

Indefinite and Definite Integration

Exercise-1: Single Choice Problems

1. $\int a^x \left(\ln x + \ln a \cdot \ln \left(\frac{x}{e} \right)^x \right) dx =$

- (a) $a^x \ln \left(\frac{e}{x} \right)^{2x} + C$ (b) $a^x \ln \left(\frac{x}{e} \right)^x + C$
(c) $a^x \ln \left(\frac{x}{e} \right)^x + C$ (d) None of these

2. The value of:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \frac{1}{\sqrt{n}\sqrt{n+3}} + \dots + \frac{1}{\sqrt{n}\sqrt{2n}} \right) \text{ is:}$$

- (a) $\sqrt{2} - 1$ (b) $2(\sqrt{2} - 1)$
(c) $\sqrt{2} + 1$ (d) $2(\sqrt{2} + 1)$

3. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is :

- (a) $(\sin \alpha, \cos \alpha)$ (b) $(\cos \alpha, \sin \alpha)$
(c) $(-\sin \alpha, \cos \alpha)$ (d) $(-\cos \alpha, \sin \alpha)$

4. The value of the integral $\int_0^2 \frac{\log(x^2+2)}{(x+2)^2} dx$ is :

- (a) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} + \frac{5}{12} \log 2 - \frac{1}{4} \log 3$
(b) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} - \frac{5}{12} \log 2 - \frac{1}{12} \log 3$
(c) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} + \frac{5}{12} \log 2 + \frac{1}{12} \log 3$
(d) $\frac{\sqrt{2}}{3} \tan^{-1} \sqrt{2} \frac{5}{12} \log 2 + \frac{1}{12} \log 3$

5. If $I_1 = \int_0^1 \frac{1+x^8}{1+x^4} dx$ and $I_2 = \int_0^1 \frac{1+x^9}{1+x^3} dx$, then

- (a) $I_1 > 1, I_2 < 1$ (b) $I_1 < 1, I_2 > 1$
(c) $1 < I_1 < I_2$ (d) $I_2 < I_1 < 1$

6. Let $f: (0, 1) \rightarrow (0, 1)$ be a differentiable function such that $f'(x) \neq 0$ for all $x \in (0, 1)$ and $f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$. Suppose for all x , $\lim_{t \rightarrow x} \left(\frac{\int_0^t \sqrt{1-(f(s))^2} ds - \int_0^x \sqrt{1-(f(s))^2} dx}{f(t)-f(x)} \right) = f(x)$. Then the value of $f\left(\frac{1}{4}\right)$ belongs to:
- (a) $\left\{ \frac{\sqrt{7}}{4}, \frac{\sqrt{15}}{4} \right\}$ (b) $\left\{ \frac{\sqrt{7}}{3}, \frac{\sqrt{15}}{3} \right\}$
 (c) $\left\{ \frac{\sqrt{7}}{2}, \frac{\sqrt{15}}{2} \right\}$ (d) $\left\{ \sqrt{7}, \sqrt{15} \right\}$
7. If $f(\theta) = \frac{4}{3}(1 - \cos^6 \theta - \sin^6 \theta)$, then
- $$\lim_{n \rightarrow \infty} \frac{1}{n} \left[\sqrt{f\left(\frac{1}{n}\right)} + \sqrt{f\left(\frac{2}{n}\right)} + \sqrt{f\left(\frac{3}{n}\right)} + \dots + \sqrt{f\left(\frac{n}{n}\right)} \right] =$$
- (a) $\frac{1-\cos 1}{2}$ (b) $1 - \cos 2$
 (c) $\frac{\sin 2}{2}$ (d) $\frac{1-\cos 2}{2}$
8. The value of $\int_0^1 \frac{(x^6-x^3)}{(2x^3+1)^3} dx$ is equal to:
- (a) $-\frac{1}{6}$ (b) $-\frac{1}{12}$
 (c) $-\frac{1}{18}$ (d) $-\frac{1}{36}$
9. $2 \int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{x} dx - \int_0^1 \frac{\tan^{-1} x}{x} dx =$
- (a) $\frac{\pi}{8} \ln 2$ (b) $\frac{\pi}{4} \ln 2$
 (c) $\frac{\pi}{2\sqrt{2}} \ln 2$ (d) $\frac{\pi}{2} \ln 2$

10. Let $f(x)$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t}f(x-t)dt$, then

$$\int_0^1 f(x)dx =$$

- (a) $\frac{1}{3}$ (b) $\frac{1}{4}$
 (c) $\frac{7}{12}$ (d) $\frac{5}{12}$

11. If $f'(x) = f(x) + \int_0^1 f(x)dx$ and given $f(0) = 1$, then $\int f(x)dx$ is equal to:

- (a) $\frac{2}{3-e} e^x + \left(\frac{3-e}{1-e}\right)x + C$ (b) $\frac{2}{3-e} e^x + \left(\frac{1-e}{3-e}\right)x + C$
 (c) $\frac{2}{2-e} e^x + \left(\frac{1+e}{3+e}\right)x + C$ (d) $\frac{2}{2-e} e^x + \left(\frac{1-e}{3+e}\right)x + C$

12. For any $x \in \mathbb{R}$, and f be a continuous function. Let $I_1 = \int_{\sin^2 x}^{1+\cos^2 x} tf(t(2-t))dt$,

$$I_1 = \int_{\sin^2 x}^{1+\cos^2 x} f(t(2-t)) dt, \text{ then } I_1 =$$

13. If the integral $\int \frac{5 \tan x \, dx}{\tan x - 2} = x + a \ln |\sin x - 2 \cos x| + C$, then 'a' is equal to :

14. $\int \frac{(2+\sqrt{x})dx}{(x+1+\sqrt{x})^2}$ is equal to :

- (a) $\frac{x}{x+\sqrt{x+1}} + C$ (b) $\frac{2x}{x+\sqrt{x+1}} + C$
 (c) $\frac{-2x}{x+\sqrt{x+1}} + C$ (d) $\frac{-x}{x+\sqrt{x+1}} + C$

15. Evaluate $\int \frac{(\sqrt[3]{x+\sqrt{2-x^2}})(\sqrt[6]{1-\sqrt{2-x^2}})}{\sqrt[3]{1-x^2}} dx$; $x \in (0, 1)$:

- (a) $2^{\frac{1}{6}}x + C$ (b) $2^{\frac{1}{12}}x + C$
(c) $2^{\frac{1}{3}}x + C$ (d) None of these

16. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}} = \frac{1}{\lambda} \sin^{-1}(\lambda \sin x) + C$, then $\lambda =$

- (a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 2 (d) $\sqrt{5}$

17. $\int \frac{dx}{\sqrt[3]{x^{5/2}(x+1)^{7/2}}}$ is equal to:

- (a) $-\left(\frac{x+1}{x}\right)^{\frac{1}{6}} + C$ (b) $6\left(\frac{x+1}{x}\right)^{-\frac{1}{6}} + C$
(c) $\left(\frac{x}{x+1}\right)^{\frac{5}{6}} + C$ (d) $-\left(\frac{x}{x+1}\right)^{\frac{5}{6}} + C$

18. If $I_n = \int (\sin x)^n dx$; $n \in N$, then $5I_4 - 6I_6$ is equal to :

- (a) $\sin x \cdot (\cos x)^5 + C$
(b) $\sin 2x \cos 2x + C$
(c) $\frac{\sin 2x}{8} [1 + \cos^2 2x - 2 \cos 2x] + C$
(d) $\frac{\sin 2x}{8} [1 + \cos^2 2x + 2 \cos 2x] + C$

19. $\int \frac{x^2}{(a+bx)^2} dx$ equals to :

- (a) $\frac{1}{b^3} \left(a + bx - a \ln|a+bx| - \frac{a^2}{a+bx} \right) + C$
(b) $\frac{1}{b^3} \left(a + bx - 2a \ln|a+bx| - \frac{a^2}{a+bx} \right) + C$
(c) $\frac{1}{b^3} \left(a + bx + 2a \ln|a+bx| - \frac{a^2}{a+bx} \right) + C$
(d) $\frac{1}{b^3} \left(a + bx - 2a \ln|a+ax| - \frac{a^2}{a+bx} \right) + C$

20. $\int \frac{8x^{43} + 13x^{38}}{(x^{13} + x^5 + 1)^4} dx =$

(a) $\frac{x^{39}}{3(x^{13} + x^5 + 1)^3} + C$ (b) $\frac{x^{39}}{(x^{13} + x^5 + 1)^3} + C$

(c) $\frac{x^{39}}{5(x^{13} + x^5 + 1)^5} + C$ (d) None of these

21. $\int \left(\frac{\cos 6x + 6 \cos 4x + 15 \cos 2x + 10}{10 \cos^2 x + 5 \cos x \cos 3x + \cos x \cos 5x} \right) dx = f(x) + C$, then $f(10)$ is equal to :

- (a) 20 (b) 10
 (c) $2 \sin 10$ (d) $2 \cos 10$

22. $\int (1 + x - x^{-1}) e^{x+x^{-1}} dx =$

- (a) $(x + 1)e^{x+x^{-1}} + C$ (b) $(x - 1)e^{x+x^{-1}} + C$
 (c) $-e^{x+x^{-1}} + C$ (d) $xe^{x+x^{-1}} + C$

23. If $\int e^x \left(\frac{2 \tan x}{1 + \tan x} + \operatorname{cosec}^2 \left(x + \frac{\pi}{4} \right) \right) dx = e^x \cdot g(x) + K$, then $g\left(\frac{5\pi}{4}\right) =$

- (a) 0 (b) 1
 (c) -1 (d) 2

24. $\int e^{x \sin x + \cos x} \left(\frac{x^4 \cos^3 x - x \sin x + \cos x}{x^2 \cos^2 x} \right) dx =$

- (a) $e^{x \sin x + \cos x} \left(x - \frac{1}{\cos x} \right) + C$ (b) $e^{x \sin x + \cos x} \left(x - \frac{1}{x \cos x} \right) + C$
 (c) $e^{x \sin x + \cos x} \left(1 - \frac{1}{x \cos x} \right) + C$ (d) $e^{x \sin x + \cos x} \left(1 - \frac{1}{\cos x} \right) + C$

25. The value of the definite integral $\int_0^1 \frac{1+x+\sqrt{x+x^2} dx}{\sqrt{x}+\sqrt{1+x}} dx$ is :

- (a) $\frac{1}{3}(2^{1/2} - 1)$ (b) $\frac{2}{3}(2^{1/2} - 1)$
 (c) $\frac{2}{3}(2^{3/2} - 1)$ (d) $\frac{1}{3}(2^{3/2} - 1)$

26. $\int x^{x^2+1} (2 \ln x + 1) dx$

- (a) $x^{2x} + C$ (b) $x^2 \ln x + C$
 (c) $x^{(x^x)} + C$ (d) $(x^x)^x + C$

27. If $\int \frac{\cosec^2 x - 2010}{\cos^{2010} x} dx = -\frac{f(x)}{(g(x))^{2010}} + C$; where $f\left(\frac{\pi}{4}\right) = 1$; then the number of solutions of the equation $\frac{f(x)}{g(x)} = \{x\}$ in $[0, 2\pi]$ is/are : (where $\{.\}$ represents fractional part function)

- (a) 0 (b) 1
- (c) 2 (d) 3

28. $\int x^x \left((\ln x)^2 + \ln x + \frac{1}{x} \right) dx$ is equal to :

- (a) $x^x \left((\ln x)^2 - \frac{1}{x} \right) + C$ (b) $x^x (\ln x - x) + C$
- (c) $x^x \frac{(\ln x)^2}{2} + C$ (d) $x^x \ln x + C$

29. $I = \int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}}$ is equal to :

- (a) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + C$ (b) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$
- (c) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + C$ (d) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$

30. $I = \int \left(\frac{\ln x - 1}{(\ln x)^2 + 1} \right)^2 dx$ is equal to :

- (a) $\frac{x}{x^2 + 1} + C$ (b) $\frac{\ln x}{(\ln x)^2 + 1} + C$
- (c) $\frac{x}{1 + (\ln x)^2} + C$ (d) $e^x \left(\frac{x}{x^2 + 1} \right) + C$

31. $I = \int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}} = k \sqrt[4]{\frac{x-1}{x+2}} + C$, then 'k' is equal to :

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
- (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

32. $\int \frac{1-x^7}{x(1+x^7)} dx = P \log|x| + Q \log|x^7 + 1| + C$, then :

- (a) $2P - 7Q = 0$ (b) $2P + 7Q = 0$
- (c) $7P + 2Q = 0$ (d) $7P - 2Q = 1$

33. $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$ is equal to :

- | | |
|------------------------------|-----------------------------|
| (a) $\sin 2x + C$ | (b) $\frac{\sin 2x}{2} + C$ |
| (c) $\frac{-\sin 2x}{2} + C$ | (d) $-2 \sin 2x + C$ |

34. $I = \int \frac{(\sin 2x)^{1/3} d(\tan^{1/3} x)}{\sin^{2/3} x + \cos^{2/3} x} =$

- | | |
|---|---|
| (a) $\frac{1}{2^{2/3}} \ln(1 + \tan^{1/3} x) + C$ | (b) $\ln(1 + \tan^{2/3} x) + C$ |
| (c) $2^{1/3} \ln(1 + \tan^{2/3} x) + C$ | (d) $\frac{1}{2^{2/3}} \ln(1 + \tan^{2/3} x) + C$ |

35. $\int \sqrt{\frac{(2012)^{2x}}{1 - (2012)^{2x}}} (2012)^{\sin^{-1}(2012)x} dx =$

- | |
|---|
| (a) $(\log_{2012} e)^2 (2012)^{\sin^{-1}(2012)x} + C$ |
| (b) $(\log_{2012} e)^2 (2012)^{x+\sin^{-1}(2012)x} + C$ |
| (c) $(\log_{2012} e)^2 (2012)^{\sin^{-1}(2012)x} + C$ |
| (d) $\frac{(2012)^{\sin^{-1}(2012)x}}{(\log_{2012} e)^2}$ |

(where C denotes arbitrary constant.)

36. $\int \frac{(x+2)dx}{(x^2+3x+3)\sqrt{x+1}}$ is equal to :

- | | |
|--|---|
| (a) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3(x+1)}} \right) + C$ | (b) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\sqrt{\frac{x}{3(x+1)}} \right) + C$ |
| (c) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\sqrt{x}}{\sqrt{3(x+1)}} \right) + C$ | (d) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3(x+1)}} \right) + C$ |

(where C is arbitrary constant.)

37. $\int \left(\frac{f(x)g'(x) - f'(x)g(x)}{f(x)g(x)} \right) (\log(g(x)) - \log(f(x))) dx$ is equal to :

- | | |
|--|--|
| (a) $\log \left(\frac{g(x)}{f(x)} \right) + C$ | (b) $\frac{1}{2} \left(\frac{g(x)}{f(x)} \right)^2 + C$ |
| (c) $\frac{1}{2} \left(\log \left(\frac{g(x)}{f(x)} \right) \right)^2 + C$ | (d) $\log \left(\left(\frac{g(x)}{f(x)} \right)^2 \right) + C$ |

38. $\int \left(\int e^x \left(\ln x + \frac{2}{x} - \frac{1}{x^2} \right) dx \right) dx =$
- (a) $e^x \ln x + C_1 x + C_2$ (b) $e^x \ln x + \frac{1}{x} + C_1 x + C_2$
 (c) $\frac{\ln x}{x} + C_1 x + C_2$ (d) None of these
39. Maximum value of the function $f(x) = \pi^2 \int_0^1 t \sin(x + \pi t) dt$ over all real number $x :$
- (a) $\sqrt{\pi^2 + 1}$ (b) $\sqrt{\pi^2 + 2}$
 (c) $\sqrt{\pi^2 + 3}$ (d) $\sqrt{\pi^2 + 4}$
40. Let 'f' is a function, continuous on $[0, 1]$ such that $f(x) \leq \sqrt{5} \quad \forall x \in [0, 1]$ and $f(x) \leq \frac{2}{x} \quad \forall x \in \left[\frac{1}{2}, 1\right]$ then the smallest 'a' for which $\int_0^1 f(x) dx \leq a$ holds for all 'f' is :
- (a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{2} + 2 \ln 2$
 (c) $2 + \ln\left(\frac{\sqrt{5}}{2}\right)$ (d) $2 + 2 \ln\left(\frac{\sqrt{5}}{2}\right)$
41. Let $I_n = \int_1^{e^2} (\ln x)^n d(x^2)$, then the value of $2I_n + nI_{n-1}$ equals to :
- (a) 0 (b) $2e^2$
 (c) e^2 (d) 1
42. Let a function $f: R \rightarrow R$ be defined as $f(x) = x + \sin x$. The value of $\int_0^{2\pi} f^{-1}(x) dx$ will be:
- (a) $2\pi^2$ (b) $2\pi^2 - 2$
 (c) $2\pi^2 + 2$ (d) π^2
43. The value of the definite integral $\int_{-1}^1 e^{-x^4} (2 + \ln(x + \sqrt{x^2 + 1}) + 5x^3 - 8x^4) dx$ is equal to :
- (a) $4e$ (b) $\frac{4}{e}$
 (c) $2e$ (d) $\frac{2}{e}$

44. $\int_{-10}^0 \frac{\left| \frac{2[x]}{3x-[x]} \right|}{\frac{2[x]}{3x-[x]}} dx$ is equal to (where $[*]$ denotes greatest integer function.)

- (a) $\frac{28}{3}$
- (b) $\frac{1}{3}$
- (c) 0
- (d) None of these

45. If $f(x) = \frac{x}{1+(\ln x)(\ln x)\dots\infty} \forall x \in [1, \infty)$ then $\int_1^{2e} f(x)dx$ equals to :

- (a) $\frac{e^2-1}{2}$
- (b) $\frac{e^2+1}{2}$
- (c) 0
- (d) None of these

46. $\int_0^4 \frac{(y^2-4y+5)\sin(y-2)}{(2y^2-8y+11)} dy$ is equal to :

- (a) 0
- (b) 2
- (c) -2
- (d) None of these

47. Let $\frac{d}{dx} F(x) = \left(\frac{e^{\sin x}}{x} \right), x > 0$. If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(1)$, then one of the possible values of k, is :

- (a) 15
- (b) 16
- (c) 63
- (d) 64

48. Value of $\lim_{h \rightarrow 0} \frac{0}{he^{-1/h}}$ is equal to :

- (a) $\pi(1 - \pi^2)e^{-\pi^2}$
- (b) $2\pi(1 - \pi^2)e^{-\pi^2}$
- (c) $\pi(1 - \pi)e^{-\pi}$
- (d) $\pi^2 e^{-\pi^2}$

49. Let $f: R^+ \rightarrow R$ be a differentiable function with $f(1) = 3$ and satisfying :

$$\int_1^{xy} f(t)dt = y \int_1^x f(t)dt + x \int_1^y f(t)dt \quad \forall x, y \in R^+, \text{ then } f(e) =$$

- (a) 3
- (b) 4
- (c) 1
- (d) None of these

50. If $[.]$ denotes the greatest integer function, then the integral

$$\int_0^{\pi/2} \frac{e^{\sin x - [\sin x]} d(\sin^2 x - [\sin^2 x])}{\sin x - [\sin x]} \text{ is } \lambda, \text{ then } [\lambda - 1] \text{ is equal to :}$$

- (a) 0
- (b) 1
- (c) 2
- (d) 3

51. Calculate the reciprocal of the limit $\lim_{x \rightarrow \infty} \int_0^x xe^{t^2-x^2} dt$

52. Let $L = \lim_{n \rightarrow \infty} \left(\frac{(2.1+n)}{1^2+n.1+n^2} + \frac{(2.2+n)}{2^2+n.2+n^2} + \frac{(2.3+n)}{3^2+n.3+n^2} + \dots + \frac{(2.n+n)}{3n^2} \right)$ then value of e^L is:

53. The value of the definite integral $\int_0^2 (\sqrt{1+x^3} + \sqrt[3]{x^2+2x}) dx$ is:

- (a) 4
 - (b) 5
 - (c) 6
 - (d) 7

54. The value of the definite integral $\int_0^{\infty} \frac{\ln x}{x^2+4} dx$ is:

- (a) $\frac{\pi \ln 3}{2}$ (b) $\frac{\pi \ln 2}{3}$
 (c) $\frac{\pi \ln 2}{4}$ (d) $\frac{\pi \ln 4}{3}$

55. The value of the definite integral $\int_0^{10} ((x - 5) + (x - 5)^2 + (x - 5)^3) dx$ is :

- (a) $\frac{125}{3}$ (b) $\frac{250}{3}$
 (c) $\frac{125}{6}$ (d) $\frac{250}{4}$

56. The value of definite integral $\int_0^{\infty} \frac{dx}{(1+x^9)(1+x^2)}$ equals to :

- (a) $\frac{\pi}{16}$ (b) $\frac{\pi}{8}$
 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

57. The value of the definite integral $\int_0^{\pi/2} \left(\frac{1+\sin 3x}{1+2\sin x} \right) dx$ equals to :

- (a) $\frac{\pi}{2}$ (b) 1
 (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$

58. The value of $\lim_{x \rightarrow \infty} \frac{\int_0^x (\tan^{-1}x)^2 dx}{\sqrt{x^2+1}} =$
- (a) $\frac{\pi^2}{16}$
 - (b) $\frac{\pi^2}{4}$
 - (c) $\frac{\pi^2}{2}$
 - (d) None of these

59. If $\int_0^1 \left(\sum_{r=1}^{2013} \frac{x}{x^2+r^2} \right) (\prod_{r=1}^{2013} (x^2 + r^2)) dx = \frac{1}{2} [(\prod_{r=1}^{2013} (1 + r^2)) - k^2]$ then $k =$
- (a) 2013
 - (b) $2013!$
 - (c) 2013^{2013}
 - (d) 2013^{2013}

60. $f(x) = 2x - \tan^{-1}x - \ln(x + \sqrt{1 + x^2})$
- (a) strictly increases $\forall x \in R$
 - (b) strictly increases only in $(0, \infty)$
 - (c) strictly decreases $\forall x \in R$
 - (d) strictly decreases in $(0, \infty)$ and strictly increases in $(-\infty, 0)$

61. The value of the definite integral $\int_0^{\pi/2} \frac{dx}{\tan x + \cot x + \operatorname{cosec} x + \sec x}$ is :
- (a) $1 - \frac{\pi}{4}$
 - (b) $\frac{\pi}{4} + 1$
 - (c) $\pi + \frac{1}{4}$
 - (d) None of these

62. The value of the definite integral $\int_3^7 \frac{\cos x^2}{\cos x^2 + \cos(10-x)^2} dx$ is :
- (a) $\frac{3}{2}$
 - (b) $\frac{5}{2}$
 - (c) $\frac{1}{2}$
 - (d) None of these

63. The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\ln x}{x} \right| dx$ is :
- (a) $\frac{3}{2}$
 - (b) $\frac{5}{2}$
 - (c) 3
 - (d) 5

64. The value of $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\int_2^{\cosec^2 x} \tg(t) dt}{x^2 - \frac{\pi^2}{16}}$ is :

- (a) $\frac{2}{\pi} g(2)$
 (c) $-\frac{16}{\pi} g(2)$

- (b) $-\frac{4}{\pi} g(2)$
 (d) $-4 g(2)$

65. The value of $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n-k}{n^2} \cos \frac{4k}{n}$ equals :

- (a) $\frac{1}{4} \sin 4 + \frac{1}{16} \cos 4 - \frac{1}{16}$
 (c) $\frac{1}{16} (1 - \sin 4)$

- (b) $\frac{1}{4} \sin 4 + \frac{1}{16} \cos 4 + \frac{1}{16}$
 (d) $\frac{1}{16} (1 - \cos 4)$

66. For each positive integer n , define a function f_n on $[0, 1]$ as follows:

$$f_n(x) = \begin{cases} 0 & \text{if } x = 0 \\ \sin \frac{\pi}{2n} & \text{if } 0 < x \leq \frac{1}{n} \\ \sin \frac{\pi}{2n} & \text{if } \frac{1}{n} < x \leq \frac{2}{n} \\ \sin \frac{3\pi}{2n} & \text{if } \frac{2}{n} < x \leq \frac{3}{n} \\ \sin \frac{3\pi}{2n} & \text{if } \frac{3}{n} < x \leq \frac{4}{n} \end{cases}$$

Then the value of $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx$ is :

- (a) π
 (c) $\frac{1}{\pi}$
- (b) $\frac{\pi}{2}$
 (d) $\frac{2}{\pi}$

67. Let n be a positive integer, then

$\int_0^{n+1} \min\{|x-1|, |x-2|, |x-3|, \dots, |x-n|\} dx$ equals

- (a) $\frac{(n+1)}{4}$
 (c) $\frac{(n+3)}{4}$
- (b) $\frac{(n+2)}{4}$
 (d) $\frac{(n+4)}{4}$

68. For positive integers $k = 1, 2, 3, \dots, n$, let S_k denotes the area of ΔAOB_k (where 'O' is origin) such that $\angle AOB_k = \frac{k\pi}{2n}$, $OA = 1$ and $OB_k = k$. The value of the $\lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{k=1}^n S_k$ is :

- (a) $\frac{2}{\pi^2}$ (b) $\frac{4}{\pi^2}$
 (c) $\frac{8}{\pi^2}$ (d) $\frac{1}{2\pi^2}$

69. If $A = \int_0^1 \prod_{r=1}^{2014} (r - x) dx$ and $B = \int_0^1 \prod_{r=0}^{2013} (r + x) dx$ then :

- (a) $A = 2B$ (b) $2A = B$
(c) $A + B = 0$ (d) $A = B$

70. If $f(x) = \left[\frac{x}{120} + \frac{x^3}{30} \right]$ defined in $[0, 3]$, then $\int_0^1 (f(x) + 2) dx =$

(where $[.]$ denotes greatest integer function)

71. If $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$, $g(x) = \int_0^{\cos x} (1 + \sin t)^2 dt$, then the value of $f' \left(\frac{\pi}{2} \right)$ is equal to :

72. Let $f(x) = \frac{1}{x^2} \int_0^x (4t^2 - 2f'(t))dt$, find $9f'(4)$

73. Evaluate $\lim_{n \rightarrow \infty} \left(\frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \frac{3^2}{n^3+3^3} + \dots + \frac{4}{9n} \right)$

- (a) $\frac{\ln 3}{3}$ (b) $\frac{\ln 9}{3}$
 (c) $\frac{\ln 4}{3}$ (d) $\frac{\ln 6}{3}$

74. The value of $\int_0^{2\pi} \cos^{-1} \left(\frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}} \right) dx$ is :

- (a) π^2
- (b) $\frac{\pi^2}{2}$
- (c) $2\pi^2$
- (d) π^3

75. Given a function 'g' continuous everywhere such that $\int_0^1 g(t)dt = 2$ and $g(1) = 5$.

If $f(x) = \frac{1}{2} \int_0^x (x-t)^2 g(t)dt$, then the value of $f'(1)-f''(1)$ is :

- (a) 0
- (b) 1
- (c) 2
- (d) 3

76. If $\int_0^{\pi} \frac{x^3 \cos^4 x \sin^2 x}{\pi^2 - 3\pi x + 3x^2} dx = \lambda \int_0^{\pi/2} \sin^2 x dx$, then the value of λ is :

- (a) $\frac{\pi}{12}$
- (b) $\frac{\pi}{8}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{3}$

77. $\int_0^{\sqrt{3}} \left(\frac{1}{2} \frac{d}{dx} \left(\tan^{-1} \frac{2x}{1-x^2} \right) \right) dx$ equals to :

- (a) $\frac{\pi}{3}$
- (b) $-\frac{\pi}{6}$
- (c) $\frac{\pi}{2}$
- (d) None of these

78. Let $y = \{x\}^{[x]}$ then the value of $\int_0^3 y dx$ equals to :

(where $\{.\}$ and $[.]$ denote fractional part and greatest integer function respectively.)

- (a) 1
- (b) $\frac{11}{6}$
- (c) 3
- (d) $\frac{5}{6}$

79. $\int_0^1 \frac{\tan^{-1} x}{x} dx =$

- (a) $\int_0^{\pi/4} \frac{\sin x}{x} dx$
- (b) $\int_0^{\pi/2} \frac{x}{\sin x} dx$
- (c) $\frac{1}{2} \int_0^{\pi/2} \frac{x}{\sin x} dx$
- (d) $\frac{1}{2} \int_0^{\pi/4} \frac{x}{\sin x} dx$

80. The value of $\int_0^{4/\pi} \left(3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x} \right) dx$ is :

- | | |
|--------------------------------|--------------------------------|
| (a) $\frac{8\sqrt{2}}{\pi^3}$ | (b) $\frac{24\sqrt{2}}{\pi^3}$ |
| (c) $\frac{32\sqrt{2}}{\pi^3}$ | (d) None of these |

81. The number of values of x satisfying the equation :

$$\int_{-1}^x \left(8t^2 + \frac{28t}{3} + 4 \right) dt = \frac{\frac{3}{2}x+1}{\log(x+1)\sqrt{x+1}}, \text{ is :}$$

- | | |
|-------|-------|
| (a) 0 | (b) 1 |
| (c) 2 | (d) 3 |

82. $\lim_{n \rightarrow \infty} \frac{1+2^4+3^4+\dots+n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5}$ is:

- | | |
|--------------------|-------------------|
| (a) $\frac{1}{30}$ | (b) zero |
| (c) $\frac{1}{4}$ | (d) $\frac{1}{5}$ |

83. The value of $\lim_{x \rightarrow 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t) dt}{2x - \sin 2x}$ is equal to :

- | | |
|-------------------|--------------------|
| (a) 0 | (b) -1 |
| (c) $\frac{2}{3}$ | (d) $-\frac{1}{4}$ |

84. Consider a parabola $y = \frac{x^2}{4}$ and the point $F(0, 1)$.

Let $A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3), \dots, A_n(x_n, y_n)$ are 'n' points on the parabola such $x_k > 0$ and $\angle OFA_k = \frac{k\pi}{2n}$ ($k = 1, 2, 3, \dots, n$). Then the value of

$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n FA_k$, is equal to :

- | | |
|---------------------|---------------------|
| (a) $\frac{2}{\pi}$ | (b) $\frac{4}{\pi}$ |
| (c) $\frac{8}{\pi}$ | (d) None of these |

85. The minimum value of $f(x) = \int_0^4 e^{|x-t|} dt$ where $x \in [0, 3]$ is :

- | | |
|------------------|---------------|
| (a) $2e^2 - 1$ | (b) $e^4 - 1$ |
| (c) $2(e^2 - 1)$ | (d) $e^2 - 1$ |

86. If $\int_0^\infty \frac{\cos x}{x} dx = \frac{\pi}{2}$, then $\int_0^\infty \frac{\cos^3 x}{x} dx$ is equals to :

- | | |
|---------------------|----------------------|
| (a) $\frac{\pi}{2}$ | (b) $\frac{\pi}{4}$ |
| (c) π | (d) $\frac{3\pi}{2}$ |

87. $\int \sqrt{1 + \sin x} \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right) dx =$:

- | | |
|---------------------------------------|--------------------------|
| (a) $\frac{1 + \sin x}{2} + C$ | (b) $(1 + \sin x)^2 + C$ |
| (c) $\frac{1}{\sqrt{1 + \sin x}} + C$ | (d) $\sin x + C$ |

88. If $I_n = \int_0^\pi \frac{\sin(2nx)}{\sin 2x} dx$, then the value of $I_{n+\frac{1}{2}}$ is equal to ($n \in I$):

- | | |
|----------------------|-----------|
| (a) $\frac{n\pi}{2}$ | (b) π |
| (c) $\frac{\pi}{2}$ | (d) 0 |

89. The value of function $f(x) = 1 + x + \int_1^x (\ln^2 t + 2 \ln t) dt$ where $f'(x)$ vanishes is:

- | | |
|-------------------|-----------------------|
| (a) $\frac{1}{e}$ | (b) 0 |
| (c) $\frac{2}{e}$ | (d) $1 + \frac{2}{e}$ |

90. Let f be a differentiable function on \mathbb{R} and satisfies $f(x) = x^2 + \int_0^x e^{-t} f(x-t) dt$;

then $\int_0^1 f(x) dx$ is equal to :

- | | |
|--------------------|--------------------|
| (a) $\frac{1}{3}$ | (b) $\frac{1}{4}$ |
| (c) $\frac{7}{12}$ | (d) $\frac{5}{12}$ |

91. The value of the definite integral $\int_{-(\pi/2)}^{\pi/2} \frac{\cos^2 x}{1+5^x} dx$ equals to :

- | | |
|----------------------|---------------------|
| (a) $\frac{3\pi}{2}$ | (b) π |
| (c) $\frac{\pi}{2}$ | (d) $\frac{\pi}{4}$ |

92. $\int \left(\frac{x^2 - x + 1}{x^2 + 1} \right) e^{\cot^{-1}(x)} dx = f(x) \cdot e^{\cot^{-1}(x)} + C$

Where C is constant of integration. Then $f(x)$ is equal to :

- | | |
|----------|------------------|
| (a) $-x$ | (b) $\sqrt{1-x}$ |
| (c) x | (d) $\sqrt{1+x}$ |

93. $\lim_{n \rightarrow \infty} \frac{1}{n^3} (\sqrt{n^2 + 1} + 2\sqrt{n^2 + 2^2} + \dots + n\sqrt{n^2 + n^2}) = :$

- (a) $\frac{3\sqrt{2}-1}{2}$
- (b) $\frac{2\sqrt{2}-1}{3}$
- (c) $\frac{3\sqrt{3}-1}{3}$
- (d) $\frac{4\sqrt{2}-1}{2}$

94. $\int \frac{(x^3 - 1)}{(x^4 + 1)(x + 1)} dx$, is :

- (a) $\frac{1}{4} \ln(1 + x^4) + \frac{1}{3} \ln(1 + x^3) + c$
- (b) $\frac{1}{4} \ln(1 + x^4) - \frac{1}{3} \ln(1 + x^3) + c$
- (c) $\frac{1}{4} \ln(1 + x^4) - \ln(1 + x) + c$
- (d) $\frac{1}{4} \ln(1 + x^4) + \ln(1 + x) + c$

95. The value of $\lim_{x \rightarrow 0^+} \frac{\int_1^{\cos x} (\cos^{-1} t) dt}{2x - \sin 2x}$ is equal to :

- (a) 0
- (b) -1
- (c) $\frac{2}{3}$
- (d) $-\frac{1}{4}$

96. Let $f(x) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n^2 + n + 2k} \right) =$

- (a) $\tan(\sin 1)$
- (b) $\sin(\tan 1)$
- (c) 0
- (d) $\sin\left(\frac{\tan 1}{2}\right)$

97. The value of $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n^2 + n + 2k} \right) =$

- (a) $\frac{1}{4}$
- (b) $\frac{1}{3}$
- (c) $\frac{1}{2}$
- (d) 1

98. The value of $\lim_{y \rightarrow 1^+} \frac{\int_1^y |t-1| dt}{\tan(y-1)}$ is :

- (a) 0
- (b) 1
- (c) 2
- (d) does not exist

99. Given that $\int \frac{dx}{(1+x^2)^n} = \frac{x}{2(n-1)(1+x^2)^{n-1}} + \frac{(2n-3)}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}}$. Find the value of $\int_0^1 \frac{dx}{(1+x^2)^4}$: (you may or may not use reduction formula given)

- | | |
|---------------------------------------|---------------------------------------|
| (a) $\frac{11}{48} + \frac{5\pi}{64}$ | (b) $\frac{11}{48} + \frac{5\pi}{32}$ |
| (c) $\frac{1}{24} + \frac{5\pi}{64}$ | (d) $\frac{1}{96} + \frac{5\pi}{32}$ |

100. Find the value of $\int_0^{\pi/4} (\sin x)^4 dx$:

- | | |
|-------------------------------------|-------------------------------------|
| (a) $\frac{3\pi}{16}$ | (b) $\frac{3\pi}{32} - \frac{1}{4}$ |
| (c) $\frac{3\pi}{32} - \frac{3}{4}$ | (d) $\frac{3\pi}{16} - \frac{7}{8}$ |

101. $\int \frac{\cos 9x + \cos 6x}{2 \cos 5x - 1} dx = A \sin 4x + B \sin x + C$, then $A + B$ is equal to:

(Where C is constant of integration)

- | | |
|-------------------|-------------------|
| (a) $\frac{1}{2}$ | (b) $\frac{3}{4}$ |
| (c) 2 | (d) $\frac{5}{4}$ |

102. $\int \frac{dx}{x^{2014}+x} = \frac{1}{p} \ln \left(\frac{x^q}{1+x^r} \right) + C$ where $p, q, r \in \mathbb{N}$ then the value of $(p + q + r)$ equals

(Where C is constant of integration)

- | | |
|----------|----------|
| (a) 6039 | (b) 6048 |
| (c) 6047 | (d) 6021 |

103. If $\int_0^1 e^{-x^2} dx = a$, then $\int_0^1 x^2 e^{-x^2} dx$ is equal to :

- | | |
|----------------------------|----------------------------|
| (a) $\frac{1}{2e}(ea - 1)$ | (b) $\frac{1}{2e}(ea + 1)$ |
| (c) $\frac{1}{e}(ea - 1)$ | (d) $\frac{1}{e}(ea + 1)$ |

104. If $f(x)$ is a continuous function for all real values of x and satisfies

$$\int_n^{n+1} f(x) dx = \frac{n^2}{2} \forall n \in I, \text{ then } \int_{-3}^5 f(|x|) dx \text{ is equal to :}$$

- | | |
|--------------------|--------------------|
| (a) $\frac{19}{2}$ | (b) $\frac{35}{2}$ |
| (c) $\frac{17}{2}$ | (d) $\frac{37}{2}$ |

105. $\int \frac{dx}{x^4(1+x^3)^2} = a \ln \left| \frac{1+x^3}{x^3} \right| + \frac{b}{x^3} + \frac{c}{1+x^3} + d$, then

(where d is arbitrary constant)

- | | |
|---|--|
| (a) $a = \frac{1}{3}, b = \frac{1}{3}, x = \frac{1}{3}$ | (b) $a = \frac{2}{3}, b = \frac{1}{3}, c = \frac{1}{3}$ |
| (c) $a = \frac{2}{3}, b = -\frac{1}{3}, c = -\frac{1}{3}$ | (d) $a = \frac{2}{3}, b = \frac{1}{3}, c = -\frac{1}{3}$ |

106. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \dots + \frac{1}{\sqrt{n}\sqrt{4n}}$ is equal to :

- | | |
|-----------------------|---------------------|
| (a) 2 | (b) 4 |
| (c) $2(\sqrt{2} - 1)$ | (d) $2\sqrt{2} - 1$ |

107. Let $(x) = \int_x^2 \frac{dy}{\sqrt{1+y^3}}$. The value of the integral $\int_0^2 xf(x) dx$ is equal to :

- | | |
|-------------------|-------------------|
| (a) 1 | (b) $\frac{1}{3}$ |
| (c) $\frac{4}{3}$ | (d) $\frac{2}{3}$ |

108. The value of $\int_0^{\pi/3} \ln(1 + \sqrt{3} \tan x) dx$ equals

- | | |
|-----------------------------|---------------------------|
| (a) $\frac{\pi}{3} \ln 2$ | (b) $\frac{\pi}{3}$ |
| (c) $\frac{\pi^2}{6} \ln 2$ | (d) $\frac{\pi}{2} \ln 2$ |

109. If $\int_0^{100} f(x) dx = a$, then $\sum_{k=0}^n \frac{x^{k+2} 2^k}{k!} dx$ is :

- | | |
|----------|---------|
| (a) 100a | (b) a |
| (c) 0 | (d) 10a |

110. The value of $\int_0^1 \lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{x^{k+2} 2^k}{k!} dx$:

- (a) $e^2 - 1$ (b) 2
 (c) $\frac{e^2 - 1}{2}$ (d) $\frac{e^2 - 1}{4}$

111. Evaluate : $\int x^5 \sqrt{1+x^3} dx$.

- (a) $\frac{1}{15}(1+x^3)^{5/2} - \frac{1}{9}(1+x^3)^{3/2} + c$

(b) $\frac{2}{15}(1+x^3)^{5/2} - \frac{1}{9}(1+x^3)^{3/2} + c$

(c) $\frac{2}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

(d) $\frac{1}{15}(1+x^3)^{5/2} - \frac{2}{9}(1+x^3)^{3/2} + c$

112. If $f(x) = \int_0^x \frac{\sin t}{t} dt$, which of the following is true?

- (a) $f(0) > f(1.1)$
 - (b) $f(0) < f(1.1) > f(2.1)$
 - (c) $f(0) < f(1.1) < f(2.1) > f(3.1)$
 - (d) $f(0) < f(1.1) < f(2.1) < f(3.1) > f(4.1)$

113. Evaluate : $\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)} dx.$

- (a) $\ln|x^2 + 3| + 3\tan^{-1}x + c$ (b) $\frac{1}{2}\ln|x^2 + 3| + \tan^{-1}x + c$
 (c) $\frac{1}{2}\ln|x^2 + 3| + 3\tan^{-1}x + c$ (d) $\ln|x^2 + 3| - \tan^{-1}x + c$

114. $\int \frac{\sqrt{\sec^5 x}}{\sqrt{\sin^3 x}} dx$ equals to :

- (a) $(\tan x)^{3/2} - \sqrt{\tan x} + C$ (b) $2 \left(\frac{1}{3} (\tan x)^{3/2} - \frac{1}{\sqrt{\tan x}} \right) + C$
 (c) $\frac{1}{3} (\tan x)^{3/2} - \sqrt{\tan x} + C$ (d) $\sqrt{\sin x} + \sqrt{\cos x} + C$

115. $\lim_{x \rightarrow 0} \int_0^x \frac{e^{\sin(tx)}}{x} dt$ equals to :

- | | |
|-------|--------------------|
| (a) 1 | (b) 2 |
| (c) e | (d) Does not exist |

116. If $A = \int_0^\pi \frac{\sin x}{x^2} dx$, then $\int_0^{\pi/2} \frac{\cos 2x}{x} dx$ is equal to :

- | | |
|-------------|-----------------------|
| (a) $1 - A$ | (b) $\frac{3}{2} - A$ |
| (c) $A - 1$ | (d) $1 + A$ |

Answer

1.	(b)	2.	(b)	3.	(b)	4.	(d)	5.	(d)	6.	(a)	7.	(d)	8.	(d)	9.	(b)	10.	(d)
11.	(b)	12.	(a)	13.	(b)	14.	(b)	15.	(a)	16.	(a)	17.	(b)	18.	(c)	19.	(b)	20.	(a)
21.	(a)	22.	(d)	23.	(b)	24.	(b)	25.	(c)	26.	(d)	27.	(a)	28.	(d)	29.	(d)	30.	(c)
31.	(d)	32.	(b)	33.	(c)	34.	(d)	35.	(c)	36.	(a)	37.	(c)	38.	(a)	39.	(d)	40.	(d)
41.	(b)	42.	(a)	43.	(b)	44.	(a)	45.	(a)	46.	(a)	47.	(d)	48.	(d)	49.	(d)	50.	(c)
51.	(c)	52.	(b)	53.	(c)	54.	(c)	55.	(a)	56.	(c)	57.	(b)	58.	(b)	59.	(b)	60.	(a)
61.	(a)	62.	(a)	63.	(b)	64.	(c)	65.	(d)	66.	(d)	67.	(a)	68.	(d)	69.	(d)	70.	(b)
71.	(d)	72.	(b)	73.	(a)	74.	(d)	75.	(b)	76.	(a)	77.	(b)	78.	(c)	79.	(c)	80.	(c)
81.	(b)	82.	(d)	83.	(d)	84.	(b)	85.	(c)	86.	(a)	87.	(d)	88.	(d)	89.	(d)	90.	(d)
91.	(d)	92.	(c)	93.	(b)	94.	(c)	95.	(d)	96.	(b)	97.	(c)	98.	(a)	99.	(a)	100.	(b)
101.	(d)	102.	(a)	103.	(a)	104.	(b)	105.	(c)	106.	(a)	107.	(d)	108.	(a)	109.	(b)	110.	(d)
111.	(c)	112.	(d)	113.	(c)	114.	(b)	115.	(a)	116.	(c)								

Exercise-2: One or More than One Answer is/are Correct

$$1. \int \frac{dx}{(1+\sqrt{x})^8} = -\frac{1}{3(1+\sqrt{x})^{k_1}} + \frac{2}{7(1+\sqrt{x})^{k_2}} + C, \text{ then :}$$

2. If $\int_{-\alpha}^{\alpha} (e^x + \cos x \ln(x + \sqrt{1+x^2})) dx > \frac{3}{2}$, then possible value of α can be :

3. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}} dx = A \sin^{-1} \left(\frac{x^{3/2}}{B} \right) + C$, where C is any arbitrary constant, then :

- (a) $A = \frac{2}{3}$ (b) $B = a^{\frac{3}{2}}$
 (c) $A = \frac{1}{3}$ (d) $B = a^{\frac{1}{2}}$

4. Let $\int x \sin x \cdot \sec^3 x \, dx = \frac{1}{2}(x \cdot f(x) - g(x)) + k$, then :

- (a) $f(x) \notin (-1,1)$
 - (b) $g(x) = \sin x$ has 6 solution for $x \in [-\pi, 2\pi]$
 - (c) $g'(x) = f(x), \forall x \in \mathbb{R}$
 - (d) $f(x) = g(x)$ has no solution

5. If $\int (\sin 30 + \sin \theta) \cos \theta e^{\sin \theta} d\theta = (A \sin^3 \theta + B \cos^2 \theta + C \sin \theta + D \cos \theta + E) e^{\sin \theta} + F$, then :

6. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}} dx = A \sin^{-1} \left(\frac{x^{3/2}}{B} \right) + C$, where C is any arbitrary constant, then :

- | | |
|-----------------------|-------------------|
| (a) $A = \frac{2}{3}$ | (b) $B = a^{3/2}$ |
| (c) $A = \frac{1}{3}$ | (d) $B = a^{1/2}$ |

7. If $f(\theta) = \lim_{n \rightarrow \infty} \sum_{r=0}^{n\theta} \frac{2r}{n\sqrt{(3\theta n - 2r)(n\theta + 2r)}}$ then :

- | | |
|--|--|
| (a) $f(1) = \frac{\pi}{6}$ | (b) $f(\theta) = \frac{\theta}{2} \int_0^\theta \frac{dx}{\sqrt{\theta^2 - (x - \frac{\theta}{2})^2}}$ |
| (c) $f(\theta)$ is a constant function | (d) $y = f(\theta)$ is invertible |

8. If $f(x+y) = f(x)f(y)$ for all x, y and $f(0) \neq 0$, and $F(x) = \frac{f(x)}{1+(f(x))^2}$ then :

- | |
|--|
| (a) $\int_{-2010}^{2011} F(x)dx = \int_0^{2011} F(x)dx$ |
| (b) $\int_{-2010}^{2011} F(x)dx = \int_0^{2010} F(x)dx = \int_0^{2011} F(x)dx$ |
| (c) $\int_{-2010}^{2011} F(x)dx = 0$ |
| (d) $\int_{-2010}^{2011} (2F(-x) - F(x))dx = 2 \int_0^{2010} F(x)dx$ |

9. Let $J = \int_{-1}^2 \left(\cot^{-1} \frac{1}{x} + \cot^{-1} x \right) dx$, $K = \int_{-2\pi}^{7\pi} \frac{\sin x}{|\sin x|} dx$. Then which of the following alternative(s) is/are correct ?

- | | |
|----------------------|---------------------------------|
| (a) $2J + 3K = 8\pi$ | (b) $4J^2 + K^2 = 26\pi^2$ |
| (c) $2J - K = 3\pi$ | (d) $\frac{J}{K} = \frac{2}{5}$ |

10. Which of the following function(s) is/are even ?

- | |
|--|
| (a) $f(x) = \int_0^x \ln(t + \sqrt{1+t^2}) dt$ |
| (b) $g(x) = \int_0^x \frac{(2^{t+1})t}{2^t - 1} dt$ |
| (c) $h(x) = \int_0^x (\sqrt{1+t+t^2} - \sqrt{1-t+t^2}) dt$ |
| (d) $l(x) = \int_0^x \ln\left(\frac{1-t}{1+t}\right) dt$ |

11. Let $l_1 = \lim_{x \rightarrow \infty} \sqrt{\frac{x - \cos^2 x}{x + \sin x}}$ and $l_2 = \lim_{h \rightarrow 0^+} \int_{-1}^1 \frac{h dx}{h^2 + x^2}$. Then :

- (a) Both l_1 and l_2 are less than $22/7$
- (b) One of the two limits is rational and other irrational
- (c) $l_2 > l_1$
- (d) l_2 is greater than 3 times of l_1

12. For $a > 0$, if $I = \int \sqrt{\frac{x}{a^3 - x^3}} dx = A \sin^{-1} \left(\frac{x^{3/2}}{B} \right) + C$, where C is any arbitrary constant, then :

- | | |
|-----------------------|-------------------|
| (a) $A = \frac{2}{3}$ | (b) $B = a^{3/2}$ |
| (c) $A = \frac{1}{3}$ | (d) $B = a^{1/2}$ |

13. If $\int \frac{dx}{1 - \sin^4 x} = a \tan x + b \tan^{-1}(c \tan x) + D$, then :

- | | |
|-----------------------|-------------------------------|
| (a) $a = \frac{1}{2}$ | (b) $b = \sqrt{2}$ |
| (c) $c = \sqrt{2}$ | (d) $b = \frac{1}{2\sqrt{2}}$ |

14. The value of definite integral :

$\int_{-2014}^{2014} \frac{dx}{1 + \sin^{2015}(x) + \sqrt{1 + \sin^{4030}(x)}}$ equals :

- | | |
|----------------|----------|
| (a) 0 | (b) 2014 |
| (c) $(2014)^2$ | (d) 4028 |

15. Let $L = \lim_{n \rightarrow \infty} \int_a^\infty \frac{n dx}{1 + n^2 x^2}$ where $a \in R$ then L can be :

- | | |
|-----------|---------------------|
| (a) π | (b) $\frac{\pi}{2}$ |
| (c) 0 | (d) $\frac{\pi}{3}$ |

16. Let $I = \int_0^1 \sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} dx$ and $J = \int_0^1 \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ then correct statement(s) is/are :

- | | |
|---------------------------|---------------------------|
| (a) $I + J = 2$ | (b) $I - J = \pi$ |
| (c) $I = \frac{2+\pi}{2}$ | (d) $J = \frac{4-\pi}{2}$ |

Answers

1.	(b, c)	2.	(a, b, c, d)	3.	(a, b)	4.	(a, c, d)	5.	(a, b, c)	6.	(a, b)
7.	(a, b, d)	8.	(b, d)	9.	(a, b)	10.	(a, b, c, d)	11.	(a, b, c, d)	12.	(a, b)
13.	(a, c)	14.	(b)	15.	(a, b, c,)	16.	(b, c)				