcyclist start from point A and reach at point B. What is the minimum time in which it can cover if distance between A and B is 1.5 km. (Given : that motorcycle comes to rest at B)

(A) 30 sec (B) 15 sec (C) 10 sec (D) 5 sec

7. The acceleration of free fall at a planet is determined by timing the fall of a steel ball photo –electrically. The ball passes B and C at times t_1 and t_2 after release from A. The acceleration of free fall is given by



(A) $\dfrac{2h}{t_2 - t_1}$ (B) $\dfrac{h}{t_2^2 - t_1^2}$ (C) $\dfrac{2h}{t_2^2 - t_1^2}$ (D) $\dfrac{2h}{t_2^2 + t_1^2}$

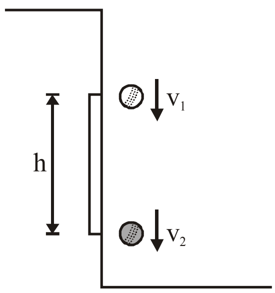
8. A particle has an initial velocity of 9 m/s due east and a constant acceleration of 2 m/s^2 due west. The distance covered by the particle in the fifth second of its motion is :-

(A) 0 (B) 0.5 m (C) 2 m (D) none of these

9. A physics teacher finds a scrap of paper on which one of his students has written the following equation: $0^2 - 5^2 = 2 \times (-9.8) \times x$; of which of the following problem would this equation be part of the correct solution?

(A) Find the speed of an object 5 seconds after it was dropped from rest.
(B) Find the distance of an object has fallen 5 seconds after it was released from rest on Earth.
(C) Find the height from which a ball when released will strike the ground with a speed of 5 m/s.
(D) Find the maximum height to which a ball will rise if it is thrown upward with an initial speed of 5 m/s.

10. A ball dropped from the top of a building passes past a window of height h in time t. If its speeds at the top and the bottom edges of the window are denoted by v_1 and v_2 respectively, which of the following set of equations are correct?



(A) $v_2 - v_1 = gt$ and $(v_2 - v_1)t = h$ (B) $v_2 - v_1 = gt$ and $(v_2 + v_1)t = 2h$
(C) $v_2 + v_1 = gt$ and $(v_2 - v_1)t = h$ (D) None of the above.

11. A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of :

(A) 3 s (B) 5 s (C) 7 s (D) 9 s

12. A ball is thrown vertically upward with initial velocity 30 m/sec. What will be its position vector at time t = 5 sec taking origin at the point of projection, vertical up as positive y-axis and horizontal as x-axis:-

(A) (0, 25) (B) (0, 20) (C) (0, 45) (D) (0, 5)

13. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is [g=10 m/s^2]

(A) 1 : 1 (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{3}{2}}$

14. A body of mass 'm' is travelling with a velocity 'u'. When a constant retarding force 'F' is applied, it comes to rest after travelling a distance 's_1'. If the initial velocity is '2u', with the same force 'F', the distance travelled before it comes to rest is 's_2'. Then

(A) $s_2 = 2s_1$ (B) $s_2 = \frac{s_1}{2}$ (C) $s_2 = s_1$ (D) $s_2 = 4\ s_1$

15. A ball is thrown vertically upward with initial velocity 30 m/sec. What will be its position vector at time t = 5 sec, taking origin at 45 m above the point of projection, vertical up as positive y-axis and horizontal as x-axis :-

(A) (0, –25) (B) (0, –20) (C) (0, –45) (D) (0, –5)

Motion with variable acceleration & calculus

16. If $s = 2t^3 + 3t^2 + 2t + 8$ then the time at which acceleration is zero, is :-

(A) $t = \frac{1}{2}$ (B) t = 2 (C) $t = \frac{1}{2\sqrt{2}}$ (D) Never

17. Velocity of a particle varies with time as v = 4t. The displacement of particle between t = 2 to t = 4 sec, is :-

(A) 12 m (B) 36 m (C) 24 m (D) 6 m

18. A point mass moves with velocity $v = (5t - t^2)\ ms^{-1}$ in a straight line. Find the distance travelled (i.e. $\int v\,dt$) in fourth second.

(A) $\frac{31}{6}$ m (B) $\frac{29}{6}$ m (C) $\frac{37}{6}$ m (D) None of these

19. A particle is projected with velocity v_0 along x–axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e., $a = -\alpha x^2$. The distance at which the particle stops is:–

(A) $\sqrt{\frac{3v_0}{2\alpha}}$ (B) $\left(\frac{3v_0}{2\alpha}\right)^{\frac{1}{3}}$ (C) $\sqrt{\frac{3v_0^2}{2\alpha}}$ (D) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

20. The acceleration vector along x–axis of a particle having initial speed v_0 changes with distance as $a = \sqrt{x}$. The distance covered by the particle, when its speed becomes twice that of initial speed is:–

(A) $\left(\frac{9}{4}v_0\right)^{\frac{4}{3}}$ (B) $\left(\frac{3}{2}v_0\right)^{\frac{4}{3}}$ (C) $\left(\frac{2}{3}v_0\right)^{\frac{4}{3}}$ (D) $2v_0$

21. For a particle moving in a straight line the position of the particle at time (t) is given by $x = \frac{t^3}{6} - t^2 - 9t + 18$ m. What is the velocity of the particle when its acceleration is zero :-

(A) 18 m/s (B) –9 m/s (C) –11 m/s (D) 6 m/s

22. A particle moves along a straight line such that at time t its displacement from a fixed point O on the line is $3t^2 - 2$. The velocity of the particle when t = 2 is:

(A) 8 ms^{-1} (B) 4 ms^{-1} (C) 12 ms^{-1} (D) 0

23. Temperature of a body varies with time as $T = (T_0 + \alpha t^2 + \beta \sin t)K$, where T_0 is the temperature in Kelvin at t = 0 sec. & $\alpha = 2/\pi$. K/s^2 & $\beta = -4$ K, then rate of change of temperature at $t = \pi$sec. is

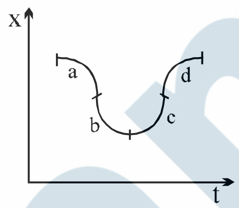
(A) 8 K (B) 8^0K (C) 8K/sec (D) 8^0K/sec

24. The velocity of a particle moving on the x-axis is given by $v = x^2 + x$ where v is in m/s and x is in m. Find its acceleration in m/s^2 when passing through the point x = 2m

(A) 0 (B) 5 (C) 11 (D) 30

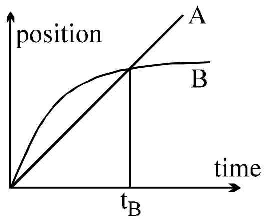
Question based on graph

25. The graph shown is a plot of position versus time. For which labeled region is the velocity positive and the acceleration negative?



(A) a (B) b (C) c (D) d

26. The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?



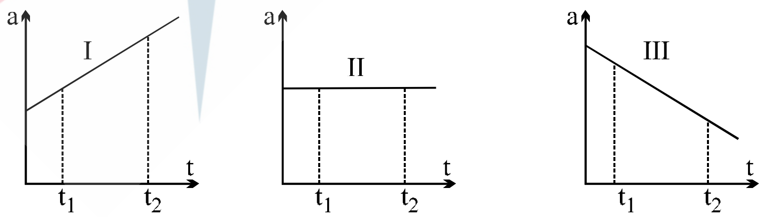
(A) At time t_B, both trains have the same velocity.

(B) Both trains have the same velocity at some time after t_B

(C) Both trains have the same velocity at some time before t_B.

(D) Somewhere on the graph, both trains have the same acceleration.

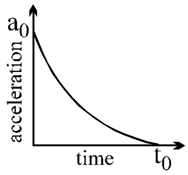
27. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?

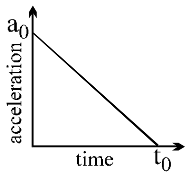


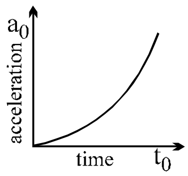
(A) graph I, only (B) graphs I and II, only

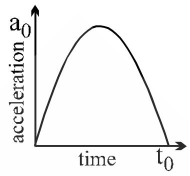
(C) graphs I and III, only (D) graphs I, II, and III

28. Acceleration versus time graphs for four objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?

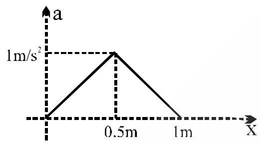
(A) 

(B) 

(C) 

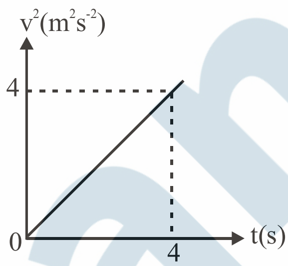
(D) 

29. A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure. The maximum velocity of particle is :-



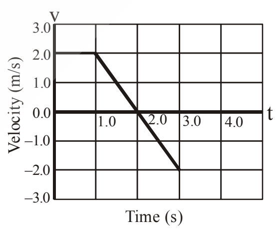
(A) 1 m/s (B) 6 m/s (C) 2 m/s (D) none

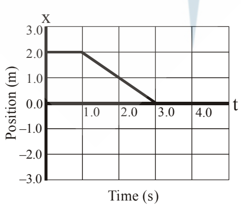
30. A particle is moving along a straight line such that square of its velocity varies with time as shown in the figure. What is the acceleration of the particle at t = 4 s?

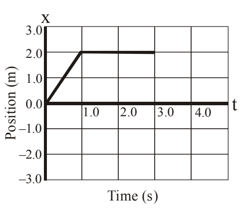


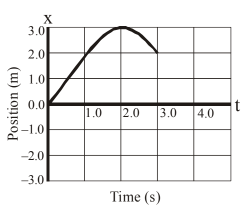
(A) 4 m/s^2 (B) 1/4 m/s^2 (C) 1/2 m/s^2 (D) 0

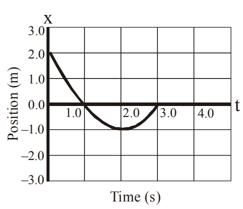
31. The graph below shows the velocity of a particle moving in a straight line. At t = 0, the particle is located at x = 0. Which of the following graphs shows the position of the particle with respect to time, x(t) ?



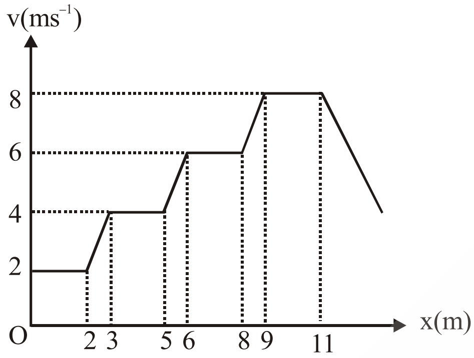
(A) 

(B) 

(C) 

(D) 

32. The velocity of a particle that moves in the positive x-direction varies with its position as shown in figure. The acceleration of the particle when x = 5.5 m is-



(A) 0 (B) 5 ms^{-2} (C) 10 ms^{-2} (D) 20 ms^{-2}

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 33 and 34

A particle is moving in a straight line along positive y-axis. Its displacement from origin at any time t is given by $y = 5t^2 - 10t + 5$ where y is in meters and t is in seconds.

33. The velocity at t = 2s will be :

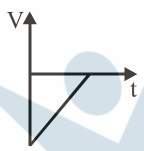
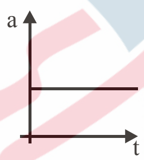
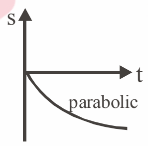
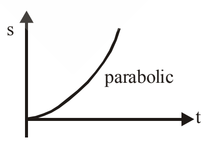
(A) 20 ms^{-1} (B) 10 ms^{-1} (C) 5 ms^{-1} (D) 15 ms^{-1}

34. Displacement of particle when its velocity is zero, is

(A) 2.5 m (B) 1.25 m (C) 5m (D) 0m

MATRIX MATCH TYPE QUESTION

35. v, a, s and t denote velocity, acceleration, displacement and time respectively. Match the columns :-

Column-I		**Column-II**
(A) 	(P)	Velocity of the particle is in positive direction, acceleration in negative direction
(B) 	(Q)	Both velocity and acceleration of the particle are in negative directions.
(C) 	(R)	Velocity of the particle is in negative direction and acceleration in positive direction
(D) 	(S)	Velocity and acceleration both in positive direction
	(T)	Acceleration is constant

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

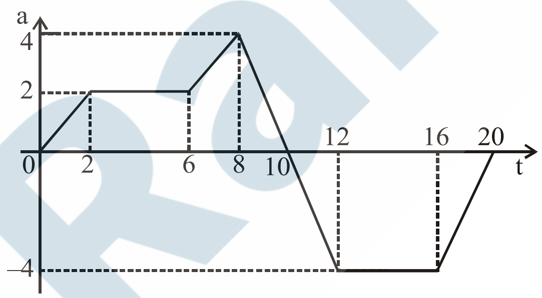
1. A parachutist jumps out of an airplane and accelerates with gravity for 6 seconds. He then pulls the parachute cord and after a 4 s deceleration period, descends at 10 m/s for 60 seconds, reaching the ground. From what height did the parachutist jump? Assume acceleration due to gravity to be 10 m/s^2 throughout the motion.

(A) 840 m (B) 920 m (C) 980 m (D) 1020 m

2. A train moving with a speed of 60 km/hr is slowed down uniformly to 30 km/hr for repair purposes during running. After this it was accelerated uniformly to reach to its original speed. If the distance covered during constant retardation be 2 km and that covered during constant acceleration be 1 km, find the time lost in the above journey

(A) 1 min (B) 2 min (C) 4 min (D) 5 min

3. If initial velocity of particle is 2 m/s, the maximum velocity of particle from t = 0 to t = 20 sec is :



(A) 20 m/s (B) 18 m/s (C) 22 m/s (D) 24 m/s

ASSERTION & REASON

These questions contains, Statement-1 (assertion) and Statement-2 (reason).

(A) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true ; Statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

(E) Both Statement-1 and Statement-2 are false.

4. **Statement I :** When velocity of a particle is zero then acceleration of particle is zero.

and

Statement II : Acceleration is equal to rate of change of velocity.

5. **Statement-I :** A particle moves in a straight line with constant acceleration. The average velocity of this particle cannot be zero in any time interval.

and

Statement-II : For a particle moving in straight line with constant acceleration, the average velocity in a time interval is $\frac{u+v}{2}$, where u and v are initial and final velocity of the particle of the given time interval.

6. A particle moves in a straight line, according to the law $x = 4a\,[t + a\sin\left(\frac{t}{a}\right)]$, where x is its position in meters, t in sec. & a is some constants, then the velocity is zero at :-

(A) $x = 4a^2\pi$ meters (B) $t = \pi$ sec. (C) $t = 0$ sec (D) none

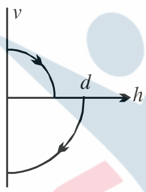
7. A particle moving on the x-axis with constant acceleration has displacements of 6 m from t = 4 s to t = 7 s and 3 m from t = 5 s to t = 8 s. The distance covered from t = 6 s to t = 9 s is

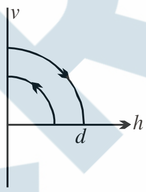
(A) 1.75 m (B) 2.25 m (C) 3.0 m (D) 0

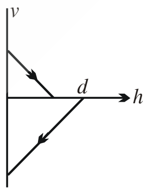
8. A point moves in a straight line so that its displacement is x m at time t sec, given by $x^2 = t^2 + 1$. Its acceleration in m/s^2 at time t sec is :

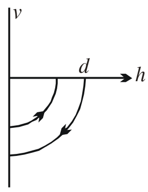
(A) $\frac{1}{x}$ (B) $\frac{1}{x} - \frac{1}{x^2}$ (C) $-\frac{t}{x^2}$ (D) $\frac{1}{x^3}$

9. A ball is dropped vertically from a height *d* above the ground, hits the ground and bounces up vertically to a height *d*/2. Neglecting subsequent motion and air resistances, its velocity *v* varies with the height *h* above the ground as :- **[IIT-JEE'2000 (Scr)]**

(A) 

(B) 

(C) 

(D) 

MULTIPLE CORRECT TYPE QUESTIONS

10. A particle moving along a straight line with uniform acceleration has velocities 7m/s at A and 17 m/s at C. B is the mid point of AC. Then

(A) The velocity at B is 12 m/s.

(B) The average velocity between A and B is 10 m/s.

(C) The ratio of the time to go from A to B to that from B to C is 3 : 2.

(D) The average velocity between B and C is 15 m/s.

11. A particle moves along the X–axis as $x = u(t - 2s) + a(t - 2s)^2$

(A) The initial velocity of the particle is u (B) The acceleration of the particle is a

(C) The acceleration of the particle is 2a (D) At t = 2s particle is at the origin.

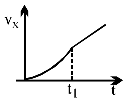
12. The position of a particle with time is given by

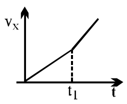
$(x, y) = (8t^2, 3)$ for $t \le t_1$

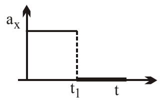
$= (8t\,t_1, 3)$ for $t > t_1$

Choose the **CORRECT** alternative.

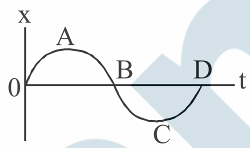
(A) Particle moves along a straight line parallel to x axis.

(B) 

(C) 

(D) 

13. A particle has a rectilinear motion and the figure gives its displacement as a function of time. Which of the following statements are true with respect to the motion



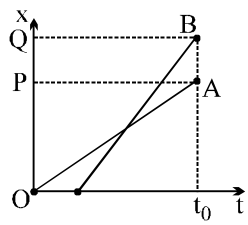
(A) in the motion between O and A the velocity is positive and acceleration is negative

(B) between A and B the velocity and acceleration are positive

(C) between B and C the velocity is negative and acceleration is positive

(D) between C and D the acceleration is positive

14. The position–time (x–t) graphs for two children A and B returning from their school O to their homes P and Q respectively along straight line path (taken as x axis) are shown in figure below.



Choose the **CORRECT** statement (s):

(A) A lives closer to the school than B

(B) A starts from the school earlier than B

(C) A and B have equal average velocities from 0 to t_0.

(D) B overtakes A on the way

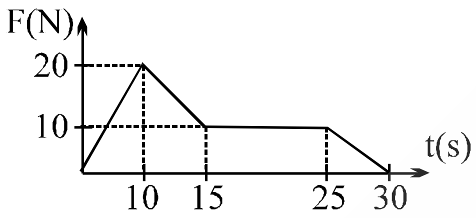
15. A ball is dropped from a building. Somewhere down it crosses a window of length 4 m in 0.5 sec. Speed of ball at top of window is v_1 and at bottom v_2, then choose the **CORRECT** option(s) ($g = 10$ m/s^2) :-

(A) $v_2 - v_1 = 5$ m/s (B) $v_2 + v_1 = 16$ m/s (C) $\frac{v_2}{v_1} = 9$ (D) $\frac{v_2}{v_1} = \frac{21}{11}$

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 16 to 19

In figure shown, the graph shows the variation of a unidirectional force F acting on a body of mass 10kg (in gravity free space), with time t. The velocity of the body at t = 0 is zero. (Area under F-t curve gives change in momentum)



16. The velocity of the body at t = 30 s is

(A) 30 m/s (B) 20 m/s (C) 40 m/s (D) none

17. The power of the force at t = 12 s is (Power = force × velocity)

(A) 225.0 W (B) 217.6 W (C) 226.7 W (D) none

18. The average acceleration of the body from t = 0 to t = 15 s is :-

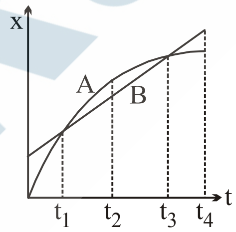
(A) 1.25 m/s^2 (B) 4/7 m/s^2 (C) 5/6 m/s^2 (D) 7/6 m/s^2

19. The change in momentum of the body between the time t = 10 s to 15 s is :-

(A) 100 kg.m/s (B) 75 kg.m/s (C) 125 kg.m/s (D) none

Paragraph for question nos. 20 to 23

The graph given shows the **POSITION** of two cars, A and B, as a function of time. The cars move along the x-axis on parallel but separate tracks, so that they can pass each other's position without colliding.



20. At which instant in time is car-A overtaking the car-B ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4

21. At time t_3, which car is moving faster?

(A) car A (B) car B (C) same speed (D) None of these

22. At which instant do the two cars have the same velocity ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4

23. Which one of the following best describes the motion of car A as shown on the graphs?

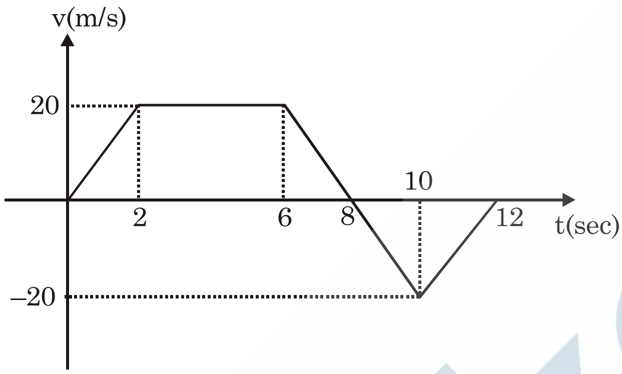
(A) speeding up (B) constant velocity

(C) slowing down (D) first speeding up, then slowing down

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT (THREE COLUMNS AND FOUR ROWS)

Answer Q.24, Q.25 and Q.26 by appropriately matching the information given in the three columns of the following table.

The velocity-time graph of an object moving along a straight line is given below.



	Column–I Time interval		Column-II Average velocity		Column-III Average acceleration
(I)	0 to 2 sec	(i)	10 m/s	(P)	zero
(II)	2 to 6 sec	(ii)	$-\frac{10}{3}\,m/s$	(Q)	$-2\ m/s^2$
(III)	0 to 10 sec	(iii)	15 m/s	(R)	$-\frac{10}{3}\,m/s^2$
(IV)	6 to 12 sec	(iv)	20 m/s	(S)	$-4\ m/s^2$

24. Which of the following combination is correctly matched :-

(A) (III) (iv) (S) (B) (III) (i) (R) (C) (III) (i) (S) (D) (III) (i) (Q)

25. Which of the following combination is correctly matched :-

(A) (II) (iv) (P) (B) (I) (i) (Q) (C) (II) (iv) (Q) (D) (I) (i) (S)

26. Which of the following combination is correctly matched :-

(A) (IV) (ii) (S) (B) (IV) (iii) (R) (C) (IV) (i) (S) (D) (IV) (ii) (R)

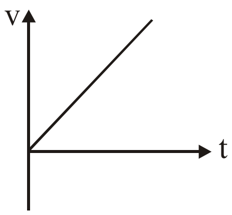
MATRIX MATCH TYPE QUESTION

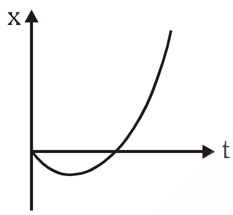
27. A balloon rises up with constant net acceleration of $10 m/s^2$. After 2 s a particle drops from the balloon. After further 2 s match the following : ($g = 10\ m/s^2$)

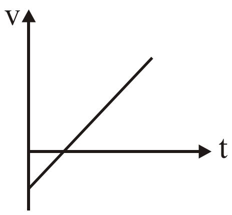
	Column-I		Column-II
(A)	Height of particle from ground	(P)	Zero
(B)	Speed of particle	(Q)	10 SI units
(C)	Displacement of Particle	(R)	40 SI units
(D)	Acceleration of particle	(S)	20 SI units

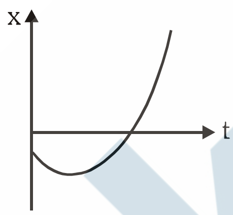
28. In the first column of the given table, some velocity-time (v-t) graphs and in the second column some position-time (x-t) graphs are shown. Sugest suitable match or matches.

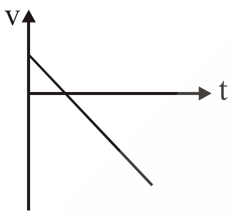
Column - I | **Column - II**

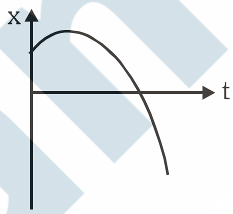
(A) 

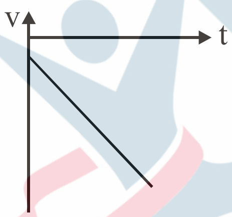
(P) 

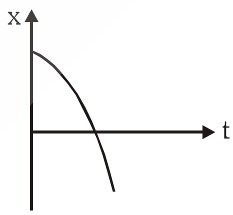
(B) 

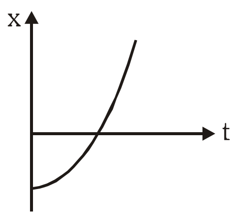
(Q) 

(C) 

(R) 

(D) 

(S) 

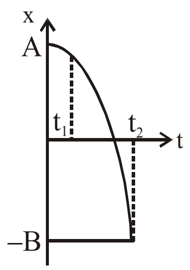
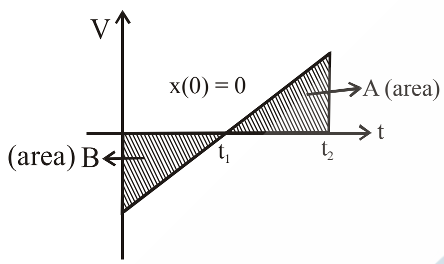
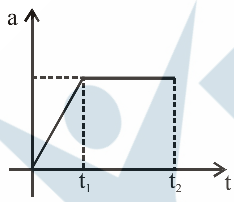
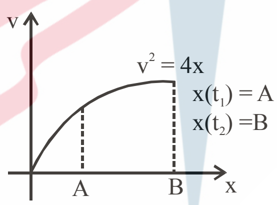
(T) 

29. Match the column :-

Column–I : Shows graph of One Dimension motion of a particle. Symbols have their usual meaning such as x(0) = initial position, $x(t_1)$ = position at $t = t_1$, v(0) = initial velocity.

Column-II : Shows physical quantities. Displacement and distance are asked for $0 < t < t_2$, and average values are asked for $0 < t < t_2$

V_0, A & B are positive constant

Column-I	Column-II
(A) 	(P) |Displacement| = Distance
(B) 	(Q) |Instantaneous velocity| = |Instantaneous speed|
(C)  $x(0) = 0$, $x(t_1) = A$, $x(t_2) = B$, $v(0) = v_0$	(R) |Average velocity| ≤ Average speed
(D) 	(S) Instantaneous acceleration = Average acceleration
	(T) Displacement = A – B and Distance = A + B

EXERCISE (JM)

1. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10s is:-

[AIEEE-2009]

(1) 7 units (2) 8.5 units (3) 10 units (4) $7\sqrt{2}$ units

2. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt}=-2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :-

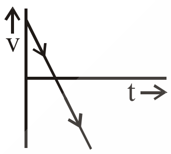
[AIEEE-2011]

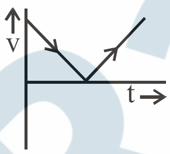
(1) 4 s (2) 8 s (3) 1 s (4) 2 s

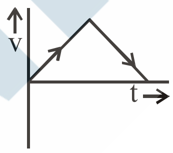
3. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is : **[JEE-Main-2014]**

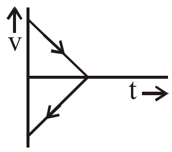
(1) $2g\,H = nu^2(n-2)$ (2) $g\,H = (n-2)u^2$ (3) $2g\,H = n^2u^2$ (4) $g\,H = (n-2)^2u^2$

4. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[JEE-Main-2017]**

(1) 

(2) 

(3) 

(4) 

EXERCISE (J-A)

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the *instantaneous* density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to : **[JEE Advanced-2017]**

(A) R^3 (B) $\frac{1}{R}$ (C) R (D) $R^{2/3}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) $2\sqrt{2}\frac{v}{\pi}$, (b) $\frac{2v}{\pi}$, (c) $\frac{2\sqrt{2}v}{3\pi}$

2. Ans.

v_0	a
—	+

Because particle is slowing down so velocity & acceleration are in opposite direction.

3. Ans. 3

4. Ans. 260 m

5. Ans. (a) 4 m/s^2, (b) 200, (c) 1000 m

6. Ans. 7

7. Ans. 18.5 m

8. Ans. Zero

9. Ans. 2 sec. after body is dropped

10. Ans. $\frac{45}{4}$ m

11. Ans. 7

12. Ans. 50

13. Ans. 30

14. Ans. 5h/3

15. Ans. 4

16. Ans. 6

17. Ans. (a) 27 m/s, (b) 108 m, (c) –100 m, (d) 316 m

18. Ans. 1 kg m/sec.

19. Ans. 186.25 N

20. Ans. 2π rad/sec.

21. Ans. 23 amp

22. Ans. 13 rad/s

23. Ans. –72, –16, 24

24. Ans. (a) 0, 8 sec (b) 0, 4, 8 sec

25. Ans. 1.815×10^5 ergs.

26. Ans. 100 kgm/s

27. Ans. (a) 7, $\frac{11}{2}$ (b) 3, –3

28. Ans. 3

29. Ans. 2

30. Ans. $\left(27\hat{i}+2\hat{k}\right) m/s^2$

31. Ans. 3

32. Ans. 3

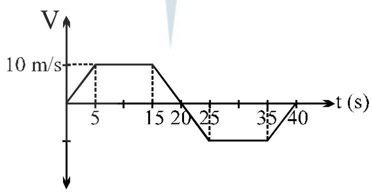
33. Ans. (a) 3 m; (b) –3 m/s^2; (c) 1m/s; (d) 3/2 m/s

34. Ans. (i) section (a) as slope = v = $\frac{dx}{dt}$ is positive and increasing.

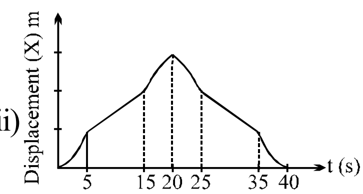
(ii) section (d) as slope = v = $\frac{dx}{dt}$ is negative and constant.

35. Ans. (a) – 2 ms^{-1} (b) 5 m/s

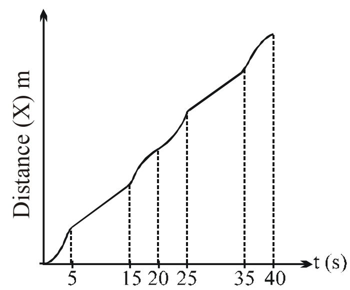
36. Ans. (i)



(ii)



(iii)



EXERCISE (S-2)

1. Ans. 25m **2. Ans.** 42km/hr **3. Ans.** 5m

4. Ans. (a) $\sqrt{60}\text{m}/\text{sec}$ (b) $\sqrt{\frac{3}{5}}\text{sec}$ (c) $\frac{3}{\sqrt{60}}\text{sec}$ (d) $\frac{9}{4}\text{m}$ **5. Ans.** 12 km

6. Ans. 80 m/sec **7. Ans.** (i) $t=\frac{\sqrt{x^2+(9)^2}}{4}+\frac{15-x}{5}$ (ii) 3 km from the camp.

8. Ans. 10 sec ; 700m **9. Ans.** 2 **10. Ans.** 012 **11. Ans.** 7

EXERCISE (O-1)

1. Ans. (B) **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (C)** **5. Ans. (A)** **6. Ans. (A)**
7. Ans. (C) **8. Ans. (B)** **9. Ans. (D)** **10. Ans. (B)** **11. Ans. (B)** **12. Ans. (A)**
13. Ans. (B) **14. Ans. (D)** **15. Ans. (B)** **16. Ans. (D)** **17. Ans. (C)** **18. Ans. (A)**
19. Ans. (D) **20. Ans. (B)** **21. Ans. (C)** **22. Ans. (C)** **23. Ans. (C)** **24. Ans. (D)**
25. Ans. (D) **26. Ans. (C)** **27. Ans. (D)** **28. Ans. (D)** **29. Ans. (A)** **30. Ans. (B)**
31. Ans. (C) **32. Ans. (C)** **33. Ans. (B)** **34. Ans. (D)**
35. Ans. (A) → (R,T) ; (B) → (T) ; (C) → (R, T) ; (D) → (S, T)

EXERCISE (O-2)

1. Ans. (B) **2. Ans. (A)** **3. Ans. (C)** **4. Ans. (D)** **5. Ans. (D)** **6. Ans. (A)**
7. Ans. (B) **8. Ans. (D)** **9. Ans. (A)** **10. Ans. (B,C,D)** **11. Ans. (C,D)**
12. Ans. (A,D) **13. Ans. (A,C,D)** **14. Ans. (A,B,D)**
15. Ans. (A, B, D) **16. Ans. (A)** **17. Ans. (B)** **18. Ans. (D)** **19. Ans. (B)**
20. Ans. (A) **21. Ans. (B)** **22. Ans. (B)** **23. Ans. (C)** **24. Ans. (D)** **25. Ans. (A)**
26. Ans. (D) **27. Ans. (A) - (R); (B) - (P) ; (C) - (S) ; (D) - (Q)**
28. Ans. (A) → (T); (B) → (P,Q); (C) → (R); (D) → (S)
29. Ans. (A) → (P,Q,R) ; (B) → (Q,R,S,T) ; (C) → (P,Q,R) ; (D) → (P,Q,R,S)

EXERCISE (JM)

1. Ans. (4) **2. Ans. (4)** **3. Ans. (1)** **4. Ans. (1)**

EXERCISE (J-A)

1. Ans. (C)