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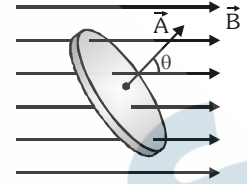
ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

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ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

MAGNETIC FLUX

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector) then $\phi = \vec{B} \cdot \vec{A} = BA \cos \theta$



FLUX LINKAGE

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is $\phi = BA \cos \theta$. If the coil has N turns, the total flux linkage $\phi = NBA \cos \theta$

- Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = B \times \text{area} = \left[\frac{F}{IL} \right] [L^2] \quad \therefore B = \frac{F}{IL \sin \theta} \quad [\because F = B I L \sin \theta]$$

$$\therefore [\phi] = \left[\frac{MLT^{-2}}{AL} \right] [L^2] = [ML^2 T^{-2} A^{-1}]$$

SI UNIT of magnetic flux :

$\therefore [ML^2 T^{-2}]$ corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{weber (Wb)}$$

or $T-m^2$ (as tesla = Wb/m^2) $\left[\text{ampere} = \frac{\text{coulomb}}{\text{second}} \right]$

- For a given area flux will be maximum :

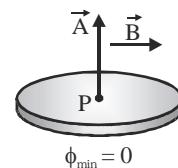
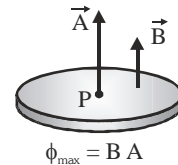
when magnetic field \vec{B} is normal to the area

$$\theta = 0^\circ \Rightarrow \cos \theta = \text{maximum} = 1 \quad \phi_{\max} = B A$$

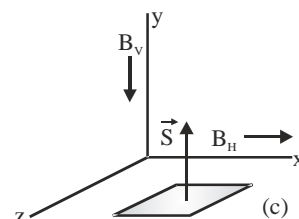
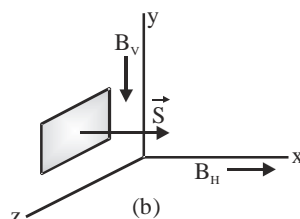
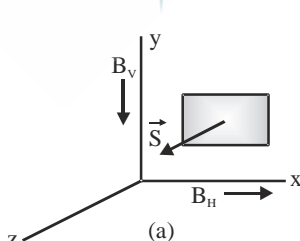
For a given area flux will be minimum :

when magnetic field \vec{B} is parallel to the area

$$\theta = 90^\circ \Rightarrow \cos \theta = \text{minimum} = 0 \quad \phi_{\min} = 0$$



Ex. At a given plane, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S , if the area S is in (a) x - y plane (b) y - z plane and (c) z - x plane ?



Sol. $\vec{B} = \hat{i} B_H - \hat{j} B_V = \text{constant}$, so $\phi = \vec{B} \cdot \vec{S}$ [$\vec{B} = \text{constant}$]

(a) For area in x-y plane $\vec{S} = S \hat{k}$, $\phi_{xy} = (\hat{i} B_H - \hat{j} B_V) \cdot (\hat{k} S) = 0$

(b) For area S in y-z plane $\vec{S} = S \hat{i}$, $\phi_{yz} = (\hat{i} B_H - \hat{j} B_V) \cdot (\hat{i} S) = B_H S$

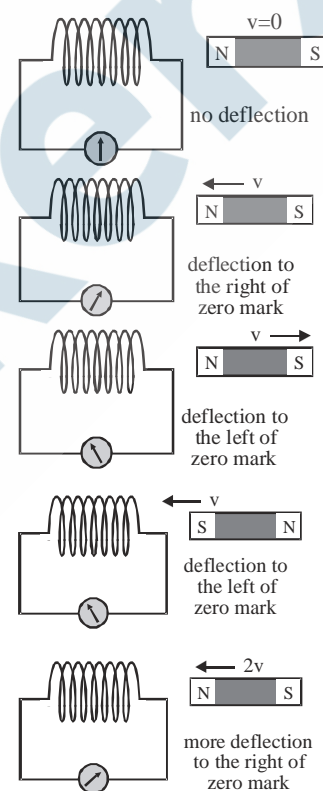
(c) For area S in z-x plane $\vec{S} = S \hat{j}$, $\phi_{zx} = (\hat{i} B_H - \hat{j} B_V) \cdot (\hat{j} S) = -B_V S$

Negative sign implies that flux is directed vertically downwards.

FARADAY'S EXPERIMENT

Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are :

- When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.
- When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.
- When the N-pole of a strong bar magnet is moved away from the coil, the galvanometer shows a deflection left to the zero mark.
- If the above experiments are repeated by bringing the S-pole of the, magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.
- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slow.



CONCLUSIONS

Whenever there is a relative motion between the source of magnetic field (magnet) and the coil, an emf is induced in the coil. When the magnet and coil move towards each other then the flux linked with the coil increases and emf is induced. When the magnet and coil move away from each other the magnetic flux linked with the coil decreases, again an emf is induced. This emf lasts so long the flux is changing. Due to this emf an electric current start to flow and the galvanometer shows deflection.

The deflection in galvanometer last as long the relative motion between the magnet and coil continues. Whenever relative motion between coil and magnet takes place an induced emf produced in coil. If coil is in closed circuit then current and charge is also induced in the circuit. This phenomenon is called electro magnetic induction.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Faraday's law of induction may be stated as follows:

The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux:

$$\varepsilon = - \frac{d\Phi_B}{dt}$$

For a coil that consists of N loops, the total induced emf would be N times as large:

$$\varepsilon = -N \frac{d\Phi_B}{dt}$$

Thus, we see that an emf may be induced in the following ways:

- (i) by varying the magnitude of \vec{B} with time (illustrated in Figure)

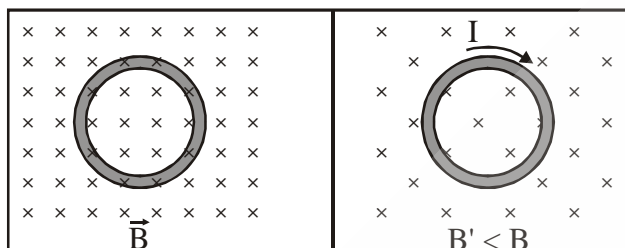


Figure : Inducing emf by varying the magnetic field strength

- (ii) by varying the magnitude of \vec{A} , i.e., the area enclosed by the loop with time (illustrated in Figure)

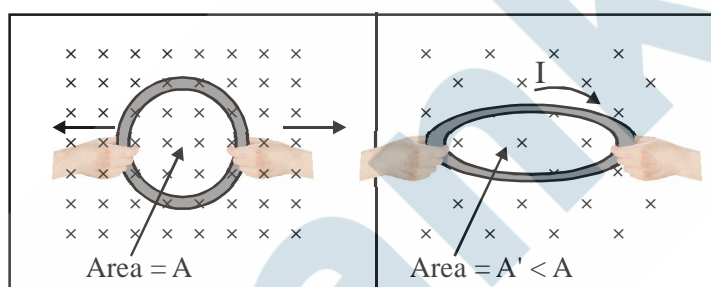


Figure : Inducing emf by changing the area of the loop

- (iii) varying the angle between \vec{B} and the area vector \vec{A} with time (illustrated in Figure)

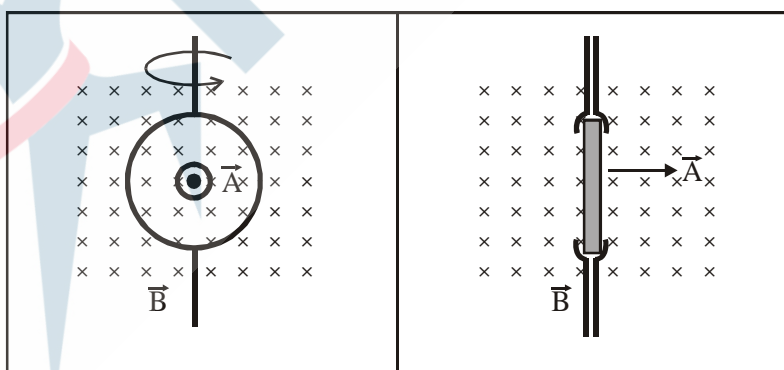


Figure : Inducing emf by varying the angle between \vec{B} and \vec{A}

LENZ'S LAW

Lenz's Law :

The direction of the induced current is determined by Lenz's law:

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector \vec{A} .
2. Assuming that \vec{B} is uniform, take the dot product of \vec{B} and \vec{A} . This allows for the determination of the sign of the magnetic flux Φ_B .
3. Obtain the rate of flux change $d\Phi_B / dt$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt} \begin{cases} > 0 \Rightarrow \text{induced emf } \varepsilon < 0 \\ < 0 \Rightarrow \text{induced emf } \varepsilon > 0 \\ = 0 \Rightarrow \text{induced emf } \varepsilon = 0 \end{cases}$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of \vec{A} , curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure.

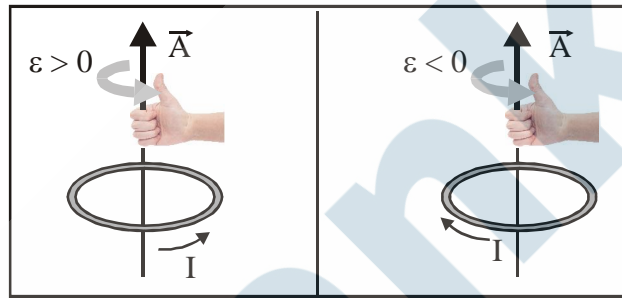


Figure : Determination of the direction of induced current by the right-hand rule
In Figure we illustrate the four possible scenarios of time-varying magnetic flux and show how Lenz's law is used to determine the direction of the induced current I .

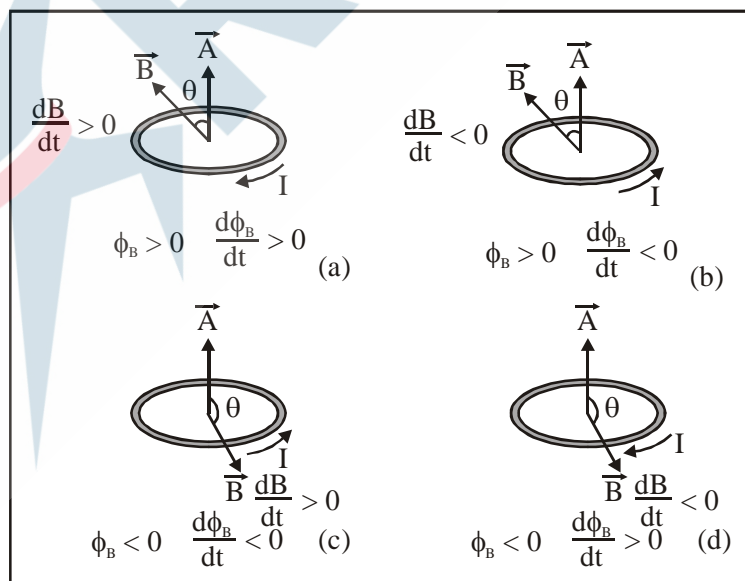


Figure : Direction of the induced current using Lenz's law

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The above situations can be summarized with the following sign convention:

Φ_B	$d\Phi_B/dt$	ε	I
+	+	−	−
	−	+	+
−	+	−	−
	−	+	+

The positive and negative signs of I correspond to a counter clockwise and clockwise currents, respectively.

Ex. The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m² acts perpendicular to the plane of the coil. What will be the radius of the coil when the induced e.m.f. in the $1\mu\text{V}$

Sol. Induced emf $e = \frac{d(BA)}{dt} = \frac{Bd(\pi r^2)}{dt} = 2B\pi r \frac{dr}{dt}$ radius of coil $r = \frac{e}{2B\pi \left(\frac{dr}{dt}\right)} = \frac{1 \times 10^{-6}}{2 \times 10^{-3} \times \pi \times 10^{-2}} = \frac{5}{\pi} \text{ cm}$

Ex. The ends of a search coil having 20 turns, area of cross-section 1 cm^2 and resistance 2 ohms are connected to a ballistic galvanometer of resistance 40 ohms. If the plane of search coil is inclined at 30° to the direction of a magnetic field of intensity 1.5 Wb/m^2 , coil is quickly pulled out of the field to a region of zero magnetic field, calculate the charge passed through the galvanometer.

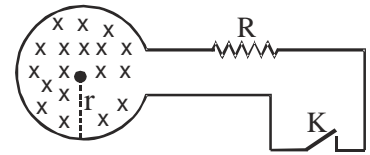
Sol. The total flux linked with the coil having turns N and area A is

$$\phi_1 = N(\vec{B} \cdot \vec{A}) = NBA \cos\theta = NBA \cos(90^\circ - 30^\circ) = \frac{NBA}{2}$$

when the coil is pulled out, the flux becomes zero, $\phi_2 = 0$ so change in flux is $\Delta\phi = \frac{NBA}{2}$

the charge flowed through the circuit is $q = \frac{\Delta\phi}{R} = \frac{NBA}{2R} = \frac{20 \times 1.5 \times 10^{-4}}{2 \times 42} = 0.357 \times 10^{-4} \text{ C}$

Ex. Shown in the figure is a circular loop of radius r and resistance R . A variable magnetic field of induction $B = B_0 e^{-t}$ is established inside the coil. If the key (K) is closed. Then calculate the electrical power developed right after closing the key.



Sol. Induced emf $e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A \frac{dB}{dt} = \pi r^2 B_0 \frac{d}{dt}(e^{-t}) = -\pi r^2 B_0 e^{-t}$

At $t = 0$, $e_0 = B_0 e^{-0} \cdot \pi r^2 = B_0 \pi r^2$

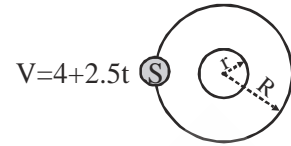
The electric power developed in the resistor R just at the instant of closing the key is $P = \frac{e_0^2}{R} = \frac{B_0^2 \pi^2 r^4}{R}$

Ex. Two concentric coplanar circular loops made of wire, resistance per unit length $10^{-4} \Omega\text{m}^{-1}$, have diameters 0.2 m and 2 m . A time-varying potential difference $(4 + 2.5 t)$ volt is applied to the larger loop. Calculate the current in the smaller loop.

Sol. The magnetic field at the centre O due to the current in the larger loop is $B = \frac{\mu_0 I}{2R}$

If ρ is the resistance per unit length, then

$$I = \frac{\text{potential difference}}{\text{resistance}} = \frac{4 + 2.5t}{2\pi R \cdot \rho}$$



$$\therefore B = \frac{\mu_0}{2R} \cdot \frac{4 + 2.5t}{2\pi R \rho}$$

$\therefore r \ll R$, so the field B can be taken almost constant over the entire area of the smaller loop.

$$\therefore \text{the flux linked with the smaller loop is } \phi = B \times \pi r^2 = \frac{\mu_0}{2R} \cdot \frac{4 + 2.5t}{2\pi R \rho} \cdot \pi r^2$$

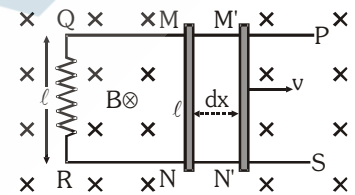
$$\text{Induced emf } e = \frac{d\phi}{dt} = \frac{\mu_0 r^2}{4R^2 \rho} \times 2.5$$

The corresponding current in the smaller loop is I' then

$$I' = \frac{e}{R} = \frac{\mu_0 r^2}{4R^2 \rho} \times 2.5 \times \frac{1}{2\pi r \rho} = \frac{2.5 \mu_0 r}{8\pi R^2 \rho^2} = \frac{2.5 \times 4\pi \times 10^{-7} \times 0.1}{8\pi \times (1)^2 \times (10^{-4})^2} = 1.25 \text{ A}$$

Induced emf by changing the area of the coil

A U shaped frame of wire, PQRS is placed in a uniform magnetic field B perpendicular to the plane and vertically inward. A wire MN of length ℓ is placed on this frame. The wire MN moves with a speed v in the direction shown. After time dt the wire reaches to the position $M'N'$ and distance covered = dx . The change in area $\Delta A = \text{Length} \times \text{area} = \ell dx$. Change in the magnetic flux linked with the loop in the dt is $d\phi = B \times \Delta A = B \times \ell dx$



$$\text{induced emf } e = \frac{d\phi}{dt} = B \ell \frac{dx}{dt} = B \ell v \because \left[v = \frac{dx}{dt} \right]$$

If the resistance of circuit is R and the circuit is closed then the current through the circuit

$$I = \frac{e}{R} \Rightarrow I = \frac{Bv\ell}{R}$$

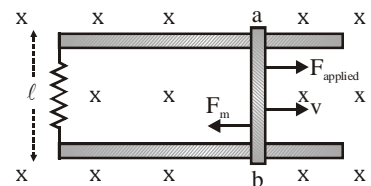
A magnetic force acts on the conductor in opposite direction of velocity is

$$F_m = i \ell B = \frac{B^2 \ell^2 v}{R}$$

So, to move the conductor with a constant velocity v an

equal and opposite force F has to be applied in the conductor.

$$F = F_m = \frac{B^2 \ell^2 v}{R}$$



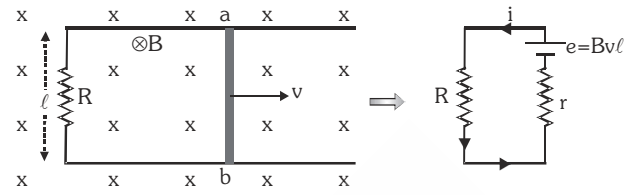
The rate at which work is done by the applied force is, $P_{\text{applied}} = Fv = \frac{B^2 \ell^2 v^2}{R}$

and the rate at which energy is dissipated in the circuit is, $P_{\text{dissipated}} = i^2 R = \left[\frac{Bv\ell}{R} \right]^2 R = \frac{B^2 \ell^2 v^2}{R}$

This is just equal to the rate at which work is done by the applied force.

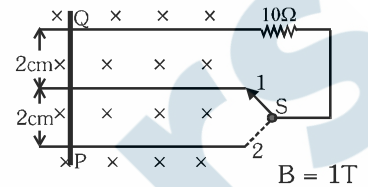
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- In the figure shown, we can replace the moving rod ab by a battery of emf $Bv\ell$ with the positive terminal at a and the negative terminal at b . The resistance r of the rod ab may be treated as the internal resistance of the battery.



Hence, the current in the circuit is $i = \frac{e}{R+r} = \frac{Bv\ell}{R+r}$

- Ex.** Wire PQ with negligible resistance slides on the three rails with 5 cm/sec . Calculate current in 10Ω resistance when switch S is connected to (a) position 1 (b) position 2



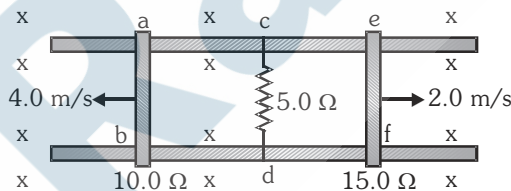
- Sol.** (a) For position 1

$$\text{Induced current } I = \frac{e}{R} = \frac{Bv\ell}{R} = \frac{1 \times 5 \times 10^{-2} \times 2 \times 10^{-2}}{10} = 0.1 \text{ mA}$$

- (b) For position 2

$$\text{Induced current } I = \frac{e}{R} = \frac{Bv(2\ell)}{R} = \frac{1 \times 5 \times 10^{-2} \times 4 \times 10^{-2}}{10} = 0.2 \text{ mA}$$

- Ex.** Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0Ω resistor. The circuit also contains two metal rods having resistance of 10.0Ω and 15.0Ω along the rails (fig). The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.0Ω resistor.



- Sol.** Two conductors are moving in uniform magnetic field, so motional emf will be induced across them.

The rod ab will act as a source of emf $e_1 = Bv\ell = (0.01)(4.0)(0.1) = 4 \times 10^{-3} \text{ V}$

and internal resistance $r_1 = 10.0 \Omega$

Similarly, rod ef will also act as source of emf $e_2 = (0.01)(2.0)(0.1) = 2 \times 10^{-3} \text{ V}$

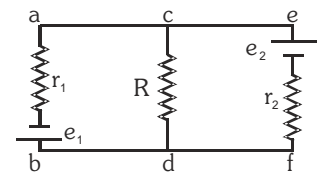
and internal resistance $r_2 = 15.0 \Omega$

From right hand rule: $V_b > V_a$ and $V_e > V_f$ Also $R = 5.0 \Omega$,

$$E_{eq} = \frac{e_1 r_2 - e_2 r_1}{r_1 + r_2} = \frac{6 \times 10^{-3} - 20 \times 10^{-3}}{15 + 10} = \frac{40}{25} \times 10^{-3} = 1.6 \times 10^{-3} \text{ volt}$$

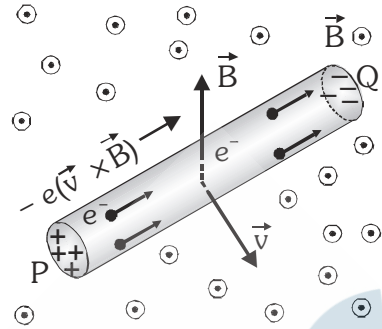
$$r_{eq} = \frac{15 \times 10}{15 + 10} = 6 \Omega \quad \text{and}$$

$$I = \frac{E_{eq}}{r_{eq} + R} = \frac{1.6 \times 10^{-3}}{6 + 6} = \frac{1.6}{11} \times 10^{-3} = \frac{8}{55} \times 10^{-3} \text{ amp from d to c}$$



MOTIONAL EMF FROM LORENTZ FORCE

A conductor PQ is placed in a uniform magnetic field \vec{B} , directed normal to the plane of paper outwards. PQ is moved with a velocity \vec{v} , the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force, $\vec{F}_m = (e\vec{v} \times \vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative charge accumulates at Q and a positive charge at P. An electric field \vec{E} is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = e\vec{E}$



The accumulation of charge at the two ends continues till these two forces balance each other.

$$\text{so } \vec{F}_m = -\vec{F}_e \Rightarrow e(\vec{v} \times \vec{B}) = -e\vec{E} \Rightarrow \vec{E} = -(\vec{v} \times \vec{B})$$

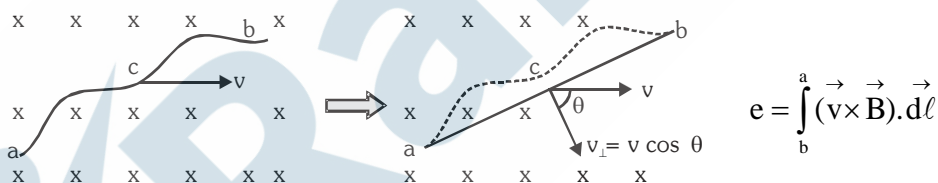
The potential difference between the ends P and Q is $V = \vec{E} \cdot \vec{\ell} = (\vec{v} \times \vec{B}) \cdot \vec{\ell}$. It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\mathcal{E} = B \ell v$ (for $\vec{B} \perp \vec{v} \perp \vec{\ell}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $d\vec{\ell}$ of conductor the contribution de to the emf is the magnitude

$d\ell$ multiplied by the component of $\vec{v} \times \vec{B}$ parallel to $d\vec{\ell}$, that is $de = (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$

For any two points a and b the motional emf in the direction from b to a is,

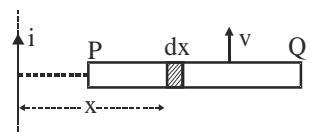


Motional emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab.

Thus, $e_{acb} = e_{ab} = (\text{length of } ab)(v_{\perp})(B)$, v_{\perp} is the component of velocity perpendicular to both \vec{B} and ab. From **right hand rule** : b is at higher potential and a at lower potential. Hence, $V_{ba} = V_b - V_a = (ab)(v \cos \theta)(B)$

Ex. A rod PQ of length L moves with a uniform velocity v parallel to a long straight wire carrying a current i , the end P remaining at a distance r from the wire. Calculate the emf induced across the rod. Take $v = 5.0 \text{ m/s}$, $i = 100 \text{ amp}$, $r = 1.0 \text{ cm}$ and $L = 19 \text{ cm}$.

Sol. The rod PQ is moving in the magnetic field produced by the current-carrying long wire. The field is not uniform throughout the length of the rod (changing with distance). Let us consider a small element of length dx at distance x from wire. if magnetic field at the position of dx is B then emf induced



$$d\mathcal{E} = B v dx = \frac{\mu_0 i}{2\pi x} v dx$$

\therefore emf \mathcal{E} is induced in the entire length of the rod PQ is $\mathcal{E} = \int d\mathcal{E} = \int_P^Q \frac{\mu_0 i}{2\pi x} v dx$

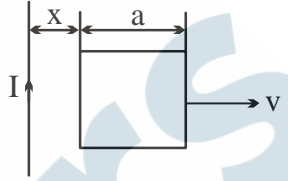
Now $x = r$ at P, and $x = r + L$ at Q. hence

$$\mathcal{E} = \frac{\mu_0 i v}{2\pi} = \int_r^{r+L} \frac{dx}{x} = \frac{\mu_0 i v}{2\pi} [\log_e x]_r^{r+L} = \frac{\mu_0 i v}{2\pi} [\log_e (r + L) - \log_e r] = \frac{\mu_0 i v}{2\pi} \log \frac{r + L}{r}$$

Putting the given values :

$$\mathcal{E} = (2 \times 10^{-7}) (100) (5.0) \log_e \frac{1.0 + 19}{1.0} = 10^{-4} \log_e 20 \text{ Wb/s} = 3 \times 10^{-4} \text{ volt}$$

Ex. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Fig. The frame translates to the right with a constant velocity v . Find the emf induced in the frame as a function of distance x .



Ans. $\xi_i = \frac{\mu_0}{4\pi} \frac{2Ia^2v}{x(x+a)}$

Sol. Field, due to the current carrying wire, at a perpendicular distance x from it is given by,

$$B(x) = \frac{\mu_0}{2\pi} \frac{i}{x}$$

Motional emf is given by $\left| \int -(\vec{v} \times \vec{B}) \cdot d\vec{\ell} \right|$

There will be no induced emf in the segments (2) and (4) as, $\vec{v} \uparrow \uparrow d\vec{\ell}$ and magnitude of emf induced 1 and 3, will be

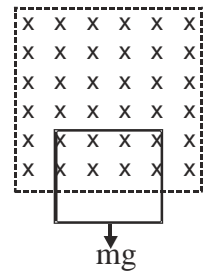
$$\xi_1 = v \left(\frac{\mu_0}{2\pi} \frac{i}{x} \right) a \text{ and } \xi_2 = v \left(\frac{\mu_0}{2\pi} \frac{i}{(a+x)} \right) a,$$

respectively, and their sense will be in the direction of $(\vec{v} \times \vec{B})$. So, emf induced in the network

$$\begin{aligned} &= \xi_1 - \xi_2 \text{ [as } \xi_1 > \xi_2] \\ &= \frac{av\mu_0 i}{2\pi} \left[\frac{1}{x} - \frac{1}{a+x} \right] = \frac{va^2\mu_0 i}{2\pi x(a+x)} \end{aligned}$$

Ex. A horizontal magnetic field B is produced across a narrow gap between square iron pole-pieces as shown. A closed square wire loop of side ℓ , mass m and resistance R is allowed to fall with the top of the loop in the field. Show that the loop attains a terminal velocity given by

$$v = \frac{Rmg}{B^2 \ell^2} \text{ while it is between the poles of the magnet.}$$

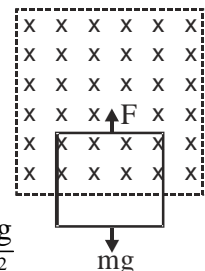


Sol. As the loop falls under gravity, the flux passing through it decreases and so an induced emf is set up in it. Then a force F which opposes its fall. When this force becomes equal to the gravity force mg , the loop attains a terminal velocity v .

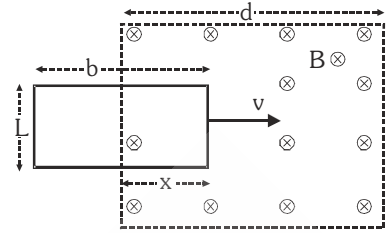
The induced emf $e = B v \ell$, and the induced current is $i = \frac{e}{R} = \frac{B v \ell}{R}$

The force experienced by the loop due to this current is $F = B \ell i = \frac{B^2 v \ell^2}{R}$

When v is the terminal (constant) velocity $F = mg$ or $\frac{B^2 v \ell^2}{R} = mg$ or $v = \frac{Rmg}{B^2 \ell^2}$



Ex. Figure shows a rectangular conducting loop of resistance R , width L , and length b being pulled at constant speed v through a region of width d in which a uniform magnetic field B is set up by an electromagnet. Let $L = 40 \text{ mm}$, $b = 10 \text{ cm}$, $d = 15 \text{ cm}$, $R = 1.6 \Omega$, $B = 2.0 \text{ T}$ and $v = 1.0 \text{ m/s}$



- Plot the flux ϕ through the loop as a function of the position x of the right side of the loop.
- Plot the induced emf as a function of the position of the loop.
- Plot the rate of production of thermal energy in the loop as a function of the position of the loop.

Sol. (i) When the loop is not in the field :

The flux linked with the loop $\phi = 0$

When the loop is entirely in the field :

Magnetic flux linked with the loop

$$\phi = B L b$$

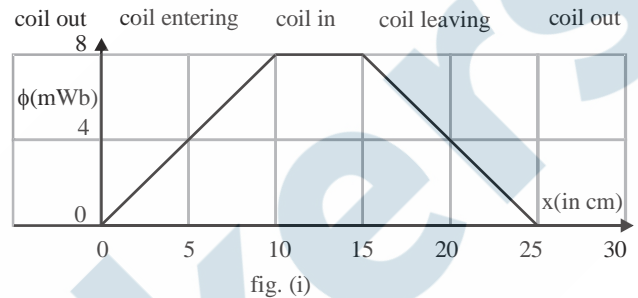
$$= 2 \times 40 \times 10^{-3} \times 10^{-1} = 8 \text{ mWb}$$

When the loop is entering the field :

The flux linked with the loop $\phi = B L x$

When the loop is leaving the field :

The flux $\phi = B L [b - (x - d)]$



- Induced emf is $e = -\frac{d\phi}{dt} = -\frac{d\phi}{dx} \frac{dx}{dt} = -\frac{d\phi}{dx} v$

$= -\text{slope of the curve of figure (i)} \times v$

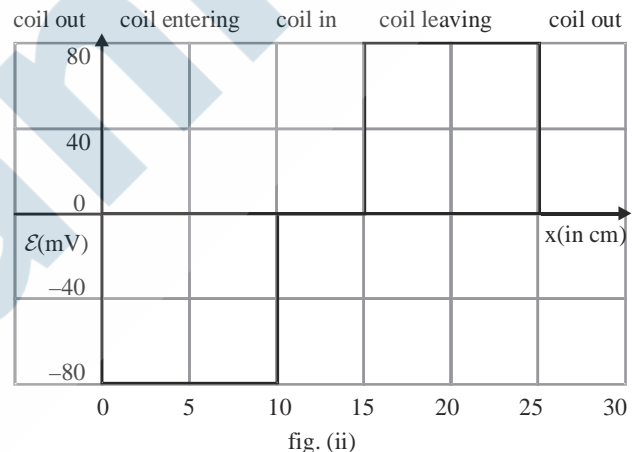
The emf for 0 to 10 cm :

$$e = -\frac{(8-0) \times 10^{-3}}{(10-0) \times 10^{-2}} \times 1 = -80 \text{ mV}$$

The emf for 10 to 15 cm : $e = 0 \times 1 = 0$

The emf for 15 to 25 cm :

$$e = -\frac{(0-8) \times 10^{-3}}{(25-15) \times 10^{-2}} \times 1 = +80 \text{ mV}$$

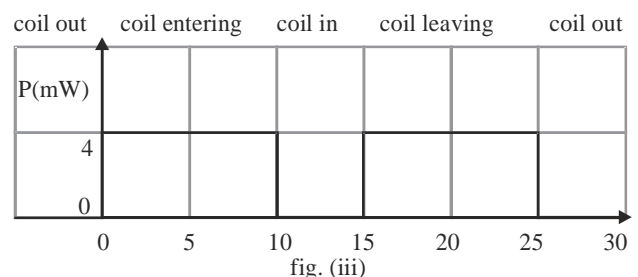


- The rate of thermal energy production is $P = \frac{e^2}{R}$

$$\text{for } 0 \text{ to } 10 \text{ cm : } P = \frac{(80 \times 10^{-3})^2}{1.6} = 4 \text{ mW}$$

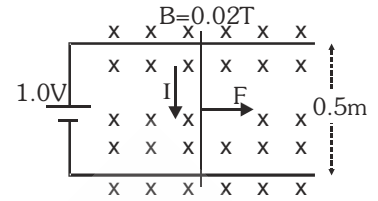
$$\text{for } 10 \text{ to } 15 \text{ cm : } P = 0$$

$$\text{for } 15 \text{ to } 25 \text{ cm : } P = \frac{(80 \times 10^{-3})^2}{1.6} = 4 \text{ mW}$$



Ex. Two long parallel wires of zero resistance are connected to each other by a battery of 1.0 V . The separation between the wires is 0.5 m . A metallic bar, which is perpendicular to the wires and of resistance 10Ω , moves on these wires. When a magnetic field of 0.02 tesla is acting perpendicular to the plane containing the bar and the wires. Find the steady-state velocity of the bar. If the mass of the bar is 0.002 kg then find its velocity as a function of time.

Sol. The current in the 10Ω bar is $I = \frac{1.0 \text{ V}}{10\Omega} = 0.1 \text{ A}$



The current carrying bar is placed in the magnetic field \vec{B} (0.2 T) perpendicular to the plane of paper and directed downwards.

The magnetic force of the bar is $F = B I \ell = 0.02 \times 0.5 \times 0.1 = 1 \times 10^{-3} \text{ N}$

The moving bar cuts the lines of force of \vec{B} . If v be the instantaneous velocity of the bar, then the emf induced in the bar is $\mathcal{E} = B\ell v = 0.02 \times 0.5 \times v = 0.01 v$ volt. By Lenz's law, \mathcal{E} will oppose the motion of the bar which will ultimately attain a steady velocity. In this state, the induced emf \mathcal{E} will be equal

to the applied emf (1.0 volt). $\therefore 0.01 v = 1.0$ or $v = \frac{1.0}{0.01} = 100 \text{ ms}^{-1}$

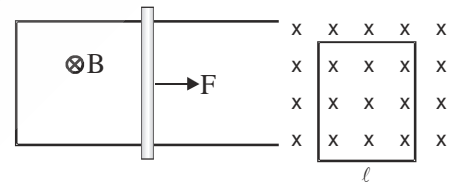
Again, a magnetic force F acts on the bar. If m be the mass of the bar, the acceleration of the rod is

$$\frac{dv}{dt} = \frac{F}{m} \Rightarrow dv = \frac{F}{m} \cdot dt \quad \text{Integrating, } \int dv = \int \frac{F}{m} dt \Rightarrow v = \frac{F}{m} t + C \text{ (constant)}$$

If at $t = 0$, $v = 0$ then $C = 0$. $\therefore v = \frac{F}{m} t$ But $F = 1 \times 10^{-3} \text{ N}$, $m = 0.002 \text{ kg}$

$$\therefore v = \frac{1 \times 10^{-3}}{0.002} t = 0.5 t$$

Ex. In figure, a rod closing the circuit moves along a U-shaped wire at a constant speed v under the action of the force F . The circuit is in a uniform magnetic field perpendicular to its plane. Calculate F if the rate generation of heat is P .

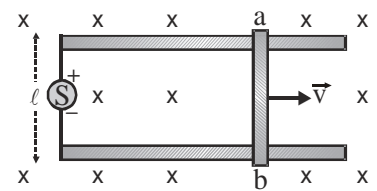


Sol. The emf induced across the ends of the rod, $\mathcal{E} = B\ell v$

Current in the circuit, $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$ Magnetic force on the conductor, $F' = I\ell B$, towards left

$$\therefore \text{acceleration is zero } F' = F \Rightarrow B I \ell = F \text{ or } I = \frac{F}{B\ell} \therefore P = \mathcal{E} I = B \ell v \times \frac{F}{B\ell} = Fv \therefore F = \frac{P}{v}$$

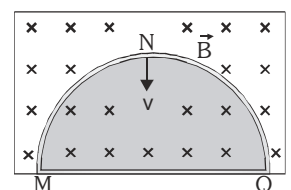
Ex. The diagram shows a wire ab of length ℓ and resistance R sliding on a smooth pair of rails with a velocity v towards right. A uniform magnetic field of induction B acts normal to the plane containing the rails and the wire inwards. S is a current source providing a constant I in the circuit. Determine the potential difference between a and b .



Sol. The wire ab which is moving with a velocity v is equivalent to an emf source of value $B v \ell$ with its positive terminal towards a .

$$\therefore \text{Potential difference } V_a - V_b = Bv\ell - IR$$

Ex. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction \vec{B} (fig.). At the position MNQ , the speed of the ring is v . What is the potential difference developed across the ring at the position MNQ ?



Sol. Let semiconductor ring falls through an infinitesimally small distance dx from its initial position MNQ to $M'Q'N'$ in time dt (fig).

decrease in area of the ring inside the magnetic field,

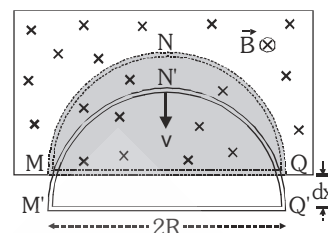
$$dA = -MQQ'M' = -M'Q' \times QQ' = -2R dx$$

\therefore change in magnetic flux linked with the ring,

$$d\phi = B \times dA = B \times (-2R dx) = -2BR dx$$

The potential difference developed across the ring, $e = -\frac{d\phi}{dt} = -\left[-2BR \frac{dx}{dt}\right] = 2BRv$

the speed with which the ring is falling $v = \frac{dx}{dt}$



Ex. A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity (Fig.). At the top the bars are interconnected through a resistance R . The separation between the bars is equal to l . The system is located in a uniform magnetic field of induction B , perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.

Ans. $v = \frac{mgR \sin \alpha}{B^2 l^2}$

Sol. From Lenz's law, the current through the connector is directed from A to B. Here $\xi_{in} = vBl$ between A and B.

where v is the velocity of the rod at any moment.

For the rod, from $F_x = mw_x$

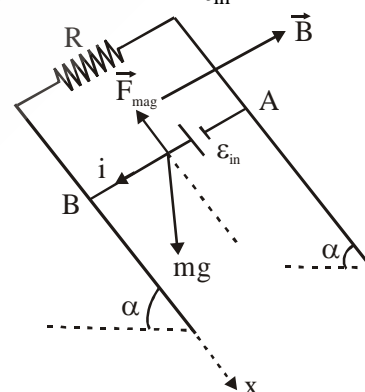
$$\text{or, } mg \sin \alpha - i\ell B = mw$$

For steady state, acceleration of the rod must be equal to zero.

$$\text{Hence, } mg \sin \alpha = i\ell B \quad \dots\dots\dots (1)$$

$$\text{But, } i = \frac{\xi_{in}}{R} = \frac{vBl}{R}$$

$$\text{from (1) and (2) } v = \frac{mg \sin \alpha R}{B^2 \ell^2}$$



INDUCED E.M.F. DUE TO ROTATION OF A CONDUCTOR ROD IN A UNIFORM MAGNETIC FIELD

Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.

Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

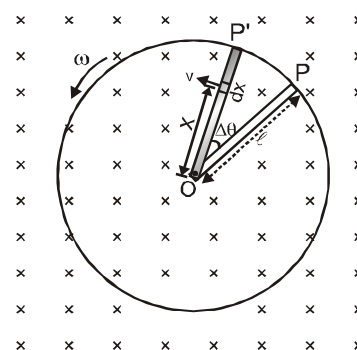
So, according to Lorentz's formula induced e.m.f. across this small element

$$d\epsilon = B v \cdot dx$$

\therefore This small element dx is at distance x from O (axis of rotation)

\therefore Linear velocity of this element dx is $v = \omega x$

substitute of value of v in eqⁿ (i) $d\epsilon = B \omega x dx$



Electromagnetic induction & Alternating current

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

So, net induced e.m.f. across conducting rod $\varepsilon = \int d\varepsilon = \int_0^{\ell} B\omega \, x \, dx = \omega B \left(\frac{x^2}{2} \right)_0^{\ell}$

or $\varepsilon = \frac{1}{2} B\omega \ell^2$ $\varepsilon = \frac{1}{2} B \times 2\pi f \times \ell^2$ [f = frequency of rotation]

$= B f (\pi \ell^2)$ area traversed by the rod $A = \pi \ell^2$ or $\varepsilon = BAf$

Ex. A wheel with 10 metallic spokes each 0.5 m long is rotated with angular speed of 120 revolutions per minute in a plane normal to the earth's magnetic field. If the earth's magnetic field at the given place is 0.4 gauss, find the e.m.f. induced between the axle and the rim of the wheel.

Sol. $\omega = 2\pi n = 2\pi \times \frac{120}{60} = 4\pi$, $B = 0.4 \text{ G} = 4 \times 10^{-5} \text{ T}$, length of each spoke = 0.5 m

induced emf $e = \frac{1}{2} B\omega \ell^2 = \frac{1}{2} \times 4 \times 10^{-5} \times 4\pi \times (0.5)^2 = 6.28 \times 10^{-5} \text{ volt}$

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

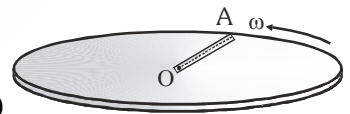
Ex. A horizontal copper disc of diameter 20 cm, makes 10 revolutions/sec about a vertical axis passing through its centre. A uniform magnetic field of 100 gauss acts perpendicular to the plane of the disc. Calculate the potential difference its centre and rim in volts.

Sol. $B = 100 \text{ gauss} = 100 \times 10^{-4} \text{ Wb/m}^2 = 10^{-2}$,
 $r = 10 \text{ cm} = 0.10 \text{ m}$, frequency of rotation = 10 rot/sec

The emf induced between centre and rim $\varepsilon = \frac{1}{2} B\omega r^2 = \frac{1}{2} B\omega r^2$ ($\because r = \ell$)

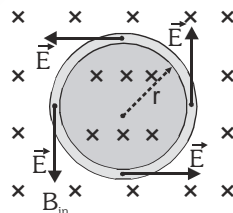
$\omega = 2\pi f = 2 \times 3.14 \times 10 = 62.8 \text{ s}^{-1}$

$\therefore \varepsilon = \frac{1}{2} \times 10 \times 62.8 \times (0.1)^2 = 3.14 \times 10^{-3} \text{ V} = 3.14 \text{ mV}$.



INDUCED ELECTRIC FIELD

When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux.



Important properties of induced electric field :

- (i) It is non conservative in nature. The line integral of \vec{E} around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf.

Hence $\oint \vec{E} \cdot d\vec{\ell} = e = -\frac{d\phi}{dt} \dots(i)$

This equation is valid only if the path around which we integrate is stationary.

- (ii) Due to of symmetry, the electric field \vec{E} has the same magnitude at every point on the circle and it is tangential at each point (figure).
- (iii) Being nonconservative field, so the concept of potential has no meaning for such a field.
- (iv) This field is different from the conservative electrostatic field produced by stationary charges.
- (v) The relation $\vec{F} = q \vec{E}$ is still valid for this field. (vi) This field can vary with time.

- For symmetrical situations $E\ell = \left| \frac{d\phi}{dt} \right| = A \left| \frac{dB}{dt} \right|$

ℓ = the length of closed loop in which electric field is to be calculated

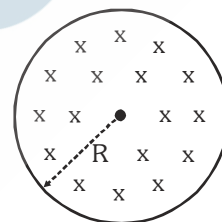
A = the area in which magnetic field is changing.

Direction of induced electric field is the same as the direction of included current.

Ex. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate

$\alpha \frac{\text{tesla}}{\text{second}}$. Find the magnitude of electric field as a function of r , the

distance from the geometric centre of the region.



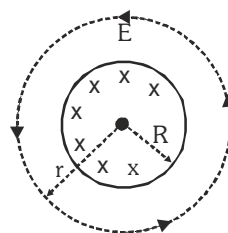
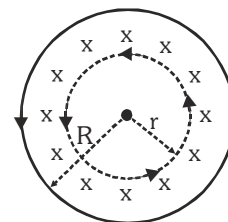
Sol. For $r \leq R$:

$$\therefore E\ell = A \left| \frac{dB}{dt} \right|$$

$$\therefore E(2\pi r) = (\pi r^2) \alpha \Rightarrow E = \frac{r\alpha}{2} \Rightarrow E \propto r$$

E - r graph is straight line passing through origin.

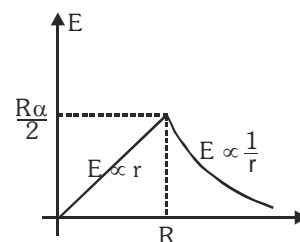
At $r = R$, $E = \frac{R\alpha}{2}$



For $r \geq R$:

$$\therefore E\ell = A \left| \frac{dB}{dt} \right|$$

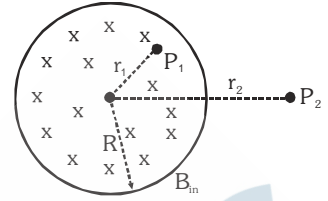
$$\therefore E(2\pi r) = (\pi R^2) \alpha \Rightarrow E = \frac{\alpha R^2}{2r} \Rightarrow E \propto \frac{1}{r}$$



Ex. For the situation described in figure the magnetic field changes with time according to,

$$B = (2.00 t^3 - 4.00 t^2 + 0.8) \text{ T and } r_2 = 2R = 5.0 \text{ cm}$$

- Calculate the force on an electron located at P_2 at $t = 2.00 \text{ s}$
- What are the magnetude and direction of the electric field at P_1 when $t = 3.00 \text{ s}$ and $r_1 = 0.02 \text{ m}$.



Sol. $E \ell = A \left| \frac{dB}{dt} \right| \Rightarrow E = \frac{\pi R^2}{2\pi r_2} \frac{d}{dt} (2t^3 - 4t^2 + 0.8) = \frac{R^2}{2r_2} (6t^2 - 8t)$

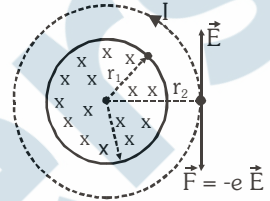
- Force on electron at P_2 is $F = eE$

$$\therefore \text{ at } t = 2 \text{ s } F = \frac{1.6 \times 10^{-19} \times (2.5 \times 10^{-2})^2}{2 \times 5 \times 10^{-2}} \times [6(2)^2 - 8(2)]$$

$$= \frac{1.6}{4} \times 2.5 \times 10^{-21} \times (24 - 16) = 8 \times 10^{-21} \text{ N at } t = 2 \text{ s,}$$

$\frac{dB}{dt}$ is positive so it is increasing.

- direction of induced current and E are as shown in figure and hence force of electron having charge $-e$ is right perpendicular to r_2 downwards



- For $r_1 = 0.02 \text{ m}$ and at $t = 3 \text{ s}$, $E = \frac{\pi r_1^2}{2\pi r_1} (6t^2 - 8t) = \frac{0.02}{2} \times [6(3)^2 - 8(3)]$

$$= 0.3 \text{ V/m at } t = 3 \text{ sec, } \frac{dB}{dt}$$

is positive so B is increasing and hence direction of E is same as in case (a) and it is left perpendicular to r_1 upwards.

Generators :

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

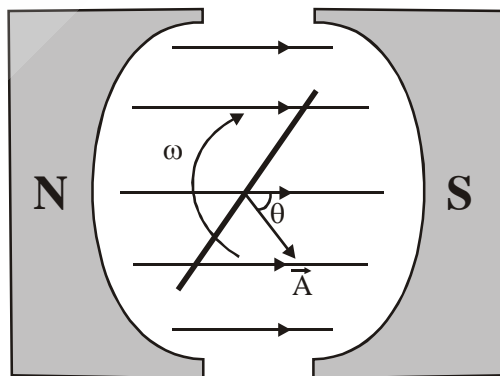
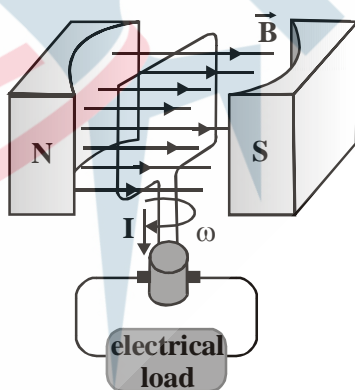


Figure : (a) A simple generator. (b) The rotating loop as seen from above.

Figure (a) is a simple illustration of a generator. It consists of an N -turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure (b), we see that the magnetic flux through the loop may be written as

$$\Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta = BA \cos \omega t$$

The rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = -BA \omega \sin \omega t$$

Since there are N turns in the loop, the total induced emf across the two ends of the loop is

$$\varepsilon = -N \frac{d\Phi_B}{dt} = NBA \omega \sin \omega t$$

If we connect the generator to a circuit which has a resistance R, then the current generated in the

circuit is given by
$$I = \frac{|\varepsilon|}{R} = \frac{NBA\omega}{R} \sin \omega t$$

The current is an alternating current which oscillates in sign and has an amplitude $I_0 = NBA\omega / R$. The power delivered to this circuit is

$$P = I |\varepsilon| = \frac{(NBA\omega)^2}{R} \sin^2 \omega t$$

On the other hand, the torque exerted on the loop is

$$\tau = \mu B \sin \theta = \mu B \sin \omega t$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau \omega = \mu B \omega \sin \omega t$$

Since the dipole moment for the N-turn current loop is

$$\mu = NIA = \frac{N^2 A^2 B \omega}{R} \sin \omega t$$

the above expression becomes

$$P_m = \left(\frac{N^2 A^2 B \omega}{R} \sin \omega t \right) B \omega \sin \omega t = \frac{(NAB\omega)^2}{R} \sin^2 \omega t$$

As expected, the mechanical power put in is equal to the electric power output.

SELF INDUCTION

When the current through the coil changes, the magnetic flux linked with the coil also changes. Due to this change of flux a current induced in the coil itself according to lenz concept it opposes the change in magnetic flux. This phenomenon is called self induction and a factor by virtue of coil shows opposition for change in magnetic flux called coefficient of self inductance of coil.

Considering this coil circuit in two cases.

Case - I Current through the coil is constant

If $I \rightarrow B \rightarrow \phi$ (constant) \Rightarrow No EMI

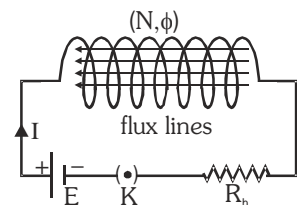
total flux of coil $(N\phi) \propto$ current through the coil

$$N\phi \propto I \Rightarrow N\phi = LI \quad L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$$

where L = coefficient of self inductance of coil

S I unit of L : $1 \frac{Wb}{amp} = 1 \text{ Henry} = 1 \frac{N.m}{A^2} = 1 \frac{J}{A^2}$ **Dimensions :** $[M^1 L^2 T^{-2} A^{-2}]$

Note : L is constant of coil it **does not depends on current** flow through the coil.



Case - II Current through the coil changes w.r.t. time

$$\text{If } \frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow \text{Static EMI} \Rightarrow N\phi = LI$$

$$-N \frac{d\phi}{dt} = -L \frac{dI}{dt}, \quad (-N \frac{d\phi}{dt}) \text{ called total self induced emf of coil 'e}_s\text{'}$$

$$e_s = -L \frac{dI}{dt} \quad \text{S.I. unit of } L \rightarrow \frac{V \cdot s}{A}$$

SELF-INDUCTANCE OF A PLANE COIL

Total magnetic flux linked with N turns,

$$\phi = NBA = N \left(\frac{\mu_0 NI}{2r} \right) A = \frac{\mu_0 N^2 I}{2r} A = \frac{\mu_0 N^2 I}{2r} \times \pi r^2 = \frac{\mu_0 \pi N^2 r}{2} I \quad \text{But } \phi = LI \therefore L = \frac{\mu_0 \pi N^2 r}{2}$$

Ex. Self-Inductance of a Solenoid :

Compute the self-inductance of a solenoid with turns, length ℓ , and radius NR with a current I flowing through each turn, as shown in Figure.

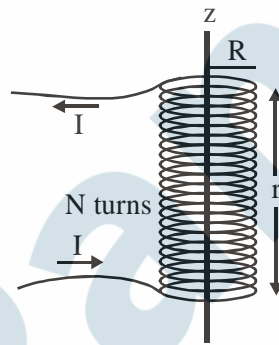


Figure : Solenoid

Solution:

Ignoring edge effects and applying Ampere's law, the magnetic field inside a solenoid is given by Eq. :

$$\vec{B} = \frac{\mu_0 NI}{\ell} \hat{k} = \mu_0 nI \hat{k}$$

where $n = N / \ell$ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0 nI \cdot (\pi R^2) = \mu_0 nI \pi R^2$$

$$\text{Thus, the self-inductance is } L = \frac{N\Phi_B}{I} = \mu_0 n^2 \pi R^2 \ell$$

We see that L depends only on the geometrical factors (n , R and ℓ) and is independent of the current I .

Ex. Self-Inductance of a Toroid :

Calculate the self-inductance of a toroid which consists of N turns and has a rectangular cross section, with inner radius a , outer radius b and height h , as shown in Figure (a).

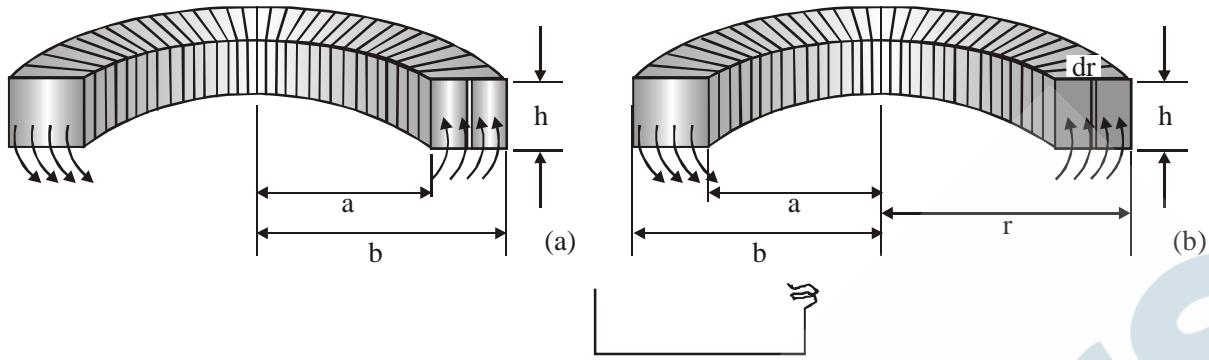


Figure : A toroid with N turns

Solution : According to Ampere's law discussed in section, the magnetic field is given by

$$\oint \vec{B} \cdot d\vec{s} = \oint B ds = B \oint ds = B (2\pi r) = \mu_0 NI$$

or
$$B = \frac{\mu_0 NI}{2\pi r}$$

The magnetic flux through one turn of the toroid may be obtained by integrating over the rectangular cross section, with $dA = h dr$ as the differential area element (figure-b)

$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \int_a^b \left(\frac{\mu_0 NI}{2\pi r} \right) h dr = \frac{\mu_0 NI h}{2\pi} \ln \left(\frac{b}{a} \right)$$

The total flux is $N\Phi_B$. Therefore, the self-inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln \left(\frac{b}{a} \right)$$

Again, the self-inductance L depends only on the geometrical factors. Let's consider the situation where $a \gg b - a$. In this limit, the logarithmic term in the equation above may be expanded as

$$\ln \left(\frac{b}{a} \right) = \ln \left(1 + \frac{b-a}{a} \right) \approx \frac{b-a}{a}$$

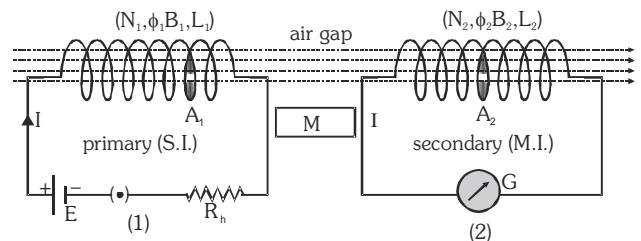
and the self-inductance becomes

$$L \approx \frac{\mu_0 N^2 h}{2\pi} \cdot \frac{b-a}{a} = \frac{\mu_0 N^2 A}{2\pi a} = \frac{\mu_0 N^2 A}{\ell}$$

where $A = h(b - a)$ is the cross-sectional area, and $\ell = 2\pi a$. We see that the self inductance of the toroid in this limit has the same form as that of a solenoid.

MUTUAL INDUCTION

Whenever the current passing through primary coil or circuit change then magnetic flux neighbouring secondary coil or circuit will also change. Acc. to Lenz for opposition of flux change, so an emf induced in the neighbouring coil or circuit.



This phenomenon called as 'Mutual induction'. In case of mutual inductance for two coils situated close to each other, flux linked with the secondary due to current in primary.

Due to Air gap always $\phi_2 < \phi_1$ and $\phi_2 = B_1 A_2$ ($\theta = 0^\circ$).

Case - I When current through primary is constant

Total flux of secondary is directly proportional to current flow through the primary coil

$$N_2 \phi_2 \propto I_1 \Rightarrow N_2 \phi_2 = M I_1, \quad M = \frac{N_2 \phi_2}{I_1} = \frac{N_2 B_1 A_2}{I_1} = \frac{(\phi_1)_s}{I_1} \text{ where } M : \text{ is coefficient of mutual induction.}$$

Case - II When current through primary changes with respect to time

$$\text{If } \frac{dI_1}{dt} \rightarrow \frac{dB_1}{dt} \rightarrow \frac{d\phi_1}{dt} \rightarrow \frac{d\phi_2}{dt}$$

$$\Rightarrow \text{Static EMI} \quad N_2 \phi_2 = M I_1 - N_2 \frac{d\phi_2}{dt} = -M \frac{dI_1}{dt}, \quad \left[-N_2 \frac{d\phi}{dt} \right]$$

$$e_m = -M \left(\frac{dI_1}{dt} \right)$$

Secondary ← → Primary

called total mutual induced emf of secondary coil e_m .

- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

'M' depends on :

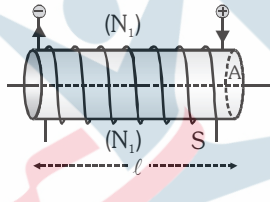
- Number of turns (N_1, N_2).
- Area of cross section.
- Distance between two coils (As $d \downarrow = M \uparrow$).
- **Coupling factor 'K'** between two coils.
- Coefficient of self inductance (L_1, L_2).
- Magnetic permeability of medium (μ_r).
- Orientation between two coils.

DIFFERENT COEFFICIENT OF MUTUAL INDUCTANCE

- In terms of their number of turns
- In terms of their coefficient of self inductances
- **In terms of their nos of turns (N_1, N_2)**

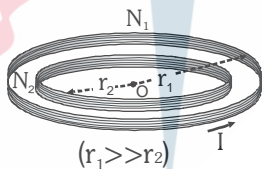
(a) **Two co-axial solenoids :-** ($M_{s_1 s_2}$)

Coefficient of mutual inductance between two solenoids



$$M_{s_1 s_2} = \frac{N_2 B_1 A}{I_1} = \frac{N_2}{I_1} \left[\frac{\mu_0 N_1 I_1}{l} \right] A \Rightarrow M_{s_1 s_2} = \left[\frac{\mu_0 N_1 N_2 A}{l} \right]$$

(b) **Two plane concentric coils** ($M_{c_1 c_2}$)



$$M_{c_1 c_2} = \frac{N_2 B_1 A_2}{I_1} \text{ where } B_1 = \frac{\mu_0 N_1 I_1}{2r_1}, \quad A_2 = \pi r_2^2$$

$$M_{c_1 c_2} = \frac{N_2}{I_1} \left[\frac{\mu_0 N_1 I_1}{2r_1} \right] (\pi r_2^2) \Rightarrow M_{c_1 c_2} = \frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$$

Two concentric loop :

Two concentric square loops :

A square and a circular loop

$$M \propto \frac{r_2^2}{r_1} (r_1 \gg r_2)$$

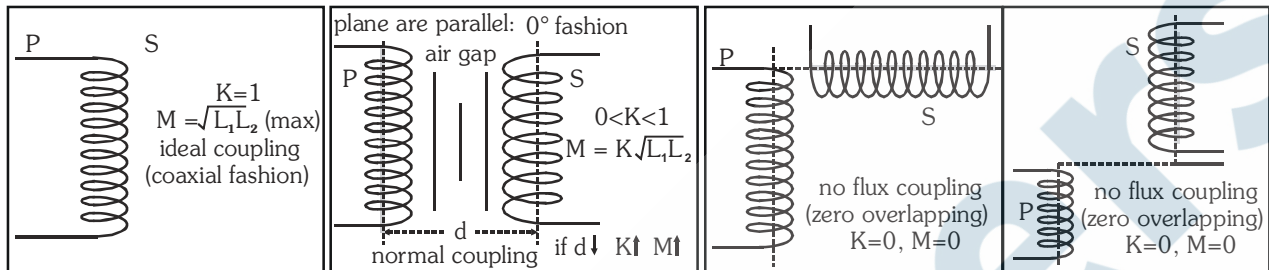
$$M \propto \frac{b^2}{a}$$

$$M \propto \frac{r^2}{a}$$

In terms of L_1 and L_2 : For two magnetically coupled coils :-

$M = K\sqrt{L_1 L_2}$ here 'K' is coupling factor between two coils and its range $0 \leq K \leq 1$

- For ideal coupling $K_{\max} = 1 \Rightarrow M_{\max} = \sqrt{L_1 L_2}$ (where M is geometrical mean of L_1 and L_2)
- For real coupling ($0 < K < 1$) $M = K\sqrt{L_1 L_2}$
- Value of coupling factor 'K' decided from fashion of coupling.
- **Different fashion of coupling**



'K' also defined as $K = \frac{\phi_s}{\phi_p} = \frac{\text{mag. flux linked with secondary (s)}}{\text{mag. flux linked with primary (p)}}$

INDUCTANCE IN SERIES AND PARALLEL

Two coil are connected in series : Coils are lying close together (M)

If $M = 0$, $L = L_1 + L_2$

If $M \neq 0$ $L = L_1 + L_2 + 2M$

(a) When current in both is in the same direction Then $L = (L_1 + M) + (L_2 + M)$

(b) When current flow in two coils are mutually in opposite directions.

$L = L_1 + L_2 - 2M$

Two coils are connected in parallel :

(a) If $M = 0$ then $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1 L_2}{L_1 + L_2}$ (b) If $M \neq 0$ then $\frac{1}{L} = \frac{1}{(L_1 + M)} + \frac{1}{(L_2 + M)}$

Ex. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely wound wires in series carrying current in the opposite sense. What do you expect about its self-inductance ? Will it be larger or small ?

Sol. As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

This situation is similar to two coils connected in series and producing fluxes in opposite directions. Therefore, their equivalent inductance must be $L_{eq} = L + L - 2M = L + L - 2L = 0$

Ex. A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is $1.2 \times 10^{-3} \text{ m}^2$. Around its central section a coil of 300 turns is closely wound. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Sol. $M = \frac{\mu_0 N_1 N_2 A}{\ell} = \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3} = 3 \times 10^{-3} \text{ H}$

$$\mathcal{E} = -M \frac{dI}{dt} = -3 \times 10^{-3} \left[\frac{-2 - 2}{0.25} \right] = 48 \times 10^{-3} \text{ V} = 48 \text{ mV}$$

ENERGY STORED IN INDUCTOR

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field. An increasing current in an inductor causes an emf between its terminals.

$$\text{Power } P = \text{The work done per unit time} = \frac{dW}{dt} = -ei = -\left[L \frac{di}{dt}\right]i = -L i \frac{di}{dt}$$

here i = instantaneous current and L = inductance of the coil

$$\text{From } dW = -dU \text{ (energy stored)} \quad \text{so } \frac{dW}{dt} = -\frac{dU}{dt} \quad \therefore \frac{dU}{dt} = Li \frac{di}{dt} \Rightarrow dU = Li di$$

The total energy U supplied while the current increases from zero to final value i is,

$$U = L \int_0^i i di = \frac{1}{2} L (i^2)_0^i \quad \therefore U = \frac{1}{2} L I^2$$

the energy stored in the magnetic field of an inductor when a current I is $= \frac{1}{2} L I^2$.

The source of this energy is the external source of emf that supplies the current.

- After the current has reached its final steady state value I , $\frac{di}{dt} = 0$ and no more energy is input to the inductor.
- When the current decreases from i to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2} L i^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch the current decreases very rapidly, the induced emf is very large and the energy may be dissipated in an arc the switch.

MAGNETIC ENERGY PER UNIT VOLUME OR ENERGY DENSITY

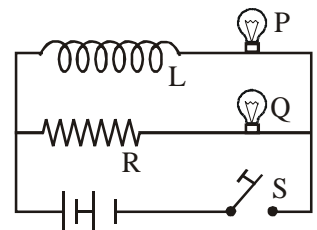
- The energy in an inductor is actually stored in the magnetic field within the coil. For a long solenoid its magnetic field can be assumed completely within the solenoid.

The energy U stored in the solenoid when a current I is,

$$U = \frac{1}{2} L I^2 = \frac{1}{2} (\mu_0 n^2 V) I^2 \quad (L = \mu_0 n^2 V) \quad (V = \text{Volume} = A\ell)$$

$$\text{The energy per unit volume } u = \frac{U}{V} = \frac{1}{2} \mu_0 n^2 I^2 = \frac{(\mu_0 n I)^2}{2 \mu_0} = \frac{B^2}{2 \mu_0} \quad (B = \mu_0 n I) \therefore u = \frac{1}{2} \frac{B^2}{\mu_0}$$

Ex. Figure shows an inductor L a resistor R connected in parallel to a battery through a switch. The resistance of R is same as that of the coil that makes L . Two identical bulb are put in each arm of the circuit.



- Which of two bulbs lights up earlier when S is closed?
- Will the bulbs be equally bright after some time?

- Sol.**
- When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of lamp P so lamp Q light up earlier.
 - Yes. At steady state inductive effect becomes meaningless so both lamps become equally bright after some time.

Ex. A very small circular loop of area $5 \times 10^{-4} \text{ m}^2$, resistance 2 ohm and negligible inductance is initially coplanar and concentric with a much larger fixed circular loop of radius 0.1 m. A constant current of 1 ampere is passed in the bigger loop and the smaller loop is rotated with angular velocity ω rad/s about a diameter. Calculate (a) the flux linked with the smaller loop (b) induced emf and induced current in the smaller loop as a function of time.

Sol. (a) The field at the centre of larger loop $B_1 = \frac{\mu_0 I}{2R} = \frac{2\pi \times 10^{-7}}{0.1} = 2\pi \times 10^{-6} \text{ Wb/m}^2$

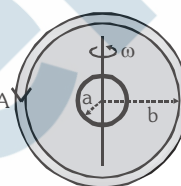
is initially along the normal to the area of smaller loop. Now as the smaller loop (and hence normal to its plane) is rotating at angular velocity ω , with respect to \vec{B} so the flux linked with the smaller loop at time t is, $\phi_2 = B_1 A_2 \cos \theta = (2\pi \times 10^{-6}) (5 \times 10^{-4}) \cos \omega t$
i.e., $\phi_2 = \pi \times 10^{-9} \cos \omega t \text{ Wb}$

(b) The induced emf in the smaller loop

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(\pi \times 10^{-9} \cos \omega t)$$

$$= \pi \times 10^{-9} \omega \sin \omega t \text{ volt}$$

(c) The induced current in the smaller loop is, $I_2 = \frac{e_2}{R} = \frac{1}{2} \pi \omega \times 10^{-9} \sin \omega t \text{ ampere}$



R-L DC CIRCUIT

Current Growth

(i) **emf equation** $E = IR + L \frac{dI}{dt}$

(ii) **Current at any instant**

When key is closed the current in circuit increases exponentially with respect to time. The current in

circuit at any instant 't' given by $I = I_0 \left[1 - e^{-\frac{t}{\lambda}} \right]$

$t = 0$ (just after the closing of key) $\Rightarrow I = 0$

$t = \infty$ (some time after closing of key) $\Rightarrow I \rightarrow I_0$

(iii) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero.

Open circuit, $t = 0, I = 0$

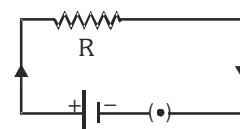
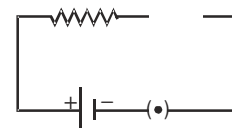
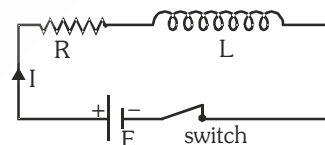
Inductor provide infinite resistance

(iv) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and current in circuit is constant.

Short circuit, $t \rightarrow \infty, I \rightarrow I_0$, Inductor provide zero resistance $I_0 = \frac{E}{R}$

(Final, steady, maximum or peak value of current) or ultimate current

Note : Peak value of current in circuit does not depends on self inductance of coil.



(v) Time constant of circuit (λ)

$\lambda = \frac{L}{R_{\text{sec}}}$ It is a time in which current increases up to 63% or 0.63 times of peak current value.

(vi) Half life (T)

It is a time in which current increases upto 50% or 0.50 times of peak current value.

$$I = I_0 (1 - e^{-t/\lambda}), t = T, I = \frac{I_0}{2} \Rightarrow \frac{I_0}{2} = I_0 (1 - e^{-T/\lambda}) \Rightarrow e^{-T/\lambda} = \frac{1}{2} \Rightarrow e^{T/\lambda} = 2$$

$$\frac{T}{\lambda} \log_e e = \log_e 2 \Rightarrow T = 0.693 \lambda \Rightarrow T = 0.693 \frac{L}{R_{\text{sec}}}$$

(vii) Rate of growth of current at any instant :-

$$\left[\frac{dI}{dt} \right] = \frac{E}{L} (e^{-t/\lambda}) \Rightarrow t = 0 \Rightarrow \left[\frac{dI}{dt} \right]_{\text{max}} = \frac{E}{L} \quad t = \infty \Rightarrow \left[\frac{dI}{dt} \right] \rightarrow 0$$

Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.

Current Decay

(i) Emf equation $IR + L \frac{dI}{dt} = 0$

(ii) Current at any instant

Once current acquires its final max steady value, if suddenly switch is put off then current start decreasing exponentially wrt to time. At switch put off condition $t = 0, I = I_0$, source emf E is cut off from circuit $I = I_0 (e^{-t/\lambda})$

Just after opening of key $t = 0 \Rightarrow I = I_0 = \frac{E}{R}$

Some time after opening of key $t \rightarrow \infty \Rightarrow I \rightarrow 0$

(iii) Time constant (λ)

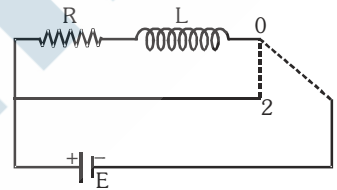
It is a time in which current decreases up to 37% or 0.37 times of peak current value.

(iv) Half life (T)

It is a time in which current decreases upto 50% or 0.50 times of peak current value.

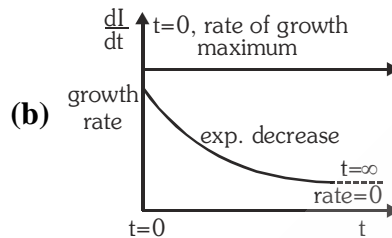
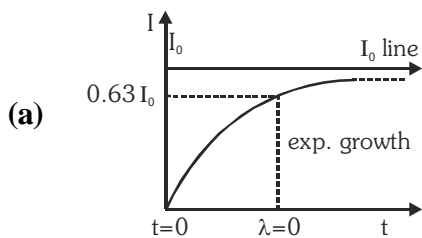
(v) Rate of decay of current at any instant

$$\left[-\frac{dI}{dt} \right] = \left[\frac{E}{L} \right] e^{-t/\lambda} \quad t = 0, \left[-\frac{dI}{dt} \right]_{\text{max}} = \frac{E}{L} \quad t \rightarrow \infty \Rightarrow \left[-\frac{dI}{dt} \right] \rightarrow 0$$

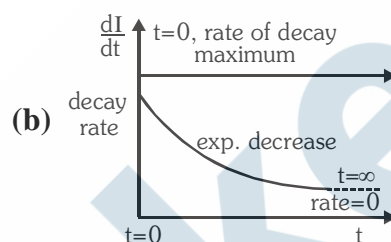
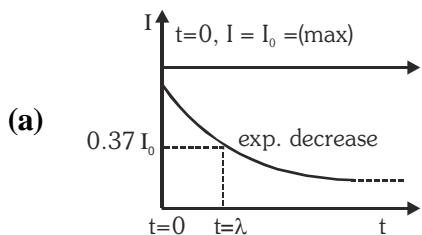


Graph for R-L circuit :-

Current Growth :-



Current decay :-



LC Oscillations :

Consider an *LC* circuit in which a capacitor is connected to an inductor, as shown in Figure.

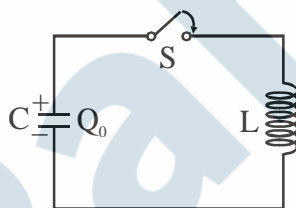


Figure *LC* Circuit

Suppose the capacitor initially has charge Q_0 . When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.

The total energy in the *LC* circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt} = \frac{d}{dt} \left(\frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} LI^2 \right) = \frac{Q}{C} \frac{dQ}{dt} + LI \frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L \frac{d^2Q}{dt^2} = 0$$

where $I = -dQ / dt$ (and $dI / dt = -d^2Q/dt^2$). Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of change in the capacitor plate immediately after the switch has been closed. The same equation can be obtained by applying the modified Kirchhoff's loop rule clockwise.

$$\frac{Q}{C} - L \frac{dI}{dt} = 0$$

followed by our definition of current.

The general solution to equation is $Q(t) = Q_0 \cos(\omega_0 t + \phi)$

where Q_0 is the amplitude of the charge and ϕ is the phase. The angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I(t) = -\frac{dQ}{dt} = \omega_0 Q_0 \sin(\omega_0 t + \phi) = I_0 \sin(\omega_0 t + \phi)$$

where $I_0 = \omega_0 Q_0$. From the initial conditions $Q(t = 0) = Q_0$ and $I(t = 0) = 0$, the phase ϕ can be determined to $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$$Q(t) = Q_0 \cos \omega_0 t$$

$$\text{and } I(t) = I_0 \sin \omega_0 t$$

The time dependence of $Q(t)$ and $I(t)$ are depicted in figure.

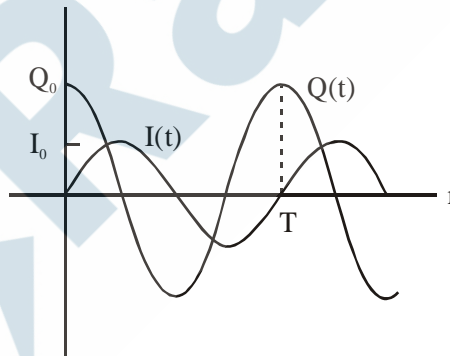


Figure : Charge and current in the LC circuit as a function of time

Using Eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E = U_E = \frac{Q^2(t)}{2C} = \left(\frac{Q_0^2}{2C} \right) \cos^2 \omega_0 t$$

$$\text{and } U_B = \frac{1}{2} LI^2(t) = \frac{LI_0^2}{2} \sin^2 \omega_0 t = \frac{L(-\omega_0 Q_0)^2}{2} \sin^2 \omega_0 t = \left(\frac{Q_0^2}{2C} \right) \sin^2 \omega_0 t = \frac{Q_0^2}{2C}$$

The electric and magnetic energy oscillation is illustrated in figure.

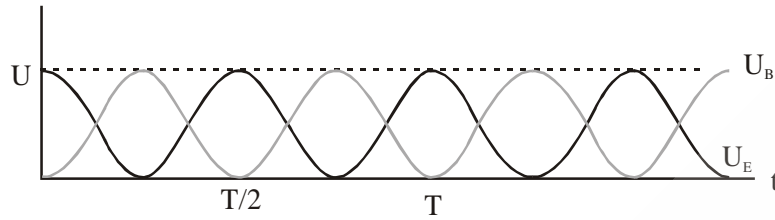


Figure : Electric and magnetic energy oscillations

The mechanical analog of the *LC* oscillations is the mass-spring system, shown in Figure.

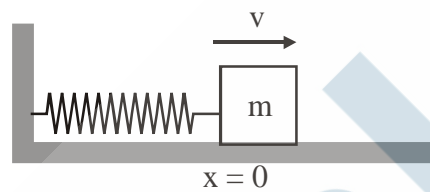


Figure : Mass-spring oscillations

If the mass is moving with a speed v and the spring having a spring constant k is displaced from its equilibrium by x , then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively.

In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt} \left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2 \right) = mv \frac{dv}{dt} + kx \frac{dx}{dt} = 0$$

Using $v = dx/dt$ and $dv/dt = d^2x/dt^2$, the above equation may be rewritten as

$$m \frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$

is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

$$U = \frac{1}{2} m x_0^2 \omega_0^2 \sin^2 (\omega_0 t + \phi) + \frac{1}{2} k x_0^2 \cos^2 (\omega_0 t + \phi)$$

$$= \frac{1}{2} k x_0^2 [\sin^2 (\omega_0 t + \phi) + \cos^2 (\omega_0 t + \phi)] = \frac{1}{2} k x_0^2$$

In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).

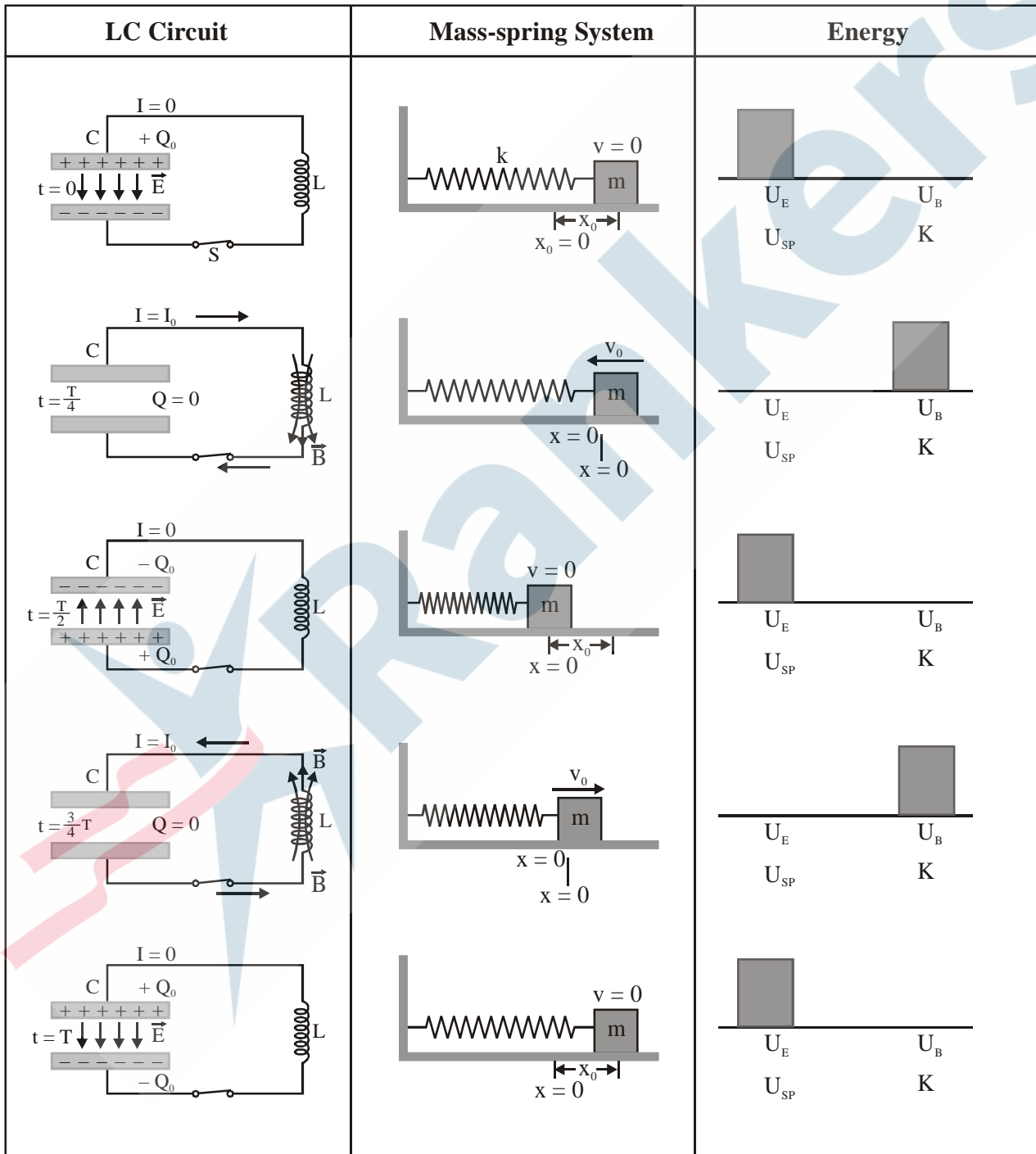


Figure : Energy oscillations in the *LC* Circuit and the mass-spring system

LC Circuit :

Ex. Consider the circuit shown in Figure. Suppose the switch which has been connected to point *a* for a long time is suddenly thrown to *b* at $t = 0$.

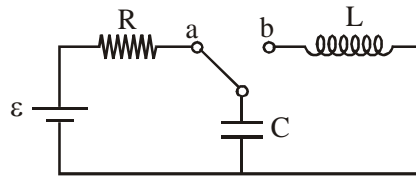


Figure : LC circuit

Find the following quantities :

- (a) the frequency of oscillation of the *LC* circuit.
- (b) the maximum charge that appears on the capacitor.
- (c) the maximum current in the inductor.
- (d) the total energy the circuit possesses at any time t .

Solution :

- (a) The (angular) frequency of oscillation of the *LC* circuit is given by $\omega = 2\pi f = 1/\sqrt{LC}$. Therefore, the frequency is :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

- (b) The maximum charge stored in the capacitor before the switch is thrown to *b* is
 $Q = C\varepsilon$

- (c) The energy stored in the capacitor before the switch is thrown is :

$$U_E = \frac{1}{2} C\varepsilon^2$$

On the other hand, the magnetic energy stored in the inductor is :

$$U_B = \frac{1}{2} LI^2$$

Thus, when the current is at its maximum, all the energy originally stored in the capacitor is now in the inductor :

$$\frac{1}{2} C\varepsilon^2 = \frac{1}{2} LI_0^2$$

This implies a maximum current

$$I_0 = \varepsilon \sqrt{\frac{C}{L}}$$

- (d) At any time, the total energy in the circuit would be equal to the initial energy that the capacitance stored, that is

$$U = U_E + U_B = \frac{1}{2} C\varepsilon^2$$

ALTERNATING CURRENT

ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it is change continuously in magnitude and periodically in direction with time. It can be represented by a sine curve or cosine curve

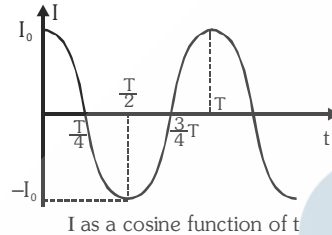
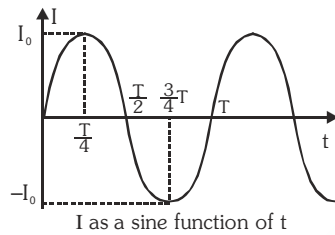
$$I = I_0 \sin \omega t \quad \text{or} \quad I = I_0 \cos \omega t$$

where I = Instantaneous value of current at time t ,

I_0 = Amplitude or peak value

$$\omega = \text{Angular frequency} \quad \omega = \frac{2\pi}{T} = 2\pi f$$

T = time period f = frequency



AMPLITUDE OF AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0 . Peak to peak value = $2I_0$

PERIODIC TIME

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

FREQUENCY

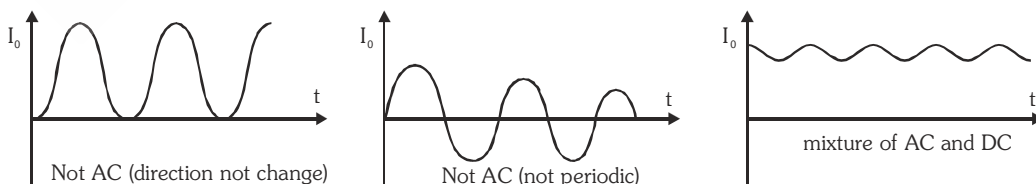
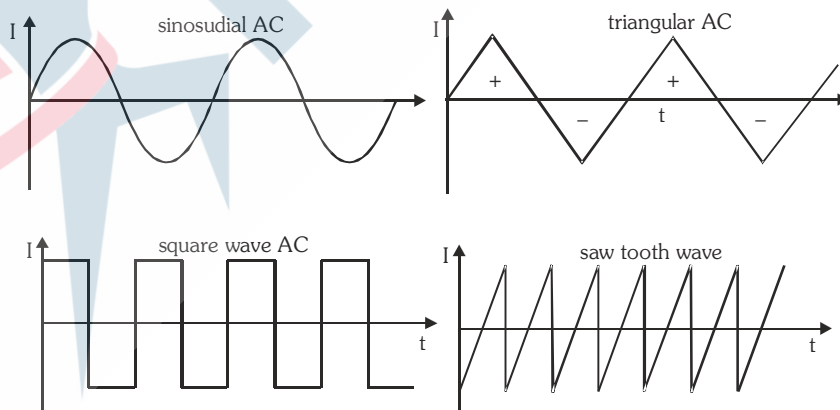
The number of cycle completed by an alternating current in one second is called the frequency of the current.

UNIT : cycle/s ; (Hz)

In India : $f = 50$ Hz , supply voltage = 220 volt **In USA** : $f = 60$ Hz , supply voltage = 110 volt

CONDITION REQUIRED FOR CURRENT/ VOLTAGE TO BE ALTERNATING

- Amplitude is constant
 - Alternate half cycle is positive and half negative
- The alternating current continuously varies in magnitude and periodically reverses its direction.



AVERAGE VALUE OR MEAN VALUE

The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

$$\text{average value of current for half cycle } \langle I \rangle = \frac{\int_0^{T/2} I dt}{\int_0^{T/2} dt}$$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{T/2} I_0 \sin \omega t dt}{\int_0^{T/2} dt} = \frac{2I_0}{\omega T} [-\cos \omega t]_0^{T/2} = \frac{2I_0}{\pi}$$

$$\begin{aligned} \langle \sin \theta \rangle &= \langle \sin 2\theta \rangle = 0 \\ \langle \cos \theta \rangle &= \langle \cos 2\theta \rangle = 0 \\ \langle \sin \theta \cos \theta \rangle &= 0 \\ \langle \sin^2 \theta \rangle &= \langle \cos^2 \theta \rangle = \frac{1}{2} \end{aligned}$$

- For symmetric AC, average value over full cycle = 0,
Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(-ve) half cycle
0	$\frac{2I_0}{\pi}$	$-\frac{2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

MAXIMUM VALUE

- $I = a \sin \theta \Rightarrow I_{Max.} = a$
- $I = a + b \sin \theta \Rightarrow I_{Max.} = a + b$ (if a and $b > 0$)
- $I = a \sin \theta + b \cos \theta \Rightarrow I_{Max.} = \sqrt{a^2 + b^2}$
- $I = a \sin^2 \theta \Rightarrow I_{Max.} = a$ ($a > 0$)

ROOT MEAN SQUARE (rms) VALUE

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} \quad \text{rms value} = \text{Virtual value} = \text{Apparent value}$$

rms value of $I = I_0 \sin \omega t$:

$$\begin{aligned} I_{rms} &= \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T} \int_0^T \sin^2 \omega t dt} \\ &= I_0 \sqrt{\frac{1}{T} \int_0^T \left[\frac{1 - \cos 2\omega t}{2} \right] dt} = I_0 \sqrt{\frac{1}{T} \left[\frac{t}{2} - \frac{\sin 2\omega t}{2 \times 2\omega} \right]_0^T} = \frac{I_0}{\sqrt{2}} \end{aligned}$$

- If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular frequency
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2} \sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin \omega t + I_0 \cos \omega t$	0	$\sqrt{2} I_0$	I_0	ω

- For above varieties of current $rms = \frac{\text{Peak value}}{\sqrt{2}}$

Ex. If $I = 2\sqrt{t}$ ampere then calculate average and rms values over $t = 2$ to 4 s

Sol. $\langle I \rangle = \frac{\int_2^4 2\sqrt{t} dt}{\int_2^4 dt} = \frac{4 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right)_2^4}{(t)_2^4} = \frac{2}{3} [8 - 2\sqrt{2}]$ and $I_{rms} = \sqrt{\frac{\int_2^4 (2\sqrt{t})^2 dt}{\int_2^4 dt}} = \sqrt{\frac{\int_2^4 4t dt}{2}} = \sqrt{2 \left[\frac{t^2}{2} \right]_2^4} = 2\sqrt{3} \text{ A}$

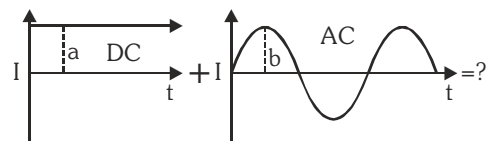
Ex. Find the time required for 50Hz alternating current to change its value from zero to rms value.

Sol. $\because I = I_0 \sin \omega t \therefore \frac{I_0}{\sqrt{2}} = I_0 \sin \omega t \Rightarrow \sin \omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \frac{\pi}{4}$
 $\Rightarrow \left(\frac{2\pi}{T} \right) t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8} = \frac{1}{8 \times 50} = 2.5 \text{ ms}$

Ex. If $E = 20 \sin (100\pi t)$ volt then calculate value of E at $t = \frac{1}{600} \text{ s}$

Sol. At $t = \frac{1}{600} \text{ s}$ $E = 20 \sin \left[100\pi \times \frac{1}{600} \right] = 20 \sin \left[\frac{\pi}{6} \right] = 20 \times \frac{1}{2} = 10 \text{ V}$

Ex. If a direct current of value a ampere is superimposed on an alternating current $I = b \sin \omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?



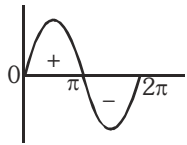
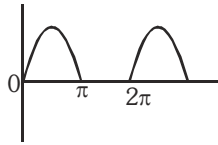
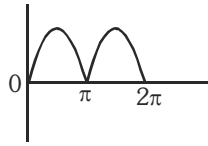
Sol. As current at any instant in the circuit will be,

$$I = I_{DC} + I_{AC} = a + b \sin \omega t$$

$$\therefore I_{eff} = \sqrt{\frac{1}{T} \int_0^T I^2 dt} = \sqrt{\frac{1}{T} \int_0^T (a + b \sin \omega t)^2 dt} = \sqrt{\frac{1}{T} \int_0^T (a^2 + 2ab \sin \omega t + b^2 \sin^2 \omega t) dt}$$

but as $\frac{1}{T} \int_0^T \sin \omega t dt = 0$ and $\frac{1}{T} \int_0^T \sin^2 \omega t dt = \frac{1}{2}$ $\therefore I_{eff} = \sqrt{a^2 + \frac{1}{2}b^2}$

SOME IMPORTANT WAVE FORMS AND THEIR RMS AND AVERAGE VALUE

Nature of wave form	Wave-form	RMS Value	Average or mean Value
Sinusoidal		$\frac{I_0}{\sqrt{2}}$ $= 0.707 I_0$	$\frac{2I_0}{\pi}$ $= 0.637 I_0$
Half wave rectified		$\frac{I_0}{2}$ $= 0.5 I_0$	$\frac{I_0}{\pi}$ $= 0.318 I_0$
Full wave rectified		$\frac{I_0}{\sqrt{2}}$ $= 0.707 I_0$	$\frac{2I_0}{\pi}$ $= 0.637 I_0$

PHASE AND PHASE DIFFERENCE

(a) Phase

$$I = I_0 \sin (\omega t \pm \phi)$$

Initial phase = ϕ (it does not change with time)

Instantaneous phase = $\omega t \pm \phi$ (it changes with time)

- Phase decides both value and sign. © UNIT: radian

(b) Phase difference

$$\text{Voltage } V = V_0 \sin (\omega t + \phi_1)$$

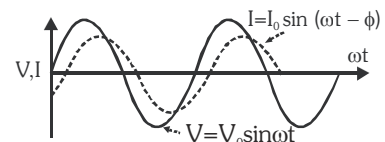
$$\text{Current } I = I_0 \sin (\omega t + \phi_2)$$

- Phase difference of I w.r.t. V $\phi = \phi_2 - \phi_1$
- Phase difference of V w.r.t. I $\phi = \phi_1 - \phi_2$

LAGGING AND LEADING CONCEPT

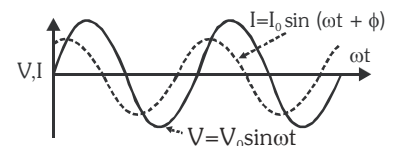
(a) V leads I or I lags V → It means, V reach maximum before I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin (\omega t - \phi)$
 and if $V = V_0 \sin (\omega t + \phi)$ then $I = I_0 \sin \omega t$



(b) V lags I or I leads V → It means V reach maximum after I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin (\omega t + \phi)$
 and if $V = V_0 \sin (\omega t - \phi)$ then $I = I_0 \sin \omega t$

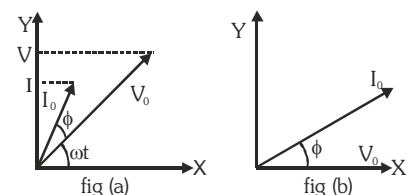


PHASOR AND PHASOR DIAGRAM

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angle between them is called phasor diagram.

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin (\omega t + \phi)$

In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity



Ex. The Equation of current in AC circuit is $I = 4\sin\left[100\pi t + \frac{\pi}{3}\right]$ A. Calculate.

(i) RMS Value (ii) Peak Value (iii) Frequency (iv) Initial phase (v) Current at $t = 0$

Sol. (i) $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ A (ii) Peak value $I_0 = 4$ A

(iii) $\therefore \omega = 100\pi$ rad/s \therefore frequency $f = \frac{100\pi}{2\pi} = 50$ Hz

(iv) Initial phase = $\frac{\pi}{3}$ (v) At $t = 0$, $I = 4\sin\left[100\pi \times 0 + \frac{\pi}{3}\right] = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ A

Ex. If $I = I_0 \sin \omega t$, $E = E_0 \cos\left[\omega t + \frac{\pi}{3}\right]$. Calculate phase difference between E and I

Sol. $I = I_0 \sin \omega t$ and $E = E_0 \sin\left[\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right]$ \therefore phase difference = $\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

Ex. If $E = 500 \sin(100\pi t)$ volt then calculate time taken to reach from zero to maximum.

Sol. $\therefore \omega = 100\pi \Rightarrow T = \frac{2\pi}{100\pi} = \frac{1}{50}$ s, time taken to reach from zero to maximum = $\frac{T}{4} = \frac{1}{200}$ s

Ex. Show that average heat produced during a cycle of AC is same as produced by DC with $I = I_{\text{rms}}$.

Sol. For AC, $I = I_0 \sin \omega t$, the instantaneous value of heat produced (per second) in a resistance R is,
 $H = I^2 R = I_0^2 \sin^2 \omega t \times R$ the average value of heat produced during a cycle is :

$$H_{\text{av}} = \frac{\int_0^T H dt}{\int_0^T dt} = \frac{\int_0^T (I_0^2 \sin^2 \omega t \times R) dt}{\int_0^T dt} = \frac{1}{2} I_0^2 R \left[\because \int_0^T I_0^2 \sin^2 \omega t dt = \frac{1}{2} I_0^2 T \right]$$

$$\Rightarrow H_{\text{av}} = \left(\frac{I_0}{\sqrt{2}} \right)^2 R = I_{\text{rms}}^2 R \dots (i)$$

However, in case of DC, $H_{\text{DC}} = I^2 R \dots (ii)$ $\therefore I = I_{\text{rms}}$ so from equation (i) and (ii) $H_{\text{DC}} = H_{\text{av}}$
 AC produces same heating effects as DC of value $I = I_{\text{rms}}$. This is also why AC instruments which are based on heating effect of current give rms value.

DIFFERENT TYPES OF AC CIRCUITS

In order to study the behaviour of A.C. circuits we classify them into two categories :

- Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.
- Complicated circuit containing any two of the three circuit elements R, L and C or all of three elements.

AC CIRCUIT CONTAINING PURE RESISTANCE

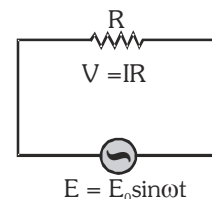
Let at any instant t the current in the circuit = I.

Potential difference across the resistance = I R.

with the help of kirchoff's circuital law $E - I R = 0$

$$\Rightarrow E_0 \sin \omega t = I R$$

$$\Rightarrow I = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t \quad (I_0 = \frac{E_0}{R} = \text{peak or maximum value of current})$$



Alternating current developed in a pure resistance is also of sinusoidal nature. In an a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. as shown in figure.

In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.

i.e. $I = I_0 \sin \omega t$ and $E = E_0 \sin \omega t$.

Since $I_0 = \frac{E_0}{R}$, hence $\frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} \Rightarrow I_{\text{rms}} = \frac{E_{\text{rms}}}{R}$

AC CIRCUIT CONTAINING PURE INDUCTANCE

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf.

Let the alternating e.m.f. $E = E_0 \sin \omega t$

When a.c. flows through the circuit, emf induced across inductance $= -L \frac{dI}{dt}$

Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other than inductance so with the help of

Kirchoff's circuital law $E + \left(-L \frac{dI}{dt}\right) = 0 \Rightarrow E = L \frac{dI}{dt}$ so we get $I = \frac{E_0}{\omega L} \sin \left(\omega t - \frac{\pi}{2}\right)$

Maximum current $I_0 = \frac{E_0}{\omega L} \times 1 = \frac{E_0}{\omega L}$, Hence, $I = I_0 \sin \left(\omega t - \frac{\pi}{2}\right)$

In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$.

or alternating emf leads the a. c. by a phase angle of $\frac{\pi}{2}$.

Expression $I_0 = \frac{E_0}{\omega L}$ resembles the expression $\frac{E}{I} = R$.

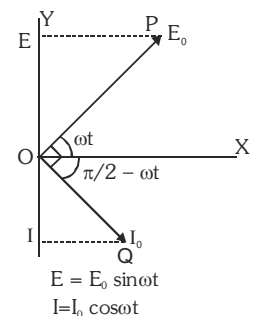
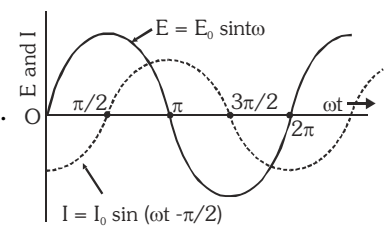
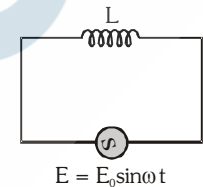
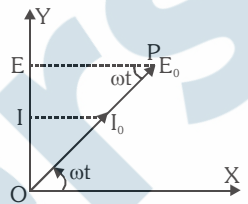
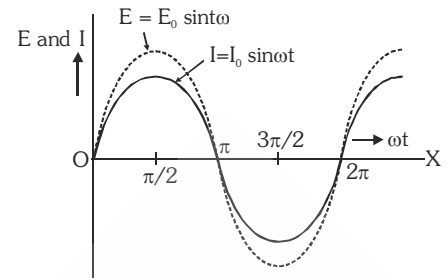
This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circuit.

$X_L = \omega L = 2 \pi f L$ where f = frequency of A.C.

Unit of X_L : ohm

$(\omega L) = \text{Unit of } L \times \text{Unit of } \omega = \text{henry} \times \text{sec}^{-1}$

$= \frac{\text{Volt}}{\text{Ampere / sec}} \times \text{sec}^{-1} = \frac{\text{Volt}}{\text{Ampere}} = \text{ohm}$

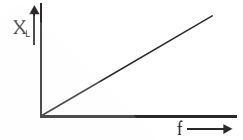


Inductive reactance $X_L \propto f$

Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.

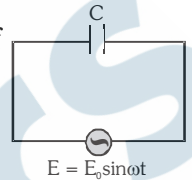
For d.c. circuit, $f = 0$ $\therefore X_L = \omega L = 2\pi f L = 0$

Hence, inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c.



AC CIRCUIT CONTAINING PURE CAPACITANCE

A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin \omega t$. When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.



The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time. Let at any instant t charge on the capacitor = q

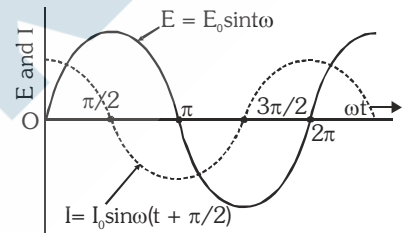
Instantaneous potential difference across the capacitor $E = \frac{q}{C}$

$$\Rightarrow q = C E \Rightarrow q = C E_0 \sin \omega t$$

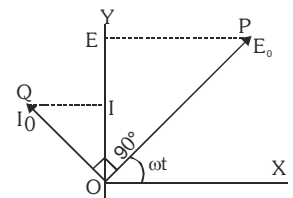
The instantaneous value of current

$$I = \frac{dq}{dt} = \frac{d}{dt}(C E_0 \sin \omega t) = C E_0 \omega \cos \omega t$$

$$\Rightarrow I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where } I_0 = \omega C V_0$$



In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\pi/2$. The alternating emf lags behind the alternating current by a phase angle of $\pi/2$.



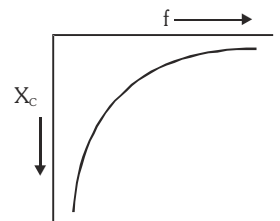
IMPORTANT POINTS

$\frac{E}{I}$ is the resistance R when both E and I are in phase, in present case they

differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor,

the capacitor offer opposition to the flow of A.C. This non-resistive opposition

to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C . $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$



Unit of X_C : ohm

Capacitive reactance X_C is inversely proportional to frequency of A.C. X_C decreases as the frequency increases.

This is because with an increase in frequency, the capacitor charges and discharges rapidly following the flow of current.

For d.c. circuit $f = 0$ $\therefore X_c = \frac{1}{2\pi fC} = \infty$ but has a very small value for a.c.

This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

INDIVIDUAL COMPONENTS (R or L or C)

TERM	R	L	C
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	$R = \text{Resistance}$	$X_L = \text{Inductive reactance.}$	$X_C = \text{Capacitive reactance.}$
Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with f			
G, S_L, S_C (mho, seiman)	R does not depend on f $G = 1/R = \text{conductance.}$	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f} \right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

Ex. A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz Calculate its reactance.

Sol. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 10^3 \times 50 \times 10^{-12}} = \frac{10^7}{\pi} \Omega$

Ex. In given circuit applied voltage $V = 50\sqrt{2} \sin 100\pi t$ volt and ammeter reading is 2A then calculate value of L

Sol. $V_{rms} = I_{rms} X_L \quad \because \text{Reading of ammeter} = I_{rms}$

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{\sqrt{2} I_{rms}} = \frac{50\sqrt{2}}{\sqrt{2} \times 2} = 25 \Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi} \text{ H}$$

Ex. A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp ?

Sol. \because resistance of the lamp $R = \frac{V_s^2}{W} = \frac{(100)^2}{50} = 200 \Omega$ and the maximum current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2} \text{ A}$

\therefore when the lamp is put in series with a capacitance and run at 200 V AC, from $V = IZ$

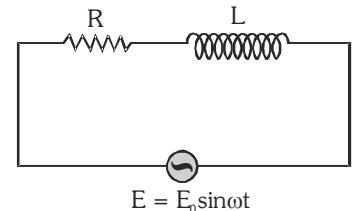
$$Z = \frac{V}{I} = \frac{200}{\frac{1}{2}} = 400 \Omega \quad \text{Now as in case of C-R circuit } Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}},$$

$$\Rightarrow R^2 + \frac{1}{(\omega C)^2} = (400)^2 \quad \Rightarrow \frac{1}{(\omega C)^2} = 16 \times 10^4 - (200)^2 = 12 \times 10^4 \quad \Rightarrow \frac{1}{\omega C} = \sqrt{12} \times 10^2$$

$$\Rightarrow C = \frac{1}{100\pi \times \sqrt{12} \times 10^2} \text{ F} = \frac{100}{\pi \sqrt{12}} \mu\text{F} = 9.2 \mu\text{F}$$

RESISTANCE AND INDUCTANCE IN SERIES (R-L CIRCUIT)

A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.



PHASOR DIAGRAM FOR L-R CIRCUIT

Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin \omega t$.

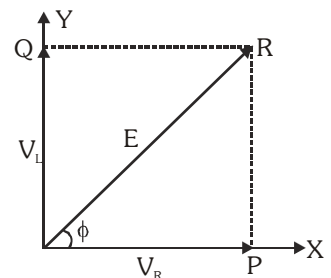
As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.

Then $V_L = IX_L$ and $V_R = IR$

Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$.

So V_R and V_L are mutually perpendicular (Note : $E \neq V_R + V_L$)

The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).



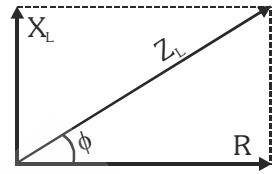
The resultant of V_R and V_L = the magnitude of vector OR $E = \sqrt{V_R^2 + V_L^2}$

$$\text{Thus } E^2 = V_R^2 + V_L^2 = I^2 (R^2 + X_L^2) \Rightarrow I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind

the e.m.f. E by a phase angle ϕ $\tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R}$

$$\Rightarrow \phi = \tan^{-1} \left(\frac{\omega L}{R} \right)$$



Inductive Impedance Z_L :

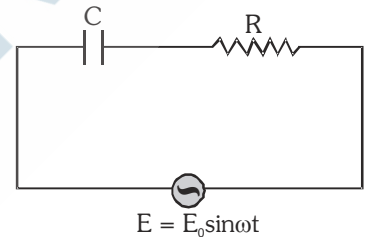
In L-R circuit the maximum value of current $I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$ Here $\sqrt{R^2 + \omega^2 L^2}$ represents the

effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L . $Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi fL)^2}$ The reciprocal of

impedance is called admittance $Y_L = \frac{1}{Z_L} = \frac{1}{\sqrt{R^2 + \omega^2 L^2}}$

RESISTANCE AND CAPACITOR IN SERIES (R-C CIRCUIT)

A circuit containing a series combination of a resistance R and a capacitor C , connected with a source of e.m.f. of peak value E_0 as shown in fig.



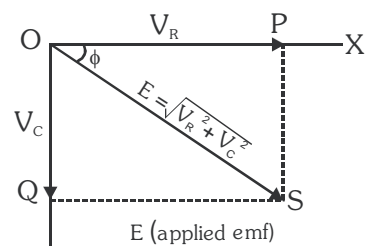
PHASOR DIAGRAM FOR R-C CIRCUIT

Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$$V_C = I X_C \text{ and } V_R = I R$$

where X_C = capacitive reactance and I = instantaneous current.

Now, V_R is in phase with I , while V_C lags behind I by 90° .



The phasor diagram is shown in fig.

The vector OP represents V_R (which is in phase with I)

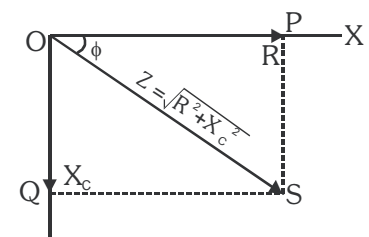
and the vector OQ represents V_C (which lags behind I by $\frac{\pi}{2}$).

The vector OS represents the resultant of V_R and

V_C = the applied e.m.f. E .

$$\text{Hence } V_R^2 + V_C^2 = E^2 \Rightarrow E = \sqrt{V_R^2 + V_C^2}$$

$$\Rightarrow E^2 = I^2 (R^2 + X_C^2) \Rightarrow I = \frac{E}{\sqrt{R^2 + X_C^2}}$$



The term $\sqrt{R^2 + X_C^2}$ represents the effective resistance of the R-C circuit and called the capacitive

impedance Z_C of the circuit. Hence, in C-R circuit $Z_C = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

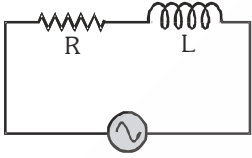
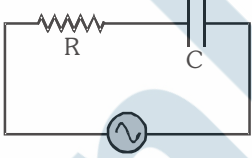
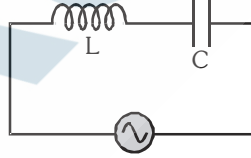
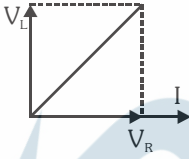
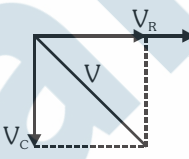
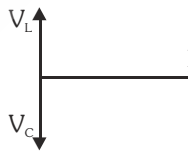
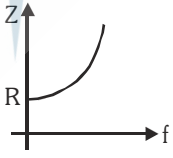
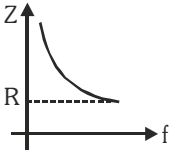
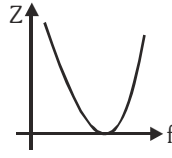
Capacitive Impedance Z_C :

In R-C circuit the term $\sqrt{R^2 + X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C

The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan \phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}, \quad \tan \phi = \frac{X_C}{R} = \frac{1}{\omega CR} \Rightarrow \phi = \tan^{-1} \left(\frac{1}{\omega CR} \right)$$

COMBINATION OF COMPONENTS (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit			
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C \text{ (} V_L > V_C \text{)}$ $V = V_C - V_L \text{ (} V_C > V_L \text{)}$
Phase difference in between V and I	V leads I ($\phi = 0$ to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0) V leads I ($\phi = +\frac{\pi}{2}$, if $X_L > X_C$)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$)
Variation of Z with f	$Z = \sqrt{R^2 + X_L^2}$ as $f \uparrow, Z \uparrow$ 	$Z = X_L - X_C $ as $f \uparrow, Z \downarrow$ 	as $f \uparrow, Z$ first \downarrow then \uparrow 
At very low f	$Z \simeq R \text{ (} X_L \rightarrow 0 \text{)}$	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R \text{ (} X_C \rightarrow 0 \text{)}$	$Z \simeq X_L$

Ex. Calculate the impedance of the circuit shown in the figure.

Sol. $Z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50 \Omega$

Ex. If $X_L = 50 \Omega$ and $X_C = 40 \Omega$ Calculate effective value of current in given circuit.

Sol. $Z = X_L - X_C = 10 \Omega$

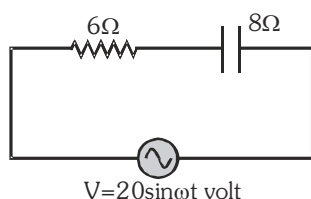
$$I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4A \Rightarrow I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2} A$$

Ex. In given circuit calculate, voltage across inductor

Sol. $\because V^2 = V_R^2 + V_L^2 \quad \therefore V_L^2 = V^2 - V_R^2$

$$V_L \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (60)^2} = \sqrt{6400} = 80 V$$

Ex. In given circuit find out (i) impedance of circuit (ii) current in circuit



Sol. (i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{(6)^2 + (8)^2} = 10 \Omega$

(ii) $V = IZ \Rightarrow I = \frac{V_0}{Z} = \frac{20}{10} = 2A$ so $I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2} A$

Ex. When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Sol. For 10 V D.C. $\because V = IR \quad \therefore$ Resistance of coil $R = \frac{10}{2.5} = 4\Omega$ For 10 V A.C. $\Leftarrow V = IZ$

$$\Rightarrow Z = \frac{V}{I} = \frac{20}{10} = 5\Omega$$

$$\because Z = \sqrt{R^2 + X_L^2} = 5 \Rightarrow R^2 + X_L^2 = 25 \Rightarrow X_L^2 = 5^2 - 4^2 \Rightarrow X_L = 3 \Omega$$

Ex. When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by $\pi/2$ radians.

(a) Name the devices X and Y.

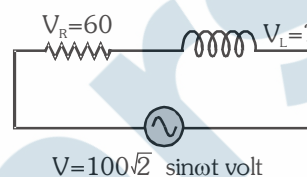
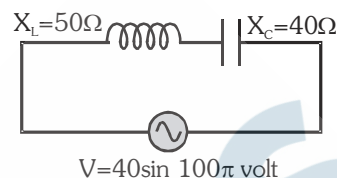
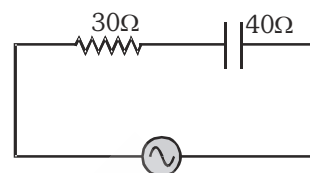
(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Sol. (a) X is resistor and Y is a capacitor

(b) Since the current in the two devices is the same (0.5A at 220 volt)

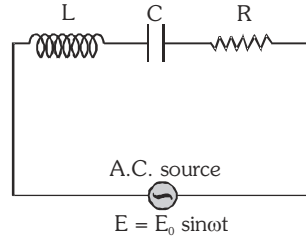
When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440 \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35A$$



INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES (L-C-R SERIES CIRCUIT)

A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0 , as shown in fig.



PHASOR DIAGRAM FOR SERIES L-C-R CIRCUIT

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$. As L, C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.

Let at any instant of time t the current in the circuit is I

Let at this time t the potential differences across L, C, and R

$$V_L = I X_L, V_C = I X_C \text{ and } V_R = I R$$

Now, V_R is in phase with current I but V_L leads I by 90°

While V_C lags behind I by 90° .

The vector OP represents V_R (which is in phase with I) the vector

OQ represent V_L (which leads I by 90°)

and the vector OS represents V_C (which lags behind I by 90°)

V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT.

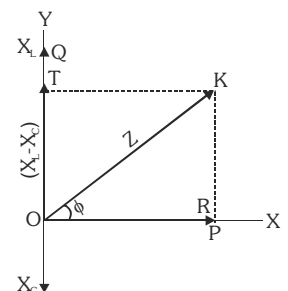
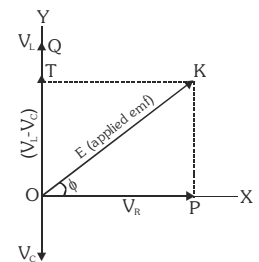
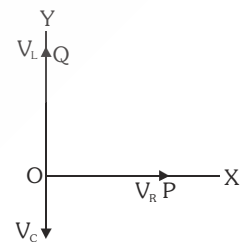
Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.

$$\text{Thus } E = \sqrt{V_R^2 + (V_L - V_C)^2} = I \sqrt{R^2 + (X_L - X_C)^2} \Rightarrow I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\text{Impedance } Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

The phasor diagram also shown that in LCR circuit the applied e.m.f.

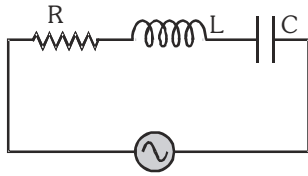
leads the current I by a phase angle ϕ $\tan \phi = \frac{X_L - X_C}{R}$



SERIES LCR AND PARALLEL LCR COMBINATION

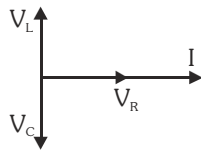
SERIES L-C-R CIRCUIT

1. Circuit diagram

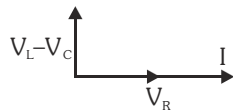


I same for R, L & C

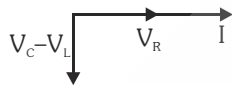
2. Phasor diagram



(i) If $V_L > V_C$ then



(ii) If $V_C > V_L$ then

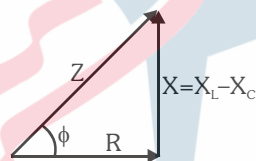


(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

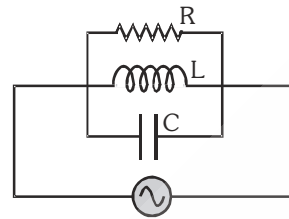
$$\text{Impedance } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$$

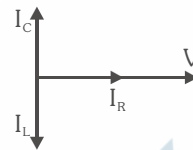
(iv) Impedance triangle



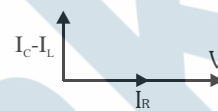
PARALLEL L-C-R CIRCUIT



V same for R, L and C
V same for R, L & C



(i) if $I_C > I_L$ then



(ii) if $I_L > I_C$ then

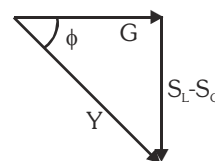


(iii) $I = \sqrt{I_R^2 + (I_L - I_C)^2}$

$$\text{Admittance } Y = \sqrt{G^2 + (S_L - S_C)^2}$$

$$\tan \phi = \frac{S_L - S_C}{G} = \frac{I_L - I_C}{I_R}$$

(iv) Admittance triangle



GOLDEN KEY POINTS

Series

- if $X_L > X_C$ then V leads I, ϕ (positive)
circuit nature inductive
- if $X_C > X_L$ then V lags I, ϕ (negative)
circuit nature capacitive

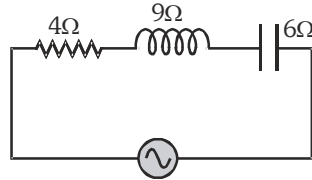
Parallel

- if $S_L > S_C$ ($X_L < X_C$) then V leads I, ϕ (positive)
circuit nature inductive
- if $S_C > S_L$ ($X_C < X_L$) then V lags I, ϕ (negative)
circuit nature capacitive

Electromagnetic induction & Alternating current

- In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and on R it never greater than source voltage or current.
- In parallel A.C. circuit phase difference between I_L and I_C is π

Ex. Find out the impedance of given circuit.



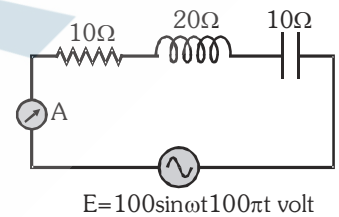
Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{4^2 + (9 - 6)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5\Omega$
 $(\because X_L > X_C \therefore \text{Inductive})$

Ex. Find out reading of A C ammeter and also calculate the potential difference across, resistance and capacitor.

Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\sqrt{2} \Omega \Rightarrow I_0 = \frac{V_0}{Z} = \frac{100}{10\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ A}$

\therefore ammeter reads RMS value, so its reading $= \frac{10}{\sqrt{2} \sqrt{2}} = 5 \text{ A}$

so $V_R = 5 \times 10 = 50 \text{ V}$ and $V_C = 5 \times 10 = 50 \text{ V}$



Ex. In LCR circuit with an AC source $R = 300 \Omega$, $C = 20 \mu\text{F}$, $L = 1.0 \text{ H}$, $E_{\text{rms}} = 50 \text{ V}$ and $f = 50/\pi \text{ Hz}$. Find RMS current in the circuit.

Sol. $I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{E_{\text{rms}}}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C} \right]^2}} = \frac{50}{\sqrt{300^2 + \left[2\pi \times \frac{50}{\pi} \times 1 - \frac{1}{20 \times 10^{-6} \times 2\pi \times \frac{50}{\pi}} \right]^2}}$

$\Rightarrow I_{\text{rms}} = \frac{50}{\sqrt{(300)^2 + \left[100 - \frac{10^3}{2} \right]^2}} = \frac{50}{100\sqrt{9+16}} = \frac{50}{100} = 0.5 \text{ A}$

RESONANCE

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance : (i) Series Resonance (ii) Parallel Resonance

SERIES RESONANCE

(a) At Resonance

- (i) $X_L = X_C$ (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)
 (iv) $Z_{\min} = R$ (impedance minimum) (v) $I_{\max} = \frac{V}{R}$ (current maximum)

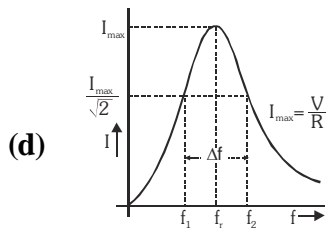
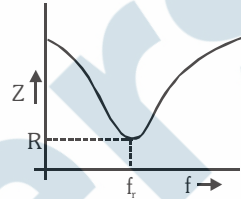
(b) Resonance frequency

$$\because X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f

- (i) If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)
 (ii) At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, $\phi = \text{zero}$
 (iii) If $f > f_r$ then $X_L > X_C$ circuit nature is inductive, ϕ (positive)

Variation of I with f as f increase, Z first decreases then increase



(d) as f increase, I first increase then decreases

- At resonance impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

Half power frequencies

The frequencies at which, power become half of its maximum value called half power frequencies

$$\text{Band width} = \Delta f = f_2 - f_1$$

Quality factor Q : Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

- (i) It represents the sharpness of resonance.
 (ii) It is unit less and dimension less quantity

$$(iii) Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{\text{band width}}$$

Magnification

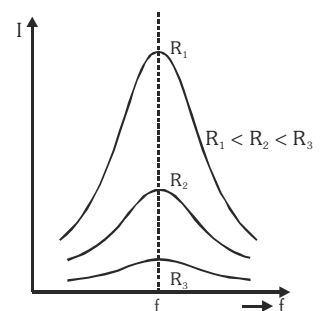
At resonance V_L or $V_C = QE$ (where E = supplied voltage)

So at resonance Magnification factor = Q-factor

Sharpness

Sharpness \propto Quality factor \propto Magnification factor

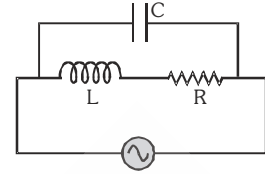
R decrease \Rightarrow Q increases \Rightarrow Sharpness increases



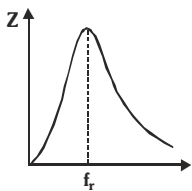
PARALLEL RESONANCE

(a) **At resonance**

- (i) $S_L = S_C$
- (ii) $I_L = I_C$
- (iii) $\phi = 0$
- (iv) $Z_{\max} = R$ (impedance maximum)
- (v) $I_{\min} = \frac{V}{R}$ (current minimum)

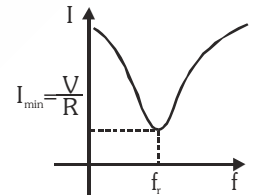


(b) **Resonant frequency** $f_r = \frac{1}{2\pi\sqrt{LC}}$

(c)  **Variation of Z with f** as f increases, Z first increases then decreases

- ⊙ If $f < f_r$ then $S_L > S_C$, ϕ (positive), circuit nature is inductive
- ⊙ If $f > f_r$ then $S_C > S_L$, ϕ (negative), circuit nature capacitive.

(d) **Variation of I with f** as f increases, I first decreases then increases

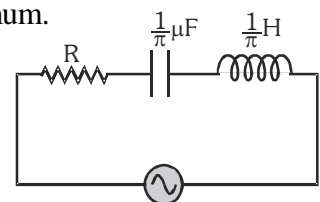


Note : For this circuit $f_r = \frac{1}{2\pi\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} \Rightarrow Z_{\max} = \frac{L}{RC}$ For resonance $\frac{1}{LC} > \frac{R^2}{L^2}$

Ex. For what frequency the voltage across the resistance R will be maximum.

Sol. It happens at resonance

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi} \times 10^{-6} \times \frac{1}{\pi}}} = 500 \text{ Hz}$$

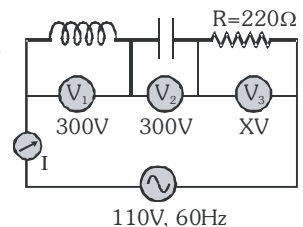


Ex. A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also calculate the value of X and I.

Sol. At resonance

$$\omega L = \frac{1}{\omega C}, \quad C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (60)^2 \times 40 \times 10^{-3}} = 176 \mu\text{F}$$

$$V = V_R \Rightarrow X = 110 \text{ V and } I = \frac{V}{R} = \frac{110}{220} = 0.5 \text{ A}$$



Ex. A coil, a capacitor and an A.C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a battery of emf 12 V, and internal resistance 4Ω , then calculate the current through the coil.

Sol. At resonance current is maximum. $I = \frac{V}{R} \Rightarrow$ Resistance of coil $R = \frac{V}{I} = \frac{24}{6} = 4\Omega$

When coil is connected to battery, suppose I current flow through it then

$$I = \frac{E}{R+r} = \frac{12}{4+4} = 1.5 \text{ A}$$

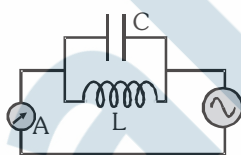
Ex. Radio receiver recives a message at 300m band, If the available inductance is 1 mH, then calculate required capacitance

Sol. Radio recives EM waves. (velocity of EM waves $c = 3 \times 10^8 \text{ m/s}$)

$$\therefore c = f\lambda \Rightarrow f = \frac{3 \times 10^8}{300} = 10^6 \text{ Hz}$$

$$\text{Now } f = \frac{1}{2\pi\sqrt{LC}} = 1 \times 10^6 \Rightarrow C = \frac{1}{4\pi^2 L \times 10^{12}} = 25 \text{ pF}$$

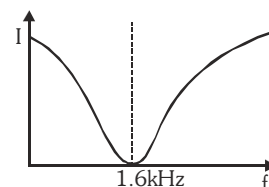
Ex. In a L–C circuit parallel combination of inductance of 0.01 H and a capacitor of 1 μF is connected to a variable frequency alternating current source as shown in figure. Draw a rough sketch of the current variation as the frequency is changed from 1kHz to 3kHz.



Sol. L and C are connected in parallel to the AC source,

$$\text{so resonance frequency } f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 10^{-6}}} = \frac{10^4}{2\pi} \approx 1.6 \text{ kHz}$$

In case of parallel resonance, current in L–C circuit at resonance is zero, so the I–f curve will be as shown in figure.



POWER IN AC CIRCUIT

The average power dissipation in LCR AC circuit

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin (\omega t - \phi)$

Instantaneous power $P = (V_0 \sin \omega t)(I_0 \sin (\omega t - \phi)) = V_0 I_0 \sin \omega t (\sin \omega t \cos \phi - \sin \phi \cos \omega t)$

$$\text{Average power } \langle P \rangle = \frac{1}{T} \int_0^T (V_0 I_0 \sin^2 \omega t \cos \phi - V_0 I_0 \sin \omega t \cos \omega t \sin \phi) dt$$

$$= V_0 I_0 \left[\frac{1}{T} \int_0^T \sin^2 \omega t \cos \phi dt - \frac{1}{T} \int_0^T \sin \omega t \cos \omega t \sin \phi dt \right] = V_0 I_0 \left[\frac{1}{2} \cos \phi - 0 \times \sin \phi \right]$$

$$\Rightarrow \langle P \rangle = \frac{V_0 I_0 \cos \phi}{2} = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

Electromagnetic induction & Alternating current

Instantaneous power / Average power / actual power / Virtual power / apparent power / Peak power

power	dissipated	power/power loss	Power/rms	Power
$P = VI$	$P = V_{rms} I_{rms}$	$I_{rms} \cos \phi$	$P = V_{rms} I_{rms}$	$P = V_0 I_0$

- $I_{rms} \cos \phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms} \sin \phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

Power factor :

Average power $\bar{P} = E_{rms} I_{rms} \cos \phi = \text{rms power} \times \cos \phi$

$$\text{Power factor } (\cos \phi) = \frac{\text{Average power}}{\text{rms Power}} \quad \text{and} \quad \cos \phi = \frac{R}{Z}$$

Power factor : (i) is leading if I leads V (ii) is lagging if I lags V

GOLDEN KEY POINTS

$P_{av} \leq P_{rms}$				
Power factor varies from 0 to 1				
Pure/Ideal R	$\phi = 0$	V, I same Phase	Power factor = $\cos \phi = 1$ (maximum)	Average power $V_{rms} \cdot I_{rms}$
L	$+\frac{\pi}{2}$	V leads I	0	0
C	$-\frac{\pi}{2}$	V lags I	0	0
Choke coil	$+\frac{\pi}{2}$	V leads I	0	0

- At resonance power factor is maximum ($\phi = 0$ so $\cos \phi = 1$) and $P_{av} = V_{rms} I_{rms}$

Ex. A voltage of 10 V and frequency 10^3 Hz is applied to $\frac{1}{\pi}$ μ F capacitor in series with a resistor of 500Ω .

Find the power factor of the circuit and the power dissipated.

Sol. $\therefore X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^3 \times \frac{10^{-6}}{\pi}} = 500\Omega \quad \therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2} \Omega$

Power factor $\cos \phi = \frac{R}{Z} = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}$,

Power dissipated $= V_{rms} I_{rms} \cos \phi = \frac{V_{rms}^2}{Z} \cos \phi = \frac{(10)^2}{500\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{10} \text{ W}$

Ex. If $V = 100 \sin 100 t$ volt and $I = 100 \sin (100 t + \frac{\pi}{3})$ mA for an A.C. circuit then find out

- (a) phase difference between V and I (b) total impedance, reactance, resistance
(c) power factor and power dissipated (d) components contains by circuits

Sol. (a) Phase difference $\phi = -\frac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \frac{V_0}{I_0} = \frac{100}{100 \times 10^{-3}} = 1k\Omega$

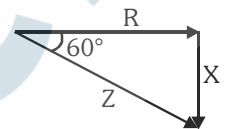
Now resistance $R = Z \cos 60^\circ = 1000 \times \frac{1}{2} = 500\Omega$

reactance $X = Z \sin 60^\circ = 1000 \times \frac{\sqrt{3}}{2} = \frac{500}{\sqrt{3}}\Omega$

(c) $\phi = -60^\circ \Rightarrow$ Power factor = $\cos \phi = \cos (-60^\circ) = 0.5$ (leading)

Power dissipated $P = V_{rms} I_{rms} \cos \phi = \frac{100}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \times \frac{1}{2} = 2.5 \text{ W}$

(d) Circuit must contains R as $\phi \neq \frac{\pi}{2}$ and as ϕ is negative
so C must be their, (L may exist but $X_C > X_L$)



Ex. If power factor of a R-L series circuit is $\frac{1}{2}$ when applied voltage is $V = 100 \sin 100\pi t$ volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Sol. $\cos \phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{Z} \Rightarrow Z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R \Rightarrow X_L = \sqrt{3} R$

$\omega L = \sqrt{3} R \Rightarrow L = \frac{\sqrt{3} R}{\omega} = \frac{\sqrt{3} \times 200}{100\pi} = \frac{2\sqrt{3}}{\pi} \text{ H}$

Ex. A circuit consisting of an inductance and a resistanc joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

Sol. Apparent power = $200 \times 10 = 2000 \text{ W}$

\therefore Power factor $\cos \phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$

Wattless current = $I_{rms} \sin \phi = 10 \sqrt{1 - \left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4} \text{ A}$

Ex. A coil has a power factor of 0.866 at 60 Hz. What will be power factor at 180 Hz.

Sol. Given that $\cos \phi = 0.866$, $\omega = 2\pi f = 2\pi \times 60 = 120\pi$ rad/s,

$$\omega' = 2\pi f' = 2\pi \times 180 = 360\pi \text{ rad/s}$$

$$\text{Now, } \cos \phi = R/Z \Rightarrow R = Z \cos \phi = 0.866 Z$$

$$\text{But } Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow \omega L = \sqrt{Z^2 - R^2} = \sqrt{Z^2 - (0.866 Z)^2} = 0.5 Z$$

$$\therefore L = \frac{0.5Z}{\omega} = \frac{0.5Z}{120\pi}$$

When the frequency is changed to $\omega' = 2\pi \times 180 = 3 \times 120\pi = 300$ rad/s, then inductive reactance $\omega' L = 3 \omega L = 3 \times 0.5 Z = 1.5 Z$

$$\therefore \text{New impedance } Z' = \sqrt{[R + (\omega' L)^2]} = \sqrt{(0.866 Z)^2 + (1.5 Z)^2} = Z \sqrt{[(0.866)^2 + (1.5)^2]} = 1.732Z$$

$$\therefore \text{New power factor} = \frac{R}{Z'} = \frac{0.866 Z}{1.732 Z} = 0.5$$

CHOKE COIL

In a direct current circuit, current is reduced with the help of a resistance. Hence there is a loss of electrical energy $I^2 R$ per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced. It also known as ballast.

Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I = \frac{E}{Z}$ with $Z = \sqrt{(R + r)^2 + (\omega L)^2}$ So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r ,

$$\text{The power loss in the choke } P_{av} = V_{rms} I_{rms} \cos \phi \rightarrow 0 \quad \because \cos \phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \rightarrow 0$$

Ex. A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil.

$$\text{Sol. } V = \sqrt{V_R^2 + V_L^2} \Rightarrow V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120 \text{ V}$$

Ex. An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

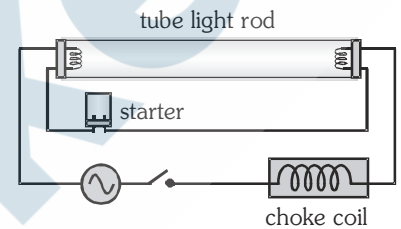
$$\text{Sol. Resistance of lamp } R = \frac{V}{I} = \frac{80}{10} = 8\Omega$$

Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run

$$\text{on } 100 \text{ Volt a.c. then. } Z = \frac{V}{I} = \frac{100}{10} = 10 \Omega \text{ but } Z = \sqrt{R^2 + (\omega L)^2}$$

$$\Rightarrow (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36$$

$$\Rightarrow \omega L = 6 \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02 \text{ H}$$



Ex. Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)

Sol. (a) Maximum voltage across lamp = 60V

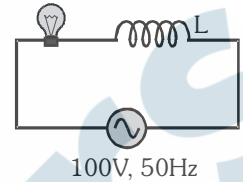
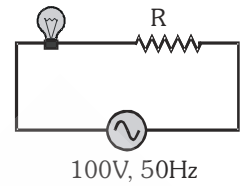
$$\therefore V_{\text{Lamp}} + V_R = 100 \quad \therefore V_R = 40V$$

$$\text{Now current through Lamp is} = \frac{\text{Wattage}}{\text{voltage}} = \frac{10}{60} = \frac{1}{6} \text{ A}$$

$$\text{But } V_R = IR \Rightarrow 40 = \frac{1}{6}(R) \Rightarrow R = 240 \Omega$$

(b) Now in this case $(V_{\text{Lamp}})^2 + (V_L)^2 = (V)^2$
 $(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80 \text{ V}$

$$\text{Also } V_L = IX_L = \frac{1}{6} X_L \text{ so } X_L = 80 \times 6 = 480 \Omega = L(2\pi f) \Rightarrow L = 1.5 \text{ H}$$



A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

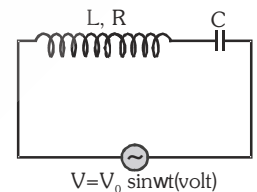
Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Ex. A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c.

voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.

(a) Find out relation between ω_0 , L and C

(b) What is phase difference between V and I at resonance, is it changes when resistance of choke coil is zero.



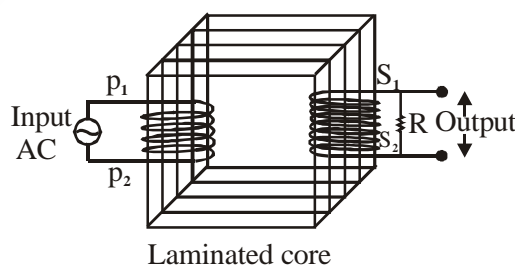
Sol. (a) At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

(b) $\therefore \cos \phi = \frac{R}{Z} = \frac{R}{R} = 1 \quad \therefore \phi = 0^\circ$ No, It is always zero.

Transformers

One of the great advantages of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. The necessary conversion is accomplished by a static device called transformer using the principle of mutual induction.

The figure shows an idealised transformer which consists of two coils or windings, electrically insulated from each other but wound on the same core. The winding to which power is supplied is called primary, the winding from which power is delivered is called the secondary.



Electromagnetic induction & Alternating current

The ac source causes an alternating current in the primary which sets up an alternating flux in the core and this induces an emf in each winding of secondary in accordance with Faraday's law. For ideal transformer we assume that primary has negligible resistance and all the flux in core links both primary and secondary. The primary winding has N_1 turns and secondary has N_2 turns. When the magnetic flux changes because of changing currents in the two coils, the resulting induced emf are

$$E_1 = -N_1 \frac{d\phi_B}{dt} \text{ and } E_2 = -N_2 \frac{d\phi_B}{dt}$$

The flux per turn B is same in both primary and the secondary so that the emf per turn is same in each. The ratio of secondary emf E_2 to the primary emf E_1 is therefore equal at any instant to the ratio of secondary to primary turns.

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

If the windings have zero resistance, the induced emf 1 and 2 are equal to the terminal voltage across the primary and the secondary respectively, hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

If $N_2 > N_1$ then $V_2 > V_1$ and we have step up transformer, if $N_2 < N_1$ then $V_2 < V_1$ and we have a step down transformer.

If the transformer is assumed to be 100% efficient (no energy losses) the power input is equal to the power output i.e.

$$I_2 V_2 = I_1 V_1 \quad \therefore \quad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

All the currents and voltages derived above have same frequency as that of source. The equations obtained above apply to ideal transformers, although some energy is lost but well designed transformers have efficiency more than 95%, this is a good approximation. The causes of energy losses and their rectification is given below

Cause	Rectification
1. Due to poor design and air gaps in the core, all the flux due to primary does not pass through the secondary.	1. By winding the primary and secondary coil one over the other.
2. Resistance of windings causes $I^2 R$ loss.	2. In high current, low voltage, these are minimised by using thick wire.
3. The alternating magnetic flux induces eddy currents in the core and causes heating.	3. By using laminated core it can be reduced.
4. Alternating magnetising of core causes hysteresis loss.	4. It is kept minimum by using a magnetic material having low hysteresis loss. (e.g. soft iron)

Damped Harmonic Oscillator:

In the previous section we have considered several examples of oscillatory systems. In each case the amplitude of the oscillation does not depend on time and the system, once set to oscillate, will continue to do so forever. Physical systems are never as ideal as that. Even in the most controlled case there will be dissipative elements.

In each of these cases the amplitudes of the oscillation will gradually decrease and the motion will be 'damped'.

Most of the time we are interested in the motion of an object through air or other viscous fluids. The motion of the object is then subject to a resistive force called aerodynamic or viscous drag. Experimentally the resistive force is found to be proportional to the velocity of the body for low relative speed with respect to the medium.

A laboratory model for a damped spring-mass system is shown in figure. Where a vertical spring has at its end a mass m which is connected to massless piston which moves through a liquid. For low speeds it is reasonable to write the frictional force as

$$F_f = -\alpha v \quad (\alpha \text{ in kg/s})$$

where α is a positive constant known as damping constant. The equation of motion of a harmonic oscillator becomes

$$m \frac{d^2 x}{dt^2} = -kx - \alpha \frac{dx}{dt}$$

which can be rewritten as

$$\frac{d^2 x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = 0$$

where $2\gamma = \alpha/m > 0$ and ω_0^2 , as before is equal to k/m . γ is known as damping coefficient

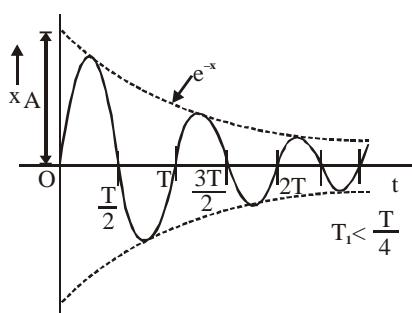
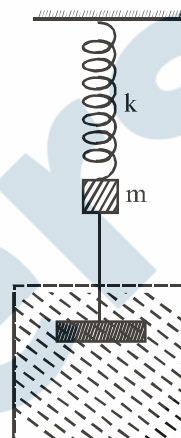
Damped Oscillations

If the dissipative forces are small, i.e. if $\gamma^2 < \omega_0^2$, the dominant force acting on the mass is the conservation restoring force. The system therefore shows oscillations, even though the amplitudes of oscillation continuously decrease with time.

we get $x = Ae^{-\gamma t} \sin(\Omega t + \phi)$

where the new angular frequency of oscillation Ω is given by $\Omega^2 = \omega_0^2 - \gamma^2$

i.e the frequency shift to a lower value due to damping. The amplitude of oscillation is no longer constant but decreases with time in an exponential fashion. The variation of x with time is shown in figure.



It may be seen that $\sin(\Omega t + \phi)$ is a periodic function with period $2\pi/\Omega$. when this is multiplied with $e^{-\gamma t}$ the zeroes of the product would still be at the same place where $\sin(\Omega t + \phi)$ becomes zero. The maxima however does not fall midway between the two minima because of the exponential $e^{-\gamma t}$ but occurs at a slightly earlier time.

Power Loss in a Damped Oscillation-Q-factor

Because of the work done against the nonconservative forces, the energy stored in an oscillator continuously decreases. The fraction of energy lost in a period with respect to its average value for that period is a measure of the quality of the oscillator. The less the value of this fraction is, the more is the ability of the oscillator to sustain periodic motion. Quantitatively, the quality of an oscillator is measured through a Q-factor defined as

$$Q = 2\pi \times \frac{\text{Average energy stored in one period}}{\text{average loss of energy in that period}}$$

If the damping is light i.e. $\gamma \ll \omega_0$, The instantaneous energy of the oscillator is

$$\begin{aligned} E(t) &= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}mA^2\omega_0^2 e^{-2\gamma t} \end{aligned}$$

where we have replaced k by $m\omega_0^2$. At $t = 0$ the energy of the oscillator is $E_0 = mA^2\omega_0^2/2$ so that

$$E(t) = E_0 e^{-2\gamma t}$$

$$P(t) = \frac{dE}{dt} = -2\gamma E_0 e^{-2\gamma t} = 2\gamma E(t)$$

Energy lost in one period can be approximately written as

$$\langle P(t) \rangle \frac{2\pi}{\Omega} = 2\gamma \langle E(t) \rangle \frac{2\pi}{\Omega}$$

where the angular bracket $\langle \dots \rangle$ denotes the time average value during a period at time t . The quality or Q-factor for an oscillator is

$$2\pi \frac{\langle E(t) \rangle}{2\gamma \langle E(t) \rangle \frac{2\pi}{\Omega}} = \frac{\Omega}{2\gamma} \cong \frac{\omega_0}{2\gamma}$$

It may be noted that Q is a dimensionless quantity, greater than unity by our assumption that $\gamma \ll \omega_0$.

Time to reach extreme:

The natural logarithm of two successive maxima is called logarithmic decrement and is denoted by λ . From Equation it follows that

$$\lambda = \frac{2\pi\gamma}{\Omega} = \frac{\pi}{Q}$$

Overdamped Motion

In equation if $\gamma > \omega_0$ we do not get oscillatory solution

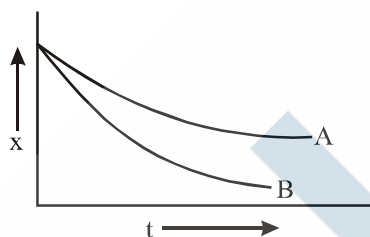
Critical Damping ($\gamma = \omega_0$)

The solution for the displacement is therefore

$$x = (A + Bt)e^{-\gamma t}$$

$$x' = -\frac{mg}{k}(1 + \gamma t)e^{-\gamma t}$$

It may be seen (hat once again there are no oscillation. As $t \rightarrow \infty$, $x \rightarrow 0$ so that the particle takes an infinite time before reaching the equilibrium position. The difference with the overdamped case is that the approach to the equilibrium is faster. The variation of displacement with time is shown in figure (curve B.)



Forced Oscillations and Resonance

The oscillations in a system can be indefinitely maintained by supplying energy continuously. In mechanical system this can be done by subjecting it to an external force which itself has harmonic time dependence. An interesting phenomenon occurs if the frequency of the external source is equal or nearly equal to the natural frequency of the system. The amplitude of oscillations is found to increase many folds in such cases. This is called resonance.

Forced Damped Oscillations

In the presence of resistive forces proportional to velocity, equation be comes

$$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m} \cos \omega t$$

Let us therefore try a solution of the type

$$x = A \cos \omega t + B \sin \omega t + Ce^{-\lambda t}$$

The steady - state displacement is then given by

$$x = C \cos (\omega t + \delta)$$

with

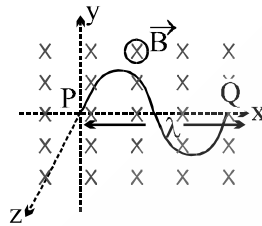
$$C = \frac{F_0}{m} \sqrt{\frac{1}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\gamma^2}}$$

$$\text{and } \tan \delta = \frac{2\omega\gamma}{\omega^2 - \omega_0^2}$$

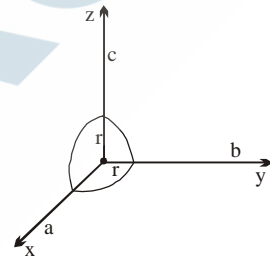
EXERCISE (S-1)

Faraday's law & Motional emf.

1. A wire forming one cycle of sine curve is moved in x-y plane with velocity $\vec{V} = V_x \hat{i} + V_y \hat{j}$. There exist a magnetic field $\vec{B} = -B_0 \hat{k}$. Find the motional emf develop across the ends PQ of wire.

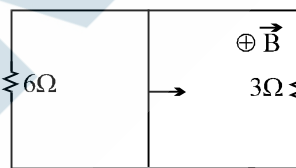


2. A wire is bent into 3 circular segments of radius $r = 10$ cm as shown in figure. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane & ca lying in the zx plane.

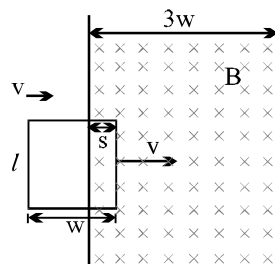


- (i) if a magnetic field B points in the positive x direction, what is the magnitude of the emf developed in the wire, when B increases at the rate of 3 mT/s ?
 (ii) what is the direction of the current in the segment bc.

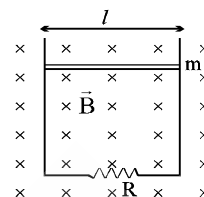
3. A rectangular loop with a sliding connector of length $l = 1.0$ m is situated in a uniform magnetic field $B = 2\text{T}$ perpendicular to the plane of loop. Resistance of connector is $r = 2\Omega$. Two resistances of 6Ω and 3Ω are connected as shown in figure. Find the external force required to keep the connector moving with a constant velocity $v = 2\text{m/s}$.



4. A rectangular loop of dimensions l & w and resistance R moves with constant velocity V to the right as shown in the figure. It continues to move with same speed through a region containing a uniform magnetic field B directed into the plane of the paper & extending a distance $3w$. Sketch the flux, induced emf & external force acting on the loop as a function of the distance.



5. A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in figure. The wire has a mass m and length l and the resistance of the circuit is R . If a uniform magnetic field B is directed perpendicular to the frame, then find the terminal speed of the wire as it falls under the force of gravity.

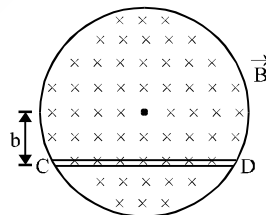


6. It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction. The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC . The combined resistance of the coil and the galvanometer is 0.50Ω . Estimate the field strength of magnet. (NCERT)

Induced electric field

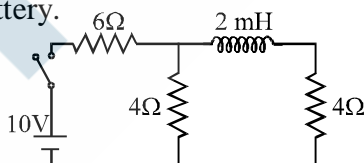
7. There exists a uniform cylindrically symmetric magnetic field directed along the axis of a cylinder but varying with time as $B = kt$. If an electron is released from rest in this field at a distance of ' r ' from the axis of cylinder, its acceleration, just after it is released would be (e and m are the electronic charge and mass respectively)
8. An air-cored solenoid of length 30 cm , area of cross-section 25 cm^2 and number of turns 500 , carries a current of 2.5 A . The current is suddenly switched off in a brief time of 10^{-3} s . How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid. (NCERT)

9. A uniform magnetic field \vec{B} fills a cylindrical volumes of radius R . A metal rod CD of length l is placed inside the cylinder along a chord of the circular cross-section as shown in the figure. If the magnitude of magnetic field increases in the direction of field at a constant rate dB/dt , find the magnitude and direction of the EMF induced in the rod.

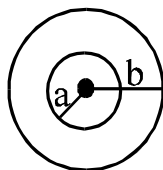


Inductance

10. In the given circuit, find the ratio of i_1 to i_2 where i_1 is the initial (at $t = 0$) current and i_2 is steady state (at $t = \infty$) current through the battery.

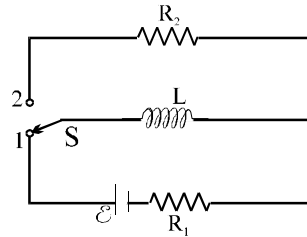


11. Two concentric and coplanar circular coils have radii a and $b (> a)$ as shown in figure. Resistance of the inner coil is R . Current in the outer coil is increased from 0 to i , then find the total charge circulating the inner coil.

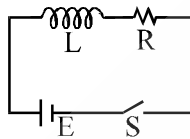


12. Find the dimension of the quantity $\frac{L}{RCV}$, where symbols have usual meaning.

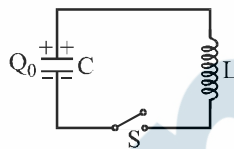
13. In the circuit shown, initially the switch is in position 1 for a long time. Then the switch is shifted to position 2 for a long time. Find the total heat produced in R_2 .



14. An emf of 15 volt is applied in a circuit containing 5 H inductance and $10\ \Omega$ resistance. Find the ratio of the currents at time $t = \infty$ and $t = 1$ second.
15. In the circuit shown in figure switch S is closed at time $t = 0$. Find the charge which passes through the battery in one time constant.



16. A capacitor C with a charge Q_0 is connected across an inductor through a switch S. If at $t = 0$, the switch is closed, then find the instantaneous charge q on the upper plate of capacitor.



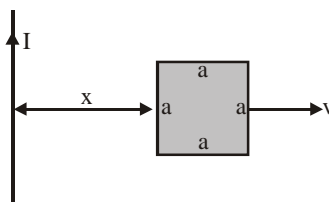
17. An inductor of inductance 2.0mH , is connected across a charged capacitor of capacitance $5.0\mu\text{F}$ and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor, and I the current in the circuit. It is found that the maximum value of Q is $200\mu\text{C}$.

- (a) when $Q = 100\mu\text{C}$, what is the value of $|dI/dt|$?
- (b) when $Q = 200\mu\text{C}$, what is the value of I ?
- (c) Find the maximum value of I .
- (d) when I is equal to one half its maximum value, what is the value of $|Q|$

18. A pair of adjacent coils has a mutual inductance of 1.5 H . If the current in one coil changes from 0 to 20 A in 0.5 s , what is the change of flux linkage with the other coil ? (NCERT)

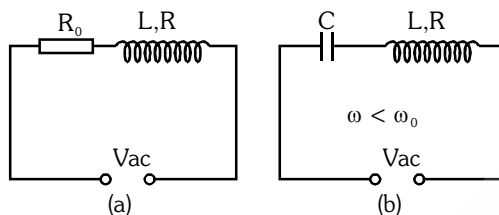
19. (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in figure.

- (b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, $v = 10\text{ m/s}$. Calculate the induced emf in the loop at the instant when $x = 0.2\text{m}$. Take $a = 0.1\text{ m}$ and assume that the loop has a large resistance. (NCERT)

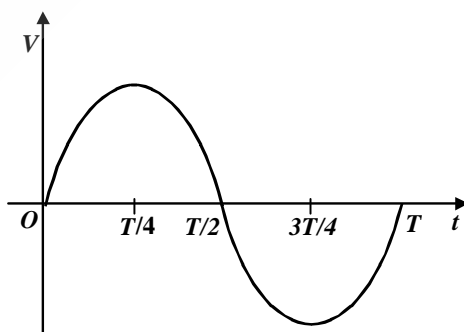


Alternating current

20. Draw the approximate voltage vector diagrams in the electric circuits shown in Fig. a, b. The external voltage V is assumed to be alternating harmonically with frequency ω .

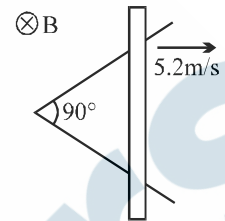


21. A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12V, 50 rad/s ac source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.
22. An LCR series circuit with 100Ω resistance is connected to an ac source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60° . When only the inductance is removed, the current leads the voltage by 60° . Calculate the current and the power dissipated in the LCR circuit.
23. A series LCR circuit containing a resistance of 120Ω has angular resonance frequency $4 \times 10^5 \text{ rad s}^{-1}$. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C . At what frequency the current in the circuit lags the voltage by 45° ?
24. Find the value of an inductance which should be connected in series with a capacitor of $5\mu\text{F}$, a resistance of 10Ω and an ac source of 50 Hz so that the power factor of the circuit is unity.
25. In an L-R series A.C circuit the potential difference across an inductance and resistance joined in series are respectively 12 V and 16V. Find the total potential difference across the circuit.
26. In an LR series circuit, a sinusoidal voltage $V = V_0 \sin \omega t$ is applied. It is given that $L = 35 \text{ mH}$, $R = 11 \Omega$, $V_{\text{rms}} = 220 \text{ V}$, $\frac{\omega}{2\pi} = 50 \text{ Hz}$ and $\pi = 22/7$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph.
- [JEE 2004]**

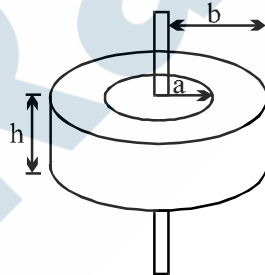
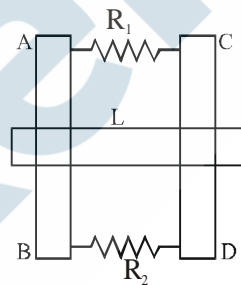


EXERCISE (S-2)

1. Two straight conducting rails form a right angle where their ends are joined. A conducting bar contact with the rails starts at vertex at the time $t = 0$ & moves symmetrically with a constant velocity of 5.2 m/s to the right as shown in figure. A 0.35 T magnetic field points out of the page. Calculate:
 - (i) The flux through the triangle by the rails & bar at $t = 3.0 \text{ s}$.
 - (ii) The emf around the triangle at that time.
 - (iii) In what manner does the emf around the triangle vary with time.



2. Two parallel vertical metallic rails AB & CD are separated by 1 m . They are connected at the two ends by resistance R_1 & R_2 as shown in the figure. A horizontally metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails, it is observed that when the terminal velocity is attained, the power dissipated in R_1 & R_2 are 0.76 W & 1.2 W respectively. Find the terminal velocity of bar L & value R_1 & R_2 .
3. A long straight wire is arranged along the symmetry axis of a toroidal coil of rectangular cross-section, whose dimensions are given in the figure. The number of turns on the coil is N , and relative permeability of the surrounding medium is unity. Find the amplitude of the emf induced in this coil, if the current $i = i_m \cos \omega t$ flows along the straight wire.



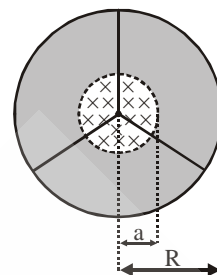
4. A metal rod of resistance 20Ω is fixed along a diameter of a conducting ring of radius 0.1 m and lies on x - y plane. There is a magnetic field $\vec{B} = (50\text{T})\hat{k}$. The ring rotates with an angular velocity $\omega = 20 \text{ rad/sec}$ about its axis. An external resistance of 10Ω is connected across the centre of the ring and rim. Find the current through external resistance.
5. A uniform but time varying magnetic field $B = Kt - C$; ($0 \leq t \leq C/K$), where K and C are constants and t is time, is applied perpendicular to the plane of the circular loop of radius ' a ' and resistance R . Find the total charge that will pass around the loop.
6. A charged ring of mass $m = 50 \text{ gm}$, charge 2 coulomb and radius $R = 2 \text{ m}$ is placed on a smooth horizontal surface. A magnetic field varying with time at a rate of $(0.2 \text{ t}) \text{ Tesla/sec}$ is applied on to the ring in a direction normal to the surface of ring. Find the angular speed attained in a time $t_1 = 10 \text{ sec}$. Assume that the magnetic field is cylindrically symmetric and covering the entire ring.

7. A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R . The wheel has light nonconducting spokes and is free to rotate without friction about its axis see figure. A uniform magnetic field extends over a circular region within the rim. It is given by,

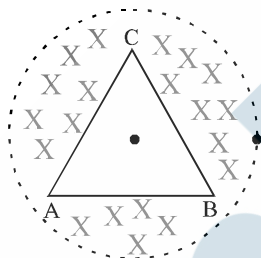
$$\mathbf{B} = -B_0 \mathbf{k} \quad (r \leq a; a < R)$$

$$= 0 \quad (\text{otherwise})$$

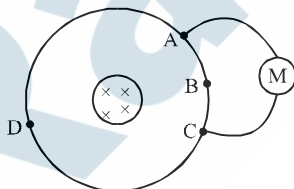
What is the angular velocity of the wheel after the field is suddenly switched off ?
(NCERT)



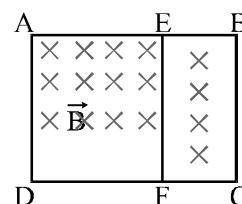
8. A triangular wire frame (each side = 2m) is placed in a region of time variant magnetic field having $dB/dt = \sqrt{3}$ T/s. The magnetic field is perpendicular to the plane of the triangle. The base of the triangle AB has a resistance 1Ω while the other two sides have resistance 2Ω each. The magnitude of potential difference between the points A and B will be



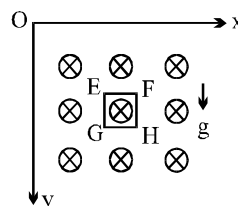
9. A variable magnetic field creates a constant emf \mathcal{E} in a conductor ABCDA. The resistances of portion ABC, CDA and AMC are R_1 , R_2 and R_3 respectively. What current will be shown by meter M? The magnetic field is concentrated near the axis of the circular conductor.



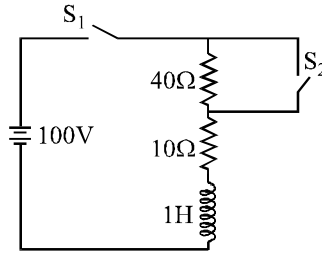
10. A rectangular frame ABCD made of a uniform metal wire has a straight connection between E & F made of the same wire as shown in the figure. AEFD is a square of side 1 m & $EB = FC = 0.5$ m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the plane of the paper & normal to it. The rate of change of the magnetic field is 1 T/s, the resistance per unit length of the wire is $1 \Omega/\text{m}$. Find the current in segments AE, BE & EF.



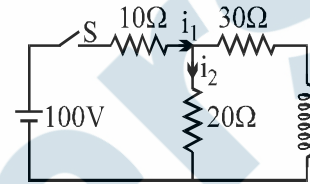
11. A magnetic field $\mathbf{B} = (B_0 y / a) \hat{k}$ is into the plane of paper in the $+z$ direction. B_0 and a are positive constants. A square loop EFGH of side a , mass m and resistance R , in x - y plane, starts falling under the influence of gravity. Note the directions of x and y axes in the figure. Find
- the induced current in the loop and indicate its direction,
 - the total Lorentz force acting on the loop and indicate its direction,
 - an expression for the speed of the loop, $v(t)$ and its terminal value.



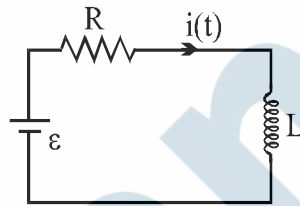
12. In the circuit shown in the figure the switches S_1 and S_2 are closed at time $t = 0$. After time $t = (0.1) \ln 2$ sec, switch S_2 is opened. Find the current in the circuit at time $t = (0.2) \ln 2$ sec.



13. Find the values of i_1 and i_2
- immediately after the switch S is closed.
 - long time later, with S closed.
 - immediately after S is open.
 - long time after S is opened.

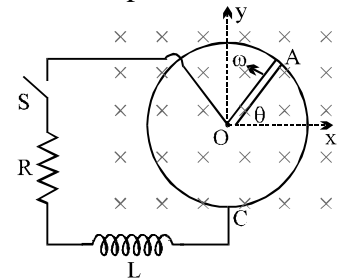


14. Suppose the emf of the battery in the circuit shown varies with time t so the current is given by $i(t) = 3 + 5t$, where i is in amperes & t is in seconds. Take $R = 4\Omega$, $L = 6H$ & find an expression for the battery emf as function of time.



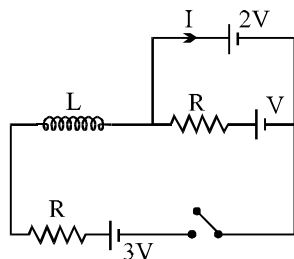
15. A metal rod OA of mass m & length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O . The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform & constant magnetic induction \vec{B} is applied perpendicular & into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O & a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.

- What is the induced emf across the terminals of the switch ?
- Obtain an expression for the current as a function of time after switch S is closed.
 - Obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X -axis at $t = 0$.

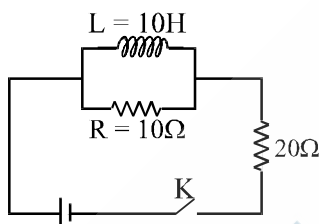


16. A zero resistance coil of inductance L connects the upper ends of two vertical parallel long conductors. A horizontal sliding conductor, free to slide up and down, always maintaining contact with the vertical conductors, starts falling from rest at $t = 0$, due to its own weight mg . A uniform magnetic field of magnitude B exists in the region horizontally and perpendicular to the plane of the conductors. The distance between the vertical conductors is ' l '. After what time does the conductor come back to its starting position? Also find maximum speed achieved.

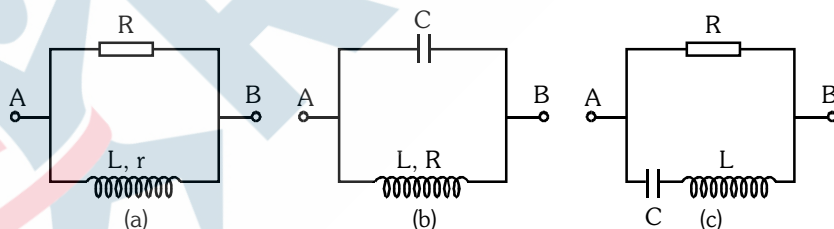
17. In the LR circuit shown, what is the variation of the current I as a function of time? The switch is closed at time $t = 0$ sec.



18. Two resistors of 10Ω and 20Ω and an ideal inductor of $10H$ are connected to a $2V$ battery as shown. The key K is shorted at time $t = 0$. Find the initial ($t = 0$) and final ($t \rightarrow \infty$) currents through battery.

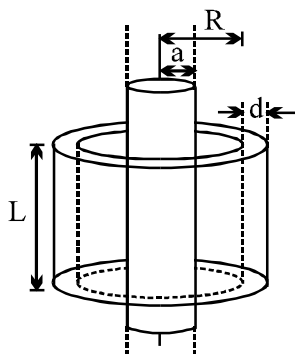


19. Two coils, 1 & 2, have a mutual inductance $= M$ and resistances R each. A current flows in coil 1, which varies with time as: $I_1 = kt^2$, where k is a constant and ' t ' is time. Find the total charge that has flown through coil 2, between $t = 0$ and $t = T$.
20. A box P and a coil Q are connected in series with an ac source of variable frequency. The emf of source is $10 V$. Box P contains a capacitance of $1\mu F$ in series with a resistance of 32Ω while coil Q has a self-inductance $4.9 mH$ and a resistance of 68Ω series. The frequency is adjusted so that the maximum current flows in P and Q . Find the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively.
21. Draw the approximate vector diagrams of currents in the circuits shown in Fig. The voltage applied across the points A and B is assumed to be sinusoidal; the parameters of each circuit are so chosen that the total current I_0 lags in phase behind the external voltage by an angle ϕ .



22. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R , thickness d ($d \ll R$) and length L . A variable current $i = i_0 \sin \omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ , find the induced current in the shell.

[JEE 2005]

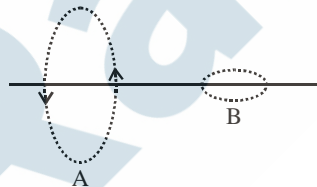


EXERCISE (O-1)

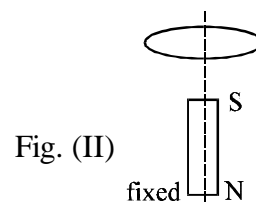
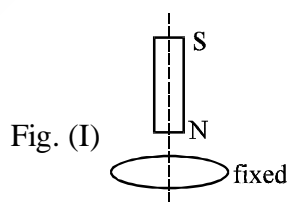
SINGLE CORRECT TYPE QUESTIONS

Faraday's law & motional emf.

1. A square of side 2 meters lies in the x - y plane in a region, where the magnetic field is given by $\vec{B} = B_0 (2\hat{i} + 3\hat{j} + 4\hat{k})T$, where B_0 is constant. The magnitude of flux passing through the square is:-
 (A) $8 B_0 \text{ Wb.}$ (B) $12 B_0 \text{ Wb.}$ (C) $16 B_0 \text{ Wb.}$ (D) $\sqrt{4 \times 29} B_0 \text{ Wb}$
2. **Statement-1 :** When a magnet is made to fall freely through a closed coil, its acceleration is always less than acceleration due to gravity.
and
Statement-2 : Current induced in the coil opposes the motion of the magnet, as per Lenz's law.
 (A) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1
 (B) Statement-1 is true, Statement-2 is true, Statement-2 is NOT a correct explanation for Statement-1
 (C) Statement-1 is True, Statement-2 is False
 (D) Statement-1 is False, Statement-2 is True
3. In the given figure the centre of a small conducting circular loop B lies on the axis of bigger circular loop A and their axis are mutually perpendicular. An anticlockwise (when viewed from the side of B) current in the loop A start increasing then :-

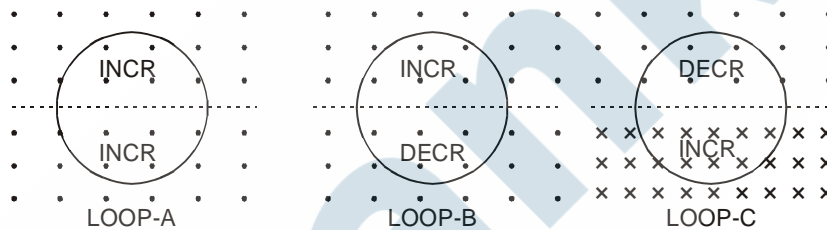
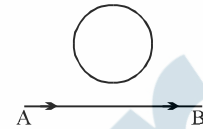


- (A) current induced in the loop B is in clockwise direction (when viewed from above the B)
 - (B) current induced in the loop B is in anti-clockwise direction (when viewed from above the B)
 - (C) current must induced in the loop B but its direction can not be predicted
 - (D) no current is induced in the loop B
4. A vertical bar magnet is dropped from position on the axis of a fixed metallic coil as shown in fig - I. In fig. II the magnet is fixed and horizontal coil is dropped. The acceleration of the magnet and coil are a_1 and a_2 respectively then :-

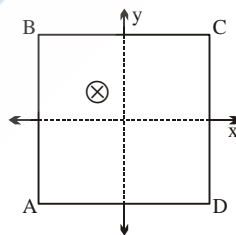


- (A) $a_1 > g, a_2 > g$ (B) $a_1 > g, a_2 < g$ (C) $a_1 < g, a_2 < g$ (D) $a_1 < g, a_2 > g$

5. Two identical coaxial circular loops carry a current i each circulating in the same direction. If the loops approach each other
- (A) the current in each will decrease
 - (B) the current in each will increase
 - (C) the current in each will remain the same
 - (D) the current in one will increase and in other will decrease
6. In the arrangement shown in given figure current from A to B is increasing in magnitude. Induced current in the loop will
- (A) have clockwise direction
 - (B) have anticlockwise direction
 - (C) be zero
 - (D) oscillate between clockwise and anticlockwise
7. Three identical conducting circular loops are placed in uniform magnetic fields. Inside each loop, there are two magnetic field regions, separated by dashed line that coincides with a diameter, as shown. Magnetic fields may either be increasing (marked as INCR) or decreasing (marked as DECR) in magnitude at the same rates. If I_A , I_B and I_C are the magnitudes of the induced currents in the loops A, B and C respectively then choose the **CORRECT** relation :-



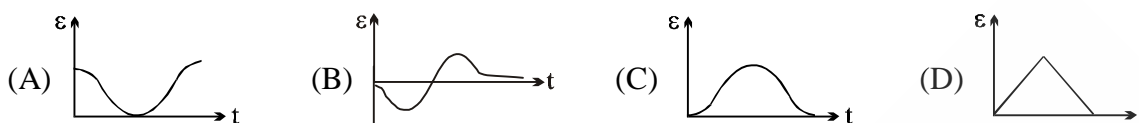
- (A) $I_A > I_B = I_C$
 - (B) $I_A = I_C > I_B$
 - (C) $I_A = I_B = I_C$
 - (D) $I_C > I_A > I_B$
8. A square coil ABCD is placed in x-y plane with its centre at origin. A long straight wire, passing through origin, carries a current in negative z-direction. Current in this wire increases with time. The induced current in the coil is :



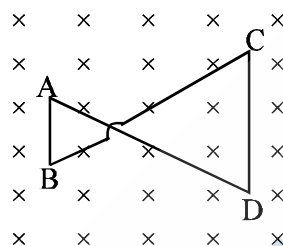
- (A) clockwise
 - (B) anticlockwise
 - (C) zero
 - (D) alternating
9. A short circuited coil is kept on the ground and a magnet is dropped on it as shown. The coil shows (when viewed from top)
- (A) anticlockwise current that increases in magnitude
 - (B) anticlockwise current that remains constant
 - (C) clockwise current that remains constant
 - (D) clockwise current that increases in magnitude



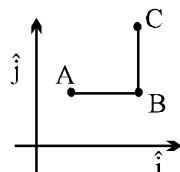
10. The variation of induced emf (ϵ) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as [JEE 2004(Scr)]



11. A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



- (A) B to A and D to C
(B) A to B and C to D
(C) A to B and D to C
(D) B to A and C to D
12. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time ' t ', as $R = R_0 + t$. The e.m.f induced in the loop is
- (A) $2\pi(R_0 + t)B$ clockwise
(B) $\pi(R_0 + t)B$ clockwise
(C) $2\pi(R_0 + t)B$ anticlockwise
(D) zero
13. A thin wire of length $2m$ is perpendicular to the xy plane. It is moved with velocity $\vec{v} = (2\hat{i} + 3\hat{j} + \hat{k}) \text{ m/s}$ through a region of magnetic induction $\vec{B} = (\hat{i} + 2\hat{j}) \text{ Wb/m}^2$. Then potential difference induced between the ends of the wire :
- (A) 2 volts
(B) 4 volts
(C) 0 volts
(D) none of these
14. A square loop of side a and resistance R is moved in the region of uniform magnetic field B (loop remaining completely inside field), with a velocity v through a distance x . The work done is :
- (A) $\frac{B\ell^2 vx}{R}$
(B) $\frac{2B^2 \ell^2 vx}{R}$
(C) $\frac{4B^2 \ell^2 vx}{R}$
(D) 0
15. There is a uniform magnetic field B normal to the xy plane. A conductor ABC has length $AB = l_1$, parallel to the x -axis, and length $BC = l_2$, parallel to the y -axis. ABC moves in the xy plane with velocity $v_x \hat{i} + v_y \hat{j}$. The potential difference between A and C is proportional to :-



- (A) $v_x l_1 + v_y l_2$
(B) $v_x l_2 + v_y l_1$
(C) $v_x l_2 - v_y l_1$
(D) $v_x l_1 - v_y l_2$

16. A uniform but time variant magnetic field exists in a cylindrical region directed along the axis of cylinder of radius R . The graph of induced electric field at a given time v/s . r is (r = distance from axis)



17. A metal disc rotates freely, between the poles of a magnet in the direction indicated. Brushes P and Q make contact with the edge of the disc and the metal axle. What current, if any, flows through R ?

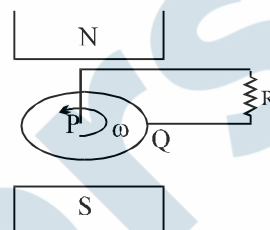
(A) a current from P to Q

(B) a current from Q to P

(C) no current, because the emf in the disc is opposed by the back emf

(D) no current, because the emf induced in one side of the disc is opposed by the emf induced in the other side.

(E) no current, because no radial emf is induced in the disc



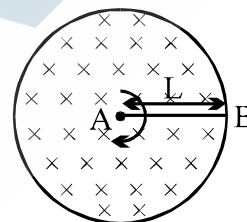
18. A copper rod AB of length L , pivoted at one end A, rotates at constant angular velocity ω , at right angles to a uniform magnetic field of induction B . The e.m.f developed between the mid point C of the rod and end B is

(A) $\frac{B\omega L^2}{4}$

(B) $\frac{B\omega L^2}{2}$

(C) $\frac{3B\omega L^2}{4}$

(D) $\frac{3B\omega L^2}{8}$



19. The e.m.f. induced in a coil of wire, which is rotating in a magnetic field, does not depend on

(A) the angular speed of rotation

(B) the area of the coil

(C) the number of turns on the coil

(D) the resistance of the coil

Induced electric field

20. A ring of resistance 10Ω , radius 10cm and 100 turns is rotated at a rate 100 revolutions per second about its diameter is perpendicular to a uniform magnetic field of induction 10mT . The amplitude of the current in the loop will be nearly (Take : $\pi^2 = 10$)

(A) 200A

(B) 2A

(C) 0.002A

(D) none of these

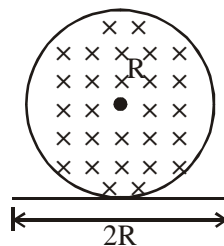
21. A uniform but time varying magnetic field is present in a circular region of radius R . The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α . There is a straight conducting rod of length $2R$ placed as shown in figure. The magnitude of induced emf across the rod is

(A) $\pi R^2 \alpha$

(B) $\frac{\pi R^2 \alpha}{2}$

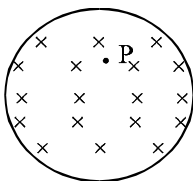
(C) $\frac{R^2 \alpha}{\sqrt{2}}$

(D) $\frac{\pi R^2 \alpha}{4}$



Electromagnetic induction & Alternating current

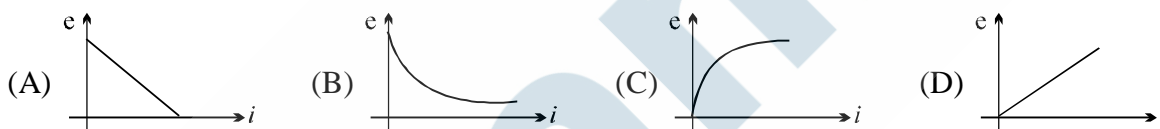
22. Figure shows a uniform magnetic field B confined to a cylindrical volume and is increasing at a constant rate. The instantaneous acceleration experienced by an electron placed at P is



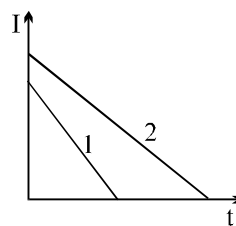
- (A) zero (B) towards right (C) towards left (D) upwards
23. **Statement-1** : For a charged particle moving from point P to point Q the net work done by an induced electric field on the particle is independent of the path connecting point P to point Q .
Statement-2 : The net work done by a conservative force on an object moving along closed loop is zero.
- (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
 (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1
 (C) Statement-1 is true, statement-2 is false.
 (D) Statement-1 is false, statement-2 is true.

Inductance

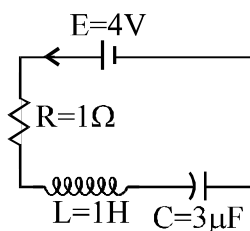
24. In an L-R circuit connected to a battery of constant e.m.f. E switch S is closed at time $t = 0$. If e denotes the magnitude of induced e.m.f. across inductor and i the current in the circuit at any time t . Then which of the following graphs shows the variation of e with i ?



25. A current of $2A$ is increasing at a rate of $4 A/s$ through a coil of inductance $2H$. The energy stored in the inductor per unit time is :-
 (A) $2 J/s$ (B) $1 J/s$ (C) $16 J/s$ (D) $4 J/s$
26. Two identical inductance carry currents that vary with time according to linear laws (as shown in figure). In which of two inductance is the self induction emf greater?
 (A) 1
 (B) 2
 (C) same
 (D) data are insufficient to decide

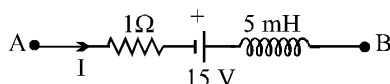


27. The current in the given circuit is increasing with a rate $a = 4 \text{ amp/s}$. The charge on the capacitor at an instant when the current in the circuit is 2 amp will be :

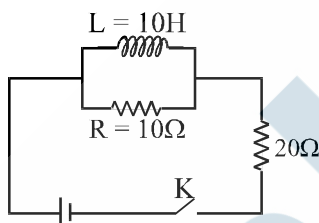


- (A) $4\mu C$ (B) $5\mu C$ (C) $6\mu C$ (D) none of these

28. A long solenoid of N turns has a self inductance L and area of cross section A . When a current i flows through the solenoid, the magnetic field inside it has magnitude B . The current i is equal to:
 (A) BAN/L (B) $BANL$ (C) BN/AL (D) B/ANL
29. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5 A and it is decreasing at a rate of 10^3 As^{-1} then $V_B - V_A$ equals



- (A) 20 V (B) 15 V (C) 10 V (D) 5 V
30. In **Problem 29**, if I is reversed in direction, then $V_B - V_A$ equals
 (A) 5 V (B) 10 V (C) 15 V (D) 20 V
31. Two resistors of 10Ω and 20Ω and an ideal inductor of 10H are connected to a 2V battery as shown. The key K is shorted at time $t = 0$. Find the initial ($t = 0$) and final ($t \rightarrow \infty$) currents through battery.



- (A) $\frac{1}{15} \text{ A}, \frac{1}{10} \text{ A}$ (B) $\frac{1}{10} \text{ A}, \frac{1}{15} \text{ A}$ (C) $\frac{2}{15} \text{ A}, \frac{1}{10} \text{ A}$ (D) $\frac{1}{15} \text{ A}, \frac{2}{25} \text{ A}$
32. An inductor coil stores U energy when i current is passed through it and dissipates energy at the rate of P . The time constant of the circuit, when this coil is connected across a battery of zero internal resistance is :-
 (A) $\frac{4U}{P}$ (B) $\frac{U}{P}$ (C) $\frac{2U}{P}$ (D) $\frac{2P}{U}$
33. A small coil of radius r is placed at the centre of a large coil of radius R , where $R \gg r$. The coils are coplanar. The coefficient of mutual inductance between the coils is :-
 (A) $\frac{\mu_0 \pi r}{2R}$ (B) $\frac{\mu_0 \pi r^2}{2R}$ (C) $\frac{\mu_0 \pi r^2}{2R^2}$ (D) $\frac{\mu_0 \pi r}{2R^2}$
34. A long straight wire is placed along the axis of a circular ring of radius R . The mutual inductance of this system is :-
 (A) $\frac{\mu_0 R}{2}$ (B) $\frac{\mu_0 \pi R}{2}$ (C) $\frac{\mu_0}{2}$ (D) 0
35. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon-
 (A) the rates at which currents are changing in the two coils
 (B) relative position and orientation of the two coils
 (C) the materials of the wires of the coils
 (D) the currents in the two coils

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36. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($L \gg l$). The loop are coplanar & their centres coincide. The mutual inductance of the system is proportional to :

(A) $\frac{l}{L}$ (B) $\frac{l^2}{L}$ (C) $\frac{L}{l}$ (D) $\frac{L^2}{l}$

37. L , C and R represent physical quantities inductance, capacitance and resistance. The combination which has the dimensions of frequency is

(A) $\frac{1}{RC}$ and $\frac{R}{L}$ (B) $\frac{1}{\sqrt{RC}}$ and $\sqrt{\frac{R}{L}}$ (C) \sqrt{LC} (D) $\frac{C}{L}$

38. A coil of inductance $5H$ is joined to a cell of emf $6V$ through a resistance 10Ω at time $t = 0$. The emf across the coil at time $t = \ln \sqrt{2}$ s is:

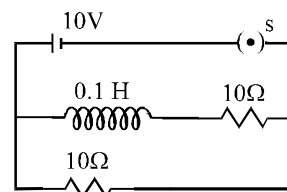
(A) $3V$ (B) $1.5 V$ (C) $0.75 V$ (D) $4.5 V$

39. For L-R circuit, the time constant is equal to

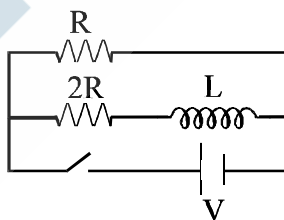
- (A) twice the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance
(B) ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance
(C) half the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance
(D) square of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance

40. In the adjoining circuit, initially the switch S is open. The switch ' S ' is closed at $t = 0$. The difference between the maximum and minimum current that can flow in the circuit is

- (A) 2 Amp
(B) 3 Amp
(C) 1 Amp
(D) nothing can be concluded



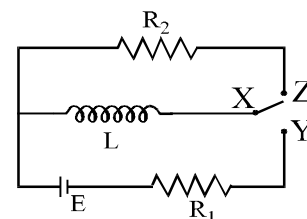
41. Find the ratio of time constant in build up and decay in the circuit as shown in figure :-



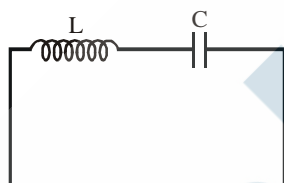
- (A) 1 : 1 (B) 3 : 2 (C) 2 : 3 (D) 1 : 3

42. In the circuit shown, X is joined to Y for a long time, and then X is joined to Z . The total heat produced in R_2 is :

(A) $\frac{LE^2}{2R_1^2}$ (B) $\frac{LE^2}{2R_2^2}$
(C) $\frac{LE^2}{2R_1R_2}$ (D) $\frac{LE^2R_2}{2R_1^2}$



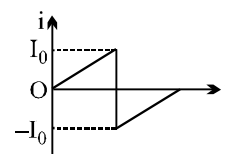
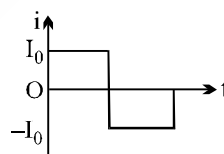
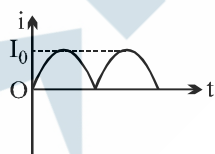
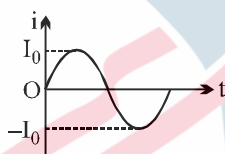
43. In a L–R decay circuit, the initial current at $t = 0$ is I . The total charge that has flown through the resistor till the energy in the inductor has reduced to one-fourth its initial value, is
 (A) LI/R (B) $LI/2R$ (C) $LI\sqrt{2}/R$ (D) None
44. The inductor in a L–C oscillation has a maximum potential difference of 16 V and maximum energy of 640 μJ . Find the value of capacitor in μF in L–C circuit.
 (A) 5 (B) 4 (C) 3 (D) 2
45. A condenser of capacity 6 μF is fully charged using a 6-volt battery. The battery is removed and a resistanceless 0.2 mH inductor is connected across the condenser. The current which is flowing through the inductor when one-third of the total energy is in the magnetic field of the inductor is :-
 (A) 0.1 A (B) 0.2 A (C) 0.4 A (D) 0.6 A
46. In an LC circuit the capacitor has maximum charge q_0 . The value of $\left(\frac{dI}{dt}\right)_{\text{max}}$ is :-



- (A) $\frac{q_0}{LC}$ (B) $\frac{q_0}{\sqrt{LC}}$ (C) $\frac{q_0}{2LC}$ (D) $\frac{2q_0}{LC}$

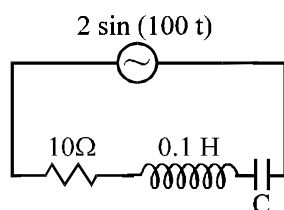
Alternating current

47. When 100 V DC is applied across a solenoid a current of 1 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are:
 (A) 100Ω , 0.93 H (B) 200Ω , 1.0 H (C) 10Ω , 0.86H (D) 200Ω , 0.55 H
48. If I_1 , I_2 , I_3 and I_4 are the respective r.m.s. values of the time varying currents as shown in the four cases I, II, III and IV. Then identify the correct relations.



- (A) $I_1 = I_2 = I_3 = I_4$ (B) $I_3 > I_1 = I_2 > I_4$ (C) $I_3 > I_4 > I_2 = I_1$ (D) $I_3 > I_2 > I_1 > I_4$

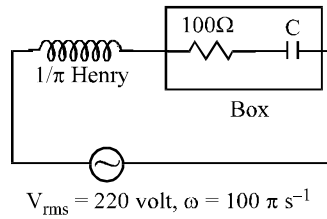
49. The power factor of the circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to



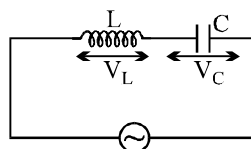
- (A) 400 μF (B) 300 μF (C) 500 μF (D) 200 μF

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50. In the circuit, as shown in the figure, if the value of R.M.S current is 2.2 ampere, the power factor of the box is



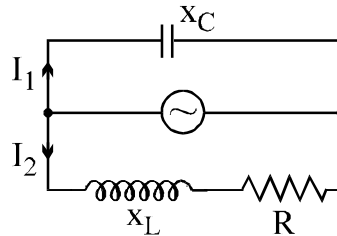
- (A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$
51. The power in ac circuit is given by $P = E_{rms} I_{rms} \cos \phi$. The value of $\cos \phi$ in series LCR circuit at resonance is:
- (A) zero (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$
52. In ac circuit when ac ammeter is connected it reads i current. If a student uses dc ammeter in place of ac ammeter the reading in the dc ammeter will be:
- (A) $\frac{i}{\sqrt{2}}$ (B) $\sqrt{2} i$ (C) $0.637 i$ (D) zero
53. The phase difference between current and voltage in an AC circuit is $\pi/4$ radian. If the frequency of AC is 50 Hz, then the phase difference is equivalent to the time difference :
- (A) 0.78 s (B) 15.7 ms (C) 0.25 s (D) 2.5 ms
54. The effective value of current $i = 2 \sin 100 \pi t + 2 \sin(100 \pi t + 30^\circ)$ is :
- (A) $\sqrt{2}$ Amp (B) $2\sqrt{2+\sqrt{3}}$ Amp (C) 4 Amp (D) None
55. In series LR circuit $X_L = 3R$. Now a capacitor with $X_C = R$ is added in series. Ratio of new to old power factor is
- (A) 1 (B) 2 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$
56. The current I , potential difference V_L across the inductor and potential difference V_C across the capacitor in circuit as shown in the figure are best represented vectorially as



- (A) (B) (C) (D)

57. In a series R-L-C circuit, the frequency of the source is half of the resonance frequency. The nature of the circuit will be
 (A) capacitive (B) inductive (C) purely resistive (D) data insufficient

58. In the shown AC circuit phase difference between currents I_1 and I_2 is



- (A) $\frac{\pi}{2} - \tan^{-1} \frac{X_L}{R}$ (B) $\tan^{-1} \frac{X_L - X_C}{R}$ (C) $\frac{\pi}{2} + \tan^{-1} \frac{X_L}{R}$ (D) $\tan^{-1} \frac{X_L - X_C}{R} + \frac{\pi}{2}$
59. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4A, then that in the secondary is
 (A) 4A (B) 2A (C) 6A (D) 10 A
60. The primary of a 1 : 3 step - up transformer is connected to a source and the secondary is connected to a resistor R. The power dissipated by R in this situation is P. If R is connected directly to the source it will dissipate a power of :
 (A) P/9 (B) P/3 (C) P (D) 3P
61. An ideal efficient transformer has a primary power input of 10kW. The secondary current when the transformer is on load is 25A. If the primary : secondary turns ratio is 8 : 1, then the potential difference applied to the primary coil is
 (A) $\frac{10^4 \times 8^2}{25}$ V (B) $\frac{10^4 \times 8}{25}$ V (C) $\frac{10^4}{25 \times 8}$ V (D) $\frac{10^4}{25 \times 8^2}$ V
62. The core of any transformer is laminated so as to -
 (A) Make it light weight
 (B) Make it robust and strong
 (C) Increase the secondary voltage
 (D) Reduce the energy loss due to eddy current
63. If the difference between the equivalent inductance in the following figures is nL then find the value of n . Given coupling coefficient is $C = \sqrt{2}$ (Where coupling coefficient is defined as $C = \frac{\sqrt{L_1 L_2}}{M}$)

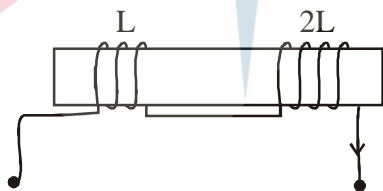


Figure (A)

(A) 2

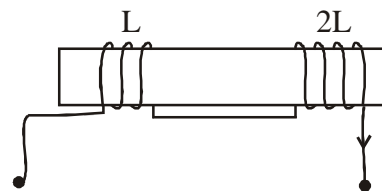


Figure (B)

(C) 4

(D) 5

(B) 3

SUPPLEMENT FOR JEE-MAINS

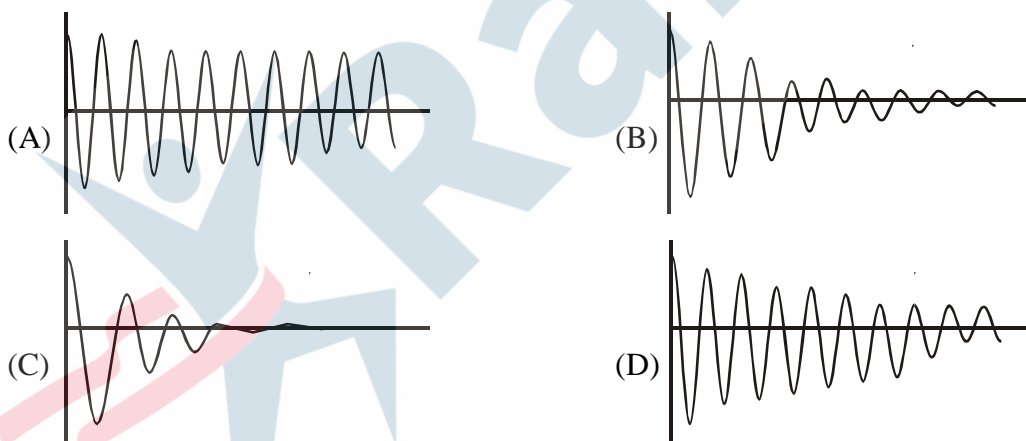
- 64.** Five particles undergo damped harmonic motion. Values for the spring constant k , the damping constant b , and the mass m are given below. Which leads to the smallest rate of loss of mechanical energy at the initial moment?
- (A) $k = 100\text{N/m}$, $m = 50\text{g}$, $b = 8\text{g/s}$ (B) $k = 150\text{N/m}$, $m = 50\text{g}$, $b = 5\text{g/s}$
 (C) $k = 150\text{N/m}$, $m = 10\text{g}$, $b = 8\text{g/s}$ (D) $k = 200\text{N/m}$, $m = 8\text{g}$, $b = 6\text{g/s}$
- 65.** An RLC circuit has a capacitance of $12\mu\text{F}$, an inductance of 25mH , and a resistance of 60Ω . The current oscillates with an angular frequency of :
- (A) $1.2 \times 10^3 \text{ rad/s}$ (B) $1.4 \times 10^3 \text{ rad/s}$ (C) $1.8 \times 10^3 \text{ rad/s}$ (D) $2.2 \times 10^3 \text{ rad/s}$
 (E) $2.6 \times 10^3 \text{ rad/s}$
- 66.** An RLC circuit has an inductance of 25mH and a capacitance of $5.0\mu\text{F}$. The charge on the capacitor does NOT oscillate but rather decays exponentially to zero. The resistance in the circuit must be:
- (A) greater than or equal to $100\sqrt{2}\Omega$ (B) less than $100\sqrt{2}\Omega$ but greater than $50\sqrt{2}\Omega$
 (C) less than $50\sqrt{2}\Omega$ but greater than $25\sqrt{2}\Omega$ (D) less than $25\sqrt{2}\Omega$ but greater than 0
- 67.** Two underdamped oscillators are known to have the same natural frequency ω_0 . The mass and damping coefficient of the first oscillator are m_1 and b_1 , and the mass and damping coefficient of the second oscillator are m_2 and b_2 , respectively. A sinusoidal driving force of $F_{\text{ext}} = F_0 \cos \omega t$ is applied to each oscillator. Starting with ω far from ω_0 the driving force is tuned in order to observe resonant behavior. If $m_1 = 4 m_2$ and $b_1 = 2b_2$, then which one of the following statements concerning the driven oscillations is correct ?
- (A) The resonant peak of the first driven oscillator is higher and narrower than that of the second oscillator.
 (B) The resonant peak of the first driven oscillator is higher and wider than that of the second oscillator.
 (C) The resonant peak of the first driven oscillator is lower and wider than that of the second oscillator.
 (D) The resonant peak of the first driven oscillator is lower and narrower than that of the second oscillator.
- 68.** A simple pendulum has a time period T if there is no air resistance. If a small air resistance is acting on the bob as it oscillates,
- (A) The time period will be initially more than T and decreases with time.
 (B) The time period will be less than T initially and increases with time
 (C) The time period will be less than T and remains constant
 (D) The time period will be more than T and remains constant.
- 69.** A block is executing damped harmonic oscillation with time period T . Choose correct statement
- (1) Time taken to go from extreme to mean position is $\frac{T}{4}$
 (2) Time taken to go from one extreme to another is $\frac{T}{2}$
 (3) Time taken to go from one extreme to another is less than $\frac{T}{2}$
 (4) Time taken to go from one extreme to another is more than $\frac{T}{2}$
- (A) 1,2 only (B) 1,2,3 only (C) 2 only (D) 1,2,4 only

70. The angular frequency of the damped oscillator is given by, $\omega = \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}$ where k is the spring constant,

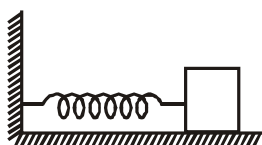
m is the mass of the oscillator and r is damping constant. If the ratio $\frac{r^2}{mk}$ is 8%, the change in time period

compared to the undamped oscillator is approximately as follows:

- (A) decreases by 8% (B) decreases by 1% (C) increases by 1% (D) increases by 8%
71. Two spheres of the same diameter but of different masses are suspended by strings of equal length. If the spheres are deflected from their positions of equilibrium, which of the two will have a greater oscillation period and which will have a greater logarithmic decrement if their oscillations occur in a real medium with viscosity?
- (A) Heavier mass has larger time period & greater logarithmic decrement
 (B) Lighter mass has larger time period & greater logarithmic decrement
 (C) Lighter mass has larger time period but lesser logarithmic decrement
 (D) Heavier mass has larger time period but lesser logarithmic decrement
72. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10 cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3, the time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbon dioxide will be close to ($\ln 5 = 1.601$, $\ln 2 = 0.693$).
- [JEE Main Online 2014]
- (A) 231 s (B) 208 s (C) 142 s (D) 161 s
73. Which graph has the highest Q factor ?



74. In the situation shown, the block can execute free oscillation (no damping) with angular frequency ω_1 . In presence of weak damping, it executes damped SHM with angular frequency ω_2 . When it is subjected to a sinusoidal force, it executes forced oscillation with maximum amplitude at angular frequency ω_3 (assume damping is present) :-

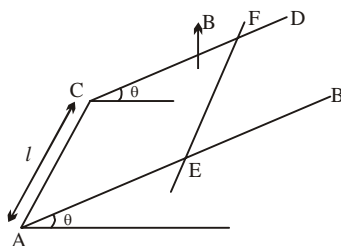
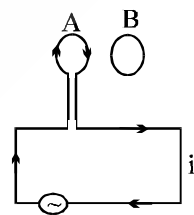


- (A) $\omega_1 > \omega_2 > \omega_3$ (B) $\omega_1 > \omega_2 = \omega_3$ (C) $\omega_1 = \omega_2 = \omega_3$ (D) $\omega_1 > \omega_3 > \omega_2$

75. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force; then – [AIEEE- 2004]
- (A) $\omega_1 = \omega_2$
 (B) $\omega_1 > \omega_2$
 (C) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
 (D) $\omega_1 < \omega_2$
76. A pendulum with time period of 1s is losing energy due to damping. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s^{-1}) is:- [JEE Mains On line 2015]
- (A) 2 (B) $\frac{1}{15} \ln 3$ (C) $\frac{1}{2}$ (D) $\frac{1}{30} \ln 3$

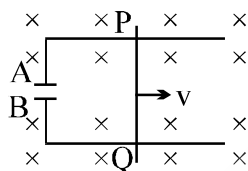
MULTIPLE CORRECT TYPE QUESTIONS

77. The dimension of the ratio of magnetic flux and the resistance is equal to that of :
 (A) induced emf (B) charge (C) inductance (D) current
78. Two circular coils A and B are facing each other as shown in figure. The current i through A can be altered
 (A) there will be repulsion between A and B if i is increased
 (B) there will be attraction between A and B if i is increased
 (C) there will be neither attraction nor repulsion when i is changed
 (D) attraction or repulsion between A and B depends on the direction of current. It does not depend whether the current is increased or decreased.
79. A bar magnet is moved along the axis of copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true?
 (A) The south pole faces the ring and the magnet moves towards it.
 (B) The north pole faces the ring and the magnet moves towards it.
 (C) The south pole faces the ring and the magnet moves away from it.
 (D) The north pole faces the ring and the magnet moves away from it.
80. AB and CD are smooth parallel rails, separated by a distance l , and inclined to the horizontal at an angle θ . A uniform magnetic field of magnitude B , directed vertically upwards, exists in the region. EF is a conductor of mass m , carrying a current i . For EF to be in equilibrium,

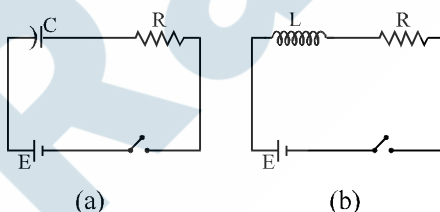


- (A) i must flow from E to F (B) $Bil = mg \tan \theta$
 (C) $Bil = mg \sin \theta$ (D) $Bil = mg$

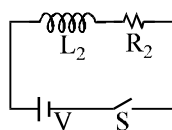
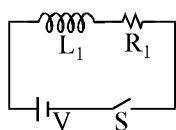
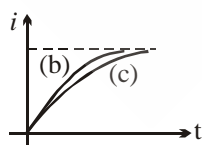
81. In the **previous question**, if B is normal to the plane of the rails
 (A) $Bil = mg \tan \theta$ (B) $Bil = mg \sin \theta$
 (C) $Bil = mg \cos \theta$ (D) equilibrium cannot be reached
82. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 20$ m/s in a uniform magnetic field $B = 4.0$ T directed into the paper. A capacitor of capacity $C = 10 \mu\text{F}$ is connected as shown in figure. Then



- (A) $q_A = +800 \mu\text{C}$ and $q_B = -800 \mu\text{C}$
 (B) $q_A = -800 \mu\text{C}$ and $q_B = +800 \mu\text{C}$
 (C) $q_A = 0 = q_B$
 (D) charge stored in the capacitor increases exponentially with time
83. An LR circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the circuit
 (A) current in the circuit
 (B) magnetic field energy in the inductor
 (C) power delivered by the battery
 (D) emf induced in the inductor
84. The switches in figures (a) and (b) are closed at $t = 0$

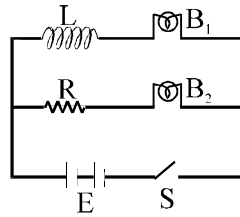


- (A) The charge on C just after $t = 0$ is EC .
 (B) The charge on C long after $t = 0$ is EC .
 (C) The current in L just after $t = 0$ is E/R .
 (D) The current in L long after $t = 0$ is E/R .
85. Current growth in two L-R circuits (b) and (c) as shown in figure (a). Let L_1 , L_2 , R_1 and R_2 be the corresponding values in two circuits. Then :-

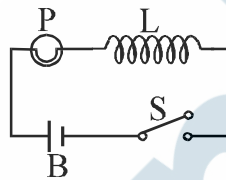


- (a) (b) (c) (D) $L_1 < L_2$
- (A) $R_1 > R_2$ (B) $R_1 = R_2$ (C) $L_1 > L_2$

86. An inductor L , a resistance R and two identical bulbs B_1 and B_2 are connected to a battery through a switch S as shown in the figure. The resistance of coil having inductance L is also R . Which of the following statement gives the correct description of the happenings when the switch S is closed?



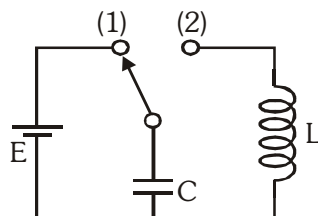
- (A) The bulb B_2 lights up earlier than B_1 and finally both the bulbs shine equally bright.
 (B) B_1 lights up earlier and finally both the bulbs acquire equal brightness.
 (C) B_2 lights up earlier and finally B_1 shines brighter than B_2 .
 (D) B_1 and B_2 light up together with equal brightness all the time.
87. In figure, a lamp P is in series with an iron-core inductor L . When the switch S is closed, the brightness of the lamp rises relatively slowly to its full brightness than it would do without the inductor. This is due to



- (A) the low resistance of P
 (B) the induced-emf in L
 (C) the low resistance of L
 (D) the high voltage of the battery B
88. Two different coils have self inductance 8mH and 2mH . The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are I_1 , V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are I_2 , V_2 and W_2 respectively. Then:

- (A) $\frac{I_1}{I_2} = \frac{1}{4}$ (B) $\frac{I_1}{I_2} = 4$ (C) $\frac{W_2}{W_1} = 4$ (D) $\frac{V_2}{V_1} = \frac{1}{4}$

89. Initially key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at $t = 0$. The time when the energy in both capacitor and inductor will be same-



- (A) $\frac{\pi\sqrt{LC}}{4}$ (B) $\frac{\pi\sqrt{LC}}{2}$ (C) $\frac{5\pi\sqrt{LC}}{4}$ (D) $\frac{5\pi\sqrt{LC}}{2}$

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 90 to 92

The fact that a changing magnetic flux produces an electric field is basic to the operation of many high energy particle accelerators. Since the principle was first successfully applied to the acceleration of electrons (or β particles) in a device called the betatron, this method of acceleration is often given that name. The general idea involved is shown in figure.

An electromagnet is used to produce a changing flux through a circular loop defined by the doughnut-shaped vacuum chamber. We see that there will be an electric field E along the circular length of the doughnut, i.e. circling the magnet poles, given by

$$2\pi aE = \frac{d\phi}{dt}$$

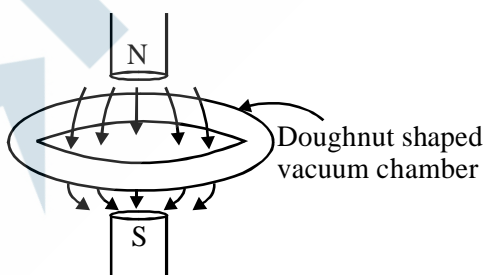
Where a is the radius of the doughnut. Any charged particle inside the vacuum chamber will experience a force qE and will accelerate. Ordinarily, the charged particle would shoot out of the vacuum chamber and becomes lost.

However, if the magnetic field at the position of the doughnut is just proper to satisfy the relation
Centripetal force = magnetic force

$$\text{or } \frac{mv^2}{a} = qvB$$

then the charge will travel in a circle within the doughnut. By proper shaping of the magnet pole pieces, this relation can be satisfied. As a result, the charge will move at high speed along the loop within the doughnut. Each time it goes around the loop, it has, in effect, fallen through a potential

difference equal to the induced, emf, namely $\varepsilon = \frac{d\phi}{dt}$. Its energy after n trips around the loop will be $q(n\varepsilon)$

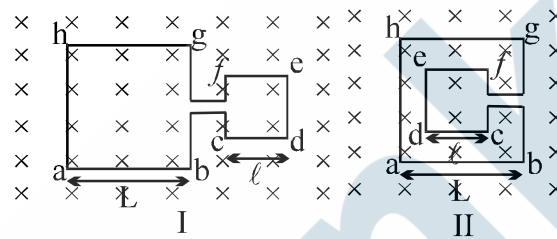


90. Working of betatron is not based upon which of the following theories :-
- (A) Changing magnetic flux induces electric field
 - (B) Charged particle at rest can be accelerated only by electric fields
 - (C) magnetic fields can apply a force on moving charges which is perpendicular to both magnetic field and motion of the particle
 - (D) Beta particles are emitted in radioactive decay process.

91. Variable magnetic flux :-
 (A) Can change sinusoidally
 (B) Should increase all the time
 (C) Must become zero when induced field is maximum
 (D) None of these
92. Magnetic field which keeps the particles in circular path must :-
 (A) Remain a constant everywhere
 (B) Increase gradually which is proportional to K.E. of the particle
 (C) Increase gradually which is proportional to speed of the particle
 (D) None of these

Paragraph for Question No. 93 to 96

The adjoining figure shows two different arrangements in which two square wire frames of same resistance are placed in a uniform constantly decreasing magnetic field B .

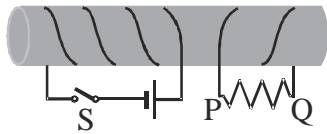


93. The value of magnetic flux in each case is given by
 (A) Case I: $\Phi = \pi(L^2 + \ell^2)B$; Case II: $\Phi = \pi(L^2 - \ell^2)B$
 (B) Case I: $\Phi = \pi(L^2 + \ell^2)B$; Case II: $\Phi = \pi(L^2 + \ell^2)B$
 (C) Case I: $\Phi = (L^2 + \ell^2)B$; Case II: $\Phi = (L^2 - \ell^2)B$
 (D) Case I: $\Phi = (L + \ell)^2 B$; Case II: $\Phi = \pi(L - \ell)^2 B$
94. The direction of induced current in the case I is
 (A) from a to b and from c to d
 (B) from a to b and from f to e
 (C) from b to a and from d to c
 (D) from b to a and from e to f
95. The direction of induced current in the case II is
 (A) from a to b and from c to d
 (B) from b to a and from f to e
 (C) from b to a and from c to d
 (D) from a to b and from d to c
96. If I_1 and I_2 are the magnitudes of induced current in the cases I and II, respectively, then
 (A) $I_1 = I_2$ (B) $I_1 > I_2$ (C) $I_1 < I_2$ (D) nothing can be said

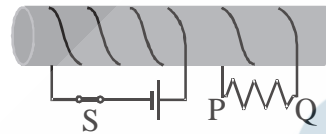
EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

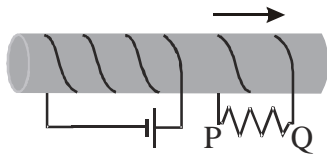
1. For each of the experiments (1, 2, 3, 4) shown in figure. Choose the **CORRECT** option(s) which shows the direction of current flow through the resistor PQ? Note that the wires are not always wrapped around the plastic tube in the same way.



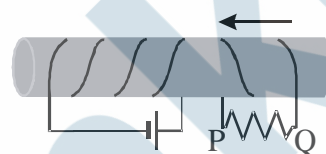
(1) S to be closed



(2) S to be opened



(3) Resistor coil PQ moves to right



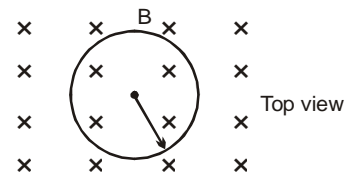
(4) Resistor coil PQ moves to left

- | | | | |
|--|--|--|--|
| <p>(1)</p> <p>(A) P to Q</p> <p>(B) P to Q</p> <p>(C) Q to P</p> <p>(D) Q to P</p> | <p>(2)</p> <p>P to Q</p> <p>Q to P</p> <p>Q to P</p> <p>Q to P</p> | <p>(3)</p> <p>P to Q</p> <p>P to Q</p> <p>Q to P</p> <p>P to Q</p> | <p>(4)</p> <p>P to Q</p> <p>Q to P</p> <p>Q to P</p> <p>P to Q</p> |
|--|--|--|--|
2. An electron is moving in a circular orbit of radius R with an angular acceleration α . At the centre of the orbit is kept a conducting loop of radius r , ($r \ll R$). The e.m.f induced in the smaller loop due to the motion of the electron is :-

- (A) zero, since charge on electron is constant (B) $\frac{\mu_0 e r^2}{4R} \alpha$
- (C) $\frac{\mu_0 e r^2}{4\pi R} \alpha$ (D) none of these

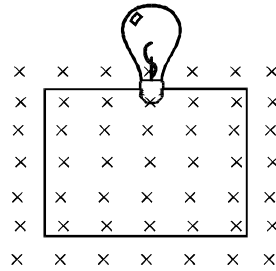
3. A non conducting ring (of mass m , radius r , having charge Q) is placed on a rough horizontal surface (in a region with transverse magnetic field). The field is increasing with time at the rate R and coefficient of friction between the surface and the ring is μ . For ring to remain in equilibrium μ should be greater than :-

- (A) $\frac{QrR}{mg}$ (B) $\frac{QrR}{2mg}$ (C) $\frac{QrR}{3mg}$ (D) $\frac{2QrR}{mg}$

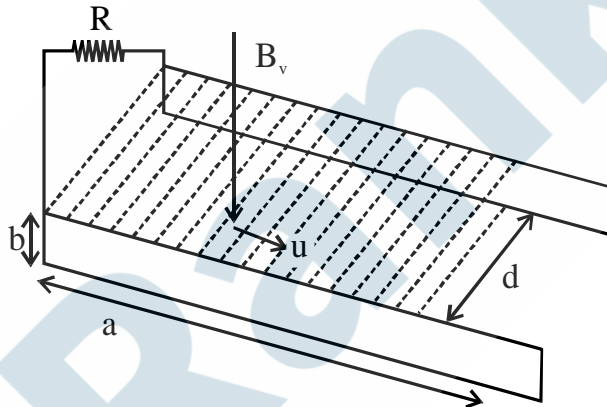


Electromagnetic induction & Alternating current

4. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 20T. A 10 V light bulb is in a series with the loop as shown in the fig. The magnetic field is decreasing steadily to zero over a time interval Δt . The bulb will shine with full brightness if Δt is equal to :-



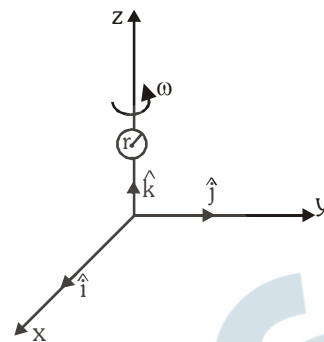
- (A) 20 ms (B) 0.02 ms (C) 2 ms (D) 0.2 ms
5. The figure shows an apparatus suggested by Faraday to generate electric current from a flowing river. Two identical conducting plates of length a and width b are placed parallel facing one another on opposite sides of the river following with velocity u at a distance d apart. Now both the plates are connected by a load resistance R . Then the current through the load R is :- (Consider vertical component of the magnetic field produced by earth is B_v and the resistivity of river water is ρ .)



- (A) $\frac{B_v ub}{R}$ (B) $\frac{B_v ub}{R + \frac{\rho d}{ab}}$ (C) $\frac{B_v ud}{R + \frac{\rho d}{ab}}$ (D) None of the above
6. The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m² acts perpendicular to the plane of the coil. The radius of the coil when the induced e.m.f. in the coil is $1\mu\text{V}$, is :-
- (A) $\frac{2}{\pi}$ cm (B) $\frac{3}{\pi}$ cm (C) $\frac{4}{\pi}$ cm (D) $\frac{5}{\pi}$ cm
7. A composite rod of length ℓ is one fourth insulator and remaining conductor is made to rotate freely with angular velocity ω , in a space free of any gravitational, electric & magnetic field. Then potential difference across the conducting region will be (rotation is about insulating end).
- (A) $\frac{3m_e \omega^2 \ell^2}{4e}$ (B) $\frac{1}{4} \frac{m_e \omega^2 \ell^2}{e}$ (C) $\frac{1}{16} \frac{m_e \omega^2 \ell^2}{e}$ (D) $\frac{15}{32} \frac{m_e \omega^2 \ell^2}{e}$

8. A circular loop wire of radius r rotates about the z -axis with angular velocity ω . The normal to the loop is always perpendicular to the z -axis. At time $t = 0$, the normal is parallel to the y -axis. An external magnetic field $\vec{B} = B_y \hat{j} + B_z \hat{k}$ is applied. The EMF $\varepsilon(t)$ induced in the loop is

- (A) $\pi r^2 \omega B_y \sin \omega t$ (B) $\pi r^2 \omega B_z \cos \omega t$
 (C) $\pi r^2 \omega B_z \sin \omega t$ (D) $\pi r^2 \omega B_y \cos \omega t$

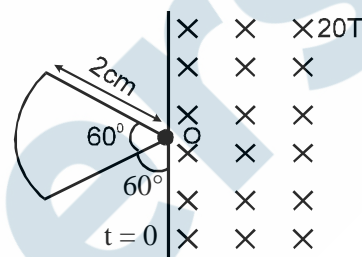


9. A uniform magnetic field 20 T exists on right side of the boundary in a gravity free space as shown in figure. The given circular arc of radius 2cm made of conducting wire of total resistance 4Ω is rotated around point O at a constant angular speed 2 rad per second. Power required

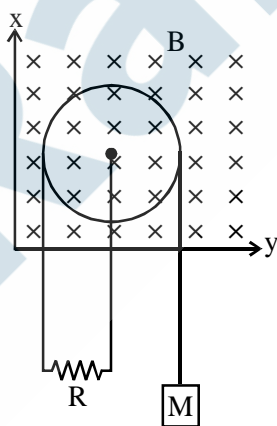
to maintain the constant angular velocity between time interval $t = \frac{\pi}{6}$ s

to $t = \frac{\pi}{3}$ s is :-

- (A) $64 \mu W$ (B) $32 \mu W$ (C) $128 \mu W$ (D) $16 \mu W$



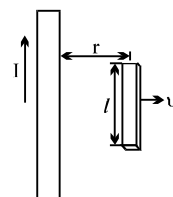
10. The block of mass (M) is connected by thread which is wound on a pulley, free to rotate about fixed horizontal axis as shown. A uniform magnetic field B exists in a horizontal plane. The disc is connected with the resistance R as shown. Calculate the terminal velocity of the block if it was released from rest. Treat pulley as uniform metallic disc of radius L .



- (A) $\frac{4mgR}{B^2 L^2}$ (B) $\frac{3mgR}{4B^2 L^2}$ (C) $\frac{2mgR}{B^2 L^2}$ (D) $\frac{3mgR}{2B^2 L^2}$

11. A conducting rod moves with constant velocity v perpendicular to the long, straight wire carrying a current I as shown. Compute the emf generated between the ends of the rod.

- (A) $\frac{\mu_0 v I l}{\pi r}$ (B) $\frac{\mu_0 v I l}{2\pi r}$
 (C) $\frac{2\mu_0 v I l}{\pi r}$ (D) $\frac{\mu_0 v I l}{4\pi r}$



12. A metallic rod of length L and mass M is moving under the action of two unequal forces F_1 and F_2 (directed opposite to each other) acting at its ends along its length. Ignore gravity and any external magnetic field. If specific charge of electrons is (e/m) , then the potential difference between the ends of the rod in steady state must be

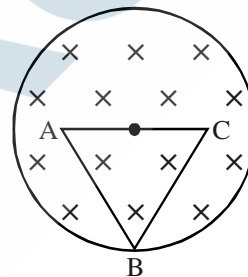
(A) $|F_1 - F_2| mL/eM$ (B) $(F_1 + F_2) mL/eM$
(C) $[mL/eM] \ln [F_1/F_2]$ (D) None

13. Two parallel long straight conductors lie on a smooth surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. They all start moving out with a constant velocity v . If r is the resistance per unit length of the wire the current in the circuit will be

(A) $\frac{Bv}{r}$ (B) $\frac{Br}{v}$ (C) Bvr (D) Bv

14. An equilateral triangle ABC of side a is placed in the magnetic field with side AC and its centre coinciding with the centre of the magnetic field. The magnetic field varies with time as $B = ct$. The emf induced across side AB is :-

(A) $\frac{\sqrt{3}}{4} a^2 c$ (B) Zero
(C) $\frac{\sqrt{3}}{8} a^2 c$ (D) $\frac{(\sqrt{2}-1)}{2} a^2 c$

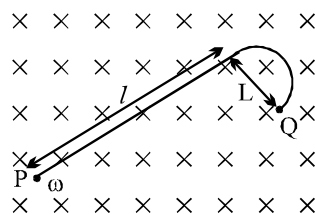


15. The magnetic field in a region is given by $\vec{B} = B_0 \left(1 + \frac{x}{a}\right) \hat{k}$. A square loop of edge - length d is

placed with its edge along x & y axis. The loop is moved with constant velocity $\vec{V} = V_0 \hat{i}$. The emf induced in the loop is

(A) $\frac{V_0 B_0 d^2}{a}$ (B) $\frac{V_0 B_0 d^2}{2a}$ (C) $\frac{V_0 B_0 a^2}{d}$ (D) None

16. When a 'J' shaped conducting rod is rotating in its own plane with constant angular velocity ω , about one of its end P , in a uniform magnetic field \vec{B} directed normally into the plane of paper then magnitude of emf induced across it will be



(A) $B\omega\sqrt{L^2 + l^2}$ (B) $\frac{1}{2} B\omega L^2$ (C) $\frac{1}{2} B\omega(L^2 + l^2)$ (D) $\frac{1}{2} B\omega l^2$

17. A rectangular coil of single turn, having area A , rotates in a uniform magnetic field B with an angular velocity ω about an axis perpendicular to the field. If initially the plane of coil is perpendicular to the field, then the average induced e.m.f. when it has rotated through 90° is :-

(A) $\frac{\omega BA}{\pi}$ (B) $\frac{\omega BA}{2\pi}$ (C) $\frac{\omega BA}{4\pi}$ (D) $\frac{2\omega BA}{\pi}$

18. In figure (a) a solenoid produces a magnetic field whose strength increases into the plane of the page. An induced emf is established in a conduction loop surrounding the solenoid, and this emf lights bulbs A and B. In figure (b) point P and Q are shorted. After the short is inserted

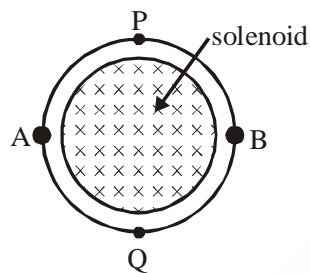


fig. (a)

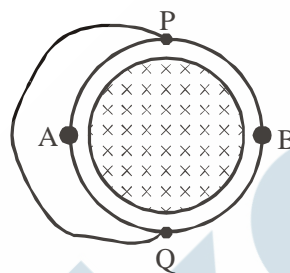
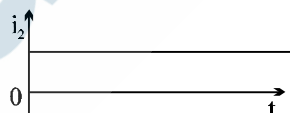
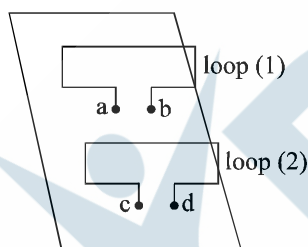
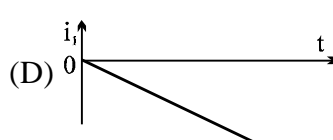
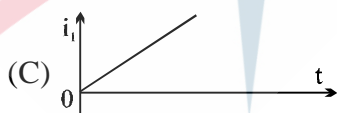
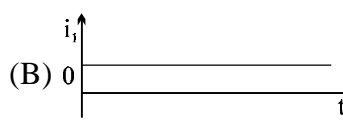


fig. (b)

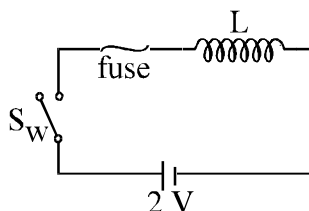
- (A) Bulb A goes out, bulb B gets brighter
(B) Bulb B goes out, bulb A gets brighter
(C) Bulb A goes out, bulb B gets dimmer
(D) Bulb B goes out, bulb A gets dimmer
19. An electric current i_1 can flow either direction through loop (1) and induced current i_2 in loop (2). Positive i_1 is when current is from 'a' to 'b' in loop (1) and positive i_2 is when the current is from 'c' to 'd' in loop (2). In an experiment, the graph of i_2 against time 't' is as shown below



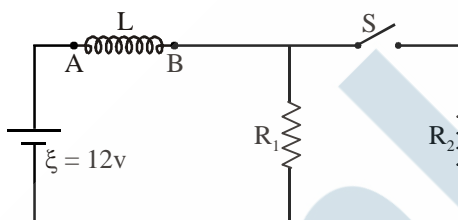
Which one(s) of the following graphs could have caused i_2 to behave as given above.



20. In the circuit shown, the cell is ideal. The coil has an inductance of 4H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5A. The switch is closed at $t = 0$. The fuse will blow :



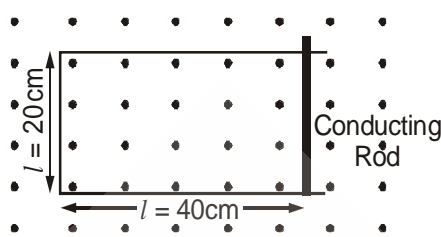
- (A) just after $t = 0$ (B) after 2s (C) after 5s (D) after 10s
21. The circuit shown has been operating for a long time. The instant after the switch in the circuit labeled S is opened, what is the voltage across the inductor V_L and which labeled point (A or B) of the inductor is at a higher potential ? Take $R_1 = 4.0 \Omega$, $R_2 = 8.0 \Omega$, and $L = 2.5 \text{ H}$.



- (A) $V_L = 12 \text{ V}$; Point A is at the higher potential
 (B) $V_L = 12 \text{ V}$; Point B is at the higher potential
 (C) $V_L = 6\text{V}$; Point A is at the higher potential
 (D) $V_L = 6\text{V}$; Point B is at the higher potential
22. When a resistance R is connected in series with an element A, the electric current is found to be lagging behind the voltage by angle θ_1 . When the same resistance is connected in series with element B, current leads voltage by θ_2 . When R , A, B are connected in series, the current now leads voltage by θ . Assume same AC source is used in all cases, then
- (A) $\theta = \theta_2 - \theta_1$ (B) $\tan\theta = \tan\theta_2 - \tan\theta_1$
 (C) $\theta = \frac{\theta_1 + \theta_2}{2}$ (D) None of these
23. An current is given by $I = I_0 + I_1 \sin \omega t$ then its rms value will be
- (A) $\sqrt{I_0^2 + 0.5 I_1^2}$ (B) $\sqrt{I_0^2 + 0.5 I_0^2}$ (C) 0 (D) $I_0/\sqrt{2}$
24. Power factor of an L-R series circuit is 0.6 and that of a C-R series circuit is 0.5. If the element (L, C, and R) of the two circuits are joined in series the power factor of this circuit is found to be 1. The ratio of the resistance in the L-R circuit to the resistance in the C-R circuit is
- (A) 6/5 (B) 5/6 (C) $\frac{4}{3\sqrt{3}}$ (D) $\frac{3\sqrt{3}}{4}$

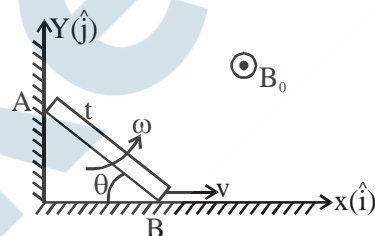
MULTIPLE CORRECT TYPE QUESTIONS

25. Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance $1 \Omega/\text{m}$. Position of the conducting rod at $t = 0$ is shown. A time dependent magnetic field $B = 2t$ Tesla is switched on at $t = 0$. After the magnetic field is switched on, the conducting rod is moved to the left perpendicular to the rails at constant speed 5 cm/s by some external agent.

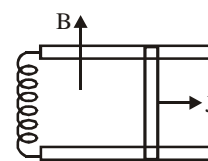


- (A) The current in the loop at $t = 0$ due to induced emf is 0.16 A , clockwise
 (B) At $t = 2 \text{ s}$, induced emf has magnitude 0.08 V
 (C) The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at $t = 2 \text{ s}$, is equal to 0.08 N
 (D) The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at $t = 2 \text{ s}$, is equal to 0.16 N

26. A thin conducting rod of length ℓ is moved such that its end B moves along the X-axis while end A moves along the Y-axis. A uniform magnetic field $B = B_0 \hat{k}$ exists in the region. At some instant, velocity of end B is v and the rod makes an angle of $\theta = 60^\circ$ with the X-axis as shown in the figure. Then, at this instant



- (A) angular speed of rod AB is $\omega = \frac{2v}{\sqrt{3}\ell}$ (B) angular speed of rod AB is $\omega = \frac{\sqrt{3}v}{2\ell}$
 (C) e.m.f. induced in rod AB is $B\ell v\sqrt{3}$ (D) e.m.f. induced in rod AB is $B\ell v/2\sqrt{3}$
27. Two parallel resistanceless rails are connected by an inductor of inductance L at one end as shown in the figure. A magnetic field B exists in the space which is perpendicular to the plane of the rails. Now a conductor of length ℓ and mass m is placed transverse on the rails and given an impulse J towards the rightward direction. Then choose the **CORRECT** option (s).



- (A) Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\frac{3J^2 L}{4B^2 \ell^2 m}}$
 (B) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\frac{3J^2}{4Lm}}$

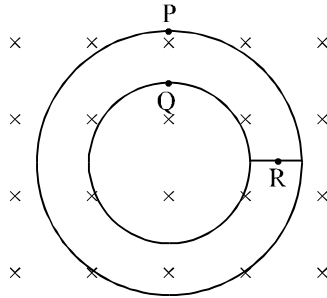
- (C) Velocity of the conductor is half of the initial velocity after a displacement of the conductor

$$d = \sqrt{\frac{3J^2 L}{B^2 \ell^2 m}}$$

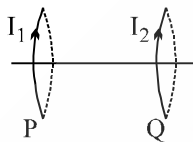
- (D) Current flowing through the inductor at the instant when velocity of the conductor is half of the

$$\text{initial velocity is } i = \sqrt{\frac{3J^2}{mL}}$$

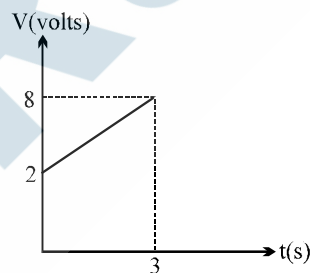
28. Figure shown plane figure made of a conductor located in a magnetic field along the inward normal to the plane of the figure. The magnetic field starts diminishing. Then the induced current



- (A) at point P is clockwise
(B) at point Q is anticlockwise
(C) at point Q is clockwise
(D) at point R is zero
29. Two circular coils P & Q are fixed coaxially & carry currents I_1 and I_2 respectively



- (A) if $I_2 = 0$ & P moves towards Q, a current in the same direction as I_1 is induced in Q
(B) if $I_1 = 0$ & Q moves towards P, a current in the opposite direction to that of I_2 is induced in P.
(C) when $I_1 \neq 0$ and $I_2 \neq 0$ are in the same direction then the two coils tend to move apart.
(D) when $I_1 \neq 0$ and $I_2 \neq 0$ are in opposite directions then the coils tends to move apart.
30. A circuit element is placed in a closed box. At time $t = 0$, constant current generator supplying a current of 1 amp, is connected across the box. Potential difference across the box varies according to graph shown in figure. The element in the box is :

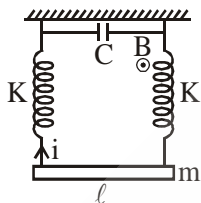


- (A) resistance of 2Ω
(B) battery of emf 6V
(C) inductance of 2H
(D) capacitance of 0.5F
31. Two coils A and B have coefficient of mutual inductance $M = 2\text{H}$. The magnetic flux passing through coil A changes by 4 Weber in 10 seconds due to the change in current in B. Then
- (A) the change in current in B in this time interval is 0.5 A
(B) the change in current in B in this time interval is 2A
(C) the change in current in B in this time interval is 8A
(D) a change in current of 1A in coil A will produce a change in flux passing through B by 4 Weber.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 32 and 33

In the figure shown a uniform conducting rod of mass m and length ℓ is suspended in vertical plane by two conducting springs of spring constant K . Upper end of spring are connected to each other by capacitor of capacitance C . A uniform horizontal magnetic field (B_0) perpendicular to plane of spring exists in space. Initially rod is in equilibrium but if centre of rod is pulled down and released, it performs SHM. Assume that the spring is small and neglect the magnetic force of interaction between circular section of springs & self inductance of rod.



32. Find time period of oscillation of rod :-

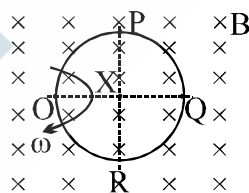
- (A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{B^2\ell^2C}{K}}$ (C) $\pi\sqrt{\frac{m + B^2\ell^2C}{K}}$ (D) $2\pi\sqrt{\frac{B^2\ell^2C + m}{2K}}$

33. Choose correct options from following :-

- (A) Electrical energy stored in capacitor is maximum when rod is at its lower extreme position
 (B) Electrical energy stored in capacitor is maximum when rod is at its mean position
 (C) Current in rod is maximum at mean position of rod
 (D) If magnetic field is switched off then mean position of rod will change

Paragraph for Question No. 34 to 36

A conducting ring of radius a is rotated about a point O on its periphery as shown in the figure in a plane perpendicular to uniform magnetic field B which exists everywhere. The rotational velocity is ω .



34. Choose the correct statement(s) related to the potential of the points P , Q and R

- (A) $V_P - V_O > 0$ and $V_R - V_O < 0$ (B) $V_P = V_R > V_O$
 (C) $V_O > V_P = V_Q$ (D) $V_Q - V_P = V_P - V_O$

35. Choose the correct statement(s) related to the magnitude of potential differences

- (A) $V_P - V_O = \frac{1}{2} B\omega a^2$ (B) $V_P - V_Q = \frac{1}{2} B\omega a^2$
 (C) $V_Q - V_O = 2B\omega a^2$ (D) $V_P - V_R = 2B\omega a^2$

36. Choose the correct statement(s) related to the induced current in the ring
- (A) Current flows from $Q \longrightarrow P \longrightarrow O \longrightarrow R \longrightarrow Q$
(B) Current flows from $Q \longrightarrow R \longrightarrow O \longrightarrow P \longrightarrow Q$
(C) Current flows from $Q \longrightarrow P \longrightarrow O$ and from $Q \longrightarrow R \longrightarrow O$
(D) No current flows

Paragraph for question nos. 37 to 39

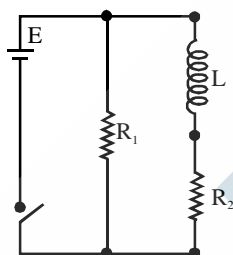
In a series L-R circuit, connected with a sinusoidal ac source, the maximum potential difference across L and R are respectively 3 volts and 4 volts.

37. At an instant the potential difference across resistor is 2 volts. The potential difference in volt, across the inductor at the same instant will be :
- (A) $3 \cos 30^\circ$ (B) $3 \cos 60^\circ$ (C) $3 \cos 45^\circ$ (D) None of these
38. At the same instant, the magnitude of the potential difference in volt, across the ac source may be
- (A) $4 + 3\sqrt{3}$ (B) $\frac{4 + 3\sqrt{3}}{2}$ (C) $1 + \frac{\sqrt{3}}{2}$ (D) $2 + \frac{\sqrt{3}}{2}$
39. If the current at this instant is decreasing the magnitude of potential difference at that instant across the ac source is
- (A) Increasing (B) Decreasing (C) Constant (D) Cannot be said

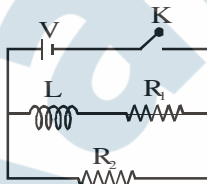
EXERCISE-JM

1. An inductor of inductance $L = 400 \text{ mH}$ and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is:-

[AIEEE - 2009]



- (1) $6(1 - e^{-t/0.2})\text{V}$ (2) $12e^{-5t}\text{V}$ (3) $6e^{-5t}\text{V}$ (4) $\frac{12}{t}e^{-3t}\text{V}$
2. In the circuit show below, the key K is closed at $t = 0$. The current through the battery is : [AIEEE - 2010]



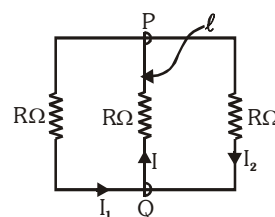
- (1) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ (2) $\frac{VR_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
- (3) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$ (4) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1 R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$
3. A rectangular loop has a sliding connector PQ of length ℓ and resistance $R\Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1 , I_2 and I are :- [AIEEE - 2010]

(1) $I_1 = I_2 = \frac{B\ell v}{6R}$, $I = \frac{B\ell v}{3R}$

(2) $I_1 = -I_2 = \frac{B\ell v}{R}$, $I = \frac{2B\ell v}{R}$

(3) $I_1 = I_2 = \frac{B\ell v}{3R}$, $I = \frac{2B\ell v}{3R}$

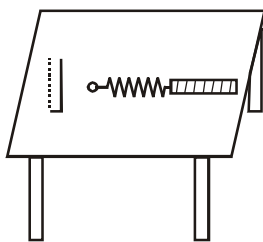
(4) $I_1 = I_2 = I = \frac{B\ell v}{R}$



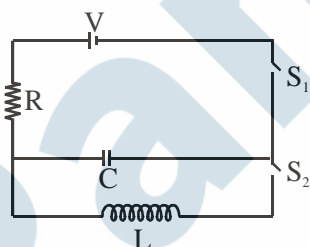
Electromagnetic induction & Alternating current

4. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30° . On taking out the inductor from the circuit the current leads the voltage by 30° . The power dissipated in the LCR circuit is : **[AIEEE - 2010]**
- (1) 242 W (2) 305 W (3) 210 W (4) 0 W
5. A boat is moving due east in a region where the earth's magnetic field is $5.0 \times 10^{-5} \text{ NA}^{-1} \text{ m}^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1} , the magnitude of the induced emf in the wire of aerial is :- **[AIEEE - 2011]**
- (1) 0.50 mV (2) 0.15 mV (3) 1 mV (4) 0.75 mV
6. A horizontal straight wire 20 m long extending from east to west is falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field $0.30 \times 10^{-4} \text{ Wb/m}^2$. The instantaneous value of the e.m.f. induced in the wire will be :- **[AIEEE - 2011]**
- (1) 6.0 mV (2) 3 mV (3) 4.5 mV (4) 1.5 mV
7. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is :- **[AIEEE - 2011]**
- (1) $2\pi\sqrt{LC}$ (2) \sqrt{LC} (3) $\pi\sqrt{LC}$ (4) $\frac{\pi}{4}\sqrt{LC}$
8. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :- **[AIEEE - 2012]**
- (1) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping
(2) Development of air current when the plate is placed
(3) Induction of electrical charge on the plate
(4) Shielding of magnetic lines of force as aluminium is a paramagnetic material
9. If a simple pendulum has Significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average lifetime of the pendulum is (assuming damping is small) in seconds: **[AIEEE-2012]**
- (1) $\frac{1}{b}$ (2) $\frac{2}{b}$ (3) $\frac{0.693}{b}$ (4) b
10. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to a times its original magnitude, where a equals: **[JEE Main-2013]**
- (1) 0.81 (2) 0.729 (3) 0.6 (4) 0.7

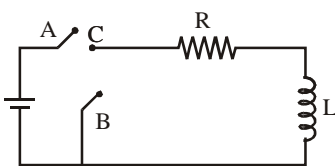
11. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field ' B ' in the region, the e.m.f. induced across the ends of the rod is : [JEE Main-2013]



- (1) $\frac{2B\omega l^2}{2}$ (2) $\frac{3B\omega l^2}{2}$ (3) $\frac{4B\omega l^2}{2}$ (4) $\frac{5B\omega l^2}{2}$
12. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is :- [JEE Main-2013]
- (1) 9.1×10^{-11} weber (2) 6×10^{-11} weber (3) 3.3×10^{-11} weber (4) 6.6×10^{-9} weber
13. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open, (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which of the following statement is correct? [JEE Main-2013]

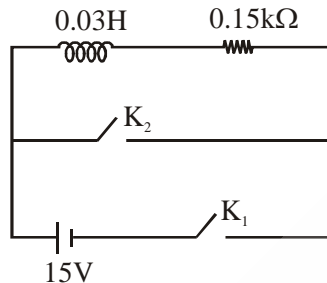


- (1) Work done by the battery is half of the energy dissipated in the resistor
 (2) At $t = \tau$, $q = CV/2$
 (3) At $t = 2\tau$, $q = CV(1 - e^{-2})$
 (4) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$
14. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to : [JEE Main-2014]

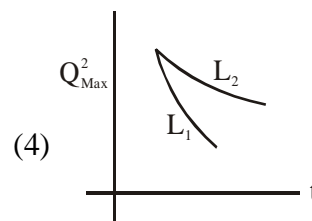
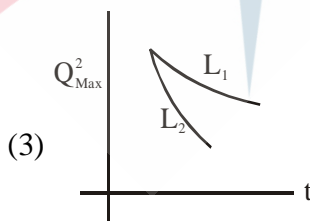
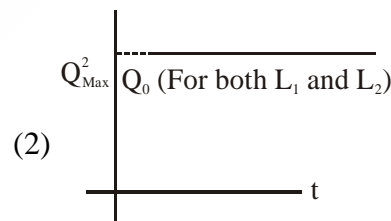
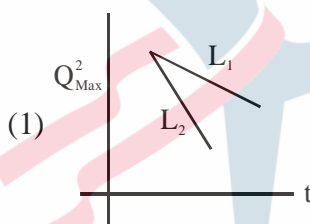
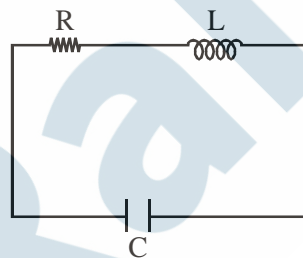


- (1) -1 (2) $\frac{1-e}{e}$ (3) $\frac{e}{1-e}$ (4) 1

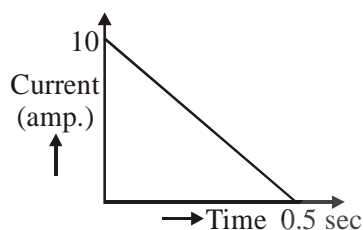
15. An inductor ($L = 0.03 \text{ H}$) and a resistor ($R = 0.15 \text{ k}\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1\text{ms}$, the current in the circuit will be ($e^5 \cong 150$):-
[JEE Main-2015]



- (1) 6.7 mA (2) 0.67 mA (3) 100 mA (4) 67 mA
16. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below. If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)
[JEE Main-2015]



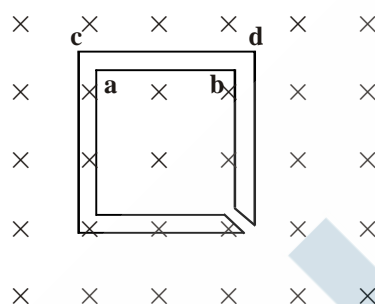
17. An arc lamp requires a direct current of 10A at 80V to function. If it is connected to a 220V (rms), 50Hz AC supply, the series inductor needed for it to work is close to :- **[JEE Main-2016]**
 (1) 0.065 H (2) 80 H (3) 0.08 H (4) 0.044 H
18. In a coil of resistance $100\ \Omega$, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :- **[JEE Main-2017]**



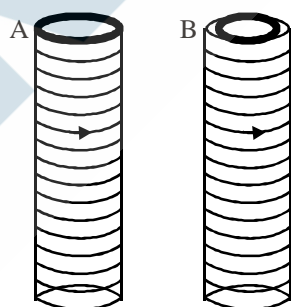
- (1) 250 Wb (2) 275 Wb (3) 200 Wb (4) 225 Wb
19. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by :- **[JEE Main-2018]**
 (1) $\frac{\omega_0 R}{L}$ (2) $\frac{R}{(\omega_0 C)}$ (3) $\frac{CR}{\omega_0}$ (4) $\frac{\omega_0 L}{R}$
20. In an a. c. circuit, the instantaneous e.m.f. and current are given by
 $e = 100 \sin 30 t$
 $i = 20 \sin \left(30t - \frac{\pi}{4} \right)$
 In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively. **[JEE Main-2018]**
 (1) $\frac{1000}{\sqrt{2}}, 10$ (2) $\frac{50}{\sqrt{2}}, 0$ (3) 50, 0 (4) 50, 10

EXERCISE - JA

1. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then, [JEE-2009]



- (A) $I_1 > I_2$
 (B) $I_1 < I_2$
 (C) I_1 is in the direction **ba** and I_2 is in the direction **cd**
 (D) I_1 is in the direction **ab** and I_2 is in the direction **dc**
2. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B , are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B , respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are) [JEE-2009]



- (A) $\rho_A > \rho_B$ and $m_A = m_B$
 (B) $\rho_A < \rho_B$ and $m_A = m_B$
 (C) $\rho_A > \rho_B$ and $m_A > m_B$
 (D) $\rho_A < \rho_B$ and $m_A < m_B$
3. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased [JEE 2010]
- (A) the bulb glows dimmer
 (B) the bulb glows brighter
 (C) total impedance of the circuit is unchanged
 (D) total impedance of the circuit increases

4. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is :-

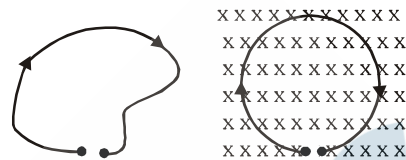
[JEE 2010]

(A) IBL

(B) $\frac{IBL}{\pi}$

(C) $\frac{IBL}{2\pi}$

(D) $\frac{IBL}{4\pi}$



5. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in **Column I**. Match the two

[JEE 2010]

Column I

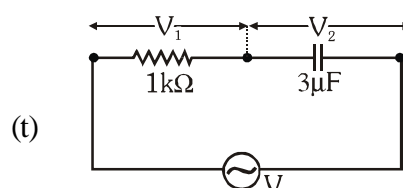
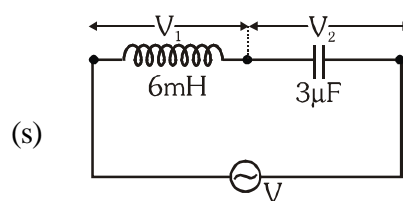
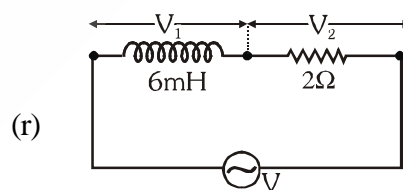
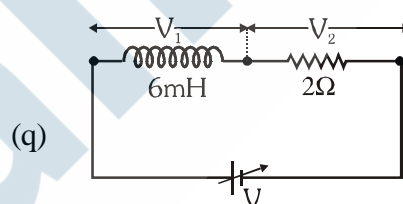
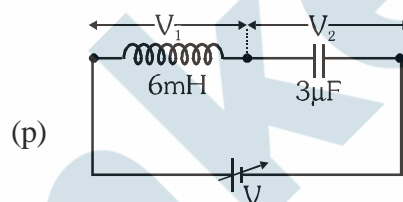
(A) $I \neq 0$, V_1 is proportional to I

(B) $I \neq 0$, $V_2 > V_1$

(C) $V_1 = 0$, $V_2 = V$

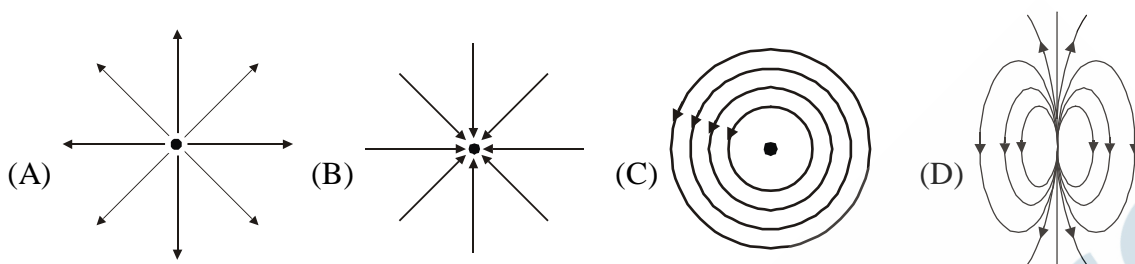
(D) $I \neq 0$, V_2 is proportional to I

Column II



6. Which of the field patterns given below is valid for electric field as well as for magnetic field?

[JEE 2011]



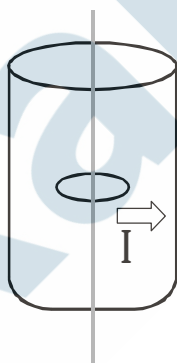
7. A series R-C circuit is connected to AC voltage source. Consider two cases ; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

[JEE 2011]

- (A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$ (C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

8. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then 'N' is

[JEE 2011]



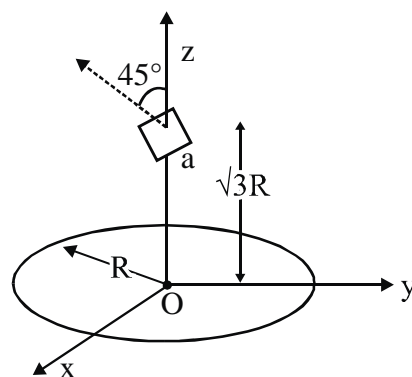
9. A series R-C combination is connected to an AC voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is :-

[JEE 2011]

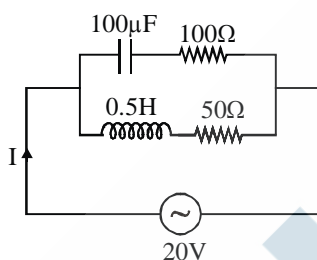
10. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side a ($a \ll R$) having two turns is placed with its centre at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance

between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is :-

[JEE 2012]



11. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are) [JEE 2012]
- (A) The emf induced in the loop is zero if the current is constant.
 (B) The emf induced in the loop is infinite if the current is constant.
 (C) The emf induced in the loop is zero if the current decreases at a steady rate.
 (D) The emf induced in the loop is finite if the current decreases at a steady rate.
12. In the given circuit, the AC source has $\omega = 100 \text{ rad/s}$. Considering the inductor and capacitor to be ideal, the correct choice (s) is(are) [JEE 2012]



- (A) The current through the circuit, I is 0.3 A.
 (B) The current through the circuit, i is $0.3\sqrt{2} \text{ A}$.
 (C) The voltage across 100Ω resistor = $10\sqrt{2} \text{ V}$.
 (D) The voltage across 50Ω resistor = 10V.

Paragraph for Questions 13 and 14

A point charge Q is moving in a circular orbit of radius R in the x - y plane with an angular velocity ω .

This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z -axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ .

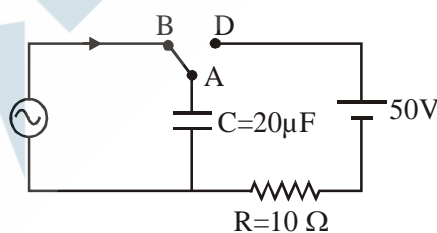
13. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change is [JEE Advance-2013]
- (A) $-\gamma BQR^2$ (B) $-\gamma \frac{BQR^2}{2}$ (C) $\gamma \frac{BQR^2}{2}$ (D) γBQR^2
14. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is
- (A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$ (C) BR (D) $2BR$

Paragraph for Questions 15 and 16

A thermal power plant produces electric power of 600 kW and 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values. **[JEE Advance-2013]**

15. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is
(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1
16. If the direct transmission method with a cable of resistance $0.4 \Omega \text{ km}^{-1}$ is used, the power dissipation (in %) during transmission is
(A) 20 (B) 30 (C) 40 (D) 50
17. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1 \text{ A}$ and $\omega = 500 \text{ rad s}^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20 \mu\text{F}$, $R = 10 \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement (s).

[JEE Advance-2014]

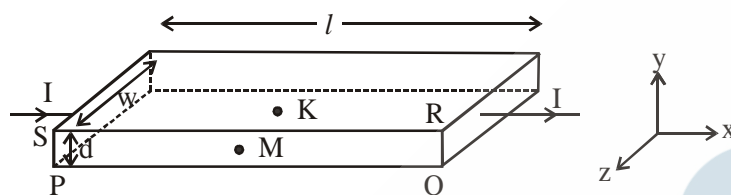


- (A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1 \times 10^{-3} \text{ C}$.
- (B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.
- (C) Immediately after A is connected to D, the current in R is 10A
- (D) $Q = 2 \times 10^{-3} \text{ C}$

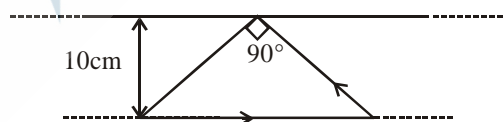
Paragraph for Question No. 18 and 19

In a thin rectangular metallic strip a constant current I flows along the positive x -direction, as shown in the figure. The length, width and the thickness of the strip are l , w and d , respectively.

A uniform magnetic field \vec{B} is applied on the strip along the positive y -direction. Due to this, the charge carriers experience a net deflection along the z -direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the z -direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons. [JEE Advance-2015]

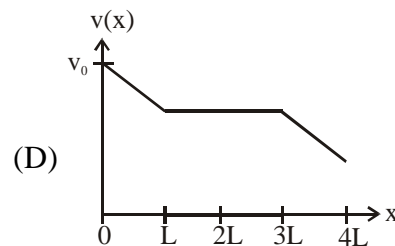
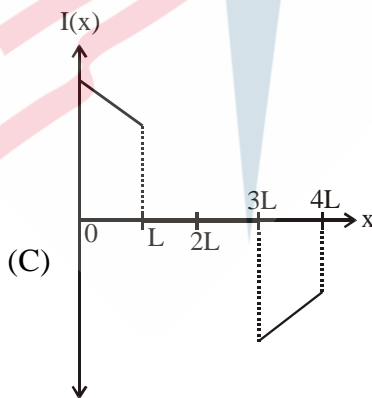
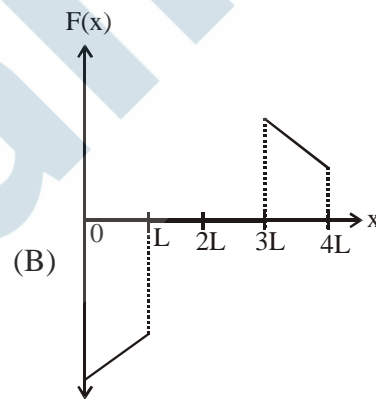
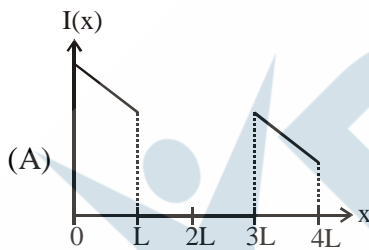
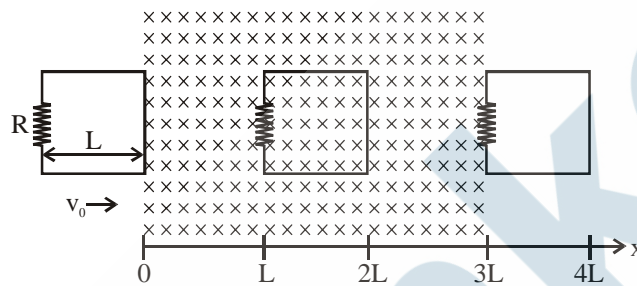


18. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2 , respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x - y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B , the correct statement(s) is(are)
- (A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$ (B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$
 (C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$ (D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$
19. Consider two different metallic strips (1 and 2) of same dimensions (length l , width w and thickness d) with carrier densities n_1 and n_2 , respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2 , both along positive y -direction. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)
- (A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$ (B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$
 (C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$ (D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$
20. A conducting loop in the shape of right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at constant rate of 10 A s^{-1} . Which of the following statement(s) is(are) true? [JEE Advance-2016]

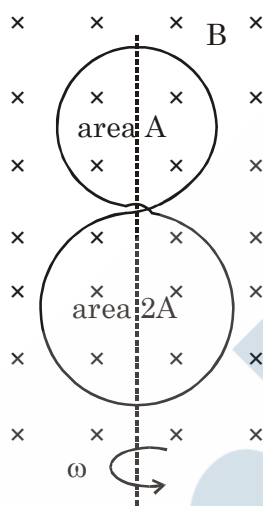


- (A) The induced current in the wire is in opposite direction to the current along the hypotenuse.
 (B) There is a repulsive force between the wire and the loop
 (C) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire
 (D) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

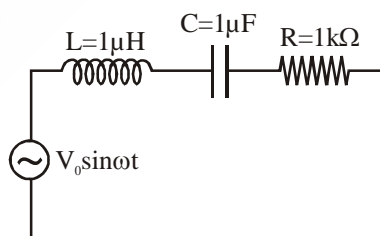
21. Two inductors L_1 (inductance 1 mH, internal resistance $3\ \Omega$) and L_2 (inductance 2mH, internal resistance $4\ \Omega$), and a resistor R (resistance $12\ \Omega$) are all connected in parallel across a 5V battery. The circuit is switched on at time $t = 0$. The ratio of the maximum to the minimum current (I_{\max}/I_{\min}) drawn from the battery is. **[JEE Advance-2016]**
22. A rigid wire loop of square shape having side of length L and resistance R is moving along the x -axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters a region of length $3L$ where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0 , the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x . Counter-clockwise current is taken as positive. Which of the following schematic plot(s) is(are) correct ? (Ignore gravity) **[JEE Advance-2016]**



23. A circular insulated copper wire loop is twisted to form two loops of area A and $2A$ as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field \vec{B} points into the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct? [JEE Advance-2017]



- (A) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper
 (B) The net emf induced due to both the loops is proportional to $\cos \omega t$
 (C) The emf induced in the loop is proportional to the sum of the areas of the two loops
 (D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone
24. In the circuit shown, $L = 1 \mu\text{H}$, $C = 1 \mu\text{F}$ and $R = 1 \text{ k}\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct ? [JEE Advance-2017]



- (A) The frequency at which the current will be in phase with the voltage is independent of R .
 (B) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero
 (C) At $\omega \gg 10^6 \text{ rad.s}^{-1}$, the circuit behaves like a capacitor.
 (D) The current will be in phase with the voltage if $\omega = 10^4 \text{ rad.s}^{-1}$.

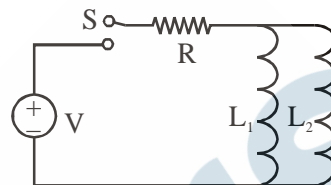
25. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct? **[JEE Advance-2017]**

(A) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)

(B) After a long time, the current through L_1 will be $\frac{V}{R} \frac{L_2}{L_1 + L_2}$

(C) After a long time, the current through L_2 will be $\frac{V}{R} \frac{L_1}{L_1 + L_2}$

(D) At $t = 0$, the current through the resistance R is $\frac{V}{R}$



26. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t$$

$$V_Y = V_0 \sin \left(\omega t + \frac{2\pi}{3} \right) \text{ and}$$

$$V_Z = V_0 \sin \left(\omega t + \frac{4\pi}{3} \right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be:- **[JEE Advance-2017]**

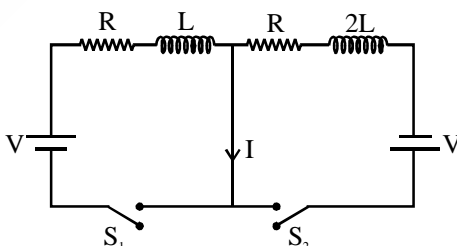
(A) $V_{XY}^{rms} = V_0$

(B) $V_{YZ}^{rms} = V_0 \sqrt{\frac{1}{2}}$

(C) Independent of the choice of the two terminals

(D) $V_{XY}^{rms} = V_0 \sqrt{\frac{3}{2}}$

27. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statement(s) is (are) true? **[JEE Advance-2018]**



(A) $I_{max} = \frac{V}{2R}$

(B) $I_{max} = \frac{V}{4R}$

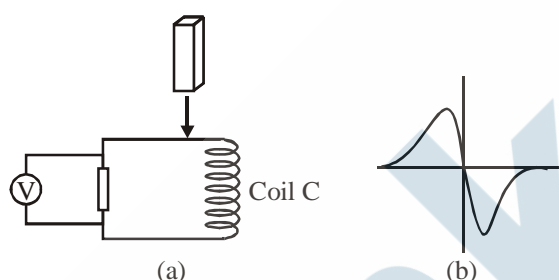
(C) $\tau = \frac{L}{R} \ln 2$

(D) $\tau = \frac{2L}{R} \ln 2$

ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

CBSE Previous Year's Questions

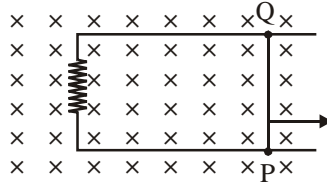
1. A solenoid with an iron core and a bulb are connected to a dc. source. How does the brightness of the bulb change, when the imp core is removed from the solenoid? [1; CBSE-2004]
2. Peak value of emf of an a.c. source is E_0 . What is its r.m.s. value? [1; CBSE-2004]
3. A bar magnet M is dropped so that it falls vertically through the coil C. The graph obtained for voltage produced across the coil vs time is shown in figure (b).
(i) Explain the shape of the graph.
(ii) Why is the negative peak longer than the positive peak? [2; CBSE-2004]



4. What is induced emf? Write Faraday's law of electromagnetic induction Express it mathematically. A conducting rod of length ℓ with one end pivoted is rotated with a uniform angular speed in a vertical plane, normal to a uniform magnetic field 'B'. Deduce an expression for the emf induced in this rod. In India, domestic power supply is at 220 V, 50 Hz, while in USA it is 110 V, 50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply. [5; CBSE-2004]
5. A bulb and a capacitor are connected in series to an a.c. source of variable frequency. How will the brightness of the bulb change on increasing the frequency of the a.c. source? Give reason. [1; CBSE-2005]
6. A circular coil of radius 8 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 s^{-1} in a uniform horizontal magnetic field of magnitude $3 \times 10^{-2} \text{ T}$. Find the maximum and average value of the emf induced in the coil. [2; CBSE-2005]
7. State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing variation of current with frequency of a.c. source in a series LCR circuit [2; CBSE-2005]
8. Define self-inductance and give its S. I. unit. Derive an expression for self- inductance of a long, air-cored solenoid of length ℓ , radius r , and having N number of turns. [3; CBSE-2005]
9. An alternating voltage of frequency f is applied across a series LCR circuit. Let f_r be the resonance frequency for the circuit. Will the current in the circuit lag, lead or remain in phase with the applied voltage when (i) $f > f_r$, (ii) $f < f_r$? Explain your answer in each case. [2; CBSE-2006]
10. When an inductor L and a resistor R in series are connected across a 12 V, 50Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by $\pi/3$ radian. Calculate the value of R . [3; CBSE-2006]

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11. A 0.5 long metal rod PQ completes the circuit as shown in the figure. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the total circuit is 3Ω , calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 ms^{-1} . [3; CBSE-2006]



12. What are eddy currents. How are these produced? in what sense are eddy currents considered undesirable in a transformer and how are these reduced in such a device? [3; CBSE-2006]
13. In a series LCR circuit, the voltages across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is the phase difference between the applied voltage and the current in the circuit ?
14. Calculate the current drawn by the primary of a transformer which steps down 200 V to 20 V to operate a device of resistance 20Ω . Assume the efficiency of the transformer to be 80%. [1; CBSE-2007]
15. An a.c. voltage of 100 V, 50 Hz is connected across a 20 ohm resistor and mH inductor in series. Calculate (i) impedance of the circuit, (ii) rms current in the circuit. [2; CBSE-2007]
16. Explain the term 'inductive reactance'. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage. An a.c. voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance L. Show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\pi/2$. [3; CBSE-2007]
17. Explain the term 'capacitive reactance'. Show graphically the variation of capacitive reactance with frequency of the applied alternating voltage. An a.c. voltage $E = E_0 \sin \omega t$ is applied across a pure capacitor of capacitance C. Show mathematically that the current flowing through it leads the applied voltage by a phase angle of $\pi/2$. [3; CBSE-2007]
18. Prove that an ideal capacitor, in an a. c. circuit does not dissipate power. [2; CBSE-2008]
19. Derive an expression for the impedance of a.c. circuit consisting of an inductor and a resistor. [2; CBSE-2008]
20. A metallic rod of length ℓ is rotated at a constant angular speed ω , normal to a uniform magnetic field B. Derive an expression for the current induced in the rod, if the resistance of the rod is R. [3; CBSE-2008]
21. An inductor 200mH, capacitor 500 μF , resistor 10Ω are connected in series with a 100 V, variable frequency ac. source. Calculate the [3; CBSE-2008]
(i) frequency at which the power factor of the circuit is unity
(ii) current amplitude at this frequency
(iii) Q-factor
22. (a) Define self inductance. Write is S.I, units.
(b) Derive an expression for self inductance of a long solenoid of length ℓ , cross-sectional area A having N number of turns. [3; CBSE-2009]

23. (a) Derive an expression for the average power consumed in a series LCR circuit connected to a.c. source, in which the phase difference between the voltage and the current in the circuit is ϕ . (b) Define the quality factor in an ac. circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends. [5; CBSE-2009]

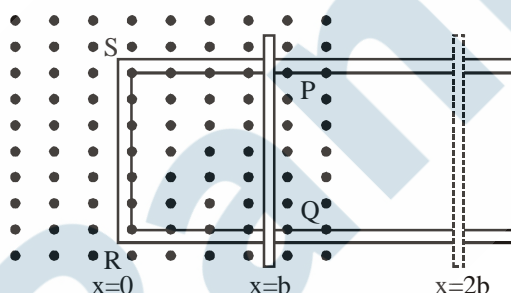
24. (a) Derive the relationship between the peak, and the rms value of current in an ac. circuit, (b) Describe briefly, with the help of a labeled diagram, working of a step - up transformer. A step - up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy? Explain.

25. Define self-inductance of a coil. Write its S.I. units. [1; CBSE-2010]

26. Two identical loops, one of copper and the other of aluminium, are rotated with the same angular speed in the same magnetic field. Compare (i) the induced emf and (ii) the current produced in the two coils. Justify your answer. [2; CBSE-2010]

27. State Faraday's law of electromagnetic induction.

Figure shows a rectangular conductor PQRS in which the conductor PQ is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from $x = 0$ to $x = b$ and is zero for $x > b$. Assume that only the arm PQ possesses resistance r . When the arm PQ is pulled outward from $x=0$ with constant speed v , obtain the expressions for the flux and the induced emf. Sketch the variations of these quantities with distance $0 \leq X \leq 2b$. [5; CBSE-2010]



28. Draw a schematic diagram of a step-up transformer. Explain its working principle. Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this ratio related to the currents in the two coils? How is the transformer used in large scale transmission and distribution of electrical energy over long distances? [5; CBSE-2010]

29. What are eddy currents? Write any two applications of eddy currents. [2; CBSE-2011]

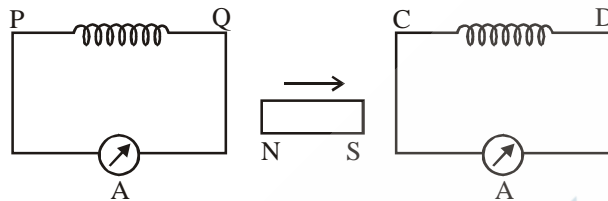
30. State the working of a.c. generator with the help of a labelled diagram. The coil of an a.c. generator having N turns, each of area A , is rotated with a constant angular velocity ω . Deduce the expression for the alternating e.m.f. generated in the coil. What is the source of energy generation in this device? [5; CBSE-2011]

31. Two bar magnetic are quickly moved towards a metallic loop connected across a capacitor 'C' as shown in the figure. Predict the polarity of the capacitor. [1; CBSE-2011]



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32. (a) Show that in an a.c. circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase.
(b) A horizontal straight wire of length L extending from east to west is falling with speed v at right angles to the horizontal component of Earth's magnetic field B .
(i) Write the expression for the instantaneous value of the e.m.f. induced in the wire,
(ii) What is the direction of the e.m.f. ?
(iii) Which end of the wire is at the higher potential? [5; CBSE-2011]
33. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil. [1; CBSE-2012]

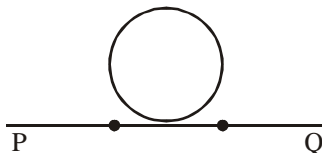


34. Mention the two characteristic properties of the material suitable for making core of a transformer. [1; CBSE-2012]
35. State the underlying principle of a transformer. How is the large scale transmission of electric energy over a long distances done with the use of transformers? [2; CBSE-2012]
36. A light bulb is rated 100 W for 220 V ac supply of 50 Hz. Calculate [2; CBSE-2012]
(i) The resistance of the bulb (ii) The rms current through the bulb

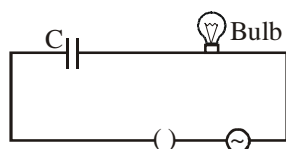
OR

An alternative voltage given by $V = 140 \sin 314t$ is connected across a pure resistor of 50Ω . Find

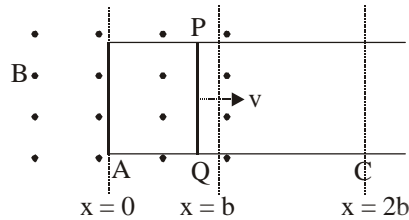
- (i) the frequency of the source (ii) the rms current through the resistor
37. A series LCR circuit is connected to an ac source. Using the phasor diagram, derive the expression for the source, explaining the nature of its variation. [3; CBSE-2012]
38. How does the mutual inductance of a pair of coils change when [CBSE-2013]
(i) distance between the coils is increased and
(ii) number of turns in the coils is increased?
39. The motion of copper plate is damped when it is allowed to oscillate between the two poles of a magnet. What is the cause of this damping? [CBSE-2013]
40. (a) For a given a.c, $i = i_m \sin \omega t$, show that the average power dissipated in a resistor R over a complete cycle is $\frac{1}{2} i_m^2 R$ (CBSE-2013)
- (b) A light bulb is rated at 100 W for a 220V a.c. supply. Calculate the resistance of the bulb.
41. A conducting loop is held above a current carrying wire 'PQ' as shown in the figure. Depict the direction of the current induced in the loop when the current in the wire PQ is constantly increasing. [CBSE-2014]



42. Why is the use of ac. voltage preferred over dc. voltage ? Give two reasons. [CBSE-2014]
43. A voltage $V = V_0 \sin \omega t$ is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle. [CBSE-2014]
Under what condition is (i) no power dissipated even though the current flows through the circuit, (ii) maximum power dissipated in the circuit ?
44. Define the term self-inductance of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance L to build up a current I through it. [CBSE-2014]
45. A planar loop of rectangular shape is moved within the region of a uniform magnetic field acting perpendicular to its plane. What is the direction and magnitude of the current induced in it ? [1; CBSE-2015]
46. Sunita and her friends visited an exhibition. The policeman asked them to pass through a metal detector. Sunita's friends were initially scared of it. Sunita, however, explained to them the purpose and working of the metal detector. [4; CBSE-2015]
Answer the following questions :
- On what principle does a metal detector work ?
 - Why does the detector emit sound when a person carrying any metallic object walks through it ?
 - State any two qualities which Sunita displayed while explaining the purpose of walking through the detector.
47. (a) State Faraday's law of electromagnetic induction. [5; CBSE-2015]
(b) Explain, with the help of a suitable example, how we can show that Lenz's law is a consequence of the principle of conservation of energy.
(c) Use the expression for Lorentz force acting on the charge carriers of a conductor to obtain the expression for the induced emf across the conductor of length l moving with velocity v through a magnetic field B acting perpendicular to its length.
- OR**
- (a) Using phasor diagram, derive the expression for the current flowing in an ideal inductor connected to an a.c. source of voltage, $v = v_0 \sin \omega t$. Hence plot graphs showing variation of (i) applied voltage and (ii) the current as a function of ωt .
(b) Derive an expression for the average power dissipated in a series LCR circuit.
48. (i) When an AC source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero. [3; CBSE-2016]
(ii) A bulb is connected in series with a variable capacitor and an A.C. source as shown. What happens to the brightness of the bulb when the key is plugged in and capacitance of the capacitor is gradually reduced ?



49. (b) Sketch the change in flux, emf and force when a conducting rod PQ of resistance R and length ℓ moves freely to and fro between A and C with speed v on a rectangular conductor placed in uniform magnetic field as shown in the figure. [5; CBSE-2016]

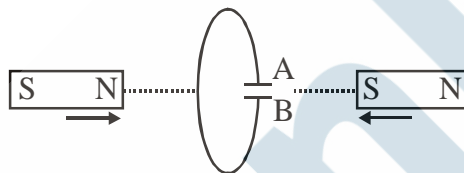


OR

In a series LCR circuit connected to an a.c. source of voltage $v = v_m \sin \omega t$, use phasor diagram to derive an expression for the current in the circuit.

Hence obtain the expression for the power dissipated in the circuit. Show that power dissipated at resonance is maximum.

50. Predict the polarity of the capacitor in the situation described below : [2; CBSE-2017]

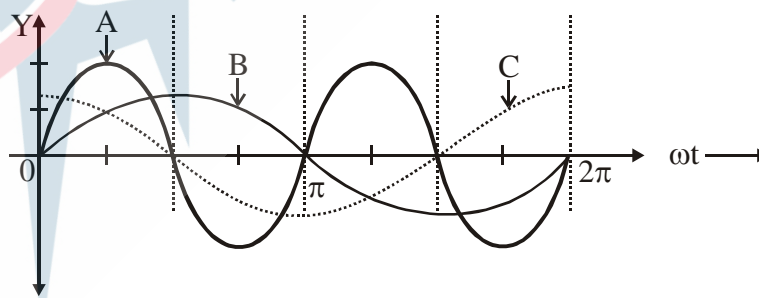


51. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. [3; CBSE-2017]

OR

Define self-inductance of a coil. Obtain the expression for the energy stored in an inductor L connected across a source of emf.

52. A device 'X' is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph : [5; CBSE-2017]



- Identify the device 'X'.
- Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.
- How does its impedance vary with frequency of the ac source? Show graphically.
- Obtain an expression for the current in the circuit and its phase relation with ac voltage.

OR

- (a) Draw a labelled diagram of an ac generator. Obtain the expression for the emf induced in the rotating coil of N turns each of cross-sectional area A , in the presence of a magnetic field \vec{B} .
- (b) A horizontal conducting rod 10 m long extending from east to west is falling with a speed 5.0 ms^{-1} at right angles to the horizontal component of the Earth's magnetic field, $0.3 \times 10^{-4} \text{ Wb m}^{-2}$. Find the instantaneous value of the emf induced in the rod.
- 53.** The teachers of Geeta's school took the students on a study trip to a power generating station, located nearly 200 km away from the city. The teacher explained that electrical energy is transmitted over such a long distance to their city, in the form of alternating current (ac) raised to a high voltage. At the receiving end in the city, the voltage is reduced to operate the devices. As a result, the power loss is reduced. Geeta listened to the teacher and asked questions about how the ac is converted to a higher or lower voltage.
- (a) Name the device used to change the alternating voltage to a higher or lower value. State one cause for power dissipation in this device.
- (b) Explain with an example, how power loss is reduced if the energy is transmitted over long distances as an alternating current rather than a direct current.
- (c) Write two values each shown by the teachers and Geeta. **[4; CBSE-2018]**
- 54.** (a) State the principle of an ac generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A , rotating with a constant angular speed ' ω ' in a magnetic field \vec{B} , directed perpendicular to the axis of rotation.
- (b) An aeroplane is flying horizontally from west to east with a velocity of 900 km/hour. Calculate the potential difference developed between the ends of its wings having a span of 20 m. The horizontal component of the Earth's magnetic field is $5 \times 10^{-4} \text{ T}$ and the angle of dip is 30° .

OR

A device X is connected across an ac source of voltage $V = V_0 \sin \omega t$. the current through X is given

as
$$I = I_0 \sin \left(\omega t + \frac{\pi}{2} \right).$$

- (a) Identify the device X and write the expression for its reactance.
- (b) Draw graphs showing variation of voltage and current with time over one cycle of ac, for X .
- (c) How does the reactance of the device X vary with frequency of the ac? Show this variation graphically.
- (d) Draw the phasor diagram for the device X . **[5; CBSE-2018]**

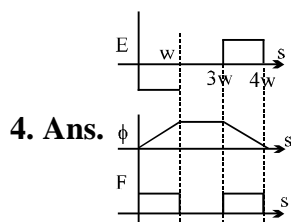
ANSWER KEY

EXERCISE (S-1)

1. Ans. $\lambda V_y B_0$

2. Ans. (i) $2.4 \times 10^{-5} \text{ V}$ (ii) from c to b

3. Ans. 2 N



5. Ans. $\frac{mgR}{B^2 \ell^2}$

6. Ans. 0.75 T

7. Ans. $\frac{erk}{2m}$ directed along tangent to the circle of radius r , whose centre lies on the axis of cylinder.

8. Ans. 6.5 V

9. Ans. $\frac{l}{2} \frac{dB}{dt} \sqrt{R^2 - \frac{l^2}{4}}$

10. Ans. 0.8

11. Ans. $\frac{\mu_0 i a^2 \pi}{2Rb}$

12. Ans. I^{-1}

13. Ans. $\frac{LE^2}{2R_1^2}$

14. Ans. $\frac{e^2}{e^2 - 1}$

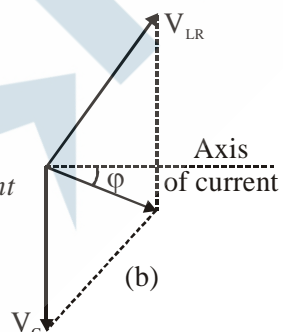
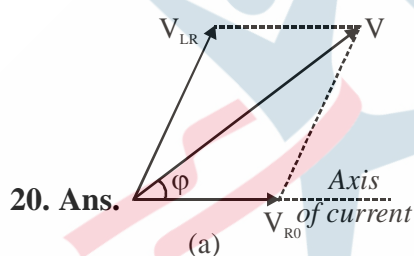
15. Ans. $\frac{EL}{eR^2}$

16. Ans. $q = Q_0 \sin\left(\sqrt{\frac{1}{LC}} t + \frac{\pi}{2}\right)$

17. Ans. (a) 10^4 A/s (b) 0 (c) 2 A (d) $100\sqrt{3} \mu\text{C}$

18. Ans. 30 Wb.

19. Ans. $\varepsilon = 1.7 \times 10^{-5} \text{ V}$



21. Ans. 0.08 H, 17.28 W

22. Ans. 2 A, 400 W

23. Ans. 0.2 mH, $\frac{1}{32} \mu\text{F}$, $8 \times 10^5 \text{ rad/s}$

24. Ans. $\frac{20}{\pi^2} \cong 2 \text{ H}$

25. Ans. 20 V

26. Ans. 20 A, $\pi/4$,

EXERCISE (S-2)

1. Ans. (i) 85.22 Tm^2 ; (ii) 56.8 V ; (iii) linearly

2. Ans. $V = 1 \text{ ms}^{-1}$, $R_1 = 0.47 \Omega$, $R_2 = 0.30 \Omega$

3. Ans. $\frac{\mu_0 h \omega_i m N}{2\pi} \ln \frac{b}{a}$

4. Ans. $\frac{1}{3} A$

5. Ans. $C \pi a^2 / R$

6. Ans. 200 rad/sec

7. Ans. $-\frac{B \pi a^2 \lambda}{MR} \hat{k}$

8. Ans. 0.4 V

9. Ans. $\frac{E R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$

10. Ans. $I_{EA} = \frac{7}{22} A$; $I_{BE} = \frac{3}{11} A$; $I_{FE} = \frac{1}{22} A$

11. Ans. (a) $i = \frac{B_0 a v}{R}$ in anticlockwise direction, $v = \text{velocity at time } t$, (b) $F_{\text{net}} = B_0^2 a^2 v / R$,

$$(c) V = \frac{mgR}{B_0^2 a^2} \left(1 - e^{-\frac{B_0^2 a^2 t}{mR}} \right)$$

12. Ans. $67/32 A$

13. Ans. (i) $i_1 = i_2 = 10/3 A$, (ii) $i_1 = 50/11 A$; $i_2 = 30/11 A$, (iii) $i_1 = 0$, $i_2 = 20/11 A$, (iv) $i_1 = i_2 = 0$

14. Ans. $42 + 20t \text{ volt}$

15. Ans. (a) $E = \frac{1}{2} B \omega r^2$ (b) (i) $I = \frac{B \omega r^2 [1 - e^{-Rt/L}]}{2R}$, (ii) $\tau = \frac{mgr}{2} \cos \omega t + \frac{\omega B^2 r^4}{4R} (1 - e^{-Rt/L})$

16. Ans. $2\pi \frac{\sqrt{mL}}{lB}$, $g \frac{\sqrt{mL}}{lB}$

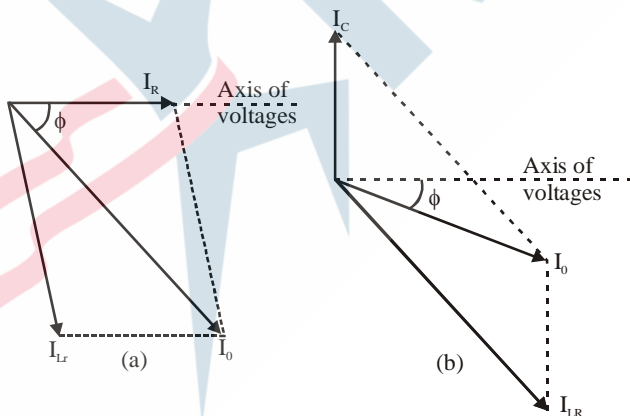
17. Ans. $-\frac{V}{R} e^{-\frac{Rt}{L}}$

18. Ans. $\frac{1}{15} A$, $\frac{1}{10} A$

19. Ans. $kMT^2/(R)$

20. Ans. 77Ω , 97.6Ω , 7.7 V , 9.76 V

21. Ans.



22. Ans. $I = \frac{(\mu_0 n i_0 \omega \cos \omega t) \pi a^2 (Ld)}{\rho 2\pi R}$

EXERCISE (O-1)

- | | | | | | |
|-----------------|--------------|-----------------|--------------|--------------|--------------------|
| 1. Ans. (C) | 2. Ans. (A) | 3. Ans. (D) | 4. Ans. (C) | 5. Ans. (A) | 6. Ans. (A) |
| 7. Ans. (B) | 8. Ans. (C) | 9. Ans. (A) | 10. Ans. (B) | 11. Ans. (A) | 12. Ans. (C) |
| 13. Ans. (A) | 14. Ans. (D) | 15. Ans. (C) | 16. Ans. (A) | 17. Ans. (A) | 18. Ans. (D) |
| 19. Ans. (D) | 20. Ans. (B) | 21. Ans. (D) | 22. Ans. (B) | 23. Ans. (D) | 24. Ans. (A) |
| 25. Ans. (C) | 26. Ans. (A) | 27. Ans. (C) | 28. Ans. (A) | 29. Ans. (B) | 30. Ans. (C) |
| 31. Ans. (A) | 32. Ans. (C) | 33. Ans. (B) | 34. Ans. (D) | 35. Ans. (B) | 36. Ans. (B) |
| 37. Ans. (A) | 38. Ans. (A) | 39. Ans. (A) | 40. Ans. (C) | 41. Ans. (B) | 42. Ans. (A) |
| 43. Ans. (B) | 44. Ans. (A) | 45. Ans. (D) | 46. Ans. (A) | 47. Ans. (D) | 48. Ans. (B) |
| 49. Ans. (C) | 50. Ans. (A) | 51. Ans. (B) | 52. Ans. (D) | 53. Ans. (D) | 54. Ans. (D) |
| 55. Ans. (D) | 56. Ans. (D) | 57. Ans. (A) | 58. Ans. (C) | 59. Ans. (B) | 60. Ans. (A) |
| 61. Ans. (B) | 62. Ans. (D) | 63. Ans. (C) | 64. Ans. (B) | 65. Ans. (B) | 66. Ans. (A) |
| 67. Ans. (D) | 68. Ans. (D) | 69. Ans. (C) | 70. Ans. (C) | 71. Ans. (B) | 72. Ans. (D) |
| 73. Ans. (A) | 74. Ans. (A) | 75. Ans. (A) | 76. Ans. (D) | 77. Ans. (B) | 78. Ans. (A) |
| 79. Ans. (B, C) | | 80. Ans. (A, B) | 81. Ans. (B) | 82. Ans. (A) | 83. Ans. (D) |
| 84. Ans. (B, D) | | 85. Ans. (B, D) | 86. Ans. (A) | 87. Ans. (B) | 88. Ans. (A, C, D) |
| 89. Ans. (A, C) | | 90. Ans. (D) | 91. Ans. (B) | 92. Ans. (C) | 93. Ans. (C) |
| 94. Ans. (C) | | 95. Ans. (B) | 96. Ans. (B) | | |

EXERCISE (O-2)

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|--------------------|--------------|-----------------|-----------------|--------------------|--------------|
| 1. Ans. (A) | 2. Ans. (B) | 3. Ans. (B) | 4. Ans. (A) | 5. Ans. (C) | 6. Ans. (D) |
| 7. Ans. (D) | 8. Ans. (A) | 9. Ans. (D) | 10. Ans. (A) | 11. Ans. (B) | 12. Ans. (A) |
| 13. Ans. (A) | 14. Ans. (C) | 15. Ans. (A) | 16. Ans. (C) | 17. Ans. (D) | 18. Ans. (A) |
| 19. Ans. (D) | 20. Ans. (D) | 21. Ans. (D) | 22. Ans. (B) | 23. Ans. (A) | 24. Ans. (D) |
| 25. Ans. (A, B, C) | | 26. Ans. (A, D) | 27. Ans. (A, B) | 28. Ans. (A, C, D) | |
| 29. Ans. (B, D) | | 30. Ans. (D) | 31. Ans. (B) | 32. Ans. (D) | 33. Ans. (B) |
| 34. Ans. (B, D) | | 35. Ans. (C) | 36. Ans. (D) | 37. Ans. (A) | 38. Ans. (B) |
| 39. Ans. (A) | | | | | |

EXERCISE-JM

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|--------------|--------------|--------------|--------------|--------------|--------------|
| 1. Ans. (2) | 2. Ans. (3) | 3. Ans. (3) | 4. Ans. (1) | 5. Ans. (2) | 6. Ans. (2) |
| 7. Ans. (4) | 8. Ans. (1) | 9. Ans. (2) | 10. Ans. (2) | 11. Ans. (4) | 12. Ans. (1) |
| 13. Ans. (3) | 14. Ans. (1) | 15. Ans. (2) | 16. Ans. (3) | 17. Ans. (1) | 18. Ans. (1) |
| 19. Ans. (4) | 20. Ans. (1) | | | | |

EXERCISE-JA

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|---|----------------|------------------------|-----------------|----------------------|-----------------|
| 1. Ans. (D) | 2. Ans. (B, D) | 3. Ans. (B) | 4. Ans. (C) | | |
| 5. Ans. (A)-R, S, T; (B)-Q, R, S, T; (C)-P, Q; (D)-Q, R, S, T | | | | 6. Ans. (C) | 7. Ans. (B, C) |
| 8. Ans. 6 | 9. Ans. 4 | 10. Ans. 7 | 11. Ans. (A, C) | 12. Ans. (C) or (AC) | |
| 13. Ans. (B) | 14. Ans. (B) | 15. Ans. (A) | 16. Ans. (B) | 17. Ans. (C, D) | 18. Ans. (A, D) |
| 19. Ans. (A, C) | | 20. Ans. (B, D) | 21. Ans. 8 | 22. Ans. (C, D) | 23. Ans. (A, D) |
| 24. Ans. (A, B) | | 25. Ans. (A), (B), (C) | | 26. Ans. (C), (D) | 27. Ans. (B, D) |