Solved Paper 2017*

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

	No. of Questions
Part I Physics	1–40
Part II Chemistry	41–80
Part III a. English Proficiency	81–95
b. Logical Reasoning	96–105
Part IV Mathematics	106–150

- All questions are Multiple Choice Questions having four options out of which **only one** is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.
- Time allotted to complete this paper is 3 hrs.

PART I

Physics

1. If temperature of a black body increases from 300 K to 900 K, then the rate of energy radiation increases by

a. 81 *b.* 3
c. 9 *d.* 2

2. A whistle of frequency 500 Hz tied to the end of a string of length 1.2 m revolves at 400 rev/min. A listener standing some distance away in the plane of rotation of whistle hears frequencies in the range.

(Speed of sound = 340 m/s)

a. 436 Hz to 574 Hz *b.* 426 Hz to 586 Hz
c. 426 Hz to 574 Hz *d.* 436 Hz to 586 Hz

3. The focal length of a thin convex lens for red and blue rays are 100 cm and 96.8 cm, respectively. Then, the dispersive power of the material of the lens is

a. 0.968 *b.* 0.98
c. 0.0325 *d.* 0.325

4. Two metal plates having a potential difference of 800 V are 2 cm apart. It is found that a particle of mass 1.96×10^{-15} kg remains suspended in the region between the plates. The charge on the particle must be (e = elementary charge).

a. $2e$ *b.* $3e$ *c.* $6e$ *d.* $8e$

5. At what angle θ to the horizontal should an object is projected, so that the maximum height reached is equal to the horizontal range?

a. $\tan^{-1}(2)$ *b.* $\tan^{-1}(4)$
c. $\tan^{-1}\left(\frac{2}{3}\right)$ *d.* $\tan^{-1}(3)$

6. A body of mass 1 kg is executing simple harmonic motion. Its displacement y at t seconds is given by $y = 6\sin\left(100t + \frac{\pi}{4}\right)$ cm. Its maximum kinetic energy is

a. 6 J *b.* 18 J *c.* 24 J *d.* 36 J

7. A positive charge q is projected in magnetic field of width $\frac{mv}{\sqrt{2}\,qB}$ with velocity v. Then, the time taken by charged particle to emerge from the magnetic field is

a. $\frac{m}{\sqrt{2}\,qB}$ *b.* $\frac{\pi m}{4qB}$

c. $\frac{\pi m}{2qB}$ *d.* $\frac{\pi m}{\sqrt{2}\cdot qB}$

8. In Young's double slit experiment, the slits are 2 mm apart and are illuminated by photons of two wavelengths $\lambda_1 = 12000$ Å and $\lambda_2 = 10000$ Å. At what minimum distance from the common central bright fringe on the screen 2m from the slit will a bright fringe from one interference pattern coincide with a bright fringe from the other?

a. 8 mm *b.* 6 mm

c. 4 mm *d.* 3 mm

9. Two blocks A and B are placed one over the other on a smooth horizontal surface. The maximum horizontal force that can be applied on lower block B, so that A and B move without separation is 49 N. The coefficient of friction between A and B is

a. 0.2 *b.* 0.3

c. 0.5 *d.* 0.8

10. An aeroplane is flying in a horizontal direction with a velocity u and at a height of 2000 m. When it is vertically below a point A on the ground a food packet is released from it. The packet strikes the ground at point B. If $AB = 3$ km and $g = 10\text{ m/s}^2$, then the value of u is

a. 54 km/h *b.* 540 km/h

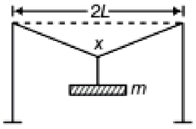
c. 150 km/h *d.* 300 km/h

11. A conducting circular loop is placed in a uniform magnetic field, $B = 0.025$ T with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm/s. The induced emf when the radius is 2 cm, is

a. $2\pi\,\mu$V *b.* $\pi\,\mu$V

c. $\frac{\pi}{2}\,\mu$V *d.* $2\,\mu$V

12. A mild steel wire of length $2L$ and cross-sectional area A is stretched, well with in the elastic limit, horizontally between two pillars as shown in figure. A mass m is suspended from the mid-point of the wire, strain in the wire is



a. $\frac{x^2}{2L^2}$ *b.* $\frac{x}{L}$ *c.* $\frac{x^2}{L}$ *d.* $\frac{x^2}{2L}$

13. The resistance of a wire at 20 °C is 20Ω and 500 °C is 60 Ω. At which temperature, its resistance will be 25Ω?

a. 50 °C *b.* 60 °C

c. 70 °C *d.* 80 °C

14. The de-Broglie wavelength of a proton (charge $= 1.6\times 10^{-19}$ C, $m = 1.6\times 10^{-27}$ kg) accelerated through a potential difference of 1 kV is

a. 600 Å *b.* 0.9×10^{-12} m

c. 7 Å *d.* 0.9 nm

15. An ice-berg of density 900 kgm^{-3} is floating in water of density 1000 kgm^{-3}. The percentage of volume of ice-berg outside the water is

a. 20% *b.* 35%

c. 10% *d.* 11%

16. The total energy of an electron in the first excited state of hydrogen is about − 3.4 eV. Its kinetic energy in this state is

a. − 3.4 eV *b.* − 6.8 eV *c.* 6.8 eV *d.* 3.4 eV

17. A common emitter amplifier has a voltage gain of 50, an input impedance of 100Ω and an output impedance of 200Ω. The power gain of the amplifier is

a. 500 *b.* 1000

c. 1250 *d.* 50

18. The horizontal range and maximum height attained by a projectile are R and H, respectively. If a constant horizontal acceleration $a = \frac{g}{4}$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

a. $(R + H), \frac{H}{2}$ *b.* $\left(R + \frac{H}{2}\right), 2H$

c. $(R + 2H), H$ *d.* $(R + H), H$

19. A balloon is filled at 27° C and 1 atm pressure by 500 m^3 He. At −3° C and 0.5 atm pressure, the volume of He will be

a. 700 m^3 *b.* 900 m^3
c. 1000 m^3 *d.* 500 m^3

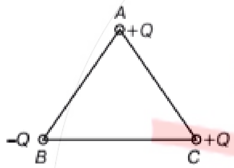
20. The ratio of intensity at the centre of a bright fringe to the intensity at a point distance one-fourth of the distance between two successive bright fringes will be

a. 4 *b.* 3 *c.* 2 *d.* 1

21. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a vertical displacement from equilibrium, it undergoes oscillation with a time period T. Then,

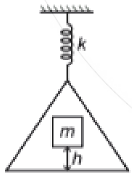
a. $T \propto \sqrt{\rho}$ *b.* $T \propto \frac{1}{\sqrt{A}}$
c. $T \propto \sqrt{A}$ *d.* $T \propto \frac{1}{\sqrt{m}}$

22. Three charges are placed at the three vertices of an equilateral triangle of side a as shown in the figure. The force experienced by the charge placed at the vertex A in a direction normal to BC is



a. $\frac{Q^2}{4\pi\varepsilon_0 a^2}$ *b.* $-\frac{Q^2}{4\pi\varepsilon_0 a^2}$ *c.* zero *d.* $\frac{Q^2}{2\pi\varepsilon_0 a^2}$

23. A load of mass m falls from a height h on the scale pan hung from a spring as shown. If the spring constant is k and the mass of the scale pan is zero and the mass m does not bounce relative to the pan, then the amplitude of vibration is



a. $\frac{mg}{k}$ *b.* $\frac{mg}{k}\sqrt{1+\frac{2hk}{mg}}$
c. $\frac{mg}{k}+\frac{mg}{k}\sqrt{\frac{1+2hk}{mg}}$ *d.* None of these

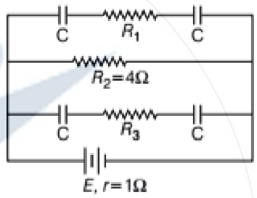
24. The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and $\frac{N_0}{e}$ counts per minute at $t = 5$ min. The time (in minutes) at which the activity reduces to half its value is

a. $\log_e \frac{2}{5}$ *b.* $\frac{5}{\log_e 2}$
c. $5 \log_{10} 2$ *d.* $5 \log_e 2$

25. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If lenses are made of different materials of refractive indices μ_1 and μ_2, R is the radius of curvature of the curved surface of the lenses, then the focal length of combination is

a. $\frac{R}{2(\mu_1+\mu_2)}$ *b.* $\frac{R}{2(\mu_1-\mu_2)}$
c. $\frac{R}{(\mu_1-\mu_2)}$ *d.* $\frac{2R}{\mu_1-\mu_2}$

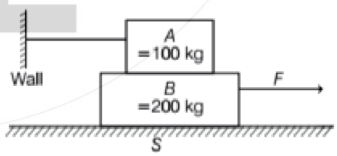
26. In the given circuit diagram,
$E = 5$ V, $r = 1\,\Omega$, $R_2 = 4\Omega$, $R_1 = R_3 = 1\,\Omega$ and $C = 3\mu F$



Then, what will be the numerical value of charge on each plates of the capacitor?

a. 24 μC *b.* 12 μC
c. 6 μC *d.* 3 μC

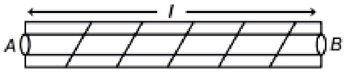
27. A block A of mass 100 kg rests on another block B of mass 200 kg and is tied to a wall as shown in the figure. The coefficient of friction between A and B is 0.2 and that between B and ground is 0.3. The minimum force required to move the block B is $(g = 10\ ms^{-2})$



a. 900 N *b.* 200 N
c. 1100 N *d.* 700 N

28. A uniform rod of length l and mass m is free to rotate in a vertical plane about A. The rod initially in horizontal position is released. The initial angular acceleration of the rod is

(Moment of inertia of rod about A is $\frac{ml^2}{3}$)



a. $\frac{3g}{2l}$ b. $\frac{2l}{3g}$ c. $\frac{3g}{2l^2}$ d. $mg\frac{l}{2}$

29. Monochromatic radiation of wavelength λ is incident on a hydrogen sample in ground state. Hydrogen atom absorbs a friction of light and subsequently emits radiations of six different wavelengths. The wavelength λ is

a. 97.2 nm b. 121.6 nm
c. 110.3 nm d. 45.2 nm

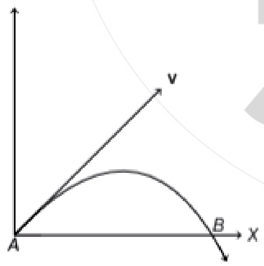
30. A coil in the shape of an equilateral triangle of side l is suspended between the pole pieces of a permanent magnet such that B is in plane of the coil. If due to a current i in the triangle, a torque τ rests on it, the side l of the triangle is

a. $2\left(\frac{\tau}{\sqrt{3}\,Bi}\right)^{\frac{1}{2}}$ b. $\frac{2}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)$
c. $2\left(\frac{\tau}{Bi}\right)^{\frac{1}{2}}$ d. $\frac{1}{\sqrt{3}}\left(\frac{\tau}{Bi}\right)$

31. Work done in increasing the size of a soap bubble from radius of (3 to 5) cm is nearly (surface tension of soap solution = 0.03 Nm^{-1})

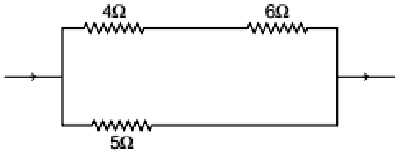
a. $0.2\,\pi$ mJ b. 2π mJ
c. 0.4 mJ d. 0.4π mJ

32. The velocity of a projectile at the initial point A is $(2\hat{i}+3\hat{j})$ m/s. Its velocity (in m/s) at point B is



a. $-2\hat{i}-3\hat{j}$ b. $-2\hat{i}+3\hat{j}$
c. $2\hat{i}-3\hat{j}$ d. $2\hat{i}+3\hat{j}$

33. In the circuit shown, the heat produced in 5Ω resistor is 10 cal s^{-1}. The heat produced per sec in 4Ω resistor will be



a. 1 cal b. 2 cal
c. 3 cal d. 4 cal

34. An α-particle after passing through potential difference of V volt collides with a nucleus. If the atomic number of the nucleus is Z, then distance of closest approach is

a. $14.4\cdot\frac{Z}{V}$ Å
b. $14.4\frac{Z}{V}$ m
c. $14.4\frac{V}{Z}$ m
d. $14.4\frac{V}{Z}$ Å

35. Two simple pendulums of lengths 5m and 20m respectively are given small displacement in one direction at the same time. They will again be in the same sense when the pendulum of shorter length has completed n oscillations. Then, n is

a. 5 b. 1 c. 2 d. 3

36. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between the plates is d. The space between the plates is now filled with two dielectrics constant $K_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $K_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

a. 1.8 pF b. 45 pF
c. 40.5 pF d. 20.25 pF

37. A particle moving along X-axis has acceleration f, at time t given by $f = f_0\left(1-\frac{t}{T}\right)$, where f_0 and T are constants. The particle at $t = 0$ and the instant when $f = 0$, the particle's velocity v_X is

a. f_0T b. $\frac{1}{2}f_0T^2$
c. f_0T^2 d. $\frac{1}{2}f_0T$

38. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a sky satellite orbiting a few 100 km above the earth's surface ($R = 6400$ km) will approximately be

a. $\frac{1}{2}$ h *b.* 1 h *c.* 2 h *d.* 4 h

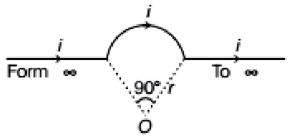
39. A transverse wave propagating on a stretched string of linear density 3×10^{-4} kgm^{-1} is represented by the equation

$$y = 0.2 \sin(1.5x + 60t)$$

where, x in metres and t is in seconds. The tension in the string (in Newton) is

a. 0.24 *b.* 0.48
c. 1.20 *d.* 1.80

40. What is the magnetic field at the centre of arc in the figure below?



a. $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}[\sqrt{2} + \pi]$

b. $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}\left[\sqrt{2} + \frac{\pi}{4}\right]$

c. $\frac{\mu_0}{4\pi} \cdot \frac{i}{r}[\sqrt{2} + \pi]$

d. $\frac{\mu_0}{4\pi} \cdot \frac{i}{r}\left[\sqrt{2} + \frac{\pi}{4}\right]$

PART II

Chemistry

41. 4 g of copper was dissolved in conc. HNO_3. The copper nitrate thus obtained gave 5g of its oxide on strong heating. The equivalent weight of copper is

a. 23 *b.* 32
c. 12 *d.* 20

42. Choose the incorrect statement.

a. Sodium borohydride reacts very slowly with cold water.

b. Sodium borohydride reacts violently with cold water to give $H_2(g)$.

c. Solubility of sodium borohydride in water at 25°C is 10.05 g/mL.

d. Melting point of sodium borohydride is 500°C.

43. The orbital angular momentum of an electron in $2s$ orbital is

a. $\frac{h}{4\pi}$ *b.* zero
c. $\frac{h}{2\pi}$ *d.* $\sqrt{2}\cdot\frac{h}{2\pi}$

44. Which of the following are isoelectronic species?

I. CH_3^+ II. NH_2^-
III. NH_4^+ IV. NH_3

a. I, II and III *b.* II, III and IV
c. I, II and IV *d.* II and I

45. Two elements A and B have electronegativities 1.2 and 3.0 respectively. The nature of bond between A and B would be

a. ionic *b.* covalent
c. coordinate *d.* metallic

46. In the compound,

$$\underset{(I)}{CH_2}=\underset{(II)}{CH}-C\equiv\underset{(III)}{CH}$$

The most acidic hydrogen atom is

a. Only I *b.* Only II
c. Only III *d.* All are equally acidic

47. Reductive ozonolysis of $(CH_3)_2C=C(CH_3)_2$ followed by hydrolysis gives

a. only one type of ketone
b. only one type of aldehyde
c. two types of ketone
d. two types of aldehyde

48. Which of the following reactions does not involved absorption energy?

I. $O(g)+e^- \longrightarrow O^-(g)$

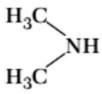
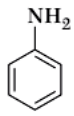
II. $S(g)+e^- \longrightarrow S^-(g)$

III. $O^-(g)+e^- \longrightarrow O^{2-}(g)$

IV. $Cl(g)+e^- \longrightarrow Cl^-(g)$

a. Only II *b.* I and III
c. I, II and III *d.* I, II and IV

49. The most reactive amine towards reaction with dil. HCl is

a. CH_3-NH_2 *b.*  *c.* $(CH_3)_3N$ *d.* 

50. Blocks of magnesium are fixed to the bottom of a ship to

a. block hole in the ship
b. acidity of sea water
c. make the ship lighter
d. prevent the action of water and salt

51. On electrolysis of water, a total of 1 mole of gases is evolved. The amount of water decomposed is

a. 1 mol *b.* 2 mol *c.* $\frac{1}{3}$ mol *d.* $\frac{2}{3}$ mol

52. How many moles of Fe^{2+} ions are formed when excess of iron react with 500 mL 0.4 NHCl, under inert atmosphere?
(assume no change in volume)

a. 0.4 *b.* 0.1 *c.* 0.2 *d.* 0.8

53. Sodium sulphate is soluble in water but barium sulphate is insoluble because

a. hydration energy of Na_2SO_4 is more than of its lattice energy

b. lattice energy of $BaSO_4$ is more than its hydration energy

c. Both (a) and (b)

d. None of the above

54. In the reaction,

$$CH_3CH_2Cl \xrightarrow{KCN} (A) \xrightarrow{LiAlH_4} (P)$$; end product (P) is

a. $CH_3CH_2NO_2$ *b.* $CH_3CH_2CH_2NO_2$
c. $CH_3CH_2NH_2$ *d.* $CH_3CH_2CH_2NH_2$

55. Which of the following reactions is an example of calcination process?

a. $2Ag + 2HCl + [O] \longrightarrow 2AgCl + H_2O$
b. $2Zn + O_2 \longrightarrow 2ZnO$
c. $2ZnS + 3O_2 \longrightarrow 2ZnO + 2SO_2$
d. $MgCO_3 \xrightarrow{\Delta} MgO + CO_2$

56. For an endothermic reaction, where ΔH represent the enthalpy of reaction in kJ/mol, the minimum value for energy of activation (for forward reaction) will be

a. less than ΔH *b.* zero
c. more than ΔH *d.* equal to ΔH

57. Which of the following metal is leached by cyanide process?
a. Ag *b.* Na *c.* Al *d.* Cu

58. Which of the following is a diamagnetic complex?
a. $[Co(NH_3)_6]^{3+}$ *b.* $[NiCl_4]^{2-}$
c. $[CuCl_4]^{2-}$ *d.* $[Fe(H_2O)_6]^{3+}$

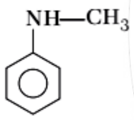
59. Neoprene is a
a. monomer of rubber *b.* synthetic rubber
c. a natural rubber *d.* vulcanised rubber

60. Among the following, which have highest melting point?
a. Ionic solids *b.* Pseudo solids
c. Molecular solids *d.* Amorphous solids

61. The night-blindness is developed due to deficiency of vitamin
a. B_6 *b.* C *c.* B_{12} *d.* A

62. The transfer RNA anticodon for the messenger RNA codon G – C – A is
a. C – G – U *b.* C – C – U
c. U – C – C *d.* G – U – C

63. 0.765 g of an acid gives 0.535 g of CO_2 and 0.138 g of H_2O. Then, the ratio of percentage of carbon and hydrogen is
a. 19 : 2 *b.* 18 : 11 *c.* 20 : 17 *d.* 1 : 7

64. Maximum pK_b value of
a. 

b. $(CH_3CH_2)_2NH$
c. $(CH_3)_2NH$
d. 

65. Which of the following is an incorrect set of quantum members?
a. $n = 2, l = 0, m = 0$ *b.* $n = 1, l = 0, m = 0$
c. $n = 3, l = 3, m = 0$ *d.* $n = 2, l = 1, m = 1$

66. The most acidic oxide for nitrogen is
a. NO_2 *b.* N_2O *c.* NO *d.* N_2O_5

67. Which of the following show maximum bond order?
a. O_2 *b.* O_2^- *c.* O_2^+ *d.* O_2^{2-}

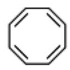
68. Which of the following show an increase in entropy?
I. Boiling of water II. Melting of ice
III. Freezing of water
IV. Formation of hydrogen gas from water
a. (I) and (II) *b.* Only (III)
c. (I), (II) and (IV) *d.* (III) and (IV)

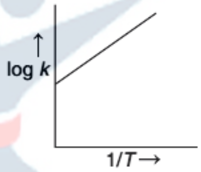
69. BF_3 is an acid, according to
a. Lewis *b.* Arrhenius
c. Bronsted and Lowery *d.* All of these

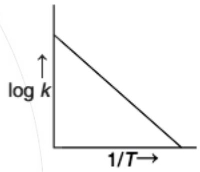
70. For the reaction,
$$N_2O_4(g) \longrightarrow 2NO_2(g)$$
a. $\Delta H > \Delta E$ *b.* $\Delta H < \Delta E$ *c.* $\Delta H = \Delta E$ *d.* $\Delta H = 0$

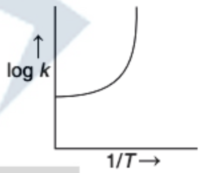
71. Which of the following elements mostly form covalent compounds?
a. Cs *b.* Rb *c.* K *d.* Li

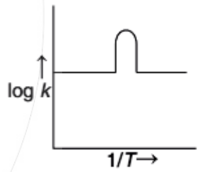
72. When aqueous solution of borax is acidified with HCl, we get
a. B_2H_6 *b.* H_3BO_3 *c.* B_2O_3 *d.* All of these

73. Which of the following compound does not follow Huckel's rule?
a.  *b.*  *c.*  *d.* 

74. A graph is ploted between log k *versus* $\frac{1}{T}$ for calculation of activation energy (E_a).
The correct plot is
a. 

b. 

c. 

d. 

75. The hybridisation of Fe in $[K_3Fe(CN)_6]$ is
a. sp^3 *b.* dsp^3 *c.* sp^3d^2 *d.* d^2sp^3

76. Which of the following shows maximum magnetic moment?
a. Mg^{2+} *b.* Ti^{3+} *c.* V^{3+} *d.* Fe^{2+}

77. Consider the following radioactive decays.
I. ${}_{92}U \xrightarrow{-\alpha} {}_{90}Th$ and II. ${}_{90}Th \xrightarrow{-\alpha} {}_{88}Ra$
In which case group of parent and daughter elements remain unchanged?
a. In (I) *b.* In (II)
c. Both in (I) and (II) *d.* None of these

78. Chlorobenzene $\xrightarrow{\text{Reaction }(X)}$ Phenol $\xrightarrow{\text{Reaction }(Y)}$ Salicylaldehyde.
The reaction(s) '*X*' and '*Y*' respectively are
a. Fries rearrangement and Kolbe
b. Cumene and Reimer-Tiemann
c. Dow and Reimer-Tiemann
d. Dow and Sandmeyer

79. Which of the following has largest number of moles?

a. 8 g of oxygen atoms
b. 16 g of oxygen gas
c. 14 g of nitrogen gas (N_2)
d. All have same number of moles

80. One mole each of four ideal gases are kept as follows.

I. 5 L of gas (*A*) at 2 atm pressure
II. 2.5 L of gas (*B*) at 2 atm pressure
III. 1.25 L of gas (*C*) at 2 atm pressure
IV. 2.5 L of gas (*D*) at 2.5 atm pressure

Which of the above gases is kept at highest temperature?

a. Gas (*A*) *b.* Gas (*B*) *c.* Gas (*C*) *d.* Gas (*D*)

PART III

a. English Proficiency

81. Along the northern frontier of India (a) is seen (b) the Himalayas mighty in their splendour. (c) No error (d)

82. The father with the son were (a) mysteriously missing (b) from the house. (c) No error (d)

83. It is not advisable (a) to take heavy luggages (b) while on journey (c) these days. No error (d)

Directions (Q. Nos. 84-85) *Fill in the blanks with suitable preposition from the alternatives given under each sentence.*

84. The problem of communal harmony cannot be glossed .... by the government.

a. at ***b.*** on
c. over ***d.*** for

85. She could not muster .... courage to stand against the maltreatment.

a. to ***b.*** up
c. about ***d.*** on

Directions (Q. Nos. 86-88) *The following sentences consist of a word or a phrase which is written in italicised letters. Each sentence is followed by four words or phrases. Select the word or the phrase which is closest to the opposite in meaning of the italicised word or phrase.*

86. Philosophers say that the world is *an illusion*.

a. a fact ***b.*** a reality
c. an actuality ***d.*** a truth

87. She used to *disparage* her neighbours every now and then.

a. please ***b.*** praise
c. belittle ***d.*** denigrate

88. The momentum of the movement *slackened* in course of time.

a. stopped
b. quickened
c. multiplied
d. recovered

Directions (Q. Nos. 89-90) *In the following sentences, a word or a phrase is written in italicised letters. For each italicised part, four words/phrases are listed below each sentence. Choose the word nearest in meaning to the italicised word/phrase.*

89. The opposition criticised the ruling party for the *deteriorating* law and order situation in the state.

a. disrupting
b. worsening
c. crumbling
d. eroding

90. The two opposing parties have reached *stalemate*.

a. dilemma
b. deadlock
c. exhaustion
d. settlement

Directions (Q. Nos. 91-95) *Read the passage given below and answer the questions that follow.*

A pioneering scheme has been started recently in Southampton of England's south coast to educate tourists who have been convicted of drunken driving.

The penalty for drunken driving might be the loss of the driving licence and a heavy fine. But under the new scheme, convicted drivers do not pay the fine. Instead they have to attend eight training sessions; one a week organised by the local authority probation service.

Designed to demonstrate the damage alcohol can do, the scheme was devised by senior probation officer John Cook. He said that about a quarter of the people who came to him had a drinking problem, and had not realised how much they were drinking. One way of getting the message across was to make the drivers pour out their usual ration of alcohol and then measure it. Almost everyone poured out not a single measure, but a double atleast, an example of how easy it is to have more than just one drink and to encourage other people to do the same. The instructors on the course are giving clinical evidence of the effects of alcohol on the body and brain. The sober truth is that drinking badly affects driving skills, although the drinker might like to believe otherwise.

91. The southampton scheme requires convicted drivers

a. to pay a heavy fine
b. to attend eight driving sessions–one a week
c. to undergo a probation service
d. to surrender their driving licence

92. John Cook devised the scheme
a. as a demonstration technique for driving
b. to deny the harmful effects of alcohol
c. to show that Southampton was concerned about drivers
d. to prove that alcohol does influence driving

93. The problem with a quarter of the people who went to John Cook was that they
a. did not want to stop drinking
b. were unaware of the fact that they could get drunk
c. would not admit that they had a drinking problem
d. did not know how much they were drinking

94. Most drivers start off with atleast
a. a double measure
b. a single measure
c. a little less than a single measure
d. two doubles

95. The truth is that alcohol
a. does not affect the body but only the brain
b. affects only the brain
c. affects the body and the brain
d. has no effect on the body or the brain

b. Logical Reasoning

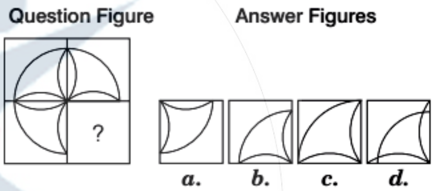
96. 'Shoes' is related to 'Leather', in the same way as 'Rubber' is related to
a. Plastic *b.* Polythene
c. Latex *d.* Chappal

97. Find the odd one from the following options
a. 81 : 243 *b.* 25 : 75
c. 64 : 192 *d.* 16 : 64

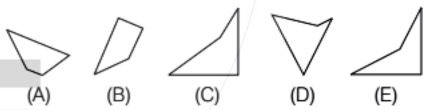
98. Complete the series by replacing '?' mark.
4, 11, 30, 67, 128, ?
a. 219 *b.* 228
c. 237 *d.* 240

99. Lakshmi is elder than Meenu. Leela is elder than Meenu but younger than Lakshmi. Latha is younger than both Meenu and Hari but Hari is younger than Meenu. Who is the youngest?
a. Lakshmi *b.* Meenu
c. Leela *d.* Latha

100. In the following question a part of problem figure is missing. Find out from the given answer figures *a*, *b*, *c* and *d* that can replace the question mark (?) to complete the figure.

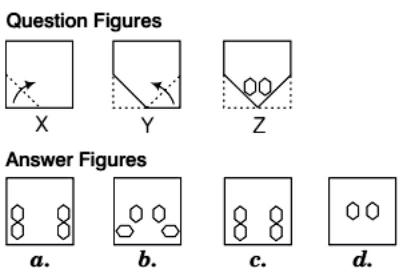


101. In the following question, five figures are given. Out of them find the three figures that can be joined to form a square.



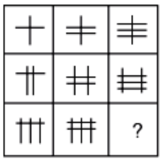
a. ACD *b.* BCD
c. BDE *d.* CDE

102. The three problem figures marked X, Y and Z show the manner in which a piece of paper is folded step by step and then cut. From the answer figures *a*, *b*, *c* and *d* select the one showing the unfolded pattern of the paper after the cut.

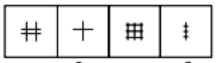


103. Choose the answer figure which completes the problem figure matrix.

Question Figures



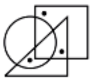
Answer Figures



a. ***b.*** ***c.*** ***d.***

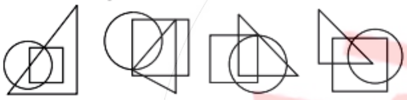
104. In the following question, one or more dots are place in the figure marked as (A). This figure is followed by four alternatives marked as *a*, *b*, *c* and *d*. One out of these four options contains region(s) common to circle, square and triangles, similar to that marked by the dot in figures (A). Find that figure

Question Figure



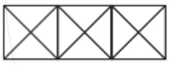
(A)

Answer Figures



a. ***b.*** ***c.*** ***d.***

105. How many different triangles are there in the figures shown below?



a. 28 ***b.*** 24
c. 20 ***d.*** 16

PART IV

Mathematics

106. The coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+\ldots+(1+x)^{30}$ is

a. ${}^{51}C_5$ **b.** ${}^{9}C_5$
c. ${}^{31}C_6-{}^{21}C_6$ **d.** ${}^{30}C_5+{}^{20}C_5$

107. If $z=a+ib$ satisfies $\arg(z-1)=\arg(z+3i)$, then $(a-1):b=$

a. $2:1$ **b.** $1:3$
c. $-1:3$ **d.** None of these

108. If p and p' denote the lengths of the perpendicular from a focus and the centre of an ellipse with semi-major axis of length a, respectively, on a tangent to the ellipse and r denotes the focal distance of the point, then

a. $ap=rp'$ **b.** $rp=ap'$
c. $ap=rp'+1$ **d.** $ap'+rp=1$

109. The value of $\sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to

a. $5(2n-9)$ **b.** $10n$
c. $9(n-4)$ **d.** None of these

110. The numbers $3^{2\sin 2\alpha-1}$, 14 and $3^{4-2\sin 2\alpha}$ form first three terms of an AP, its fifth term is

a. -25 **b.** -12 **c.** 40 **d.** 53

111. For the equation $3x^2+px+3=0$, $p>0$, if one of the roots is square of the other, then p is equal to

a. 1/2 **b.** 1 **c.** 3 **d.** 2/3

112. If $a=\log_2 3$, $b=\log_2 5$ and $c=\log_7 2$, then $\log_{140} 63$ in terms of a, b, c is

a. $\frac{2ac+1}{2c+abc+1}$ **b.** $\frac{2ac+1}{2a+c+a}$
c. $\frac{2ac+1}{2c+ab+a}$ **d.** None of these

113. If $\cos(x-y)$, $\cos x$ and $\cos(x+y)$ are in HP, then $\cos x \sec(y/2)$ is equal to

a. $\pm\sqrt{2}$ **b.** $\pm 1/\sqrt{2}$
c. ± 2 **d.** None of these

114. Let $A=\{1, 2, 3, 4, 5\}$ and R be a relation defined by
$R=\{(x, y): x, y \in A, x+y=5\}$. Then, R is

a. reflexive and symmetric but not transitive
b. an equivalence relation
c. symmetric but neither reflexive nor transitive
d. neither reflexive nor symmetric but transitive

115. The number of times the digit 5 will be written when listing the integers from 1 to 1000, is

a. 271 **b.** 272
c. 300 **d.** None of these

116. Let A and B be two sets such that $A\cap X=B\cap X=\phi$ and $A\cup X=B\cup X$ for same set X. Then,

a. $A=B$ **b.** $A=X$
c. $B=X$ **d.** $A\cup B=X$

117. Let $A=[-1, 1]$ and $f: A\to A$ be defined as $f(x)=x|x|$ for all $x\in A$, then $f(x)$ is

a. many-one and into function
b. one-one and into function
c. many-one and into function
d. one-one and onto function

118. The general solution of
$\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x$
is

a. $n\pi+\frac{\pi}{8}$ **b.** $\frac{n\pi}{2}+\frac{\pi}{8}$
c. $(-1)^n\frac{n\pi}{2}+\frac{\pi}{8}$ **d.** $2n\pi+\cos^{-1}\frac{3}{2}$

119. Two equal sides of an isosceles triangle are $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point $(1, -10)$. Find the equation of the third side

a. $x-3y=-31$ **b.** $x-3y=31$
c. $x+3y=31$ **d.** $x+3y=-31$

120. If two distinct chords drawn from the point (p, q) on the circle $x^2+y^2=px+qy$ (where $pq\neq 0$) are bisected by the X-axis, then

a. $p^2=q^2$ **b.** $p^2=8q^2$
c. $p^2<8q^2$ **d.** $p^2>8q^2$

121. The length of perpendicular drawn from the point (2, 3, 4) to line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$, is

a. $\frac{3}{7}\sqrt{101}$ **b.** $\frac{2}{7}\sqrt{101}$
c. $\frac{2}{7}\sqrt{103}$ **d.** $\frac{3}{7}\sqrt{103}$

122. The image of the point $(1, 6, 3)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ is

a. $(-1, 0, 7)$ **b.** $(-1, 0, -7)$
c. $(1, 0, 7)$ **d.** $(2, 0, 7)$

123. The distances of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along a straight line $x = y = z$ is $2\sqrt{3}k$, then the value of k is

a. 5 **b.** 6
c. $\sqrt{3}$ **d.** 4

124. $\lim_{n \to \infty} \sin[\pi\sqrt{n^2+1}]$ is equal to

a. ∞ **b.** 0
c. does not exist **d.** None of these

125. If $f(x) = \begin{cases} e^x, & x \le 0 \\ |1-x|, & x > 0 \end{cases}$, then

a. $f(x)$ is differentiable at $x = 0$
b. $f(x)$ is continuous at $x = 0, 1$
c. $f(x)$ is differentiable at $x = 1$
d. None of these

126. If a function $f: R \to R$ satisfy the equation $f(x+y) = f(x) + f(y)$, $\forall\, x, y$ and the function $f(x)$ is continuous at $x = 0$, then

a. $f(x)$ is continuous for all positive real values of x
b. $f(x)$ is continuous for all x
c. $f(x) = 0$ for all x
d. None of these

127. The value of $f(0)$, so that the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, is

a. $\frac{1}{3}$ **b.** $-\frac{1}{3}$ **c.** $\frac{2}{3}$ **d.** $-\frac{2}{3}$

128. Consider the greatest integer function, defined by $f(x) = [x]$, $0 \le x < 2$. Then,

a. f is derivable at $x = 1$
b. f is not derivable at $x = 1$
c. f is derivable at $x = 2$
d. None of these

129. Let $f(x) = -2x^3 + 21x^2 - 60x + 41$, then

a. $f(x)$ is decreasing in $(-\infty, 1)$
b. $f(x)$ is decreasing in $(-\infty, 2)$
c. $f(x)$ is increasing in $(-\infty, 1)$
d. $f(x)$ is increasing in $(-\infty, 2)$

130. Rolle's theorem is not applicable for the function $f(x) = |x|$ in the interval $[-1, 1]$ because

a. $f'(1)$ does not exist
b. $f'(-1)$ does not exist
c. $f(x)$ is discontinuous at $x = 0$
d. $f'(0)$ does not exist

131. If the curve $y = a^x$ and $y = b^x$ intersect at angle α, then $\tan\alpha$ is equal to

a. $\frac{a-b}{1+ab}$ **b.** $\frac{\log a - \log b}{1 + \log a \log b}$
c. $\frac{a+b}{1-ab}$ **d.** $\frac{\log a + \log b}{1 - \log a \log b}$

132. The expression $\frac{\int_0^n [x]\,dx}{\int_0^n \{x\}\,dx}$, where $[x]$ and $\{x\}$ are integral and fractional part of x and $n \in N$, is equal to

a. $\frac{1}{n-1}$ **b.** $\frac{1}{n}$ **c.** n **d.** $n-1$

133. The maximum value of $f(x) = x + \sin 2x$, $x \in [0, 2\pi]$ is

a. $\frac{\pi}{2}$ **b.** 2π **c.** $\frac{3\pi}{4}$ **d.** $\frac{3\pi}{2}$

134. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$ and above X-axis, is

a. $2\sqrt{2}$ **b.** $2\sqrt{2} - 2$ **c.** $2\sqrt{2} + 2$ **d.** 0

135. The solution of $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$, is

a. $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] + C = 0$
b. $\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + C$
c. $\log\left[1 - \tan\left(\frac{x+y}{2}\right)\right] = x + C$
d. None of these

136. The area enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is

a. $\frac{5}{3}$ sq units **b.** $\frac{5}{4}$ sq units
c. $\frac{5}{12}$ sq units **d.** $\frac{12}{5}$ sq units

137. If $\lim_{x \to \infty}\left(an - \frac{1+n^2}{1+n}\right) = b$, where a is finite number, then

a. $a = 2$ **b.** $a = 0$ **c.** $b = 1$ **d.** $b = -1$

138. If the papers of 4 students can be checked by anyone of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teachers, is equal to

a. 12/49 **b.** 6/49 **c.** 9/49 **d.** 15/49

139. If equation $(10x-5)^2+(10y-4)^2 = \lambda^2(3x+4y-1)^2$ represents a hyperbola, then

a. $-2<\lambda<2$
b. $\lambda>2$
c. $\lambda<-2$ or $\lambda>2$
d. $0<\lambda<2$

140. Let $\hat{a}$ and $\hat{b}$ be two non-collinear unit vectors. If $u=\hat{a}-(\hat{a}\cdot\hat{b})\hat{b}$ and $v=\hat{a}\times\hat{b}$, then $|v|$ is equal to

a. $|u|$ *b.* $|u|+|v\cdot\hat{a}|$
c. $2|v|$ *d.* $|v|+u\cdot(\hat{a}+\hat{b})$

141. If the variance of the observations $x_1, x_2, \ldots\ldots, x_n$ is σ^2, then the variance of $\alpha x_1, \alpha x_2, \ldots\ldots, \alpha x_n, \alpha \neq 0$ is

a. σ^2 *b.* $\alpha\sigma^2$
c. $\alpha^2\sigma^2$ *d.* $\frac{\sigma^2}{\alpha^2}$

142. Coefficient of variation of two distributions are 50 and 60 and their arithmetic means are 30 and 25, respectively. Difference of their standard deviation is

a. 0 *b.* 1
c. 1.3 *d.* 2.5

143. The maximum value of $z=9x+13y$ subject to constraints $2x+3y\le 18$, $2x+y\le 10$, $x\ge 0$, $y\ge 0$ is

a. 130 *b.* 81
c. 79 *d.* 99

144. A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss atleast 4 occassions is

a. $\frac{1}{4}$ *b.* $\frac{5}{8}$
c. $\frac{1}{2}$ *d.* $\frac{1}{6}$

145. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ and $f(1)=2$, $f(p+q)=f(p)\cdot f(q)$, $\forall\, p, q \in R$, then $x^{f(1)}+y^{f(2)}+z^{f(3)}-\frac{(x+y+z)}{x^{f(1)}+y^{f(2)}+z^{f(3)}}$ is equal to

a. 0 *b.* 1
c. 2 *d.* 3

146. The value of $\frac{2}{1!}+\frac{2+4}{2!}+\frac{2+4+6}{3!}+\ldots$ is

a. e *b.* $2e$
c. $3e$ *d.* None of these

147. If z_1, z_2 and z_3 represent the vertices of an equilateral triangle such that $|z_1|=|z_2|=|z_3|$, then

a. $z_1+z_2=z_3$ *b.* $z_1+z_2+z_3=0$
c. $z_1z_2=\frac{1}{z_3}$ *d.* $z_1-z_2=z_3-z_2$

148. If $\int \frac{(\sqrt{x})^5\,dx}{(\sqrt{x})^7+x^6}=\lambda\log\left(\frac{x^9}{x^9+1}\right)+C$, then $a+\lambda$ equal to

a. 2 *b.* >2
c. <2 *d.* >3

149. Line joining the points $(0, 3)$ and $(5, -2)$ is a tangent to the curve $y=\frac{ax}{1+x}$, then

a. $a=1\pm\sqrt{3}$ *b.* $a\in\phi$
c. $a=-1\pm\sqrt{3}$ *d.* $a=-2\pm2\sqrt{3}$

150. The shortest distance between the parabolas $y^2=4x$ and $y^2=2x-6$ is

a. 2 *b.* $\sqrt{5}$
c. 3 *d.* None of these

Answers

Physics

1. (a)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (b)
11. (b)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (d)	**17.** (c)	**18.** (d)	**19.** (b)	**20.** (c)
21. (b)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (c)	**28.** (a)	**29.** (a)	**30.** (a)
31. (d)	**32.** (c)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (c)	**37.** (d)	**38.** (c)	**39.** (b)	**40.** (b)

Chemistry

41. (b)	**42.** (b)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (a)	**48.** (d)	**49.** (b)	**50.** (d)
51. (d)	**52.** (b)	**53.** (c)	**54.** (d)	**55.** (d)	**56.** (c)	**57.** (a)	**58.** (a)	**59.** (b)	**60.** (a)
61. (d)	**62.** (a)	**63.** (a)	**64.** (d)	**65.** (c)	**66.** (d)	**67.** (c)	**68.** (c)	**69.** (a)	**70.** (a)
71. (d)	**72.** (b)	**73.** (d)	**74.** (b)	**75.** (d)	**76.** (d)	**77.** (a)	**78.** (c)	**79.** (d)	**80.** (a)

English Proficiency

81. (b)	**82.** (a)	**83.** (b)	**84.** (c)	**85.** (b)	**86.** (b)	**87.** (b)	**88.** (b)	**89.** (b)	**90.** (b)
91. (b)	**92.** (d)	**93.** (d)	**94.** (a)	**95.** (c)					

Logical Reasoning

96. (c)	**97.** (d)	**98.** (a)	**99.** (d)	**100.** (a)	**101.** (a)	**102.** (b)	**103.** (c)	**104.** (c)	**105.** (a)

Mathematics

106. (c)	**107.** (b)	**108.** (a)	**109.** (a)	**110.** (d)	**111.** (c)	**112.** (d)	**113.** (a)	**114.** (c)	**115.** (c)
116. (a)	**117.** (d)	**118.** (b)	**119.** (b)	**120.** (d)	**121.** (a)	**122.** (c)	**123.** (a)	**124.** (b)	**125.** (b)
126. (b)	**127.** (a)	**128.** (b)	**129.** (b)	**130.** (d)	**131.** (b)	**132.** (d)	**133.** (b)	**134.** (b)	**135.** (b)
136. (c)	**137.** (c)	**138.** (b)	**139.** (c)	**140.** (a)	**141.** (c)	**142.** (a)	**143.** (c)	**144.** (c)	**145.** (c)
146. (c)	**147.** (b)	**148.** (b)	**149.** (b)	**150.** (b)					

Hints & Solutions

Physics

1. ***(a)*** According to Stefan's law, $\frac{E}{t} = \sigma AeT^4$

So, we can write as

$$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$$

$$\frac{E_2}{E_1} = \left(\frac{900}{300}\right)^4 \Rightarrow \frac{E_2}{E_1} = (3)^4$$

$$E_2 = 81\ E_1$$

$$\Rightarrow \quad \frac{E_2}{E_1} = 81$$

2. *(d)* $v_s = r\omega = 1.2 \times 2\pi f \qquad [\because \omega = 2\pi f]$

$$= 1.2 \times 2 \times 3.14 \times \left(\frac{400}{60}\right) = 50.24$$

$$\simeq 50 \text{ m/s}$$

$$\nu_{min} = \frac{v}{v + v_s} \cdot \nu$$

$$= \frac{340}{340 + 50} \times 500 = 436 \text{ Hz}$$

$$\nu_{max} = \frac{v}{v - v_s} \cdot \nu = \frac{340}{340 - 50} \times 500$$

$$= 586 \text{ Hz}$$

3. ***(c)*** We have, $f_R - f_B = \omega f_Y$

$$\omega = \frac{f_R - f_B}{f_Y}$$

Here, $f_R = 100$ cm

$f_B = 96.8$ cm

$f_Y = \sqrt{f_B \times f_R}$

$= \sqrt{96.8 \times 100}$

$= 98.4$ cm

$\therefore$ Dispersive power

$$\omega = \frac{f_R - f_B}{f_Y}$$

$$\Rightarrow \quad = \frac{100 - 96.8}{98.4}$$

$$\Rightarrow \quad \omega = \frac{3.2}{98.4} = 0.0325$$

4. ***(b)*** V = 800 V

$$\therefore \quad E = \frac{V}{d} = \frac{800}{2 \times 10^{-2}} \text{ V/m}$$

$$= 4 \times 10^4 \text{ V/m}$$

Force due to E is balanced by the weight of particle, i.e.,

$$qE = mg$$

$$\Rightarrow \quad q = \frac{mg}{E}$$

$$= \frac{1.96 \times 10^{-15} \times 9.8}{4 \times 10^4}$$

$$= 4.8 \times 10^{-19}$$

$$= 3 \times 1.6 \times 10^{-19}$$

$$= 3e$$

5. ***(b)*** Given, $H = R$

$$\text{or} \quad \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$\text{or} \quad \tan\theta = 4$$

$$\Rightarrow \quad \theta = \tan^{-1}(4)$$

6. ***(b)*** Given, $A = 6 \text{ cm} = 6 \times 10^{-2}$ m

$\omega = 100$ rad/s, $m = 1$ kg

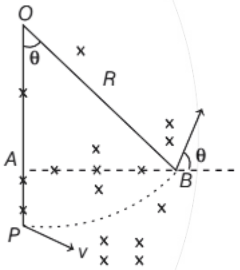
$$K_{max} = \frac{1}{2} m\omega^2 A^2$$

$$= \frac{1}{2} \times 1 \times (100)^2 \times (6 \times 10^{-2})^2$$

$$= 18 \text{ J}$$

7. ***(b)*** When a charge q is projected in a perpendicular magnetic field B with velocity v, then radius of path followed is given by

$$R = \frac{mv}{qB}$$



As width $\left(\frac{mv}{\sqrt{2}qB}\right)$ of field is less than radius, so path will be a part of circle.

In ΔOAB, $\sin\theta = \frac{AB}{OB}$

$$\sin\theta = \frac{\frac{mv}{\sqrt{2}qB}}{\frac{mv}{qB}} \qquad \left[\because OB = R = \frac{mv}{qB} \text{ and } AB = \frac{mv}{\sqrt{2}qB}\right]$$

$$\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

$\therefore$ Time taken to cover 2π angle is $T = \frac{2\pi m}{qB}$

Hence, time taken to cover $\frac{\pi}{4}$ angle, is

$$t = \frac{\left(\frac{2\pi m}{qB}\right)}{2\pi} \times \frac{\pi}{4} = \frac{\pi m}{4qB}$$

8. ***(b)*** Given, $\lambda_1 = 12000$ Å

and $\lambda_2 = 10000$ Å

$D = 2$ m

$d = 2$ mm $= 2 \times 10^{-3}$ m

We have,

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = \frac{12000}{10000} = \frac{6}{5}$$

5th and 6th fringes will coincide, respectively. The minimum distance is given as

$$X = \frac{n_1 \lambda_1 D}{d} \qquad (n_1 = 5)$$

$$= \frac{5 \times 12000 \times 10^{-10} \times 2}{2 \times 10^{-3}}$$

$$= 5 \times 1.2 \times 10^4 \times 10^{-10} \times 10^3 = 6 \text{ mm}$$

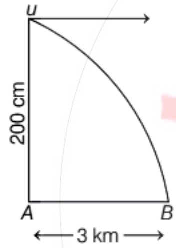
9. ***(c)*** Equation of motion of lower block is

$$F = \mu(M_A + M_B)\,g$$

$$\mu = \frac{F}{(M_A + M_B)g}$$

$$\Rightarrow \quad \mu = \frac{49}{(3+7)9.8} = \frac{1}{2} = 0.5$$

10. ***(b)*** Given, $R = 3$ km $= 3000$ m



Range, $R = u\sqrt{\frac{2h}{g}}$

$$\Rightarrow \quad u = R\sqrt{\frac{g}{2h}} \quad \text{or} \quad u = 3000 \times \sqrt{\frac{10}{2 \times 2000}}$$

$$= 3000 \times \frac{1}{20}$$

$$= 150 \text{ m/s}$$

$$= 150 \times \frac{18}{5} \text{ km/h}$$

$$= 540 \text{ km/h}$$

11. ***(b)*** Here, magnetic field, $B = 0.025$ T

$\therefore$ Magnetic flux $\phi = B \cdot A = B \cdot \pi r^2$

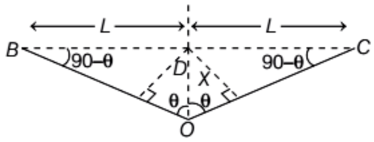
Induced emf $|e| = \frac{d\phi}{dt}$

$$\Rightarrow \quad = B\pi\, 2r \cdot \frac{dr}{dt}$$

$$\Rightarrow \quad = 0.025 \times \pi \times 2 \times 2 \times 10^{-2} \times 1 \times 10^{-3}$$

$$\Rightarrow \quad e = \pi\mu \text{ V}$$

12. ***(a)*** Increase in length,



$$\Delta L = BO + OC - (BC)$$

$[\because BO = OC$ and $BC = 2BD = 2L]$

$$= 2BO - 2BD$$

$$\Rightarrow \quad \Delta L = 2BO - 2L$$

$$= 2[L^2 + x^2]^{\frac{1}{2}} - 2L$$

$$\text{or} \quad \Delta L = 2L\left[1 + \frac{x^2}{L^2}\right]^{\frac{1}{2}} - 2L$$

$[\because$ using Bionomial theorem$]$

$$\Rightarrow \quad \Delta L \approx 2L\left[1 + \frac{1}{2}\frac{x^2}{L^2} - 1\right]$$

$$= \frac{x^2}{L} \qquad [\because x << L]$$

$$\therefore \text{ Strain} = \frac{\Delta L}{2L} = \frac{\frac{x^2}{L}}{2L} = \frac{x^2}{2L^2}$$

13. ***(d)*** Given, $t_1 = 20°$C

$R_1 = 20\Omega$

$t_2 = 500°$C

$R_2 = 60\Omega$

We have, $R_t = R_0(1 + \alpha t)$

Here, $20 = R_0(1 + 20\alpha)$ ...(i)

$60 = R_0(1 + 500\alpha)$ ...(ii)

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{60}{20} = \frac{R_0(1+500\alpha)}{R_0(1+20\alpha)}$$

$$\Rightarrow \quad 3 = \frac{1+500\alpha}{1+20\alpha}$$

$$\Rightarrow \quad 3(1 + 20\alpha) = 1 + 500\alpha$$

$$\Rightarrow \quad 3 + 60\alpha = 1 + 500\alpha$$

$$\Rightarrow \quad 440\alpha = 2$$

$$\Rightarrow \quad \alpha = \frac{2}{440} = \frac{1}{220} °\text{C}$$

Also given $R_t = 25\ \Omega$, $t = ?$

$$\text{Now,} \quad \frac{20}{25} = \frac{R_0\left(1 + \frac{20}{220}\right)}{R_0\left(1 + \frac{t}{220}\right)} \qquad [\text{Using Eq. (i)}]$$

$$\Rightarrow \quad 4\left(1 + \frac{t}{220}\right) = 5\left(1 + \frac{20}{220}\right)$$

On solving, we get

$$t = 80° \text{ C}$$

14. ***(b)*** de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2\,mqV}}$$

$$\Rightarrow \quad \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-19} \times 1000}}$$

$$\Rightarrow \quad \lambda = \frac{6.6 \times 10^{-34}}{7.16 \times 10^{-22}} = 0.9 \times 10^{-12} \text{ m}$$

15. ***(c)*** Let the volume of ice-berg is V and its density is ρ. If this ice-berg floats in water with volume V_{in} inside it, then

$$V_{in}\sigma g = V\rho g \quad [\sigma = \text{density of water}]$$

$$\Rightarrow \quad V_{in} = \left(\frac{\rho}{\sigma}\right)\cdot V$$

$$\Rightarrow \quad V_{out} = V - V_{in}$$

$$\Rightarrow \quad = \left[\frac{\sigma - \rho}{\sigma}\right]V = \frac{1000 - 900}{1000}V$$

$$\Rightarrow \quad V_{out} = \frac{V}{10}$$

$$\Rightarrow \quad V_{out}/V = 0.1 = 10\%$$

16. ***(d)*** We have, kinetic energy of electron,

$$K = \frac{Ze^2}{8\pi\varepsilon_0 \cdot r}$$

also, potential energy of electron,

$$U = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r}$$

$\therefore$ Total energy,

$$E = K + U = \frac{Ze^2}{8\pi\varepsilon_0 \cdot r} - \frac{Ze^2}{4\pi\varepsilon_0 \cdot r}$$

$$E = -\frac{Ze^2}{8\pi\varepsilon_0 r} \text{ or } K = -E = -(-3.4)$$

$$= 3.4 \text{ eV}$$

17. ***(c)*** Voltage gain $= \beta \times$ impedance gain

$$\Rightarrow \quad 50 = \beta \times \frac{200}{100}$$

$$\left[\because \text{Impedance gain} = \frac{\text{Output impedance}}{\text{Input impedance}}\right]$$

$$\Rightarrow \quad \beta = 25$$

Also power gain $= \beta^2 \times$ impedance gain

$$= (25)^2 \times \frac{200}{100} = 1250$$

18. ***(d)*** Time of flight, $T = \frac{2u_y}{g}$

Maximum height, $H = \frac{u_y^2}{2g}$

and horizontal range, $R = u_x \times T$

When a horizontal acceleration is given to the projectile, then u_y, T and H will remains unchanged while range will become

$$R' = u_x \times T + \frac{1}{2}aT^2$$

$$= R + \frac{1}{2}\frac{g}{4}\left(\frac{4u_y^2}{g^2}\right) = R + H \qquad \left[\because a = \frac{g}{4}\right]$$

19. ***(b)*** We have, $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ $\qquad \left[\because \frac{pV}{T} = \text{constant}\right]$

$$\text{or } V_2 = \frac{p_1V_1T_2}{p_2T_1}$$

$$= \frac{1\times 500\times(273-3)}{0.5\times(273+27)} = \frac{1\times 500\times 270}{0.5\times 300}$$

$$V_2 = 900 \text{ m}^3$$

20. ***(c)*** Intensity at centre of bright fringe,

$$I_0 = I + I + 2\sqrt{II}\cdot\cos 0$$

$$I_0 = 2I + 2I \qquad [\because \cos 0° = 1]$$

$$\Rightarrow \quad I_0 = 4I$$

Similarly, intensity at point distance one-fourth of the distance

(with phase difference $= \frac{2\pi}{4} = \frac{\pi}{2}$)

$$I' = I + I + 2\sqrt{II}\cos\frac{\pi}{2} = 2I + 2\sqrt{II}\times 0$$

$$I' = 2I$$

$$\therefore \quad \frac{I_0}{I'} = \frac{4I}{2I} = 2$$

21. ***(b)*** Let block is displaced through x m, then weight of displaced water or upthrust (upwards)

$$= -Ax\rho g$$

where, A is area of a cross-section of the block and ρ is density. This must be equal to force ($= ma$) applied where m is mass of the block and a is acceleration.

$$\therefore \quad ma = -Ax\rho g$$

$$\text{or} \quad a = -\frac{A\rho g}{m}x = -\omega^2 x$$

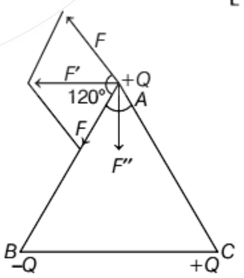
This is equation of SHM.

So, the time period of oscillation is given as

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{A\rho g}} \Rightarrow T \propto \frac{1}{\sqrt{A}}$$

22. ***(c)*** Resultant force,

$$F' = \sqrt{F^2 + F^2 + 2FF\cos 120°} = F \qquad \left[\because \cos 120° = \frac{-1}{2}\right]$$



Now, from figure

$$F = \sqrt{F'^2 + F''^2 + 2F'F''\cos 90°}$$

Now, the force normal to BC at vertex A is

$$F'' = \sqrt{F^2 - F'^2} = 0 \qquad (\because F' = F)$$

23. ***(b)*** From the conservation principle,

$$mgh = \frac{1}{2}kX_0^2 - mgX_0$$

where, X_0 is maximum elongation in spring.

$$\Rightarrow \quad \frac{1}{2}kX_0^2 - mgX_0 - mgh = 0$$

$$\Rightarrow \quad X_0^2 - \frac{2mg}{k}X_0 - \frac{2mg}{k}h = 0$$

$$X_0 = \frac{2\frac{mg}{2} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + 4 \times \frac{2mg}{k}h}}{2}$$

Amplitude = elongation in spring for lowest extreme position – elongation in spring for equilibrium position

$$= X_0 - X_1 = \frac{mg}{k}\sqrt{1 + \frac{2hk}{mg}} \qquad \left[\because X_1 = \frac{mg}{k}\right]$$

24. ***(d)*** Fraction remains after n half-lives

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

Given, $\quad N = \frac{N_0}{e} \quad \Rightarrow \quad \frac{N_0}{eN_0} = \left(\frac{1}{2}\right)^{\frac{5}{T}}$

$$\Rightarrow \quad \frac{1}{e} = \left(\frac{1}{2}\right)^{\frac{5}{T}}$$

Taking log on both sides, we get

$$\log 1 - \log e = \frac{5}{T}\log\frac{1}{2}$$

$$\Rightarrow \quad -1 = \frac{5}{T}(-\log 2)$$

$$\Rightarrow \quad T = 5\log_e 2$$

Now, let t' be the time after which activity reduces to half

$$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{t'/5\log_e 2}$$

$$\Rightarrow \quad t' = 5\log_e 2$$

25. ***(c)*** Focal length of combination,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{...(i)}$$

Using lens Maker's formula,

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = -\frac{(\mu_2 - 1)}{R}$$

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R}$$

or $\quad \frac{1}{f_1} = \frac{(\mu_1 - 1)}{R}$

$$\frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R}$$

Putting these values in Eq (i), we get

$$\Rightarrow \quad \frac{1}{f} = \frac{(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{(\mu_1 - 1 - \mu_2 + 1)}{R}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{\mu_1 - \mu_2}{R}$$

$$\Rightarrow \quad f = \frac{R}{(\mu_1 - \mu_2)}$$

26. ***(c)*** Current will flow only through the branch containing resistance R_2. (Because at stedy state, capacitor works as open circuit, hence branch containing resistances R_1 and R_2 is ineffective).

$$\therefore \quad i = \frac{E}{R_2 + r} = \frac{5}{4 + 1} = 1\text{ A}$$

Potential difference across R_2

$$= 1 \times 4 = 4\text{ V}$$

If q be the charge on each plate of capacitor, then

$$\frac{q}{C} + \frac{q}{C} = 4$$

or $\quad \frac{2q}{C} = 4$

or $\quad \frac{2q}{3 \times 10^{-6}} = 4$

$$q = 6\,\mu\text{C}$$

27. ***(c)*** Friction force between A and B and between block B and surface S will oppose F

$$\therefore \quad F = F_{AB} + F_{BS}$$

$$= \mu_{AB}m_A g + \mu_{BS}(m_A + m_B)g$$

$$= 0.2 \times 100 \times 10 + 0.3(100 + 200) \times 10$$

$$= 200 + 900 = 1100\text{ N}$$

28. ***(a)*** The moment of inertia of the uniform rod about an axis through one end and perpendicular to its length is

$$= \frac{ml^2}{3}$$

where, m is the mass and l its length.

Torque $(\tau = I \cdot \alpha)$ acting on centre of gravity of rod is given by

$$\tau = mg\frac{l}{2}$$

also, $\quad \frac{ml^2}{3} \cdot \alpha = mg\frac{l}{2}$

$$\Rightarrow \quad \alpha = \frac{3g}{2l}$$

29. ***(a)*** As H-atom emits 6 spectral lines, so $\frac{n(n-1)}{2} = 6$

$$\therefore \quad n = 4$$

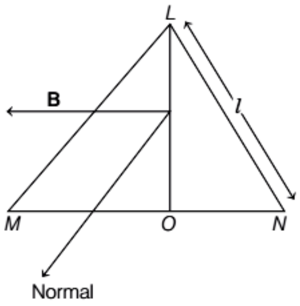
Now, $\frac{1}{\lambda} = R\left[1 - \frac{1}{4^2}\right]$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left[\frac{15}{16}\right] \Rightarrow \lambda = \frac{16}{109677 \times 15}\text{ cm}$$

$$\Rightarrow \quad \lambda = 97.2\text{ nm}$$

30. ***(a)*** Torque acting on equilateral triangle in a magnetic field B is

$\tau = i\,AB\sin\theta$



Area of triangle LMN,

$$A = \frac{\sqrt{3}}{2}l^2 \text{ and } \theta = 90^\circ$$

Substituting the given values in expression for torque, we get

$$\tau = i \times \frac{\sqrt{3}}{4}l^2 B\sin 90^\circ$$

$$\tau = \frac{\sqrt{3}}{4}il^2 B \qquad (\because \sin 90^\circ = 1)$$

Here, $l = 2\left(\dfrac{\tau}{\sqrt{3}\,Bi}\right)^{1/2}$

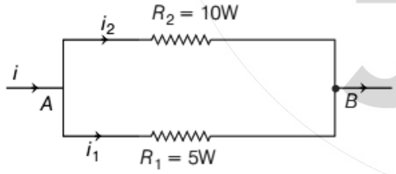
31. ***(d)*** Work done = Change in surface energy

$$r_2 = 5\text{ cm}$$
$$r_1 = 3\text{ cm}$$

$$\Rightarrow \quad W = 2T \times 4\pi(r_2^2 - r_1^2)$$
$$= 2 \times 0.03 \times 4\pi[(5)^2 - (3)^2] \times 10^{-4}$$
$$= 0.4\pi \text{ mJ}$$

32. ***(c)*** From the figure, the X-component remains unchanged while the Y-component is reverse. Then, the velocity at point B is $(2\hat{i} - 3\hat{j})$ m/s.

33. ***(b)*** The equivalent circuits diagram is



Potential drop is same for both braveles

$$\therefore \quad i_1R_1 = i_2R_2$$
$$i_1 \times 5 = i_2 \times 10$$
$$\Rightarrow \quad \frac{i_1}{i_2} = 2$$

Now, heat produced is given by

$$H = i^2RT$$

$$\therefore \quad \frac{H_1}{H_2} = \left(\frac{i_1}{i_2}\right)^2 \times \left(\frac{R_1}{R_2}\right) \qquad [\because t = 1s]$$

$$\text{or} \quad \frac{10}{H_2} = (2)^2 \times \left(\frac{5}{4}\right) \qquad [\because R_2 = 4\Omega]$$

$$\Rightarrow \quad H_2 = 2\text{ cal}$$

34. ***(a)*** Given KE of α-particle = 2 eV

$$r = \frac{(Ze)(e)}{4\pi\varepsilon_0 \cdot (\text{KE})}$$

$$= \frac{2Ze \times 9 \times 10^9}{2V}$$

$$\Rightarrow \quad r = \frac{2 \times Z \times 1.6 \times 10^{-19} \times 9 \times 10^9}{2V}$$

$$\Rightarrow \quad r = 14.4 \cdot \frac{Z}{V}\text{Å}$$

35. ***(c)*** According to question,

Time period of oscillation of a simple series is

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$$

$\therefore$ Ratio to time period, $\dfrac{T_1}{T_2} = \dfrac{1}{2}$

$$\Rightarrow \quad T_2 = 2T_1$$

Thus, the shorter pendulum would have completely exactly 2 oscillations when the longer pendulum complete first oscillation.

36. ***(c)*** The two capacitors formed are in series, hence capacitance of the combination

$$C = \frac{C_1C_2}{C_1 + C_2} \qquad \text{...(i)}$$

where, $\quad C_1 = \dfrac{K_1\varepsilon_0 A}{\frac{d}{3}} \qquad \left(\because C = \dfrac{\varepsilon_0 A}{d}\right)$ ...(ii)

$$C_2 = \frac{K_2\varepsilon_0 A}{\frac{2d}{3}} \qquad \text{...(iii)}$$

$$C_{eq} = \frac{\dfrac{3K_1\varepsilon_0 A}{d} \times \dfrac{K_2\varepsilon_0 A \cdot 3}{2d}}{\dfrac{3K_1\varepsilon_0 A}{d} + \dfrac{3K_2\varepsilon_0 A}{2d}}$$

[Using Eq. (i) and (ii)]

$$\Rightarrow \quad C_{eq} = \frac{9}{2}\frac{K_1K_2}{(6K_1 + 3K_2)} \cdot \left(\frac{\varepsilon_0 A}{d}\right)$$

It is given

$$\frac{\varepsilon_0 A}{d} = 9\text{ pF}$$

Using given values, $\quad C_{eq} = 40.5\text{ pF}$

37. ***(d)*** Acceleration

$$f = \frac{dv}{dt} = f_0\left(1 - \frac{t}{T}\right)$$

or $$dv = f_0\left(1 - \frac{t}{T}\right) \cdot dt \qquad \text{...(i)}$$

Integrating Eq. (i) on both sides, we get

$$v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} + C \qquad \text{...(ii)}$$

After applying boundary conditions

$v = 0$ at $t = 0$

We get, $C = 0$

$$\Rightarrow \quad v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} \qquad \text{...(iii)}$$

As $f = f_0\left(1 - \frac{t}{T}\right)$

When $f_0 = 0, t = T$

Substituting $t = T$ in Eq. (iii), then velocity

$$v_X = f_0 T - \frac{f_0}{T} \cdot \frac{T^2}{2} = \frac{1}{2} f_0 T$$

38. ***(c)*** By Kepler's law of planetary motion,

$T^2 \propto r^3$, hence $T_1^2 \propto r_1^3$

Simiarly $T_2^2 \propto r_2^3$

Given, $r_2 = 6400$ km

$r_1 = 36000$ km

For a geostationary satellite, $T = 24$ h

$\therefore \quad T_1^2 \propto (36000)^3$

$T_2^2 \propto (6400 + h)^3$

Therefore,

$$(T_2)^2 = T_1^2 \left(\frac{6400 + h}{36000}\right)^3$$

$$T_2 = T_1\left(\frac{6400}{36000}\right)^{3/2} \text{ Neglecting } h \ [\because h <<< R]$$

$$\therefore \quad T_2 = (24)\left(\frac{64}{360}\right)^{3/2} \Rightarrow T_2 = 1.8 \text{ h}$$

$T_2 \simeq 2$ hr

39. ***(b)*** Equation of wave,

$$y = 0.2 \sin [1.5x + 60t]$$

Comparing with standard equation,

$$y = A \sin [kx + \omega t]$$

we get $k = 1.5 = \frac{2\pi}{\lambda}$ and $\omega = 60 = \frac{2\pi}{T}$

$\therefore$ Velocity of wave, $v = \frac{\omega}{k} = \frac{60}{1.5} = 40$ m/s

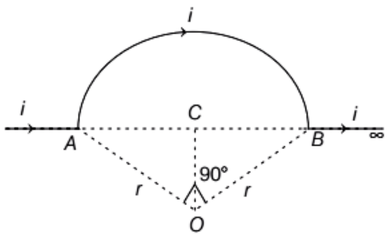
Velocity of wave in stretched string, $v = \sqrt{T/m}$,

where, m is the linear density, T is tension in the string.

So, $T = v^2 m = (40)^2 \times 3 \times 10^{-4}$

$= 0.48$ N

40. ***(b)*** Given, radius of circular loop is r



In ΔAOB

$$AB^2 = AO^2 + OB^2$$

$$AB^2 = r^2 + r^2$$

$$\Rightarrow \quad AB = \sqrt{2}r$$

In ΔAOC,

$$AO^2 = OC^2 + AC^2$$

$$\Rightarrow \quad OC^2 = r^2 - \left(\frac{r}{\sqrt{2}}\right)^2 \qquad \left[\because AC = \frac{AB}{2} = \frac{r}{\sqrt{2}}\right]$$

$$\Rightarrow \quad OC = AC = \frac{r}{\sqrt{2}}$$

Now, magnetic field due to straight wire is

$$B_1 = \frac{\mu_0 i}{4\pi a}[\sin\phi_1 + \sin\phi_2] = \frac{\mu_0 i}{4\pi \times \frac{r}{\sqrt{2}}}$$

[sin 90°+sin 90°]

$$\Rightarrow \quad B_1 = \frac{\mu_0 i \times \sqrt{2}}{2\pi r} \qquad \text{...(i)}$$

And magnetic field due to circular path is

$$B_2 = \frac{\mu_0 i}{2r} \times \frac{1}{4} \qquad \text{...(iii)}$$

[$\because$ Circular path is quadrant of circle]

Net magnetic field at point O is $B = B_1 + B_2$

$$\Rightarrow \quad B = \frac{\sqrt{2}\mu_0 i}{2\pi r} + \frac{\mu_0 i}{8r} = \frac{\mu_0}{4\pi} \cdot \frac{2i}{r}\left[\sqrt{2} + \frac{\pi}{4}\right]$$

Chemistry

41. ***(b)*** ∵ 4 g of copper gave 5g of its oxide means,

1 g of oxygen combine with 4 g of copper.

∵ Eq. wt of oxygen = 8,

Therefore, 8g of oxygen combine with 4×8 g of copper = 32 g

Hence, equivalent weight of copper = 32

42. ***(b)*** Option (b) is the incorrect statement and can be corrected as, reaction of $NaBH_4$ with cold water is very slow. It is not violent. All other are correct statements.

43. ***(b)*** ∵ For *s*-subshell, $l = 0$

Hence, orbital angular momentum of an electron in 2*s*

orbital is $= \sqrt{l(l+1)} \cdot h/2\pi = \sqrt{0(0+1)^{\frac{h}{2\pi}}} = 0$

Thus, the angular orbital momentum of an electron in 2*s* orbital is 0.

44. ***(b)*** I. Number of electrons in $\overset{+}{C}H_3 = 6 + 3 - 1 = 8$

II. Number of electrons in $NH_2^- = 7 + 2 + 1 = 10$

III. Number of electrons in $NH_4^+ = 7 + 4 - 1 = 10$

IV. Number of electrons in $NH_3 = 7 + 3 = 10$

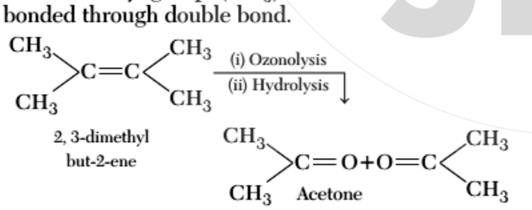
Since, species with same number of electrons are called isoelectronic species. Hence, II, III and IV are isoelectronic.

45. ***(a)*** ∵ Difference of electronegativity between (*A*) and (*B*) $= 3.0 - 1.2 = 1.8$ and a bond having electronegativity difference greater than 1.65 is of ionic nature.

Hence, bond between (*A*) and (*B*) would be of ionic nature.

46. ***(c)*** The hydrogen, attached with *sp*-hybrid C-atom is most acidic because of highest electronegativity of *sp* hybrid carbon atom, H atom becomes relatively more acidic.

47. ***(a)*** Reductive ozonolysis of $(H_3C)_2C = C(CH_3)_2$ followed by hydrolysis gives only one type of ketones because of symmetry across the $C = C$ and bonding of two same alkyl groups (CH_3) with both the carbons bonded through double bond.



48. ***(d)*** Reaction (I), (II) and (IV) are of exothermic nature, thus do not involve absorption energy.

49. ***(b)*** More be the ability of N-atom of —NH_2 group to donate lone pair of electrons to HCl (dil.), more be the reactivity of amine.

In (b) two alkyl groups are attached to the —NH_2 (having electron releasing nature) which increases the electron donating ability of N-atom of —NH_2.

Hence (b) is most reactive.

Note (i) (b) i.e. $(CH_3)_3N$ has three alkyl groups attached with the N-atom, but due to steric-effect, N-atom cannot able to donate lone pair of electrons.

(ii) In (d), due to resonance, lone pair of electrons get delocalised.

50. ***(d)*** Blocks of magnesium prevents the action of water and salt of iron (of ship) by sacrificing itself.

This method of protection of ship (i.e. of iron) is known as sacrificial protection.

51. ***(d)*** ∵ Only one mole of total gases are evolved and H_2O decomposes as

$$H_2O \xrightarrow{\text{Decomposition}} H_2 + \frac{1}{2}O_2$$

i.e. total moles $= 1 + \frac{1}{2} = \frac{3}{2}$ mole by one mole of H_2O.

Now,

∵ 3/2 moles of gases are evolved by = 1 mole of H_2O

∴ 1 mole of gases are evolved by $= \frac{1}{3/2} = \frac{2}{3}$ mole of H_2O.

52. ***(b)*** ∵ Reaction between Fe and HCl is as follows

$$Fe + 2HCl \longrightarrow \underset{(+2)}{FeCl_2} + H_2$$

n (moles of Fe) = n (moles of HCl)

n(Fe) = Normality (HCl) × Volume of HCl

$$n(Fe) = \frac{0.4 \times 500}{1000} = 0.2 \text{ mol}$$

As each Fe consumer (2 electrons) to change the Fe^{2+} ion.

∴ Number of Fe^{2+} ions produced $= \frac{0.2}{2} = 0.1$ mol

53. ***(c)*** For dissolution hydration energy must be more higher (–ve) than of lattice energy. Thus, (c) is the correct answer.

54. ***(d)*** In the given reaction the end product is propyl amine. It is obtained as follows

$$CH_3CH_2Cl \xrightarrow{KCN} \underset{(A)}{CH_3CH_2CN} \xrightarrow[\text{(Reduction)}]{LiAlH_4} \underset{\text{propyl amine }(P)}{CH_3CH_2CH_2NH_2}$$

55. ***(d)*** Conversion of carbonate ore into its oxide (by heating in absence of air) is known as calcination.

$$MgCO_3 \xrightarrow{\Delta} MgO + CO_2(g)$$

56. ***(c)*** We know that, $\Delta H = E_f - E_b$

where, E_b and E_f = activation energy for backward and forward respectively.

Thus, $E_f = \Delta H + E_b$, means $E_f > \Delta H$.

57. ***(a)*** Ag is leached by cyanide process by making the complex using NaCN/KCN as follows

$$Ag_2S + 4NaCN \longrightarrow \underset{\text{(Soluble)}}{2Na[Ag(CN)_2]} + Na_2S$$

58. ***(a)*** Among the given option (a) is a diamagnetic complex.

(a) $[Co(NH_3)_6]^{3+} \Rightarrow Co\ (27) \Rightarrow Co^{3+} = 3d^6 4s^0$. Thus, has no unpaired electrons and is a diamagnetic complex.

(b) $[NiCl_4]^{2-} \Rightarrow Ni\ (28) \Rightarrow Ni^{2+} = 3d^9 4s^0$

Thus, has two unpaired electron and is paramagnetic.

(c) $[CuCl_4]^{2-} \Rightarrow Cu\ (29) \Rightarrow Cu^{2+} = 3d^9 4s^0$

Thus, has one unpaired electron and is paramagnetic.

(d) $[Fe(H_2O)_6]^{3+} \Rightarrow Fe\ (26) \Rightarrow Fe^{3+} = 3d^5 4s^0$

Thus, has five unpaired electrons and is paramagnetic.

59. ***(b)*** Neoprene is a synthetic rubber. It is a polymer of chloroprene (2-chlorobuta,1, 3-diene).

60. ***(a)*** Ionic solids have highest melting point due to strong electrostatic force of attraction.

61. ***(d)*** Night-blindness is developed due to deficiency of vitamin A.

62. ***(a)*** The transfer RNA anticodon for messanger RNA codon G – C – A is

C – G – U, because in anticodon of RNA,

G replaced by C, C replaced by G and A replaced by U

63. ***(a)*** For % of CO_2,

$\because$ 0.765 g of acid gives $CO_2 = 0.535$ g

$\therefore$ 100 g of acid gives $CO_2 = \dfrac{0.535 \times 100}{0.765}$

% of $CO_2 = 70.00$

For % of carbon,

$\because$ 44 g of CO_2 gives, C = 12 g

$\therefore$ 70 g of CO_2 gives, $C = \dfrac{12 \times 70}{44} = 19.09$ g

$\therefore$ % of carbon = 19.00

Similarly, for % of $H_2O = \dfrac{0.138 \times 100}{0.765}$

$= 18.03 = 18.00$

% of hydrogen $= \dfrac{2 \times 18}{18} = 2.00$

Hence, ratio of carbon and hydrogen is 19 : 2.

64. ***(d)*** Least basic has maximum value of pK_b.
Since, in option (d), lone pair of electrons over N-atom are delocalised to two benzene rings, thus basic.

65. ***(c)*** For principal quantum number (n), l (azimuthal quantum number) can be upto $(n-1)$ and thus, $m = \pm l$, i.e. $+l, 0, -l$

(a) $n = 2, l = 0, m = 0$ (correct)

(b) $n = 1, l = 0, m = 0$ (correct)

(c) $n = 3, l = 3, m = 0$ (not correct)

(d) $n = 2, l = 1, m = 1$ (correct)

66. ***(d)*** More higher be the positive oxidation number of N-atom in its oxide more acidic be the nature of oxide. Thus, N_2O_5 is most acidic oxide.

Oxide	Oxidation state of N
NO_2	+4
N_2O	1
NO	2
N_2O_5	5

67. ***(c)*** $\because$ Bond order (BO) $= \dfrac{N_b - N_a}{2}$

Thus, for O_2, BO $= \dfrac{10-6}{2} = 2$

for O_2^-, BO $= \dfrac{10-7}{2} = 1.5$; for O_2^+, BO $= \dfrac{10-5}{2} = 2.5$

for O_2^{2-}, BO $= \dfrac{10-8}{2} = 1.0$

Hence, maximum value of BO is for O_2^+.

68. ***(c)*** Option (c) is the correct option as in (I), (II) and (IV) the state changes towards more random state, thus entropy increases.

69. ***(a)*** BF_3 is a Lewis acid, as it accepts lone pair of electrons. It is not an Arrhenius acid because, it does not furnish H^+ ion in its aqueous solution. It is also not a Bronsted Lowry acid as it does not give H^+ ions in any solvent.

70. ***(a)*** $\because \Delta H = \Delta E + \Delta n_g RT$

and Δn_g (for the given reaction) $= 2 - 1 = +1$

$\therefore \Delta H > \Delta E$

71. ***(d)*** First member of 1st group i.e. lithium, mostly form covalent compounds due to its small size and comparatively high I.E. (with respect to other members of its group).

72. ***(b)*** When aqueous solution of borax $(Na_2B_4O_7)$ is acidified with HCl, we get H_3BO_3 and NaCl as follows

$$\underset{\text{Borox}}{Na_2B_4O_7} + 2HCl + 5H_2O \longrightarrow 2NaCl + \underset{\text{Boric acid}}{4H_3BO_3}$$

73. ***(d)*** An aromatic compound can follow Huckel's rule, if it has $(4n+2)\pi$ electrons.

(where, n = integer 0, 1, 2, 3, ... etc.)

Thus,

(a) $n = 0$; number of π-electrons $= 4 \times 0 + 2 = 2\pi$

$\therefore$ Follow Huckel's rule.

(b) $n = 1$; number of π-electrons $= 6\pi$

$\therefore$ Follows Huckel's rule.

(c) $n = 1$; number of π-electrons $= 6\pi$

$\therefore$ Follows Huckel's rule.

(d) $n = 1$, number of π-electrons $= 8\pi$

$(4n + 2 = 4 \times 1 + 2 = 6)$

But this compound has 8 electrons which distorts the plane and hence does not follows Huckel's rule.

74. ***(b)*** According to Arrhenius equation's

$$\log k = \log A - \frac{E_a}{2.303R} \cdot \frac{1}{T}$$

On comparing it with the equation of straight line with negative slope, $y = mx + c$

The value of $\log k$ should increase uniformly with T or decreases with $\frac{1}{T}$.

Thus, plot (b) is the correct answer.

75. ***(d)*** CN^- is a strong ligand and form six coordinate bonds with Fe element. Thus, Fe shows d^2sp^3 hybridisation in $[K_3Fe(CN)_6]$.

76. ***(d)*** More the number of unpaired electrons more is the magnetic moment.

Number of unpaired electrons in $Mg^{2+} = 0$ [Ne]

Number of unpaired electrons in $Ti^{3+} = 1\,[Ar]3d^1$

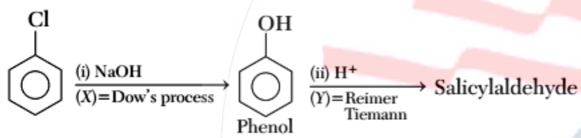
Number of unpaired electrons in $V^{3+} = 2\,\,[Ar]3d^2$

Number of unpaired electrons in $Fe^{2+} = 4\,\,[Ar]3d^6$

Thus, Fe^{2+} show maximum value of magnetic moment (μ). Also, $\mu = \sqrt{n(n+2)}$ BM(where, n = number of unpaired electrons).

77. ***(a)*** Elements with atomic number 90 to 103 belongs to same group, i.e. III B group (i.e. actinoids).

78. ***(c)***



79. ***(d)*** $\because n = \frac{10}{m} = \frac{\text{actual mass}}{\text{molar mass}}$

For

(a) $n = \frac{w}{m} = \frac{8}{16} = \frac{1}{2}$ mol

(b) $n = \frac{16}{32} = \frac{1}{2}$ mol

(c) $n = \frac{14}{28} = \frac{1}{2}$ mol

(d) Thus, all have same number of moles.

80. ***(a)*** $\because$ For ideal gas,

$$pV = nRT \quad \text{or} \quad T = \frac{pV}{nR}$$

$\because \quad n = 1 = \text{constant}$ (Given)

R = gas constant

$\therefore \quad T \propto pV$

Thus, the gas that shows highest product of $p \times V$, has highest temperature.

For gas $(A) = p \times V = 2 \times 5 = 10$

$(B) = p \times V = 2 \times 2.5 = 5$

$(C) = p \times V = 2 \times 1.25 = 2.5$

$(D) = p \times V = 2.5 \times 2.5 = 6.25$

Hence, gas (A) shows highest temperature.

a. English Proficiency

81. ***(b)*** 'Are' should be used in place of 'is'.

82. ***(a)*** 'Was' should be used in place of 'were'.

83. ***(b)*** 'Luggage' should be used in place of 'luggages'.

84. ***(c)*** Gloss over something is used.

85. ***(b)*** Preposition ***up*** is used with muster.

86. ***(b)*** **Illusion** means a false idea or belief; a deceptive appearance or impression. Its antonym will be **reality.**

87. ***(b)*** **Disparage** means to criticise someone in a way that shows that you do not respect or value him. Its antonym will be **praise.**

88. ***(b)*** **Slackened** means reduce or decrease in speed or intensity. Its antonym will be **quickened**.

89. ***(b)*** **Deteriorating** means becoming progressively worse. So, 'worsening' is its nearest meaning word.

90. ***(b)*** **Stalemate** means a situation in which neither group involved in an argument or an act can win or get an advantage and no action can be taken.

91. ***(b)*** The southampton scheme requires convicted drivers to attend eight driving sessions-one a week.

92. ***(d)*** John Cook devised the scheme to prove that alcohol does influence driving.

93. ***(d)*** The problem with a quarter of the people who went to John Cook was that they did not know how much they were drinking.

94. ***(a)*** Most drivers start off with atleast a double measure.

95. ***(c)*** The truth is that alcohol affects the body and the brain.

b. Logical Reasoning

96. ***(c)*** 'Leather' is a raw material used to make 'Shoes'. Same as, 'Rubber' is made using 'Latex' which is a raw material.

97. ***(d)*** Here, $81 \times 3 = 243$

$25 \times 3 = 75$

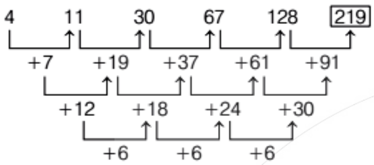
$\Rightarrow \quad 64 \times 3 = 192$

but $\quad 16 \times 3 = 48 \neq 64$

We can see that in all options second number is three times the first number except 16 : 64.

Hence, 16 : 64 is odd one.

98. ***(a)*** The sequence to given series is as follows



Hence, 219 will come in place of question mark'?'.

99. ***(d)*** From given information, we have

Lakshmi > Meenu ...(i)

Lakshmi > Leela > Meenu ...(ii)

Meenu > Hari > Latha ...(iii)

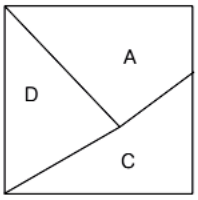
After arranging the equations,

Lakshmi > Leela > Meenu >Hari > Latha

Hence, Latha is the youngest.

100. ***(a)*** Here, all the three equal parts have same design. So, we can obtain the answer figure for the missing portion by rotating question figure by 90° clockwise. Hence, option(a) is the right answer.

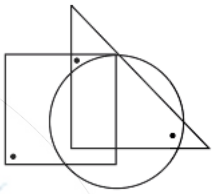
101. ***(a)*** Out of five given figures, we can take figure (A), (C) and (D) to make a square.



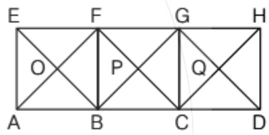
102. ***(b)*** After folding, paper step by step and then cut as shown in the question, it will show as an option (b).

103. ***(c)*** Moving row wise the number of horizontal lines increases by one in each step continuing the process, we will get figure of option (c) which will replace the question mark'?'.

104. ***(c)*** Required figure given in option (c).



105. ***(a)*** Naming the figure,



Clearly, there are 28 triangles in the given figure namely, ΔEOF, ΔAOE, ΔAOB, ΔBOF, ΔABF, ΔBEF, ΔABE, ΔAEF, ΔBPF, ΔFPG, ΔCPG, ΔBPC, ΔBFG, ΔBCG, ΔCFG, ΔBCF, ΔGQC, ΔCDQ, ΔDQH, ΔGQH, ΔGDC, ΔGDH, ΔGHC, ΔCDH, ΔAFC, ΔBGD, ΔEBG and ΔFCH.

Mathematics

106. ***(c)*** $(1+x)^{21}+(1+x)^{22}+......+(1+x)^{30}$

$$=(1+x)^{21}[1+(1+x)^1+....+(1+x)^9]$$

It forms a GP series with $r=(1+x)$

$$=(1+x)^{21}\left[\frac{(1+x)^{10}-1}{(1+x)-1}\right]=\frac{1}{x}[(1+x)^{31}-(1+x)^{21}]$$

$\therefore$ Coefficient of x^5 in the given expression

$$=\text{Coefficient of } x^5 \text{ in } \frac{1}{x}[(1+x)^{31}-(1+x)^{21}]$$

$$=\text{Coefficient of } x^6 \text{ in } [(1+x)^{31}-(1+x)^{21}]$$

$$={}^{31}C_6-{}^{21}C_6$$

107. ***(b)*** We have,

$\arg(z-1)=\arg(z+3i)$ $\quad[\because z=a+ib]$

$\Rightarrow \arg((a-1)+ib)=\arg(a+(b+3)i)$

$\Rightarrow \tan^{-1}\left(\frac{b}{a-1}\right)=\tan^{-1}\left(\frac{b+3}{a}\right)$

$\Rightarrow$ ttt $\frac{b}{a-1}=\frac{b+3}{a} \Rightarrow ab=(a-1)(b+3)$

$\Rightarrow 3(a-1)=b$

$\Rightarrow (a-1):b=1:3$

108. ***(a)*** Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and let $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ ...(i)

be a tangent to it at point $(a\cos\theta, b\sin\theta)$.

Then, p = length of the perpendicular from $S(ae, 0)$ on Eq. (i)

$$\Rightarrow p=\left|\frac{e\cos\theta-1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\right| \Rightarrow ap=\frac{ae\cos\theta-a}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

p' = length of the perpendicular from $O(0, 0)$ on Eq.(i)

$$\Rightarrow p'=\left|\frac{1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\right| \text{ and } r=ae\cos\theta-a$$

$$\Rightarrow rp'=\frac{ae\cos\theta-a}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

Clearly, $rp'=ap$

109. ***(a)*** We have,

$$\sum_{r=1}^{10} r\cdot\frac{{}^nC_r}{{}^nC_{r-1}}=\sum_{r=1}^{10}(n-r+1)$$

$$\Rightarrow \sum_{r=1}^{10} r\cdot\frac{{}^nC_r}{{}^nC_{r-1}}=\sum_{r=1}^{10}\{(n+1)-r\}$$

$$\Rightarrow \sum_{r=1}^{10} r\cdot\frac{{}^nC_r}{{}^nC_{r-1}}=10(n+1)-\sum_{r=1}^{10}r$$

$$\Rightarrow \sum_{r=1}^{10} r\cdot\frac{{}^nC_r}{{}^nC_{r-1}}=10(n+1)-55$$

$$=10n-45=5(2n-9)$$

110. ***(d)*** Since, $3^{2\sin 2\alpha-1}$, 14 and $3^{4-2\sin 2\alpha}$ are in AP.

Therefore,

$$2\times 14=3^{2\sin 2\alpha-1}+3^{4-2\sin 2\alpha}$$

$\Rightarrow 28=\frac{a}{3}+\frac{3^4}{a}$, where $a=3^{2\sin 2\alpha}$

$\Rightarrow a^2-84a+243=0$

$\Rightarrow (a-81)(a-3)=0 \Rightarrow a=81, a=3$

$\Rightarrow 3^{2\sin 2\alpha}=3^4$ or $3^{2\sin 2\alpha}=3$

$\Rightarrow 2\sin 2\alpha=1$ $\quad[\because 2\sin 2\alpha\neq 4]$

$\Rightarrow \sin 2\alpha=1/2 \Rightarrow 2\alpha=30°$ $\quad[\because \sin 30°=1/2]$

Thus, the first three terms of the AP are 1, 14, 27.

Hence, its fifth term $a_5=a_1+(5-1)d$

$$=1+4\times 13=1+52=53$$

111. ***(c)*** Let α, α^2 be the roots of $3x^2+px+3$

$\therefore$ Product of the roots,

$$\alpha\cdot\alpha^2=3/3 \Rightarrow \alpha^3=1$$

$\Rightarrow \alpha=1, \omega, \omega^2$

Where, $\omega=-\frac{1+\sqrt{3}i}{2}$ and $\omega^2=\frac{-1-\sqrt{3}i}{2}$

Again, $\alpha+\alpha^2=-\frac{p}{3}$

$\Rightarrow 1+1=-\frac{p}{3}$ (if $\alpha=1$)

$\Rightarrow p=-6$

But $p>0$

$\therefore \alpha=1$ is not possible.

If $\alpha=\omega$, then $\alpha+\alpha^2=\omega+\omega^2=-1$

$\{\because 1+\omega+\omega^2=0\}$

$\therefore -1=-\frac{p}{3} \Rightarrow p=3$

Again, if $\alpha=\omega^2$, then

$\alpha+\alpha^2=\omega^2+\omega^4=\omega^2+\omega=-1$

$[\because \omega^3=1, \omega^4=\omega.\omega^3=\omega]$

$\therefore -1=-\frac{p}{3} \Rightarrow p=3$

112. ***(d)*** We have,

$$\log_{140}63=\log_{2^2\times5\times7}(3\times3\times7)$$

$$=\frac{\log_2(3\times3\times7)}{\log_2(2^2\times5\times7)}=\frac{\log_2 3+\log_2 3+\log_2 7}{2\log_2 2+\log_2 5+\log_2 7}$$

$$=\frac{2a+\frac{1}{c}}{2+b+\frac{1}{c}}=\frac{2ac+1}{2c+bc+1}\quad\left[\because \log_2 7=\frac{1}{\log_7 2}\right]$$

113. ***(a)*** Given that, $\cos(x-y)$, $\cos x$, $\cos(x+y)$ are in HP.

Then, $\cos x = \dfrac{2\cos(x-y)\cos(x+y)}{\cos(x-y)+\cos(x+y)}$ $\left[\begin{array}{l}\because a, b, c \text{ are in HP} \\ b = \dfrac{2ac}{a+c}\end{array}\right]$

$\Rightarrow \cos x = \dfrac{2(\cos^2 x - \sin^2 y)}{2\cos x \cos y}$

$\Rightarrow \cos^2 x \cos y = \cos^2 x - \sin^2 y$

$\Rightarrow \cos^2 x(\cos y - 1) = -\sin^2 y$

$\Rightarrow \cos^2 x(1-\cos y) = 1-\cos^2 y$

$\Rightarrow \cos^2 x(1-\cos y) = (1-\cos y)(1+\cos y)$

$\Rightarrow \cos^2 x = 1+\cos y$

$\Rightarrow \cos^2 x = 2\cos^2(y/2)$

$\Rightarrow \cos^2 x \sec^2(y/2) = 2$

$\Rightarrow \cos x \sec(y/2) = \pm\sqrt{2}$

114. ***(c)*** We first write the elements of the set R.

i.e. $R = \{(1,4), (4,1), (2,3), (3,2)\}$

Since, $(1,1) \notin R \Rightarrow R$ is not reflexive.

Now as, $(1,4) \in R \Rightarrow (4,1) \in R$ and $(2,3) \in R$

$\Rightarrow (3,2) \in R$

So, R is symmetric

Now, $(1,4) \in R$ and $(4,1) \in R$

$\not\Rightarrow (1,1) \in R$

Thus, R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

115. ***(c)*** Any number from 1 to 999 is of the form abc when $0 \le a, b, c \le 9$. Let us first count the number in which 5 occurs exactly once.

Since, 5 can occur at one place in $1 \times {}^3C_1 \times 9 \times 9 = 243$ ways, next 5 can occur in exactly two places in ${}^3C_2 \times 9 = 27$. Lastly, 5 can occur in all three digits in only one way.

Hence, the number of times 5 occurs

$= 1 \times 243 + 27 \times 2 + 1 \times 3$

$= 243 + 54 + 3 = 300$

116. ***(a)*** Given, that, $A \cup X = B \cup X$

$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$

[Using distributive law]

$\Rightarrow A \cup \phi = (A \cap B) \cup \phi [\because A \cap X = \phi]$

$\Rightarrow A = A \cap B$ ...(i)

Again, consider $A \cup X = B \cup X$

$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$

[Using distributive law]

$\Rightarrow (B \cap A) \cup \phi = B \cup \phi$ $[B \cap X = \phi]$

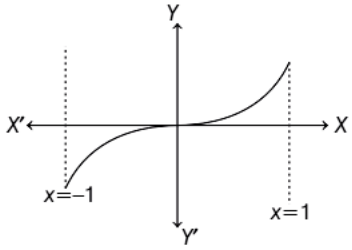
$\Rightarrow A \cap B = B$ ...(ii)

Thus from Eq. (i) and (ii), we get

$A = B$

117. ***(d)*** $f(x) = x|x| = \begin{cases} x^2, & x \ge 0 \\ -x^2, & x < 0 \end{cases}$

and $f[-1,1] \to [-1,1]$



Since, $-1 \le x \le 1$, therefore $-1 \le f(x) \le 1$

$\Rightarrow$ codomain range.

$\therefore$ Function is one-one and onto.

118. ***(b)*** We have,

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow \sin x + \sin 3x - 3\sin 2x = \cos x + \cos 3x - 3\cos 2x$

$\Rightarrow 2\sin 2x \cos x - 3\sin 2x - 2\cos 2x \cos x + 3\cos 2x = 0$

$\left[\because \sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}\right.$

$\left.\text{and } \cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}\right]$

$\Rightarrow \sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$

$\Rightarrow (\sin 2x - \cos 2x)(2\cos x - 3) = 0$

$\Rightarrow \sin 2x = \cos 2x$ $[\because \cos x \ne 3/2]$

$\because \tan 2x = 1$

$\Rightarrow \tan 2x = \tan\dfrac{\pi}{4}$

$\Rightarrow 2x = n\pi \pm \dfrac{\pi}{4}$

$\Rightarrow x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}$ [$\because$ neglect – ve sign]

119. ***(b)*** The third side is parallel to a bisector of the angle between equal sides.

The bisectors are $7x - y + 3 = \pm 5(x + y - 3)$

$\Rightarrow 2x - 6y + 18 = 0$ or $12x + 4y - 12 = 0$

$\Rightarrow x - 3y + 9 = 0$ or $3x + y - 3 = 0$

Let the third side be $x - 3y = k$ or $3x + y = L$

It passes through $(1, -10)$.

$k = 31, L = -7$

Hence, required lines are $x - 3y = 31$, $3x + y = -7$

120. ***(d)*** Let (t, m) be the other end of the chord drawn from the point (p, q) on the circle

$x^2 + y^2 = px + qy$

Their mid-point is $\left(\dfrac{t+p}{2}, \dfrac{m+q}{2}\right)$

Since, mid-point lies on X-axis i.e. $y = 0$

$m + q = 0$ ...(i)

Also, (t, m) lies on the circle.

$\therefore t^2 + m^2 - pt - qm = 0$ ...(ii)

From Eqs. (i) and (ii), we get $t^2 - pt + 2q^2 = 0$

Which is quadratic in t such that, Discriminant > 0

$\Rightarrow p^2 - 8q^2 > 0 \Rightarrow p^2 > 8q^2$

121. ***(a)*** Let P be the foot of the perpendicular drawn from $A(2, 3, 4)$ to the given line l.

$$\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}$$

Now, any point on the line l is given by

$$x = 4-2\lambda,\ y = 6\lambda,\ z = 1-3\lambda$$

The coordinates of P are $(4-2\lambda,\ 6\lambda,\ 1-3\lambda)$

The direction ratios of AP are

$(4-2\lambda-2,\ 6\lambda-3,\ 1-3\lambda-4)$

i.e. $(2-2\lambda,\ 6\lambda-3,\ -3-3\lambda)$

And the direction ratios of l are -2, 6 and -3.

Given, $AP \perp l$

$\therefore\ -2(2-2\lambda)+6(6\lambda-3)-3(-3-3\lambda)=0$

$\Rightarrow \quad \lambda = \frac{13}{49}$

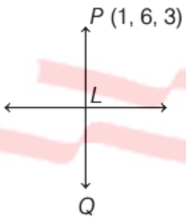
$\therefore \quad AP^2 = (4-2\lambda-2)^2+(6\lambda-3)^2+(1-3\lambda-4)^2$

$= 22-26\lambda+49\lambda^2$

Put $\quad \lambda = \frac{13}{49}$, we get

$$AP^2 = \frac{909}{49} \quad \Rightarrow \quad AP = \frac{3}{7}\sqrt{101}$$

122. ***(c)*** Let $P(1, 6, 3)$ be the given point and L be the foot of perpendicular from P to the given line. The coordinates of a general point on the given line are



$$\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda \text{ (say)}$$

i.e. $x = \lambda$

$y = 2\lambda+1, \quad z = 3\lambda+2$

If the coordinates of L are $(\lambda,\ 2\lambda+1,\ 3\lambda+2)$, then the direction ratios of PL are $(\lambda-1,\ 2\lambda-5,\ 3\lambda-1)$.

Since, the direction ratios of given line which is perpendicular to PL, are 1, 2 and 3. Therefore, $(\lambda-1)1+(2\lambda-5)2+(3\lambda-1)3=0$, which gives $\lambda = 1$.

Hence, coordinates of L are $(1, 3, 5)$.

Let $Q(x_1, y_1, z_1)$ be the image of $P(1, 6, 3)$ in the given line. Then, L is the mid-point of PQ.

Therefore, $\frac{x_1+1}{2}=1,\ \frac{y_1+6}{2}=3,\ \frac{z_1+3}{2}=5$

$\Rightarrow \quad x_1 = 1,\ y = 0,\ z = 7$

Hence, the image of $(1, 6, 3)$ in the given line is $(1, 0, 7)$.

123. ***(a)*** Given equation of plane is $x-y+z=5$.

The distance measured along the line $x = y = z$.

$\Rightarrow \quad x/1 = y/1 = z/1$

Direction ratios of the given line is $(1, 1, 1)$.

So, the equation of line PQ is [where, $P = (1,-5,9)$]

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$$

$\Rightarrow \quad x = \lambda+1,\ y = \lambda-5,\ z = \lambda+9$

Since, it lies on the plane $x+y+z=5$

$\therefore\ (\lambda+1)-(\lambda-5)+(\lambda+9)=5 \Rightarrow \lambda = -10$

The coordinate of Q is $(-9, -15, -1)$ and the coordinate of P is $(1, -5, 9)$

$$PQ = \sqrt{(10)^2+(10)^2+(10)^2} = 10\sqrt{3}$$

$\therefore \quad 2\sqrt{3}k = 10\sqrt{3} \Rightarrow k = 5$

124. ***(b)*** $\lim_{n\to\infty} \sin\left\{n\pi\left(1+\frac{1}{n^2}\right)^{1/2}\right\}$

$= \lim_{n\to\infty} \sin\left\{n\pi\left(1+\frac{1}{2n^2}-\frac{1}{8n^4}+\dots\right)\right\}$

$= \lim_{n\to\infty} \sin\left\{n\pi+\frac{\pi}{2n}-\frac{\pi}{8n^3}+\dots\right\}$

$= \lim_{n\to\infty} (-1)^n \sin\pi\left(\frac{1}{2n}-\frac{1}{8n^3}+\dots\right) = 0$

125. ***(b)*** $f(x) = \begin{cases} e^x, & x \le 0 \\ 1-x, & 0 < x \le 1 \\ x-1, & x > 1 \end{cases}$

At $x = 0$

$\lim_{x\to 0} f(x) = \lim_{x\to 0^+}(1-x) = 1$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^x = k = f(0)$

$\Rightarrow f(x)$ is continuous at $x = 0$

At $x = 1$

$\lim_{x\to 1^+}(x-1) = 1-1 = 0$

$\lim_{x\to 1^-}(1-x) = 1-1 = 0$

$f(1) = 1-1 = 0$

$f(x)$ is continuous at $x = 1$

$Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim_{h\to 0} \frac{1+h-1-0}{h} = 1$

$Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{1-(1-h)-0}{-h}$

$= \lim_{h\to 0} \frac{h}{-h} = -1$

$\therefore \quad Rf'(1) \ne Lf'(1)$

$\Rightarrow f(x)$ is not differentiable at $x = 1$

$Rf'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} -\left[\frac{1-h-1}{h}\right] = -1$

$Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0} \frac{e^{-h}-1}{-h} = 1$

So, it is not differentiable at $x = 0$.

Similarly, it is not differentiable at $x = 1$ but it is continuous at $x = 0$ and 1.

126. ***(b)*** Since, $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0)$

Let a be any point

Now, at $x = a$, $\lim_{x\to a} f(x) = \lim_{h\to 0} f(a+h)$

$= \lim_{h\to 0} f(a) + \lim_{h\to 0} f(h)$

$= f(a)+f(0) = f(a+0) = f(a)$

$\therefore\ f(x)$ is continuous at $x = a$, where a is any arbitrary point.

Hence, $f(x)$ is continuous for all x.

127. ***(a)*** Since, x, $\sin^{-1} x$, $\tan^{-1} x$ are continuous functions, so the function f is clearly continuous at each point of its domain except possibly at $x = 0$.

So, for f to be continuous at $x = 0$

$$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{2 - \frac{\sin^{-1} x}{x}}{2 + \frac{\tan^{-1} x}{x}} = \frac{1}{3}$$

$$\left[\because \lim_{x\to 0} \frac{\sin^{-1} x}{x} = 1, \lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1\right]$$

128. ***(b)*** The given function is

$$f(x) = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$$

Now, LHD $= \lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^{-1}} \frac{0-1}{x-1}$

Which does not exist

$\therefore$ f is not derivable at $x = 1$.

129. ***(b)*** Given, $f(x) = -2x^3 + 21x^2 - 60x + 41$ ...(i)

On differentiating Eq. (i) w.r.t. x, we get

$f'(x) = -6x^2 + 42x - 60$

$= -6(x^2 - 7x + 10) = -6(x-2)(x-5)$

If $x < 2$, $f'(x) < 0$, i.e., $f(x)$ is decreasing.

130. ***(d)*** Rolle's theorem is applicable only if function is continuous and differentiable. But $f(x) = |x|$ is not differentiable at $x = 0$.

i.e. $f'(0)$ does not exist.

131. ***(b)*** Clearly, the point of intersection of curves is $(0, 1)$.

Now, slope of tangent of first curve

$$m_1 = \frac{dy}{dx} = a^x \log a$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = m_1 = \log a$$

Slope of tangent of second curve,

$$m_2 = \frac{dy}{dx} = b^x \log b$$

$$\Rightarrow m_2 = \left(\frac{dy}{dx}\right)_{(0,1)} = \log b$$

$$\therefore \tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\log a - \log b}{1 + \log a \log b}$$

132. ***(d)*** We have, $\int_0^n [x]\, dx$

$= \int_0^1 0\, dx + \int_1^2 1\, dx + \int_2^3 2\, dx + ... + \int_{n-1}^n (n-1)\, dx$

$= 1(2-1) + 2(3-2) + 3(4-3) + ... + (n-1)\{n-(n-1)\}$

$= 1 + 2 + 3 + ... + (n-1) = \frac{n(n-1)}{2}$

and $\int_0^n \{x\}\, dx = \int_0^n (x - [x])\, dx = \int_0^n x dx - \int_0^n [x] dx$

$= \frac{n^2}{2} - \frac{n(n-1)}{2} = \frac{n}{2}$

$$\therefore \frac{\int_0^n [x]\, dx}{\int_0^n \{x\}\, dx} = n - 1$$

133. ***(b)*** Given, $f(x) = x + \sin 2x$

$\Rightarrow f'(x) = 1 + 2\cos 2x$

For maximum or minimum value, $f'(x) = 0$

$\Rightarrow 1 + 2\cos 2x = 0 \Rightarrow \cos 2x = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

Find $f(0), f\left(\frac{2\pi}{3}\right), f\left(\frac{4\pi}{3}\right), f\left(\frac{5\pi}{3}\right), f(2\pi)$

$\Rightarrow f(0) = 0, f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} - 0.8$

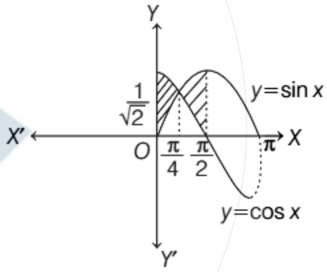
$\Rightarrow f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + 0.8, f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} - 0.8$

$\Rightarrow f(2\pi) = 2\pi + 0 = 2\pi$

$\therefore$ Maximum value of $f(x) = 2\pi$.

134. ***(b)*** Given, $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$

$$= \begin{cases} \cos x - \sin x, & 0 \le x \le \frac{\pi}{4} \\ \sin x - \cos x, & \frac{\pi}{4} \le x \le \frac{\pi}{2} \end{cases}$$



Required area $= \int_0^{\pi/2} |\cos x - \sin x|\, dx$

$= \int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$

$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$

$= (\sqrt{2} - 1) - (1 - \sqrt{2}) = 2\sqrt{2} - 2$

135. ***(b)*** Put $x + y = v$ and $1 + \frac{dy}{dx} = \frac{dv}{dx}$

Therefore, the differential equation reduces to

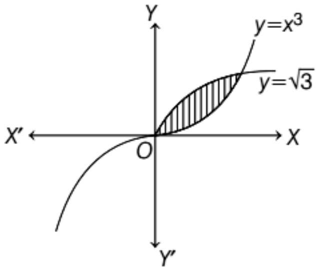
$\frac{dv}{dx} = (1 + \cos v) + \sin v = 2\cos^2\frac{v}{2} + 2\sin\frac{v}{2}\cos\frac{v}{2}$

$= 2\cos^2\frac{v}{2}\left(1 + \frac{\tan v}{2}\right)$

$$\Rightarrow \int \frac{\sec^2\frac{v}{2}}{2\left[1 + \tan\left(\frac{v}{2}\right)\right]} dv = \int dx$$

$$\therefore \log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + C$$

136. ***(c)*** Since, the intersection of two curves $y = x^3$ and $y = \sqrt{x}$ are $x = 0$ and $x = 1$.



$$\therefore \quad A = \left| \int_0^1 (x^3 - \sqrt{x})\, dx \right| = \left| \left[\frac{x^4}{4} - \frac{2x^{3/2}}{3} \right]_0^1 \right|$$

$$= \left| \left[\frac{1}{4} - \frac{2}{3} \right] \right| = \frac{5}{12} \text{ sq units}$$

137. ***(c)*** $\lim_{n \to \infty} \frac{an(1+n) - (1+n^2)}{1+n}$

$$= \lim_{n \to \infty} \frac{(a-1)n^2 + an - 1}{n+1} = \infty,$$

If $a - 1 \neq 0$ limit does not exist and if $a - 1 = 0$, then

$$\lim_{n \to \infty} \frac{an - 1}{n+1} = a = b \Rightarrow a = b = 1$$

138. ***(b)*** Total ways in which papers can be checked is equal to 7^4. Now, two teachers who have to check all the papers can be selected in $^7C_2$ ways and papers can be checked by them is $(2^4 - 2)$ favourable ways.

Thus, required probability $= \frac{^7C_2(2^4 - 2)}{7^4} = \frac{6}{49}$

139. ***(c)*** Given, equation of hyperbola is $(10x - 5)^2 + (10y - 4)^2 = \lambda^2 (3x + 4y - 1)^2$ can be rewritten as

$$\frac{\sqrt{\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{2}{5}\right)^2}}{\left| \frac{3x + 4y - 1}{5} \right|} = \left| \frac{\lambda}{2} \right|$$

This is of the form of $\frac{PS}{PM} = e$

Where, P is any point on the hyperbola and S is a focus and M is the point of directrix.

Here, $\left| \frac{\lambda}{2} \right| > 1 \Rightarrow |\lambda| > 2$ $\qquad (\because e > 1)$

$\Rightarrow \quad \lambda < -2$ or $\lambda > 2$

140. ***(a)*** $v = \hat{a} \times \hat{b} = |a||b| \sin\theta\, \hat{n} = \sin\theta\, \hat{n}$

[$\because \hat{a}$ and $\hat{b}$ are unit vectors]

Where, θ is the angle between a and b.

[$\because |a| = 1 = |b|$]

$\therefore \quad |u| = \sin\theta$

Now, $u = \hat{a} - (\hat{a} \cdot \hat{b})\hat{b} = \hat{a} - (\cos\theta)\hat{b}$

$\Rightarrow |u|^2 = (\hat{a} - (\cos\theta)\hat{b}) \cdot (\hat{a} - \cos\theta\, \hat{b})$

$= \mathbf{a}^2 + \cos^2\theta\, \mathbf{b}^2 - 2\cos\theta(\hat{a} \cdot \hat{b})$

$[|\mathbf{a}|^2 = |\mathbf{b}|^2 = |\mathbf{a}||\mathbf{b}| = 1]$

$= 1 + \cos^2\theta - 2\cos^2\theta \quad [\because \hat{a} \cdot \hat{b} = |\hat{a}||\hat{b}|\cos\theta]$

$= 1 - \cos^2\theta = \sin^2\theta$

$\therefore \quad |u| = \sin\theta$

Thus $|v| = |u|$

141. ***(c)*** Variance $= \frac{1}{n}\Sigma(x - \bar{x})^2 = \sigma^2$

New variance $= \frac{1}{n}\Sigma(\alpha x - \alpha\bar{x})^2$

$= \alpha^2 \frac{1}{n}\Sigma(x - \bar{x})^2 = \alpha^2\sigma^2$

142. ***(a)*** Given, coefficient of variation $C_1 = 50$ and coefficient of variation $C_2 = 60$.

We have, $\bar{x}_1 = 30$ and $\bar{x}_2 = 25$

$\because \quad C = \frac{\sigma}{\bar{x}} \times 100$

$\Rightarrow \quad 50 = \frac{\sigma_1}{30} \times 100 \Rightarrow \sigma_1 = 15$

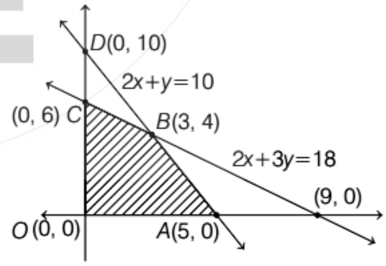
and $\quad 60 = \frac{\sigma_2}{25} \times 100 \Rightarrow \sigma_2 = 15$

$\therefore$ Required difference $\sigma_1 - \sigma_2 = 15 - 15 = 0$

143. ***(c)*** Given, constraints are $2x + 3y \le 18$, $2x + y \le 10$, $x \ge 0$, $y \ge 0$.

The feasible region is $OABCO$.

Also, $z = 9x + 13y$



At $O(0, 0)$, $z = 0$

At $A(5, 0)$, $z = 45$

At $B(3, 4)$, $z = 27 + 52 = 79$

At $C(0, 6)$, $z = 78$

$\therefore$ Maximum value of z is 79.

Probability of getting head = 1/2

144. ***(c)*** The man has to win atleast 4 times.

$\because$ Probability of getting head = 1/2

$\therefore$ Required probability

$$= {}^7C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 + {}^7C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2 + {}^7C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right) + {}^7C_7\left(\frac{1}{2}\right)^7$$

$$= ({}^7C_4 + {}^7C_5 + {}^7C_6 + {}^7C_7)\times\frac{1}{2^7} = \frac{64}{2^7} = \frac{1}{2}$$

145. ***(c)*** $\because \frac{-\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}, \frac{-\pi}{2} \le \sin^{-1} y \le \frac{\pi}{2}$

and $\frac{-\pi}{2} \le \sin^{-1} z \le \frac{\pi}{2}$

Given that,

$$\sin^{-1}x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

Which is possible only when

$$\sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$$

$\Rightarrow \quad x = y = z = 1$

Also given that $f(p+q) = f(p)\cdot f(q) \forall\ p, q \in R$

$[\because f(1) = 2]$

Put $p = q = 1$

Then, $f(2) = f(1)\ f(1) = 2 \times 2 = 4$

and put $p = 1, q = 2$

then, $f(3) = f(1)\ f(2) = 2\cdot 2^2 = 8$

$$\therefore \quad x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$$

$$= 1 + 1 + 1 - \frac{3}{1+1+1} = 3 - 1 = 2$$

146. ***(c)*** Here,

$$T_n = \frac{n(n+1)}{n!} = \frac{n-1+2}{(n-1)!} = \frac{1}{(n-2)!} + \frac{2}{(n-1)!}$$

$$\therefore \quad S = \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty}\frac{1}{(n-2)!} + 2\sum_{n=1}^{\infty}\frac{1}{(n-1)!}$$

$$= e + 2e = 3e$$

147. ***(b)*** Since, $|z_1| = |z_2| = |z_3|$

$\therefore$ 0 is the circumcentre of an equilateral ΔABC.

$$\therefore \frac{x_1 + x_2 + x_3}{3} = 0 = \frac{y_1 + y_2 + y_3}{3}$$

where, $z_1 = x_1 + iy_1$

$$\Rightarrow \frac{x_1 + x_2 + x_3}{3} + i\left(\frac{y_1 + y_2 + y_3}{3}\right) = 0$$

$\Rightarrow (x_1 + iy_1) + (x_2 + iy_2) + (x_3 + iy_3) = 0$

$\Rightarrow \quad z_1 + z_2 + z_3 = 0$

148. ***(b)*** Let $I = \int \frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7} = \int \frac{dx}{(\sqrt{x})^7\left(\frac{1}{(\sqrt{x})^5} + 1\right)}$

Put $1 + \frac{1}{(\sqrt{x})^5} = t$

$$\Rightarrow -\frac{5}{2}\cdot\frac{1}{(\sqrt{x})^7}\,dx = dt$$

$$\therefore\ I = -\frac{2}{5}\int\frac{dt}{t} = \frac{-2}{5}\log t + C$$

$$= \frac{-2}{5}\log\left[1 + \frac{1}{(\sqrt{x})^5}\right] + C = -\frac{2}{5}\log\left(\frac{x^{5/2}+1}{x^{5/2}}\right) + C$$

$\Rightarrow\ \lambda = \frac{-2}{5}$ and $a = \frac{5}{2}$

$\therefore\ \ a + \lambda = 2.1 > 2$

149. ***(b)*** Equation of line joining the points $(0, 3)$ and $(5, -2)$ is

$$y - 3 = \frac{-2-3}{5-0}(x - 0).$$

$\Rightarrow \quad y - 3 = -x \Rightarrow y = 3 - x$

If this line is tangent to $y = \frac{ax}{(x+1)}$, then

$(3 - x)(x - 11) = ax$ should have equal roots.

$\Rightarrow \quad x^2 + (a-2)x - 3 = 0$

$D = 0$ for equal roots

Thus, $\quad (a-2)^2 + 12 = 0$

$\Rightarrow$ No value of $a \Rightarrow a \in \phi$

150. ***(b)*** Shortest distance between two curves occured along the common normal.

$\therefore$ Normal to $y^2 = 4x$ at $(m^2, 2m)$ is

$\Rightarrow \quad y - 2m = -m(x - m^2)$

$y + mx - 2m - m^3 = 0$

Normal to $y^2 = 2(x-3)$ at $\left(\frac{m^2}{2} + 3, m\right)$ is

$$y - m = -m\left(x - \frac{1}{2}m^2 - 3\right)$$

$$y + m(x-3) - m - \frac{m^3}{2} = 0$$

Both normals are same, if

$$-2m - m^3 = -4m - \frac{1}{2}m^3 \Rightarrow m = 0, 2, -2$$

So, points will be $(4, 4)$ and $(5, 2)$ or $(4, -4)$ and $(5, -2)$.

Hence, shortest distance will be

$$\sqrt{(1+4)} = \sqrt{5}$$