CONTENTS

ERRORS IN MEASUREMENTS & INSTRUMENTS

ERRORS

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity. (1) random and (2) systematic errors.

1. Random errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of random error can be some what reduced by taking the average of measured values. Random errors have no fixed sign or size.

2. Systematic errors occur due to error in the procedure, or miscalibration of the instrument etc. Such errors have same size and sign for all the measurements. Such errors can be determined.

A measurement with relatively small random error is said to have high precision. A measurement with small random error and small systematic error is said to have high accuracy.

The experimental error [uncertainty] can be expressed in several standard ways.

Error limits $Q \pm \Delta Q$ is the measured quantity and ΔQ is the magnitude of its limit of error. This expresses the experimenter's judgment that the 'true' value of Q lies between $Q - \Delta Q$ and $Q + \Delta Q$. This entire interval within which the measurement lies is called the range of error. Random errors are expressed in this form.

Absolute Error

Error may be expressed as absolute measures, giving the size of the error in a quantity in the same units as the quantity itself.

Least Count Error :– If the instrument has known least count, the absolute error is taken to be equal to the least count unless otherwise stated.

Relative (or Fractional) Error

Error may be expressed as relative measures, giving the ratio of the quantity's error to the quantity itself. In general,

$$\text{relative error} = \frac{absolute\ error\ in\ a\ measurement}{size\ of\ the\ measurement}$$

We should know the error in the measurement because these errors propagate through the calculations to produce errors in results.

A. ***Systematic errors*** **:**

They have a known sign. The systematic error is removed before beginning calculations. Bench error and zero error are examples of systematic error.

B. ***Random error*** **:**

They have unknown sign. Thus they are represented in the form A ± a.

Here we are only concerned with *limits of error*. We must assume a “worst-case” combination. In the case of substraction, A – B, the worst-case deviation of the answer occurs when the errors are either + a and – b or – a and + b. In either case, the maximum error will be (a + b).

For example in the experiment on finding the focal length of a convex lens, the object distance (u) is found by subtracting the positions of the object needle and the lens. If the optical bench has a least count of 1 mm, the error in each position will be 0.5 mm. So, the error in the value of u will be 1 mm.

1. ***Addition and subtraction rule*** **:** The absolute random errors add.

   Thus if $R = A + B$, $r = a + b$ and if $R = A - B$, $r = a + b$

2. ***Product and quotient rule*** **:** The relative random errors add.

   Thus if $R = AB$, $\frac{r}{R} = \frac{a}{A} + \frac{b}{B}$ and if $R = \frac{A}{B}$, then also $\frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

3. ***Power rule*** **:** When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q. This also holds for negative powers. If $R = Q^P$, $\frac{r}{R} = P \times \frac{q}{Q}$

4. The quotient rule is not applicable if the numerator and denominator are dependent on each other.

   e.g if $R = \frac{XY}{X+Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows $\frac{1}{R} = \frac{1}{X} + \frac{1}{Y}$. Differentiating both the sides, we get $-\frac{dR}{R^2} = -\frac{dX}{X^2} - \frac{dY}{Y^2}$.

   Thus $\frac{r}{R^2} = \frac{x}{X^2} + \frac{y}{Y^2}$

Examples

A student finds the constant acceleration of a slowly moving object with a stopwatch. The equation used is $S = (1/2)AT^2$. The time is measured with a stopwatch, the distance, S with a meter stick. What is the acceleration and its estimated error?

$S = 2 \pm 0.005$ meter. $T = 4.2 \pm 0.2$ second.

Sol : We use capital letters for quantities, lower case for errors. Solve the equation for the result, a. $A = 2S/T^2$. Its random-error equation is $\frac{a}{A} = 2\frac{t}{T} + \frac{s}{S}$. Thus $A = 0.23 \pm 0.02$ m/s^2.

SIGNIFICANT DIGITS

Significant figures are digits that are statistically significant. There are two kinds of values in science:

1. Measured Values
2. Computed Values

The way that we identify the proper number of significant figures in science are different for these two types.

MEASURED VALUES

Identifying a measured value with the correct number of significant digits requires that the instrument's calibration be taken into consideration. The last significant digit in a measured value will be the first estimated position. For example, a metric ruler is calibrated with numbered calibrations equal to 1 cm. In addition, there will be ten unnumbered calibration marks between each numbered position. (each equal to 0.1 cm). Then one could with a little practice estimate between each of those marking. (each equal to 0.05 cm). That first estimated position would be the last significant digit reported in the measured value. Let's say that we were measuring the length of a tube, and it extended past the fourteenth numbered calibration half way between the third and fourth unnumbered mark. The metric ruler was a meter stick with 100 numbered calibrations. The reported measured length would be 14.35 cm. Here the total number of significant digits will be 4.

COMPUTED VALUE

The other type of value is a computed value. The proper number of significant figures that a computed value should have is decided by a set of conventional rules. However before we get to those rules for computed values we have to consider how to determine how many significant digits are indicated in the numbers being used in the math computation.

A. Rules for determining the number of significant digits in number with indicated decimals.

1. All nonzero digits (1-9) are to be counted as significant.
2. Zeros that have any nonzero digits anywhere to the LEFT of them are considered significant zeros.
3. All other zeros not covered in rule (2) above are NOT be considered significant digits.

For example : 0.0040000

The 4 is obviously to be counted significant (Rule-1), but what about the zeros? The first three zeros would not be considered significant since they have no nonzero digits anywhere to their left (Rule-3). The last four zeros would all be considered significant since each of them has the nonzero digit 4 to their left (Rule-2). Therefore the number has a total of five significant digits.

Here is another example : 120.00420

The digit 1, 2, 4 and 2 are all considered significant (Rule-1). All zeros are considered significant since they have non-zero digits somewhere to their left (Rule-2). So there are a total of eight significant digits. If in the question, we are given a number like 100, we will treat that the number has only one significant digit by convention.

B. Determining the number of significant digits if number is not having an indicated decimal.

The decimal indicated in a number tells us to what position of estimation the number has been indicated. But what about 1,000,000?

Notice that there is no decimal indicated in the number. In other words, there is an ambiguity concerning the estimated position. This ambiguity can only be clarified by placing the number in exponential notation.

For example : If I write the number above in this manner.

1.00×10^6

I have indicated that the number has been recorded with three significant digits. On the other hand, if I write the same number as : 1.0000×10^6

I have identified the number to have 5 significant digits. Once the number has been expressed in exponential notation form then the digits that appear before the power of ten will all be considered significant. So for example : 2.0040×10^4 will have five significant digits. This means that unit conversion will not change the number of significant digits.

Thus 0.000010 km = 1.0 cm = 0.010 m = 1.0×10^{-2} m = 1.0×10^{-5} km

Rule for expressing proper number of significant digits in an answer from multiplication or division

For multiplication AND division there is the following rule for expressing a computed product or quotient with the proper number of significant digits.

The product or quotient will be reported as having as many significant digits as the number involved in the operation with the least number of significant digits.

For example : $0.000170 \times 100.40 = 0.017068$

The product could be expressed with no more that three significant digits since 0.000170 has only three significant digits, and 100.40 has five. So according to the rule the product answer could only be expressed with three significant digits. Thus the answer should be 0.0171 (after rounding off)

Another example : $2.000 \times 10^4 / 6.0 \times 10^{-3} = 0.33 \times 10^7$

The answer could be expressed with no more that two significant digits since the least digit number involved in the operation has two significant digits.

Sometimes this would required expressing the answer in exponential notation.

For example : $3.0 \times 800.0 = 2.4 \times 10^3$

The number 3.0 has two significant digits and then number 800.0 has four. The rule states that the answer can have no more than two digits expressed. However the answer as we can all see would be 2400. How do we express the answer 2400 while obeying the rules? The only way is to express the answer in exponential notation so 2400 could be expressed as : 2.4×10^3

Rule for expressing the correct number of significant digits in an addition or substraction :

The rule for expressing a sum or difference is considerably different than the one for multiplication of division. The sum or difference can be no more precise than the least precise number involved in the mathematical operation. Precision has to do with the number of positions to the RIGHT of the decimal. The more position to the right of the decimal, the more precise the number. So a sum or difference can have no more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

For example : 160.45 + 6.732 = 167.18 (after rounding off)

The answer could be expressed only to two positions to the right of the decimal, since 160.45 is the least precise.

Another example : 45.621 + 4.3 – 6.41 = 43.5 (after rounding off)

The answer could be expressed only to one position to the right of the decimal, since the number 4.3 is the least precise number (i.e. having only one position to the right of its decimal). Notice we aren't really determining the total number of significant digits in the answer with this rule.

Rules for rounding off digits :

There are a set of conventional rules for rounding off.

1. Determine according to the rule what the last reported digit should be.
2. Consider the digit to the right of the last reported digit.
3. If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.
4. If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.
5. If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

For example if we wish to round off the following number to 3 significant digits : 18.3682

The last reported digits would be the 3. The digit to its right is a 6 which is greater than 5. According to the Rule-4 above, the digit 3 is increased by one and the answer is : 18.4

Another example : Round off 4.565 to three significant digits.

The last reported digit would be the 6. The digit to the right is a 5 followed by nothing. Therefore according to Rule-5 above since the 6 is even it remains so and the answer would be 4.56.

(i) Measurement of length

The simplest method measuring the length of a straight line is by means of a meter scale. But there exists some limitation in the accuracy of the result:

(i) the dividing lines have a finite thickness.

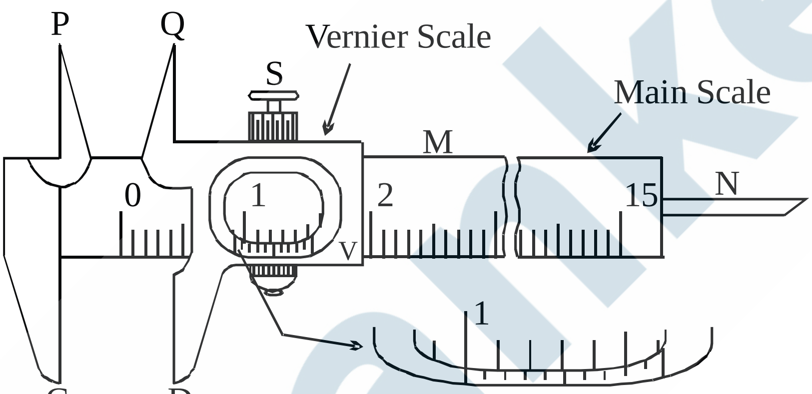
(ii) naked eye cannot correctly estimate less than 0.5 mm

For greater accuracy devices like

(a) Vernier callipers

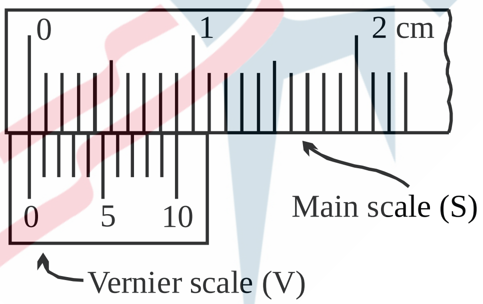
(b) micrometer scales (screw gauge) are used.

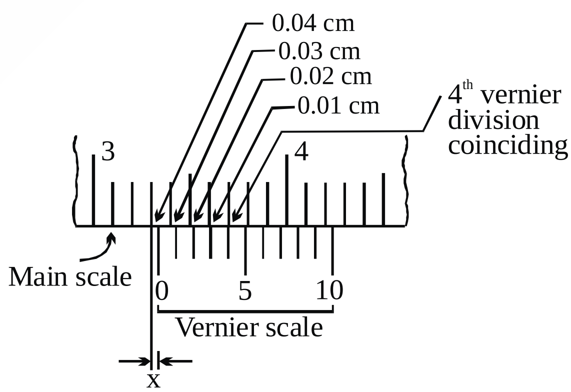
VERNIER CALLIPERS:



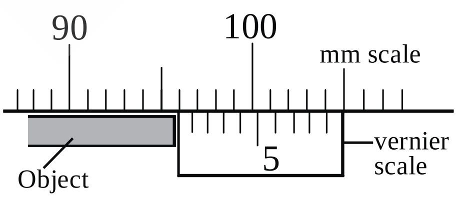
Vernier Callipers

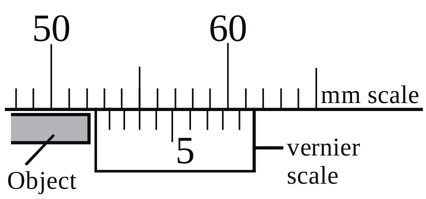
Principle of Vernier





Reading a vernier with 4th division coinciding





It consists of a main scale graduated in cm/mm over which an auxiliary scale (or Vernier scale) can slide along the length. The division of the Vernier scale being shorter than the divisions of the main scale.

Least count of Vernier Callipers

The least count or Vernier constant (v. c) is the minimum value of correct estimation of length without eye estimation. If N division of vernier coincides with (N-1) division of main scale, then

$$N\,(VS) = (N-1)\text{ ms} \quad \Rightarrow 1VS = \frac{N-1}{N}\text{ ms}$$

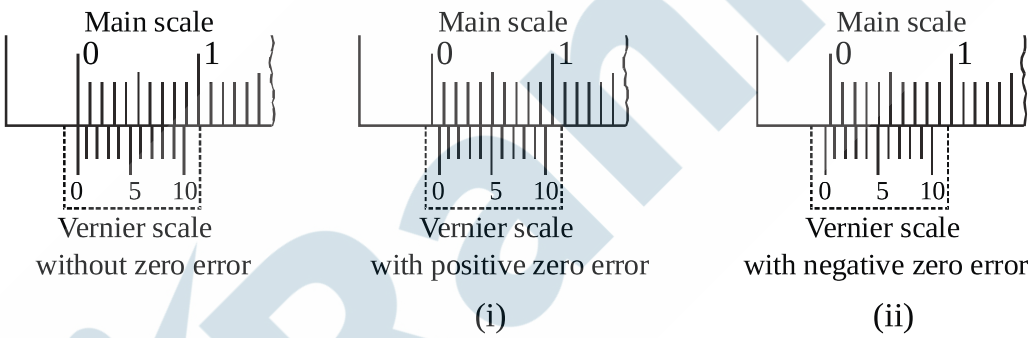
Vernier constant = 1 ms – 1 vs = $\left(1-\frac{N-1}{N}\right)$ ms = $\frac{1\text{ms}}{N}$, which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

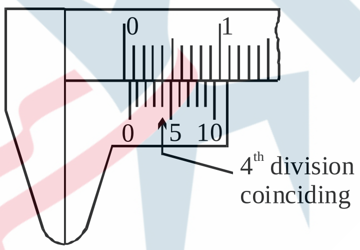
Length as measured by Vernier Callipers

The formula for measuring the length is L = main scale reading + least count of vernier scale × Vernier scale division coinciding with a main scale division

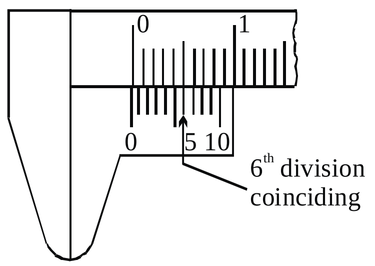
Main scale reading is given by the zeroth division of the vernier scale as shown in the figure.

Zero error:





Positive zero error (+0.04 cm) and its correction



Negative zero error = (–0.04 cm) and its correction

If the zero marking of main scale and vernier callipers do not coincide, necessary correction has to be made for this error which is known as zero error of the instrument.

If the zero of the vernier scale is to the right of the zero of the main scale the zero error is said to be positive and the correction will be negative and vice versa.

The zero error is always subtracted from the reading to get the corrected value.

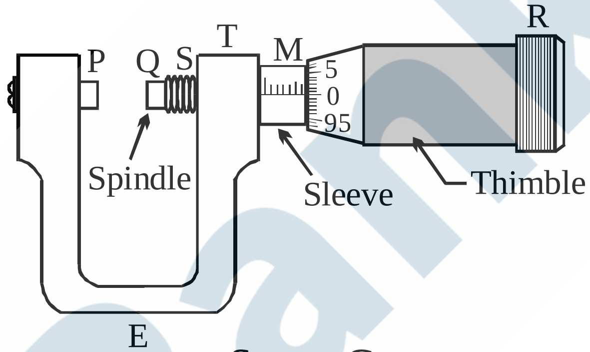
If the zero error is positive, its value is calculated as we take any normal reading. If the zero error negative (the zero of vernier scale lies to the left of the zero of main scale),

negative zero error = – [Total no. of vsd – vsd coinciding] × L.C.

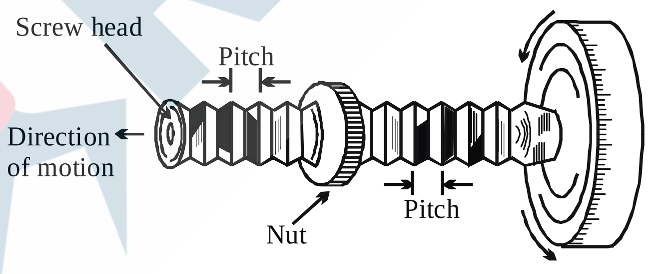
Do not try to read the main scale at the point where the lines match best. This has no meaning. Read from the vernier scale instead. Sometimes it is difficult to tell whether the best match of lines is for vernier marks 9, 0, or 1. Make your best estimate, but realize that the final result including the vernier must round off to the result you would choose if there was no vernier. If the mark is close to 3.20 on the main scale, but the vernier reading is 9, the length is 3.19 cm. If the mark is close to 3.2 on the main scale and the vernier is 1, the length is 3.21 cm.

SCREW GAUGE (OR MICROMETER SCREW)

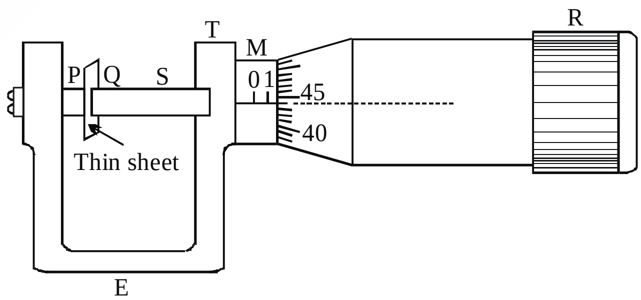
In general vernier callipers can measure accurately upto 0.01 em and for greater accuracy micrometer screw devices e.g. screw gauge, spherometer are used. These consist of accurately cut screw which can be moved in a closely fitting fixed nut by tuming it axially. The instrument is provided with two scales:

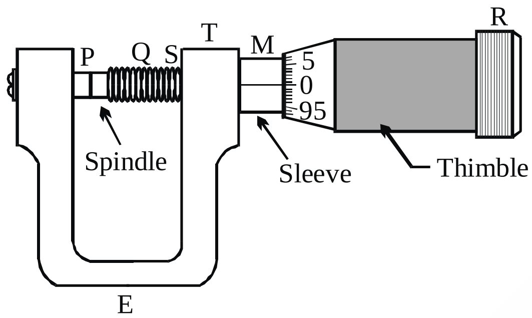


Screw Gauge

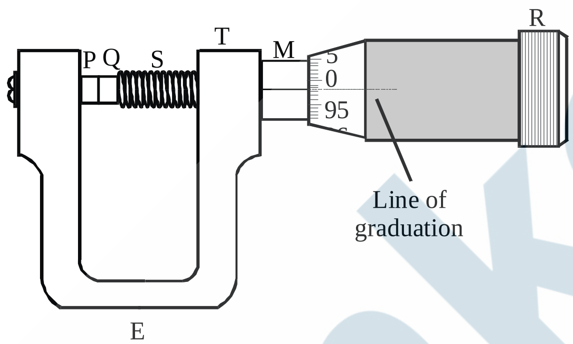


Principle of a micrometer

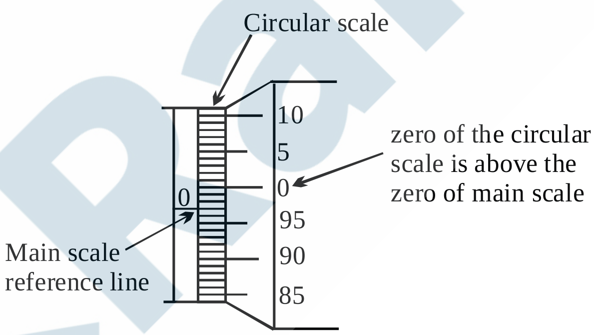


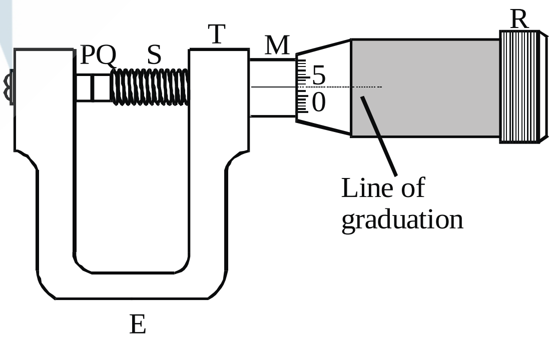


Screw gauge with no zero error

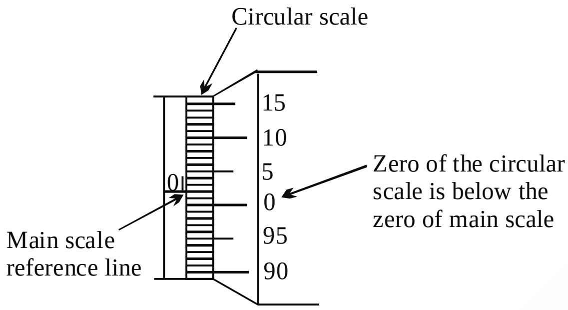


Negative zero error
(3 division error) i.e., - 0.003 cm





Positive zero error
(2 division error) i.e., + 0.002 cm



(i) The main scale or pitch scale M graduated along the axis of the screw.

(ii) The cap-scale or head scale H round the edge of the screw head.

Constants of the Screw Gauge

(a) Pitch : The translational motion of the screw is directly proportional to the total rotation of the head. The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for 10 rotation of cap = 5 mm, then pitch = 0.5 mm

(b) Least count : In this case also, the minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions. Thus in the aforesaid Illustration:, if the total cap division is 100, then least count = 0.5mm/100 = 0.005 mm

(c) Measurement of length by screw gauge :

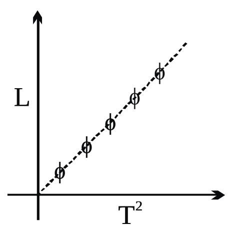
$L = n \times \text{pitch} + f \times \text{least count}$, where n = main scale reading & f = caps scale reading

Zero Error : In a perfect instrument the zero of the head scale coincides with the line of graduation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line or reference line of the cap lies **below** the line of graduation and versa. The corresponding corrections will be just opposite.

(ii) Measurement of g using a simple pendulum

A small spherical bob is attached to a cotton thread and the combination is suspended from a point A. The length of the thread (L) is read off on a meter scale. A correction is added to L to include the finite size of the bob and the hook. The corrected value of L is used for further calculation. The bob is displaced slightly to one side and is allowed to oscillate, and the total time taken for 50 complete oscillations is noted on a stop-watch.

The time period (T) of a single oscillation is now calculated by division.



Observations are now taken by using different lengths for the cotton thread (L) and pairs of values of L and T are taken. A plot of L v/s T^2, on a graph, is linear. g is given by $g = 4\pi^2 \dfrac{L}{T^2}$

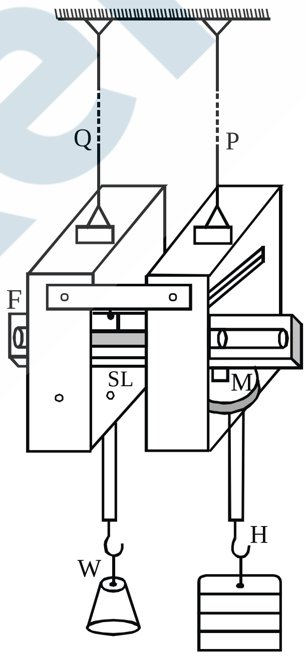
The major errors in this experiment are

(a) **Systematic :** Error due to finite amplitude of the pendulum (as the motion is not exactly SHM). This may be corrected for by using the correct numerical estimate for the time period. However the practice is to ensure that the amplitude is small.

(b) **Statistical :** Errors arising from measurement of length and time. $\frac{\delta g}{g} = \frac{\delta L}{L} + 2\left(\frac{\delta T}{T}\right)$

The contributions to δL, δT are both statistical and systematic. These are reduced by the process of averaging. The systematic error in L can be reduced by plotting several values of L vs T^2 and fitting to a straight line. The slope of this fit gives the correct value of L/T^2

(iii) **Determination of Young's Modulus by Searle's Method**

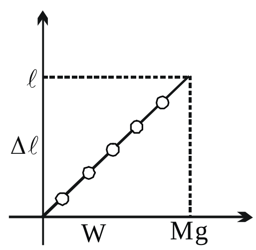


The experimental set up consists of two identical wires P and Q of uniform cross section suspended from a fixed rigid support. The free ends of these parallel wires are connected to a frame F as shown in the figure. The length of the wire Q remains fixed while the load L attached to the wire P through the frame F is varied in equal steps so as to produce extension along .the length. The extension thus produced is measured with the help of spirit level SL and micrometer screw M attached to the F frame on the side of the experimental wire. On placing the slotted weights on the hanger H upto a permissible value (half of the breaking force) the wire gets extended by small amount and the spirit level gets disturbed from horizontal setting. This increase in length is measured by turning the micrometer screw M upwards so as to restore the balance of the spirit level. If n be the number of turns of the micrometer screw and f be the difference in the cap reading, the increase in length *M* is obtained by $\Delta l = n \times \text{pitch} + f \times \text{least count}$

In some situations, the change in length is obtained by vernier arrangement instead of the screw gauge.

The load on the hanger is reduced in the same steps and spirit level is restored to horizontal position. The mean of these two observations gives the true increase in length of the wire corresponding to the given value of load. This is to eliminate the effect of hysteresis.

From the data obtained, a graph showing extension (Δl) against the load (W) is plotted which is obtained as a straight line passing through the origin.



The slope of the line gives $\tan\theta = \frac{l}{W} = \frac{l}{Mg}$

Now, stress $= \frac{Mg}{\pi r^2}$ and strain $= \frac{l}{L}$

$$Y = \text{Stress/ strain} = \frac{MgL}{\pi r^2 l} = \frac{L}{\pi r^2 \tan\theta}$$

With known values of initial length L, radius r of the experimental wire and $\tan\theta$, Young's modulus Y can be calculated.

(iv) Specific Heat of a liquid using a calorimeter:

The principle is to take a known quantity of liquid in an insulated calorimeter and heat it by passing a known current (i) through a heating coil immersed within the liquid for a known length of time (t).

The mass of the calorimeter (m_1) and, the combined mass of the calorimeter and the liquid (m_2) are measured. The potential drop across the heating coil is V and the maximum temperature of the liquid is measured to θ_2.

The specific heat of the liquid (S_l) is found by using the relation

$$(m_2 - m_1)S_l(\theta_2 - \theta_0) + m_1S_c(\theta_2 - \theta_0) = i.\ V.\ t$$

or, $$(m_2 - m_1)S_l + m_1S_c = i.\ V.\ t/(\theta_2 - \theta_0) \qquad \text{..... (1)}$$

Here, θ_0 is the room temperature, while S_c is the specific heat of the material of the calorimeter and the stirrer. If S_c is known, then S_l can be determined.

On the other hand, if S_c is unknown: one can either repeat the experiment with water or a different mass of the liquid and use the two equations to eliminate m_1S_c.

The sources of error in this experiment are errors due to improper connection of the heating coil, radiation, apart from statistical errors in measurement.

Error analysis :

After correcting for systematic errors, equation (1) is used to estimate the remaining errors.

(v) Focal length of a concave mirror and a convex lens using the u-v method.

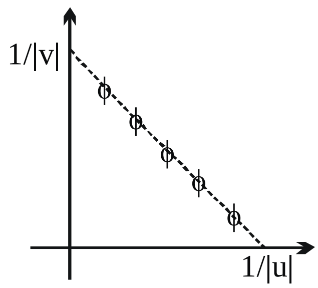
In this method one uses an optical bench and the convex lens (or the concave mirror) is placed on the holder.

The position of the lens is noted by reading the scale at the bottom of the holder. A bright object (a filament lamp or some similar object) is placed at a fixed distance (u) in front of the lens (mirror).

The position of the image (v) is determined by moving a white screen behind the lens until a sharp image is obtained (for real images).

For the concave mirror, the position of the image is determined by placing a sharp object (a pin) on the optical bench such that the parallax between the object pin and the image is nil.

A plot of |u| versus |v| gives a rectangular hyperbola. A plot of $\frac{1}{|v|}$ vs $\frac{1}{|u|}$ gives a straight line. The intercepts are equal to $\frac{1}{|f|}$, where *f* is the focal length.



Error : The systematic error in this experiment is mostly due to improper position of the object on the holder. This error maybe eliminated by reversing the holder (rotating the holder by 180° about the vertical) and then taking the readings again. Averages are then taken.

The equation for random errors gives: $\frac{\delta f}{f^2} = \frac{\delta u}{u^2} + \frac{\delta v}{v^2}$

The errors δu, δv correspond to the error in the measurement of u and v. Actually, we know the errors in the object position, lens position & image position. So, the errors in u & v are too be estimated as described before.

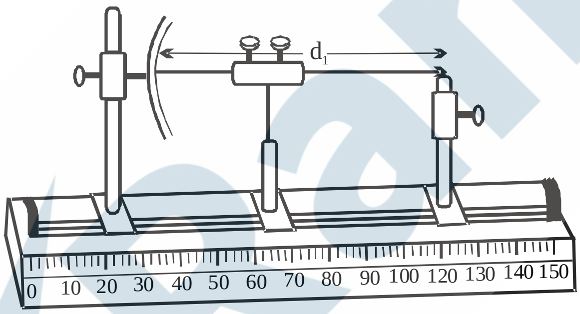
Index Error or Bench Error and its correction:

In an experiment using an optical bench we are required to measure the object and image distances from the pole or vertex on the mirror. The distance between the tip of the needles and the pole of the mirror is the actual distance. But we practically measure distances between the indices with the help of the scale engraved on the bench. These distances are called the observed distances. The actual distances may not be equal to the observed distances and due to this reason an error creeps in the measurement of the distances. This error is called the index or the bench error. This error is estimated with the help of a needle of known length placed horizontally between the tip of the needle and the pole.

Index Error = Observed distance – actual distance and

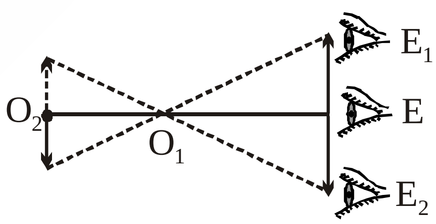
Index Correction = Actual – observed distance

Note: Index correction whether positive or negative, is always added algebraically to the observed distance to get the corrected distance.



Parallax

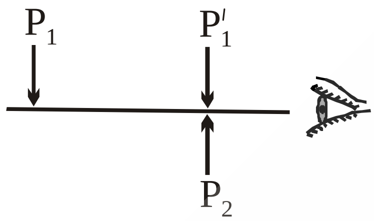
When two objects O_1 and O_2 are placed in such a way that both of them lie in the same line of sight as shown in figure, then the object nearer to the eye covers the object farther from it. Their images on the retina are superimposed and therefore, it is impossible to decide which is the nearer object. To identify this fact, the observer displaces his eye to a position E_1 or E_2 until he is able to see two distinct objects.



The more distant object O_2 apparently moves in the direction opposite to the displacement of the observer's eye with respect to the nearer object O_1. This relative shift in the position of two objects due to the shift in the position of the observer's eye is called parallax.

Parallax between the two objects disappear if they are at the same position.

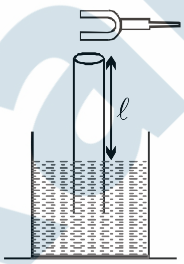
The figure shows the tips of two pins P_1 and P_2 kept in the upright positions. The parallax between P_1 and P_2 is removed by shifting the position of observer's eye sideways. As the farther pin P_1 is displaced towards the pin P_2 the relative shift (parallax) between their positions decreases as the position of eye is displaced sideways. The relative shift vanishes when the pin P_1 occupies the position P'_1, that is , when the tips of the two are just coincident. At this position there is no parallax between the tips of the two pins.



(vi) Speed of sound using resonance column

A tuning fork of known frequency (f) is held at the mouth of a long tube, which is dipped into water as shown in the figure. The length (l_1) of the air column in the tube is adjusted until it resonates with the tuning fork. The air temperature and humidity are noted. The length of the tube is adjusted again until a second resonance length (l_2) is found (provided the tube is long)

Then, $l_2 - l_1 = \lambda / 2$, provided l_1, l_2 are resonance lengths for adjacent resonances.



$\therefore\ \lambda = 2(l_2 - l_1)$, is the wavelength of sound.

Since the frequency f, is known; the velocity of sound in air at the temperature (θ) and humidity (h) is given by $C = f\lambda = 2(l_2 - l_1)f$

It is also possible to use a single measurement of the resonant length directly, but, then it has to be corrected *for* the "end effect":

$$\lambda(\text{fundamental}) = 4(l_1 + 0.3d), \text{ where } d = \text{diameter}$$

Errors :

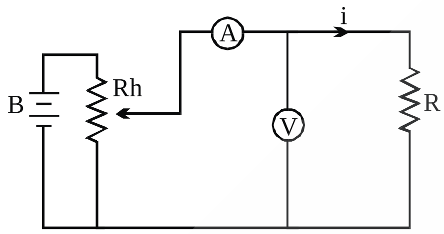
The major systematic errors introduced are due to end effects in (end correction) and also due to excessive humidity.

Random errors are given by $\dfrac{\delta C}{C} = \dfrac{\delta(l_2 - l_1)}{l_2 - l_1} = \dfrac{\delta l_2 + \delta l_1}{l_2 - l_1}$

(vii) Verification of Ohm's law using voltmeter and ammeter

A voltmeter (V) and an ammeter (A) are connected in a circuit along with a resistance R as shown in the figure, along with a battery B and a rheostat, Rh

Simultaneous readings of the current i and the potential drop V are taken by changing the resistance in the rheostat (Rh). A graph of *V* vs *i* is plotted and it is found to be linear (within errors). The magnitude of R is determined by either



(a) taking the ratio $\frac{V}{i}$ and then (b) fitting to a straight line: V = iR, and determining the slope R.

Errors :

Systematic errors in this experiment arise from the current flowing through V (finite resistance of the voltmeter), the Joule heating effect in the circuit and the resistance of the connecting wires/ connections of the resistance. The effect of Joule heating may be minimized by switching on the circuit for a short while only, while the effect of finite resistance of the voltmeter can be overcome by using a high resistance instrument or a potentiometer. The lengths of connecting wires should be minimized as much as possible.

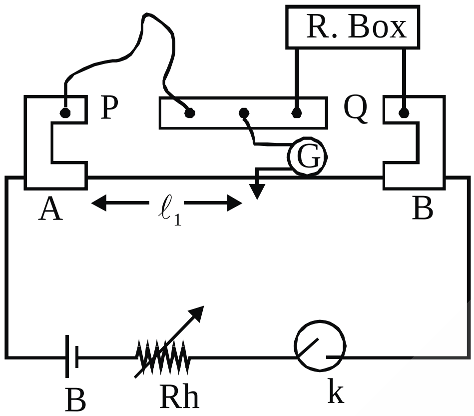
Error analysis :

The error in computing the ratio $R = \frac{V}{i}$ is given by $\left|\frac{\delta R}{R}\right| = \left|\frac{\delta V}{V}\right| + \left|\frac{\delta i}{i}\right|$

where δV and δi are of the order of the least counts of the instruments used.

(viii) Specific resistance of the material of a wire using a meter bridge :

A known length (*l*) of a wire is connected in one of the gaps (P) of a metre bridge, while a Resistance Box is inserted into the other gap (Q). The circuit is completed by using a battery (B), a Rheostat (Rh), a Key (K) and a galvanometer (G).The balance length (*l*) is found by closing key K and momentarily connecting the galvanometer until it gives zero deflection (null point). Then, $\frac{P}{Q} = \frac{l}{100-l}$ ....(1)



using the expression for the meter bridge at balance. Here P represents the resistance of the wire while Q represents the resistance in the resistance box. The key K is open when the circuit is not in use.

The resistance of the wire, $P = \rho \dfrac{L}{\pi r^2} \Rightarrow \rho = \dfrac{\pi r^2}{L} P$ ....(2)

where r is the radius of wire and L is the length of the wire, r is measured using a screw gauge while L is measured with a scale.

Errors :

The major systematic errors in this experiment are due to the heating effect, end corrections introduced due to shift of the zero of the scale at A and B, and stray resistances in P and Q, and errors due to non-uniformity of the meter bridge wire.

Error analysis :

End corrections can be estimated by including known resistances P_1 and Q_1 in the two ends and finding the null point:

$$\frac{P_1}{Q_1} = \frac{l_1 + \alpha}{100 - l_1 + \beta} \qquad \text{..... (2),}$$

where α and β are the end corrections.

When the resistance Q_1 is placed in the left gap and P_1 in the right gap,

$$\frac{Q_1}{P_1} = \frac{l_2 + \alpha}{100 - l_2 + \beta} \qquad \text{..... (3)}$$

which give two linear equation for finding α and β.

In order that α and β be measured accurately, P_1 and Q_1 should be as different from each other as possible. For the actual balance point, $\dfrac{P}{Q} = \dfrac{l + \alpha}{100 - l + \beta} = \dfrac{l_1'}{l_2'}$,

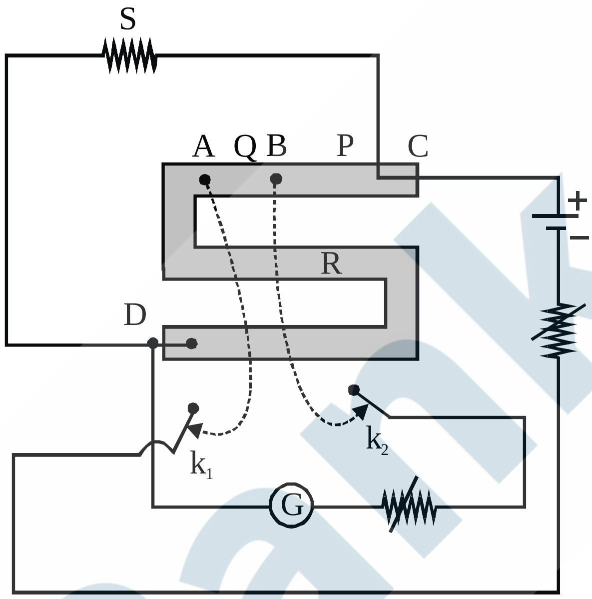
Errors due to non-uniformity of the meter bridge wire can be minimized by interchanging the resistances in the gaps P and Q. $\therefore \dfrac{\delta P}{P} = \left|\dfrac{\delta l_1'}{l_1'}\right| + \left|\dfrac{\delta l_2'}{l_2'}\right|$

where, $\delta l'_1$ and $\delta l'_2$ are of the order of the least count of the scale.

The error is, therefore, minimum if $l'_1 = l'_2$ i.e. when the balance point is in the middle of the bridge. The error in ρ is $\frac{\delta\rho}{\rho} = \frac{2\delta r}{r} + \frac{\delta L}{L} + \frac{\delta P}{P}$

(ix) Measurement of unknown resistance using a P.O. Box

A P.O. Box can also be used to measure an unknown resistance. It is a Wheatstone Bridge with three arms P, Q and R; while the fourth arm(s) is the unknown resistance. P and Q are known as the ratio arms while R is known at the rheostat arm. At balance, the unknown resistance



$$S = \left(\frac{P}{Q}\right)R \qquad .....(1)$$

The ratio arms are first adjusted so that they carry 100 Ω each. The resistance in the rheostat arm is now adjusted so that the galvanometer deflection is in one direction, if $R = R_0$ (Ohm) and in the opposite direction when $R = R_0 + 1$ (ohm).

This implies that the unknown resistance, S lies between R_0 and $R_0 + 1$ (ohm). Now, the resistance in P and Q are made 100 Ω and 1000 Ω respectively, and the process is repeated.

Equation (1) is used to compute S.

The ratio P/Q is progressively made 1:10, and then 1 :100. The resistance S can be accurately measured.

Errors :

The major sources of error are the connecting wires, unclear resistance plugs, change in resistance due to Joule heating, and the insensitivity of the Wheatstone bridge.

These may be removed by using thick connecting wires, clean plugs, keeping the circuit on for very brief periods (to avoid Joule heating) and calculating the sensitivity.

In order that the sensitivity is maximum, the resistance in the arm P is close to the value of the resistance S.

METER BRIDGE

Wheatstone Bridge :

Wheatstone bridge is a special electric network in which four resistors P, Q, R and X are connected to form the four arms of a quadrilateral ABCD. A cell with key K_1 is connected across the diagonal AC and a galvanometer with key K_2 across another diagonal BD. The arrangement may be used to measure resistance of a conductor accurately.

The resistances P and Q are kept in a fixed ratio and hence they are said to form the ratio arms of the bridge. Unknown resistance is placed for resistance X and R is a resistance box to provide variable known resistance in steps.

After closing keys K_1 and K_2 resistance in resistance box R may be adjusted to obtain null deflection in galvanometer. In this situation bridge is said to be balanced. When bridge is balanced, B and D must be at same potential.

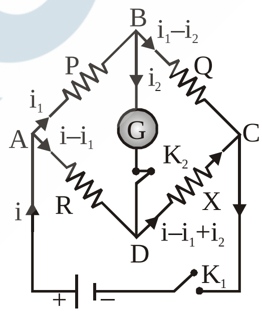
$$V_B = V_D$$

no current flows through the galvanometer i.e., $i_2 = 0$

$\therefore \quad V_A - V_B = V_A - V_D$ and $V_B - V_C = V_D - V_C$

$\Rightarrow \quad Pi_1 = R(i - i_1)$ ....(i) and $Q(i_1 - i_2) = X(i - i_1 + i_2)$ ....(ii)

$Qi_1 = X(i - i_1)$ .... (iii)



Dividing (i) and (iii), we get $\frac{P}{Q} = \frac{R}{X}$

This, is the condition of balanced bridge. [Wheatstone bridge]

In balancing condition of the bridge, unknown resistance can be calculated.

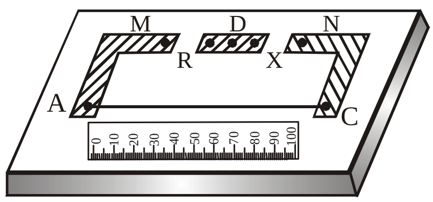
$$X = R\frac{Q}{P}$$

Two practical versions of Wheatstone bridge are

(a) Post office box, and (b) Meter bridge or slide wire bridge.

Meter Bridge:

A meter bridge consists of a one meter long wire AC, made of a homogeneous material constantan or eureka alloy and having a uniform cross-section. The wire is stretched on a wooden board. A meter scale is fixed on the board parallel to the wire. The ends A and C of the wire are fixed to two thick copper strips M and N respectively. Another strip D is fixed between strips M and N to form two gaps for introducing resistances R and X. Connecting terminals are attached at the ends of the copper strips and also in the middle of strip D for making connections.

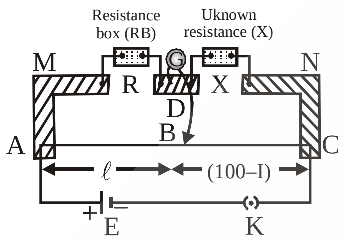


EXPERIMENT

Object :- To find resistance of a given wire using a metre bridge and hence determine the specific resistance of its material.

Apparatus required : A resistance box, a wire about 1 metre long (of the material whose specific resistance is to be determined), a metre bridge, a jockey, one- way key, a galvanometer, a battery eliminator or a cell, thick connecting wires, sand paper, screw gauge etc.

Required formula : Unknown resistance $X = R\left(\frac{100-\ell}{\ell}\right)$ and specific resistance of the material of the given wire, $\rho = \frac{XA}{L} = \frac{X(\pi r^2)}{L}$, where r and L are the radius and length of the given wire respectively.



Procedure :
1. Arrange the meter bridge and the various component as shown in figure and make tight connections. Putting RB in right gap and X in left gap
2. Take out the plug from RB to introduce a suitable resistance say, R = 2Ω and close the key K.
3. Now, touch the jockey on meter-bridge wire at different places to obtain such a position, where there is no deflection in the galvanometer. At this condition
   AB = ℓ and BC = $(100 - \ell)$
   Putting RB in left gap and X in right gap
4. Repeat step 2 and 3. But in this balancing condition BC = ℓ and AB = $(100 - \ell)$
5. Now disconnect the unknown resistance wire from the circuit. Straighten it by stretching and remove any kinks and measure its length with the help of a metre scale.
6. Measure the diameter of the wire with the help of a screw gauge.

Precautions :
1. The connections should be neat, clean and tight.
2. All the plugs in the resistance box should be tight
3. While moving the jockey to locate the balance point, the jockey should be lifted again and again and should not be pressed and slided on the wire throughout.
4. The plug in key K should be inserted only when the observations are to be taken.
5. The balance point should always be obtained near the midpoint or in between from 30 cm to 70 cm.
6. Diameter of wire should be measured in two mutually perpendicular direction at one place.

Source and of error :
1. The plugs may not be clean.
2. The meter bridge wire may not have uniform cross-section.
3. Connections may loose.
4. The screw gauge may have faulty calibration or backlash error.

OHM'S LAW

OHM'S LAW :

According to this law the ratio of the potential difference across any conductor to the electric current flowing through it is constant, under constant physical conditions i.e. temperature, pressure etc. If V is the potential difference between the two points along a conductor and I is current flowing through it, then $\frac{V}{I}$ = constant. This constant is known as resistance of the conductor represented by R.

So, V = IR

Resistance :

Obstruction in the flow of charge through a conductor is called a resistance. Its S.I. unit is Ohm (Ω).

Conductance :

The inverse of resistance is known as conductance. Its S.I. unit is $(\text{ohm})^{-1}$ or mho or siemen.

Specific resistance or resistivity :

For conductors of given material at given temperature, the resistance is directly proportional to the length ℓ and inversely proportional to the area of cross-section A, i.e.,

$$R \propto \frac{\ell}{A} \Rightarrow R = \frac{\rho\ell}{A}$$

where ρ is a constant which depends upon the nature of the material and is called resistivity or specific resistance of the conductor. For unit length and unit area of cross-section, $\rho = R$; so specific resistance is the resistance of a conductor of unit length and of unit area of cross-section. Its S.I. unit is ohm-metre.

Temperature dependenc of resistivity :

When temperature of a conductor increases, ρ increases as; $\rho_\theta = \rho_0 (1 + \alpha\theta)$. Resistance of metals increase with rise in temperature. $R_\theta = R_0 (1 + \alpha\theta)$, where α is coefficient of resistance given as

$$\alpha = \frac{R_\theta - R_0}{R_0.\theta} (°C)^{-1}$$

Electrical conductivity :

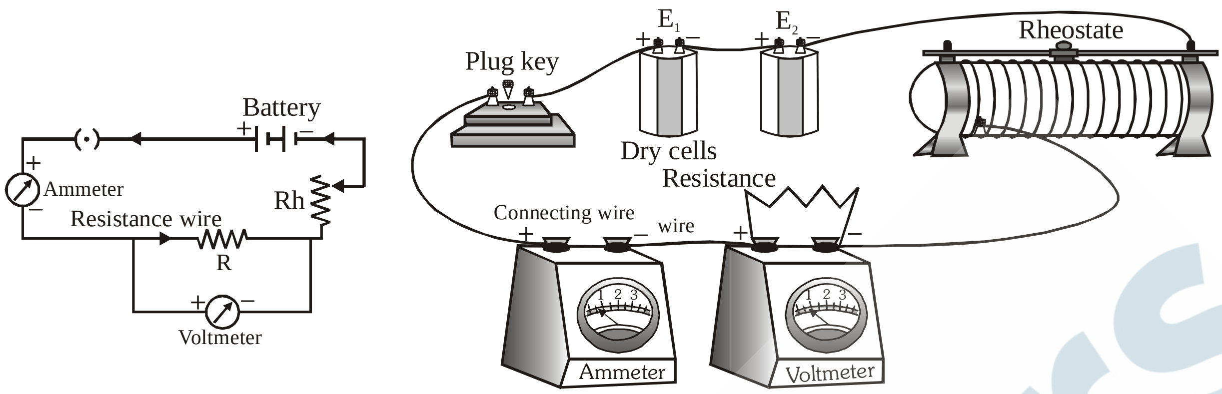
Electrical conductivity is the inverse of term resistivity and is denoted by sigma (σ) given, as,

$\sigma = \frac{1}{\rho} = \frac{\ell}{RA}$ (S.I. unit of σ is $\text{ohm}^{-1}\text{m}^{-1}$)

EXPERIMENT

Object :- To determine the resistance of a given wire by plotting a graph of potential difference versus current.

Apparatus required :- A given resistance wire in the form of a coil, a battery eliminator or an accumulator or two dry cells, a dc voltmeter, a dc ammeter, a rheostat, one plug key, thick connecting wire, sand paper etc.



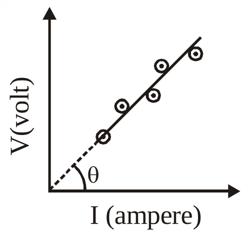
Circuit Diagram

Procedure:

1. Draw the circuit diagram as shown in figure.
2. Note the range, the least count, and the zero error of voltmeter as well as that of the ammeter.
3. Insert the plug in key K. Slide the rheostate contact to the extreme right figure. So that the current passing through the resistance wire is minimum.
4. Adjust the rheostate and record the readings of the ammeter and the voltmeter. Then shift the rheostate contact to increase the current and note readings again. Take similarly ten observations.
5. Cut the resistance wire at the ends just coming out of voltmeter and measure its length 'ℓ', using metre scale.

Graph :

Using the readings of voltmeter (V) and ammeter (I) draw a graph as straight line best fitting all the points.



Calculation :

Slope of V–I curve $= \frac{\Delta V}{\Delta I} = \tan\theta = R$

Precautions :

1. Connections should be neat, clean and tight.
2. Connecting wires should be of thick copper wires.
3. Voltmeter and ammeter should be of proper range.
4. The key/plug should be inserted only while taking observations, otherwise current gives unnecessary heating of wires.
5. The unknown resistance should not be too low (than internal resistance of battery)

Sources of error :

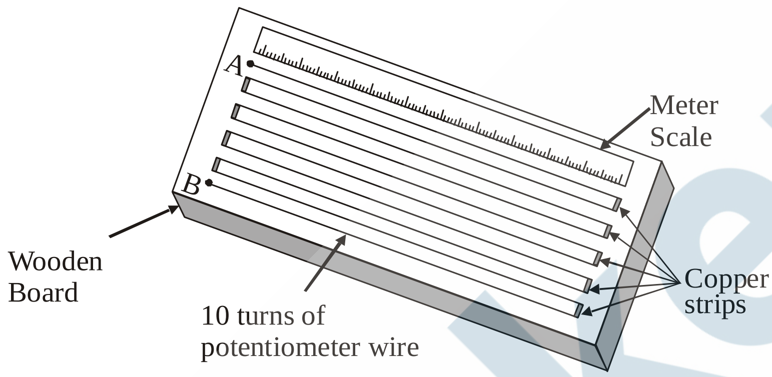
1. Connections may be loose.
2. Rheostate may have very high resistance.
3. The unknown resistance may be too low.
4. Thick connecting wires may not be available.

POTENTIOMETER

Potentiometer :

1. It is a device used to measure potential difference between two point in an electric circuit or emf of a cell with greater accuracy.
2. It behaves like an ideal voltmeter means its effective resistance is infinity.
3. It uses the concept of zero deflection during measurement.

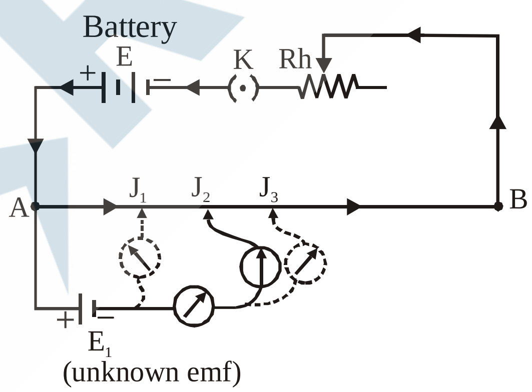
Construction :-



A thin, long, straight and uniform wire AB of constantan or manganin, which has a low temperature coefficient of resistance and high resistivity, is stretched on a wooden board along a meter-scale. The longer wire (taken long to increase accuracy) is arranged in the form of parallel wires and its ends are connected by thick strips of copper, so that the current flows through the wires in series.

Principle :

When a steady current flows through the potentiometer wire from the end A to B, as shown in figure, by a battery E then an electric potential falls uniformly along the whole length of the wire. This fall in potential per unit length of the wire, is known s "potential gradient".



Due to the constant current through potentiometer wire potential difference between any two points on the wire is directly proportional to the distance between the points, means

$V \propto \ell$

$\Rightarrow \quad V = x\ell \quad$ or $\quad \frac{V}{\ell} = x$ = potential gradient

By comparing the unknown emf, say as E_1 in the figure, with the help of a galvanometer to the uniformly distributed potential over the wire, we can find the value of unknown. So, now suppose the jockey is made to touch a point J_1 on the wire such that the potential difference between A and J_1 is lower than the emf of E_1, then a resultant current flows through the secondary circuit or galvanometer and the needle of the galvanometer deflects in one direction.

On the other hand, if the jockey is made to touch a point J_2 on the wire such that the potential difference between A and J_2 is higher than the emf of the cell E_1, then a resultant current flows in opposite direction than previous one in the galvanometer and its needle deflects in the opposite side. It is evident that in between J_1 and J_2, there will be a point J such that when the jockey is made to touch on J, there will be no deflection in the galvanometer. This point J is called **balancing/null point** and the distance AJ is called **balancing length.** At this situation.

p.d. between A and J = emf of the cell

$\Rightarrow \quad x\ell = E_1 \quad \Rightarrow \quad E_1 = x\ell$

where x - potential gradient

ℓ - balancing length

E_1 - unknown emf

Sensitivity of Potentiometer :

If on displacing the jockey slightly from the null point position, the galvanometer shows a large deflection, then the potentiometer is said to be sensitive. The sensitivity of the potentiometer depends upon the potential **gradient as :** sensitivity of potentiometer $\propto \dfrac{1}{\text{potential gradient}}$

The potential gradient can be reduced by employing a long wire in the potentiometer.

EXPERIMENT

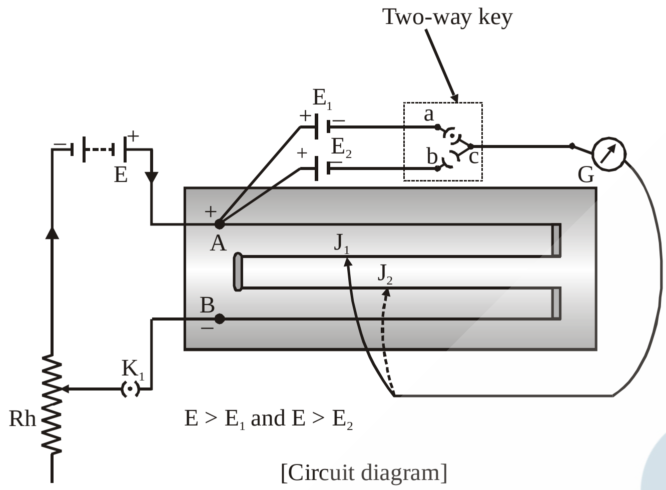
Object :- To compare the emf of two given primary cells (Daniel and Lechlanche Cell) using a potentiometer.

Apparatus required : A battery eliminator, a low resistance box about 20Ω, a one way key, two primary cells (Daniel and Lechlanche), one two way key, a resistance box plug type, a galvanometer, a potentiometer with a sliding jockey, connecting wires etc.

Formula required : $\dfrac{E_1}{E_2} = \dfrac{\ell_1}{\ell_2}$

where E_1 and E_2 are the emf's of the given primary cells and ℓ_1 and ℓ_2 are their respective balancing lengths on potentiometer wire.

Circuit diagram :



[Circuit diagram]

Procedure :

1. Arrange the circuit as shown in figure and plug key K_1.
2. In this circuit, by inserting the plug in the gap ac, cell E_1 comes in the secondary circuit. Introduce minimum resistance by Rh (rheostate) and then press the jockey at one end of the wire and note the direction of deflection in the galvanometer. Now repeat the same process with the jockey pressed near the other end of the wire and note the direction of deflection. The connections are correct if two deflections are in opposite directions.
3. If deflections are obtained in same direction, check connections and assure that $E > E_1$ and $E > E_2$.
4. After this, touch the jockey at different places on the wire and obtain a balance point, where on pressing the jockey, there is no deflection in the galvanometer note the length AJ_1 as ℓ_1 .
5. Introduce E_2 in the S circuit by transferring the plug ac to bc. Obtain the balancing point J_2 and note the length AJ_2 as ℓ_2.
6. Repeat the experiment by shifting the contact point of rheostate.

Precautions :

1. The plugs should be introduced in the keys only when the observations are to be taken.
2. The positive poles of the battery and cells E_1 and E_2 should all be connected to the terminal at zero of the wires.
3. The emf of the battery should be greater than the emfs of either of the two cells.
4. The jockey should not be rubbed along the wire. It should touch the wire gently.
5. The current should not be passed for a long time so as to avoid any heating of the wires resulting into change of resistance.
6. The balancing poing should always be obtained at large distance and if possible it should be obtained at the last wire.

Source of error :

1. The battery (E) may not be fully charged.
2. By rubbing the jockey on the wire, it may not have uniform cross-section throughout its length.
3. The resistance of copper strips may not be zero.
4. Due to the current passed through the wire for a long time, its resistance may change.

EXPERIMENT

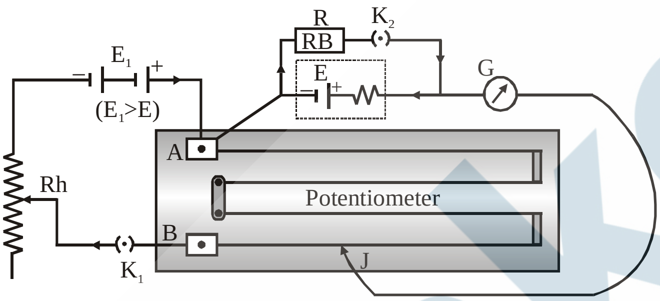
Object : To determine the internal resistance of a given primary cell by using a potentiometer.

Apparatus required : Battery eliminator or cells in series, connecting wires, a rheostate, a galvanometer, a lechlanche cell, a resistance box, two one-way keys, a potentiometer with a sliding jockey etc.

Formula required : Internal resistance of a (primary) cell is $r = R\left(\frac{\ell_1}{\ell_2} - 1\right)$

where R–resistance used across the cell, ℓ_1 and ℓ_2 are the balancing lengths for the cell when it is in open and closed circuit respectively.

Circuit diagram :



[Circuit diagram]

Procedure :

1. Make the circuit as shown and ensure that the connections are tight and clean.
2. Keeping the key K_2 open, insert the plug in key K_1 and put the jockey near the end A and press it. Note the direction of deflection in galvanometer.
   Again note the direction of deflection by moving the jockey to the other end B and pressing.
   The deflection in these cases must be in opposite directions. If it is not so, then check the connections and adjust the rheostate so that the above condition is achieved.
3. Keeping K_2 open and K_1 close, obtain the balancing length for the emf of cell and note it as ℓ_1.
4. Introduce some resistance (R) from the resistance box and close K_2 now and then obtain balancing length again and record it as ℓ_2.
5. Repeat the experiment for the different values of R and obtain different readings of ℓ_2.

Precautions :

1. Reading should be taken quickly as possible.
2. The cell should not be disturbed during the course of experiment.
3. The positive terminals of battery and cell should be connected at one point A.
4. Emf of the battery should be greater than the emf of lechanche cells.
5. All connections should be tight, neat and clean
6. The jockey should not be rubbed along the wire. It should touch the wire gently.

Source of error :

1. The battery (E) may not be fully charged.
2. By rubbing the jockey on the wire, it may not have uniform cross-section throughout its length.
3. The resistance of copper strips may not be zero.
4. Due to the current passed through the wire for a long time, its resistance may change.

GALVANOMETER

Galvanometer :

- It is a device which is used to detect the presence of current or to measure weak current in the circuit.
- It is based on the principle that a current carrying coil placed in a uniform magnetic field experiences a torque.
- Basically, galvanometers are of two types :
  (i) Moving coil type
  (ii) Moving Magnet type (Tangent Galvanometer)

Principle of a moving coil galvanometer

When a current I is flowing through a coil in a uniform magnetic field, the coil experiences first deflecting torque τ =NAIB and then restoring torque $\tau = C\theta$. Because both the torque oppose each other, therefore in equilibrium.

$$\tau_{\text{deflecting}} = \tau_{\text{restoring}}$$

$$NIAB = c\theta \Rightarrow \quad I = \left(\frac{C}{BAN}\right)\theta = k\theta$$

$\Rightarrow \quad I \propto \theta \quad$ or $\quad$ current α deflection

where $k = \dfrac{C}{BAN}$ = Galvanometer's constant

i.e. deflection of the needle in the galvanometer is directly proportional to the current flowing through it.

(i) **Figure of Merit :** Current required to produce a deflection of one division is called figure of merit of galvanometer.

$$F = \frac{I}{\theta} = k$$

(ii) **Current Sensitivity :** Deflection per unit current is called current sensitivity of galvanometer.

$$S_C = \frac{\theta}{I} = \frac{1}{k} = \frac{1}{F} = \text{reciprocal of figure of merit}$$

(iii) **Voltage sensitivity :** Deflection per unit voltage i.e. $S_V = \dfrac{\theta}{V} = \dfrac{\theta}{IR} = \dfrac{S_C}{R}$

Sp. Note :

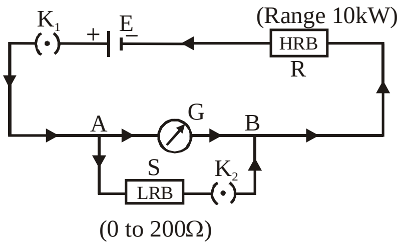
Suspension type moving coil galvanometers are used in advanced laboratories in colleges and universities for very accurate measurements whereas in school laboratories pivoted type (weston) galvanometers are used.

EXPERIMENT

Object : (a) To determine the resistance of a galvanometer by half deflection method and
(b) To find its figure of merit.

Apparatus required : A galvanometer, a battery or accumulator, a low resistance box (LRB), a high resistance box (HRB), two one-way keys, connecting wires etc.

Circuit diagram :

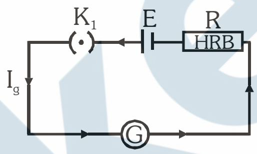


[Circuit diagram]

Theory :

(a) **Resistance of galvanometer by half deflection method :** When a high resistance R is applied in the circuit with K_1 closed and K_2 open, the galvanometer draws a current I_g and shows a deflection θ such that

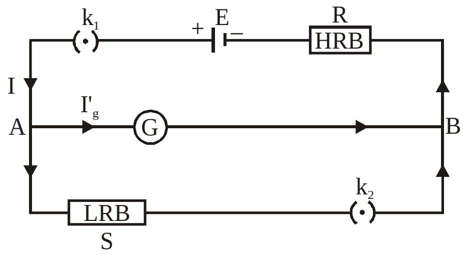
$$I_g = \frac{E}{R+G} \qquad .....(i)$$



where E, is emf of the battery and G is resistance of the galvanometer.

Now K_2 is closed. Adjust the resistance in LRB such that galvanometer deflection becomes equal to $\frac{\theta}{2}$. Now the galvanometer draw the current.

$$I'_g = \frac{IS}{G+S} = \frac{ES}{R(G+S)+GS} \qquad \left(\text{where } I = \frac{E}{R+\frac{GS}{G+S}}\right)$$



Also $\qquad I'_g = \frac{1}{2} I_g$

$\therefore \qquad \frac{ES}{R(G+S)+GS} = \frac{1}{2} \cdot \frac{E}{R+G}$

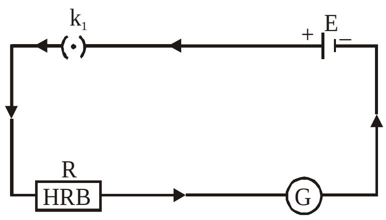
gives $\qquad G = \frac{RS}{R-S}$

Knowing R and S, G can be calculated.

Also if R > > S, S can be dropped in comparison to R and then $\boxed{G \simeq S}$.

(b) **Figure of merit of galvanometer** is defined as the current required per division of deflection in galvanometer. It is denoted by k. $\qquad$ Figure merit $k = \frac{I}{\theta}$

The circuit diagram for determination of figure of merit (k) of a galvanometer is shown in the figure. When a high resistance R is introduced in the circuit



through HRB, a small current I_g is drawn by it and it shows a deflection θ such that

$$I_g = k\theta = \frac{E}{R+G}$$

$\Rightarrow$ Figure of merit $k = \frac{1}{\theta}\left(\frac{E}{R+G}\right)$ ....(ii)

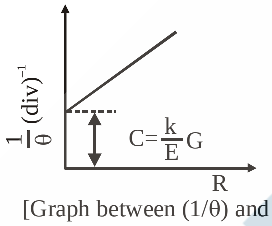
Maximum current measured by galvanometer or full scale deflection current for galvanometer

I_g = Number of division on one side of galvanometer scale × figure of merit

From equation (ii)

$$\frac{1}{\theta} = \frac{k}{E}R + \frac{k}{E}G \quad .....(iii)$$

$\therefore$ Graph between $\frac{1}{\theta}$ and R is as shown.



[Graph between (1/θ) and R]

Procedure :

1. In the circuit, introduce a high resistance R in HRB and then insert the plug in the key K_1. Adjust the value of R to get deflection θ in even number of divisions.
2. Now close the key K_2 also and adjust the shunt resistance S from low resistance box (LRB) to get a deflection exactly half of θ in galvanometer.
3. Repeat experiment for different values of R and θ.

Precautions :

1. All connections in the circuit should be neat, clean and tight.
2. All the plugs in HRB and LRB should be tight.
3. A very high resistance R from HRB should be introduced first and then key K_1 should be closed to avoid any over current damage in galvanometer.
4. The emf of the cell or battery should be constant.
5. The deflection in the galvanometer should be as large as possible and should be even number of divisions.

Sources of error

1. The plugs of HRB and LRB may not be clean.
2. The emf of the battery may not be constant.
3. The galvanometer divisions may not be uniform.

EXERCISE (S)

1. How many significant figures are given in the following quantities ?

(A) 343 g (B) 2.20 (C) 1.103 N (D) 0.4142 s

(E) 0.0145 m (F) 1.0080 V (G) 9.1×10^4 km (H) 1.124×10^{-3} V

2. Perform the following operations:

(A) 703 + 7 + 0.66 (B) 2.21×0.3 (C) 12.4×84 (D) 14.28/0.714

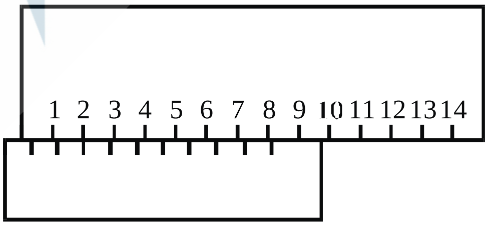
3. Solve with due regard to significant digits

(i) $\sqrt{6.5-6.32}$ (ii) $\dfrac{2.91\times 0.3842}{0.080}$

4. The main scale of a vernier calipers reads in millimeter and its vernier is divided into 10 divisions which coincide with 9 divisions of the main scale. When the two jaws of the instrument touch each other the seventh division of the vernier scale coincide with a scale division and the zero of the vernier lies to the right of the zero of main scale. Furthermore, when a cylinder is tightly placed along its length between the two jaws, the zero of the vernier scale lies slightly to the left of 3.2 cm; and the fourth vernier division coincides with a scale division. Calculate the measured length of the cylinder.

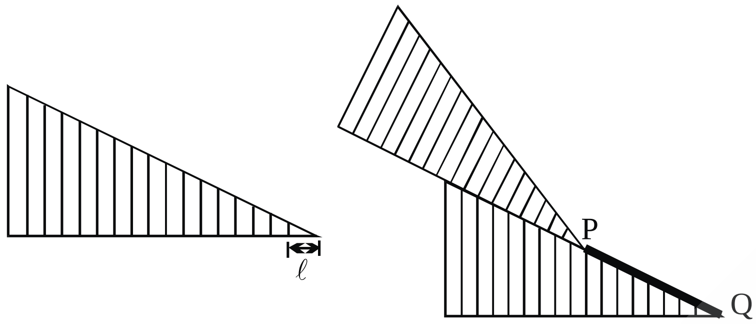
5. The VC shown in the diagram has zero error in it (as you can see). It is given that 9 msd = 10 vsd.

(i) What is the magnitude of the zero error? (1msd = 1 mm)



(ii) The observed reading of the length of a rod measured by this VC comes out to be 5.4 mm. If the vernier had been error free then reading of main scale would be ___ and the coinciding division of vernier scale would be _____.

6. Consider a home made vernier scale as shown in the figure.



In this diagram, we are interested in measuring the length of the line PQ. If both the inclines are identical and their angles are equal to θ then what is the least count of the instrument.

7. The pitch of a screw gauge is 0.5 mm and there are 50 divisions on the circular scale. In measuring the thickness of a metal plate, there are five divisions on the pitch scale (or main scale) and thirty fourth division coincides with the reference line. Calculate the thickness of the metal plate.

8. The pitch of a screw gauge is 1 mm and there are 50 divisions on its cap. When nothing is put in between the studs, 44^{th} division of the circular scale coincides with the reference line zero of the main scale is not visible. When a glass plate is placed between the studs, the main scale reads three divisions and the circular scale reads 26 divisions. Calculate the thickness of the plate.

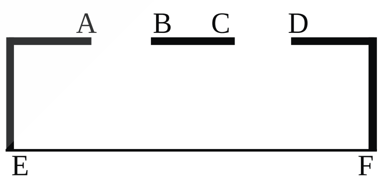
9. In a given optical bench, a needle of length 10 cm is used to estimate bench error. The object needle, image needle & lens holder have their reading as shown.

$x_0 = 1.1$ cm $\quad x_I = 21.0$ cm $\quad x_L = 10.9$ cm

Estimate the bench errors which are present in image needle holder and object needle holder. Also find the focal length of the convex lens when.

$x_0 = 0.6$ cm $\quad x_I = 22.5$ cm $\quad x_L = 11.4$ cm

10. Make the appropriate connections in the meter bridge set up shown. Resistance box is connected between ____. Unknown resistance is connected between____. Battery is connected between _______.



11. A body travels uniformly a distance of (13.8 ± 0.2)m in time (4.0 ± 0.3) sec. Calculate its velocity.

12. Consider S = x cos (θ) for x = (2.0 ± 0.2) cm, θ = 53 ± 2 °. Find S.

13. Two resistance R_1 and R_2 are connected in (i) series and (ii) parallel. What is the equivalent resistance with limit of possible percentage error in each case of $R_1 = 5.0 \pm 0.2\ \Omega$ and $R_2 = 10.0 \pm 0.1\ \Omega$.

14. 5.74 gm of a substance occupies a volume of 1.2 cm^3. Calculate its density with due regard for significant figures.

15. The time period of oscillation of a simple pendulum is given by T = $2\pi\sqrt{l/g}$. The length of the pendulum is measured as l = 10.0 ± 0.1cm and the time period as T = 0.50 ± 0.02 s. Determine percentage error in the value of g.

16. In a Searle's experiment, the diameter of the wire as measured by a screw gauge of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data. **[JEE 2004]**

17. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47^{th} division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in appropriate number of significant figures. **[JEE 2004]**

18. A physical quantity P is related to four observables A, B, C and D as P = $4\pi^2A^3B^2/(\sqrt{C}\,D)$

The percentage error of the measurement in A, B, C and D are 1%, 3% and 2%, 4% respectively. Determine the percentage error & absolute error in the quantity P. Value of P is calculated 3.763.

19. A glass prism of angle A = 60° gives minimum angle of deviation θ = 30° with the max. error of 1^0 when a beam of parallel light passed through the prism during an experiment. Find the permissible error in the measurement of refractive index μ of the material of the prism.

20. In a vernier calipers the main scale and the vernier scale are made up of different materials. When the room temperature increases by ΔT°C, it is found the reading of the instrument remains the same. Earlier it was observed that the front edge of the wooden rod placed for measurement crossed the N^{th} main scale division and (N + 2) msd coincided with the 2^{nd} vsd. Initially, 10 vsd coincided with 9 msd. If coefficient of linear expansion of the main scale is α_1 and that of the vernier scale is α_2 then what is the value of α_1/α_2? (Ignore the expansion of the rod on heating)

21. In a vernier callipers, n divisions of its main scale match with (n + 1) divisions on its vernier scale. Each division of the main scale is a units. Using the vernier principle, calculate its least count. **[JEE 2003]**

22. The period of oscillation of a simple pendulum in an experiment is recorded as 2.63 sec, 2.56 sec, 2.42 sec, 2.71 sec and 2.80 sec respectively. Find the time period, the absolute error in each observation, average absolute error and the percentage error.

23. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures. **[JEE 2005]**

EXERCISE (O)

1. A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.06 cm. The maximum percentage error in the measurement of its density is :- **[JEE 2004]**

(A) 1 (B) 2 (C) 3 (D) 4

2. The edge of a cube is $a = 1.2 \times 10^{-2}$ m. Then its volume will be recorded as : **[JEE 2003]**

(A) 1.7×10^{-6} m^3 (B) 1.70×10^{-6} m^3

(C) 1.70×10^{-7} m^3 (D) 1.78×10^{-6} m^3

3. A vernier callipers having 1 main scale division = 0.1 cm is designed to have a least count of 0.02 cm. If n be the number of divisions on vernier scale and m be the length of vernier scale, then :-

(A) n = 10, m = 0.5 cm (B) n = 9, m = 0.4 cm

(C) n = 10, m = 0.8 cm (D) n = 10, m = 0.2 cm

4. In the Searle's experiment, after every step of loading , why should we wait for two minutes before taking the readings? (More than one correct.)

(A) So that the wire can have its desired change in length.

(B) So that the wire can attain room temperature.

(C) So that vertical oscillations can get subsided.

(D) So that the wire has no change in its radius.

5. In a meter bridge set up, which of the following should be the properties of the one meter long wire?

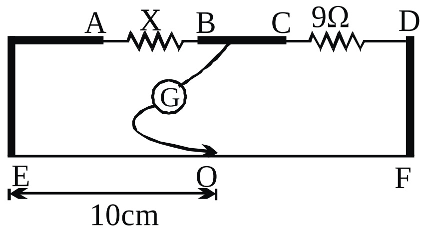
(A) High resistivity and low temperature coefficient

(B) Low resistivity and low temperature coefficient

(C) Low resistivity and high temperature coefficient

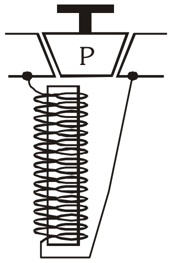
(D) High resistivity and high temperature coefficient

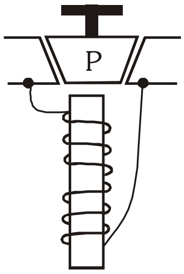
6. Consider the MB shown in the diagram, let the resistance X have temperature coefficient α_1 and the resistance from the RB have the temperature coefficient α_2. Let the reading of the meter scale be 10cm from the LHS. If the temperature of the two resistance increase by small temperature ΔT then what is the shift in the position of the null point? Neglect all the other changes in the bridge due to temperature rise.

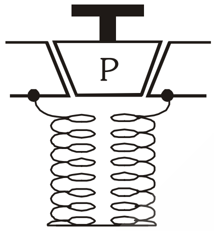


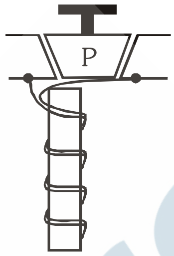
(A) $9(\alpha_1 - \alpha_2)\Delta T$ (B) $9(\alpha_1 + \alpha_2)\Delta T$ (C) $\frac{1}{9}(\alpha_1 + \alpha_2)\Delta T$ (D) $\frac{1}{9}(\alpha_1 - \alpha_2)\Delta T$

7. Identify which of the following diagrams represent the internal construction of the coils wound in a resistance box or PO box?

(A) 

(B) 

(C) 

(D) 

8. In a meter bridge experiment, we try to obtain the null point at the middle. This

(A) reduces systematic error as well as random error.

(B) reduces systematic error but not the random error.

(C) reduces random error but not the systematic error

(D) reduces neither systematic error nor the random error

9. An approximate value of number of seconds in an year is $\pi \times 10^7$. Determine the % error in this value

(A) 0.5% (B) 8% (C) 4% (D) 15%

10. In a Searle's experiment for determination of Young's Modulus, when a load of 50 kg is added to a 3 meter long wire micrometer screw having pitch 1 mm needs to be given a quarter turn in order to restore the horizontal position of spirit level. Young's modulus of the wire if its cross sectional area is 10^{-5} m^2 is

(A) $6\times 10^{11} N/m^2$ (B) 1.5×10^{11} N/m^2 (C) 3×10^{11} N/m^2 (D) None

11. On the basis of detail given about two measuring instruments, select the correct statement.

(i) Vernier callipers having main scale division = 0.05 cm and Vernier scale division = $\frac{2.45}{50}$ cm.

(ii) Screw gaugae having pitch 0.5 mm and its circular scale division measures 0.01 mm.

(A) Both the instruments have same least count.

(B) Least count of screw gauge is lesser than that of vernier callipers.

(C) Both the instruments have same least count but screw gauge is more precise.

(D) Both the instruments have different least count and screw gauge is more precise.

12. A student obtained following observations in an experiment of meter bridge to find unknown resistance of given wire :

S.No.	**R**	ℓ	**100 – ℓ**	$S=\left(\frac{100-\ell}{\ell}\right)R$
1	2Ω	43	57	2.65
2	3Ω	52	48	2.77
3	4Ω	58	42	2.89
4	5Ω	69	31	2.25

The most accurate value of unknown resistance will be

(A) 2.65 Ω (B) 2.77 Ω (C) 2.89 Ω (D) 2.25 Ω

13. In which of the following instruments used in the lab there exists an error of random category known as back lash error

(i) Screw gauge (ii) Spherometer (iii) Searle's apparatus (iv) Vernier callipers

(A) (i) & (ii) only (B) (i), (ii) & (iii) only (C) (i) only (D) all four

14. In Searle's apparatus, when experimental wire is loaded and unloaded, the air bubble in spirit level gets shifted.

(A) towards reference wire while loading & towards experimental wire while unloading

(B) towards experimental wire while loading & towards reference wire while unloading

(C) towards experimental wire, both the times, during loading & unloading

(D) towards reference wire, both the times during loading & unloading

15. Accuracy and precision are ______(i)________ and these are respectively linked with ______(ii) & _____(iii)______ .Fill the blanks above in correct order.

(A) (i) same, (ii) systematic error, (iii) random error

(B) (i) different, (ii) systematic error (iii) random error

(C) (i) same, (ii) random error, (iii) systematic error

(D) (i) different, (ii) random error, (iii) systematic error

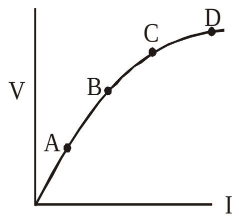
16. The vernier of a circular scale is divided in to 30 divisions, which coincides with 29 main scale divisions. If each main scale division is $(1/2)^o$, the least count of the instrument is

(A) 0.1' (B) 1' (C) 10' (D) 30'

17. When the gap is closed without placing any object in the screw gauge whose least count is 0.005 mm, the 5^{th} division on its circular scale coincides with the reference line on main scale, and when a small sphere is placed reading on main scale advances by 4 divisions, whereas circular scale reading advances by five times to the corresponding reading when no object was placed. There are 200 divisions on the circular scale. The radius of the sphere is

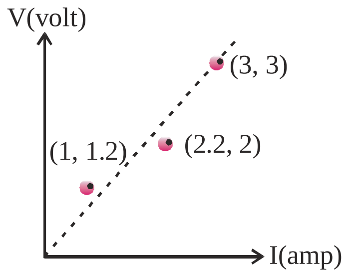
(A) 4.10 mm (B) 4.05 mm (C) 2.10 mm (D) 2.05 mm

18. Variation of current passing through a conductor as the voltage supplied across its ends as varied is shown in the adjoining diagram. If the resistance (R) is determined at the points A, B, C and D we will find that -



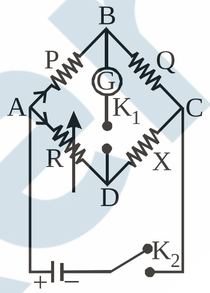
(A) $R_C = R_D$ (B) $R_B > R_A$ (C) $R_C > R_B$ (D) $R_A > R_C$

19. In the measurement of resistance of a wire using Ohm's law, the plot between V and I is drawn as shown. The resistance of the wire is -



(A) 0.833 Ω (B) 0.9 Ω (C) 1 Ω (D) None of these

20. In Wheatstone bridge experiment as shown in figure –



(A) Key K_1 should be pressed first and then K_2

(B) Key K_2 should be pressed first and then K_1

(C) any key can be pressed in any order

(D) both keys should be pressed simultaneously.

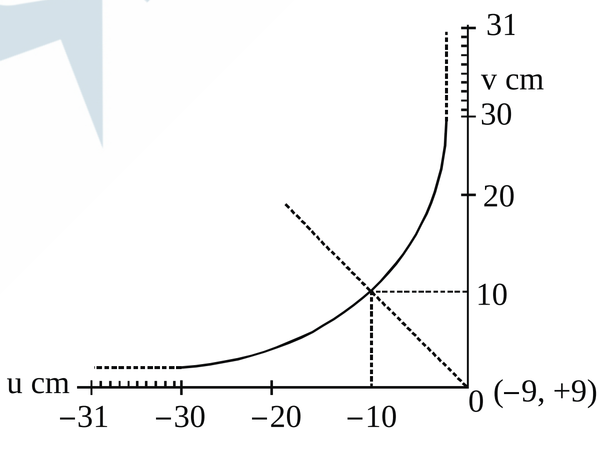
21. In a metre bridge experiment null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If X < Y, then where will be the new position of the null point from the same end, if one decide to balance a resistance of 4X against Y -

(A) 50 cm (B) 80 cm (C) 40 cm (D) 70 cm

22. In a resonance column method, resonance occurs at two successive level of l_1=30.7 cm and l_2 = 63.2 cm using a tuning fork of f = 512 Hz. What is the maximum error in measuring speed of sound using relations $v = f\lambda$ & $\lambda = 2(l_2 - l_1)$ **[JEE 2005]**

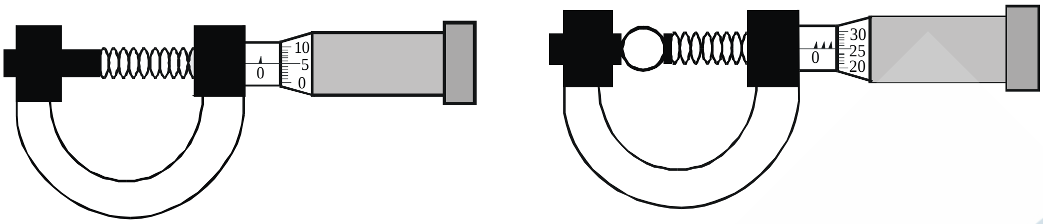
(A) 256 cm/sec (B) 92 cm/sec (C) 128 cm/sec (D) 102.4 cm/sec

23. Graph of position of image vs position of point object from a convex lens is shown. Then, focal length of the lens is **[JEE 2006]**



(A) 0.50 ± 0.05 cm (B) 0.50 ± 0.10 cm (C) 5.00 ± 0.05 cm (D) 5.00 ± 0.10 cm

24. The circular divisions of shown screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball is **[JEE 2006]**



(A) 2.25 mm (B) 2.20 mm (C) 1.20 mm (D) 1.25 mm

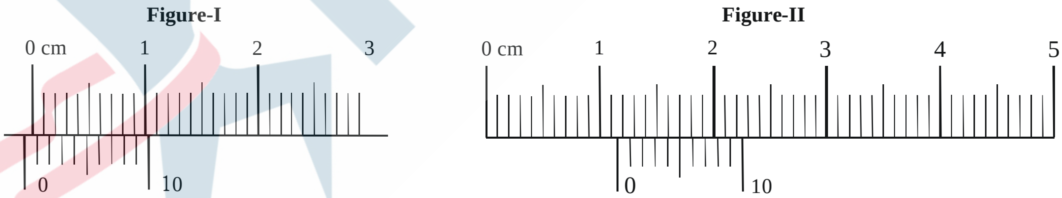
25. A student performs an experiment for determination of $g\left(=\frac{4\pi^2 l}{T^2}\right)$ $l \approx 1$m and he commits an error of Δl. For the experiment takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1sec. For which of the following data, the measurement of g will be most accurate? **[JEE 2006]**

	Δl	ΔT	n	Amplitude of oscillation
(A)	5 mm	0.2 sec	10	5 mm
(B)	5 mm	0.2 sec	20	5 mm
(C)	5 mm	0.1 sec	20	1 mm
(D)	1 mm	0.1 sec	50	1 mm

26. In an experiment to determine the focal length (f) of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then, **[JEE 2007]**

(A) $x < f$ (B) $f < x < 2f$ (C) $x = 2f$ (D) $x > 2f$

27. In ordinary Vernier calipers, 10th division of the Vernier scale coincides with 9th division of the main scale. In a specially designed Vernier calipers the Vernier scale is so constructed that 10th division on it coincides with 11th division on the main scale. Each division on the main scale equals to 1 mm. The calipers have a zero error as shown in the figure-I. When the Vernier caliper is used to measure a length, the concerned portion of its scale is shown in figure-II.



(A) Zero error in the calipers has magnitude 0.7 mm.
(B) The length being measured is 1.08 cm.
(C) The length being measured is 1.22 cm.
(D) Zero error in the calipers has magnitude 0.3 mm

28. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained from the reading is **[JEE 2007]**

(A) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2 (B) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2
(C) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2 (D) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

EXERCISE-JM

1. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are meausred by **[AIEEE - 2008]**

(1) a vernier scale provided on the microscope

(2) a stanard laboratory scale

(3) a meter scale provided on the microscope

(4) a screw gauge provided on the microscope

2. Two full turns of the circular scale of gauge cover a diastance of 1 mm on scale. The total number of divisions on circular scale is 50. Further, it is found that screw gauge has a zero error of –0.03 mm. While measuring the diameter of a thin wire a student notes the main scale reading of 3 mm and the number of circular scale division in line, with the main scale as 35. The diameter of the wire is **[AIEEE - 2008]**

(1) 3.32 mm (2) 3.73 mm (3) 3.67 mm (4) 3.38 mm

3. In an experiment the angles are required to be measured using an instrument 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (=0.5°), then the least count of the instrument is :-**[AIEEE - 2009]**

(1) One degree (2) Half degree (3) One minute (4) Half minute

4. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be :-**[AIEEE - 2009]**

(1) (f, f) (2) $(4f, 4f)$ (3) $(2f, 2f)$ (4) $\left(\frac{f}{2}, \frac{f}{2}\right)$

5. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are :- **[AIEEE - 2010]**

(1) 4, 4, 2 (2) 5, 1, 2 (3) 5, 1, 5 (4) 5, 5, 2

6. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm.

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above date is :- **[AIEEE - 2011]**

(1) 0.026 cm (2) 0.005 cm (3) 0.52 cm (4) 0.052 cm

7. A spectrometer gives the following reading when used to measure the angle of a prism. Main scale reading : 58.5 degree

Vernier scale reading : 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data : **[AIEEE - 2012]**

(1) 59 degree (2) 58.59 degree (3) 58.77 degree (4) 58.65 degree

8. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is :-**[AIEEE - 2012]**

(1) 3% (2) 6% (3) zero (4) 1%

9. The current voltage relation of diode is given by I = $(e^{1000V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm$ 0.01V while measuring the current of 5 mA at 300 K, what will be error in the value of current in mA ? **[JEE-Main 2014]**

(1) 0.5 mA (2) 0.05 mA (3) 0.2 mA (4) 0.02 mA

10. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it ? **[JEE-Main 2014]**

(1) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

(2) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

(3) A meter scale

(4) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

11. The period of oscillation of a simple pendulum is T = $2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is : **[JEE-Main 2015]**

(1) 1% (2) 5% (3) 2% (4) 3%

12. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement , it is found that when the two jaws of the screw gauge are brought in contact, the 45^{th} division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25^{th} division coincides with the main scale line ? **[JEE-Main 2016]**

(1) 0.50 mm (2) 0.75 mm (3) 0.80 mm (4) 0.70 mm

13. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be :- **[JEE-Main 2016]**

(1) 92 $\pm$ 3 s (2) 92 $\pm$ 2 s (3) 92 $\pm$ 5.0 s (4) 92 $\pm$ 1.8 s

14. To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection then G, R and S are related by the equation : **[JEE-Mains (Online) 2016]**

(1) 2S (R+G) = RG (2) S (R+G) = RG (3) 2G = S (4) 2S = G

15. The following observations were taken for determining surface tensiton T of water by capillary method:

Diameter of capilary, D = 1.25×10^{-2} m

rise of water, h = 1.45×10^{-2} m

Using g = 9.80 m/s^2 and the simplified relation $T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to : **[JEE-Main 2017]**

(1) 2.4% (2) 10% (3) 0.15% (4) 1.5%

16. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is :- **[JEE-Main 2018]**

(1) 3.5 % (2) 4.5 % (3) 6 % (4) 2.5 %

EXERCISE - JA

1. Students I, II and III perform an experiment for measuring the acceleration due to gravity (g) using pendulum. They use different lengths of the pendulum and/or record time for different number of oscillations. The observations are shown in the table. **[JEE 2008]**

Least count for length = 0.1 cm   Least count for time = 0.1 s

Student	Length of the Pendulum (cm)	Number of oscilltions (n)	Total time for (n) oscillations (s)	Time period (s)
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage error in g, i.e., $\left(\frac{\Delta g}{g}\times 100\right)$ for students I, II and III, respectively,

(A) $E_I = 0$ (B) E_I is minimum

(C) $E_I = E_{II}$ (D) E_{II} is maximum

2. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stopwatch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true? **[JEE 2010]**

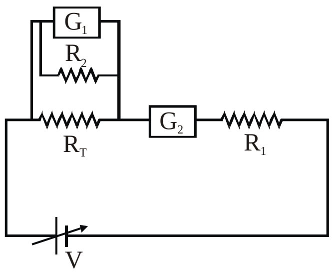
(A) Error ΔT in measuring T, the time period is 0.05 seconds

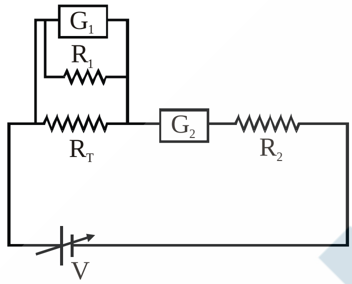
(B) Error ΔT in measuring T, the time period is 1 second

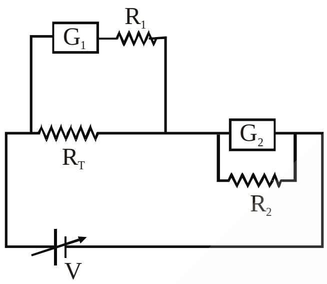
(C) Percentage error in the determination of g is 5%

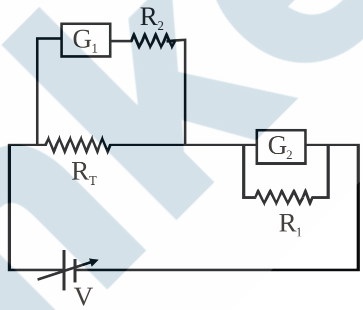
(D) Percentage error in the determination of g is 2.5%

3. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is :– **[JEE 2010]**

(A)


(B)


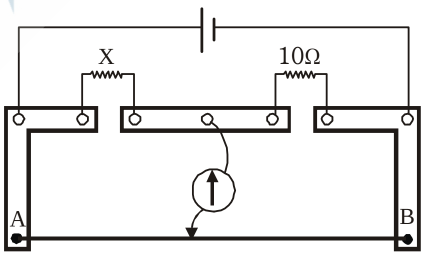
(C)


(D)


4. A Vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is **[JEE 2010]**

(A) 0.02 mm (B) 0.05 mm (C) 0.1 mm (D) 0.2 mm

5. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is **[JEE 2011]**



(A) 10.2 ohm (B) 10.6 ohm (C) 10.8 ohm (D) 11.1 ohm

6. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is **[JEE 2011]**

(A) 0.9% (B) 2.4% (C) 3.1% (D) 4.2%

7. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi \ell d^2}\right)$ by using Searle's method, a wire of length L = 2m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension l = 0.25 mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and l are the same.

(B) due to the error in the measurement of d is twice that due to the error in the measurement of l.

(C) due to the error in the measurement of l is twice that due to the error in the measurement of d.

(D) due to the error in the measurement of d is four times that due to the error in the measurement of l. **[JEE 2012]**

8. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24^{th} division of the Vernier scale excatly coincides with one of the main scale divisions. The diameter of the cylinder is :- **[JEE-Advance 2013]**

(A) 5.112 cm (B) 5.124 cm (C) 5.136 cm (D) 5.148 cm

9. Using the expression 2d sin θ = λ, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0° :- **[JEE-Advance 2013]**

(A) the absolute error in d remains constant (B) the absolute error in d increases

(C) the fractional error in d remains constant (D) the fractional error in d decreases

10. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2}m of the main scale. The 20^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45^{th} division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is. **[JEE-Advance 2014]**

11. Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then :

[JEE-Advance 2015]

(A) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

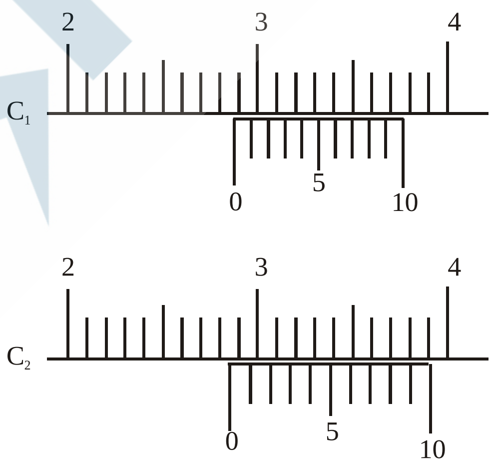
(B) If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

(D) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

12. The energy of a system as a function of time t is given as $E(t) = A^2\exp(-\alpha t)$, where $\alpha = 0.2s^{-1}$. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of E(t) at t = 5 s is. **[JEE-Advance 2015]**

13. There are two vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively, are **[JEE-Advance 2016]**



(A) 2.87 and 2.86 (B) 2.87 and 2.87 (C) 2.87 and 2.83 (D) 2.85 and 2.82

14. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true ?

(A) The error in the measurement of r is 10% **[JEE-Advance 2016]**

(B) The error in the measurement of T is 3.57 %

(C) The error in the measurement of T is 2%

(D) The error in the determined value of g is 11%

15. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is δT = 0.01 second and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity g = 10 ms^{-2} and the velocity of sound is 300 ms^{-1}. Then the fractional error in the measurement, δL/L, is closest to **[JEE-Advance 2017]**

(A) 0.2 % (B) 5 % (C) 3 % (D) 1 %

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation z = x/y. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

[JEE-Advance 2018]

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in Δy/y, is $1 \mp (\Delta y / y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\frac{\Delta x}{x} << 1, \frac{\Delta y}{y} << 1$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on Paragraph "A", the question given below is one of them)

16. Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is Δa(Δa/a << 1), then what is the error Δr in determining r ?

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$ (C) $\frac{2\Delta a}{(1-a^2)}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

PARAGRAPH "A"

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation z = x/y. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}.$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in Δy/y, is $1 \mp (\Delta y / y)$. The relative errors in independent variables are always added. So the error in z will be

$$\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).$$

The above derivation makes the assumption that $\frac{\Delta x}{x} << 1, \frac{\Delta y}{y} << 1$. Therefore, the higher powers of these quantities are neglected.

(There are two questions based on Paragraph "A", the question given below is one of them)

17. In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| << 1$, In (1 + x) = x up to first power in x. The error Δλ, in the determination of the decay constant λ, in s^{-1}, is :-

(A) 0.04 (B) 0.03 (C) 0.02 (D) 0.01

18. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2kg, the vernier scale division which coincides with a main scale division is....... Take g = 10 ms^{-2} and π = 3.2. **[JEE-Advance 2018]**

ANSWER KEY

EXERCISE (S)

1. Ans. (A) 3, (B) 3, (C) 4, (D) 4, (E) 3, (F) 5, (G) 2, (H) 4

2. Ans. (A) 711, (B) 0.7, (C) 1.0×10^3, (D) 20.0

3. Ans. (i) 0.4, (ii) 14

4. Ans. 3.07 cm

5. Ans. (i) x = –0.7 msd, (ii) 6, 1

6. Ans. L.C. = $l\left[\frac{1-\cos\theta}{\cos\theta}\right]$

7. Ans. 2.84 mm

8. Ans. $R_t = 3.64$ mm

9. Ans. 5.5 $\pm$ 0.05 cm

10. Ans. CD, AB, EF

11. Ans. v = (3.4 ± 0.4) m/s

12. Ans. S = (1.2 $\pm$ 0.18) cm

13. Ans. $R_8 = 15.0\ \Omega \pm 2\%$, $R_p = 3.3\ \Omega \pm 3\%$

14. Ans. 4.8 g/cm^3

15. Ans. 9%

16. Ans. $\Delta Y = 0.0489Y = 1.1 \times 10^{10}\ N/m^2$

17. Ans. 2.6 cm^2 (in two significant figures)

18. Ans. 14%, 0.53

19. Ans. $5\pi/18\%$

20. Ans. 1.8 /(N+2)

21. Ans. $\frac{a}{n+1}$

22. Ans. T = 2.62 s. 0.01 s, – 0.06 s, – 0.20 s, 0.09 s, 0.18 s, Average absolute error = 0.11 s, 4.2 %

23. Ans. 2.66 g/cm^3

EXERCISE (O)

1. Ans. (D)	**2. Ans. (A)**	**3. Ans. (C)**	**4. Ans. (A,B,C)**	**5. Ans. (A)**	**6. Ans. (A)**
7. Ans. (D)	**8. Ans. (A)**	**9. Ans. (A)**	**10. Ans. (A)**	**11. Ans. (A)**	**12. Ans. (B)**
13. Ans. (B)	**14. Ans. (A)**	**15. Ans. (B)**	**16. Ans. (B)**	**17. Ans. (D)**	**18. Ans. (D)**
19. Ans. (C)	**20. Ans. (B)**	**21. Ans. (A)**	**22. Ans. (D)**	**23. Ans. (C)**	**24. Ans. (C)**
25. Ans. (D)	**26. Ans. (B)**	**27. Ans. (A,C)**	**28. Ans. (A, B)**		

EXERCISE-JM

1. Ans. (1)	**2. Ans. (4)**	**3. Ans. (3)**	**4. Ans. (3)**	**5. Ans. (2)**	**6. Ans. (4)**
7. Ans. (4)	**8. Ans. (2)**	**9. Ans. (3)**	**10. Ans. (4)**	**11. Ans. (4)**	**12. Ans. (3)**
13. Ans. (2)	**14. Ans. (2)**	**15. Ans. (4)**	**16. Ans. (2)**		

EXERCISE - JA

1. Ans. (B)	**2. Ans. (A,C)**	**3. Ans. (C)**	**4. Ans. (D)**	**5. Ans. (B)**	**6. Ans. (C)**
7. Ans. (A)	**8. Ans. (B)**	**9. Ans. (D)**	**10. Ans. 4**	**11. Ans. (B,C)**	**12. Ans. 4**
13. Ans. (C)	**14. Ans. (A, B, D)**		**15. Ans. (D)**	**16. Ans. (B)**	**17. Ans. (C)**

18. Ans. 3 [2.99, 3.01]