## ~齐Rankers

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## ELECTROSTATICS

## ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of charge. SI unit of charge : ampere $\times$ second i.e. Coulomb Dimension : [A T]
Practical units of charge are ampere $\times$ hour ( $=3600 \mathrm{C}$ ) and faraday (= 96500 C )

- Millikan calculated quanta of charge by 'Highest common factor' (H.C.F.) method and it is equal to charge of electron.
- $1 \mathrm{C}=3 \times 10^{9}$ state coulomb, 1 absolute - coulomb $=10 \mathrm{C}, 1$ Faraday $=96500 \mathrm{C}$.


## SPECIFIC PROPERTIES OF CHARGE

- Charge is a scalar quantity : It represents excess or deficiency of electrons.
- Charge is transferable : If a charged body is put in contact with an another body, then charge can be transferred to another body.
- Charge is always associated with mass

Charge cannot exist without mass though mass can exist without charge.

- So the presence of charge itself is a convincing proof of existence of mass.
- In charging, the mass of a body changes.
- When body is given positive charge, its mass decreases.
- When body is given negative charge, its mass increases.
- Charge is quantized

The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e . Thus charge q of a body is always given by

$$
\mathrm{q}=\mathrm{ne}
$$

$\mathrm{n}=$ positive integer or negative integer
The quantum of charge is the charge that an electron or proton carries.
Note : Charge on a proton $=(-)$ charge on an electron $=1.6 \times 10^{-19} \mathrm{C}$

- Charge is conserved

In an isolated system, total charge does not change with time, though individual charge may change i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.

- Charge is invariant

Charge is independent of frame of reference. i.e. charge on a body does not change whatever be its speed.

- Attraction - Repulsion

Similar charges repel each other while dissimilar attract

## METHODS OF CHARGING

- Friction: If we rub one body with other body, electrons are transferred from one body to the other.
- Electrostatic induction

If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'.

- Charging a body by induction (in four successive steps)

|  |  | uncharged body is disconnected from the earth step-3 |  |
| :---: | :---: | :---: | :---: |

Some important facts associated with induction-
(i) Inducing body neither gains nor loses charge
(ii) The nature of induced charge is always opposite to that of inducing charge
(iii) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

## - Conduction

The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

- The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)
- The charge gained by the uncharged body is always lesser than initial charge present on the charged body.


## COULOMB'S LAW

The electrostatic force of interaction between two static point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.
Force of electrostatic interaction depends on the nature of medium between the charges. If two points charges $q_{1}$ and $q_{2}$ separated by a distance $r$. Let $F$ be the electrostatic force between these two charges. According to Coulomb's law.

$$
\begin{aligned}
& \mathrm{F} \propto \mathrm{q}_{1} \mathrm{q}_{2} \text { and } \mathrm{F} \propto \frac{1}{\mathrm{r}^{2}} \\
& \mathrm{~F}_{\mathrm{e}}=\frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\mathrm{r}^{2}}
\end{aligned}
$$

where $\left[\mathrm{k}=\frac{1}{4 \pi \epsilon_{0}}=9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right] \mathrm{k}=$ coulomb's constant or electrostatic force constant

## Coulomb's law in vector form

$$
\begin{aligned}
& \overrightarrow{\mathrm{F}}_{12}=\text { force on } \mathrm{q}_{1} \text { due to } \mathrm{q}_{2}=\frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\mathrm{r}^{2}} \hat{\mathrm{r}}_{21} \\
& \overrightarrow{\mathrm{~F}}_{21}=\frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\mathrm{r}^{2}} \hat{\mathrm{r}}_{12} \text { (here } \hat{\mathrm{r}}_{12} \text { is unit vector from } \mathrm{q}_{1} \text { to } \mathrm{q}_{2} \text { ) }
\end{aligned}
$$



## Coulomb's law in terms of position vector

$$
\overrightarrow{\mathrm{F}}_{12}=\frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\left|\overrightarrow{\mathrm{r}}_{1}-\overrightarrow{\mathrm{r}}_{2}\right|^{3}}\left(\overrightarrow{\mathrm{r}}_{1}-\overrightarrow{\mathrm{r}}_{2}\right)
$$



## Principle of superposition

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually
$\mathrm{F}=\frac{\mathrm{kq}_{0} \mathrm{q}_{1}}{\mathrm{r}_{1}^{2}}+\frac{\mathrm{kq}_{0} \mathrm{q}_{2}}{\mathrm{r}_{2}^{2}}+\ldots+\frac{\mathrm{kq}_{0} \mathrm{q}_{\mathrm{i}}}{\mathrm{r}_{\mathrm{i}}^{2}}+\ldots \frac{\mathrm{kq}_{0} \mathrm{q}_{\mathrm{n}}}{\mathrm{r}_{\mathrm{n}}^{2}}$
in vector form $\vec{F}=\mathrm{kq}_{0} \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{q}_{\mathrm{i}}}{\mathrm{r}_{\mathrm{i}}^{2}} \hat{\mathrm{r}}_{\mathrm{i}}$

## Some important points regarding Coulomb's law and electric force

- The law is based on physical observations and is not logically derivable from any other concept. Experiments till today reveal its universal nature.
- The force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to number of point charges is the resultant of forces due to individual point charges, i.e., $\overrightarrow{\mathrm{F}}_{1}=\overrightarrow{\mathrm{F}}_{12}+\overrightarrow{\mathrm{F}}_{13}+\ldots$.
- The net Coulomb's force on two charged particles in free space and in a medium filled upto infinity are

$$
\mathrm{F}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}^{2}} \text { and } \mathrm{F}^{\prime}=\frac{1}{4 \pi \varepsilon} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}^{2}} \text {. So } \frac{\mathrm{F}}{\mathrm{~F}^{\prime}}=\frac{\varepsilon}{\varepsilon_{0}}=\mathrm{K},
$$

- Dielectric constant ( K ) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium filled upto infinity.
- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherical charged body, that too for external point.

Although net electric force on both particles change in the presence of dielectric but force due to one charge particle on another charge particle does not depend on the medium between them.

- Equilibrium of suspended point charge system

For equilibrium position
$\mathrm{T} \cos \theta=\mathrm{mg}$ and $\mathrm{T} \sin \theta=\mathrm{F}_{\mathrm{e}}=\frac{\mathrm{kQ}^{2}}{\mathrm{x}^{2}} \Rightarrow \tan \theta=\frac{\mathrm{F}_{\mathrm{e}}}{\mathrm{mg}}=\frac{\mathrm{kQ}^{2}}{\mathrm{x}^{2} \mathrm{mg}}$

- If $\theta$ is small then

$$
\begin{aligned}
& \tan \theta \approx \sin \theta=\frac{\mathrm{x}}{2 \ell} \Rightarrow \frac{\mathrm{x}}{2 \ell}=\frac{\mathrm{kQ}^{2}}{\mathrm{x}^{2} \mathrm{mg}} \Rightarrow \mathrm{x}^{3}=\frac{2 \mathrm{kQ} \mathrm{Q}^{2} \ell}{\mathrm{mg}} \Rightarrow \mathrm{x}=\left[\frac{\mathrm{Q}^{2} \ell}{2 \pi \epsilon_{0} \mathrm{mg}}\right. \\
& \text { - If whole set up is taken into an artificial satellite }\left(\mathrm{g}_{\text {eff }} \simeq 0\right) \mathrm{T}=\mathrm{F}_{\mathrm{e}}=\frac{\mathrm{kq}^{2}}{4 \ell^{2}}
\end{aligned}
$$



Ex. For the system shown in figure find Q for which resultant force on q is zero.
Sol. For force on $q$ to be zero, charges $q$ and Q must be of opposite of nature. Net attraction force on q due to charges $\mathrm{Q}=$ Repulsion force on q due to q $\sqrt{2} F_{A}=F_{R} \Rightarrow \sqrt{2} \frac{k Q q}{a^{2}}=\frac{k q^{2}}{(\sqrt{2} a)^{2}} \Rightarrow q=2 \sqrt{2}$ Q Hence $q=-2 \sqrt{2} Q$


Ex. Given a cube with point charges $q$ on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.
Sol. The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb's law

$$
\overrightarrow{\mathrm{F}}=\frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\left|\overrightarrow{\mathrm{r}}_{1}-\overrightarrow{\mathrm{r}}_{2}\right|^{3}}\left(\overrightarrow{\mathrm{r}}_{1}-\overrightarrow{\mathrm{r}}_{2}\right)
$$

From the figure the different forces acting on A are given as

$$
\overrightarrow{\mathrm{F}}_{\mathrm{A}_{1}}=\frac{\mathrm{kq}^{2}(-\mathrm{a} \hat{\mathrm{k}})}{\mathrm{a}^{3}}
$$

$$
\overrightarrow{\mathrm{F}}_{\mathrm{A}_{2}}=\frac{k q^{2}(-\mathrm{a} \hat{j}-\mathrm{a} \hat{\mathrm{k}})}{(\sqrt{2} \mathrm{a})^{3}}, \overrightarrow{\mathrm{~F}}_{\mathrm{A}_{3}}=\frac{k q^{2}(-\mathrm{a} \hat{\mathrm{i}}-\mathrm{a} \hat{\mathrm{j}}-\mathrm{a} \hat{\mathrm{k}})}{(\sqrt{3} \mathrm{a})^{3}},
$$


$\vec{F}_{A_{4}}=\frac{k q^{2}(-a \hat{j}-a \hat{k})}{(\sqrt{2} a)^{3}}, \vec{F}_{A_{5}}=\frac{k q^{2}(-a \hat{j})}{a^{3}}, \vec{F}_{A_{6}}=\frac{k q^{2}(-a \hat{i}-a \hat{j})}{(\sqrt{2} a)^{3}}, \vec{F}_{A_{7}}=\frac{k q^{2}(-a \hat{j})}{a^{3}}$
The net force experienced by A can be given as

$$
\overrightarrow{\mathrm{F}}_{\text {net }}=\overrightarrow{\mathrm{F}}_{\mathrm{A}_{1}}+\overrightarrow{\mathrm{F}}_{\mathrm{A}_{2}}+\overrightarrow{\mathrm{F}}_{\mathrm{A}_{3}}+\overrightarrow{\mathrm{F}}_{\mathrm{A}_{4}}+\overrightarrow{\mathrm{F}}_{\mathrm{A}_{5}}+\overrightarrow{\mathrm{F}}_{\mathrm{A}_{6}}+\overrightarrow{\mathrm{F}}_{\mathrm{A}_{7}}=\frac{-\mathrm{kq}^{2}}{\mathrm{a}^{2}}\left[\left(\frac{1}{3 \sqrt{3}}+\frac{1}{\sqrt{2}}+1\right)(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}})\right]
$$

## ELECTRIC FIELD

In order to explain 'action at a distance', i.e., 'force without contact' between charges it is assumed that a charge or charge distribution produces a field in space surrounding it. So the region surrounding a charge (or charge distribution) in which its electrical effects are perceptible is called the electric field of the given charge. Electric field at a point is characterized either by a vector function of position $\overrightarrow{\mathrm{E}}$ called 'electric intensity' or by a scalar function of position V called 'electric potential'. The electric field in a certain space is also visualized graphically in terms of 'lines of force.' So electric intensity, potential and lines of force are different ways of describing the same field.

## Intensity of electric field due to point charge

Electric field intensity is defined as force on unit test charge.

$$
\overrightarrow{\mathrm{E}}=\operatorname{Lim}_{\mathrm{q}_{0} \rightarrow 0} \frac{\overrightarrow{\mathrm{~F}}}{\mathrm{q}_{0}}=\frac{\mathrm{kq}}{\mathrm{r}^{2}} \hat{\mathrm{r}}=\frac{\mathrm{kq}}{\mathrm{r}^{3}} \overrightarrow{\mathrm{r}}
$$



Note : Test charge $\left(q_{0}\right)$ is a fictitious charge that exerts no force on nearby charges but experiences forces due to them.
Unit: N/C, V/m Dimensions: $\left[\mathrm{MLT}^{-3} \mathrm{~A}^{-1}\right]$

## Due to discrete distribution of charge

Field produced by a charge distribution for discrete distribution:-
By principle of superposition intensity of electric field due to ith charge $\overrightarrow{\mathrm{E}}_{\mathrm{ip}}=\frac{\mathrm{kq}}{\mathrm{r}_{\mathrm{i}}{ }^{3}} \overrightarrow{\mathrm{r}}_{\mathrm{i}}$
$\therefore$ Net electric field due to whole distribution of charge $\overrightarrow{\mathrm{E}}_{\mathrm{p}}=\sum_{\mathrm{i}=1} \overrightarrow{\mathrm{E}}_{\mathrm{i}}$


## Continuous distribution of charge

Treating a small element as particle $\overrightarrow{\mathrm{E}}=\frac{1}{4 \pi \epsilon_{0}} \int \frac{\mathrm{dq}}{\mathrm{r}^{3}} \overrightarrow{\mathrm{r}}$
Due to linear charge distribution $E=k \int_{\ell} \frac{\lambda \mathrm{d} \ell}{\mathrm{r}^{2}} \quad[\lambda=$ charge per unit length $]$
Due to surface charge distribution $\mathrm{E}=\mathrm{k} \int_{\mathrm{s}} \frac{\sigma \mathrm{ds}}{\mathrm{r}^{2}}$ [ $\sigma=$ charge per unit area]
Due to volume charge distribution $\mathrm{E}=\mathrm{k} \int_{\mathrm{v}} \frac{\rho \mathrm{dv}}{\mathrm{r}^{2}}$ [ $\rho=$ charge per unit volume ]

## Key points

- Charged particle in an electric field always experiences a force either it is at rest or in motion.
- In presence of a dielectric , electric field decreases and becomes $\frac{1}{\epsilon_{\mathrm{r}}}$ times of its value in free space.
- Test charge is always a unit (+ve) charge. $\overrightarrow{\mathrm{E}}=\frac{\overrightarrow{\mathrm{F}}_{\text {test }}}{\text { test charge }}$
- If identical charges are placed on each vertices of a regular polygon, then $\vec{E}$ at centre $=$ zero.


## Electric field strength at a general point due to a uniformly charged rod

As shown in figure, if P is any general point in the surrounding of rod, to find electric field strength at $P$, we consider an element on rod of length $d x$ at a distance $x$ from point $O$ as shown in figure. Now if $d E$ be the electric field at P due to the element, then
$\mathrm{dE}=\frac{\mathrm{kdq}}{\left(\mathrm{x}^{2}+\mathrm{r}^{2}\right)}$ Here $\mathrm{dq}=\frac{\mathrm{Q}}{\mathrm{L}} \mathrm{dx}$


Electric field strength in x -direction due to dq at P is
$\mathrm{dE}_{\mathrm{x}}=\mathrm{dE} \sin \theta=\left[\frac{\mathrm{kdq}}{\left(\mathrm{x}^{2}+\mathrm{r}^{2}\right)}\right] \sin \theta=\frac{\mathrm{kQ} \sin \theta}{\mathrm{L}\left(\mathrm{x}^{2}+\mathrm{r}^{2}\right)} \mathrm{dx}$
Here we have $x=r \tan \theta$ and $d x=r \sec ^{2} \theta d \theta$
Thus $\mathrm{dE}_{\mathrm{x}}=\frac{\mathrm{kQ}}{\mathrm{L}} \frac{\mathrm{r} \sec ^{2} \theta \mathrm{~d} \theta}{\mathrm{r}^{2} \sec ^{2} \theta} \sin \theta$, Strength $=\frac{\mathrm{kQ}}{\mathrm{Lr}} \sin \theta \mathrm{d} \theta$


$$
\mathrm{E}_{\mathrm{x}}=\int \mathrm{dE}_{\mathrm{x}}=\frac{\mathrm{kQ}}{\mathrm{Lr}} \int_{-\theta_{2}}^{+\theta} \sin \theta \mathrm{d} \theta=\frac{\mathrm{kQ}}{\mathrm{Lr}}[-\cos \theta]_{-\theta_{2}}^{+\theta_{1}}=\frac{\mathrm{kQ}}{\mathrm{Lr}}\left[\cos \theta_{2}-\cos \theta_{1}\right]
$$

Similarly, electric field strength at point P due to dq in y -direction is

$$
\mathrm{dE}_{\mathrm{y}}=\mathrm{dE} \cos \theta=\frac{\mathrm{kQdx}}{\mathrm{~L}\left(\mathrm{r}^{2}+\mathrm{x}^{2}\right)} \times \cos \theta
$$

Again we have $x=r \tan \theta$ and $d x=r \sec ^{2} \theta d \theta$.
Thus we have $\mathrm{dE}_{\mathrm{y}}=\frac{\mathrm{kQ}}{\mathrm{L}} \cos \theta \times \frac{\mathrm{r} \sec ^{2} \theta}{\mathrm{r}^{2} \sec ^{2} \theta} \mathrm{~d} \theta=\frac{\mathrm{kQ}}{\mathrm{Lr}} \cos \theta \mathrm{d} \theta$
Net electric field strength at P due to dq in y -direction is

$$
\mathrm{E}_{\mathrm{y}}=\int \mathrm{dE}_{\mathrm{y}}=\frac{\mathrm{kQ}}{\mathrm{Lr}} \int_{-\theta_{2}}^{+\theta_{1}} \cos \theta \mathrm{~d} \theta=\frac{\mathrm{kQ}}{\mathrm{Lr}}[+\sin \theta]_{-\theta_{2}}^{+\theta_{1}}=\frac{\mathrm{kQ}}{\mathrm{Lr}}\left[\sin \theta_{1}+\sin \theta_{2}\right]
$$

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles $\theta_{1}$ and $\theta_{2}$ at the two corners of rod can be given as
in x-direction : $\mathrm{E}_{\mathrm{x}}=\frac{\mathrm{kQ}}{\mathrm{Lr}}\left(\cos \theta_{2}-\cos \theta_{1}\right)$ and in y -direction $\mathrm{E}_{\mathrm{y}}=\frac{\mathrm{kQ}}{\mathrm{Lr}}\left(\sin \theta_{1}-\sin \theta_{2}\right)$

## Electric field due to a uniformly charged ring

## Case - I : At its centre

Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. The electric field strength at centre due to segment $A B$ is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is $\mathrm{E}_{0}=0$


Case II : At a point on the axis of Ring
Here we'll find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this we consider a small section of length $\mathrm{d} \ell$ on ring as shown. The charge on this elemental section is $\mathrm{dq}=\frac{\mathrm{Q}}{2 \pi \mathrm{R}} \mathrm{d} \ell[\mathrm{Q}=$ total charge of ring $]$


Due to the element $\mathrm{d} \ell$, electric field strength $d E$ at point $P$ can be given as $d E=\frac{K d q}{\left(\mathrm{R}^{2}+\mathrm{x}^{2}\right)}$
The component of this field strength $\mathrm{dE} \sin \alpha$ which is normal to the axis of ring will be cancelled out due to the ring section opposite to $\mathrm{d} \ell$. The component of electric field strength along the axis of ring $\mathrm{dE} \cos \alpha$ due to all the sections will be added up. Hence total electric field strength at point P due to the ring is

$$
\begin{aligned}
\mathrm{E}_{\mathrm{p}} & =\int \mathrm{dE} \cos \alpha=\int_{0}^{2 \pi \mathrm{R}} \frac{\mathrm{kdq}}{\left(\mathrm{R}^{2}+\mathrm{x}^{2}\right)} \times \frac{\mathrm{x}}{\sqrt{\mathrm{R}^{2}+\mathrm{x}^{2}}}=\int_{0}^{2 \pi \mathrm{R}} \frac{\mathrm{kQx}}{2 \pi \mathrm{R}\left(\mathrm{R}^{2}+\mathrm{x}^{2}\right)} \mathrm{d} \ell=\frac{\mathrm{kQx}}{2 \pi \mathrm{R}\left(\mathrm{R}^{2}+\mathrm{x}^{2}\right)^{3 / 2}} \int_{0}^{2 \pi \mathrm{R}} \mathrm{~d} \ell \\
& =\frac{\mathrm{kQ}_{\mathrm{x}}}{2 \pi \mathrm{R}\left(\mathrm{R}^{2}+\mathrm{x}^{2}\right)^{3 / 2}}[2 \pi \mathrm{R}]=\frac{\mathrm{kQx}}{\left(\mathrm{R}^{2}+\mathrm{x}^{2}\right)^{3 / 2}}
\end{aligned}
$$

## Electric field strength due to a charged circular arc at its centre :

Figure shows a circular arc of radius R which subtend an angle $\phi$ at its centre. To find electric field strength at C , we consider a polar segment on arc of angular width $\mathrm{d} \theta$ at an angle $\theta$ from the angular bisector XY as shown. The length of elemental segment is $R \mathrm{~d} \theta$, the charge on this element $\mathrm{d} \ell$ is $\mathrm{dq}=\left(\frac{\mathrm{Q}}{\phi}\right) \cdot \mathrm{d} \theta$

Due to this dq, electric field at centre of $\operatorname{arc} \mathrm{C}$ is given as $\mathrm{dE}=\frac{\mathrm{kdq}}{\mathrm{R}^{2}}$


Now electric field component due to this segment $\mathrm{dE} \sin \theta$ which is perpendicular to the angular bisector gets cancelled out in integration and net electric field at centre will be along angular bisector
which can be calculated by integrating dEcos $\theta$ within limits from $-\frac{\phi}{2}$ to $\frac{\phi}{2}$.
Hence net electric field strength at centre $C$ is $E_{C}=\int d E \cos \theta$

$$
=\int_{-\phi / 2}^{+\phi / 2} \frac{\mathrm{kQ}}{\phi \mathrm{R}^{2}} \cos \theta \mathrm{~d} \theta=\frac{\mathrm{kQ}}{\phi \mathrm{R}^{2}} \int_{-\phi / 2}^{+\phi / 2} \cos \theta \mathrm{~d} \theta=\frac{\mathrm{kQ}}{\phi \mathrm{R}^{2}}[\sin \theta]_{-\phi / 2}^{+\phi / 2}=\frac{\mathrm{kQ}}{\phi \mathrm{R}^{2}}\left[\sin \frac{\phi}{2}+\sin \frac{\phi}{2}\right]=\frac{2 \mathrm{kQ} \sin \left(\frac{\phi}{2}\right)}{\phi \mathrm{R}^{2}}
$$

## Electric field strength due to a uniformly surface charged disc :

If there is a disc of radius $R$, charged on its surface with surface charge density $\sigma$, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.
To find electric field at point P due to this disc, we consider an
 elemental ring of radius $y$ and width dy in the disc as shown in figure. The charge on this elemental ring $\mathrm{dq}=\sigma .2 \pi y \mathrm{dy}$ [Area of elemental ring $\mathrm{ds}=2 \pi \mathrm{y} \mathrm{dy}$ ]

Now we know that electric field strength due to a ring of radius $R$, charge $Q$, at a distance $x$ from its centre on its axis can be given as $E=\frac{k Q x}{\left(\mathrm{x}^{2}+\mathrm{R}^{2}\right)^{3 / 2}}$

Here due to the elemental ring electric field strength dE at point P can be given as

$$
\mathrm{dE}=\frac{\mathrm{kdqx}}{\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)^{3 / 2}}=\frac{\mathrm{k} \sigma 2 \pi \mathrm{ydyx}}{\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)^{3 / 2}}
$$

Net electric field at point P due to this disc is given by integrating above expression from 0 to R as $\mathrm{E}=\int \mathrm{dE}=\int_{0}^{\mathrm{R}} \frac{\mathrm{k} \sigma 2 \pi \mathrm{xydy}}{\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)^{3 / 2}}=\mathrm{k} \sigma \pi \mathrm{x} \int_{0}^{\mathrm{R}} \frac{2 \mathrm{ydy}}{\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)^{3 / 2}}=2 \mathrm{k} \sigma \pi \mathrm{x}\left[-\frac{1}{\sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}}}\right]_{0}^{\mathrm{R}}=\frac{\sigma}{2 \epsilon_{0}}\left[1-\frac{\mathrm{x}}{\sqrt{\mathrm{x}^{2}+\mathrm{R}^{2}}}\right]$

Ex. Calculate the electric field at origin due to infinite number of charges as shown in figures below.


Sol. (a) $\mathrm{E}_{0}=\mathrm{kq}\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+----\right]=\frac{\mathrm{kq} \cdot 1}{(1-1 / 4)}=\frac{4 \mathrm{kq}}{3}\left[\because \mathrm{~S}_{\infty}=\frac{\mathrm{a}}{1-\mathrm{r}}, \mathrm{a}=1\right.$ and $\left.\mathrm{r}=\frac{1}{4}\right]$
(b)

$$
\mathrm{E}_{0}=\mathrm{kq}\left[\frac{1}{1}-\frac{1}{4}+\frac{1}{16}-\cdots\right]=\frac{\mathrm{kq} \cdot 1}{(1-(-1 / 4))}=\frac{4 \mathrm{kq}}{5}
$$

## ELECTRIC LINES OF FORCE

Electric lines of electrostatic field have following properties
(i) Imaginary
(ii) Can never cross each other
(iii) Can never be closed loops

$q_{A}>q_{B}$
(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system $\left(1 / \varepsilon_{0}\right)$ electric lines are associated with unit charge, so if a body encloses a charge q , total lines of force associated with it (called flux) will be $\mathrm{q} / \varepsilon_{\mathrm{o}}$.
(v) If there is no electric field there will be no lines of force.
(vi) Lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field.
(vii)Tangent to the line of force at a point in an electric field gives the direction of intensity. So a positive charge free to move follow the line of force.

- Lines of force starts from (+ve) charge and ends on (-ve) charge.


## Electric flux ( $\phi$ )

The word "flux" comes from a Latin word meaning "to flow" and you can consider the flux of a vector field to be a measure of the flow through an imaginary fixed element of surface in the field.

Electric flux is defined as $\phi_{\mathrm{E}}=\int \overrightarrow{\mathrm{E}} \cdot \mathrm{d} \overrightarrow{\mathrm{A}}$
This surface integral indicates that the surface in question is to be divided into infinitesimal elements of area $d \vec{A}$ and the scalar quantity $\overrightarrow{\mathrm{E}} \cdot \mathrm{d} \overrightarrow{\mathrm{A}}$ is to be evaluated for each element and summed over the entire surface.

## Electric flux :

(i) It is a scalar quantity
(ii) Units (V-m) and $\mathrm{N}-\mathrm{m}^{2} / \mathrm{C}$

Dimensions: $\left[\mathrm{ML}^{3} \mathrm{~T}^{-3} \mathrm{~A}^{-1}\right]$
(iii) The value of $\phi$ does not depend upon the distribution of charges and the distance between them inside the closed surface.

## Electric Flux through a circular Disc :

Figure shows a point charge q placed at a distance $\ell$ from a disc of radius R. Here we wish to find the electric flux through the disc surface due to the point charge $q$. We know a point charge $q$ originates electric flux in radially outward direction. The flux is originated in cone shown in figure passes through
 the disc surface.
To calculate this flux, we consider on elemental ring an disc surface of radius x and width dx as shown. Area of this ring (strip) is $\mathrm{dS}=2 \pi \mathrm{x} \mathrm{dx}$. The electric field due to q at this elemental ring is
given as $\mathrm{E}=\frac{\mathrm{kq}}{\left(\mathrm{x}^{2}+\ell^{2}\right)}$
If $\mathrm{d} \phi$ is the flux passing through this elemental ring, then

$$
\begin{aligned}
\mathrm{d} \phi & =\operatorname{EdS} \cos \theta=\frac{\mathrm{kq}}{\left(\mathrm{x}^{2}+\ell^{2}\right)} \times 2 \pi \mathrm{x} \mathrm{dx} \times \frac{\ell}{\sqrt{\mathrm{x}^{2}+\ell^{2}}}=\frac{2 \pi \mathrm{kq} \ell \mathrm{xdx}}{\left(\ell^{2}+\mathrm{x}^{2}\right)^{3 / 2}} \\
\phi & =\int \mathrm{d} \phi=\int_{0}^{\mathrm{R}} \frac{\mathrm{q} \ell}{2 \epsilon_{0}} \frac{\mathrm{xdx}}{\left(\ell^{2}+\mathrm{x}^{2}\right)^{3 / 2}}=\frac{\mathrm{q} \ell}{2 \epsilon_{0}} \int_{0}^{\mathrm{R}} \frac{\mathrm{x} \mathrm{dx}}{\left(\ell^{2}+\mathrm{x}^{2}\right)^{3 / 2}} \xrightarrow[\mathrm{q}^{\prime} \theta]{2 \epsilon_{0}}\left[-\frac{1}{\sqrt{\ell^{2}+\mathrm{x}^{2}}}\right]_{0}^{\mathrm{R}}=\frac{\mathrm{q} \ell}{2 \epsilon_{0}}\left[\frac{1}{\ell}-\frac{1}{\sqrt{\ell^{2}+\mathrm{x}^{2}}}\right]
\end{aligned}
$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's law.

## GAUSS'S LAW

It relates with the total flux of an electric field through a closed surface to the net charge enclosed by that surface and according to it, the total flux linked with a closed surface is $\left(1 / \varepsilon_{0}\right)$ times the charge enclosed by the closed surface i.e., $\int_{\mathrm{S}} \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{ds}}=\frac{\mathrm{q}}{\epsilon_{0}}$


## Regarding Gauss's law it is worth noting that :

(1) Gauss's law and Coulomb's law are equivalent, i.e., if we assume Coulomb's law we can prove Gauss's law and vice-versa. To prove Gauss's law from Coulomb's law consider a hypothetical spherical surface [called Gaussian-surface] of radius $r$ with point charge $q$ at its centre as shown in figure. By Coulomb's law intensity at a point $P$ on the surface will be, $\vec{E}=\frac{1}{4 \pi \varepsilon_{0} r^{3}} \vec{r}$ And hence electric flux linked with area $\overrightarrow{\mathrm{ds}} \Rightarrow \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{ds}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{r}^{3}} \overrightarrow{\mathrm{r}} \cdot \overrightarrow{\mathrm{ds}}$

Here direction of $\overrightarrow{\mathrm{r}}$ and $\overrightarrow{\mathrm{ds}}$ are same, i.e., $\int_{S} \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{ds}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{r}^{2}} \oint_{\mathrm{S}} \mathrm{ds}=\frac{1}{4 \pi \varepsilon_{0}} \frac{1}{\mathrm{r}^{2}}\left(4 \pi \mathrm{r}^{2}\right) \oint_{\mathrm{S}} \cdot \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{ds}}=\frac{\mathrm{q}}{\epsilon_{0}}$
Which is the required result. Though here in proving it we have assumed the surface to be spherical, it is true for any arbitrary surface provided the surface is closed.
(a) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or nonuniform) total flux linked with it will be zero.

(A)

(B)
(b) If a closed body encloses a charge $q$, then total flux linked with the body will be $\oint_{S} \vec{E} \cdot \overrightarrow{d s}=\frac{q}{\epsilon_{0}}$
(A)

(B)

(C)

(D)


From this expression it is clear that the flux linked with a closed body is independent of the shape and size of the body and position of charge inside it.[figure]

Note : So in case of closed symmetrical body with charge at its centre, flux linked with each half will be $\frac{1}{2}\left(\phi_{\mathrm{E}}\right)=\left(\frac{\mathrm{q}}{2 \varepsilon_{0}}\right)$ and the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be $\left(\frac{\phi_{\mathrm{E}}}{\mathrm{n}}\right)=\left(\frac{\mathrm{q}}{\mathrm{n} \varepsilon_{0}}\right)$
Gauss's law is a powerful tool for calculating electric intensity in case of symmetrical charge distribution by choosing a Gaussian- surface in such a way that $\overrightarrow{\mathrm{E}}$ is either parallel or perpendicular to its various faces. As an example, consider the case of a plane sheet of charge having charge density $\sigma$. To calculate E at a point P close to it consider a Gaussian surface in the form of a 'pill box' of cross-section $S$ as shown in


Sheet of charge
(A)


Conductor
(B) figure.
The charge enclosed by the Gaussian-surface $=\sigma$ S and the flux linked with the pill box $=\mathrm{ES}+0+$ $\mathrm{ES}=2 \mathrm{ES}$ (as E is parallel to curved surface and perpendicular to plane faces)

So from Gauss's law, $\phi_{\mathrm{E}}=\frac{1}{\varepsilon_{0}}(\mathrm{q}), 2 \mathrm{ES}=\frac{1}{\varepsilon_{0}}(\sigma \mathrm{~S}) \Rightarrow \mathrm{E}=\frac{\sigma}{2 \varepsilon_{0}}$
If $\overrightarrow{\mathrm{E}}=0, \phi=\oint \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{ds}}=0$, so $q=0$ but if $q=0, \oint \overrightarrow{\mathrm{E}} \cdot \overrightarrow{\mathrm{ds}}=0$ So $\overrightarrow{\mathrm{E}}$ may or may not be zero.
If a dipole is enclosed by a closed surface then, $q=0$, so $\oint \vec{E} \cdot \overrightarrow{d s}=0$, but $\vec{E} \neq 0$
Note : If instead of plane sheet of charge, we have a charged conductor, then as shown in figure (B) $\mathrm{E}_{\mathrm{in}}=0$. So $\phi_{\mathrm{E}}=\mathrm{ES}$ and hence in this case $\mathrm{E}=\frac{\sigma}{\epsilon_{0}}$. This result can be verified from the fact that intensity at the surface of a charged spherical conductor of radius $R$ is, $E=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{R^{2}}$ with $\mathrm{q}=4 \pi \mathrm{R}^{2} \sigma$
So for a point close to the surface of conductor, $\mathrm{E}=\frac{1}{4 \pi \varepsilon_{0} \mathrm{R}^{2}} \times\left(4 \pi \mathrm{R}^{2} \sigma\right)=\frac{\sigma}{\varepsilon_{0}}$
Ex. If a point charge $q$ is placed at the centre of a cube.
What is the flux linked (a) with the cube? (b) with each face of the cube?
Sol. (a) According to Gauss's law flux linked with a closed body is $\left(1 / \varepsilon_{0}\right)$ times the charge enclosed and here the closed body cube is enclosing a charge q so, $\phi_{\mathrm{T}}=\frac{1}{\varepsilon_{0}}(\mathrm{q})$
(b) Now as cube is a symmetrical body with 6 -faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_{\mathrm{F}}=\frac{1}{6}\left(\phi_{\mathrm{T}}\right)=\frac{\mathrm{q}}{6 \varepsilon_{0}}$
Note: (i) Here flux linked with cube or one of its faces is independent of the side of cube.
(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

Ex. Consider $\overrightarrow{\mathrm{E}}=3 \times 10^{3} \hat{\mathrm{i}}(\mathrm{N} / \mathrm{C})$ then what is the flux through the square of 10 cm side, if the normal of its plane makes $60^{\circ}$ angle with the X axis.
Sol. $\phi=\mathrm{ES} \cos \theta=3 \times 10^{3} \times\left[10 \times 10^{-2}\right]^{2} \times \cos 60^{\circ}=3 \times 10^{3} \times 10^{-2} \times 1 / 2=15 \mathrm{Nm}^{2} / \mathrm{C}$
Ex. Find the electric field due to an infinitely long cylindrical charge distribution of radius R and having linear charge density $\lambda$ at a distance half of the radius from its axis.

Sol. $r=\frac{R}{2}$ point will be inside so $E=\frac{2 k \lambda r}{R^{2}}=\frac{2 k \lambda}{R^{2}}\left(\frac{R}{2}\right)=\frac{\lambda}{4 \pi \epsilon_{0} R}$

## ELECTRIC FIELD DUE TO SOLID CONDUCTING OR HOLLOW SPHERE

- For outside point ( $\mathbf{r}>\mathbf{R}$ )

Using Gauss's theorem $\oint \overrightarrow{\mathrm{E}} . \mathrm{d} \overrightarrow{\mathrm{s}}=\frac{\Sigma \mathrm{q}}{\epsilon_{0}}$
$\because$ At every point on the Gaussian surface $\vec{E} \| \mathrm{d} \overrightarrow{\mathrm{s}}$;
$\Rightarrow \overrightarrow{\mathrm{E}} \cdot \mathrm{ds}=\mathrm{E}$ ds $\cos 0^{\circ}=\mathrm{Eds}$

$\therefore \oint \mathrm{E} . \mathrm{ds}=\frac{\Sigma q}{\epsilon_{0}}[\mathrm{E}$ is constant over the gaussian surface $] \Rightarrow \mathrm{E} \times 4 \pi \mathrm{r}^{2}=\frac{\mathrm{q}}{\epsilon_{0}} \Rightarrow \mathrm{E}_{\mathrm{p}}=\frac{\mathrm{q}}{4 \pi \epsilon_{0} \mathrm{r}^{2}}$

- For surface point $\mathbf{r}=\mathbf{R}: \mathrm{E}_{\mathrm{S}}=\frac{\mathrm{q}}{4 \pi \epsilon_{0} \mathrm{R}^{2}}$
- For Inside point $(\mathbf{r}<\mathbf{R})$ : Because charge inside the conducting sphere or hollow is zero.
(i.e. $\Sigma q=0$ ) So $\oint \vec{E} \cdot d \vec{s}=\frac{\Sigma q}{\epsilon_{0}}=0 \Rightarrow E_{\text {in }}=0$


## ELECTRIC FIELD DUE TO SOLID NON CONDUCTING SPHERE

- Outside ( $\mathbf{r}>\mathbf{R}$ )

From Gauss's theorem

$$
\oint_{\mathrm{s}} \overrightarrow{\mathrm{E} . \mathrm{d} \overrightarrow{\mathrm{~s}}}=\frac{\Sigma \mathrm{q}}{\epsilon_{0}} \Rightarrow \mathrm{E} \times 4 \pi \mathrm{r}^{2}=\frac{\mathrm{q}}{\epsilon_{0}} \Rightarrow \mathrm{E}_{\mathrm{P}}=\frac{\mathrm{q}}{4 \pi \epsilon_{0} \mathrm{r}^{2}}
$$

- At surface $(\mathbf{r}=\mathbf{R})$


$$
E_{S}=\frac{q}{4 \pi \epsilon_{0} R^{2}} \text { Put } r=R
$$

- Inside ( $\mathbf{r}<\mathbf{R}$ ) :

From Gauss's theorem $\oint_{\mathrm{s}} \overrightarrow{\mathrm{E}} . \mathrm{d} \overrightarrow{\mathrm{s}}=\frac{\Sigma \mathrm{q}}{\epsilon_{0}}$
Where $\Sigma q$ charge contained within Gaussian surface of radius $r$

$$
\mathrm{E}\left(4 \pi \mathrm{r}^{2}\right)=\frac{\Sigma \mathrm{q}}{\in_{0}} \Rightarrow \mathrm{E}=\frac{\Sigma \mathrm{q}}{4 \pi \mathrm{r}^{2} \in_{0}} \ldots(\mathrm{i})
$$



As the sphere is uniformly charged, the volume charge density (charge/volume) $\rho$ is constant throughout the sphere $\rho=\frac{\mathrm{q}}{\frac{4}{3} \pi R^{3}} \Rightarrow$ charge enclosed in gaussian surface $=\rho\left(\frac{4}{3} \pi r^{3}\right)=\left(\frac{\mathrm{q}}{(4 / 3) \pi \mathrm{R}^{3}}\right)\left(\frac{4}{3} \pi \mathrm{r}^{3}\right) \Rightarrow \sum \mathrm{q}=\frac{\mathrm{qr}^{3}}{\mathrm{R}^{3}}$ put this value in equation (i) $\mathrm{E}_{\mathrm{in}}=\frac{1}{4 \pi \epsilon_{0}} \frac{\mathrm{qr}}{\mathrm{R}^{3}}$

## ELECTRIC FIELD DUE TO AN INFINITE LINE DISTRIBUTION OF CHARGE

Let a wire of infinite length is uniformly charged having a constant linear charge density $\lambda$.
P is the point where electric field is to be calculated.
Let us draw a coaxial Gaussian cylindrical surfaces of length $\ell$.
From Gauss's theorem

$$
\begin{aligned}
& \mathrm{q} \int_{s_{1}} \overrightarrow{\mathrm{E}} . \mathrm{d} \overrightarrow{\mathrm{~S}}_{1}+\int_{\mathrm{s}_{2}} \overrightarrow{\mathrm{E}} . \mathrm{d} \vec{S}_{2}+\int_{s_{3}}{\overrightarrow{\mathrm{E}} . \mathrm{d} \vec{S}_{3}=\frac{\mathrm{q}}{\epsilon_{0}}}_{\Sigma \overrightarrow{\mathrm{E}} \perp \mathrm{~d} \overrightarrow{\mathrm{~S}}_{1} \text { so } \overrightarrow{\mathrm{E}} . \mathrm{d} \overrightarrow{\mathrm{~S}}_{1}=0 \text { and } \overrightarrow{\mathrm{E}} \perp \mathrm{~d} \overrightarrow{\mathrm{~S}}_{2} \text { so } \overrightarrow{\mathrm{E}} . \mathrm{d} \overrightarrow{\mathrm{~S}}_{2}=0}^{\mathrm{E} \times 2 \pi \mathrm{r} \ell=\frac{\mathrm{q}}{\epsilon_{0}}\left[\because \overrightarrow{\mathrm{E}} \| \mathrm{d} \vec{S}_{3}\right]}
\end{aligned}
$$



Charge enclosed in the Gaussian surface $\mathrm{q}=\lambda \ell$.
So $\mathrm{E} \times 2 \pi \mathrm{r} \ell=\frac{\lambda \ell}{\epsilon_{0}} \Rightarrow \mathrm{E}=\frac{\lambda}{2 \pi \epsilon_{0} \mathrm{r}}$ or $\mathrm{E}=\frac{2 \mathrm{k} \lambda}{\mathrm{r}}$ where $\mathrm{k}=\frac{1}{4 \pi \epsilon_{0}}$
Charged cylindrical nonconductor of infinite length
Electric field at outside point $\overrightarrow{\mathrm{E}}_{\mathrm{A}}=\frac{2 \mathrm{k} \lambda}{\mathrm{r}} \hat{\mathrm{r}} \quad \mathrm{r}>\mathrm{R}$
Electric field at inside point $\overrightarrow{\mathrm{E}}_{\mathrm{B}}=\frac{\lambda \overrightarrow{\mathrm{r}}}{2 \pi \epsilon_{0} \mathrm{R}^{2}} \mathrm{r}<\mathrm{R}$


## KEY POINTS

- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ( $\mathrm{E} \neq 0$ if non zero charge is inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. $x \ggg$ R.

Ex. A charged particle is kept in equilibrium in the electric field between the plates of millikan oil drop experiment. If the direction of the electric field between the plates is reversed, then calculate acceleration of the charged particle.
Sol. Let mass of the particle $=\mathrm{m}$,
charge on particle $=\mathrm{q}$
Intensity of electric field in between plates $=\mathrm{E}$ initially $\mathrm{mg}=\mathrm{qE}$
After reversing the field $\mathrm{ma}=\mathrm{mg}+\mathrm{qE} \Rightarrow \mathrm{ma}=2 \mathrm{mg}$
$\therefore$ Acceleration of particle $\Rightarrow \mathrm{a}=2 \mathrm{~g}$

## ELECTROSTATIC POTENTIAL ENERGY

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles. Electrostatic potential energy is defined in two ways.
(i) Interaction energy of charged particles of a system
(ii) Self energy of a charged object

## INFINITE SHEET CHARGE

## - Electrostatic Interaction Energy

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to the given configuration. When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system, hence final potential energy of system will be positive. If the force between the particle is attractive, work will be done by the system and final potential energy of system will be negative.

- Interaction Energy of a system of two charged particles

Figure shows two + ve charges $q_{1}$ and $q_{2}$ separated by a distance $r$. The electrostatic interaction energy of this system can be given as work done in bringing $q_{2}$ from infinity to the given separation from q. It can be calculated as
$\mathrm{W}=\int_{\infty}^{\mathrm{r}} \overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{dx}}=-\int_{\infty}^{\mathrm{r}} \frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\mathrm{x}^{2}} d x$ [-ve sign shows that x is decreasing]

$\mathrm{W}=\frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\mathrm{r}}=\mathrm{U}$ [ interaction energy]
If the two charges here are of opposite sign, the potential energy will be negative as $U=-\frac{\mathrm{kq}_{1} \mathrm{q}_{2}}{\mathrm{r}}$

## - Interaction Energy for a system of charged particles

When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energies of all the pairs of particles. For example if a system of three particles having charges $q_{1}, q_{2}$ and $q_{3}$ is given as shown in figure. The total interaction energy of this system can be
 given as $U=\frac{k q_{1} q_{2}}{r_{3}}+\frac{k q_{1} q_{3}}{r_{2}}+\frac{k q_{2} q_{3}}{r_{1}}$

## ELECTRIC POTENTIAL

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field potential is defined as the interaction energy of a unit positive charge. If at a point in electric field a charge $\mathrm{q}_{0}$ has potential energy U , then electric potential at that point can be given as
$\mathrm{V}=\frac{\mathrm{U}}{\mathrm{q}_{0}}$ joule/coulomb
Potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces.

- Electric Potential due to a point charge in its surrounding :

The potential at a point $P$ at a distance $r$ from the charge $q, V_{P}=\frac{U}{q_{0}}$
Where $U$ is the potential energy of charge $q_{0}$ at point $p, U=\frac{k q q_{0}}{r}$


Thus potential at point $P$ is $V_{P}=\frac{k q}{r}$

## Electric Potential due to a charge Rod :

Figure shows a rod of length L, uniformly charged with a charge Q. Due to this we'll find electric potential at a point P at a distance r from one end of the rod as shown in figure.


For this we consider an element of width dx at a distance x from the point P .
Charge on this element is $d Q=\frac{Q}{L} d x$
The potential dV due to this element at point P can be given by using the result of a point charge as

$$
\mathrm{dV}=\frac{\mathrm{kdq}}{\mathrm{x}}=\frac{\mathrm{kQ}}{\mathrm{Lx}} \mathrm{dx}
$$

Net electric potential at point $P: V=\int d V=\int_{r}^{r+L} \frac{k Q}{L x} d x=\frac{k Q}{L} \ell n\left(\frac{r+L}{r}\right)$

## Electric potential due to a charged ring

## Case-I : At its centre

To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as $d V=\frac{\mathrm{kdq}}{\mathrm{R}}$

Total potential at $C$ is $V=\int d V=\int \frac{\mathrm{kdq}}{\mathrm{R}}=\frac{\mathrm{kQ}}{\mathrm{R}}$.


As all dq's of the ring are situated at same distance R from the ring centre C , simply the potential due to all dq's is added as being a scalar quantity, we can directly say that the total electric potential at ring centre is $\frac{\mathrm{kQ}}{\mathrm{R}}$. Here we can also state that even if charge Q is non-uniformly distributed on ring, the electric potential $C$ will remain same.

## Case II : At a point on axis of ring

We find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{\mathrm{x}^{2}+\mathrm{R}^{2}}$ from the point P , thus the potential at P can be given as $V_{P}=\frac{k Q}{\sqrt{R^{2}+x^{2}}}$


## Electric potential due to a uniformly charged disc :

Figure shows a uniformly disc of radius R with surface charge density $\rho \mathrm{coul} / \mathrm{m}^{2}$. To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is $d q=\sigma 2 \pi y$ dy. Due to this ring, the electric potential at point $P$ can be given as

$$
\mathrm{dV}=\frac{\mathrm{kdq}}{\sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}}}=\frac{\mathrm{k} \cdot \sigma \cdot 2 \pi \mathrm{y} \mathrm{dy}}{\sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}}}
$$



Net electric potential at Point P due to whole disc can be given as

$$
\mathrm{V}=\int \mathrm{dV}=\int_{0}^{\mathrm{R}} \frac{\sigma}{2 \epsilon_{0}} \cdot \frac{\mathrm{y} d \mathrm{y}}{\sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}}}=\frac{\sigma}{2 \epsilon_{0}}\left[\sqrt{\mathrm{x}^{2}+\mathrm{y}^{2}}\right]_{0}^{\mathrm{R}}=\frac{\sigma}{2 \epsilon_{0}}=\left[\sqrt{\mathrm{x}^{2}+\mathrm{R}^{2}}-\mathrm{x}\right]
$$

## ELECTRIC POTENTIAL DUE TO HOLLOW OR CONDUCTING SPHERE

- At outside sphere

According to definition of electric potential, electric potential at point $P$

$$
\mathrm{V}=-\int_{\infty}^{\mathrm{r}} \overrightarrow{\mathrm{E}} \cdot \mathrm{~d} \mathrm{\vec{r}}=-\int_{\infty}^{\mathrm{r}} \frac{\mathrm{q}}{4 \pi \epsilon_{0} \mathrm{r}^{2}} \mathrm{dr}\left[\because \quad \mathrm{E}_{\text {out }}=\frac{\mathrm{q}}{4 \pi \epsilon_{0} \mathrm{r}^{2}}\right] ; \mathrm{V}=-\frac{\mathrm{q}}{4 \pi \epsilon_{0}} \int_{\infty}^{\mathrm{r}} \frac{1}{\mathrm{r}^{2}} \mathrm{dr}=\frac{\mathrm{q}}{4 \pi \epsilon_{0}}\left[\frac{1}{\mathrm{r}}\right]_{\infty}^{\mathrm{r}} \frac{\mathrm{q}}{4 \pi \epsilon_{0} \mathrm{r}}
$$

- At surface
$=V=-\int_{\infty}^{R} \vec{E} \cdot d \vec{r}=-\int_{\infty}^{R} \frac{q}{4 \pi \epsilon_{0} r^{2}} d r\left[\because E_{\text {out }}=\frac{q}{4 \pi \varepsilon_{0} r^{2}}\right] ; V=-\frac{q}{4 \pi \epsilon_{0}} \int_{\infty}^{R}\left(\frac{1}{r^{2}}\right) d r=\frac{q}{4 \pi \epsilon_{0}}\left[\frac{1}{r}\right]_{\infty}^{R} \Rightarrow V=\frac{q}{4 \pi \epsilon_{0} R}$


## - Inside the surface :

$\because$ Inside the surface $E=0\left[E=-\frac{d V}{d r}\right] \frac{d V}{d r}=0$ or $V=$ constant so $V=\frac{q}{4 \pi \epsilon_{0} R}$

## ELECTRIC POTENTIAL DUE TO SOLID NON CONDUCTING SPHERE

- At outside sphere
- At Surface
- Inside the sphere

$$
\begin{array}{ll}
V=-\int_{\infty}^{r} \vec{E} \cdot d \vec{r} & \Rightarrow V=-\left[\int_{\infty}^{R} E_{1} d r+\int_{R}^{r} E_{2} d r\right] \\
V=-\left[\int_{\infty}^{R}\left(\frac{k q}{r^{2}}\right) d r+\int_{R}^{r}\left(\frac{k q r}{R^{3}}\right) d r\right] & \Rightarrow V=-\left[k q\left(-\frac{1}{r}\right)_{\infty}^{R}+\frac{k q}{R^{3}}\left(\frac{r^{2}}{2}\right)_{R}^{r}\right] \\
V=-k q\left[-\frac{1}{R}+\frac{r^{2}}{2 R^{3}}-\frac{R^{2}}{2 R^{3}}\right] & \Rightarrow V=\frac{k q}{2 R^{3}}\left[3 R^{2}-r^{2}\right]
\end{array}
$$

Same as conducting sphere.
Same as conducting sphere.

## Potential Difference Between Two points in electric field

Potential difference between two points in electric field can be defined as work done in displacing a unit positive charge from one point to another against the electric forces.
If a unit + ve charge is displaced from a point $A$ to $B$ as shown work required can be given as
$V_{B}-V_{A}=-\int_{A}^{B} \vec{E} \cdot \overrightarrow{d x}$
If a charge q is shifted from point A to B , work done against electric forces can be given as $\mathrm{W}=\mathrm{q}$ $\left(V_{B}-V_{A}\right)$
If in a situation work done by electric forces is asked, we use $\mathrm{W}=\mathrm{q}\left(\mathrm{V}_{\mathrm{A}}-\mathrm{V}_{\mathrm{B}}\right)$
If $\mathrm{V}_{\mathrm{B}}<\mathrm{V}_{\mathrm{A}}$, then charges must have tendency to move toward B (low potential point) it implies that electric forces carry the charge from high potential to low potential points. Hence we can say that in the direction of electric field always electric potential decreases.

## Equipotential surfaces

For a given charge distribution, locus of all points having same potential is called 'equipotential surface'.

- Equipotential surfaces can never cross each other (otherwise potential at a point will have two values which is absurd)
- Equipotential surfaces are always perpendicular to direction of electric field.
- If a charge is moved from one point to the other over an equipotential surface then work done

$$
\mathrm{W}_{\mathrm{AB}}=-\mathrm{U}_{\mathrm{AB}}=\mathrm{q}\left(\mathrm{~V}_{\mathrm{B}}-\mathrm{V}_{\mathrm{A}}\right)=0 \quad\left[\because \mathrm{~V}_{\mathrm{B}}=\mathrm{V}_{\mathrm{A}}\right]
$$

- Shapes of equipotential surfaces

- The intensity of electric field along an equipotential surface is always zero.


## Electric Potential Gradient

The maximum rate of change of potential at right angles to an equipotential surface in an electric field is defined as potential gradient. $\overrightarrow{\mathrm{E}}=-\vec{\nabla} \mathrm{V}=-\operatorname{grad} \mathrm{V}$
Note : Potential is a scalar quantity but the gradient of potential is a vector quantity
In cartesian co-ordinates $\vec{\nabla} \mathrm{V}=\left[\frac{\partial \mathrm{V}}{\partial \mathrm{x}} \hat{\mathrm{i}}+\frac{\partial \mathrm{V}}{\partial \mathrm{y}} \hat{\mathrm{j}}+\frac{\partial \mathrm{V}}{\partial \mathrm{z}} \hat{\mathrm{k}}\right]$
Ex. If $V=-5 x+3 y+\sqrt{15} z$ then $E(x, y, z)=$ ?
Sol. $\quad \overrightarrow{\mathrm{E}}=-\left[\frac{\partial \mathrm{V}}{\partial \mathrm{x}} \hat{\mathrm{i}}+\frac{\partial \mathrm{V}}{\partial \mathrm{y}} \hat{\mathrm{j}}+\frac{\partial \mathrm{V}}{\partial \mathrm{z}} \hat{\mathrm{k}}\right]=-(-5 \hat{\mathrm{i}}+3 \hat{\mathrm{j}}+\sqrt{15} \hat{\mathrm{k}}) \Rightarrow|\overrightarrow{\mathrm{E}}|=\sqrt{25+9+15}=\sqrt{49}=7$ unit

## ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges, given as


In some molecules, the centres of positive and negative charges do not coincide. This results in the formation of electric dipole. Atom is non - polar because in it the centres of positive and negative charges coincide. Polarity can be induced in an atom by the application of electric field. Hence it can be called as induced dipole.

- Dipole Moment : Dipole moment $\overrightarrow{\mathrm{p}}=\mathrm{q} \overrightarrow{\mathrm{d}}$
(i) Vector quantity, directed from negative to positive charge
(ii) Dimension : [LTA], Units : coulomb $\times$ metre (or Cm)

(iii) Practical unit is "debye" $\equiv$ Two equal and opposite point charges each having charge $10^{-10}$ frankline $(\simeq e)$ and separation of $1 \AA$ then the value of dipole moment $(\overrightarrow{\mathrm{p}})$ is 1 debye.

$$
1 \text { Debye }=10^{-10} \times 10^{-10} \mathrm{Fr} \times \mathrm{m}=10^{-20} \times \frac{\mathrm{C} \times \mathrm{m}}{3 \times 10^{9}} \simeq 3.3 \times 10^{-30} \mathrm{C} \times \mathrm{m}
$$

- Dipole Placed in uniform Electric Field

Figure shows a dipole of dipole moment $\overrightarrow{\mathrm{p}}$ placed at an angle $\theta$ to the direction of electric field. Here the charges of dipole experience forces
qE in opposite direction as shown. $\overrightarrow{\mathrm{F}}_{\text {net }}=[\mathrm{q} \overrightarrow{\mathrm{E}}+(-\mathrm{q}) \overrightarrow{\mathrm{E}}]=\overrightarrow{0}$


Thus we can state that when a dipole is placed in a uniform electric field, net force on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as
$\tau=$ Force $\times$ separation between lines of actions of forces $=\mathrm{qE} \times \mathrm{d} \sin \theta=\mathrm{pE} \sin \theta$

$$
\vec{\tau}=\overrightarrow{\mathrm{r}} \times \overrightarrow{\mathrm{F}}=\overrightarrow{\mathrm{d}} \times \mathrm{q} \overrightarrow{\mathrm{E}}=\mathrm{q} \overrightarrow{\mathrm{~d}} \times \overrightarrow{\mathrm{E}}=\overrightarrow{\mathrm{p}} \times \overrightarrow{\mathrm{E}}
$$

## Work done in Rotation of a Dipole in Electric field

When a dipole is placed in an electric field at an angle $\theta$, the torque on it due to electric field is $\tau=\mathrm{pE} \sin \theta$
Work done in rotating an electric dipole from $\theta_{1}$ to $\theta_{2}$ [ uniform field]

$$
\mathrm{dW}=\tau \mathrm{d} \theta \text { so } \mathrm{W}=\int \mathrm{dW}=\int \tau \mathrm{d} \theta \text { and } \mathrm{W}_{\theta_{1} \rightarrow \theta_{2}}=\mathrm{W}=\int_{\theta_{1}}^{\theta_{2}} \mathrm{pE} \sin \theta \mathrm{~d} \theta=\mathrm{pE}\left(\cos \theta_{1}-\cos \theta_{2}\right)
$$

e.g. $\mathrm{W}_{0 \rightarrow 180}=\mathrm{pE}[1-(-1)]=2 \mathrm{pE} \quad \mathrm{W}_{0 \rightarrow 90}=\mathrm{pE}(1-0)=\mathrm{pE}$

If a dipole is rotated from field direction $\left(\theta=0^{\circ}\right)$ to $\theta$ then $\mathrm{W}=\mathrm{pE}(1-\cos \theta)$

$\theta=0$
$\tau=$ minimum $=0$
$\mathrm{W}=\operatorname{minimum}=0$
$\theta=90^{\circ}$
$\theta=180^{\circ}$
$\tau=$ maximum $=\mathrm{pE}$
$\tau=$ minimum $=0$
$\mathrm{W}=$ maximum $=2 \mathrm{pE}$

## Electrostatic potential energy :

Electrostatic potential energy of a dipole placed in a uniform field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e.,
$\mathrm{W}_{90^{\circ} \rightarrow \theta}=\int_{90^{\circ}}^{\theta} \mathrm{pE} \sin \theta \mathrm{d} \theta=-\mathrm{pE} \cos \theta=-\overrightarrow{\mathrm{p}} . \overrightarrow{\mathrm{E}}$
$\overrightarrow{\mathrm{E}}$ is a conservative field so what ever work is done in rotating a dipole from $\theta_{1}$ to $\theta_{2}$ is just equal to change in electrostatic potential energy $\mathrm{W}_{\theta_{1} \rightarrow \theta_{2}}=\mathrm{U}_{\theta_{2}}-\mathrm{U}_{\theta_{1}}=\mathrm{pE}\left(\cos \theta_{1}-\cos \theta_{2}\right)$

## Work done in rotating an electric dipole in an electric field

Suppose at any instant, the dipole makes an angle $\theta$ with the electric field.
The torque acting on dipole. $\tau=\mathrm{qEd}=(\mathrm{q} 2 \ell \sin \theta) \mathrm{E}=\mathrm{pE} \sin \theta$
The work done in rotating dipole from $\theta_{1}$ to $\theta_{2}$

$$
\begin{aligned}
& \mathrm{W}=\int_{\theta_{1}}^{\theta_{2}} \tau \mathrm{~d} \theta=\int_{\theta_{1}}^{\theta_{2}} \mathrm{pE} \sin \theta \mathrm{~d} \theta \\
& \mathrm{~W}=\mathrm{pE}\left(\cos \theta_{1}-\cos \theta_{2}\right)=\mathrm{U}_{2}-\mathrm{U}_{1} \quad(\because \mathrm{U}=-\mathrm{pE} \cos \theta)
\end{aligned}
$$



## Force on an electric dipole in Non-uniform electric field :

If in a non-uniform electric field dipole is placed at a point where electric field is E , the interaction energy of dipole at this point $U=-\vec{p} . \vec{E}$. Now the force on dipole due to electric field $F=-\frac{\Delta U}{\Delta x}$ For unidirectional variation in electric field, $F=-\frac{d}{d x}(\vec{p} \cdot \vec{E})$

If dipole is placed in the direction of electric field then $F=-p \frac{d E}{d x}$

Ex. Calculate force on a dipole in the surrounding of a long charged wire as shown in the figure.


Sol. In the situation shown in figure, the electric field strength due to the wire, at the position of dipole as $\mathrm{E}=\frac{2 \mathrm{k} \lambda}{\mathrm{r}}$

Thus force on dipole is $F=-p . \frac{d E}{d r}=-p\left[-\frac{2 \mathrm{k} \lambda}{\mathrm{r}^{2}}\right]=\frac{2 \mathrm{kp} \lambda}{\mathrm{r}^{2}}$
Here -ve charge of dipole is close to wire hence net force an dipole due to wire will be attractive.

## ELECTRIC POTENTIAL DUE TO DIPOLE

## - At axial point

Electric potential due to $+q$ charge $V_{1}=\frac{\mathrm{kq}}{(\mathrm{r}-\ell)}$
Electric potential due to -q charge $\mathrm{V}_{2}=\frac{-\mathrm{kq}}{(\mathrm{r}+\ell)}$


Net electric potential $\mathrm{V}=\mathrm{V}_{1}+\mathrm{V}_{2}=\frac{\mathrm{kq}}{(\mathrm{r}-\ell)}+\frac{-\mathrm{kq}}{(\mathrm{r}+\ell)}=\frac{\mathrm{kq} \times 2 \ell}{\left(\mathrm{r}^{2}-\ell^{2}\right)}=\frac{\mathrm{kp}}{\mathrm{r}^{2}-\ell^{2}}$
If $\mathrm{r} \ggg \ell \Rightarrow \mathrm{V}=\frac{\mathrm{kp}}{\mathrm{r}^{2}}$

- At equatorial point

Electric potential of P due to +q charge $\mathrm{V}_{1}=\frac{\mathrm{kq}}{\mathrm{x}}$
Electric potential of $P$ due to $-q$ charge $V_{2}=-\frac{k q}{x}$
Net potential $\mathrm{V}=\mathrm{V}_{1}+\mathrm{V}_{2}=\frac{\mathrm{kq}}{\mathrm{x}}-\frac{\mathrm{kq}}{\mathrm{x}}=0 \therefore \mathrm{~V}=0$


- At general point

$$
\mathrm{V}=\frac{\mathrm{p} \cos \theta}{4 \pi \varepsilon_{0} \mathrm{r}^{2}}=\frac{\overrightarrow{\mathrm{p}} \cdot \overrightarrow{\mathrm{r}}}{4 \pi \varepsilon_{0} \mathrm{r}^{3}} \overrightarrow{\mathrm{p}}=\mathrm{q} \overrightarrow{\mathrm{a}} \text { electric dipole moment }
$$

## Electric field due to an electric dipole

Figure shows an electric dipole placed on x -axis at origin. Here we wish to find the electric field and potential at a point $O$ having coordinates ( $\mathrm{r}, \theta$ ). Due to the positive charge of dipole electric field at $O$ is in radially outward direction and due to the negative charge it is radially inward as shown in figure.


$$
\mathrm{E}_{\mathrm{r}}=-\frac{\partial \mathrm{V}}{\partial \mathrm{r}}=\frac{2 \mathrm{kp} \cos \theta}{\mathrm{r}^{3}} \text { and } \mathrm{E}_{\theta}=-\frac{1}{\mathrm{r}} \frac{\partial \mathrm{~V}}{\partial \theta}=\frac{\mathrm{kp} \sin \theta}{\mathrm{r}^{3}}
$$

Thus net electric field at point $\mathrm{O}, \mathrm{E}_{\text {net }}=\sqrt{\mathrm{E}_{\mathrm{r}}^{2}+\mathrm{E}_{\theta}^{2}}=\frac{\mathrm{kp}}{\mathrm{r}^{3}} \sqrt{1+3 \cos ^{2} \theta}$
If the direction of $E_{\text {net }}$ is at an angle $\alpha$ from radial direction, then $\alpha=\tan ^{-1} \frac{E_{\theta}}{E_{r}}=\left(\frac{1}{2} \tan \theta\right)$
Thus the inclination of net electric field at point O is $(\theta+\alpha)$

## - At a point on the axis of a dipole :



Electric field due to $+q$ charge $E_{1}=\frac{k q}{(r-\ell)^{2}}$
Electric field due to -q charge $\mathrm{E}_{2}=\frac{\mathrm{kq}}{(\mathrm{r}+\ell)^{2}}$
Net electric field $\mathrm{E}=\mathrm{E}_{1}-\mathrm{E}_{2}=\frac{\mathrm{kq}}{(\mathrm{r}-\ell)^{2}}-\frac{\mathrm{kq}}{(\mathrm{r}+\ell)^{2}}=\frac{\mathrm{kq} \times 4 \mathrm{r} \ell}{\left(\mathrm{r}^{2}-\ell^{2}\right)^{2}}[\because \mathrm{p}=\mathrm{q} \times 2 \ell=$ Dipole moment $]$

$$
\mathrm{E}=\frac{2 \mathrm{kpr}}{\left(\mathrm{r}^{2}-\ell^{2}\right)^{2}} \text { If } \mathrm{r} \ggg \ell \text { then } \mathrm{E}=\frac{2 \mathrm{kp}}{\mathrm{r}^{3}}
$$

## - At a point on equatorial line of dipole :

Electric field due to $+q$ charge $E_{1}=\frac{k q}{x^{2}}$; Electric field due to $-q$ charge $E_{2}=\frac{k q}{x^{2}}$
Vertical component of $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ will cancel each other and horizontal components will be added


So net electric field at P

$$
\begin{aligned}
& \mathrm{E}=\mathrm{E}_{1} \cos \theta+\mathrm{E}_{2} \cos \theta\left[\because \mathrm{E}_{1}=\mathrm{E}_{2}\right] \\
& \mathrm{E}=2 \mathrm{E}_{1} \cos \theta=\frac{2 \mathrm{kq}}{\mathrm{x}^{2}} \cos \theta \because \cos \theta=\frac{\ell}{\mathrm{x}} \text { and } \mathrm{x}=\sqrt{\mathrm{r}^{2}+\ell^{2}} \\
& \mathrm{E}=\frac{2 \mathrm{kq} \ell}{\mathrm{x}^{3}}=\frac{2 \mathrm{kq} \ell}{\left(\mathrm{r}^{2}+\ell^{2}\right)^{3 / 2}}=\frac{\mathrm{kp}}{\left(\mathrm{r}^{2}+\ell^{2}\right)^{3 / 2}} \text { If } \mathrm{r} \ggg \ell
\end{aligned}
$$

then $E=\frac{k p}{r^{3}}$ or $\vec{E}=\frac{-k \vec{p}}{r^{3}}$
Ex. A short electric dipole is situated at the origin of coordinate axis with its axis along x -axis and equator along $y$-axis. It is found that the magnitudes of the electric intensity and electric potential due to the dipole are equal at a point distant $\mathrm{r}=\sqrt{5} \mathrm{~m}$ from origin. Find the position vector of the point.

Sol. $\because\left|\mathrm{E}_{\mathrm{P}}\right|=\left|\mathrm{V}_{\mathrm{P}}\right| \therefore \frac{\mathrm{kp}}{\mathrm{r}^{3}} \sqrt{1+3 \cos ^{2} \theta}=\frac{\mathrm{kp} \sin \theta}{\mathrm{r}^{2}} \Rightarrow 1+3 \cos ^{2} \theta=5 \cos ^{2} \theta \Rightarrow \cos \theta=\frac{1}{\sqrt{2}} \Rightarrow \theta=45^{\circ}$
Position vector $\overrightarrow{\mathrm{r}}$ of point $P$ is $\overrightarrow{\mathrm{r}}=\frac{\sqrt{5}}{2}(\hat{\mathrm{i}}+\hat{\mathrm{j}})$
Ex. Prove that the frequency of oscillation of an electric dipole of moment p and rotational inertia I for small amplitudes about its equilibrium position in a uniform electric field strength E is $\frac{1}{2 \pi} \sqrt{\left(\frac{\mathrm{pE}}{\mathrm{I}}\right)}$

Sol. Let an electric dipole (charge q and -q at a distance 2 a apart) placed in a uniform external electric field of strength $E$.

Restoring torque on dipole

$\tau=-\mathrm{pE} \sin \theta=-\mathrm{pE} \theta$ (as $\theta$ is small)
Here - ve sign shows the restoring tendency of torque.
$\because \tau=\mathrm{I} \alpha \therefore$ angular acceleration $=\alpha=\frac{\tau}{\mathrm{I}}=\frac{\mathrm{PE}}{\mathrm{I}} \theta$
For SHM $\alpha=\omega^{2} \theta$ comparing we get $\omega=\sqrt{\frac{\mathrm{pE}}{\mathrm{I}}}$
Thus frequency of oscillations of dipole $n=\frac{\omega}{2 \pi}=\frac{1}{2 \pi} \sqrt{\left(\frac{\mathrm{pE}}{\mathrm{I}}\right)}$

## ELECTROSTATIC PRESSURE

Force due to electrostatic pressure is directed normally outwards to the surface. Force on small element ds of charged conductor
$\mathrm{dF}=($ Charge on ds $) x$ Electric field $=(\sigma \mathrm{ds}) \frac{\sigma}{2 \epsilon_{0}} \mathrm{dF}=\frac{\sigma^{2}}{2 \epsilon_{0}} \mathrm{ds}$


Inside $E_{1}-E_{2}=0 \Rightarrow E_{1}=E_{2}$


Just outside $\mathrm{E}=\mathrm{E}_{1}+\mathrm{E}_{2}=2 \mathrm{E}_{2} \Rightarrow \mathrm{E}_{2}=\frac{\sigma}{2 \epsilon_{0}}$
( $E_{1}$ is field due to point charge on the surface and $E_{2}$ is field due to rest of the sphere).
The electric force acting per unit area of charged surface is defined as electrostatic pressure.

$$
\mathrm{P}_{\text {eleectrostatic }}=\frac{\mathrm{dF}}{\mathrm{dS}}=\frac{\sigma^{2}}{2 \epsilon_{0}}
$$

## EXERCISE (S-1)

## Properties of charge and Coulomb's law

1. Three charges are positioned in a straight line as depicted in the diagram. Using the information given in the diagram, determine the magnitude of charge ' $\mathrm{Q}_{1}$ ' (in $\mu \mathrm{C}$ ) if the charge q is to remain stationary.

2. Two identical balls of mass $m=0.9 \mathrm{~g}$ each are charged by the same charges, joined by a thread and suspended from the ceiling (figure). Find the charge (in $\mu \mathrm{C}$ ) that each ball should have so that the tension in both the threads are same? The distance between the centers of balls is $\mathrm{R}=3 \mathrm{~m}$.

3. Four identical charges $q$ are placed at the corners of a square of side a , what charges Q must be placed at the centre of the square so that whole system of charges is in equilibrium?
4. Six charges are kept at the vertices of a regular hexagon as shown in the figure. If magnitude of force applied by +Q on +q charge is F , then net electric force on the +Q is nF . Find the value of n .

5. Two charged balls are connected by an inextensible thread of length 3 m . Masses of balls are 2 kg and 1 kg , the charges are $+20 \mu \mathrm{C}$ and $-100 \mu \mathrm{C}$. What minimum constant external force F (in N ) must be applied to the ball of mass 1 kg so that the thread does not slack? Neglect gravity and friction.

## Electric field

6. Two point charge -4 Q and 9 Q are placed at a distance 2 m from each other. The position at which net electric field is zero from the charge $-4 Q$ is $x$ (in $m$ ). What is the value of $x$.
7. Two charges are separated as shown in the diagram A below. Each charge has a magnitude of 50 nC in diagram A. If the charge on the right is moved further to the right as depicted in diagram B, additional charge that must be removed from the left in order to maintain the initial electric field magnitude at the center (in nC ) is given by $\frac{\alpha}{9} \mathrm{nC}$. Fill the value of $\alpha$ in OMR sheet.

8. A clock face has negative charges $-q,-2 q,-3 q, \ldots . . . . .,-12 q$ fixed at the positions of the corresponding numerals on the dial. Assume that the clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction is electric field at the centre of the dial.
9. Find the magnitude of electric field (in N/C) due to a line charge of $\lambda=(2 \sqrt{2}) \mathrm{nC} / \mathrm{m}$ at a point P as shown.

10. Four uniformly charged wires of length a are arranged to form a square. Linear charge density of each wire is as shown. Electric field intensity at centre of square is $\frac{\mathrm{nk} \lambda}{\mathrm{a}}$ then value of n

11. Two concentric rings, one of radius 'a' and the other of radius ' $b$ ' have the charges +q and $-(2 / 5)^{-3 / 2} \mathrm{q}$ respectively as shown in the figure. Find the ratio $\mathrm{b} / \mathrm{a}$ if a charge particle placed on the axis at $\mathrm{z}=\mathrm{a}$ is in equilibrium.

12. The intensity of elecric field is required to exert a force on a proton equal to its weight at sea level is given by $\alpha \times 10^{-7} \mathrm{~V} / \mathrm{m}$. Fill the value of $\alpha$ (Given $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$, mass of proton $=1.6 \times 10^{-27} \mathrm{~kg}$ and charge on proton $=1.6 \times 10^{-19} \mathrm{C}$ ).
13. A thin insulating uniformly charged (linearly charged density $\lambda$ ) rod is hinged about one of its ends. It can rotate in vertical plane. If rod is in equilibrium by applying vertical electric field E as shown in figure. Find the value of $\mathrm{E}(\mathrm{in} \mathrm{N} / \mathrm{C})$. (Given that mass of $\operatorname{rod} 2 \mathrm{~kg}, \lambda=10 \mathrm{C} / \mathrm{m}, \ell=1 \mathrm{~m}, \mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}$ )


## Gauss' law

14. In a particular system, if number of electric field lines associated by 1 C charge is $10^{9}$. Then net number of electric field lines passing through the given closed surface is $\mathrm{n} \times 10^{3}$ find n .

15. A uniform electric field $\mathrm{E}=500 \mathrm{~N} / \mathrm{C}$ passes through a hemispherical surface of radius $\mathrm{R}=1.2 \mathrm{~m}$ as shown in figure. The net electric flux (in S.I. units) through the hemispherical surface only is $\mathrm{N} \pi$. Then find the value of N .

16. A non-uniform electric field in $x$-direction is increasing uniformly from $2 N / C$ at $x=1 \mathrm{~m}$ to $8 \mathrm{~N} / \mathrm{C}$ at $x=7 \mathrm{~m}$. The center of cube is at $\mathrm{x}=2 \mathrm{~m}$. If the charge enclosed inside a small cube of side length 10 cm is $8.85 \times 10^{-5 \mathrm{n}} \mathrm{C}$. The value of n will be.
17. $20 \mu \mathrm{C}$ charge is placed inside a closed surface then flux related to surface is $\phi$. If $80 \mu \mathrm{C}$ charge is added inside the surface then change in flux is given by $\alpha \phi$. Fill the value of $\alpha$.
18. A sphere of radius $R$ has charge density given by $\rho=\rho_{0}\left(1-\frac{\mathrm{nr}}{3 \mathrm{R}}\right)$, where $\rho_{0}$ is a constant, $r$ is distance from centre of sphere. For a spherical gaussian surface of radius R centered at the centre of sphere, the flux is zero. Find ' n '.
19. Four point charges are kept at the vertices of a regular tetrahedron of side $R$. Total electrostatic energy of the configuration is $\frac{\alpha \mathrm{kq}^{2}}{\mathrm{R}}$. The value of $\alpha$ is


## Electric potential energy and electric potential

20. Four identical free point positive charges $q(=1 \mu \mathrm{C})$ each are located at the four corners of a square of side 1 m . A negative charge is placed at the centre of the square to obtain equilibrium of all the charges. What is the total potential energy (in milli Joules) of the system assuming the reference point at infinity?
21. Consider the configuration of a system of four charges each of value $+q$. Find the work done by external agent in changing the configuration of the system from figure (i) to fig (ii).

22. Two fixed, equal positive charges, each +q are located at point A \& B separated by a distance of 6 m . A particle of mass $m$ having equal and opposite charge -q moves towards them along the perpendicular bisector line COD where O is the centre of line joining $A$ and $B$. If $-q$ charge is released from rest from point $C$, then speed of charge $-q$ at $O$ is given by $v=q \sqrt{\frac{x}{15 \pi \epsilon_{0} m}}$ find value of $x$. (Neglect gravity)

23. An electric field $(-30 \hat{i}+20 \hat{\mathrm{j}}) \mathrm{Vm}^{-1}$ exists in the space. If potential at the origin is zero then find the potential at $(5 \mathrm{~m}, 3 \mathrm{~m})$ in volts.
24. A circular ring of radius a with uniform charge density $\lambda$ is in the xy plane with centre at origin. A particle of mass $m$ and charge $q$ is projected from $P(0,0, a \sqrt{ } 3)$ on $+z$-axis towards origin with initial velocity $u$. The minimum value of the velocity so that the particle does not return to P is $\sqrt{\frac{\lambda q}{x \varepsilon_{0} m}}$. Find ' $x$ ' (neglect gravity).
25. A particle of mass $m$ carrying charge ' $q$ ' is projected with velocity (v) from point $P$ towards an infinite line charge from a distance ' $a$ '. Its speed reduces to zero momentarily at point $Q$ which is at a distance $\mathrm{a} / 2$ from the line charge. If another particle with mass m and charge -q is projected with the same velocity $v$ from point $P$ towards the line charge. Its speed is found to be $\frac{N v}{\sqrt{2}}$ at point ' $Q$ '. Find the value of N .
26. A simple pendulum of length $\ell$ and bob mass $m$ is hanging in front of a large nonconducting sheet of surface charge density $\sigma$. If suddenly a charge $+q$ is given to the bob in the position shown in figure. Find the maximum angle through which the string is deflected from vertical.
27. Outer cylinder of the coaxial nonconductor of radius ' $b$ ' is given a positive potential V relative to the inner cylinder of radius ' $a$ ' as shown in the figure (charge distribution is uniform). A charge $q$ (mass $=\mathrm{m}$ ) is set free with negligible velocity at the surface of the inner cylinder. Find the velocity (in $\mathrm{m} / \mathrm{s}$ ), when it hits the outer cylinder. [consider $\mathrm{V}=10, \mathrm{q}=-20, \mathrm{~m}=1$ all in S.I. Units]

28. A positive charge $Q$ is uniformly distributed throughout the volume of a nonconducting sphere of radius $R$. A point mass having charge $+q$ and mass $m$ is fired towards the centre of the sphere with velocity v from a point at distance $r(r>R)$ from the centre of the sphere. Find the minimum velocity $v$ so that it can penetrate $R / 2$ distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.

## Electric dipole

29. A plastic rod of length 1.0 m carries uniform positive charge $+4.0 \mu \mathrm{C}$ on half of its length and uniform negative charge $-4.0 \mu \mathrm{C}$ on the remaining half of its length. Find magnitude of it's net dipole moment in $\mu \mathrm{C}-\mathrm{m}$.
30. A dipole of dipole moment $\vec{p}=2 \hat{i}-3 \hat{j}+4 \hat{k}$ is placed at point $\mathrm{A}(2,-3,1)$. The electric potential due to this dipole at the point $\mathrm{B}(4,-1,0)$ is $(\mathrm{ab}) \times 10^{9}$ volt here 'a' represents sign (for negative answer select 0 for positive answer select 1 . Write the value of $(a+b)^{2}$. The parameters specified here are in S.I. units.
31. A dipole is placed at origin of coordinate system as shown in figure, find the electric field at point $\mathrm{P}(0, \mathrm{y})$.

32. A small rigid object carries positive and negative charge of magnitude 4 coulomb each. It is oriented so that the positive charge has coordinates $(-1.2 \mathrm{~mm}, 1.1 \mathrm{~mm})$ and negative charge has coordinates $(1.4 \mathrm{~mm},-1.3 \mathrm{~mm})$. The object is kept in an electric field of $(2500 \hat{\mathrm{i}}-5000 \hat{\mathrm{j}}) \mathrm{N} / \mathrm{C}$. Find the magnitude of torque (in $\mathrm{N}-\mathrm{m}$ ) acting on the dipole.
33. A charge $+Q$ is fixed at the origin of the coordinate system while a small electric dipole of dipole-moment $\vec{p}$ pointing away from the charge along the $x$-axis is set free from a point far away from the origin.
(a) calculate the K.E. of the dipole when it reaches to a point $(d, 0)$
[IIT-JEE 2003]
(b) calculate the force on the charge $+Q$ due to the dipole at this moment.

## Conductors

34. A conducting sphere of radius $R$ has two spherical cavities of radius $a$ and $b$. The cavities have charges $q_{a}$ and $q_{b}$ respectively at their centres. Find:
(a) The electric field and electric potential at a distance $r$
(i) r (distance from O , the centre of sphere $>\mathrm{R}$ )
(ii) $r$ (distance from $B$, the centre of cavity $b$ ) $<b$
(b) Surface charge densities on the surface of radius $R$, radius $a$ and radius $b$.
(c) What is the force on $q_{a}$ and $q_{b}$ ?

35. Two thin conducting shells of radii $R$ and $3 R$ are shown in figure. The outer shell carries a charge $+Q$ and the inner shell is neutral. The inner shell is earthed with the help of switch $S$. Find the charge attained by the inner shell.

36. A point charge $q$ is placed at a distance $2 r$ from the centre $O$ of a conducting neutral sphere of radius r. Due to the induced charge on the sphere, the electric potential at point $P$ on the surface of sphere is $x$ volt. Then find the value of $x$. (If $\frac{\mathrm{kq}}{\mathrm{r}}=18$ volt)

37. A conducting liquid bubble of radius a and thickness $t(t \ll a)$ is charged to potential V . If the bubble collapses to a droplet, find the potential on the droplet.
[IIT-JEE 2005]
38. Three uncharge conducing large plates are placed parallel to each other in a uniform electric field. Find the induced charge density on each surface of each plate.


## EXERCISE (S-2)

1. A thin long strip whose cross-section is a semicircle carries a uniform surface charge of density $\sigma$ on its inner surface. Find the electric field at a point O located midway on its axis.

2. An infinitely long non-conducting plane of charge density $\sigma$ has circular aperture of certain radius carved out from it. The electric field at a point which is at a distance ' $a$ ' from the centre of the aperture is $\sigma / 2 \sqrt{ } 2 \epsilon_{0}$. Find the radius of aperture.

3. A disc of radius $R$ is kept such that its axis coincide with the $x$-axis and its centre is at $(d, 0,0)$. The thickness of disc is $t$ and it carries a uniform volume charge density $\rho$. The external electric field in the space is given by $\vec{E}=K \vec{r}$ where $K=$ Constant and $\vec{r}$ is position vector of any point in space with respect to the origin of the coordinate system. Find the electric force on the disc.
4. Two mutually perpendicular infinite wires along $x$-axis and $y$-axis carry charge densities $\lambda_{1}$ and $\lambda_{2}$. The electric line of force at $P$ is along the line $y=\frac{1}{\sqrt{3}} x$, where $P$ is also a point lying on the same line then find $\lambda_{2} / \lambda_{1}$.

5. An electric field given by $\overrightarrow{\mathrm{E}}=4 \hat{\mathrm{i}}-3\left(\mathrm{y}^{2}+2\right) \hat{\mathrm{j}}$ pierces Gaussian's cube of side 1 m placed at origin such that its three sides represents $\mathrm{x}, \mathrm{y}$ and z axes. The net charge enclosed within the cube is given by $-n \varepsilon_{0}$. Find the value of $n$.

6. The charge $\mathrm{Q}=\pi \mathrm{C}$ is distributed on a thin semicircular ring of radius $\mathrm{R}=2 \mathrm{~m}$. There is a uniform electrostatic field $|\vec{E}|=2 N / C$ directed horizontally. The semicircular ring can rotate freely about a fixed vertical axis $A B$. Initially the ring is in static equilibrium as shown in figure. If we want to rotate it about the fixed axis by $90^{\circ}$ then minimum work required on the ring is $x \mathrm{~J}$. Find the value of x .

7. Four point charges $+8 \mu \mathrm{C},-1 \mu \mathrm{C},-1 \mu \mathrm{C}$ and $+8 \mu \mathrm{C}$, are fixed at the points, $-\sqrt{\frac{27}{2}} \mathrm{~m},-\sqrt{\frac{3}{2}} \mathrm{~m},+\sqrt{\frac{3}{2}} \mathrm{~m}$ and $+\sqrt{\frac{27}{2}} \mathrm{~m}$ respectively on the y -axis. A particle of mass $6 \times 10^{-4} \mathrm{~kg}$ and of charge $+0.1 \mu \mathrm{C}$ moves along the $-x$ direction. Its speed at $x=+\infty$ is $v_{0}$. Find the least value of $v_{0}$ for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. (Given: $\left.1 /\left(4 \pi \varepsilon_{0}\right)=9 \times 10^{9} \mathrm{Nm}^{2} / \mathrm{C}^{2}\right)$
8. The electric potential in a region is given by $V(x, y, z)=a x^{2}+a y^{2}+a b z^{2}$. ' $a$ ' is a positive constant of appropriate dimensions and b , a positive constant such that V is volts when $\mathrm{x}, \mathrm{y}, \mathrm{z}$ are in m . Let $\mathrm{b}=2$. The work done by the electric field when a point charge $+4 \mu \mathrm{C}$ moves from the point $(0,0,0.1 \mathrm{~m})$ to the origin is $50 \mu \mathrm{~J}$. The radius of the circle of the equipotential curve corresponding to $\mathrm{V}=6250$ volts and $\mathrm{z}=\sqrt{2} \mathrm{~m}$ is $\alpha \mathrm{m}$. Fill $\alpha^{2}$ in OMR sheet.
9. A nonconducting ring of mass mand radius R is charged as shown. The charged density i.e. charge per unit length is $\lambda$. It is then placed on a rough nonconducting horizontal surface plane. At time $t=0$, a uniform electric field $\overrightarrow{\mathrm{E}}=\mathrm{E}_{0} i$ is switched on and the ring start rolling without sliding.


Determine the friction force (magnitude and direction) acting on the ring, when it starts moving.
10. Small identical balls with equal charges are fixed at vertices of regular 2008-gon with side $a$. At a certain instant, one of the balls is released \& a sufficiently long time interval later, the ball adjacent to the first released ball is freed. The kinetic energies of the released balls are found to differ by $K$ at a sufficiently long distance from the polygon. Determine the charge $q$ of each ball.
11. Two concentric rings of radii $r$ and $2 r$ are placed with centre at origin. Two charges $+q$ each are fixed at the diametrically opposite points of the rings as shown in figure. Smaller ring is now rotated by an angle $90^{\circ}$ about Z -axis then it is again rotated by $90^{\circ}$ about Y -axis. Find the work done by electrostatic forces in each step. If finally larger ring is rotated by $90^{\circ}$ about X -axis, find the total work required to perform all three steps.

12. A non-conducting disc of radius a and uniform positive surface charge density $\sigma$ is placed on the ground, with its axis vertical. A particle of mass $m$ \& positive charge $q$ is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{\mathrm{q}}{\mathrm{m}}=\frac{4 \varepsilon_{0} \mathrm{~g}}{\sigma}$.
(a) Find the value of H if the particle just reaches the disc.
(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.
13. $S_{1}$ and $S_{2}$ are two conducting surfaces. Between $S_{1}$ and $S_{2}$ and inside $S_{1}$ is air. $S_{1}$ is spherical with $A$ its centre. $\mathrm{S}_{1}$ has total charge Q . $\mathrm{S}_{2}$ is uncharged. Find (if possible) :
(i) Charges induced on inner and outer surface of $\mathrm{S}_{2}$.
(ii) Total electric field at $\mathrm{A}, \mathrm{B}$.
(iii) Electric field at B due to induced charges on $\mathrm{S}_{2}$.
(iv) Electric field at C due to induced charges on inner surface of $\mathrm{S}_{2}$.
(v) Electric field produced by induced charges on outer surface of $S_{2}$ inside the body of $S_{2}$.
(vi) Can you find electric field at C easily ?
(take the required distances from A ). Which charge will produce electric field here.

14. The figure shows a conducting sphere ' A ' of radius ' a ' which is surrounded by a neutral conducting spherical shell B of radius 'b' (>a). Initially switches $S_{1}, S_{2}$ and $S_{3}$ are open and sphere 'A' carries a charge Q . First the switch 'S ${ }_{1}$ ' is closed to connect the shell B with the ground and then opened. Now the switch 'S2' is closed so that the sphere 'A' is grounded and then $\mathrm{S}_{2}$ is opened. Finally, the switch ' $\mathrm{S}_{3}$ ' is closed to connect the spheres together. Find the heat (in joule) which is produced after closing the switch $\mathrm{S}_{3}$. [Consider $\mathrm{b}=4 \mathrm{~cm}, \mathrm{a}=2 \mathrm{~cm}$ and $\mathrm{Q}=80 \mu \mathrm{C}$ ]

15. Consider a metal sphere, of radius $R$ that is cut in two along a plane whose minimum distance from the sphere's centre is h . Sphere is uniformly charged by a total electric charge Q . What force is necessary to hold the two parts of the sphere together ?

16. Two fixed charges $-2 Q$ and $Q$ are located at the points with co-ordinates $(-3 \mathrm{a}, 0)$ and $(3 \mathrm{a}, 0)$ respectively in the $x-y$ plane. (a) Show that all the points in the $x-y$ plane where the electric potential due to the two charges is zero lie on a circle. Find its radius and the location of its centre. (b) Give the expression for the potential $\mathrm{V}_{(\mathrm{x})}$ at a general point on the x -axis and sketch the function $\mathrm{V}_{(\mathrm{x})}$ on the whole $x$-axis. (c) If a particle of charge $+q$ starts from rest at the centre of the circle, show by a short qualitative argument that the particle eventually crosses the circle. Find its speed when it does so.
17. The electric field strength depends only on the $x, y$ and $z$ coordinates according to the law $\mathrm{E}=\frac{a(x \hat{i}+y \hat{j}+z \hat{k})}{\left(x^{2}+y^{2}+z^{2}\right)^{3 / 2}}$, where $\mathrm{a}=122.5$ SI unit and is a constant. Find the potential difference (in volt) between $(3,2,6)$ and $(0,3,4)$.

## EXERCISE (0-1)

## SINGLE CORRECT TYPE QUESTIONS

## Properties of charges and coulomb's law

1. If an object made of substance $A$ rubs an object made of substance $B$, then $A$ becomes positively charged and $B$ becomes negatively charged. If, however, an object made of substance $A$ is rubbed against an object made of substance C , then A becomes negatively charged. What will happen if an object made of substance $B$ is rubbed against an object made of substance $C$ ?
(A) B becomes positively charged and C becomes positively charged.
(B) B becomes positively charged and C becomes negatively charged.
(C) B becomes negatively charged and C becomes positively charged.
(D) B becomes negatively charged and C becomes negatively charged.
2. In normal cases thin stream of water bends toward a negatively charged rod. When a positively charged rod is placed near the stream, it will bend in the

(A) Opposite direction.
(B) Same direction.
(C) It won't bend at all.
(D) Can't be predicted.
3. Two charged bodies $A$ and $B$ exert repulsive forces on each other. If charge on $A$ is more then that on $B$, which of the following statement is true.
(A) Body A experiences more Colombian force then B.
(B) Body A experiences less Colombian force then B.
(C) Both of them experience Colombian forces of equal magnitude.
(D) It depends whether the bodies can be treated as point like charges or not.
4. Given are four arrangements of three fixed electric charges. In each arrangement, a point labeled $P$ is also identified - test charge, +q , is placed at point P . All of the charges are of the same magnitude, Q , but they can be either positive or negative as indicated. The charges and point $P$ all lie on a straight line. The distances between adjacent items, either between two charges or between a charge and point P , are all the same. Correct order of choices in a decreasing order of magnitude of force on P is
I. $+\odot+\underset{\mathrm{p}}{\bullet}$
II. $\odot \odot \stackrel{\mathrm{P}}{\bullet} \odot$
III.

IV. $+\odot \odot \stackrel{\mathrm{P}}{\bullet}$
(A) II $>$ I $>$ III $>$ IV
(B) I $>$ II $>$ III $>$ IV
(C) II $>$ I $>$ IV $>$ III
(D) III $>$ IV $>$ I $>$ II
5. Two point charge of $100 \mu \mathrm{C}$ and $-4 \mu \mathrm{C}$ are positioned at points $(-2 \sqrt{ } 3,3 \sqrt{ } 3,-4)$ and $(4 \sqrt{ } 3,-5 \sqrt{ } 3,6)$ respectively of a Cartesian coordinate system. Find the force vector on the $-4 \mu \mathrm{C}$ charge? All the coordinates are in meters.
(A) $9 \times 10^{-4}(3 \sqrt{3} \hat{i}-4 \sqrt{3} \hat{j}+5 \hat{k})$
(B) $9 \times 10^{-4}(-3 \sqrt{3} \hat{i}+4 \sqrt{3} \hat{j}-5 \hat{k})$
(C) $2.25 \times 10^{-4}(-3 \sqrt{3} \hat{i}+4 \sqrt{3} \hat{j}-5 \hat{\mathrm{k}})$
(D) $2.25 \times 10^{-4}(3 \sqrt{3} \hat{i}-4 \sqrt{3} \hat{j}+5 \hat{k})$

## Electric field

6. Five positive equal charges are placed at vertices of a regular hexagon and net electric field at the centre is $\mathrm{E}_{1}$. A negative charge having equal magnitude is placed sixth vertex and then net electric field is $E_{2}$. Find $\frac{E_{2}}{E_{1}}$.
(A) 2
(B) 1
(C) 3
(D) None of these
7. There are two point charges $q_{1}$ and $q_{2}$ lying on a circle of unit radius. Electric field intensity at the center of circle due to these charges is $\vec{E}$. Find the position vector of the center with respect to $\mathrm{q}_{2}$ if the position vector of the center with respect to $\mathrm{q}_{1}$ is $\vec{r}_{1}$.
(A) $\frac{\vec{E}+k q_{1} \vec{r}_{1}}{k q_{2}}$
(B) $-\left(\frac{\vec{E}+k q_{1} \vec{r}_{1}}{k q_{2}}\right)$
(C) $\frac{k q_{1} \vec{r}_{1}-\vec{E}}{k q_{2}}$
(D) $\frac{\vec{E}-k q_{1} \vec{r}_{1}}{k q_{2}}$
8. Three charges $+q,+2 q$ and $+4 q$ are connected by strings as shown in the figure. What is ratio of tensions in the strings AB and BC .

(A) $1: 2$
(B) $1: 3$
(C) $2: 1$
(D) $3: 1$
9. The variation of electric field between the two charges $q_{1}$ and $q_{2}$ along the line joining the charges is plotted against distance from $\mathrm{q}_{1}$ (taking rightward direction of electric field as positive) as shown in the figure. Then the correct statement is :-

(A) $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ are positive and $\mathrm{q}_{1}<\mathrm{q}_{2}$
(B) $q_{1}$ and $q_{2}$ are positive and $q_{1}>q_{2}$
(C) $q_{1}$ is positive and $q_{2}$ is negative and $q_{1}<q_{2}$
(D) $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ are negative and $\mathrm{q}_{1}<\mathrm{q}_{2}$
10. Particle $B$ of charge $Q$ and mass $m$ is in equilibrium under weight and electrostatics force applied by a fixed charged $A$, which is directly beneath the particle $B$ as shown in figure. When particle B is disturbed along vertical, the equilibrium is

(A) stable
(B) unstable
(C) neutral
(D) there can not be in equilibrium
11. A charge q is placed at the centroid of an equilateral triangle. Three equal charges Q are placed at the vertices of the triangle. The system of four charges will be in equilibrium if $q$ is equal to :-
(A) $\frac{-\mathrm{Q}}{\sqrt{3}}$
(B) $\frac{-Q}{3}$
(C) $-\mathrm{Q} \sqrt{3}$
(D) $\frac{\mathrm{Q}}{\sqrt{3}}$
12. A semi-infinite insulating rod has linear charge density $\lambda$. The electric field at the point $P$ shown in figure is :-

(A) $\frac{2 \lambda^{2}}{\left(4 \pi \varepsilon_{0} r\right)^{2}}$ at $45^{\circ}$ with AB
(B) $\frac{\sqrt{2} \lambda}{4 \pi \varepsilon_{0} r^{2}}$ at $45^{\circ}$ with AB
(C) $\frac{\sqrt{2} \lambda}{4 \pi \varepsilon_{0} r}$ at $45^{\circ}$ with AB
(D) $\frac{\sqrt{2} \lambda}{4 \pi \varepsilon_{0} r}$ at perpendicular to AB
13. The direction $(\theta)$ of $\overrightarrow{\mathrm{E}}$ at point P due to uniformly charged finite rod will be :-

(A) at angle $30^{\circ}$ from x -axis
(B) $45^{0}$ from x - axis
(C) $60^{\circ}$ from $x$-axis
(D) none of these
14. As shown in the figure, an insulating rod is set into the shape of a semicircle. The left half of the rod has a charge of $+Q$ uniformly distributed along its length, and the right half of the rod has a charge of -Q uniformly distributed along its length. What vector shows the correct direction of the electric field at point P , the centre of the semicircle ?
(A) A
(B) B
(C) C
(D) D
(D)

15. A nonconducting ring of radius $R$ has uniformly distributed positive charge $Q$. A small part of the ring, of length d , is removed $(d \ll R)$. The electric field at the centre of the ring will now be
(A) directed towards the gap, inversely proportional to $R^{3}$.
(B) directed towards the gap, inversely proportional to $R^{2}$.
(C) directed away from the gap, inversely proportional to $R^{3}$.
(D) directed away from the gap, inversely proportional to $R^{2}$.
16. The maximum electric field at a point on the axis a uniformly charged ring is $\mathrm{E}_{0}$. At how many points on the axis will the magnitude of electric field be $\mathrm{E}_{0} / 2$ :-
(A) 1
(B) 2
(C) 3
(D) 4
17. A particle of mass $m$, charge $-Q$ is constrained to move along the axis of a ring of radius $a$. The ring carries a uniform charge density $+\lambda$ along its circumference. Initially, the particle lies in the plane of the ring at a point where no net force acts on it. The period of oscillation of the particle when it is displaced slightly from its equilibrium position is
(A) $\mathrm{T}=4 \pi \sqrt{\frac{\varepsilon_{0} \mathrm{ma}^{2}}{\lambda \mathrm{Q}}}$
(B) $\mathrm{T}=2 \pi \sqrt{\frac{2 \varepsilon_{0} \mathrm{ma}^{2}}{\lambda \mathrm{Q}}}$
(C) $\mathrm{T}=2 \pi \sqrt{\frac{4 \varepsilon_{0} \mathrm{ma}^{2}}{\lambda \mathrm{Q}}}$
(D) $\mathrm{T}=2 \pi \sqrt{\frac{\varepsilon_{0} \mathrm{ma}^{2}}{2 \lambda \mathrm{Q}}}$
18. The surface charge density of a thin charged disc of radius $R$ is $\sigma$. The value of the electric field at the centre of the disc is $\frac{\sigma}{2 \epsilon_{0}}$. With respect to the field at the centre, the electric field along the axis at a distance $\sqrt{3} R$ from the centre of the disc :
(A) reduces by $70.7 \%$
(B) reduces by $29.3 \%$
(C) reduces by $86.6 \%$
(D) reduces by $13.4 \%$
19. A small ball of mass $m$ and charge $+q$ tied with a string of length $\ell$, is rotating in a vertical circle under gravity and a uniform horizontal electric field E as shown. The tension in the string will be minimum for:-

(A) $\theta=\tan ^{-1}\left(\frac{\mathrm{qE}}{\mathrm{mg}}\right)$
(B) $\theta=\pi$
(C) $\theta=\pi-\tan ^{-1}\left(\frac{\mathrm{qE}}{\mathrm{mg}}\right)$
(D) $\theta=\pi+\tan ^{-1}\left(\frac{\mathrm{qE}}{\mathrm{mg}}\right)$
20. A wheel having mass $m$ has charges $+q$ and $-q$ on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field $\mathrm{E}=$
(A) $\frac{\mathrm{mg}}{\mathrm{q}}$
(B) $\frac{m g}{2 q}$
(C) $\frac{m g \tan \theta}{2 q}$
(D) none
21. A negatively charged particle $p$ is placed, initially at rest, in a constant, uniform gravitational field and a constant, uniform electric field as shown in the diagram. What qualitatively, is the shape of the trajectory of the electron?


| p | g |
| :--- | :--- |
|  |  |

(A)

(B)

(C)

(D)
$\square$
22. A particle of mass $m$ and charge $q$ is attached to a light rod of length $L$. The rod can rotate freely in the plane of paper about the other end, which is hinged at P . The entire assembly lies in a uniform electric field $E$ also acting in the plane of paper as shown. The rod is released from rest when it makes an angle $\theta$ with the electric field direction. Determine the speed of the particle when the rod is parallel to the electric field.
(A) $\left(\frac{2 \mathrm{qEL}(1-\cos \theta)}{\mathrm{m}}\right)^{1 / 2}$
(B) $\left(\frac{2 q E L(1-\sin \theta)}{m}\right)^{1 / 2}$
(C) $\left(\frac{\mathrm{qEL}(1-\cos \theta)}{2 \mathrm{~m}}\right)^{1 / 2}$
(D) $\left(\frac{2 q E L \cos \theta}{m}\right)^{1 / 2}$

23. The fig. shows the distribution of three charges $-Q,+Q$ and $-Q$ on the $X$-axis. Which of the following figures shows the possible electric field lines for the distribution?

(A)

(B)

(C)

(D)


## Gauss' law

24. In the given figure flux through surface $S_{1}$ is $\phi_{1} \&$ through $S_{2}$ is $\phi_{2}$. Which is correct ?

(A) $\phi_{1}=\phi_{2}$
(B) $\phi_{1}>\phi_{2}$
(C) $\phi_{1}<\phi_{2}$
(D) None of these
25. A hemispherical surface (half of a spherical surface) of radius $R$ is located in a uniform electric field $E$ that is parallel to the axis of the hemisphere. What is the magnitude of the electric flux through the hemisphere surface?

(A) 0
(B) $4 \pi R^{2} E / 3$
(C) $2 \pi R^{2} E$
(D) $\pi R^{2} E$
26. Statement 1: A charge is outside the Gaussian sphere of radius $R$. Then electric field on the surface of sphere is zero.
and
Statement 2 : As $\oint \vec{E} \cdot d \vec{s}=\frac{q_{\text {in }}}{\varepsilon_{0}}$, for the sphere $\mathrm{q}_{\text {in }}$ is zero, so $\oint \vec{E} \cdot d \vec{s}=0$.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement- 1 is true, statement- 2 is false.
(D) Statement- 1 is false, statement-2 is true.
27. Figure shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centres of the spheres, at radial distance $R / 2$ from the center of sphere 1 . If the net electric field at point $P$ is zero and $Q_{1}$ is $64 \mu C$, what is $\mathrm{Q}_{2}$ (in $\mu \mathrm{C}$ ).

(A) 64
(B) 36
(C) 32
(D) 72
28. A sphere of radius $R$ carries charge density proportional to the square of the distance from the center: $\rho=\mathrm{Ar}^{2}$, where A is a positive constant. At a distance of $\mathrm{R} / 2$ from the center, the magnitude of the electric field is :-
(A) $A / 4 \pi \varepsilon_{0}$
(B) $\mathrm{AR}^{3} / 40 \varepsilon_{0}$
(C) $\mathrm{AR}^{3} / 24 \varepsilon_{0}$
(D) $\mathrm{AR}^{3} / 5 \varepsilon_{0}$
29. Three large parallel plates have uniform surface charge densities as shown in the figure. What is the electric field at $P$ ?
[IIT-JEE 2005 (Scr)]

(A) $-\frac{4 \sigma}{\epsilon_{0}} \hat{k}$
(B) $\frac{4 \sigma}{\epsilon_{0}} \hat{k}$
(C) $-\frac{2 \sigma}{\epsilon_{0}} \hat{k}$
(D) $\frac{2 \sigma}{\epsilon_{0}} \hat{k}$
30. A line of charge extends along a X -axis whose linear charge density varies directly as x . Imagine a spherical volume with its centre located on X -axis and is moving gradually along it. Which of the graphs shown in figure correspond to the flux $\phi$ with the x coordinate of the centre of the volume?

(A) a
(B) b
(C) c
(D) d
31. The electric field in a region is given by $\vec{E}=200 \hat{i} \mathrm{~N} / \mathrm{C}$ for $\mathrm{x}>0$ and $-200 \hat{i} \mathrm{~N} / \mathrm{C}$ for $\mathrm{x}<0$. A closed cylinder of length 2 m and cross-section area $10^{2} \mathrm{~m}^{2}$ is kept in such a way that the axis of cylinder is along X -axis and its centre coincides with origin. The total charge inside the cylinder is
[Take : $\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} \mathrm{~m}^{2} . \mathrm{N}$ )
(A) 0
(B) $1.86 \times 10^{-5} \mathrm{C}$
(C) $1.77 \times 10^{-11} \mathrm{C}$
(D) $35.4 \times 10^{-8} \mathrm{C}$
32. A point charge +Q is positioned at the center of the base of a square pyramid as shown. The flux through one of the four identical upper faces of the pyramid is :-

(A) $\frac{Q}{16 \varepsilon_{0}}$
(B) $\frac{Q}{4 \varepsilon_{0}}$
(C) $\frac{Q}{8 \varepsilon_{0}}$
(D) None of these
33. An infinite, uniformly charged sheet with surface charge density $\sigma$ cuts through a spherical Gaussian surface of radius R at a distance x from its center, as shown in the figure. The electric flux $\Phi$ through the Gaussian surface is :-

(A) $\frac{\pi R^{2} \sigma}{\varepsilon_{0}}$
(B) $\frac{2 \pi\left(\mathrm{R}^{2}-\mathrm{x}^{2}\right) \sigma}{\varepsilon_{0}}$
(C) $\frac{\pi(\mathrm{R}-\mathrm{x})^{2} \sigma}{\varepsilon_{0}}$
(D) $\frac{\pi\left(\mathrm{R}^{2}-\mathrm{x}^{2}\right) \sigma}{\varepsilon_{0}}$
34. Consider a circle of radius $R$. A point charge lies at a distance 'a' from its center and on its axis such that $R=a \sqrt{3}$. If electric flux passing through the circle is $\phi$ then the magnitude of the point charge is:-
(A) $\sqrt{3} \varepsilon_{0} \phi$
(B) $2 \varepsilon_{0} \phi$
(C) $4 \varepsilon_{0} \phi / \sqrt{3}$
(D) $4 \varepsilon_{0} \phi$

## Electric potential energy and electric potential

35. Two particles $X$ and $Y$, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u . After a long time, finally :-
(A) X will stop, Y will move with velocity u .
(B) X and Y will both move with velocities $\mathrm{u} / 2$ each.
(C) X will stop, Y will move with velocity $<\mathrm{u}$.
(D) both will move with velocities < u/2.
36. Two positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E .
(A) If Y is fixed, both p and E are conserved.
(B) If Y is fixed, E is conserved, but not p .
(C) If both are free to move, p is conserved but not E .
(D) If both are free, E is conserved, but not p .
37. Potential energy of a system comprising of point charges is $U_{1}$. When a charge $q$ is added in the system without disturbing other charges, the potential energy becomes $\mathrm{U}_{2}$. The potential of the point where the charge q is placed in the system is
(A) $\frac{U_{2}-U_{1}}{q}$
(B) $\frac{U_{1}-U_{2}}{q}$
(C) $\frac{U_{1}+U_{2}}{2 q}$
(D) $\frac{U_{2}-U_{1}}{2 q}$
38. Three charges $\mathrm{Q},+\mathrm{q}$ and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to :
[JEE 2000(Scr) 1 + 1]

(A) $\frac{-\mathrm{q}}{1+\sqrt{2}}$
(B) $\frac{-2 \mathrm{q}}{2+\sqrt{2}}$
(C) $-2 q$
(D) $+q$
39. Two fixed charges A and B of $5 \mu \mathrm{C}$ each are separated by a distance of 6 m . C is the mid point of the line joining $A$ and $B$. A charge ' Q ' of $-5 \mu \mathrm{C}$ is shot perpendicular to the line joining A and B through C with a kinetic energy of 0.06 J . The charge ' Q ' comes to rest at a point D . The distance CD is:-
(A) 3 m
(B) $\sqrt{3} \mathrm{~m}$
(C) $3 \sqrt{3} \mathrm{~m}$
(D) 4 m
40. Figure shows three circular arcs, each of radius $R$ and total charge as indicated. The net electric potential at the centre of curvature is :-

(A) $\frac{\mathrm{Q}}{4 \pi \epsilon_{0} \mathrm{R}}$
(B) $\frac{\mathrm{Q}}{2 \pi \epsilon_{0} \mathrm{R}}$
(C) $\frac{2 \mathrm{Q}}{\pi \in_{0} R}$
(D) $\frac{\mathrm{Q}}{\pi \epsilon_{0} R}$
41. The nuclear charge ( Ze ) is non-uniformly distribute within a nucleus of radius R . The charge density $\rho(\mathrm{r})$ [charge per unit volume] is dependent on the radial distance r from the centre of the nucleus as shown in figure. Select correct alternative/s.

(A) Electric field at $r=R$ is independent of $b$
(B) Electric potential at $\mathrm{r}=\mathrm{R}$ is proportional to b
(C) Electric field at $\mathrm{r}=\mathrm{R}$ is proportional to a
(D) Electric potential at $\mathrm{r}=\mathrm{R}$ is proportional to a
42. A solid sphere of radius $R$ is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre?
(A) R
(B) $\mathrm{R} / 2$
(C) R/3
(D) 2 R
43. When a negative charge is released and moves in electric field, it moves toward a position of
(A) lower electric potential and lower potential energy
(B) lower electric potential and higher potential energy
(C) higher electric potential and lower potential energy
(D) higher electric potential and higher potential energy
44. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge
(A) remains a constant because the electric field is uniform.
(B) increases because the charge moves along the electric field.
(C) decreases because the charge moves along the electric field.
(D) decreases because the charge moves opposite to the electric field.
45. The electric field intensity at all points in space is given by $\vec{E}=\sqrt{3} \hat{i}-\hat{j}$ volts/metre. The nature of equipotential lines in $x-y$ plane is given by :-

46. Equipotential at a great distance from a collection of charges whose total sum is not zero are approximately
(A) spheres.
(B) planes.
(C) paraboloids
(D) ellipsoids.
47. Three positive charges of equal value $q$ are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in
[JEE 2001 (Scr)]
(A)

(B)

(C)

(D)

48. In an electric field the potential at a point is given by the following relation $\mathrm{V}=\frac{343}{\mathrm{r}}$. The electric field at $\vec{r}=3 \hat{i}+2 \hat{j}+6 \hat{k}$ is :
(A) $21 \hat{i}+14 \hat{j}+42 \hat{k}$
(B) $3 \hat{i}+2 \hat{j}+6 \hat{k}$
(C) $\frac{1}{7}(3 \hat{i}+2 \hat{j}+6 \hat{k})$
(D) $-(3 \hat{i}+2 \hat{j}+6 \hat{k})$
49. From a point if we move in a direction making an angle $\theta$ measured from $+\mathrm{ve} x$-axis, the potential gradient is given as $\frac{d v}{d r}=2 \cos \theta$. Find the direction and magnitude of electric field at that point.
(A) $2 \hat{i}$
(B) $-2 \hat{i}$
(C) $\hat{i}+\hat{j}$
(D) $-\hat{i}+\hat{j}$
50. A uniform electric field having strength $\vec{E}$ is existing in $x-y$ plane as shown in figure. Find the p.d. between origin $\mathrm{O} \& A(\mathrm{~d}, \mathrm{~d}, 0)$

(A) $\operatorname{Ed}(\cos \theta+\sin \theta)$
(B) $-\mathrm{Ed}(\sin \theta-\cos \theta)$
(C) $\sqrt{2} \mathrm{Ed}$
(D) none of these
51. Figure shows equi-potential surfaces for a two charges system. At which of the labeled points point will an electron have the highest potential energy?

(A) Point A
(B) Point B
(C) Point C
(D) Point D
52. Figure shows some equipotential lines distributed in space. A charged object is moved from point $A$ to point B.

(A) The work done in Fig. (i) is the greatest.
(B) The work done in Fig. (ii) is least.
(C) The work done is the same in Fig. (i), Fig. (ii) and Fig. (iii).
(D) The work done in Fig. (iii) is greater than Fig. (ii)but equal to that in Fig. (i).
53. Consider the following conclusions regarding the components of an electric field at a certain point in space given by
$\mathrm{E}_{\mathrm{x}}=-K y$

$$
E_{y}=K x
$$

$\mathrm{E}_{\mathrm{z}}=0$
(I) The field is conservative.
(III) The lines of force are staright lines
(II) The field is non-conservative.
(IV) The lines of force are circles.

Of these conclusions
(A) II and IV are valid
(B) I and III are valid
(C) I and IV are valid
(D) II and III are valid

## Electric dipole

54. The drawing shows four points surrounding an electric dipole. Which one of the following expressions best ranks the electric potential at these four locations?

(1)
(4) $\cdot$
†p •(2)
(3)
(A) $1>2=4>3$
(B) $3>2>4>1$
(C) $3>2=4>1$
(D) $2=4>1=3$
55. Which of the following represents the equipotential lines of a dipole (two equal and opposite charges placed at small separation)?
(A)

(B)

(C)

(D)

56. Three point charges $2 \mathrm{q},-\mathrm{q}$ and -q are located respectively at $(0, a, a),(0, a,-a)$ and $(0,0,-a)$ as shown. The dipole moment of this distribution is :-
(A) $2 q a$ in the $y-z$ plane at $\tan ^{-1}\left(\frac{1}{4}\right)$ with $z$-axis
(B) $\sqrt{17} \mathrm{qa}$ in the $\mathrm{y}-\mathrm{z}$ plane at $\tan ^{-1}\left(\frac{1}{4}\right)$ with z -axis

(C) $\sqrt{5} \mathrm{qa}$ in the $\mathrm{x}-\mathrm{y}$ plane at $\tan ^{-1}(4)$ with y -axis
(D) $4 q a$ in the $x-y$ plane at $\tan ^{-1}$ (4) with $y$-axis
57. Point P lies on the axis of a dipole. If the dipole is rotated by $90^{\circ}$ anticlock wise, the electric field vector $\overrightarrow{\mathrm{E}}$ at P will rotate by
(A) $90^{\circ}$ clock wise
(B) $180^{\circ}$ clock wise
(C) $90^{\circ}$ anti clock wise
(D) none
58. A water molecule as shown is in a region of uniform electric field $\overrightarrow{\mathrm{E}}=1000 \hat{\mathrm{i}} \mathrm{V} / \mathrm{m}$. This molecule experiences
(A) A counterclockwise torque
(B) A clockwise torque
(C) A net force to the right
(D) A net force to the left
59. Electric field lines in which an electric dipole $\mathbf{p}$ is placed as shown.

Which of the following statements is correct?
(A) The dipole will not experience any force.
(B) The dipole will experience a force towards right.
(C) The dipole will experience a force towards left.
(D) The dipole will experience a force upwards.

60. A large sheet carries uniform surface charge density $\sigma$. A rod of length $2 \ell$ has a linear charge density $\lambda$ on one half and $-\lambda$ on the other half. The rod is hinged at mid point $O$ and makes angle $\theta$ with the normal to the sheet. The torque experienced by the rod is :-

(A) $\frac{\sigma \lambda \ell^{2}}{2 \varepsilon_{0}} \cos \theta$
(B) $\frac{\sigma \lambda \ell}{\varepsilon_{0}} \cos ^{2} \theta$
(C) $\frac{\sigma \lambda \ell^{2} \sin \theta}{2 \varepsilon_{0}}$
(D) $\frac{\sigma \lambda \ell \sin ^{2} \theta}{\varepsilon_{0}}$
61. An electric dipole ( dipole moment $p$ ) is placed at a radial distance $r \gg a$ (where $a$ is dipole length) from a infinite line of charge having linear charge density $+\lambda$. Dipole moment vector is aligned along radial vector $\overrightarrow{\mathrm{r}}$ force experienced by dipole is :-
(A) $\frac{\lambda \mathrm{p}}{2 \pi \varepsilon_{0} \mathrm{r}^{2}}$, attractive
(B) $\frac{\lambda \mathrm{p}}{2 \pi \varepsilon_{0}{ }^{3}}$, attractive
(C) $\frac{\lambda \mathrm{p}}{2 \pi \varepsilon_{0} \mathrm{r}^{2}}$, repulsive
(D) $\frac{\lambda p}{2 \pi \varepsilon_{0} \mathrm{r}^{3}}$, repulsive

## Conductors

62. Both question (a) and (b) refer to the system of charges as shown in the figure. A spherical shell with an inner radius 'a' and an outer radius ' b ' is made of conducting material. A point charge +Q is placed at the centre of the spherical shell and a total charge -q is placed on the shell.
a. Charge -q is distributed on the surfaces as

(A) $-Q$ on the inner surface, $-q$ on outer surface
(B) -Q on the inner surface, $-\mathrm{q}+\mathrm{Q}$ on the outer surface
(C) +Q on the inner surface, $-\mathrm{q}-\mathrm{Q}$ on the outer surface
(D) The charge -q is spread uniformly between the inner and outer surface.
b. Assume that the electrostatic potential is zero at an infinite distance from the spherical shell. The electrostatic potential at a distance $\mathrm{R}(\mathrm{a}<\mathrm{R}<\mathrm{b})$ from the centre of the shell is (where $\mathrm{K}=\frac{1}{4 \pi \varepsilon_{0}}$ )
(A) 0
(B) $\frac{K Q}{a}$
(C) $K \frac{Q-q}{R}$
(D) $\mathrm{K} \frac{\mathrm{Q}-\mathrm{q}}{\mathrm{b}}$
63. The electrostatic potential on the surface of a charged conducting sphere is 100 V . Two statements are made in this regard:
$\mathrm{S}_{1}$ : At any point inside the sphere, electric intensity is zero.
$\mathrm{S}_{2}$ : At any point inside the sphere, the electrostatic potential is 100 V .
Which of the following is a correct statement?
(A) $S_{1}$ is true but $S_{2}$ is false.
(B) Both $S_{1} \& S_{2}$ are false.
(C) $S_{1}$ is true, $S_{2}$ is also true and $S_{1}$ is the cause of $S_{2}$.
(D) $\mathrm{S}_{1}$ is true, $\mathrm{S}_{2}$ is also true but the statements are independent.
64. Figure shows two shells of radii $R$ and $2 R$. The inner shell (centre at $A$ ) is nonconducting and uniformly charged wih charge Q while the outer shell (centre at B ) is conducting and uncharged. the potential at the point B is :-

(A) zero
(B) $\frac{K Q}{R}$
(C) $\frac{K Q}{x}$
(D) None of these
65. Charge $\mathrm{Q}, 2 \mathrm{Q}$ and -Q are given to three concentric conducting spherical shells $\mathrm{A}, \mathrm{B}$ and C respectively. The ratio of charges on the inner and the outer surfaces of the shell ' $C$ ' will be :-

66. If the electric potential of the inner metal sphere is 10 volt \& that of the outer shell is 5 volt, then the potential at the centre will be :

(A) 10 volt
(B) 5 volt
(C) 15 volt
(D) 0
67. $n$ small drops of same size are charged to V volts each. If they coalesce to form a signal large drop, then its potential will be :-
(A) $\mathrm{V} / \mathrm{n}$
(B) Vn
(C) $\mathrm{Vn}^{1 / 3}$
(D) $\mathrm{Vn}^{2 / 3}$
68. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as :

(A) 1
(B) 2
(C) 3
(D) 4
69. A point positive charge is brought near an isolated conducting sphere. The electric field is best given by

(A) Fig (i)
(B) Fig (iii)
(C) Fig (ii)
(D) Fig (iv)
70. There are two uncharged identical metallic spheres 1 and 2 of radius $r$ separated by a distance d ( $\mathrm{d} \gg \mathrm{r}$ ). A charged metallic sphere of same radius having charge q is touched with one of the sphere. After some time it is moved away from the system. Now the uncharged sphere is earthed. Charge on earthed sphere is
(A) q
(B) -q
(C) $-\mathrm{qr} / 2 \mathrm{~d}$
(D) 0
71. Three conducting concentric spherical shells of radius $R, 2 R$ and $3 R$ have charges $Q, \frac{Q}{3}$ and $-2 Q$ respectively. The intermediate shell is now grounded. Find the charge flow into the earth.

(A) $\frac{\mathrm{Q}}{3}$
(B) $\frac{2 Q}{3}$
(C) Q
(D) 0
72. Figure shows a system of three concentric metal shells $A, B$ and $C$ with radii a, 2a and 3a respectively. Shell $B$ is earthed and shell $C$ is given a charge Q . Now if shell C is connected to shell A , then the final charge on the shell B , is equal to :
(A) $-\frac{4 Q}{13}$
(B) $-\frac{8 Q}{11}$
(C) $-\frac{5 Q}{3}$
(D) $-\frac{3 Q}{7}$

73. Figure shows two conducting thin concentric shells of radii $r$ and $3 r$. The outer shell carries charge $q$ and inner shell is neutral. The amount of charge which flows from inner shell to the earth after the key K is closed, is equal to :-

(A) $-q / 3$
(B) $q / 3$
(C) $3 q$
(D) $-3 q$
74. Statement-1 : A point charge $q$ is placed inside a cavity of conductor as shown. Another point charge Q is placed outside the conductor as shown. Now as the point charge Q is pushed away from conductor, the potential difference $\left(V_{A}-V_{B}\right)$ between two points $A$ and $B$ within the cavity of sphere remains constant.
Statement-2 : The electric field due to charges on outer surface of conductor and outside the conductor is zero at all points inside the conductor.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement- 1 is true, statement- 2 is false.
(D) Statement-1 is false, statement-2 is true.
75. A point charge $q$ is placed at a point inside a hollow conducting sphere. Which of the following electric force pattern is correct?
[IIT-JEE'2003 (scr)]
(A)

(B)

(C)

(D)

76. A point charge $q$ is placed at a distance $r$ from center of a conducting neutral sphere of radius $R(r>R)$. The potential at any point $P$ inside the sphere at a distance $r_{1}$ from point charge due to induced charge of the sphere is given by :-

(A) $\frac{\mathrm{Kq}}{\mathrm{r}_{1}}-\frac{\mathrm{Kq}}{\mathrm{R}}$
(B) $\frac{\mathrm{Kq}}{\mathrm{r}_{1}}-\frac{\mathrm{Kq}}{\mathrm{R}}$
(C) $\frac{\mathrm{Kq}}{\mathrm{r}}-\frac{\mathrm{Kq}}{\mathrm{r}_{1}}$
(D) $-\frac{\mathrm{Kq}}{\mathrm{r}_{1}}+\frac{\mathrm{Kq}}{\mathrm{R}}$
77. Consider a conductor with a spherical cavity in it. A point charge $\mathrm{q}_{0}$ is placed at the centre of cavity and a point charge Q is placed outside conductor.
Statement-1 : Total charge induced on cavity wall is equal and opposite to the charge inside.
Statement-2 : If cavity is surrounded by a Gaussian surface, where all parts of Gaussian surface are located inside the conductor, $\oint \vec{E} . d \vec{A}=0$; hence $q_{\text {induced }}=-q_{0}$
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.
78. Two non-conducting hemispherical surfaces, which are having uniform charge density $\sigma$ are placed on smooth horizontal surface as shown in figure. Assuming springs are ideal, calculate compression in each spring if both the hemispherical surface are just touching each other.

(A) $\frac{\sigma^{2} R^{2}}{2 \varepsilon_{0} k}$
(B) R
(C) $\frac{\sigma^{2} \pi R^{2}}{2 \varepsilon_{0} k}$
(D) None of these
79. Electric field in a region is found to be $E=3 y \hat{j}$. The total energy stored in electric field inside the cube shown will be
(A) $9 a^{5} \in_{0}$
(B) $3 \in_{0} \mathrm{a}^{5}$
(C) $\frac{3}{2} \in_{0} a^{5}$
(D) 0


## MULTIPLE CORRECT TYPE QUESTIONS

80. If a body is charged by rubbing it with another body then its weight :-
(A) may increase slightly
(B) may decrease slightly
(C) must increase slightly
(D) remains precisely constant
81. An electron is placed just in the middle between two long fixed line charges of charge density $+\lambda$ each. The wires are in the xy plane (Do not consider gravity)
(A) The equilibrium of the electron will be unstable along x -direction
(B) The equilibrium of the electron will be neutral along y-direction

(C) The equilibrium of the electron will be stable along $z$-direction
(D) The equilibrium of the electron will be stable along y-direction
82. Which statement(s) concerning electrostatic fields is/are CORRECT?
(A) Electric field lines never cross each other.
(B) Positive charge experiences force in the direction of electric field line.
(C) Electric field lines always start on negative charges and end on positive charges.
(D) Electric field lines that are closer indicate a stronger electric field while electric field lines that are far apart indicate a weaker electric field.
83. Two point charges $Q$ and $-\frac{Q}{4}$ are separated by distance $L$ then

(A) Potential is zero at a point on the axis which is at a distance $L / 3$ on the right side of charge $-\frac{\mathrm{Q}}{4}$
(B) Potential is zero at a point on the axis which is at a distance $L / 5$ on the left side of charge $-\frac{\mathrm{Q}}{4}$
(C) Electric field is zero at a point on the axis which is at a distance $L$ on the right side of charge $-\frac{Q}{4}$
(D) There exist two points on the axis, where electric field is zero
84. Graph shows the variation of potential (V), magnitude of electric field (E), charge enclosed $\left(q_{\text {enclosed }}\right)$ within the concentric sphere and net flux ( $\phi$ ) through a concentric spherical Gaussian surface as a function of distance (r) from centre of a uniformly positive charged solid sphere of radius R. Choose the CORRECT option(s):-
(A)

(B)

(C)

(D)

85. Figure shows three spherical shells in separate situations, with each shell having the same uniform positive net charge. Points 1, 4 and 7 are at the same radial distances from the centre of the their respective shells; so are points 2,5 and 8 ; and so are points 3,6 and 9 . With the electric potential taken equals to zero at an infinite distance, choose correct statement.



(A) Point 3 has highest potential
(B) point 1, 4 and 7 are at same potential
(C)Point 9 has lowest potential
(D) point 5 and 8 are at same potential
86. Which of the following represent(s) an electrostatic field :-
(A) $\overrightarrow{\mathrm{E}}=y \hat{\mathrm{i}}+x \hat{\mathrm{j}}$
(B) $\overrightarrow{\mathrm{E}}=y \hat{\mathrm{i}}$
(C) $\overrightarrow{\mathrm{E}}=y \hat{i}+x \hat{j}+z \hat{k}$
(D) $\overrightarrow{\mathrm{E}}=\alpha \overrightarrow{\mathrm{r}}$ ( $\alpha$ is constant)
87. Three identical, parallel conducting plates $A, B$ and $C$ are placed as shown. Switches $S_{1}$ and $S_{2}$ are open, and can connect A and C to earth when closed. +Q charge is given to B .

(A) If $S_{1}$ is closed with $S_{2}$ open, a charge of amount $Q$ will pass through $S_{1}$
(B) If $\mathrm{S}_{2}$ is closed with $\mathrm{S}_{1}$ open, a charge of amount Q will pass through $\mathrm{S}_{2}$
(C) If $S_{1}$ and $S_{2}$ are closed together, a charge of amount $Q / 3$ will pass through $S_{1}$, and a charge of amount $2 Q / 3$ will pass through $S_{2}$.
(D) All the above statements are incorrect
88. A positive charge Q is uniformly distributed along a circular ring of radius $R$. A small test charge $q$ is placed at the centre of the ring. Then

(A) If $q>0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre.
(B) If $q<0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring.
(C) If $q<0$, it will perform SHM for small displacement along the axis.
(D) $q$ at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q>0$.
89. If there were only one type of charge in the universe, then
(A) $\oint_{s} E \cdot d S \neq 0$ on any surface.
(B) $\oint_{s} E \cdot d S=0$ if the charge is outside the surface.
(C) $\oint_{s} E \cdot d S$ could not be defined.
(D) $\oint_{s} E . d S=\frac{q}{\varepsilon_{0}}$ if charges of magnitude $q$ were inside the surface.
90. Figure shows two uniform charged concentric spherical shell. Both charges are positive, Select correct statement

(A) Electric field intensity at B may be greater than electric field intensity at C .
(B) Electric field intensity at B must be greater than electric field intensity at C .
(C) Potential at A greater than potential at B
(D) If a charge moves from B to C work done by electric force must be positive.
91. Which of the following is true for the figure showing electric lines of force? ( E is electrical field, V is potential)

(A) $E_{A}>E_{B}$
(B) $\mathrm{E}_{\mathrm{B}}>\mathrm{E}_{\mathrm{A}}$
(C) $V_{A}>V_{B}$
(D) $V_{B}>V_{A}$
92. Consider a uniform electric field in the $\hat{z}$ direction. The potential is a constant
(A) in all space.
(B) for any $x$ for a given $z$.
(C) for any $y$ for a given $z$.
(D) on the $x-y$ plane for a given $z$.
93. Two large conducting sheets are kept parallel to each other as shown. In equilibrium, the charge density on facing surfaces is $\sigma_{1}$ and $\sigma_{2}$. What is the value of electric field at A .

$$
\left\|\begin{array}{c}
\sigma_{1} \\
. \\
\sigma_{2}
\end{array}\right\| \longleftrightarrow^{y}
$$

(A) $\frac{\sigma_{1}}{\varepsilon_{0}} \hat{\mathrm{i}}$
(B) $-\frac{\sigma_{2}}{\varepsilon_{0}} \hat{\mathrm{i}}$
(C) $\frac{\sigma_{1}+\sigma_{2}}{2 \varepsilon_{0}} \hat{\mathrm{i}}$
(D) $\frac{\sigma_{1}-\sigma_{2}}{2 \varepsilon_{0}} \hat{\mathrm{i}}$
94. A hollow closed conductor of irregular shape is given some charge. Which of the following statements are correct?
(A) The entire charge will appear on its outer surface.
(B) All points on the conductor will have the same potential.
(C) All points on its surface will have the same charge density.
(D) All points just outside it will have the same electric intensity.
95. A conducting body is given charge Q and charge -q has been placed in each of the cavity, which of the following statements is/are true?

(A)If $\mathrm{Q}=2 \mathrm{q}$, then conducting body will be at zero potential.
(B) If an external electric field is applied then the charge distribution on the outer surface of conductor would change.
(C) The potential of any point inside the cavity is less than that of conducting body.
(D)None of these

## COMPREHENSION TYPE QUESTIONS

## Paragraph for Question No. 96 to 98

Four metallic plates are placed as shown in the figure. Plate 2 is given a charge Q whereas all other plates are uncharged. Plates 1 and 4 are joined together. The area of each plate is same.

96. The charge appearing on the right side of plate 3 is
(A) 0
(B) $+\mathrm{Q} / 4$
(C) $-3 \mathrm{Q} / 4$
(D) $\mathrm{Q} / 2$
97. The charge appearing on right side of plate 4 is
(A) 0
(B) $-\mathrm{Q} / 4$
(C) $-3 Q / 4$
(D) $+\mathrm{Q} / 2$
98. The potential difference between plates 1 and 2 is
(A) $\frac{3}{2} \frac{\mathrm{Qd}}{\varepsilon_{0} \mathrm{~A}}$
(B) $\frac{\mathrm{Qd}}{\varepsilon_{0} \mathrm{~A}}$
(C) $\frac{3}{4} \frac{\mathrm{Qd}}{\varepsilon_{0} \mathrm{~A}}$
(D) $\frac{3 \mathrm{Qd}}{\varepsilon_{0} \mathrm{~A}}$

## Paragraph for Questions No. 99 to 101

Three charged particles each of $+Q$ are fixed at the corners of an equilateral triangle of side ' $a$ '. A fourth particle of charge -q and mass m is placed at a point on the line passing through centroid of triangle and perpendicular to the plane of triangle at a distance x from the centre of triangle.

99. Magnitude of resultant force on the fourth charged particle is
(A) $\frac{1}{4 \pi \epsilon_{0}} \frac{9 \sqrt{3} Q q x}{\left(3 x^{2}+a^{2}\right)^{3 / 2}}$
(B) $\frac{1}{4 \pi \epsilon_{0}} \frac{27 \sqrt{3} Q q x}{\left(3 x^{2}+a^{2}\right)^{3 / 2}}$
(C) $\frac{1}{4 \pi \epsilon_{0}} \frac{2 \sqrt{2} Q q x}{\left(2 x^{2}+a^{2}\right)^{3 / 2}}$
(D) $\frac{1}{4 \pi \epsilon_{0}} \frac{4 \sqrt{2} Q q x}{\left(2 x^{2}+a^{2}\right)^{3 / 2}}$
100. Value of $x$ for which the force is maximum is
(A) $\frac{\mathrm{a}}{\sqrt{3}}$
(B) $\frac{\mathrm{a}}{\sqrt{2}}$
(C) $\frac{\mathrm{a}}{\sqrt{6}}$
(D) $\frac{\mathrm{a}}{\sqrt{5}}$
101. For small oscillation the period of oscillation of fourth particle is
(A) $2 \pi \sqrt{\frac{4 \pi \epsilon_{0} m a^{3}}{9 \sqrt{3} Q q}}$
(B) $\pi \sqrt{\frac{4 \pi \epsilon_{0} m a^{3}}{9 \sqrt{3} Q q}}$
(C) $2 \pi \sqrt{\frac{2 \pi \epsilon_{0} m a^{3}}{27 \sqrt{3} Q q}}$
(D) $2 \pi \sqrt{\frac{\pi \epsilon_{0} m a^{3}}{27 \sqrt{3} Q q}}$

## Paragraph for Question No. 102 and 103

The figure applies to the following two questions. Positive and negative charges of equal magnitude lie along the symmetry axis of a cylinder. The distance from the positive charge to the left end-cap of the cylinder is the same as the distance from the negative charge to the right end -cap.

102. What is the flux of the electric field through the closed cylinder?
(A) 0
(B) $+\mathrm{Q} / \varepsilon_{0}$
(C) $+2 \mathrm{Q} / \varepsilon_{0}$
(D) $-\mathrm{Q} / \varepsilon_{0}$
103. What is the sign of the flux through the right end-cap of the cylinder ?
(A) Positive
(B) Negative
(C) There is no flux through the right end-cap.
(D) None of these

## Paragraph for Question No. 104 to 106

There is a cubical cavity inside a conducting sphere of radius R . A positive point charge Q is placed at the centre of the cube and another positive charge q is placed at a distance $l(>\mathrm{R})$ from the centre of the sphere. The sphere is earthed

104. Charge induced on the inner surface of cavity is
(A) -Q, uniformly distributed
(B) -Q, non-uniformly distributed
(C) $-(\mathrm{Q}+\mathrm{q})$ non-uniformly distributed
(D) none
105. Net charge on the outer surface of conducting sphere is
(A) +Q
(B) $\mathrm{Q}-\mathrm{qR} / l$
(C) $-q R / l$
(D) none
106. Potential at a point inside the cavity is
(A) zero
(B) positive
(C) negative
(D) can not be determined.

## MATRIX MATCH TYPE QUESTION

107. Column II corresponds to the graph of magnitude of electric field versus distance from centre of charge distribution in Column I.

## Column-I

(A) Ring along its axis
(B) Uniformly charged solid sphere
(C) Uniformly charged spherical shell
(D) Combination of charge +Q and -Q at the perpendicular bisector

## Column-II

(P)

(Q)

(R)

(S)

(T)

108. As shown in column I their are graphs of electric field (E) and potential $(V)$ along the line joining charges $Q_{1}$ and $Q_{2}$ are drawn against distance (r) on $x$-axis for charges $Q_{1}$ and $Q_{2}$. Take potential at infinity equal to zero. [Take direction of E in righward direction as positive]

## Column-I

(A)

(B)


## Column-II

(P) $\mathrm{Q}_{1}<0, \mathrm{Q}_{2}>0$
(Q) $\mathrm{Q}_{1}>0, \mathrm{Q}_{2}<0$
(R) $\left|\mathrm{Q}_{1}\right|>\left|\mathrm{Q}_{2}\right|$
(C)

(S) $\mathrm{Q}_{1}<0, \mathrm{Q}_{2}<0$
(T) $\left|\mathrm{Q}_{1}\right|=\left|\mathrm{Q}_{2}\right|$
109. Column I shows graphs of electric potential $V$ versus $x$ and $y$ in a certain region for four situations. Column II shows the range of angle which the electric field vector makes with positive x -direction.

## Column I

V-x, V-y
(A)

(B)

(C)

(D)


Column II
(Range of angle)
(P) $0 \leq \theta<45^{\circ}$
(Q) $45^{\circ} \leq \theta<90^{\circ}$
(R) $90^{\circ} \leq \theta<135^{\circ}$
(S) $135^{\circ} \leq \theta \leq 180^{\circ}$
110. A spherical metallic conductor has a spherical cavity. A positive charge is placed inside the cavity at its centre. Another positive charge is placed outside it. The conductor is initially electrically neutral.

## Column I <br> (Cause)

(A) If outside charge is shifted to other position
(B) If inside charge is shifted to other position within cavity
(C) If magnitude of charge inside cavity is increased
(D) If conductor is earthed

Column II (Effect)
(P) distribution of charge on inner surface of cavity changes
(Q) distribution of charge on outer surface of of conductor changes
(R) electric potential at centre of conductor due to charges present on outer surface of conductor changes
(S) force on the charge placed inside cavity changes
111. In the shown figure the conductor is uncharged and a charge $q$ is placed inside a spherical cavity at a distance $a$ from its centre $(C)$. Point $P$ and charge $+Q$ are as shown. $a, b, c, d$ are known.


## Column-I

(A) Electric field due to induced charges on the inner surface of cavity at point $P$
(B) Electric potential due to charges on the inner surface of cavity and $q$ at $P$
(C) Electric field due to induced charges on the outer surface of conductor and $Q$ at $C$
(D) Electric potential due to induced charges on the inner surface of cavity at $C$

## Column-II

(P) zero
(Q) non-zero
(R) value can be stated with the given data.
(S) value cannot be stated from the given data

## EXERCISE (O-2)

## SINGLE CORRECT TYPE QUESTIONS

1. A square of side $b$ centred at the origin with sides parallel to axes of $x$ and $y$ has surface charge density $\sigma(x, y)=\sigma_{0} x y$ within its boundaries. Total charge on the square is
(A) 0
(B) $\sigma_{0} \mathrm{~b}^{2}$
(C) $2 \sigma_{0} \mathrm{~b}^{2}$
(D) $4 \sigma_{0} \mathrm{~b}^{2}$
2. A uniformly charged rod is kept on $y$-axis with centre at origin, as shown. Which of the following actions will increase the electric field strength at the position of the dot ?

(A) make the rod longer without changing the charge
(B) make the rod shorter without changing the charge
(C) make the rod shorter without changing the linear charge density
(D) rotate the rod about yy'
3. A charged particle having some mass is resting in equilibrium at a height H above the centre of a uniformly charged non-conducting horizontal ring of radius R . The force of gravity acts downwards. The equilibrium of the particle will be stable
(A) for all values of H
(B) only if $\mathrm{H}>\frac{\mathrm{R}}{\sqrt{2}}$
(C) only if $\mathrm{H}<\frac{\mathrm{R}}{\sqrt{2}}$
(D) only if $\mathrm{H}=\frac{\mathrm{R}}{\sqrt{2}}$
4. Statement-1: A positive point charge initially at rest in a uniform electric field starts moving along electric lines of forces. (Neglect all other forces except electric forces)
Statement-2 : Electric lines of force represents path of charged particle which is released from rest in it.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.
5. A system consists of uniformly charged sphere of radius $R$ and a surrounding medium filled by a charge with the volume density $\rho=\frac{\alpha}{r}$, where $\alpha$ is a positive constant and r is the distance from the centre of the sphere. The charge of the sphere for which electric field intensity E outside the sphere is independent of $r$ is
(A) $\frac{\alpha}{2 \epsilon_{0}}$
(B) $\frac{2}{\alpha \epsilon_{0}}$
(C) $2 \pi \alpha \mathrm{R}^{2}$
(D) $\alpha \mathrm{R}^{2}$
6. A spherical insulator of radius R is charged uniformly with a charge Q throughout its volume and contains a point charge $\frac{\mathrm{Q}}{16}$ located at its centre. Which of the following graphs best represent qualitatively, the variation of electric field intensity E with distance r from the centre.
(A)

(B)

(C)

(D)

7. A positively charged sphere of radius $r_{0}$ carries a volume charge density $\rho_{\mathrm{E}}$ (Figure). A spherical cavity of radius $r_{0} / 2$ is then scooped out and left empty, as shown. What is the direction and magnitude of the electric field at point $B$ ?

(A) $\frac{17 \rho r_{0}}{54 \epsilon_{0}}$ left
(B) $\frac{\rho r_{0}}{6 \epsilon_{0}}$ left
(C) $\frac{17 \rho r_{0}}{54 \epsilon_{0}}$ right
(D) $\frac{\mathrm{rr}_{0}}{6 \epsilon_{0}}$ right
8. The diagram shows a uniformly charged hemisphere of radius R. It has volume charge density $\rho$. If the electric field at a point $2 R$ distance above its center is $E$ then what is the electric field at the point which is 2 R below its center?

(A) $\rho R / 6 \varepsilon_{0}+E$
(B) $\rho R / 12 \varepsilon_{0}-E$
(C) $-\rho R / 6 \varepsilon_{0}+E$
(D) $\rho \mathrm{R} / 24 \varepsilon_{0}+\mathrm{E}$
9. Consider a uniformly charged hemispherical shell shown below. Indicate the directions (not magnitude) of the electric field at the central point $\mathrm{P}_{1}$ and an off-centre point $\mathrm{P}_{2}$ on the drumhead of the shell.

(A) $\uparrow ;$
(B) $\uparrow ; \%$
(C) $\uparrow ; \uparrow$
(D) $\uparrow ; \longleftarrow$
10. Using thomson's model of the atom, consider an atom consisting of two electrons, each of charge -e , embeded in a sphere of charge +2 e and radius R . In equilibrium each electron is at distance d from the centre of the atom. What is equilibrium separation between electrons?

(A) R
(B) $\mathrm{R} / 2$
(C) R/3
(D) $\mathrm{R} / 4$
11. A non conducting semicircular disc (as shown in figure) has a uniform surface charge density $\sigma$. The electric potential at the centre of the disc :-

(A) $\frac{\sigma}{2 \pi \epsilon_{0}} \frac{\ln (b / a)}{(b-a)}$
(B) $\frac{\sigma(\mathrm{b}-\mathrm{a})}{2 \epsilon_{0}}$
(C) $\frac{\sigma(\mathrm{b}-\mathrm{a})}{4 \epsilon_{0}}$
(D) $\frac{\sigma(b-a)}{4 \pi \epsilon_{0}}$
12. The diagram shows three infinitely long uniform line charges placed on the $X, Y$ and $Z$ axis. The work done in moving a unit positive charge from $(1,1,1)$ to $(0,1,1)$ is equal to

(A) $(\lambda \ln 2) / 2 \pi \varepsilon_{0}$
(B) $(\lambda \ln$
2) $/ \pi \varepsilon_{0}$
(C) $(3 \lambda \ln 2) / 2 \pi \varepsilon_{0}$
(D) None
13. The diagram shows a small bead of mass $m$ carrying charge $q$. The bead can freely move on the smooth fixed ring placed on a smooth horizontal plane. In the same plane a charge $+Q$ has also been fixed as shown. The potential at the point P due to +Q is V . The velocity with which the bead should projected from the point P so that it can complete a circle should be greater than

(A) $\sqrt{\frac{6 q V}{m}}$
(B) $\sqrt{\frac{q V}{m}}$
(C) $\sqrt{\frac{3 q V}{m}}$
(D) none
14. A charged particle of charge $Q$ is held fixed and another charged particle of mass $m$ and charge $q$ (of the same sign) is released from a distance $r$. The impulse of the force exerted by the external agent on the fixed charge by the time distance between $Q$ and $q$ becomes $2 r$ is
(A) $\sqrt{\frac{\mathrm{Qq}}{4 \pi \epsilon_{0} \mathrm{mr}}}$
(B) $\sqrt{\frac{\mathrm{Qqm}}{4 \pi \epsilon_{0} r}}$
(C) $\sqrt{\frac{\mathrm{Qqm}}{\pi \epsilon_{0} r}}$
(D) $\sqrt{\frac{\mathrm{Qqm}}{2 \pi \epsilon_{0}} \mathrm{r}}$
15. In a certain region of space, the potential field depends on $x$ and $y$ coordinates as $V=\left(x^{2}-y^{2}\right)$. The corresponding electric field lines in $x-y$ plane are correctly represented by :
(A)

(B)

(C)

(D)

16. Two short electric dipoles are placed as shown. The energy of electric interaction between these dipoles will be :-

(A) $\frac{2 \mathrm{kP}_{1} \mathrm{P}_{2} \cos \theta}{\mathrm{r}^{3}}$
(B) $\frac{-2 \mathrm{kP}_{1} \mathrm{P}_{2} \cos \theta}{\mathrm{r}^{3}}$
(C) $\frac{-2 \mathrm{kP}_{1} \mathrm{P}_{2} \sin \theta}{\mathrm{r}^{3}}$
(D) $\frac{-4 \mathrm{kP}_{1} \mathrm{P}_{2} \cos \theta}{\mathrm{r}^{3}}$
17. A small electric dipole $\vec{P}$ is placed on the $X$ axis at the point $(1,0)$. The dipole vector forms an angle of $30^{\circ}$ with the X axis. Consider a non uniform electric field to have been applied in the region given by the vector $\overrightarrow{\mathrm{E}}=\mathrm{x}^{2} \hat{\mathrm{i}}+y^{2} \hat{\mathrm{j}}$. What is the force acting on the dipole?

(A) $2 \overrightarrow{\mathrm{P}} \cos 30^{\circ}(\hat{\mathrm{i}}+2 \hat{\mathrm{j}})$
(B) $2 \overrightarrow{\mathrm{P}} \cos 30^{\circ}(\hat{\mathrm{i}})$
(C) $2 \overrightarrow{\mathrm{P}} \cos 30^{\circ}(2 \hat{\mathrm{j}})$
(D) None
18. The density of charge at P on the conductor is $\sigma$. The resultant electric field near P will $[\hat{n}=$ unit normal vector at point $\mathrm{P} . \vec{r}=$ vector along OP] where $\mathrm{K}=1 / 4 \pi \varepsilon$

(A) $\left(\frac{\sigma}{\epsilon_{0}}\right) \hat{n}+\left(\frac{K q \vec{r}}{r^{3}}\right)$
(B) $\left(\frac{\sigma}{2 \epsilon_{0}}\right) \hat{n}+\left(\frac{K q \vec{r}}{r^{3}}\right)$
(C) $\left(\frac{\sigma}{\epsilon_{0}}\right) \hat{n}$
(D) $\left(\frac{\sigma}{2 \epsilon_{0}}\right) \hat{n}$
19. A metal sphere A of radius $\mathrm{r}_{1}$ charged to a potential $\phi_{1}$ is enveloped by a thin walled conducting spherical shell B of radius $r_{2}$. Then $\phi_{2}$ of the sphere A after it is connected by a thin wire to the shell B will be :-

(A) $\phi_{1} \frac{r_{1}}{r_{2}}$
(B) $\phi_{1}\left(\frac{r_{2}}{r_{1}}\right)$
(C) $\phi_{1}\left(1-\frac{r_{1}}{\mathrm{r}_{2}}\right)$
(D) $\phi_{1}\left(\frac{\mathrm{r}_{1} \mathrm{r}_{2}}{\mathrm{r}_{1}+\mathrm{r}_{2}}\right)$
20. A dipole having dipole moment $p$ is placed in front of a solid uncharged conducting sphere as shown in the diagram. The net potential at point A lying on the surface of the sphere is :-

(A) $\frac{k p \cos \phi}{r^{2}}$
(B) $\frac{k p \cos ^{2} \phi}{r^{2}}$
(C) 0
(D) $\frac{2 k p \cos ^{2} \phi}{r^{2}}$
21. A conducting sphere of radius $R$ and charge $Q$ is placed near a uniformly charged nonconducting infinitely large thin plate having surface charge density $\sigma$. Then find the potential at point A (on the surface of sphere) due to charge on sphere (here $K=\frac{1}{4 \pi \epsilon_{0}}, \theta_{0}=\frac{\pi}{3}$ )

(A) $K \frac{Q}{R}-\frac{\sigma}{4 \epsilon_{0}} R$
(B) $\mathrm{K} \frac{\mathrm{Q}}{\mathrm{R}}-\frac{\sigma R}{\epsilon_{0}}$
(C) $K \frac{Q}{R}$
(D) none of these
22. The intensity of an electric field depends only on the coordinates $\mathrm{x}, \mathrm{y}$ and z as follows :

$$
\overrightarrow{\mathrm{E}}=\mathrm{a} \frac{(x \hat{\mathrm{i}}+y \hat{\mathrm{j}}+\mathrm{z} \hat{\mathrm{k}})}{\left(\mathrm{x}^{2}+\mathrm{y}^{2}+\mathrm{z}^{2}\right)^{3 / 2}} \text { unit }
$$

The electrostatic energy stored between two imaginary concentric spherical shells of radii $R$ and $2 R$ with centre at origin is :-
(A) $\frac{4 \pi \varepsilon_{0} \mathrm{a}^{2}}{\mathrm{R}}$
(B) $\frac{2 \pi \varepsilon_{0} \mathrm{a}^{2}}{\mathrm{R}}$
(C) $\frac{\pi \varepsilon_{0} \mathrm{a}^{2}}{\mathrm{R}}$
(D) $\frac{\pi \varepsilon_{0} \mathrm{a}^{2}}{2 \mathrm{R}}$
23. A charged large metal sheet is placed into uniform electric field, perpendicularly to the electric field lines. After placing the sheet into the field, the electric field on the left side of the sheet is $\mathrm{E}_{1}=5 \times 10^{5}$ $\mathrm{V} / \mathrm{m}$ and on the right it is $\mathrm{E}_{2}=3 \times 10^{5} \mathrm{~V} / \mathrm{m}$. The sheet experiences a net electric force of 0.08 N . Find the area of one face of the sheet. Assume external field to remain constant after introducing the large
sheet. Use $\left(\frac{1}{4 \pi \varepsilon_{0}}\right)=9 \times 10^{9} \mathrm{Nm}^{2} / \mathrm{C}^{2}$

(A) $3.6 \pi \times 10^{-2} \mathrm{~m}^{2}$
(B) $0.9 \pi \times 10^{-2} \mathrm{~m}^{2}$
(C) $1.8 \pi \times 10^{-2} \mathrm{~m}^{2}$
(D) none
24. An ellipsoidal cavity is carved within a perfect conductor. A positive charge $q$ is placed at the center of the cavity. The points A \& B are on the cavity surface as shown in the figure. Then :

(A) electric field near A in the cavity $=$ electric field near B in the cavity
(B) charge density at $\mathrm{A}=$ charge density at B
(C) potential at $\mathrm{A}=$ potential at B
(D) total electric field flux through the surface of the cavity is $\mathrm{q} / \varepsilon_{0}$.

## MULTIPLE CORRECT TYPE QUESTIONS

25. $S$ is a solid neutral conducting sphere. A point charge $q$ of $1 \times 10^{-6} \mathrm{C}$ is placed at point A . C is the centre of sphere and $A B$ is a tangent. $B C=3 \mathrm{~m}$ and $\mathrm{AB}=4 \mathrm{~m}$.

(A) The electric potential of the conductor is 1.8 kV .
(B) The electric potential of the conductor is 2.25 kV .
(C) The electric potential at B due to induced charges on the sphere is -0.45 kV .
(D) The electric potential at B due to induced charges on the sphere is 0.45 kV .
26. Four identical particles each having mass $m$ and charge $q$ are placed at the vertices of a square of side $\ell$. All the particles are free to move without any friction and released simultaneously from rest. Then (A) At all instants, the particles remains at vertices of square whose edge length is changing
(B) The configuration is changing (not remaining square) as the time passes
(C) The speed of the particles when one of the particles get displaced by $\frac{\ell}{\sqrt{2}}$ is $\sqrt{\frac{q^{2}}{8 \pi \varepsilon_{0} m \ell}\left(2+\frac{1}{\sqrt{2}}\right)}$
(D) Speed of the particles can not be found
27. Two large thin conducting plates with small gap in between are placed in a uniform electric field $E$ (perpendicular to the plates). Area of each plate is $A$ and charges $+Q$ and $-Q$ are given to these plates as shown in the figure. If points $R, S$ and $T$ as shown in the figure are three points in space, then the

(A) field at point $R$ is $E$
(C) field at point $T$ is $\left(E+\frac{Q}{\varepsilon_{0} A}\right)$
(B) field at point $S$ is $E$
(D) field at point $S$ is $\left(E+\frac{Q}{A \varepsilon_{o}}\right)$
28. In a region of space, the electric field $\vec{E}=E_{0} x \hat{i}+E_{0} y \hat{j}$. Consider an imaginary cubical volume of edge ' $a$ ' with its edges parallel to the axes of coordinates. Now,

(A) the total electric flux through the faces 1 and 3 is $\mathrm{E}_{0} \mathrm{a}^{3}$
(B) the charge inside the cubical volume is $2 \varepsilon_{0} \mathrm{E}_{\mathrm{a}^{3}}{ }^{3}$
(C) the total electric flux through the faces 2 and 4 is $2 \mathrm{E}_{0} \mathrm{a}^{3}$
(D) the charge inside the cubical volume is $\varepsilon_{0} \mathrm{E}_{0} \mathrm{a}^{3}$
29. Equipotential surfaces :-
(A) are closer in regions of large electric fields compared to regions of lower electric fields.
(B) will be more crowded near sharp edges of a conductor.
(C) will be more crowded near regions of large charge densities.
(D) will always be equally spaced.

## COMPREHENSION TYPE QUESTIONS

## Paragraph for Question No. 30 and 31

A uniform ring of mass $m$ and radius $R$ can rotate freely about an axis passing through centre $C$ and perpendicular to plane of paper. Half of ring is positively charge and other half is negatively charge. Uniform electric field $\mathrm{E}_{0}$ is switched on along -ve x-axis (axis are shown in figure) [Magnitude of charge density $\lambda$ ]


30. The dipole moment of ring is :-
(A) $2 \lambda R^{2}$
(B) $4 \lambda \mathrm{R}^{2}$
(C) $2 \pi \lambda \mathrm{R}^{2}$
(D) $4 \pi \lambda \mathrm{R}^{2}$
31. If ring is slightly disturb from given position, find the angular speed of ring when it rotate by $\pi / 2$.
(A) $2 \sqrt{\frac{\lambda \mathrm{E}_{0}}{\mathrm{~m}}}$
(B) $\sqrt{\frac{\lambda \mathrm{E}_{0}}{\mathrm{~m}}}$
(C) $\sqrt{\frac{8 \lambda \mathrm{E}_{0}}{\mathrm{~m}}}$
(D) None

## MATCHING LIST TYPE $(4 \times 4 \times 4)$ SINGLE OPTION CORRECT (THREE COLUMNS AND FOUR ROWS)

Answer Q.32, 33 \& 34 by appropriately matching the information given in the three columns of the following table.
Consider a non conducting ring of radius $r$ and mass $m$ and a particle of same mass, both at rest in free space. The particle is on the axis of the ring and far away from the ring. An amount Q of positive charge is uniformly distributed on the ring and the particle is given a positive charge q . The particle is imparted a velocity v towards the centre of the ring. Consider the consequences given in the columns and answer the following questions.

## Column 1

(I) Maximum speed of the ring is v
(II) Maximum speed of the ring is $\mathrm{v} / 2$
(III) Maximum speed of the ring is
$\frac{\mathrm{v}}{2}\left[1+\sqrt{1-\frac{\mathrm{Qq}}{\pi \varepsilon_{0} \mathrm{mrv}^{2}}}\right]$
(IV) Maximum speed of the ring
(iv) Minimum speed of the particle is
$\frac{\mathrm{v}}{2}\left[1-\sqrt{1-\frac{\mathrm{Qq}}{\pi \varepsilon_{0} \mathrm{mrv}^{2}}}\right]$

## Column 3

(P) Final speed of the ring and particle is v and zero respectively.
(Q) Final speed of the ring and particle is zero and $v$ respectively.
(R) Final speed of the ring and particle is $\mathrm{v} / 4$ and
$3 \mathrm{v} / 4$ respectively.
(S) Final speed of the ring and particle is $v / 2$ for both.
32. Which of following options is the correct representation if $v=\sqrt{\frac{\mathrm{Qq}}{2 \pi \varepsilon_{0} \mathrm{mr}}}$
(A) (II) (i) S
(B) (I) (ii) P
(C) (III) (iv) R
(D) (IV) (iii) Q
33. Which of following options is the correct representation if $v=\sqrt{\frac{\mathrm{Qq}}{\pi \varepsilon_{0} \mathrm{mr}}}$
(A) (II) (i) S
(B) (I) (ii) P
(C) (III) (iv) R
(D) (IV) (iii) Q
34. Which of following options is the correct representation if $\mathrm{v}=\sqrt{\frac{2 \mathrm{Qq}}{\pi \varepsilon_{0} \mathrm{mr}}}$
(A) (II) (i) S
(B) (I) (ii) P
(C) (III) (iv) Q
(D) (IV) (iii) (Q)

## MATRIX MATCH TYPE QUESTION

35. Column-II shows some charge distributions and column-I has some statements about electric field at four points A, B, C, D. Match column-I with column-II.

## Column-I

(A) $\vec{E}_{A}$ has x component only
(P)

(B) $\vec{E}_{B}$ has y component only
(Q)


## Column-II

A solid non conducting sphere of radius $R$ of volumetric charge density $\rho$ with four symmetrical spherical cavities. All the five sphere's centre lie in same plane.

A very small circular filament lying in xy-plane. All points lie in same plane. A, B and D are at large distance compared to radius of circle.
(C) $\vec{E}_{C}$ has y component only
(R)

(S)

(T)


A charged spherical conductor with an empty cavity in it.

A hollow thick spherical neutral conductor with a concentric cavity. Charge $\mathrm{q}_{0}$ is placed inside at centre of cavity.

A small electric dipole $\vec{p}$ is placed at origin. $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D are four points at large distance from origin.

## EXERCISE-JM

1. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then $\frac{\mathrm{Q}}{\mathrm{q}}$ equals :-
[AIEEE - 2009]
(1) 1
(2) $-\frac{1}{\sqrt{2}}$
(3) $-2 \sqrt{2}$
(4) -1

This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
2. Statement-1 : For a charged particle moving from point $P$ to point $Q$ the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q .
Statement-2 : The net work done by a conservative force on an object moving along closed loop is zero.
[AIEEE - 2009]
(1) Statement -1 is true, Statement -2 is true; Statement -2 is not the correct explanation of Statement-1
(2) Statement-1 is false, Statement-2 is true
(3) Statement -1 is true, Statement -2 is false
(4) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1
3. Two points $P$ and $Q$ are maintained at the potential of 10 V and -4 V , respectively. The work done in moving 100 electrons from P to Q is :-
[AIEEE - 2009]
(1) $-2.24 \times 10^{-16} \mathrm{~J}$
(2) $2.24 \times 10^{-16} \mathrm{~J}$
(3) $-9.60 \times 10^{-17} \mathrm{~J}$
(4) $9.60 \times 10^{-17} \mathrm{~J}$
4. Let $\mathrm{P}(\mathrm{r})=\frac{\mathrm{Q}}{\pi \mathrm{R}^{4}} \mathrm{r}$ be the charge density distribution for a solid sphere of radius R and total charge Q . For a point ' p ' inside the sphere at distance $\mathrm{r}_{1}$ from the centre of the sphere, the magnitude of electric field is :-
[AIEEE - 2009]
(1) $\frac{\mathrm{Qr}_{1}^{2}}{4 \pi \epsilon_{0} \mathrm{R}^{4}}$
(2) $\frac{\mathrm{Qr}_{1}^{2}}{3 \pi \epsilon_{0} \mathrm{R}^{4}}$
(3) 0
(4) $\frac{\mathrm{Q}}{4 \pi \epsilon_{0} \mathrm{r}_{1}^{2}}$
5. A thin semi-circular ring of radius $r$ has a positive charge $q$ distributed uniformly over it. The net field $\overrightarrow{\mathrm{E}}$ at the centre O is :-
[AIEEE - 2010]

(1) $\frac{\mathrm{q}}{2 \pi^{2} \varepsilon_{0} \mathrm{r}^{2}} \hat{\mathrm{j}}$
(2) $\frac{\mathrm{q}}{4 \pi^{2} \varepsilon_{0} \mathrm{r}^{2}} \hat{\mathrm{j}}$
(3) $-\frac{\mathrm{q}}{4 \pi^{2} \varepsilon_{0} \mathrm{r}^{2}} \hat{\mathrm{j}}$
(4) $-\frac{\mathrm{q}}{2 \pi^{2} \varepsilon_{0} \mathrm{r}^{2}} \hat{\mathrm{j}}$
6. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(\mathrm{r})=\rho_{0}\left(\frac{5}{4}-\frac{\mathrm{r}}{\mathrm{R}}\right)$ upto $\mathrm{r}=\mathrm{R}$, and $\rho(\mathrm{r})=0$ for $\mathrm{r}>\mathrm{R}$, where r is the distance from the origin. The electric field at a distance $r(r<R)$ from the origion is given by :
[AIEEE - 2010]
(1) $\frac{\rho_{0} r}{3 \varepsilon_{0}}\left(\frac{5}{4}-\frac{r}{R}\right)$
(2) $\frac{4 \pi \rho_{0} r}{3 \varepsilon_{0}}\left(\frac{5}{3}-\frac{\mathrm{r}}{\mathrm{R}}\right)$
(3) $\frac{\rho_{0} r}{4 \varepsilon_{0}}\left(\frac{5}{3}-\frac{r}{R}\right)$
(4) $\frac{4 \rho_{0} r}{3 \varepsilon_{0}}\left(\frac{5}{4}-\frac{r}{R}\right)$
7. Two identical charged spheres suspended from a common point by two massless string of length $\ell$ are initially a distance $\mathrm{d}(\mathrm{d} \ll \ell)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v . Then as a function of distance x between them :-
[AIEEE - 2011]
(1) $\mathrm{v} \propto \mathrm{x}^{1 / 2}$
(2) $v \propto x$
(3) $v \propto x^{-1 / 2}$
(4) $\mathrm{v} \propto \mathrm{x}^{-1}$
8. The electrostatic potential inside a charged spherical ball is given by $\phi=\mathrm{ar}^{2}+\mathrm{b}$ where r is the distance from the centre; $\mathrm{a}, \mathrm{b}$ are constant. Then the charge density inside the ball is :-
[AIEEE - 2011]
(1) $-24 \pi \mathrm{a} \in_{0}$
(2) $-6 a \in_{0}$
(3) $-24 \pi \mathrm{a} \in_{0} \mathrm{r}$
(4) $-6 a \in_{0} r$
9. Two positive charges of magnitude 'q' are placed at the ends of a side (side 1 ) of a square of side ' 2 a '. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is :-
[AIEEE - 2011]
(1) $\frac{1}{4 \pi \epsilon_{0}} \frac{2 q Q}{a}\left(1-\frac{1}{\sqrt{5}}\right)$
(2) zero
(3) $\frac{1}{4 \pi \epsilon_{0}} \frac{2 q \mathrm{Q}}{\mathrm{a}}\left(1+\frac{1}{\sqrt{5}}\right)$
(4) $\frac{1}{4 \pi \epsilon_{0}} \frac{2 q \mathrm{Q}}{\mathrm{a}}\left(1-\frac{2}{\sqrt{5}}\right)$
10. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
[AIEEE - 2012]
An insulating solid sphere of radius R has a uniformaly positive charge density $\rho$. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphre and also at a point out side the sphere. The electric potential at infinity is zero.
Statement-1: When a charge ' $q$ ' is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{\mathrm{q} \rho}{3 \epsilon_{0}}$

Statement-2 : The electric field at a distance $r(r<R)$ from the centre of the sphere is $\frac{\rho r}{3 \epsilon_{0}}$
(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1.
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true
11. In a uniformly charged sphere of total charge $Q$ and radius $R$, the electric field $E$ is plotted as a function of distance from the centre. The graph which would correspond to the above will be :-
[AIEEE - 2012]
(1)

(2)

(3)

(4)

12. Let $\left[\epsilon_{0}\right]$ denote the dimensional formula of the permittivity of vacuum. If $\mathrm{M}=$ mass, $\mathrm{L}=$ Length, $\mathrm{T}=$ Time and $\mathrm{A}=$ electric current, then :
[JEE-Main-2013]
(1) $\left[\epsilon_{0}\right]=\left[\mathrm{M}^{-1} \mathrm{~L}^{-3} \mathrm{~T}^{2} \mathrm{~A}\right]$
(2) $\left[\epsilon_{0}\right]=\left[\mathrm{M}^{-1} \mathrm{~L}^{-3} \mathrm{~T}^{4} \mathrm{~A}^{2}\right]$
(3) $\left[\epsilon_{0}\right]=\left[\mathrm{M}^{-1} \mathrm{~L}^{2} \mathrm{~T}^{-1} \mathrm{~A}^{-2}\right]$
(4) $\left[\epsilon_{0}\right]=\left[\mathrm{M}^{-1} \mathrm{~L}^{2} \mathrm{~T}^{-1} \mathrm{~A}\right]$
13. Two charges, each equal to $q$, are kept at $x=-a$ and $x=a$ on the $x$-axis. A particle of mass $m$ and charge $\mathrm{q}_{0}=\frac{\mathrm{q}}{2}$ is placed at the origin. If charge $\mathrm{q}_{0}$ is given a small displacement $(\mathrm{y} \ll \mathrm{a})$ along the $y$-axis, the net force acting on the particle is proportional to
[JEE-Main-2013]
(1) $y$
(2) $-y$
(3) $\frac{1}{y}$
(4) $-\frac{1}{y}$
14. A charge $Q$ is uniformly distributed over a long rod $A B$ of length $L$ as shown in the figure. The electric potential at the point $O$ lying at a distance $L$ from the end $A$ is :-
[JEE-Main-2013]
(1) $\frac{\mathrm{Q}}{8 \pi \epsilon_{0} \mathrm{~L}}$
(2) $\frac{3 \mathrm{Q}}{4 \pi \epsilon_{0} \mathrm{~L}}$
(3) $\frac{\mathrm{Q}}{4 \pi \epsilon_{0} \mathrm{~L} \ln 2}$
(4) $\frac{\mathrm{Q} \ln 2}{4 \pi \epsilon_{0} \mathrm{~L}}$
15. Assume that an electric field $\overrightarrow{\mathrm{E}}=30 \mathrm{x}^{2} \hat{\hat{i}}$ exists in space. Then the potential difference $\mathrm{V}_{\mathrm{A}}-\mathrm{V}_{\mathrm{O}}$, where $\mathrm{V}_{\mathrm{O}}$ is the potential at the origin and $\mathrm{V}_{\mathrm{A}}$ the potential at $\mathrm{x}=2 \mathrm{~m}$ is :-
[JEE-Main-2014]
(1) -80 J
(2) 80 J
(3) 120 J
(4) -120 J
16. A uniformally charged solid sphere of radius $R$ has potential $V_{0}$ (measured with respect to $\infty$ ) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3 \mathrm{~V}_{0}}{2}, \frac{5 \mathrm{~V}_{0}}{4}, \frac{3 \mathrm{~V}_{0}}{4}$ and $\frac{\mathrm{V}_{0}}{4}$ have radius $R_{1}, R_{2}, R_{3}$ and $R_{4}$ respectively. Then
[JEE-Main-2015]
(1) $R_{1}=0$ and $R_{2}<\left(R_{4}-R_{3}\right)$
(2) $2 \mathrm{R}<\mathrm{R}_{4}$
(3) $\mathrm{R}_{1}=0$ and $\mathrm{R}_{2}>\left(\mathrm{R}_{4}-\mathrm{R}_{3}\right)$
(4) $\mathrm{R}_{1} \neq 0$ and $\left(\mathrm{R}_{2}-\mathrm{R}_{1}\right)>\left(\mathrm{R}_{4}-\mathrm{R}_{3}\right)$
17. A long cylindrical shell carries positive surface charge $\sigma$ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in: (figures are schematic and not drawn to scale)
[JEE-Main-2015]
(1)

(2)

(3)

(4)

18. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho=\frac{A}{r}$, where $A$ is a constant and $r$ is the distance from the centre. At the centre of the spheres is a point charge Q . The value of A such that the electric field in the region between the spheres will be constant, is :-
[JEE-Main-2016]
(1) $\frac{2 Q}{\pi a^{2}}$
(2) $\frac{Q}{2 \pi a^{2}}$
(3) $\frac{Q}{2 \pi\left(b^{2}-a^{2}\right)}$
(4) $\frac{2 \mathrm{Q}}{\pi\left(\mathrm{a}^{2}-\mathrm{b}^{2}\right)}$
19. An electric dipole has a fixed dipole moment $\overrightarrow{\mathrm{p}}$, which makes angle $\theta$ with respect to x -axis. When subjected to an electric field $\overrightarrow{\mathrm{E}}_{1}=\mathrm{E} \hat{\mathrm{i}}$, it experiences a torque $\overrightarrow{\mathrm{T}}_{1}=\tau \hat{\mathrm{k}}$. When subjected to another electric field $\overrightarrow{\mathrm{E}}_{2}=\sqrt{3} \mathrm{E}_{1} \hat{\mathrm{j}}$ it experiences torque $\overrightarrow{\mathrm{T}}_{2}=-\overrightarrow{\mathrm{T}}_{1}$. The angle $\theta$ is :
[JEE-Main-2017]
(1) $60^{\circ}$
(2) $90^{\circ}$
(3) $30^{\circ}$
(4) $45^{\circ}$
20. Three concentric metal shells $A, B$ and $C$ of respective radii $a, b$ and $c(a<b<c)$ have surface charge densities $+\sigma,-\sigma$ and $+\sigma$ respectively. The potential of shell $B$ is :-
[JEE-Main-2018]
(1) $\frac{\sigma}{\varepsilon_{0}}\left[\frac{\mathrm{a}^{2}-\mathrm{b}^{2}}{\mathrm{~b}}+\mathrm{c}\right]$
(2) $\frac{\sigma}{\varepsilon_{0}}\left[\frac{\mathrm{~b}^{2}-\mathrm{c}^{2}}{\mathrm{~b}}+\mathrm{a}\right]$
(3) $\frac{\sigma}{\varepsilon_{0}}\left[\frac{b^{2}-\mathrm{c}^{2}}{\mathrm{c}}+\mathrm{a}\right]$
(4) $\frac{\sigma}{\varepsilon_{0}}\left[\frac{a^{2}-b^{2}}{a}+c\right]$

## EXERCISE-(JA)

1. A disk of radius $a / 4$ having a uniformly distributed charge 6 C is placed in the $x-y$ plane with its centre at $(-a / 2,0,0)$. A rod of length $a$ carrying a uniformly distributed charge 8 C is placed on the x -axis from $x=a / 4$ to $x=5 a / 4$. Two point charges -7 C and 3 C are placed at $(a / 4,-a / 4,0)$ and $(-3 a / 4,3 a / 4,0)$, respectively. Consider a cubical surface formed by six surfaces $\mathrm{x}= \pm a / 2, \mathrm{y}= \pm a / 2$, $\mathrm{z}= \pm a / 2$. The electric flux through this cubical surface is
[IIT-JEE 2009]

(A) $\frac{-2 C}{\varepsilon_{o}}$
(B) $\frac{2 C}{\varepsilon_{o}}$
(C) $\frac{10 C}{\varepsilon_{o}}$
(D) $\frac{12 C}{\varepsilon_{o}}$
2. Three concentric metallic spherical shells of radii $R, 2 R, 3 R$, are given charges $Q_{1}, Q_{2}, Q_{3}$, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_{1}: Q_{2}: Q_{3}$, is
[IIT-JEE 2009]
(A) $1: 2: 3$
(B) $1: 3: 5$
(C) $1: 4: 9$
(D) $1: 8: 18$
3. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).
[IIT-JEE 2009]
(A) The angular momentum of the charge $-q$ is constant
(B) The linear momentum of the charge $-q$ is constant
(C) The angular velocity of the charge $-q$ is constant
(D) The linear speed of the charge $-q$ is constant
4. A solid sphere of radius $R$ has a charge $Q$ distributed in its volume with a charge density $\rho=K r^{a}$, where $K$ and a are constants and $r$ is the distance from its centre. If the electric field at $r=\frac{R}{2}$ is $\frac{1}{8}$ times that at $r=R$, find the value of $a$.
[IIT-JEE 2009]
5. A uniformly charged thin spherical shell of radius $R$ carries uniform surface charge density of $\sigma$ per unit area. It is made of two hemispherical shells, held together by pressing them with force $F$ (see figure). $F$ is proportional to
[IIT-JEE 2010]

(A) $\frac{1}{\varepsilon_{0}} \sigma^{2} R^{2}$
(B) $\frac{1}{\varepsilon_{0}} \sigma^{2} R$
(C) $\frac{1}{\varepsilon_{0}} \frac{\sigma^{2}}{R}$
(D) $\frac{1}{\varepsilon_{0}} \frac{\sigma^{2}}{R^{2}}$
6. A tiny spherical oil drop carrying a net charge q is balanced in still air with a vertical uniform electric field of strength $\frac{81 \pi}{7} \times 10^{5} \mathrm{~V} \mathrm{~m}^{-1}$. When the field is switched off, the drop is observed to fall with terminal velocity $2 \times 10^{-3} \mathrm{~m} / \mathrm{s}$. Given $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$, viscosity of the air $=1.8 \times 10^{-5} \mathrm{~N}-\mathrm{s} \mathrm{m}^{-2}$ and the density of oil $=900 \mathrm{~kg} \mathrm{~m}^{-3}$, the magnitude of $q$ is
[IIT-JEE 2010]
(A) $1.6 \times 10^{-19} \mathrm{C}$
(B) $3.2 \times 10^{-19} \mathrm{C}$
(C) $4.8 \times 10^{-19} \mathrm{C}$
(D) $8.0 \times 10^{-19} \mathrm{C}$
7. A few electric field lines for a system of two charges $Q_{1}$ and $Q_{2}$ fixed at two different points on the $x$-axis are shown in the figure. These lines suggest that
[IIT-JEE 2010]

(A) $\left|Q_{1}\right|>\left|Q_{2}\right|$
(B) $\left|Q_{1}\right|<\left|Q_{2}\right|$
(C) at a finite distance to the left of $Q_{1}$ the electric field is zero
(D) at a finite distance to the right of $Q_{2}$ the electric field is zero
8. Consider an electric field $\vec{E}=E_{0} \hat{x}$, where $E_{0}$ is a constant. The flux through the shaded area (as shown in the figure) due to this field is
[IIT-JEE 2011]

(A) $2 E_{0} a^{2}$
(B) $\sqrt{2} E_{0} a^{2}$
(C) $E_{0} a^{2}$
(D) $\frac{E_{0} a^{2}}{\sqrt{2}}$
9. A spherical metal shell $A$ of radius $R_{A}$ and a solid metal sphere $B$ of radius $R_{B}\left(<R_{A}\right)$ are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then [IIT-JEE 2011]
(A) $E_{A}^{\text {inside }}=0$
(B) $Q_{A}>Q_{B}$
(C) $\frac{\sigma_{A}}{\sigma_{B}}=\frac{R_{B}}{R_{A}}$
(D) $E_{A}^{\text {on surface }}<E_{B}^{\text {on surface }}$
10. A wooden block performs SHM on a frictionless surface with frequency, $\mathrm{v}_{0}$. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be :-
[IIT-JEE 2011]

(A) of the same frequency and with shifted mean position
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position
(D) of changed frequency and with the same mean position
11. Which of the following statement(s) is/are correct?
[IIT-JEE 2011]
(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of $r^{-2}$, then the Gauss law will still be valid.
(B) The Gauss law can be used to calculate the field distribution around an electric dipole.
(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
(D) The work done by the external force in moving a unit positive charge from point $A$ at potential $V_{A}$ to point B at potential $V_{B}$ is $\left(V_{B}-V_{A}\right)$
12. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side ' $\alpha$ '. The surface tension of the soap film is $\gamma$. The system of charges and planar film are in equilibrium, and $a=k\left[\frac{q^{2}}{\gamma}\right]^{1 / N}$, where ' k ' is a constant. Then N is
[IIT-JEE 2011]

## Paragraph for Question Nos. 13 and 14

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let ' $N$ ' be the number density of free electrons, each of mass ' $m$ '. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begins to oscillate about the positive ions with a natural angular frequency ' $\omega_{p}$ ', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency $\omega$, where a part of the energy is absorbed and a part of it is reflected. As $\omega$ approaches $\omega_{\rho}$, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.
[IIT-JEE 2011]
13. Taking the electronic charge as ' $e$ ' and the permittivity as ' $\varepsilon_{0}$ ', use dimensional analysis to determine the correct expression for $\omega_{p}$.
(A) $\sqrt{\frac{N e}{m \varepsilon_{0}}}$
(B) $\sqrt{\frac{m \varepsilon_{0}}{N e}}$
(C) $\sqrt{\frac{N e^{2}}{m \varepsilon_{0}}}$
(D) $\sqrt{\frac{m \varepsilon_{0}}{N e^{2}}}$
14. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27} \mathrm{~m}^{-3}$. Take $\varepsilon_{0} \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units
(A) 800 nm
(B) 600 nm
(C) 300 nm
(D) 200 nm
15. An infinitely long solid cylinder of radius $R$ has a uniform volume charge density $\rho$. It has a spherical cavity of radius $\mathrm{R} / 2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P , which is at a distance 2 R from the axis of the cylinder, is given by the expression $\frac{23 \rho R}{16 k \varepsilon_{0}}$. The value of k is
[IIT-JEE 2012]

16. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, -q at $(0,-\mathrm{a} / 4,0),+3 \mathrm{q}$ at $(0,0,0)$ and -q at $(0,+\mathrm{a} / 4,0)$. Choose the correct option(s).
[IIT-JEE 2012]

(A) The net electric flux crossing the plane $x=+a / 2$ is equal to the net electric flux crossing the plane $\mathrm{x}=-\mathrm{a} / 2$
(B) The net electric flux crossing the plane $y=+a / 2$ is more than the net electric flux crossing the plane $\mathrm{y}=-\mathrm{a} / 2$.
(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_{0}}$
(D) The net electric flux crossing the plane $\mathrm{z}=+\mathrm{a} / 2$ is equal to the net electric flux crossing the plane $\mathrm{x}=+\mathrm{a} / 2$.
17. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X . A proton is released at rest midway between the two plates. It is found to move at $45^{\circ}$ to the vertical JUST after release. Then X is nearly
[IIT-JEE 2012]
(A) $1 \times 10^{-5} \mathrm{~V}$
(B) $1 \times 10^{-7} \mathrm{~V}$
(C) $1 \times 10^{-9} \mathrm{~V}$
(D) $1 \times 10^{-10} \mathrm{~V}$
18. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $\mathrm{V}(\mathrm{r})$ with the distance r from the centre, is best represented by which graph?
[IIT-JEE 2012]
(A)

(B)

(C)

(D)

19. Six point charges are kept at the vertices of a regular hexagon of side $L$ and centre $O$, as shown in figure. Given that $K=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{L^{2}}$, which of the following statement(s) is(are) correct?
[IIT-JEE 2012]

(A) The electric field at O is 6 K along OD .
(B) The potential at O is zero.
(C) The potential at all points on the line PR is same.
(D) The potential at all points on the line ST is same.
20. Two non-conducting solid spheres of radii $R$ and $2 R$, having uniform volume charge densities $\rho_{1}$ and $\rho_{2}$ respectively, touch each other. The net electric field at a distance $2 R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_{1}}{\rho_{2}}$ can be
[JEE-Advance-2013]
(A) -4
(B) $-\frac{32}{25}$
(C) $\frac{32}{25}$
(D) 4
21. Two non-conducting spheres of radii $R_{1}$ and $R_{2}$ and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region :-
[JEE-Advance-2013]

(A) the electrostatic field is zero
(B) the electrostatic potential is constant
(C) the electrostatic field is constant in magnitude
(D) the electrostatic field has same direction
22. Let $E_{1}(r), E_{2}(r)$ and $E_{3}(r)$ be the respective electric fields at a distance $r$ from a point charge $Q$, an infinitely long wire with constant linear charge density $\lambda$, and an infinite plane with uniform surface charge density $\sigma$. If $\mathrm{E}_{1}\left(\mathrm{r}_{0}\right)=\mathrm{E}_{2}\left(\mathrm{r}_{0}\right)=\mathrm{E}_{3}\left(\mathrm{r}_{0}\right)$ at a given distance $\mathrm{r}_{0}$, then :-
[JEE-Advance-2014]
(A) $\mathrm{Q}=4 \sigma \pi \mathrm{r}_{0}^{2}$
(B) $r_{0}=\frac{\lambda}{2 \pi \sigma}$
(C) $\mathrm{E}_{1}\left(\mathrm{r}_{0} / 2\right)=2 \mathrm{E}_{2}\left(\mathrm{r}_{0} / 2\right)$
(D) $\mathrm{E}_{2}\left(\mathrm{r}_{0} / 2\right)=4 \mathrm{E}_{3}\left(\mathrm{r}_{0} / 2\right)$
23. Charges $\mathrm{Q}, 2 \mathrm{Q}$ and 4 Q are uniformly distributed in three dielectric solid spheres 1,2 and 3 of radii $R / 2, R$ and $2 R$ respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance $R$ from the centre of spheres 1,2 and 3 are $E_{1}, E_{2}$ and $E_{2}$ respectively, then
[JEE-Advance-2014]


Sphere 1


Sphere 2


Sphere 3
(A) $\mathrm{E}_{1}>\mathrm{E}_{2}>\mathrm{E}_{3}$
(B) $\mathrm{E}_{3}>\mathrm{E}_{1}>\mathrm{E}_{2}$
(C) $\mathrm{E}_{2}>\mathrm{E}_{1}>\mathrm{E}_{3}$
(D) $\mathrm{E}_{3}>\mathrm{E}_{2}>\mathrm{E}_{1}$
24. Four charges $Q_{1}, Q_{2}, Q_{3}$ and $Q_{4}$ of same magnitude are fixed along the $x$ axis at $x=-2 a,-a,+a$ and $+2 a$, respectively. A positive charge $q$ is placed on the positive $y$ axis at a distance $b>0$. Four options of the signs of these charges are given in List I. The direction of the forces on the charge $q$ is given in List II. Match List I with List II and select the correct answer using the code given below the lists.
[JEE-Advance-2014]


## List-I

(P) $\mathrm{Q}_{1}, \mathrm{Q}_{2}, \mathrm{Q}_{3}, \mathrm{Q}_{4}$ all positive
(Q) $\mathrm{Q}_{1}, \mathrm{Q}_{2}$ positive; $\mathrm{Q}_{3}, \mathrm{Q}_{4}$ negative
(R) $\mathrm{Q}_{1}, \mathrm{Q}_{4}$ positive; $\mathrm{Q}_{2}, \mathrm{Q}_{3}$ negative
(S) $\mathrm{Q}_{1}, \mathrm{Q}_{3}$ positive; $\mathrm{Q}_{2}, \mathrm{Q}_{4}$ negative

## List-II

(1) $+x$
(2) $-x$
(3) $+y$
(4) $-y$

## Code :

(A) P-3, Q-1, R-4, S-2
(B) P-4, Q-2, R-3, S-1
(C) P-3, Q-1, R-2, S-4
(D) P-4, Q-2, R-1, S-3
25. An infinitely long uniform line charge distribution of charge per unit length $\lambda$ lies parallel to the $y$-axis in the $y-z$ plane at $z=\frac{\sqrt{3}}{2} a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the $\mathrm{x}-\mathrm{y}$ plane with its centre at the origin is $\frac{\lambda \mathrm{L}}{\mathrm{n} \varepsilon_{0}}\left(\varepsilon_{0}=\right.$ perimittivity of free space) then the value of $n$ is.
[JEE-Advance-2015]

26. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density $\lambda$ are kept parallel to each other. In their resulting electric field, point charges $q$ and $-q$ are kept in equilibrium between them. The point charges are confined to move in the $x$ direction only. If they are given a small displacement about their equlibrium positions, then the correct statement(s) is (are) :
[JEE-Advance-2015]


(A) Both charges execute simple harmonic motion
(B) Both charges will continue moving in the direction of their displacement
(C) Charge $+q$ executes simple harmonic motion while charge $-q$ continues moving in the direction of its displacement.
(D) Charge -q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.
27. Consider a uniform spherical distirbution of radius $R_{1}$ centred at the origin $O$. In this distibution, a spherical cavity of radius $R_{2}$, centred at $P$ with distance $O P=a=R_{1}-R_{2}$ (see figure) is made. If the electric field inside the cavity at position $\overrightarrow{\mathrm{r}}$ is $\overrightarrow{\mathrm{E}}(\overrightarrow{\mathrm{r}})$, then the correct statement(s) is(are) :
[JEE-Advance-2015]
(A) $\vec{E}$ is uniform, its magnitude is independent of $R_{2}$ but its direction depends on $\vec{r}$
(B) $\vec{E}$ is uniform, its magnitude depends on $R_{2}$ and its direction depends on $\vec{r}$
(C) $\overrightarrow{\mathrm{E}}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$
(D) $\overrightarrow{\mathrm{E}}$ is uniform and both its magnitude and direction depend on $\overrightarrow{\mathrm{a}}$
28. A length-scale $(\ell)$ depends on the permittivity $(\varepsilon)$ of a dielectric material, Boltzmann constant $\mathrm{k}_{\mathrm{B}}$, the absolute temperature $T$, the number per unit volume ( $n$ ) of certain charged particles, and the charge (q) carried by each of the particles, Which of the following expressions(s) for $\ell$ is(are) dimensionally correct?
[JEE-Advance-2016]
(A) $\ell=\sqrt{\left(\frac{\mathrm{nq}^{2}}{\varepsilon \mathrm{k}_{\mathrm{B}} \mathrm{T}}\right)}$
(B) $\ell=\sqrt{\left(\frac{\varepsilon \mathrm{k}_{\mathrm{B}} \mathrm{T}}{\mathrm{nq}^{2}}\right)}$
(C) $\ell=\sqrt{\left(\frac{\mathrm{q}^{2}}{\varepsilon \mathrm{~m}^{2 / 3} \mathrm{k}_{\mathrm{B}} \mathrm{T}}\right)}$
(D) $\ell=\sqrt{\left(\frac{\mathrm{q}^{2}}{\varepsilon \mathrm{~m}^{1 / 3} \mathrm{k}_{\mathrm{B}} \mathrm{T}}\right)}$
29. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius $R$ as shown in the figure. Which of the following statements is/are correct ?
[JEE-Advance-2017]

(A) The circumference of the flat surface is an equipotential
(B) The electric flux passing through the curved surface of the hemisphere is $-\frac{\mathrm{Q}}{2 \varepsilon_{0}}\left(1-\frac{1}{\sqrt{2}}\right)$
(C) Total flux through the curved and the flat surfaces is $\frac{\mathrm{Q}}{\varepsilon_{0}}$
(D) The component of the electric field normal to the flat surface is constant over the surface.
30. An infinitely long thin non-conducting wire is parallel to the $z$-axis and carries a uniform line charge density $\lambda$. It pierces a thin non-conducting spherical shell of radius $R$ in such a way that the arc PQ subtends an angle $120^{\circ}$ at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is $\varepsilon_{0}$. Which of the following statements is (are) true ?
[JEE-Advance-2018]

(A) The electric flux through the shell is $\sqrt{3} \mathrm{R} \lambda / \varepsilon_{0}$
(B) The z -component of the electric field is zero at all the points on the surface of the shell
(C) The electric flux through the shell is $\sqrt{2} \mathrm{R} \lambda / \varepsilon_{0}$
(D) The electric field is normal to the surface of the shell at all points
31. A particle, of mass $10^{-3} \mathrm{~kg}$ and charge 1.0 C , is initially at rest. At time $\mathrm{t}=0$, the particle comes under the influence of an electric field $\overrightarrow{\mathrm{E}}(\mathrm{t})=\mathrm{E}_{0} \sin \omega \mathrm{t} \hat{\mathrm{i}}$ where $\mathrm{E}_{0}=1.0 \mathrm{~N} \mathrm{C}^{-1}$ and $\omega=10^{3} \mathrm{rad} \mathrm{s}^{-1}$. Consider the effect of only the electrical force on the particle. Then the maximum speed, in $\mathrm{ms}^{-1}$, attained by the particle at subsequent times is. $\qquad$ [JEE-Advance-2018]
32. The electric field $E$ is measured at a point $P(0,0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d . List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.
[JEE-Advance-2018]

## List-I

P. E is indpendent of $d$
Q. $\quad E \propto \frac{1}{d}$
R. $\quad \mathrm{E} \propto \frac{1}{\mathrm{~d}^{2}}$
S. $\quad \mathrm{E} \propto \frac{1}{\mathrm{~d}^{3}}$
(A) $\mathrm{P} \rightarrow 5 ; \mathrm{Q} \rightarrow 3,4 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 2$
(B) $\mathrm{P} \rightarrow 5 ; \mathrm{Q} \rightarrow 3, ; \mathrm{R} \rightarrow 1,4 ; \mathrm{S} \rightarrow 2$
(C) $\mathrm{P} \rightarrow 5 ; \mathrm{Q} \rightarrow 3, ; \mathrm{R} \rightarrow 1,2 ; \mathrm{S} \rightarrow 4$
(D) $\mathrm{P} \rightarrow 4 ; \mathrm{Q} \rightarrow 2,3 ; \mathrm{R} \rightarrow 1 ; \mathrm{S} \rightarrow 5$

## List-II

1. A point charge Q at the origin
2. A small dipole with point charges Q at $(0,0, \ell)$ and -Q at $(0,0,-\ell)$. Take $2 \ell \ll d$
3. An infinite line charge coincident with the $x$-axis, with uniform linear charge density $\lambda$.
4. Two infinite wires carrying uniform linear Charge density parallel to the x - axis. The one along ( $\mathrm{y}=0, \mathrm{z}=\ell$ ) has a charge density $+\lambda$ and the one along $(y=0, z=-\ell)$ has a charge density $-\lambda$. Take $2 \ell \ll \mathrm{~d}$
5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

## ELECTROSTATICS

## (CBSE Previous Year's Questions)

1. An electrostatic field line cannot be discontinuous. Why ?
[1; CBSE-2005]
2. Define electric field intensity. Write its S. 1 unit. Write the magnitude and direction of electric field intensity due to an electric dipole of length $2 a$ at the mid- point of the line joining the two charges.
[2; CBSE-2005]
3. State Gauss' theorem. Apply this theorem to obtain the expression for the electric field intensity at a point due to an infmitely long, thin, umformly charged straight wire.
[3; CBSE-2005]
4. Define the term electric dipole moment. Is it a scalar or a vector quantity?
[1; CBSE-2006]
5. A point charge ' $q$ ' is placed at O as shown in the figure.
[2; CBSE-2006] Is $\mathrm{V}_{\mathrm{P}}-\mathrm{V}_{\mathrm{Q}}$ positive or negative when (i) $\mathrm{q}>0$, (ii) $\mathrm{q}<0$ ? Justify your answer.

6. Using Gauss's theorem, show mathematically that for any point outside the shell, the field due to a uniformly charged thin spherical shell is the sai as if the entire charge of the shell is concentrated at the centre. Why do you expect the electric field inside the shell to be zero according to this theorem?
[3; CBSE-2006]
7. Two point charges $4 \mu \mathrm{C}$ and $-2 \mu \mathrm{C}$ are separated by a distance of 1 m in air. Calculate at what point on the line joining the two charges is the electric potential zero.
[1; CBSE-2007]
8. State Gauss's theorem in electrostatics. Apply this theorem to derive an expression for electric field intensity at a point near an infinitely long straight charged wire.
[2; CBSE-2007]
9. A $500 \mu \mathrm{C}$ Charge is at the centre of a square of side 10 cm . Find the work done in moving a charge of $10 \mu \mathrm{C}$ between two diagonally opposite points on the square.
[1; CBSE-2008]
10. Derive the expression for the electric potential at any point along the axial line of an electric dipole.
[1; CBSE-2008]
11. (i) Can two equi- potential surfaces intersect each other? Give reasons.
(ii) Two charges -q and +q are located at points $\mathrm{A}(0,0,-\mathrm{a})$ and $\mathrm{B}(0,0,+\mathrm{a})$ respectively. How much work is done in moving a test charge from point $\mathrm{P}(7,0,0)$ to $\mathrm{Q}(-3,0,0)$ ?
[2; CBSE-2009]
12. State Gauss's law in electrostatics. Using this law derive an expression for the electric field due to a uniformly charged infinite plane sheet.
[3; CBSE-2009]
13. Name the physical quantity whose S .1 . unit is $\mathrm{JC}^{-1}$. Is it a scalar or a vector quantity ?
[1; CBSE-2010]
14. Define electric dipole moment. Write its S.I. unit.
[1; CBSE-201I]
15. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V , What is the potential at the centre of the sphere?
[1; CBSE-2011]
16. A thin straight infinitely long conducting wire having charge density $\lambda$ is enclosed by a cylindrical surface of radius $r$ and length $l$, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder.
[2; CBSE-2011]
17. Plot a graph showing the variation of coulomb force $(\mathrm{F})$ versus $\left(\frac{1}{\mathrm{r}^{2}}\right)$, where r is the distance between the two charges of each pair of charges: $(1 \mu \mathrm{C}, 2 \mu \mathrm{C})$ and $(2 \mu \mathrm{C},-3 \mu \mathrm{C})$, interpret the graphs obtained.
[2; CBSE-2011]
18. Two wires of equal length, one of copper and the other of manganin have the same resistance. Which wire is thicker?
[1; CBSE-2012]
19. Acharge ' $q$ ' is moved without acceleration from $A$ to $C$ along the path from Ato $B$ and then from $B$ to C in electric field E as shown in the figure, (i) Calculate the potential difference between A and C . (ii) At which point (of the two) is the electric potential more and why?

20. An electric dipole is held in a uniform electric field.
(i) Show that the net force acting on it is zero
(ii) The dipole is aligned parallel to the field. Find the work done in rotating it through the angle of $180^{\circ}$.
[2; CBSE-2012]
21. Two charges of magnitudes $-2 Q$ and $+Q$ are located at points $(a, 0)$ and $(4 a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius '3a' with its centre at the origin?
[CBSE-2013]
22. (a) Define electric dipole moment. Is it a scalar or a vector? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.
(b) Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

## OR

Using Gauss' law deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point (i) outside and (ii) inside the shell.
Plot a graph showing variation of electric field as a function of $r>\operatorname{Rand} r<R$. ( $r$ being the distance from the centre of the shell)
[CBSE-2013]
23. Why do the electrostatic field lines not form closed loops ?
[CBSE-2014]
24. Draw a labelled diagram of Van de Graaft generator. State its working principle to show how by introducing a small charged sphere, into a larger sphere, a large amount of charge can be transferred to the outer sphere state the use of this machine and also point out its limitations.

OR
(a) Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\overrightarrow{\mathrm{E}}$.
(b) Consider two hollow concentric spheres, $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$, enclosing charges 2 Q and 4 Q respectively as shown in the figure, (i) Find out the ratio of the electric flux through them, (ii) How will the electric flux through the sphere $S_{1}$ change if a medium of dielectric constant ' $\varepsilon$ ' is introduced in the space inside $S_{1}$ in place of air ? Deduce the necessary expression.
[CBSE-2014]

25. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV strikes a nucleus of $Z=80$, stops and reverses its direction.
[2; CBSE-2015]
26. (a) State Gauss's law in electrostatics. Show, with the help of a suitable example along with the figure, that the outward flux due to a point charge ' q ', in vacuum within a closed surface, is independent of its size or shape and is given by $\mathrm{q} / \varepsilon_{0}$.
[5; CBSE-2015]

(b) Two parallel uniformly charged infinite plane sheets, ' 1 ' and ' 2 ', have charge densities $+\sigma$ and
$-2 \sigma$ respectively. Give the magnitude and direction of the net electric field at a point
(i) in between the two sheets and
(ii) outside near the sheet ' 1 '.
27. What is the amount of work done in moving a point charge Q around a circular arc of radius ' r ' at the centre of which another point charge ' $q$ ' is located?
[1; CBSE-2016]
28. Find the electric field intensity due to a uniformly charged spherical shell at a point (i) outside the shell and (ii) inside the shell. Plot the graph of electric field with distance from the centre of the shell.
[3; CBSE-2016]
29. (a) Explain why, for any charge configuration, the equipotential surface through a point is normal to the electric field at the point.
[5; CBSE-2016]
Draw a sketch of equipotential surfaces due to a single charge ( -q ), depicting the electric field lines due to the charge.
(b) Obtain an expression for the work done to dissociate the system of three charges placed at the vertices of an equilateral triangle of side 'a' as shown below.

30. (a) Derive an expression for the electric field E due to a dipole of length ' 2 a ' at a point distant r from the centre of the dipole on the axial line.
[CBSE-2017]
(b) Draw a graph of E versus r for $\mathrm{r} \gg \mathrm{a}$.
(c) If this dipole were kept in a uniform external electric field $\mathrm{E}_{0}$, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

## OR

(a) Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density $\sigma$.
(b) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r , in front of the charged plane sheet.

## ANSWER KEY

## EXERCISE (S-1)

1. Ans. 80
2. Ans. 003
3. Ans. $(1+2 \sqrt{ } 2) / 4 \mathrm{Q}$
4. Ans. 9
5. Ans. 3
6. Ans. 4
7. Ans. 400
8. Ans. 9.30
9. Ans. 3
10. Ans. 8
11. Ans. 2
12. Ans. 1
13. Ans. 2
14. Ans. 4
15. Ans. 576
16. Ans. 3
17. Ans. 4
18. Ans. 4
19. Ans. 6
20. Ans. 0
21. Ans. $-\frac{\mathrm{kq}^{2}}{\mathrm{a}}(3-\sqrt{2})$
22. Ans. 2
23. Ans. 90
24. Ans. 2
25. Ans. 2
26. Ans. $2 \tan ^{-1}\left(\frac{\sigma q}{2 \varepsilon_{0} m g}\right)$
27. Ans. 20
28. Ans. $\left[\frac{2 K Q q}{m R}\left(\frac{r-R}{r}+\frac{3}{8}\right)\right]^{1 / 2}$
29. Ans. 2
30. Ans. 4
31. Ans. $\frac{k P}{\sqrt{2} \mathrm{y}^{3}}(-\hat{\mathrm{i}}-2 \hat{\mathrm{j}})$
32. Ans. 028
33. Ans. (a) $K . E=\frac{P}{4 \pi \varepsilon_{0}} \frac{\mathrm{Q}}{\mathrm{d}^{2}}$,
(b) $\frac{Q P}{2 \pi \varepsilon_{0} \mathrm{~d}^{3}}$ along positive x -axis
34. Ans. (a) (i) $V=\frac{k\left(q_{a}+q_{b}\right)}{r}, E=\frac{k\left(q_{a}+q_{b}\right)}{r^{2}}$; (ii) $\frac{k\left(q_{a}+q_{b}\right)}{R}+\frac{k q_{b}}{r}-\frac{k q_{b}}{b} ; \frac{\mathrm{kq}_{b}}{r^{2}}$
(b) $\left.\sigma_{R}=\left(\frac{\mathrm{q}_{\mathrm{a}}+\mathrm{q}_{\mathrm{b}}}{4 \pi \mathrm{R}^{2}}\right), \sigma_{\mathrm{a}}=-\frac{\mathrm{q}_{\mathrm{a}}}{4 \pi \mathrm{a}^{2}} ; \sigma_{\mathrm{b}}=-\frac{\mathrm{q}_{\mathrm{b}}}{4 \pi \mathrm{~b}^{2}} ;(\mathrm{c}) \mathrm{f}=0\right]$
35. Ans. $-Q / 3$
36. Ans. 3
37. Ans. $V^{\prime}=\left(\frac{a}{3 t}\right)^{1 / 3} V$
38. Ans. $-\varepsilon \mathrm{E}, \varepsilon \mathrm{E}$ and so on

## EXERCISE (S-2)

1. Ans. $\frac{\sigma}{\pi \epsilon_{0}}$
2. Ans. a
3. Ans. $\pi K d t \rho R^{2}$
4. Ans. 3
5. Ans. 3
6. Ans. 8
7. Ans. $v_{0}=3 \mathrm{~m} / \mathrm{s} ;$ K.E. at the origin $=(27-10 \sqrt{6}) \times 10^{-4} \mathrm{~J}$ approx. $2.5 \times 10^{-4} \mathrm{~J}$
8. Ans. 6
9. Ans. $\lambda \mathrm{RE}_{0} \hat{\mathrm{i}}$
10. Ans. $\sqrt{4 \pi \varepsilon_{0} K a}$
11. Ans. $\mathrm{W}_{\text {first tep }}=\left(\frac{8}{3}-\frac{4}{\sqrt{5}}\right) \frac{\mathrm{Kq}^{2}}{\mathrm{r}}, \mathrm{W}_{\text {second step }}=0, \mathrm{~W}_{\text {total }}=0$
12. Ans. (a) $\mathrm{H}=\frac{4 \mathrm{a}}{3}$, (b) $\mathrm{U}=\mathrm{mg}\left[2 \sqrt{\mathrm{~h}^{2}+\mathrm{a}^{2}}-\mathrm{h}\right]$ equilibrium at $\mathrm{h}=\frac{\mathrm{a}}{\sqrt{3}}$,

13. Ans. (i) - $\mathrm{Q},+\mathrm{Q}$ (ii) $\mathrm{E}_{\mathrm{A}}=0$, can't be found (iii) can't be found (iv) can't be found (v) 0 (vi) No, induced charge on outer surface]
14. Ans. 180

$$
\text { 15. Ans. } \mathrm{F}=\frac{\mathrm{Q}^{2}}{32 \pi \varepsilon_{0} \mathrm{R}^{4}}\left(\mathrm{R}^{2}-\mathrm{h}^{2}\right)
$$

16. Ans. (a) radius $=4 \mathrm{a}$ \& center $(5 \mathrm{a}, 0)$
(b) $\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{-2 \mathrm{Q}}{|(\mathrm{x}+3 \mathrm{a})|}+\frac{\mathrm{Q}}{|(\mathrm{x}-3 \mathrm{a})|}\right]$
(c) speed $v=\sqrt{\frac{1}{4 \pi \varepsilon_{0}} \times \frac{q Q}{2 m a}}$

17. Ans. 7

## EXERCISE (O-1)



## EXERCISE (O-2)

| 1. Ans. (A) | 2. Ans. (B) | 3. Ans. (B) | 4. Ans. (C) | 5. Ans. (C) | 6. Ans. (A) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 7. Ans. (A) | 8. Ans. (B) | 9. Ans. (C) | 10. Ans. (A) | 11. Ans. (C) | 12. Ans. (B) |
| 13. Ans. (A) | 14. Ans. (B) | 15. Ans. (D) | 16. Ans. (B) | 17. Ans. (B) | 18. Ans. (C) |
| 19. Ans. (A) | 20. Ans. (B) | 21. Ans. (A) | 22. Ans. (C) | 23. Ans. (A) | 24. Ans. (C) |
| 25. Ans. (A, C) | 26. Ans. (A,C) | 27. Ans. (A,D) | 28. Ans. (A,B) | 29. Ans. (A,B,C) |  |
| 30. Ans. (B) | 31. Ans. (C) | 32. Ans. (B) | 33. Ans. (A) | 34. Ans. (D) |  |

35. Ans. (A) P,R,S (B) P,Q,R,S,T (C) Q,T (D) R,S

## EXERCISE-JM

| 1. Ans. (3) | 2. Ans. (4) | 3. Ans. (2) | 4. Ans. (1) | 5. Ans. (4) | 6. Ans. (3) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 7. Ans. (3) | 8. Ans. (2) | 9. Ans. (1) | 10. Ans. (4) | 11. Ans. (4) | 12. Ans. (2) |
| 13. Ans. (1) | 14. Ans. (4) | 15. Ans. (1) | 16. Ans. (1 or 2) 17. Ans. (3) | 18. Ans. (2) |  |
| 19. Ans. (1) | 20. Ans. (1) |  |  |  |  |


|  | EXERCISE-(JA) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1.Ans. (A) 2.Ans. (B) | 3. Ans. (A) | 4. Ans. 2 | 5. Ans. (A) | 6. Ans. (D) |
| 7. Ans. (A,D) | 8. Ans. (C) | 9. Ans. (A,B,C |  | 10. Ans. (A) |
| 11. Ans. (C,D) | 12. Ans. 3 | 13. Ans. (C) | 14. Ans. (B) | 15. Ans. 6 |
| 16. Ans. (A,C,D) | 17. Ans. (C) | 18. Ans. (D) | 19. Ans. (A, B, |  |
| 20. Ans. (B, D) | 21. Ans. (C, D) | 22. Ans. (C) | 23. Ans. (C) | 24. Ans. (A) |
| 25. Ans. 6 26. Ans. (C) | 27. Ans. (D) | 28. Ans. (B,D) | 29. Ans. (A) (B) | 30. Ans. (A,B) |
| 31. Ans. 2 [1.99, 2.01] | 32. Ans. (B) |  |  |  |

