Solved Paper 2016*

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

		No. of Questions
Part I	Physics	1–40
Part II	Chemistry	41–80
Part III	a. English Proficiency	81–95
	b. Logical Reasoning	96–105
Part IV	Mathematics	106–150

- All questions are Multiple Choice Questions having four options out of which **only one** is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.
- Time allotted to complete this paper is 3 hrs.

PART I

Physics

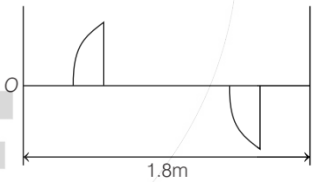
1. You are given two spheres of same material and radii 10 cm and 20 cm. They are heated to the same temperature and placed in the same environment. The ratio of their rates of cooling will be

a. 1 : 2 ***b.*** 2 : 1
c. 1 : 4 ***d.*** 4 : 1

2. A man is standing unsymmetrically between two mountains. He fires a gun and hears the first echo after 1.5 s and the second echo after 2.5 s. If the speed of sound is 340 m/s, what is the distance between the mountains?

a. 340 m ***b.*** 410 m
c. 640 m ***d.*** 680 m

3. A thin plano-convex lens of focal length f is split into two halves. One of the halves a shifted along the optical axes. The separation between object and image plane is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of lens.



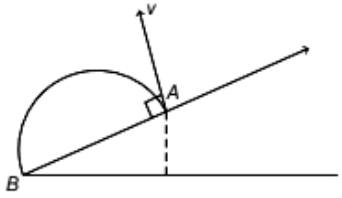
a. 0.1 m ***b.*** 0.4 m ***c.*** 0.9 m ***d.*** 1 m

4. A test charge q_0 is placed at the centre of a spherical conductor of radius R. A charge Q is placed on the spherical conductor. What will be the electrostatic force on the charge Q due to q_0?

a. Zero ***b.*** $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq_0}{R^2}$

c. $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq_0}{2R^2}$ ***d.*** $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq_0}{4R^2}$

5. A particle is projected from A at the angle of 90° with the plane AB as shown in figure. If again touches the plane at B after time T, then what is the length of AB?



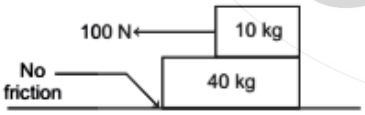
a. $Tu\sin\theta$ **b.** $Tu\cos\theta$
c. $Tu\tan\theta$ **d.** $Tu\cot\theta$

6. A pendulum is executing SHM. The velocity of the bob at the mean position is 3 cm/s. If without changing the length, the amplitude is doubled, then what will be the velocity of the bob in the mean position?
a. 1.5 cm/s **b.** 3 cm/s
c. 4.5 cm/s **d.** 6 cm/s

7. An α-particle describes a circular path of radius r_0 in a magnetic field B. What will be the radius of the circular path described by the proton of same energy in the same magnetic field?
a. $\frac{r_0}{2}$ **b.** r_0 **c.** $\sqrt{2}$ **d.** $2r_0$

8. In an interference pattern, the position of zeroth order maxima is 4.8 mm from a certain point P on the screen. The fringe width is 0.2 mm. The position of second order minima from point P is
a. 5.1 mm **b.** 5 mm
c. 40 mm **d.** 5.2 mm

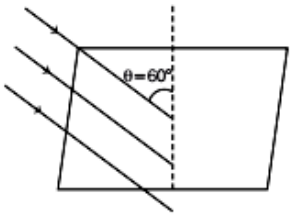
9. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab. The static coefficient of friction between the block and the slab is 0.6, while the kinetic coefficient of friction is 0.4. The 10 kg blocked is acted upon by a horizontal force of 100 N. If $g = 9.8\ ms^{-2}$, the resulting acceleration of the slab will be



a. $1.47\ ms^{-2}$ **b.** $1\ ms^{-2}$
c. $9.8\ ms^{-2}$ **d.** $0.98\ ms^{-2}$

10. A bomber plane moves horizontally with a speed of $500\ ms^{-1}$ and a bomb released from it, strikes the ground in 10 s. Angle at which it strikes the ground will be $(g = 10\ ms^{-2})$
a. $\tan^{-1}\left(\frac{1}{5}\right)$ **b.** $\tan^{-1}\left(\frac{1}{4}\right)$
c. $\tan^{-1}(1)$ **d.** $\tan^{-1}(5)$

11. A square loop of wire of side 5 cm is lying on a horizontal table. An electromagnet wave, above and to one side of the loop is turned on causing a uniform magnetic field downward at an angle of 60° to the vertical as shown in the figure. The magnetic induction is 0.5 T. The average induced emf in the loop, if the field increases from zero to its final value in 0.2 s is



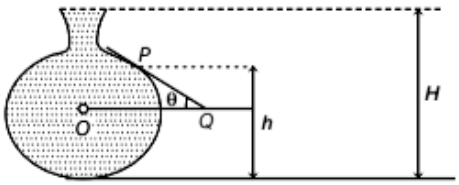
a. 5.4×10^{-3} V **b.** 3.12×10^{-3} V
c. zero **d.** 0.25×10^{-3} V

12. One end of a uniform rod of mass m_1 and uniform area of cross-section A is suspended from the roof and mass m_2 is suspended from the other end. What is the stress at the mid-point of the rod?
a. $(m_1+m_2)\,g/A$ **b.** $(m_1-m_2)\,g/A$
c. $\left[\frac{(m_1/2)+m_2}{A}\right]g$ **d.** $\left[\frac{m_1+(m_2/2)}{A}\right]g$

13. The temperature coefficient of resistance for a wire is $0.00125°C^{-1}$. At 300 K its resistance is 1 Ω. The temperature at which the resistance becomes 1.5Ω is
a. 450 K **b.** 713.5 K **c.** 454 K **d.** 900 K

14. An electron of mass m and charge e initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t (ignoring relativistic effect) is
a. $\frac{h}{eEt^2}$ **b.** $\frac{eEt}{h}$ **c.** $\frac{mh}{eEt^2}$ **d.** $-\frac{h}{eE}$

15. Figure shows the vertical cross-section of a vessel filled with a liquid of density ρ. The normal thrust per unit area on the wall of the vessel at point P, as shown will be



a. $h\rho g$ **b.** $H\rho g$
c. $(H-h)\rho g$ **d.** $(H-h)\rho g\cos\theta$

16. In the Bohr model of the hydrogen atom, let R, v and E represent the radius of the orbit, the speed of electron and the total energy of the electron, respectively. Which of the following quantities is proportional to quantum number n?

a. $\frac{R}{E}$ *b.* $\frac{E}{v}$ *c.* RE *d.* vR

17. In the common emitter configuration of n-p-n transistor 10^{10} electrons enter the emitter in 1 μs and 2% of electrons are lost to the base. What is the current gain of the amplifier?

a. 2 *b.* 98 *c.* 1 *d.* 49

18. A projectile is given an initial velocity of $(\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$ m/s, where $\hat{\mathbf{i}}$ is along the ground and $\hat{\mathbf{j}}$ is along the vertical. If $g = 10$ m/s^2, then the equation of its trajectory is

a. $y = x - 5x^2$ *b.* $y = 2x - 5x^2$

c. $4y = 2x - 5x^2$ *d.* $4y = 2x - 25x^2$

19. 1 kg of diatomic gas is at a pressure of 8×10^4 Nm^{-2}. The density of the gas is 4 kg m^{-3}. What is the energy of the gas due to its thermal motion?

a. 5×10^4 J *b.* 6×10^4 J

c. 7×10^4 J *d.* 3×10^4 J

20. In the Young's double slit experiment, the interference pattern is found to have an intensity ratio between bright and dark fringes as 9. This implies that

a. the intensities at the screen due to two slits are 5 units and 4 units respectively

b. the intensities at the screen due to two slits are 4 units and 1 unit respectively

c. the amplitude ratio is 3

d. the amplitude ratio is 2

21. The potential energy of a particle in motion along X-axis is given by $U = U_0 - U_0 \cos ax$. The period of small oscillations is

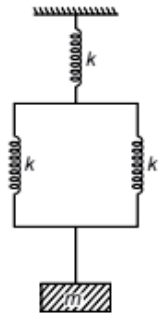
a. $2\pi\sqrt{\frac{ma}{U_0}}$ *b.* $2\pi\sqrt{\frac{U_0}{ma}}$ *c.* $\frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$ *d.* $2\pi\sqrt{\frac{m}{aU_0}}$

22. A charged particle q of mass m is released from rest in a uniform electron field E. Neglecting the effect of gravity, the kinetic energy of the charged particle after time t second is

a. $\frac{Eqm}{t}$ *b.* $\frac{E^2q^2t^2}{2m}$

c. $\frac{2E^2t^2}{mq}$ *d.* $\frac{Eq^2m}{2t^2}$

23. A body of mass m hangs from three springs each of spring constant k shown in the figure. If the mass is slightly displaced and let go, the time period of oscillations of system is



a. $2\pi\sqrt{\frac{m}{3k}}$ *b.* $2\pi\sqrt{\frac{3m}{2k}}$

c. $2\pi\sqrt{\frac{2m}{3k}}$ *d.* $2\pi\sqrt{\frac{3k}{m}}$

24. A radioactive material decays by simultaneous emission of two particles with respective half-lives T_1 and T_2. The effective half-life of material is

a. $\sqrt{T_1 T_2}$ *b.* T_1 / T_2

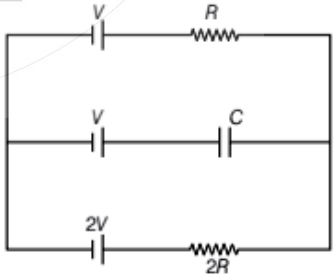
c. $\frac{T_1 T_2}{T_1 + T_2}$ *d.* $\frac{T_1 + T_2}{2}$

25. The focal length of a concave mirror is f and the distance of the object to the principal focus is p. Then, the ratio of the size of the image to the size of the object is

a. f / p *b.* p / f

c. fp *d.* $\sqrt{p / f}$

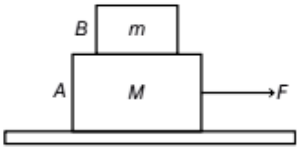
26. In the given circuit, with steady state, the potential drop across the capacitor must be



a. V *b.* $\frac{V}{2}$

c. $\frac{V}{3}$ *d.* $\frac{2V}{3}$

27. A block of mass m is placed on another block of mass M. The arrangement is placed on a horizontal table. All surfaces are smooth. If $m = 2$ kg, $M = 4$ kg and constant force $F = 3$ N, then initial acceleration of masses M and m will be (in m/s^2)



a. 0.5, 0.25 *b.* 0.5, 0
c. 0.5, −0.5 *d.* 0.75, 0

28. A thin rod has moment of inertia I_1 about an axis passing through its centre and perpendicular to length. It is bent to form a ring. The ring so formed has moment of inertia I_2 about an axis passing through centre and perpendicular to the plane of the ring. Then, the ratio I_1 / I_2 is

a. 1 *b.* $\frac{\pi}{4}$ *c.* $\frac{\pi^2}{3}$ *d.* $\frac{3}{5}$

29. In the hydrogen spectrum, λ_3 and λ_2 are the wavelengths of radiations emitted due to transition from second and first excited states to the ground state. Then, the ratio $\frac{\lambda_3}{\lambda_2}$ is

a. $\frac{27}{32}$ *b.* $\frac{32}{27}$ *c.* $\frac{4}{9}$ *d.* $\frac{9}{4}$

30. A conductive circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field B such that B is perpendicular to the plane of loop. The magnetic force acting on the loop is

a. Bir *b.* $2\pi irB$
c. zero *d.* πirB

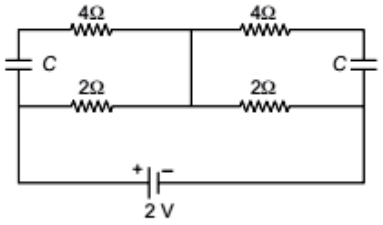
31. A soap bubble in vacuum has a radius 3 cm and another soap bubble in vacuum has radius 4 cm. If two bubbles coalesce under isothermal condition. Then, the radius of new bubble is

a. 2.3 cm *b.* 4.5 cm
c. 5 cm *d.* 7 cm

32. A projectile is thrown with an initial velocity of $(a\hat{i} + b\hat{j})$ m/s. If the range of the projectile is twice the maximum height reached by it, then

a. $b = \frac{a}{2}$ *b.* $b = a$
c. $b = 2a$ *d.* $b = 4a$

33. Find the power consumed by given circuit.



a. 1.5 W *b.* 2 W
c. 1 W *d.* insufficient data

34. An α-particle of mass m suffers one dimensional elastic collision with a nucleus of unknown mass. After the collision α-particle is scattered directly backwards losing 75% of its initial kinetic energy. Then, the mass of the nucleus is

a. m *b.* $2m$ *c.* $3m$ *d.* $5m$

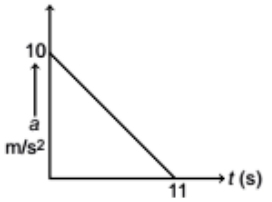
35. A meter stick of mass m, initially in vertical position, is displaced through an angle 45°, keeping upper end pivoted. What is the change in potential energy?

a. $\frac{mg}{2\sqrt{2}}$ *b.* $\frac{mg}{\sqrt{2}}$
c. $\frac{mg}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$ *d.* $mg\left(1 - \frac{1}{\sqrt{2}}\right)$

36. A parallel plate capacitor of value 1.77 μF is to be designed using a dielectric material (dielectric constant = 200) breakdown strength of 3×10^6 V m^{-1}. In order to make such a capacitor, which can withstand a potential difference of 20 V across the plates, the separation between the plates d and the area A of the plates can be
($\varepsilon_0 = 8.85 \times 10^{-12}$ C^{2}N^{-1}m^{-2})

a. $d = 10^{-6}$ m, $A = 10^{-3}$ m^2
b. $d = 10^{-5}$ m, $A = 10^{-2}$ m^2
c. $d = 10^{-4}$ m, $A = 10^{-4}$ m^2
d. $d = 10^{-4}$ m, $A = 10^{-5}$ m^2

37. A particle starting from rest undergoes a rectilinear motion with acceleration a. The variation of a with time is shown in figure. The maximum velocity attained by the particle is



a. 55 m/s *b.* 550 m/s
c. 110 m/s *d.* 650 m/s

38. A satellite of mass m is circulating around the Earth with constant angular velocity. If the radius of the orbit is R_0 and mass of the Earth is M, the angular momentum of satellite about the centre of Earth is

a. $m\sqrt{\frac{GM}{R_0}}$

b. $m\sqrt{GMR_0}$

c. $m\sqrt{\frac{Gm}{R_0}}$

d. $M\sqrt{\frac{GM}{R_0}}$

39. In a sonometer wire the tension is maintained by suspending a 50.7 kg mass from the free end of a wire. The suspended mass has a volume 0.0075 m^3. The fundamental frequency of vibration of wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency becomes

a. 480 Hz

b. 260 Hz

c. 240 Hz

d. 280 Hz

40. A current of 5 A is flowing in a wire of length 1.5 m. A force of 7.5 N acts on it. When it is placed in a uniform magnetic field of induction 2 T. The angle between the magnetic field and the direction of current is

a. 30° *b.* 45°

c. 60° *d.* 90°

PART II

Chemistry

41. A 400 mg iron capsule contains 100 mg of ferrous fumarate, $(CHCOO)_2Fe$. The percentage of iron present in it, is approximately

a. 33% *b.* 25% *c.* 14% *d.* 8%

42. The ratio of the difference between 1st and 2nd Bohr's orbit energy to that between 2nd and 3rd Bohr's orbit energy is

a. $\frac{1}{2}$ *b.* $\frac{1}{3}$ *c.* $\frac{27}{5}$ *d.* $\frac{5}{27}$

43. Hydrogen resembles halogen in many respects for which several factors are responsible. Of the following factors, which one is the most important in this respect?

a. Its tendency to lose an electron to form a cation

b. Its tendency to gain a single electron in its valence shell to attain stable electronic configuration

c. Its low negative electron gain enthalpy value

d. Its small size

44. From the following sets, which one does not contain isoelectronic species?

a. $BO_3^{3-}, CO_3^{2-}, NO_3^-$ *b.* $SO_3^{2-}, CO_3^{2-}, NO_3^-$

c. CN^-, N_2, C_2^{2-} *d.* $PO_4^{3-}, SO_4^{2-}, ClO_4^-$

45. The bond length of HCl bond is 2.29×10^{-10} m. The percentage ionic character of HCl, if measured dipole moment is 6.226×10^{-30} C-m, is

a. 8% *b.* 20%

c. 17% *d.* 50%

46. How many carbon atoms in the molecule $HOOC—(CHOH)_2—COOH$ are asymmetric?

a. 1 *b.* 2

c. 3 *d.* None of these

47. Which of the following on reductive ozonolysis gives only glyoxal?

a. Ethene *b.* Acetylene

c. Benzene *d.* Both (b) and (c)

48. Which of the following transitions involve maximum amount of energy?

a. $M^-(g) \longrightarrow M(g)$ *b.* $M(g) \longrightarrow M^+(g)$

c. $M^+(g) \longrightarrow M^{2+}(g)$ *d.* $M^{2+}(g) \longrightarrow M^{3+}(g)$

49. The boiling point of alcohols and phenols increases with increase in the number of

a. oxygen atom *b.* carbon atom

c. Both (a) and (b) *d.* hydrogen atom

50. A mixture of dilute NaOH and aluminium pieces is used to open drain because

a. the gas obtained creates higher pressure

b. they react explosively

c. they create fire

d. the given statement is not true

51. The conductivity of strong electrolyte

a. increases on dilution slightly

b. decreases on dilution

c. does not change with dilution

d. depends upon density of electrolyte itself

52. What is the freezing point of a 10% (by weight) solution of CH_3OH in water? [K_f of $CH_3OH = 1.86°$ C/m]

a. 90°C *b.* 10°C
c. 6.45°C *d.* –6.45° C

53. Why are potassium and cesium, rather than lithium used in photoelectric cells?

a. Because of their lower IE
b. Because of their lower EA
c. Both (a) and (b)
d. Because of their higher IE

54. Alkyl iodides can be prepared by

a. $RCH_2COOAg + I_2 \xrightarrow[\Delta]{CCl_4} RCH_2I$

b. $RCH_2Cl + NaI \xrightarrow[\Delta]{Acetone} RCH_2I + NaCl$

c. $R—OH + I_2 \longrightarrow RI + H_2O$

d. $CH_4 + I_2 \longrightarrow CH_3I$

55. Copper matte contains

a. Cu_2S and FeS *b.* $FeSiO_3$ and Cu_2O
c. Fe_2O_3 and Cu *d.* Cu_2O and Fe

56. For which type of reaction, order and molecularity have the same value?

a. First order
b. Bimolecular reaction
c. Trimolecular reaction
d. Elementary reaction

57. Metal which can be extracted from all the three dolomite, magnesite and carnallite is

a. Na *b.* K
c. Mg *d.* Ca

58. In the complex, $K_2Fe[Fe(CN)_6]$

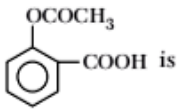
a. the complex is high spin complex
b. both Fe atoms are in the same oxidation state
c. the coordination number of iron is 4
d. both Fe atoms are in different oxidation state

59. Novolac on heating with formaldehyde undergoes cross-linking to form

a. bakelite *b.* melamine
c. dacron *d.* resin

60. Which of the following is a network solid?

a. SO_2 (solid) *b.* I_2
c. Diamond *d.* H_2O (ice)

61. The compound  is used as a/an

a. antiseptic *b.* antibiotic
c. analgesic *d.* pesticide

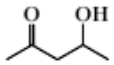
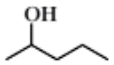
62. DNA and RNA contain four bases each. Which of the following bases is not present in RNA?

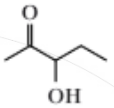
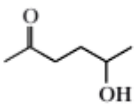
a. Adenine *b.* Thymine
c. Uracil *d.* Cytosine

63. In Duma's method, 0.3 g of an organic compound gave 45 mL of nitrogen at STP. The percentage of nitrogen is

a. 16.9 *b.* 18.7
c. 29.6 *d.* 23.2

64. Which one of the following will be most readily dehydrated in acidic condition?

a.  *b.* 

c.  *d.* 

65. The number of spherical nodes in 3*p*-orbital is

a. 0 *b.* 1
c. 2 *d.* 3

66. An element has [Ar] $3d^1\,4s^2$ configuration in its oxidation state. It's position in the periodic table is

a. period-3, group-3 *b.* period-3, group-7
c. period-4, group-3 *d.* period-3, group-9

67. The HOMO in CO is

a. π-bonding *b.* π-antibonding
c. σ-antibonding *d.* σ-bonding

68. If a gas at constant temperature and pressure expands then its

a. internal energy increases and then decreases
b. internal energy increases
c. internal energy remains the same
d. internal energy decreases

69. What is the conjugate base of OH^-?

a. O^{2-} *b.* O^- *c.* H_2O *d.* O_2

70. Adsorption is accompanied by

a. ΔH is negative
b. decrease in entropy of the system
c. $T\Delta S$ for the process is negative
d. All of the above

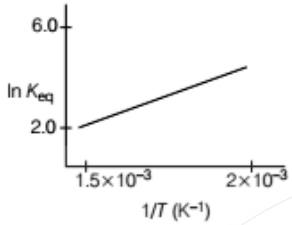
71. The difference of water molecules in gypsum and plaster of Paris is

a. $\frac{5}{2}$ *b.* 2
c. $\frac{1}{2}$ *d.* $1\frac{1}{2}$

72. Boron when treated with carbon forms
a. B_4C **b.** BC_4
c. B_4C_3 **d.** B_2C_3

73. Which of the following compounds is anti-aromatic?
a. Benzene **b.** Cyclobutadiene
c. Cyclodecapentene **d.** Cyclooctatetraene

74. A schematic plot of $\ln K_{eq}$ *versus* inverse of temperature for a reaction is shown below :



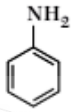
The reaction must be
a. highly spontaneous at ordinary temperature
b. one with negligible enthalpy change
c. endothermic
d. exothermic

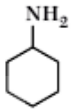
75. Clathrates are
a. non-stoichiometric compounds
b. complex compounds
c. interstitial compounds
d. ionic compounds

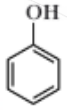
76. Transition elements are coloured
a. due to unpaired *d*-electrons
b. due to small size
c. due to metallic structure
d. All of the above

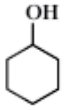
77. The colour of $_{62}Sm^{3+}$ is yellow. The expected colour of $_{66}Dy^{3+}$ is
a. yellow **b.** red
c. blue **d.** green

78. Which of the following compounds is the weakest Bronsted base?

a. 

b. 

c. 

d. 

79. Which one of the following will have the largest number of atoms?
a. 1 g Au(*s*) **b.** 1 g Na(*s*)
c. 1 g Li(*s*) **d.** 2 g $Cl_2(g)$

80. The density of neon will be maximum at
a. STP **b.** 0°C, 2 atm
c. 273°C, 1 atm **d.** 273°C, 2 atm

PART III

a. English Proficiency

Directions (Q. Nos. 81 to 83) *Choose the word which best expresses the meaning of the underlined word in the sentences.*

81. Have you <u>traced</u> the addresses that I have told you earlier?
a. Searched **b.** Found
c. Prognosised **d.** Fathomed

82. Terrorism is a <u>gargantuan</u> problem in our country.
a. Gigantic **b.** Callous
c. Spurious **d.** Lee

83. Discipline will always <u>propitiate</u> other's wishes to you.
a. Enrages **b.** Appease
c. Futilitate **d.** Reverberate

Directions (Q. Nos. 84 to Q. No. 86) *Fill in the blanks with suitable words from the options given below.*

84. A dog was run ..... by the car.
a. down
b. up
c. with
d. by

85. The old man ..... to the hospital after the car hit him near the bridge.
a. is taken **b.** are taken
c. was taken **d.** has taken

86. The rise in ..... was not limited to the prosperous agricultural zones in the rural area.
a. salary **b.** pay
c. increment **d.** wages

Directions (Q. Nos. 87 to 89) *Choose the word which is closest to the opposite in the meaning of the sentence.*

87. Why did he called a man of bad reputation to his house in the night ?

a. Gallant ***b.*** Notorious
c. Famous ***d.*** Pervert

88. The whole world, with its flora and fauna, is considered to be perishable.

a. Sustainable ***b.*** Immortal
c. Adorable ***d.*** Clinical

89. Despite his penury, he tried to teach many children on his own resources.

a. Scarcity ***b.*** Dearth
c. Opulence ***d.*** Affluence

Directions (Q. Nos. 90 to 92) *In each of the following questions, out of the four options, choose the one which can be substituted for the given words or sentences.*

90. One who does not consume alcohol.

a. Rationalist ***b.*** Atheist
c. Incalcucable ***d.*** Teetotaller

91. An animal having both male and female sex, inside single body.

a. Hermaphrodite ***b.*** Aquatic
c. Acerbic ***d.*** Camouflage

92. A text without the name of author.

a. Unnamed ***b.*** Pseudonym
c. Unanimous ***d.*** Concealed

b. Logical Reasoning

96. 'Ribs' are related to 'Lungs' in the same way 'Shell' is related to

a. Breathe ***b.*** Almond
c. Nut ***d.*** Respiration

97. Find the odd one.

a. 20, 40, 46 ***b.*** 25, 50, 56
c. 12, 24, 30 ***d.*** 18, 36, 40

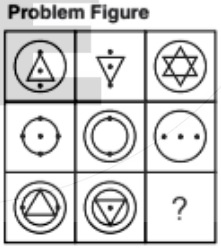
98. Complete the series by replacing '?' mark.

30, 68, 130, 222, 350, ?

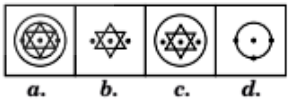
a. 420
b. 460
c. 520
d. 580

99. Choose the answer figure which completes the problem figure matrix.

Problem Figure



Answer Figures



a. ***b.*** ***c.*** ***d.***

Directions (Q. Nos. 93 to 95) *Choose the order of the sentences marked A, B, C, D, E and F to form a logical paragraph and then choose the most suitable option.*

A. The researchers in these companies claim that they could do better by allowing their employees to doze off at work place.

B. The dreams, while at work, are thus helpful to solve crucial problems.

C. Would you believe that some UK based companies are arranging for bed at the work place ?

D. The reason, they claim, could be that dreams produce creative solutions.

E. We only hope that these crucial problems in UK are different from those of ours.

F. But it is true and is considered as a step to improve quality of their products.

93. Which of the following should be the fourth sentence after rearrangement?

a. A ***b.*** B
c. C ***d.*** D

94. Which of the following should be the fifth sentence after rearrangement ?

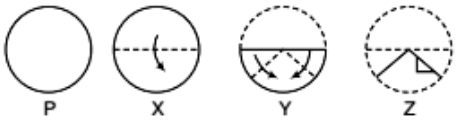
a. A ***b.*** B
c. C ***d.*** D

95. Which of the following should be the sixth sentence after rearrangement ?

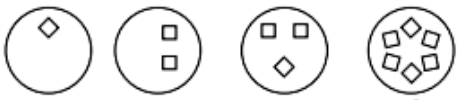
a. A ***b.*** B
c. C ***d.*** E

100. The three problem figures marked X, Y and Z show the manner in which a piece of paper P is folded step by step and then cut. Form the answer figures (a) (b) (c) and (d) select the one showing the unfolded pattern of the paper after the cut.

Problem Figure



Answer Figures

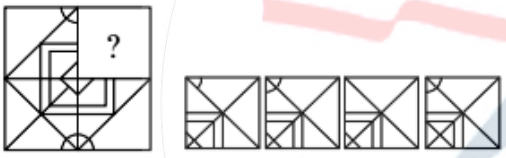


101. In the following question, five figures are given. Out of them, find the three figures that can be joined to form an equilateral triangle.

a. ABE *b.* BCE
c. ADE *d.* BDE

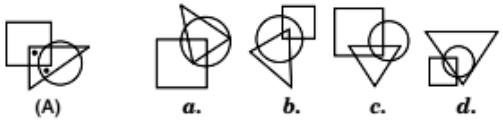
102. In the following question, a part of problem figure is missing. Find out from the given answer figures *a*, *b*, *c* and *d*, that can replace the '?' to complete the figure.

Problem Figure **Answer Figures**



103. In the following question, one or more dots are placed in the figure marked as (A). This figure is followed by four alternatives marked as *a*, *b*, *c* and *d*. One out of these four options contains region(s) common to circle, square, triangle, similar to that marked by the dot in figure (A). Find that figure.

Problem Figure **Answer Figures**

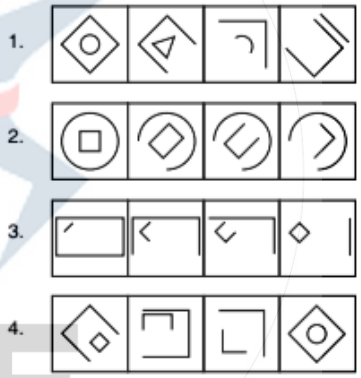


104. On a shelf , there are placed six volumes side-by-side labelled A, B, C, D, E and F. B, C, E and F have green covers while others have yellow covers. A, D, B are new volumes while the rest are old volumes. A, C, B are law reports while the rest are medical extracts. Which two volumes are old medical extracts and have green covers?

a. B, C *b.* C, D
c. C, E *d.* E, F

105. Choose the set of figures which follows the given rule.

Rule Closed figures gradually become open and open figures gradually become closed.



a. 1 *b.* 2 *c.* 3 *d.* 4

PART IV

Mathematics

106. The value of $\lim_{x \to a}\left(2-\frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$ is

a. $e^{-1/\pi}$ *b.* $e^{2/\pi}$ *c.* $e^{-2/\pi}$ *d.* $e^{1/\pi}$

107. If $|z_1|=2, |z_2|=3, |z_3|=4$ and $|2z_1+3z_2+4z_3|=4$, then $|8z_2z_3+27z_3z_1+64z_1z_2|$ is equal to

a. 24 *b.* 48 *c.* 72 *d.* 96

108. Let $\omega=\frac{-1}{2}+i\frac{\sqrt{3}}{2}$. Then, the value of the determinant $\begin{vmatrix}1 & 1 & 1\\ 1 & -1-\omega^2 & \omega^2\\ 1 & \omega^2 & \omega^4\end{vmatrix}$ is

a. 3ω *b.* $3\omega(\omega-1)$
c. $3\omega^2$ *d.* $3\omega(1-\omega)$

109. The sum of n terms of the series $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots$ is

a. $\frac{1}{2n+1}$ **b.** $\frac{2n}{2n+1}$ **c.** $\frac{n}{2n+1}$ **d.** $\frac{2n}{n+1}$

110. The period of the function $f(x)=|\sin x|-|\cos x|$, is

a. $\pi/2$ **b.** π

c. 2π **d.** None of these

111. Let $f(x)=x^3-x^2+x+1$ and $g(x)$ be a function defined by

$g(x)=\begin{cases}\text{Max}\{f(t):0\le t\le x\}, & 0\le x\le 1\\ 3-x, & 1<x\le 2\end{cases}$, then

$g(x)$ is

a. continuous and differentiable on $[0, 2]$

b. continuous but not differentiable on $[0, 2]$

c. Neither continuous nor differentiable on $[0, 2]$

d. None of the above

112. The area enclosed between the curve $y=\log_e(x+e)$ and the coordinate axes, is

a. 4 **b.** 3 **c.** 2 **d.** 1

113. Four numbers are multiplied together. Then, the probability that the product will be divisible by 5 or 10 is

a. $\frac{369}{625}$ **b.** $\frac{399}{625}$

c. $\frac{123}{625}$ **d.** $\frac{133}{625}$

114. Let X denotes the number of times head occur in n tosses of a fair coin. If $P(X=4)$, $P(X=5)$ and $P(X=6)$ are in AP, then the value of n is

a. 7, 14 **b.** 10, 14

c. 12, 7 **d.** 14, 12

115. On the set N of natural numbers, defined the relation R by $a\,R\,b$ if the GCD of a and b is 2, then R is

a. Reflexive but not symmetric

b. Symmetric only

c. Equivalence

d. Neither reflexive nor symmetric and transitive

116. The coefficient of the middle term in the binomial expansion, in powers of x, of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is same, if α equals

a. $\frac{3}{5}$ **b.** $\frac{10}{3}$

c. $\frac{-3}{10}$ **d.** $\frac{-5}{3}$

117. Greatest integral value of 'n' for which the equation $2\cos x(\sin x-\cos x)=n$ has atleast one solution is

a. -2 **b.** 0

c. 1 **d.** 2

118. Let S and S' be two foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If a circle described on SS' as diameter intersects the ellipse in real and distinct points, then the eccentricity e of the ellipse satisfies

a. $e=1/\sqrt{2}$ **b.** $e\in(1/\sqrt{2}, 1)$

c. $e\in(0, 1/\sqrt{2})$ **d.** None of these

119. It has been found that if A and B play a game 12 times, A wins 6 times, B wins 4 times and they draw twice. A and B take part in a series of 3 games. The probability that they will win alternately is

a. $\frac{5}{72}$ **b.** $\frac{5}{36}$

c. $\frac{19}{27}$ **d.** None of these

120. Let a, b, c be in AP and $|a|<1, |b|<1, |c|<1$. If

$x=1+a+a^2+\ldots$ to ∞,

$y=1+b+b^2+\ldots$ to ∞ and

$z=1+c+c^2+\ldots$ to ∞, then, x, y, z are in

a. AP **b.** GP

c. HP **d.** None of these

121. If the sides of a triangle are in GP and its larger angle is twice the smallest, then the common ratio (r) satisfies the inequality

a. $0<r>\sqrt{2}$ **b.** $1<r<\sqrt{2}$

c. $1<r<2$ **d.** None of these

122. The maximum value of $\mu=3x+4y$, subjected to the conditions $x+y\le 40$, $x+2y\le 60$, $x, y\ge 0$, is

a. 130 **b.** 140 **c.** 40 **d.** 120

123. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A\times B$ having 3 as more elements is

a. 256 **b.** 220 **c.** 219 **d.** 211

124. If $a_1, a_2, a_3, \ldots, a_n$ are in HP, then the expression $a_1a_2+a_2a_3+\ldots a_{n-1}a_n$ is equal to

a. $n(a_1-a_n)$ **b.** $(n-1)(a_1-a_n)$

c. na_1a_n **d.** $(n-1)a_1a_n$

125. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ parallel to X-axis, is

a. $y - 3z + 6 = 0$ *b.* $3y - z + 6 = 0$
c. $y + 3z + 6 = 0$ *d.* $3y - 2z + 6 = 0$

126. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$ and $x = 0$ into two parts $R_1\,(0 \le x \le b)$ and $R_2\,(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then, b equals

a. $\frac{3}{4}$ *b.* $\frac{1}{2}$ *c.* $\frac{1}{3}$ *d.* $\frac{1}{4}$

127. If three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are such that $\mathbf{a} \neq 0$ and $\mathbf{a} \times \mathbf{b} = 2(\mathbf{a} \times \mathbf{c}), |\mathbf{a}| = |\mathbf{c}| = 1, |\mathbf{b}| = 4$ and the angle between $\mathbf{b}$ and $\mathbf{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, then $\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$, where λ is equal to

a. ± 4 *b.* -2 *c.* ± 3 *d.* 2

128. z_1, z_2 and z_3 are the affixes of the vertices of a triangle having its circumcentre at the origin. If z is the affix of its orthocentre, then

a. $z_1 + z_2 + z_3 + z = 0$
b. $z_1 + z_2 + z_3 - z = 0$
c. $z_1 - z_2 + z_3 + z = 0$
d. $z_1 + z_2 - z_3 + z = 0$

129. The total number of 5-digit numbers of different digits in which digit in the middle is the highest is

a. $30 \times 3!$
b. $33 \times 3!$
c. $\sum_{n=4}^{9} {}^nC_4 \times 4!$
d. None of the above

130. If $\alpha \le \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \le \beta$, then

a. $\alpha = \frac{\pi}{4}, \beta = \frac{3\pi}{4}$ *b.* $\alpha = -\pi, \beta = 2\pi$
c. $\alpha = 0, \beta = \pi$ *d.* None of these

131. The solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)} \text{ is}$$

a. $\phi\left(\frac{y}{x}\right) = kx$ *b.* $x\,\phi\left(\frac{y}{x}\right) = k$
c. $\phi\left(\frac{y}{x}\right) = ky$ *d.* $y\,\phi\left(\frac{y}{x}\right) = k$

132. If a variable takes value 0, 1, 2, ..., n with frequencies q^n, ${}^nC_1 q^{n-1} p$, ${}^nC_2 q^{n-2} p^2$, ... ${}^nC_n p^n$, where $p + q = 1$, then the mean is

a. np *b.* nq
c. $n(p + q)$ *d.* None of these

133. If the mean and standard deviation of 10 observations $x_1, x_2, ..., x_{10}$ are 2 and 3 respectively, then the mean of $(x_1 + 1)^2$, $(x_2 + 1)^2$, ..., $(x_{10} + 1)^2$ is equal to

a. 18 *b.* 13.5
c. 14.4 *d.* 16

134. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the point $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$

a. On the left of $x = c$ *b.* On the right of $x = c$
c. At no point *d.* At all points

135. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, if

a. $k = 3$ or -3 *b.* $k = 0$ or -1
c. $k = 1$ or -1 *d.* $k = 0$ or -3

136. If the direction ratios of two lines are given by $a + b + c = 0$ and $2ab + 2ac - bc = 0$, then the angle between the lines is

a. $\frac{\pi}{3}$ *b.* $\frac{\pi}{2}$
c. $\frac{2\pi}{3}$ *d.* π

137. If the function $f(x) = x^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval $[1, 3]$ and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$, then

a. $a = 11$ *b.* $a = -11$
c. $a = 6$ *d.* $a = -6$

138. Let f be a bijection satisfying $f'(x) = f(x)$. Then, $(f^{-1})''(x)$ is equal to

a. $f^{-1}(x)$ *b.* $f(x)$
c. $-\frac{1}{x^3}$ *d.* $-\frac{1}{x^2}$

139. The interval in which $2x^3 + 5$ increases less rapidly than $9x^2 - 12x$, is

a. $(-\infty, 1)$ *b.* $(2, \infty)$
c. $(1, 2)$ *d.* None of these

140. The greatest value of the function $f(x)=\sin^{-1}x^2$ in the interval $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$, is

a. $\frac{\pi}{6}$ **b.** $\frac{\pi}{3}$ **c.** $\frac{\pi}{2}$ **d.** $-\frac{\pi}{3}$

141. Let $f:R\to R$ be given by $f(x)=[x]^2+[x+1]-3$, where $[x]$ denotes the greatest integer less than or equal to x. Then, $f(x)$ is

a. many-one and into **b.** one-one and onto
c. many-one and onto **d.** one-one and into

142. Point $(2, 4)$ is translated through a distance of $3\sqrt{2}$ units measured parallel to the line $y-x=1$, in the direction of decreasing ordinates, to reach at Q. If R is the image of Q with respect to the line $y-x=1$, then coordinates of R are

a. $(0, 0)$ **b.** $(-1, 1)$ **c.** $(6, 6)$ **d.** $(5, 7)$

143. If a chord of the circle $x^2+y^2=32$ make equal intercepts of length l on the coordinate axes, then

a. $l\in(0, 8)$ **b.** $l\in(-8, 0)$
c. $l\in(-8, 8)$ **d.** $l\in(-4\sqrt{2}, 4\sqrt{2})$

144. The value of a so that the sum of the squares of the roots of the equation $x^2-(a-2)x-a+1=0$ assume the least value, is

a. 1 **b.** 2 **c.** 3 **d.** 0

145. The slope of the tangent to the curve $y=\int_x^{x^2}\cos^{-1}t^2\,dt$, at $x=2^{-\frac{1}{4}}$ is

a. $\left(\frac{\sqrt[3]{8}}{4}-\frac{1}{3}\right)\pi$ **b.** $\left(\frac{\sqrt[4]{8}}{2}-\frac{3}{4}\right)\pi$
c. $\left(\frac{\sqrt[4]{8}}{3}-\frac{1}{4}\right)\pi$ **d.** None of these

146. If $\lim\limits_{x\to\frac{\pi}{4}}=\frac{4\sqrt{2}-(\cos x+\sin x)^5}{1-\sin 2x}$, then x is equal to

a. $\sqrt{2}$ **b.** $3\sqrt{2}$
c. $5\sqrt{2}$ **d.** None of these

147. Let f be a differentiable function satisfying $f(x)+f(y)+f(z)+f(x)\,f(y)\,f(z)=14$ for all x, $y, z\in R$. Then,

a. $f'(x)<0$ for all $x\in R$
b. $f'(x)>0$ for all $x\in R$
c. $f'(x)=0$ for all $x\in R$
d. None of the above

148. Let $f:[1,\infty)\to[2,\infty)$ be a differentiable function such that $f(1)=2$. If $6\int_1^x f(t)\,dt=3x\,f(x)-x^3$ for all $x\geq 1$, then the value of $f(2)$ is

a. 3 **b.** 4
c. 5 **d.** 6

149. If g is inverse of f and $f'(x)=\frac{1}{1+x^n}$, then $g'(x)$ equals

a. $1+x^n$ **b.** $1+[g(x)]^n$
c. $1+[f(x)]^n$ **d.** None of these

150. The value of k for which

$$f(x)=\begin{cases}\left(1+xe^{-1/x^2}\sin\frac{1}{x^4}\right)^{e^{1/x^2}}, & x\neq 0\\ k, & x=0\end{cases}$$

is continuous at $x=0$, is

a. 4 **b.** 3
c. 2 **d.** 1

Answers

Physics

1. (b)	2. (d)	3. (b)	4. (a)	5. (c)	6. (d)	7. (b)	8. (a)	9. (d)	10. (a)
11. (b)	12. (c)	13. (b)	14. (a)	15. (c)	16. (d)	17. (d)	18. (b)	19. (a)	20. (b)
21. (c)	22. (b)	23. (b)	24. (c)	25. (a)	26. (c)	27. (d)	28. (c)	29. (a)	30. (c)
31. (c)	32. (c)	33. (c)	34. (c)	35. (c)	36. (a)	37. (a)	38. (b)	39. (c)	40. (a)

Chemistry

41. (d)	42. (c)	43. (b)	44. (b)	45. (c)	46. (b)	47. (d)	48. (d)	49. (b)	50. (a)
51. (b)	52. (d)	53. (a)	54. (b)	55. (a)	56. (d)	57. (c)	58. (b)	59. (a)	60. (c)
61. (c)	62. (b)	63. (b)	64. (a)	65. (b)	66. (c)	67. (d)	68. (c)	69. (a)	70. (d)
71. (d)	72. (a)	73. (b)	74. (d)	75. (a)	76. (a)	77. (a)	78. (c)	79. (c)	80. (b)

English Proficiency

81. (a)	82. (a)	83. (b)	84. (a)	85. (c)	86. (d)	87. (c)	88. (b)	89. (c)	90. (d)
91. (a)	92. (c)	93. (d)	94. (b)	95. (d)					

Logical Reasoning

96. (c)	97. (d)	98. (c)	99. (c)	100. (b)	101. (c)	102. (b)	103. (c)	104. (d)	105. (c)

Mathematics

106. (c)	107. (d)	108. (b)	109. (c)	110. (b)	111. (b)	112. (d)	113. (a)	114. (a)	115. (b)
116. (c)	117. (b)	118. (b)	119. (b)	120. (c)	121. (b)	122. (b)	123. (c)	124. (d)	125. (a)
126. (b)	127. (a)	128. (b)	129. (d)	130. (a)	131. (a)	132. (a)	133. (a)	134. (a)	135. (d)
136. (c)	137. (a)	138. (d)	139. (c)	140. (a)	141. (a)	142. (a)	143. (c)	144. (a)	145. (c)
146. (c)	147. (c)	148. (d)	149. (b)	150. (d)					

Hints & Solutions

Physics

1. ***(b)*** Rate of cooling, $R = \frac{dT}{dt} = \frac{eA\sigma}{mc}(T^4 - T_0^4)$

where, symbols have their standard meanings.

As both spheres made from same material,

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\frac{dT_1}{dt}}{\frac{dT_2}{dt}} = \frac{\frac{A_1}{m_1}}{\frac{A_2}{m_2}} = \frac{\frac{A_1}{V_1\rho}}{\frac{A_2}{V_2\rho}} = \frac{A_1}{V_1}\cdot\frac{V_2}{A_2}$$

$$= \frac{4\pi r_1^2}{\frac{4}{3}\pi r_1^3}\cdot\frac{\frac{4}{3}\pi r_2^3}{4\pi r_2^2} = \frac{r_2}{r_1} = \frac{20}{10} \Rightarrow \frac{R_1}{R_2} = 2:1$$

2. ***(d)*** First echo is heared at time, $t_1 = 2d_1/v$

where, d_1 = distance of the man from first mountain and v = speed of sound.

Second echo is heared at time, $t_2 = \frac{2d_2}{v}$

Hence, $t_1 + t_2 = \frac{2}{v}(d_1 + d_2)$

$$\Rightarrow \quad d_1 + d_2 = \frac{v}{2}(t_1 + t_2)$$

$$= \frac{340}{2} \times 4 \quad [\because t_1 + t_2 = 1.5 + 2.5 = 4\ \text{s}]$$

$$\Rightarrow \quad d_1 + d_2 = 680\ \text{m}$$

3. ***(b)*** Given, arrangement is equivalent to focal length determination by displacement method.

Let the separation between object and image plane be a and separation between two halves be d.

So, $a = 1.8\,\text{m}$

$\therefore$ Magnification $m = \dfrac{a+d}{a-d} = \dfrac{2}{1}$

$\Rightarrow \dfrac{1.8+d}{1.8-d} = \dfrac{2}{1} \Rightarrow 1.8 + d = 3.6 - 2d$

$\Rightarrow 3d = 3.6 - 1.8 \Rightarrow d = \dfrac{1.8}{3} = 0.6\,\text{m}$

We know that, focal length of the lens, $f = \dfrac{a^2 - d^2}{4a}$

$$= \frac{(1.8)^2 - (0.6)^2}{4 \times 1.8} = \frac{3.24 - 0.36}{7.2} = \frac{2.88}{7.2} = 0.4\,\text{m}$$

4. ***(a)*** The charge on the conductor Q will behave as if it is concentrated at its centre. Both the charges are located at the same point. So, the electrostatic force between them is zero.

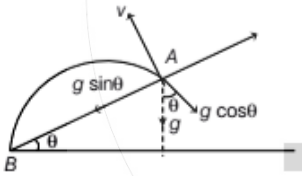
5. ***(c)*** Taking component of g, along and perpendicular to the plane, we get

Acceleration along the inclined plane (towards B) $= g\sin\theta$

Acceleration perpendicular to the inclined plane AB (in downward direction) $= g\cos\theta$

Time taken by the particle to attain maximum height is

$$t = \frac{u}{g\cos\theta}$$



$\therefore$ Time of flight, $T = 2t$

$$= \frac{2u}{g\cos\theta} \quad \ldots(\text{i})$$

$\therefore$ Distance moved by the particle along AB,

$$R = \frac{1}{2}g\sin\theta \cdot t_1^2$$

$$= \frac{1}{2}g\sin\theta \cdot \frac{4u^2}{g^2\cos^2\theta} \quad \text{[using Eq. (i), where } t_1 = T\text{]}$$

$$= \frac{\sin\theta \cdot 2u^2}{g\cos^2\theta}$$

$$= \frac{2u}{g\cos\theta} \cdot \frac{u}{\cos\theta} \cdot \sin\theta$$

$\Rightarrow R = T \cdot \tan\theta$

6. ***(d)*** Since, the length of the pendulum is not changed therefore its time period T as well as angular frequency ω remains unchanged.

Now, $v_{max} = A\omega \Rightarrow v_{max} \propto A$

Therefore, when amplitude is doubled. The maximum velocity at the mean position also gets doubled 6 cm/s.

7. ***(b)*** Radius of the circular path in magnetic field B is given by

$$r = \frac{mv}{qB}$$

$$\left[\because \text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = E \right.$$

$$\left. \Rightarrow m^2v^2 = 2mE \Rightarrow mv = \sqrt{2mE}\right]$$

$\therefore \quad r = \dfrac{\sqrt{2mE}}{qB}$

$\therefore \quad \dfrac{r_\alpha}{r_p} = \dfrac{\sqrt{2m_\alpha E}/q_\alpha B}{\sqrt{2m_p E}/q_p B}$

$$= \sqrt{\frac{m_\alpha}{m_p}} \cdot \frac{q_p}{q_\alpha} = \sqrt{\frac{4 \cdot m_p}{m_p}} \cdot \frac{q_p}{2q_p} \quad \left\{\because \begin{matrix} m_\alpha \simeq 4m_p \\ \text{and } q_\alpha = 2q_p \end{matrix}\right\}$$

$\Rightarrow \dfrac{r_\alpha}{r_p} = \dfrac{2}{2} = 1$

$\therefore \quad r_\alpha = r_p = r_0$

8. ***(a)*** The distance between zeroth order maxima and second order minima is

$$y_1 = \frac{\beta}{2} + \beta = \frac{3\beta}{2}$$

$$= \frac{3}{2} \times 0.2 = 0.3\ \text{mm}$$

$\therefore$ The distance of second order minima from point P is

$$y = (4.8 + 0.3) = 5.1\ \text{mm}$$

9. ***(d)*** Maximum static frictional force between the block and the slab is

From, $f = \mu_s mg$

$f_{ms} = 0.6 \times 10 \times 9.8\,\text{N} = 58.8\,\text{N}$

Since, the applied force is greater than f_{ms}.

Therefore, the block will be in motion. So, we should consider kinetic friction f_k.

$\therefore \quad f_k = \mu_k mg = 0.4 \times 10 \times 9.8$

$\Rightarrow \quad f_k = 4 \times 9.8$

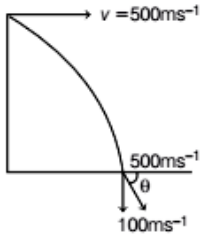
This would cause acceleration in 40 kg slab.

$\therefore$ Acceleration of the slab,

$$a = \frac{f_k}{40} = \frac{4 \times 9.8}{40} = 0.98\ \text{ms}^{-2}$$

10. ***(a)*** Horizontal component of velocity of the bomb is $v_x = 500\ \text{ms}^{-1}$ and vertical component of velocity of the bomb while striking the ground is given as

$v_y = u_y + a_y t$ [By equation of motion]
$= 0 + 10 \times 10 = 100\ \text{ms}^{-1}$



$\therefore$ Angle with which the bomb strikes the ground is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{100}{500}\right) \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

11. ***(b)*** Average value of induced emf, $\bar{e} = \frac{\Delta\phi}{\Delta t}$

$$= \frac{(NBA\cos\theta - 0)}{\Delta t}$$

$$= \frac{(1 \times 0.5 \times 25 \times 10^{-4}\cos 60° - 0)}{0.2}$$

$$\Rightarrow \qquad \bar{e} = 3.12 \times 10^{-3}\ \text{V}$$

12. ***(c)*** Stress = (weight due to mass m_2 + half of the weight of rod)/area

$$= (m_2 g + m_1 g/2)/A$$
$$= [(m_1/2) + m_2]g/A$$

13. ***(b)*** $R_{t_1} = R_0(1 + \alpha\, t_1)$ and $R_{t_2} = R_0(1 + \alpha\, t_2)$

$$\therefore \qquad \frac{R_{t_2}}{R_{t_1}} = \frac{R(1 + \alpha\, t_2)}{R(1 + \alpha\, t_1)}$$

or $$\frac{1.5}{1} = \frac{1 + 0.00125 \times t_2}{1 + 0.00125 \times 27}$$

[$\because$ Given, $t_1 = 300\ \text{K} = 300 - 273°\text{C} = 27°\text{C}$]

On solving, we get

$$t_2 = 440.5°\text{C}$$
$$= 440.5 + 273 = 713.5\ \text{K}$$

14. ***(a)*** Here, $u = 0$, $a = \frac{F}{m} = \frac{-eE}{m}$

$$\therefore \qquad v = u + at \qquad \text{(From equation of motion)}$$
$$= 0 + \frac{-eE}{m}t$$

$$\Rightarrow \qquad v = \frac{-eEt}{m}$$

de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{-m(eEt/m)} = \frac{h}{-eEt}$$

Rate of change of de-Broglie wavelength,

$$\frac{d\lambda}{dt} = \frac{d}{dt}\left(\frac{h}{-eEt}\right) = \frac{-h}{eE}\left(-\frac{1}{t^2}\right) = \frac{h}{eEt^2}$$

15. ***(c)*** Depth of point P below the free surface of water in the vessel $= (H - h)$, since the liquid exerts equal pressure in all direction at one level, hence the pressure at $P = (H - h)\rho g$.

16. ***(d)*** As, $R \propto n^2$

$$v \propto \frac{1}{n} \text{ and } E \propto \frac{1}{n^2}$$

$$\therefore \quad vR \propto \left(\frac{1}{n} \times n^2\right)$$

i.e. $vR \propto n$

17. ***(d)*** Current gain of CE amplifier, $\beta = \frac{I_C}{I_B}$

$$\therefore \quad \beta = \frac{98\%\text{ of } qt}{2\%\text{ of } qt} = \frac{98\%\text{ of net}}{2\%\text{ of net}}$$

$$= \frac{(98\%) \times 10^{10} \times 1.6 \times 10^{-19} \times 10^{-6}}{(2\%) \times 10^{10} \times 1.6 \times 10^{-19} \times 10^{-6}} = 49$$

18. ***(b)*** Initial velocity, $u = (\hat{i} + 2\hat{j})$ m/s

Magnitude of velocity, $u = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$ m/s

Equation of trajectory of projectile,

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta) \quad \left(\because \tan\theta = \frac{u_y}{u_x} = \frac{2}{1} = 2\right)$$

$$= x \times 2 - \frac{10x^2}{2(\sqrt{5})^2}[1 + (2)^2]$$

$$= 2x - \frac{10x^2}{2 \times 5}(1 + 4) = 2x - 5x^2$$

19. ***(a)*** For diatomic gas, number of degrees of freedom per molecules is $n = 5$.

Average kinetic energy of diatomic gas due to thermal motion is

$$E = \frac{5}{2}(nR)T$$
$$= \frac{5}{2}pV \qquad (\because pV = nRT)$$
$$= \frac{5}{2} \times p \times \frac{m}{\rho} \qquad \left(\because V = \frac{m}{\rho}\right)$$
$$= \frac{5}{2} \times \frac{(8 \times 10^4) \times 1}{4}$$

$$\Rightarrow \qquad E = 5 \times 10^4\ \text{J}$$

20. ***(b)*** As, $\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2} = 9$

$$\Rightarrow \qquad \frac{a+b}{a-b} = 3$$

$$\Rightarrow \qquad 3a - 3b = a + b \Rightarrow a = 2b$$

$$\therefore \qquad \frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{4b^2}{b^2} = 4:1$$

21. ***(c)*** Force acting on the particle is given by

$$F = -\frac{dU}{dx} = -\frac{d}{dx}(U_0 - U_0 \cos ax)$$

$$= -aU_0 \sin ax$$

For small oscillations,

$$\sin ax = ax$$

$$\therefore \quad F \simeq -a\,U_0(a \cdot x) = -a^2U_0x$$

$$\therefore \text{Acceleration, } \alpha = -\frac{a^2U_0}{m} \cdot x \qquad \ldots(i)$$

as $\alpha \propto -x$

Motion is simple harmonic motion.

Standard equation of SHM is $\alpha = -\omega^2 x \qquad \ldots(ii)$

∴ Comparing Eqs. (i) and (ii) we get

$$\omega^2 = \frac{a^2U_0}{m}$$

$$\therefore \text{Time period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{a^2U_0}}$$

or $$T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$$

22. ***(b)*** From work-energy theorem,

$$\text{Kinetic energy} = Fs = (qE) \cdot \left(\frac{1}{2}at^2\right) \qquad \ldots(i)$$

where, $$a = \frac{F}{m} = \frac{qE}{m}$$

Put in Eq. (i), we get

$$\therefore \quad \text{KE} = (qE) \cdot \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

or $$\text{KE} = \frac{1}{2} \cdot \frac{q^2E^2t^2}{m}$$

23. ***(b)*** Effective spring constant of the three springs is

$$k_{\text{eff}} = \frac{k \times 2k}{k + 2k} = \frac{2}{3}k$$

$$\therefore \text{Time period, } T = 2\pi\sqrt{\frac{m}{(2k/3)}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{3m}{2k}}$$

24. ***(c)*** If a sample decays simultaneously through emission of two particles, then the net decay constant

$$\lambda = \lambda_1 + \lambda_2$$

$$\frac{\log_e 2}{T} = \frac{\log_e 2}{T_1} + \frac{\log_e 2}{T_2}$$

or $$\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$

or $$T = \frac{T_1\,T_2}{T_1 + T_2}$$

25. ***(a)*** From Newton's formula,

$$x_1\,x_2 = f^2$$

Given, $$x_1 = p$$

$$\therefore \quad x_2 = \frac{f^2}{p}$$

Also, $$u = f + x_1$$

and $$v = f + x_2$$

$$\therefore \text{Magnification, } m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{v}{u} = \frac{f + x_2}{f + x_1}$$

$$= \frac{f + f^2/p}{f + p} = f/p$$

26. ***(c)*** In steady state, there is no current in capacitor branch.

Net voltage of remaining circuit = 2 V − V = V

Net resistance = $R + 2R = 3R$

Current in the circuit, $i = V/3R$

Potential across lowest row, $V' = 2V - i \cdot 2R$

$$\Rightarrow \quad V' = 2V - \frac{V}{3R} \cdot 2R = \frac{4}{3}V$$

The right end terminal being at higher potential. The potential difference across each row is same.

∴ Potential difference across capacitor,

$$V_C = V' - V$$

$$\Rightarrow \quad V_C = \frac{4}{3}V - V = \frac{V}{3}$$

27. ***(d)*** Initial acceleration of block A is

$$a_1 = \frac{F}{M} = \frac{3}{4} = 0.75 \text{ m/s}^2$$

No horizontal force acts on block B.

∴ Acceleration of block B is zero.

28. ***(c)*** Moment of inertia of the rod, $I_1 = \frac{ML^2}{12}$

Moment of inertia of the ring, $I_2 = MR^2$

$$\therefore \quad \frac{I_1}{I_2} = \frac{ML^2/12}{MR^2} = \frac{L^2}{12R^2}$$

Given, length of rod = circumference of the ring

$$L = 2\pi R$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{(2\pi R)^2}{12R^2} = \frac{\pi^2}{3}$$

29. ***(a)*** From hydrogen spectra, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For second excited state,

$$n_2 = 3, n_1 = 1, \lambda \to \lambda_3$$

$$\frac{1}{\lambda_3} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8}{9}R$$

Similarly, for first excited state,

$$n_2 = 2,\ n_1 = 1,\ \lambda \to \lambda_2$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R$$

$$\therefore \quad \frac{\lambda_3}{\lambda_2} = \frac{3/4}{8/9} = \frac{27}{32}$$

30. ***(c)*** No magnetic force acts on a closed current carrying circular coil in a uniform magnetic field.

$$\therefore \quad F = 0$$

31. ***(c)*** Let $r_1 = 3$ cm, $r_2 = 4$ cm be the radii of two bubbles.

As bubbles coalesce in vacuum, then the total pressure of air inside the bubble is same as excess pressure $p = 4T/r$.

As process is isothermal, so we have from Boyle's law

$$p_1V_1 + p_2V_2 = pV \qquad \ldots(i)$$

If r is the radius of coalesced system, then equation gives

$$\left(\frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(\frac{4T}{r_2}\right)\frac{4}{3}\pi r_2^3 = \frac{4T}{R}\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \quad r_1^2 + r_2^2 = r^2$$

or $$r = \sqrt{r_1^2 + r_2^2}$$

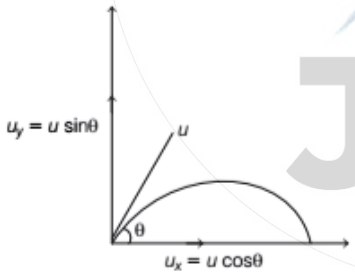
or $$r = \sqrt{3^2 + 4^2} = \sqrt{25}$$

$$\Rightarrow \quad r = 5 \text{ cm}$$

32. ***(c)*** Given, $u_x = a$

and $u_y = b$

$$\therefore \text{ Range, } \quad R = \frac{u^2 \sin 2\theta}{g}$$



$$= 2\frac{(u\cos\theta)(u\sin\theta)}{g} = \frac{2u_x \cdot u_y}{g}$$

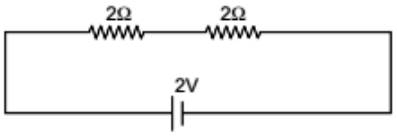
$$\Rightarrow \quad R = \frac{2ab}{g}$$

Maximum height, $H = \dfrac{u^2\sin^2\theta}{2g} \Rightarrow H = \dfrac{b^2}{2g}$

Given, $R = 2H$

$$\Rightarrow \quad \frac{2ab}{g} = 2 \times \frac{b^2}{2g} \Rightarrow b = 2a$$

33. ***(c)*** In steady state, capacitor does not allow DC current to flow, therefore equivalent circuit is as shown in figure.



$$\therefore \text{ Power, } \quad P = \frac{V^2}{R} = \frac{(2)^2}{4} \quad [R_{eq} = 2 + 2 = 4\Omega]$$

$$\Rightarrow \quad P = 1 \text{ W}$$

34. ***(c)*** From law of conservation of linear momentum,

$$mu = -mv_1 + Mv_2 \qquad \ldots(i)$$

Remaining energy of α-particle,

$$\frac{1}{2}mv_1^2 = 0.25 \times \frac{1}{2}mu^2$$

$$v_1 = 0.5\ u \qquad \ldots(ii)$$

Kinetic energy delivered to nucleus,

$$\frac{1}{2}Mv_2^2 = 0.75 \times \frac{1}{2}mu^2$$

$$v_2 = \sqrt{0.75\frac{m}{M}} \cdot u$$

Substituting v_1 and v_2 in Eq. (i), we get

$$mu = -m \times 0.5\,u + \sqrt{0.75mM} \cdot u$$

On solving, we get $(1.5\ m)^2 = 0.75mM$

$$\Rightarrow \quad M = \frac{(1.5)^2}{0.75}m = 3m$$

35. ***(c)*** The point marked on the ruler at a distance $l/2$ is the centre of mass of ruler. Now, this point is raised by a distance $\left(\dfrac{l}{2} - h\right)$ change in PE of the whole body

$= \Delta U$

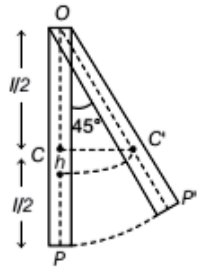
As, $$h = \frac{l\cos 45°}{2}$$

$$\Rightarrow \quad \Delta U = mg\left(\frac{l}{2} - h\right) \qquad \ldots(i)$$

$$\therefore \quad \Delta U = mg\left(\frac{l}{2} - \frac{l}{2}\cos 45°\right)$$

$$\Delta U = \frac{mg}{2}\left(1 - \frac{1}{\sqrt{2}}\right) \qquad [\because l = 1\text{m}]$$

From the given figure, C is the point of centre of mass,



36. ***(a)*** Electric field between the plates, $E = \frac{V}{d}$

$$\Rightarrow \quad d_{\min} = \frac{V}{E_{\max}} = \frac{V}{\text{Dielectric strength}}$$

$$= \frac{20}{3 \times 10^6} = 6.6 \times 10^{-6} \text{ m}$$

This shows that the value of d in alternative (a) is allowed.

Now, $\quad C = \frac{K\varepsilon_0 A}{d} \Rightarrow \frac{A}{d} = \frac{C}{K\varepsilon_0}$

$$\Rightarrow \quad \frac{A}{d} = \frac{1.77 \times 10^{-6}}{200 \times 8.85 \times 10^{-12}} = 10^3$$

or $\quad A = 10^{-3}$ m

37. ***(a)*** In given graph, $a > 0$, so velocity increases.

Change in velocity = v-u = area enclosed by graph

$$= \frac{1}{2} \times 10 \times 11 = 55 \text{ m/s}$$

As $u = 0$

So, maximum velocity, $v = 55$ m/s

38. ***(b)*** The angular momentum,

$$L = mvR_0 = m\sqrt{\frac{GM}{R_0}} \cdot R_0 = m\sqrt{\frac{GM}{R_o}} \cdot R_o^2$$

$$\therefore \quad L = m\sqrt{GMR_0}$$

39. ***(c)*** $\because$ Frequency,

$$n = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

where, T = tension in wire

and $\quad \mu$ = linear mass density.

Density of mass, $\rho = \frac{m}{v} = \frac{50.7}{0.0075} = 6760 \text{ kg/m}^3$

$$\therefore \quad \frac{n_1}{n_2} = \frac{\frac{1}{2l}\sqrt{\frac{T_1}{\mu}}}{\frac{1}{2\lambda}\sqrt{\frac{T_2}{\mu}}} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{mg}{mg - B}}$$

where, B = force of buoyancy,

$$= \sqrt{\frac{\rho Vg}{(\rho V - \rho_1 V)g}} = \sqrt{\frac{\rho}{\rho - \rho_1}}$$

where, ρ = density of mass

ρ_1 = density of water

$= 10^3 \text{ kg/m}^3$

$$\Rightarrow \quad \frac{n_1}{n_2} = \sqrt{\frac{6760}{6760 - 1000}}$$

$$= \sqrt{\frac{6760}{5760}} = \sqrt{1.1736} = 1.0833$$

$$\therefore \quad n_2 = \frac{n_1}{1.0833} = \frac{260}{1.0833} = 240 \text{ Hz}$$

40. ***(a)*** The force acting on current carrying wire,

$$F = iBl\sin\theta$$

$$\therefore \quad 7.5 = 5 \times 2 \times 1.5 \sin\theta$$

$$\sin\theta = \frac{7.5}{2 \times 5 \times 1.5} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

Chemistry

41. ***(d)*** Molecular mass of $(CHCOO)_2Fe = 170$

$\therefore$ In 100 mg $(CHCOO)_2Fe$, iron present $= \frac{56}{170} \times 100$

$= 32.9$ mg

Since, this quantity of Fe is present in 400 mg of capsule.

$\therefore$ % of Fe in capsule $= \frac{32.9}{400} \times 100 = 8.2 = 8\%$

42. ***(c)*** $\Delta E_I = E_2 - E_1$

$$= \frac{E_H}{(n_2)^2} - \frac{E_H}{(n_1)^2} = E_H\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = -\frac{E_H \times 3}{4}$$

$$\Delta E_{II} = E_3 - E_2 = E_H\left[\frac{1}{3^2} - \frac{1}{2^2}\right] = -E_H \times \frac{5}{36}$$

$$\frac{\Delta E_I}{\Delta E_{II}} = E_H \times \frac{3}{4} \times \frac{36}{E_H \times 5} = \frac{108}{20} = \frac{27}{5}$$

43. ***(b)*** Hydrogen like halogen accepts an electron readily to achieve nearest inert gas configuration.

44. ***(b)***

Species	Electrons in central element	Electrons in other element	Charge gained	Total electrons
BO_3^{3-}	5	$3 \times 8 = 24$	+ 3	32
CO_3^{2-}	6	$3 \times 8 = 24$	+ 2	32
NO_3^-	7	$3 \times 8 = 24$	+ 1	32
SO_3^{2-}	16	$3 \times 8 = 24$	+ 2	42
CN^-	6	7	+1	14
N_2	7	7	0	14
C_2^{2-}	6	6	+ 2	14
PO_4^{3-}	15	$4 \times 8 = 32$	+ 3	50
SO_4^{2-}	16	$4 \times 8 = 32$	+ 2	50
ClO_4^-	17	$4 \times 8 = 32$	+ 1	50

Thus, SO_3^{2-}, CO_3^{2-} and NO_3^- are not isoelectronic species.

45. ***(c)*** $\mu_{cal} = e \times d = 1.6 \times 10^{-19} \times 2.29 \times 10^{-10}$

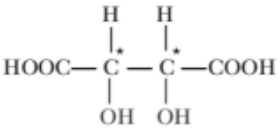
$= 3.664 \times 10^{-29}$ C-m

$$\% \text{ ionic character} = \frac{\mu_{exp}}{\mu_{cal}} \times 100$$

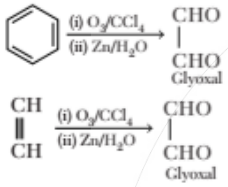
$$= \frac{6.226 \times 10^{-30}}{3.664 \times 10^{-29}} \times 100$$

$= 16.9\% \simeq 17\%$

46. ***(b)*** In the given compound, there are two chiral carbon atoms. i.e. two carbon atoms are asymmetric.



47. ***(d)***



Glyoxal

Ethene on reductive ozonolysis gives two molecules of formaldehyde.

$$H_2C{=}CH_2 \xrightarrow[(ii)\ Zn/H_2O]{(i)\ O_3/CCl_4} 2HCHO$$

48. ***(d)*** $M^{2+} \longrightarrow M^{3+}$, after the removal of second electron, the effective nuclear charge per electron increases due to which high energy is required to remove third electron.

49. ***(b)*** Boiling point of alcohols and phenols increases with increase in the number of carbon atoms (increase in van der Waals' forces). In alcohols, the boiling points decrease with increase of branching in carbon chain (because of decrease in van der Waals' forces with decrease in surface area).

50. ***(a)*** NaOH reacts with Al to evolve dihydrogen gas. The higher pressure of the hydrogen gas can be used to open drains.

$$2Al(s) + 2NaOH(aq) + 2H_2O(l) \longrightarrow 2NaAlO_2(aq) + 3H_2(g)$$

51. ***(b)*** Because the number of current carrying particles, i.e. ions present per centimetre cube of the solution become lesser on dilution. Hence, the conductivity of strong electrolytes decreases on dilution.

52. ***(d)*** 1.00 kg of solution contains 0.100 kg of CH_3OH and 0.900 kg H_2O.

Number of moles in 100 g of

$$CH_3OH = \left(\frac{100}{32.0}\right) = 3.12 \text{ mol}$$

$$\therefore \text{Molality of } CH_3OH \text{ in 0.90 kg } H_2O = \frac{3.12 \text{ mol}}{0.900 \text{ kg}}$$

$= 3.47$ m

$\Delta T_f = K_f \times m = (1.86^\circ \text{ C/m})(3.47 \text{ m}) = 6.45^\circ\text{C}$

Freezing point of the solution $(T_f) = T_f^\circ - \Delta T_f$

$= 0 - 6.45 = -6.45^\circ\text{C}$

53. ***(a)*** Metals having very high tendency to lose electrons are used in photoelectric cells. Lower the ionisation energy, higher is the tendency to lose electrons.

Potassium and cesium have much lower ionisation enthalpy (IE) than that of lithium. Therefore, these metals on exposure to light emit electrons easily but lithium does not. That's why, K and Cs rather than Li are used in photoelectric cells.

54. ***(b)*** $RCH_2Cl + NaI \xrightarrow[\Delta]{\text{Acetone}} RCH_2I + NaCl$

This reaction is known as Finkelstein reaction or halogen exchange reaction.

All other reactions are not feasible.

55. ***(a)*** Copper matte contains Cu_2S and FeS.

56. ***(d)*** If the reaction is an elementary reaction, then order and molecularity have same value.

57. ***(c)*** Dolomite = $MgCO_3 \cdot CaCO_3$

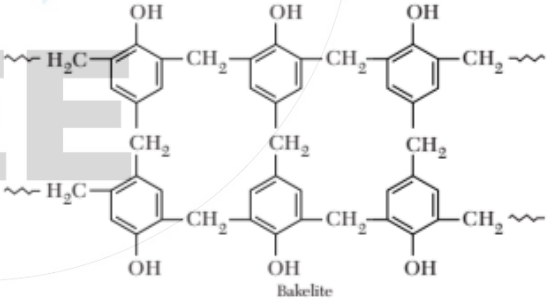
Magnesite = $MgCO_3$

Carnallite = $KCl \cdot MgCl_2 \cdot 6H_2O$

Therefore, Mg metal is extracted from the above minerals.

58. ***(b)*** In the complex, $K_2Fe[Fe(CN)_6]$, both the iron (Fe) atoms are present in the same oxidation state, i.e. +2.

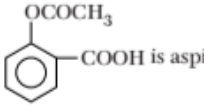
59. ***(a)*** Novolac on heating with formaldehyde undergoes cross-linking to form an infusible solid mass called bakelite.



Bakelite

60. ***(c)*** In diamond, the constituent atoms are held together by strong covalent bonds. It is a giant molecule. Thus, classified as network solid.

Note SO_2, H_2O *and* I_2 *are examples of molecular solids.*

61. ***(c)***  is aspirin. It is an acetylated product of salicylic acid and used as an analgesic.

62. ***(b)*** Thymine is not present in RNA.

As DNA contains four bases *viz.* adenine ***(A)***, guanine (G), cytosine (C) and thymine (T). RNA also contains four bases, the first three bases are same as in DNA but the fourth one is uracil (U).

63. ***(b)*** In Duma's method, % of nitrogen

$$= \frac{28 \times V \times 100}{22400 \times W}$$

Given, $W = 0.3$ g, $V = 45$ mL

$$\% \text{ Nitrogen} = \frac{28 \times 45 \times 100}{22400 \times 0.3} = 18.75$$

64. ***(a)*** Greater the conjugation, greater the stability and hence, easier the dehydration.

65. ***(b)*** Number of spherical nodes in $3p$-orbital

$$= n - l - 1 = 3 - 1 - 1 = 1$$

66. ***(c)*** Sc = [Ar] $3d^1\ 4s^2$

$n = 4$

Atomic number of element = 21

Group = total number of valence shell electrons = 3

So, the element belongs to group-3, period-4.

67. ***(d)*** CO (14 electrons)

CO

$= \sigma 1s^2, \overset{*}{\sigma}\ 1s^2, \sigma 2s^2, \overset{*}{\sigma}\ 2s^2, \pi\ 2p_x^2 = \pi 2p_y^2, \sigma\ 2p_z^2$

σ-bonding molecular orbital is HOMO.

68. ***(c)*** We know that, internal energy of a gas depends upon its pressure and temperature. Thus, if a gas expands at constant temperature and pressure then its internal energy remains same.

69. ***(a)*** Conjugate base is formed by loss of H^+.

$$OH^- \longrightarrow \underset{\text{(Conjugate base)}}{O^{2-}} + H^+$$

O^{2-} is the conjugate base of OH^-.

70. ***(d)*** In adsorption due to forces of attraction, ΔH is negative (heat is released) and as the particles come closer, entropy of the system decreases, i.e. ΔS is negative, hence $T\Delta S$ is also negative.

71. ***(d)*** Formula of gypsum = $CaSO_4 \cdot 2H_2O$

Formula of plaster of Paris = $CaSO_4 \cdot \frac{1}{2} H_2O$

Difference of water molecules = $2H_2O - \frac{1}{2} H_2O$

$$= 1\frac{1}{2} H_2O$$

72. ***(a)*** Boron when treated with carbon forms boron carbide.

$$4B + C \xrightarrow{\Delta} B_4C$$

73. ***(b)*** Cyclodecapentene and cyclooctatetraene both are non-aromatic. Both follow the Huckel rule of aromaticity but their rings are not flat. Cyclobutadiene is anti-aromatic. While benzene having 6π-electrons is aromatic.

74. ***(d)*** $\underset{(A)}{\log K_{eq}} = -\underbrace{\frac{\Delta H^\circ}{2.303\ RT} + \frac{\Delta S^\circ}{R}}_{(B)}$

Slope of the given line indicating that term A is positive, thus ΔH° is negative. Thus, reaction is exothermic.

75. ***(a)*** Clathrates are non-stoichiometric compounds where the ratio of guest and host molecules does not correspond to ideal chemical formula.

76. ***(a)*** Due to availability of unpaired d-electrons, d-d transition occurred and hence, transition elements are coloured.

77. ***(a)*** The electronic configuration of $_{62}Sm^{3+}$ is $[Xe]4f^5 5d^0 6s^0$ and that of $_{66}Dy^{3+}$ is $[Xe]4f^9 5d^0 6s^0$. The colour of f^n and f^{14-n} are often identical, i.e. yellow.

78. ***(c)*** Phenol because of the resonance stabilisation of phenoxide ion, have higher tendency to give a proton instead of accepting. It is the weakest Bronsted base or strongest Bronsted acid.

79. ***(c)***

(a) 1g Au $= \frac{1}{197}$ mol atoms of Au

$$= \frac{1}{197} \times 6.022 \times 10^{23} \text{ atoms of Au}$$

(b) 1g Na $= \frac{1}{23}$ mol atoms of Na

$$= \frac{1}{23} \times 6.022 \times 10^{23} \text{ atoms of Na}$$

(c) 1g Li $= \frac{1}{7}$ mol atoms of Li

$$= \frac{1}{7} \times 6.022 \times 10^{23} \text{ atoms of Li}$$

(d) 1g $Cl_2 = \frac{1}{71}$ mol molecules of Cl_2

$$= \frac{1}{71} \times 6.022 \times 10^{23} \text{ molecules of } Cl_2$$

$$= \frac{2}{71} \times 6.022 \times 10^{23} \text{ atoms of Cl}$$

($\because$ 1 molecule of chlorine contains 2 atoms)

Therefore, 1 g of Li has largest number of atoms.

Note If a substance exists in atomic form, it contains mol atoms and if a substance exists in molecular form, it contains mol molecules.

80. ***(b)*** We know that, density $(d) = pM/RT$

$$d \propto \frac{1}{T} \text{ and } d \propto p$$

Thus, density of neon is maximum at 0°C and 2 atm.

a. English Proficiency

81. ***(a)*** 'Traced' means 'searched'. Hence, 'searched' is the correct choice.

82. ***(a)*** 'Gargantuan' means 'gigantic' or 'colossal' which has a sense of being too big.

83. ***(b)*** 'Propitiate' means 'to appease' or 'to bring favourable remarks'.

84. ***(a)*** 'Run down' is a phrasal verb which means 'to be crushed' or 'to be trampled' which makes the sense here.

85. ***(c)*** The event is of past tense. So, the verb must be used in past tense. Hence, 'was taken' is correct choice. Also, the mode is passive.

86. ***(d)*** 'Wages' is the correct choice here. Salary, pay are not suitable usage because in agricultural context, it is improper to be used. These are basically used as corporate terms. But, 'wages' is suitable here.

87. ***(c)*** 'A man of bad reputation' is called to be 'notorious'. Hence, the opposite will be 'famous'.

88. ***(b)*** 'Perishable' means 'which has a destined end'. Hence, its opposite will be 'immortal' which means 'that will never end'.

89. ***(c)*** 'Penury' means 'the state of being poor'. Hence, the opposite will be 'opulence' which means 'having no scarcity' or 'being in plenty'.

90. ***(d)*** One who does not consume alcohol is called 'teetotaller'.

91. ***(a)*** 'Hermaphrodite' is an animal that has both sex inside its body.

92. ***(c)*** A text that doesn't bear the name of an author is called 'unanimous'.

Q. Nos. 93 to 95

On rearranging the given sentences, the correct and logical order of the sentences will be CAFDBF. Taking the reference, we can answer the questions

93. ***(d)*** D **94.** ***(b)*** B

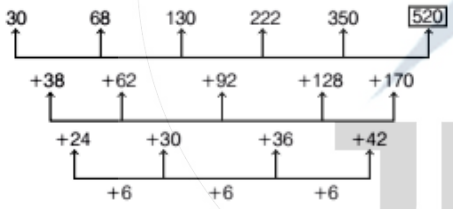
95. ***(d)*** E

b. Logical Reasoning

96. ***(c)*** 'Ribs' protect the 'Lungs'. Similarly, 'Shell' protects the 'Nut'.

97. ***(d)*** All except 18, 36, 40 have second number double the first number and third number 6 more than the second number.

98. ***(c)*** The pattern is as follows,



$\therefore$? = 520

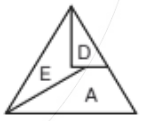
99. ***(c)*** The third figure in each row comprises of parts which are not common to the first two figures.

100. ***(b)*** Figure (X) is the first step in which a circular piece of paper is folded from upper to the lower half along the diameter. In figure (Y), both the extreme ends of the figure (X) (after folding) have been folded to form a triangle and then as given in figure (Z), a cut has been marked on the right side.

It is clear that this cut will result into two marks, one in the lower half and one in the upper half of the paper, when it is unfolded. Answer figure (b), represents the unfolded shape of the paper after the cut.

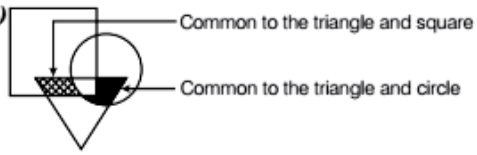
101. ***(c)*** As done in above example, we have to closely approximate the different formations. So, first we combine the figures (E) and (A) which will look like as

Now, the missing figure which will complete the above figure is figure (D). The complete figure will be as



102. ***(b)*** All the four parts are similar except for a diagonal line passing through the upper right and lower left parts. Also, each part comprises of a quarter circle in such a way that there is a semi-circle on a pair of opposite sides of the question figure. Clearly, answer figure (b) completes the pattern.

103. ***(c)***



Hence, figure shown in option (c) is correct figure.

104. ***(d)***

Volumes	Green cover	Yellow cover	New volume	Old volume	Law reports	Medical reports
A	—	✓	✓	—	✓	—
B	✓	—	✓	—	✓	—
C	✓	—	—	✓	✓	—
D	—	✓	✓	—	—	✓
E	✓	—	—	✓	—	✓
F	✓	—	—	✓	—	✓

Clearly, E and F are old volumes which have green covers and are medical extracts.

105. ***(c)*** According to question in figure 3, closed figures gradually become open and open figure gradually become closed.

Mathematics

106. ***(c)*** We have,

$$\lim_{x\to a}\left(2-\frac{a}{x}\right)^{\tan\frac{\pi x}{2a}} \qquad [\because \text{form } 1^{\infty}]$$

$$= \lim_{x\to a}\left\{1+\left(1-\frac{a}{x}\right)\right\}^{\tan\frac{\pi x}{2a}} = e^{\lim\limits_{x\to a}\left(\frac{x-a}{x}\right)\tan\frac{\pi x}{2a}}$$

$$= e^{\lim\limits_{h\to 0}\left(\frac{h}{a+h}\tan\frac{\pi}{2a}(a+h)\right)} = e^{\lim\limits_{h\to 0}\left(\frac{h}{a+h}\tan\left(\frac{\pi}{2}+\frac{\pi h}{2a}\right)\right)}$$

$$= e^{\lim\limits_{h\to 0}\left(-\frac{h}{a+h}\cot\frac{\pi h}{2a}\right)} = e^{\lim\limits_{h\to 0}\left(\frac{-h}{a\tan\pi h/2a}\right)}$$

$$= e^{\lim\limits_{h\to 0}\left(-\frac{2}{\pi}\left\{\frac{\frac{\pi h}{2a}}{\tan\pi h/2a}\right\}\right)} = e^{-2/\pi}$$

107. ***(d)*** We have,

$$|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$$

$$= \left|z_1z_2z_3\left(\frac{8}{z_1}+\frac{27}{z_2}+\frac{64}{z_3}\right)\right|$$

$$= |z_1z_2z_3|\left|\frac{8}{z_1}+\frac{27}{z_2}+\frac{64}{z_3}\right|$$

$$= |z_1||z_2||z_3|\left|\frac{8\bar{z}_1}{|z_1|^2}+\frac{27\bar{z}_2}{|z_2|^2}+\frac{64\bar{z}_3}{|z_3|^2}\right|$$

$$= 24\left|2\bar{z}_1+3\bar{z}_2+4\bar{z}_3\right|$$

$$= 24\left|\overline{2z_1+3z_2+4z_3}\right|$$

$$= 24\times 4 = 96$$

108. ***(b)*** We have,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -2-\omega^2 & \omega^2-1 \\ 1 & \omega^2-1 & \omega^4-1 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$]

$$\Rightarrow \Delta = (-2-\omega^2)(\omega-1)-(\omega^2-1)^2 \quad [\because \omega^3 = 1]$$

$$\Rightarrow \Delta = -(2\omega+\omega^3-2-\omega^2)-(\omega^4-2\omega^2+1)$$

$$\Rightarrow \Delta = -(2\omega-1-\omega^2)-(\omega-2\omega^2+1)$$

$$\Rightarrow \Delta = -(2\omega+\omega)-(-3\omega^2) = -3\omega+3\omega^2 = 3\omega(\omega-1)$$

109. ***(c)*** Let T_r be the rth term of the given series. Then,

$$T_r = \frac{1}{(2r-1)(2r+1)},\ r = 1, 2, 3 \ldots\ldots n$$

$$\Rightarrow \quad T_r = \frac{1}{2}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$$

Required sum S is given by

$$S = \sum_{r=1}^{n} T_r$$

$$\Rightarrow \quad S = \frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\right.$$

$$\left. \ldots\ldots+\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)\right]$$

$$\Rightarrow \quad S = \frac{1}{2}\left[1-\frac{1}{2n+1}\right] = \frac{n}{2n+1}$$

110. ***(b)*** We have, $f(x) = |\sin x| - |\cos x|$

$$\therefore \quad f(x+\pi) = |\sin(x+\pi)| - |\cos(x+\pi)|$$

$$\Rightarrow \quad f(x+\pi) = |-\sin x| - |-\cos x|$$

$$\Rightarrow \quad f(x+\pi) = |\sin x| - |\cos x| = f(x) \text{ for all } x \in R$$

$$\Rightarrow \quad f(x+\pi) = f(x) \text{ for all } x \in R$$

So, $f(x)$ is periodic with period π.

111. ***(b)*** We have, $f(x) = x^3 - x^2 + x + 1$

$$f'(x) = 3x^2 - 2x + 1 > 0 \text{ for all } x$$

[$\because$ Discriminant < 0 and coefficient of $x^2 > 0$]

$\Rightarrow$ $f(x)$ is increasing for all $x \in R$.

$$\therefore \quad g(x) = \begin{cases} x^3 - x^2 + x + 1, & 0 \le x \le 1 \\ 3 - x, & 1 < x \le 2 \end{cases}$$

Clearly, $g(x)$ is everywhere continuous and differentiable except possible at $x = 1$.

Clearly, $\lim\limits_{x \to 1^-} g(x) = \lim\limits_{x \to 1^+} g(x) = g(1)$

So, it is continuous at $x = 1$.

We observe that

(LHD at $x = 1$)

$$= \frac{d}{dx}(x^3 - x^2 + x + 1)\}_{x=1}$$

$$= (3x^2 - 2x + 1)_{\text{at } x=1} = 2$$

and (RHD at $x = 1$) $= \frac{d}{dx}(3 - x)_{x=1} = -1$

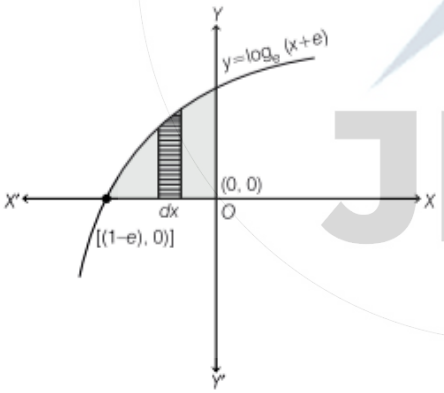
Clearly, $g(x)$ is not differentiable at $x = 1$.

112. ***(d)*** The area enclosed by $y = \log_e (x + e)$ and the coordinate axes is shaded in figure.

Let A be the required area. Then,

$$A = \int_{1-e}^{0} y\,dx = \int_{1-e}^{0} \log_e (x + e)\,dx$$

$$\Rightarrow \quad A = [x \log_e (x + e)]_{1-e}^{0} - \int_{1-e}^{0} \frac{x}{x + e}\,dx$$



We have to find the area of shaded region.

$$A = [x \log_e (x + e)]_{1-e}^{0} - [x - e \log_e (x + e)]_{1-e}^{0}$$

$\Rightarrow$ $A = 0 - [(0 - e) - (1 - e) + 0] = 1$ sq. unit

113. ***(a)*** The divisibility of the product of four numbers depends upon the value of the last digit of each number. The last digit of a number can be any one of the ten digits 0, 1, ... 9. So, the total number of ways of selecting last digit of four numbers is $10 \times 10 \times 10 \times 10 = 10^4$

If the product of the 4 numbers is not divisible by 5 or 10, then the number of choices for the last digit of each number is 8 (excluding 0 or 5).

So, favourable number of ways $= 8^4$

$\therefore$ The probability that the product is not divisible by 5 or 10 is $\left(\frac{8}{10}\right)^4$.

Hence, required probability $= 1 - \left(\frac{8}{10}\right)^4 = \frac{369}{625}$

114. ***(a)*** Clearly, X is a binomial variate with parameters n and $p = 1/2$, such that

$$P(X = r) = {}^nC_r\, P^r q^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n$$

Now,

$P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in AP.

$\Rightarrow$ $2P(X = 5) = P(X = 4) + P(X = 6)$

$\Rightarrow$ $2\,{}^nC_5 \left(\frac{1}{2}\right)^n = {}^nC_4 \left(\frac{1}{2}\right)^n + {}^nC_6 \left(\frac{1}{2}\right)^n$

$\Rightarrow$ $2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow$ $2\frac{n!}{(n-5)!5!} = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$

$\Rightarrow$ $\frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{6 \times 5}$

$\Rightarrow$ $n^2 - 21n + 98 = 0$

$\Rightarrow (n - 7)(n - 14) = 0 \Rightarrow n = 7, 14$

115. ***(b)*** For any $a \in N$, we have

(GCD of a and a) $= a$

So, R is not reflexive.

Let $(a, b) \in R$. Then,

$a\ R\ b$

$\Rightarrow$ GCD of a and b is 2.

$\Rightarrow$ GCD of b and a is 2.

$\Rightarrow$ $b\ R\ a$

So, R is symmetric.

We observe that,

GCD of 6 and 4 is 2 and GCD of 4 and 18 is also 2.

But GCD of 6 and 18 is 6.

i.e. $6\,R\,4$ and $4\,R\,18$ but $6 \not R\ 18$.

So, R is not transitive.

116. ***(c)*** We have,

In the expansion of $(1 + \alpha x)^4$, There are 5 terms.

middle term $T_3 = {}^4C_2 (\alpha x)^2 = 6\alpha^2 x^2$

In the expansion of $(1 - \alpha x)^6$, There are 7 terms.

middle term $T_4 = {}^6C_3(-\alpha x)^3 = -20\alpha^3 x^3$

Since, coefficient of middle term in $(1+\alpha x)^4$

= Coefficient of middle term in $(1-\alpha x)^6$

$\Rightarrow \quad 6\alpha^2 = -20\alpha^3$

$\Rightarrow \quad \alpha \neq 0, \alpha = \dfrac{-3}{10}$

117. ***(b)*** Given equation is $2\cos x \sin x - 2\cos^2 x = n$

$\Rightarrow \quad \sin 2x - 1 - \cos 2x = n \quad [\because \sin 2x = 2\sin x \cos x]$

$\Rightarrow \quad \sin 2x - \cos 2x = n + 1, \quad -\sqrt{2} \le n+1 \le \sqrt{2}$

$\therefore$ Greatest integral value of $n = 0$

118. ***(b)*** The equation of the circle described on SS' as a diameter is

$$(x-ae)(x+ae) + (y-0)(y-0) = 0$$

$\Rightarrow \quad x^2 - a^2e^2 + y^2 = 0 \Rightarrow x^2 + y^2 = a^2e^2$

The abscissae of the points of intersection of the ellipse and this circle are the roots of the equation.

$$\frac{x^2}{a^2} + \frac{a^2e^2 - x^2}{b^2} = 1$$

$$\Rightarrow \quad x^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = 1 - \frac{a^2e^2}{b^2}$$

$$\Rightarrow \quad x^2\left(\frac{b^2 - a^2}{a^2b^2}\right) = \frac{b^2 - a^2e^2}{b^2} \quad [\because b^2 = a^2(1-e^2)]$$

$$\Rightarrow \quad -\frac{x^2a^2e^2}{a^2} = a^2 - 2a^2e^2$$

$$\Rightarrow \quad x^2 = a^2\left(\frac{2e^2 - 1}{e^2}\right)$$

$$\Rightarrow \quad x = \pm\frac{a}{e}\sqrt{2e^2 - 1}$$

This will give distinct value of x, if $2e^2 - 1 > 0$ i.e.

$e > \dfrac{1}{\sqrt{2}}$.

Hence, $\quad e \in (1/\sqrt{2}, 1)$

119. ***(b)*** Let E be the event that A wins the game and F be the event that B wins the game. Then,

$$P(E) = \frac{6}{12} = \frac{1}{2} \text{ and } P(F) = \frac{4}{12} = \frac{1}{3}$$

$\therefore$ Required probability

$$= P\{(E \cap F \cap E) \cup (F \cap E \cap F)\}$$
$$= P\{(E \cap F \cap E) + P(F \cap E \cap F)\}$$
$$= P(E)\,P(F)\,P(E) + P(F)\,P(E)\,P(F)$$
$$= \frac{1}{2}\times\frac{1}{3}\times\frac{1}{2} + \frac{1}{3}\times\frac{1}{2}\times\frac{1}{3} = \frac{5}{36}$$

120. ***(c)*** We have,

$$x = \frac{1}{1-a},\ y = \frac{1}{1-b},\ z = \frac{1}{1-c}$$

Since, a, b, c are in AP.

$\Rightarrow \quad 1-a, 1-b, 1-c$ are in AP.

$\Rightarrow \quad \dfrac{1}{1-a}, \dfrac{1}{1-b}, \dfrac{1}{1-c}$ are in HP.

$\Rightarrow \quad x, y, z$ are in HP.

121. ***(b)*** Let the sides of the triangle be a/r, a and ar, with $a > 0$ and $r > 1$. Let α be the smallest angle, so that the largest angle is 2α. Then, α is opposite to the side a/r, and 2α is opposite to the side ar.

Applying sine rule, we get

$$\frac{a/r}{\sin\alpha} = \frac{ar}{\sin 2\alpha}$$

$$\Rightarrow \quad \frac{\sin 2\alpha}{\sin\alpha} = r^2 \Rightarrow 2\cos\alpha = r^2$$

$\Rightarrow \quad r^2 < 2$

$\Rightarrow \quad r < \sqrt{2} \quad [\because \alpha \neq 0,\ \therefore 2\cos\alpha < 2]$

Hence, $\quad 1 < r < \sqrt{2}$

122. ***(b)*** Feasible region is $OACBO$ and $\mu = 3x + 4y$.

At point $O(0, 0)$

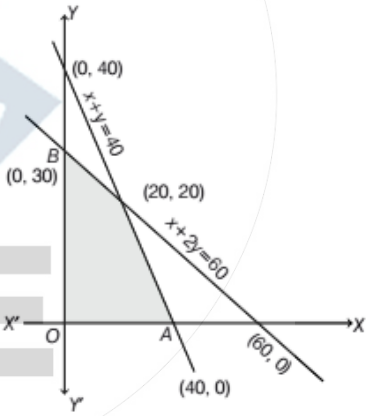
$\mu = 0 + 0 = 0$

At point $A(40, 0)$

$\mu = 3(40) + 4(0) = 120$

At point $B(0, 30)$

$\mu = 0 + 4(30) = 120$



At point $C(20, 20)$

$\mu = 3(20) + 4(20) = 140$

$\therefore$ The maximum value at the point $C(20, 20)$ is 140.

123. ***(c)*** We have, $n(A) = 2$ and $n(B) = 4$

$\therefore \quad n(A \times B) = n(A) \times n(B) = 2 \times 4 = 8$

The number of subsets at $A \times B$ having 3 as more elements

$$= {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$$
$$= ({}^8C_0 + {}^8C_1 + \ldots + {}^8C_8) - ({}^8C_0 + {}^8C_1 + {}^8C_2)$$
$$= 2^8 - (1 + 8 + 28) = 219$$

124. ***(d)*** Since, $a_1, a_2, a_3 ..... a_n$ are in HP.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ..., \frac{1}{a_n}$ are in AP.

Let d be the common difference of the AP then,

$$\frac{1}{a_2} - \frac{1}{a_1} = d, \frac{1}{a_3} - \frac{1}{a_2} = d, ...\frac{1}{a_n} - \frac{1}{a_{n-1}} = d$$

$\Rightarrow \quad a_1 - a_2 = d(a_1 a_2), a_2 - a_3 = d(a_2 a_3),$

$a_3 - a_4 = d(a_3 a_4), ..., a_{n-1} - a_n = d(a_{n-1} a_n)$

$\Rightarrow \quad (a_1 - a_2) + (a_2 - a_3) + ... + (a_{n-1} - a_n)$

$= d(a_1 a_2 + a_2 a_3 + ... + a_{n-1} a_n)$

$\Rightarrow \quad a_1 - a_n = d(a_1 a_2 + a_2 a_3 + ... + a_{n-1} a_n) \quad ...(i)$

Now, $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ..., \frac{1}{a_n}$ are in AP with common difference d.

$\Rightarrow n$th term $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$\Rightarrow \frac{1}{a_n} - \frac{1}{a_1} = (n-1)d \Rightarrow \frac{a_1 - a_n}{a_1 a_n} = (n-1)d$

$\Rightarrow \quad a_1 - a_n = (n-1)d(a_1 a_n) \quad ...(ii)$

From Eqs. (i) and (ii), we get

$$(n-1)d(a_1 a_n) = d(a_1 a_2 + a_2 a_3 + ... + a_{n-1} a_n)$$

$\Rightarrow \quad (n-1) a_1 a_n = a_1 a_2 + a_2 a_3 + ... + a_{n-1} a_n$

125. ***(a)*** The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

As, $(2\lambda + 1)x + (3\lambda + 1)y + (1 - \lambda)z + 4\lambda - 1 = 0 \quad ...(i)$

It is parallel to X-axis i.e. $\frac{x}{1} = \frac{y}{0} = \frac{z}{0} \Rightarrow$ DR's $= < 1, 0, 0 >$

$\therefore \quad 1(2\lambda + 1) + 0 \times (3\lambda + 1) + 0(1 - \lambda) = 0$

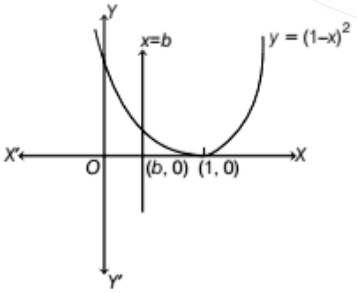
$\Rightarrow \quad \lambda = -\frac{1}{2}$

Substituting $\lambda = -\frac{1}{2}$ in Eq. (i), we get

$y - 3z + 6 = 0$ as the equation of the required plane.

126. ***(b)*** Clearly,

$$R_1 = \int_0^b (1-x)^2 dx \text{ and } R_2 = \int_b^1 (1-x)^2 dx$$



$$\Rightarrow \quad R_1 = \left[\frac{(x-1)^3}{3}\right]_0^b \text{ and } R_2 = \left[\frac{(x-1)^3}{3}\right]_b^1$$

$$\Rightarrow \quad R_1 = \frac{(b-1)^3}{3} + \frac{1}{3} \text{ and } R_2 = -\frac{(b-1)^3}{3}$$

$$\therefore \quad R_1 - R_2 = \frac{1}{4}$$

$$\Rightarrow \quad \frac{2}{3}(b-1)^3 + \frac{1}{3} = \frac{1}{4} \Rightarrow \frac{2}{3}(b-1)^3 = -\frac{1}{12}$$

$$\Rightarrow \quad (b-1)^3 = -\frac{1}{8} \Rightarrow b - 1 = -\frac{1}{2} \Rightarrow b = \frac{1}{2}$$

127. ***(a)*** We have,

$$\mathbf{a} \times \mathbf{b} = 2(\mathbf{a} \times \mathbf{c}) \Rightarrow \mathbf{a} \times (\mathbf{b} - 2\mathbf{c}) = \mathbf{0}$$

$\Rightarrow \mathbf{a}$ is parallel to $\mathbf{b} - 2\mathbf{c}$.

Now, $\quad (\mathbf{b} - 2\mathbf{c}) = \lambda \mathbf{a}$

$$\Rightarrow \quad (\mathbf{b} - 2\mathbf{c})^2 = \lambda^2 |\mathbf{a}|^2$$

$$|\mathbf{b}|^2 + 4|\mathbf{c}|^2 - 4(\mathbf{b}.\mathbf{c}) = \lambda^2 |\mathbf{a}|^2$$

$$\Rightarrow \quad 16 + 4 - 4 \times |\mathbf{b}||\mathbf{c}| \times \frac{1}{4} = \lambda^2 \Rightarrow 20 - 4 = \lambda^2$$

$$\Rightarrow \quad \lambda^2 = 16 \Rightarrow \lambda = \pm 4$$

128. ***(b)*** We know that the centroid G, circumcentre O and orthocentre O' of a triangle are collinear such that G divides OO' in the ratio 1 : 2. Since, affix at G is $\frac{z_1 + z_2 + z_3}{3}$ and O is the origin.

$$\frac{z_1 + z_2 + z_3}{3} = \frac{1 \cdot z + 2 \times 0}{1 + 2}$$

$$\Rightarrow \quad z = z_1 + z_2 + z_3$$

$$\Rightarrow \quad z_1 + z_2 + z_3 - z = 0$$

129. ***(d)*** The largest digit in the middle of the five digit number can be 4 (when number are formed with digits 0, 1, 2, 3, 4) 5 (when 0, 1, 2, 3, 4, 5 are used) ..., 9 (when 0, 1, 2, ..., 9 are used).

$\therefore$ Number of five digit numbers with largest digit 4 in the middle

$$= ({}^4C_4 \times 4! - {}^3C_3 \times 3!)$$

Number of five digit numbers with largest digit 5 in the middle

$$= ({}^5C_4 \times 4! - {}^4C_3 \times 3!) \text{ and so on.}$$

Hence, required number of numbers =

$$({}^4C_4 \times 4! - {}^3C_3 \times 3!) + ({}^5C_4 \times 4! - {}^4C_3 \times 3!)$$

$$+ ({}^6C_4 \times 4! - {}^5C_3 \times 3!) + ... + ({}^9C_4 \times 4! - {}^8C_3 \times 3!)$$

$$= 4! \sum_{n=4}^{9} {}^nC_4 - 3! \sum_{n=3}^{8} {}^nC_3 \quad = 4! {}^{10}C_5 - 3! {}^9C_4$$

$$= 3![4 \times {}^{10}C_5 - {}^9C_4] = 3! \times 882$$

130. ***(a)*** We have,

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \text{ for all } x \in [-1,1]$$

Also, $$-\frac{\pi}{4} \le \tan^{-1}x \le \frac{\pi}{4} \text{ for all } x \in [-1,1]$$

$$\therefore \quad -\frac{\pi}{4} + \frac{\pi}{2} \le \sin^{-1}x + \cos^{-1}x + \tan^{-1}x \le \frac{\pi}{4} + \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\pi}{4} \le \sin^{-1}x + \cos^{-1}x + \tan^{-1}x \le \frac{3\pi}{4}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{4} \text{ and } \beta = \frac{3\pi}{4}$$

131. ***(a)*** Given, $\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{\phi(y/x)}{\phi'(y/x)}$ ...(i)

Substituting $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ in Eq. (i), we get

$$v + x\frac{dv}{dx} = v + \frac{\phi(v)}{\phi'(v)}$$

$$\Rightarrow \quad \frac{\phi'(v)}{\phi(v)}dv = \frac{1}{x}dx$$

On integrating both sides, we get

$$\log \phi(v) = \log x + \log k$$

$$\Rightarrow \quad \phi(v) = kx \Rightarrow \phi(y/x) = kx$$

132. ***(a)*** The required mean $\bar{x}$ is given by

$$\bar{x} = \frac{0 \times {}^nC_0\, q^n p^0 + 1 \times {}^nC_1 q^{n-1} p + \ldots + n \times {}^nC_n\, q^0 p^n}{{}^nC_0 q^n p^0 + {}^nC_1 q^{n-1} p^1 + \ldots + {}^nC_n q^{n-n} p^n}$$

$$\Rightarrow \quad \bar{x} = \frac{\sum_{r=0}^{n} r \times {}^nC_r q^{n-r} p^r}{\sum_{r=0}^{n} {}^nC_r q^{n-r} p^r}$$

$$\Rightarrow \quad \bar{x} = \frac{\sum_{r=1}^{n} r \times \frac{n}{r}\, {}^{n-1}C_{r-1}\, q^{n-r} \times p \times p^{r-1}}{\sum_{r=0}^{n} {}^nC_r q^{n-r} p^r}$$

$$\Rightarrow \quad \bar{x} = \frac{np\left\{\sum_{r=1}^{n} {}^{n-1}C_{r-1}\, p^{r-1} q^{(n-1)-(r-1)}\right\}}{\sum_{r=0}^{n} {}^nC_r q^{n-r} p^r}$$

$$\Rightarrow \quad \bar{x} = \frac{np(q+p)^{n-1}}{(q+p)^n}$$

$$\Rightarrow \quad \bar{x} = np \qquad [\because q + p = 1]$$

133. ***(a)*** We have, mean = 2 and SD = 3

$$\Rightarrow \quad \frac{1}{10}\sum_{i=1}^{10} x_i = 2 \text{ and } \frac{1}{10}\sum_{i=1}^{10}(x_i^2 - 2^2) = 3^2$$

$$\Rightarrow \quad \sum_{i=1}^{10} x_i = 20 \text{ and } \sum_{i=1}^{10} x_i^2 = 130$$

Let $\bar{x}$ be the mean of $(x_1+1)^2, (x_2+1)^2, \ldots (x_{10}+1)^2$. Then,

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10}(x_i + 1)^2$$

$$\Rightarrow \quad \bar{x} = \frac{1}{10}\left(\sum_{i=1}^{10} x_i^2\right) + \frac{2}{10}\left(\sum_{i=1}^{10} x_i\right) + \frac{1}{10}\sum_{i=1}^{10} 1$$

$$\Rightarrow \quad \bar{x} = \frac{1}{10} \times 130 + \frac{2}{10} \times 20 + \frac{10}{10} = 18$$

134. ***(a)*** The equation of the tangent to the curve $y = e^x$ at (c, e^c).

Slope of tangent $= dy/dx = e^x$

$$\Rightarrow \quad y - e^c = e^c(x - c) \qquad \ldots(i)$$

The equation of the line joining $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$ is

$$y - e^{c-1} = \frac{e^c(e - e^{-1})}{2}(x - c + 1) \qquad \ldots(ii)$$

On subtracting Eq. (i) from Eq. (ii), we get

$$e^c - e^{c-1} = \frac{e^c(e - e^{-1})}{2}(x - c + 1) - e^c(x - c)$$

$$\Rightarrow e^c - e^{c-1} = (x - c)\, e^c\left(\frac{e - e^{-1}}{2} - 1\right) + e^c\left(\frac{e - e^{-1}}{2}\right)$$

$$\Rightarrow \quad x - c = \frac{(e-1)^2}{2 - (e-1)^2} < 0 \Rightarrow x < c$$

Hence, Eqs. (i) and (ii) intersect at a point on the left of $x = c$.

135. ***(d)*** We know that the lines,

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

and $$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

are coplanar, iff

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

So, the given lines will be coplanar, if

$$\begin{vmatrix} 1-2 & 4-3 & 5-4 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad k^2 + 3k = 0 \Rightarrow k = 0, -3$$

136. ***(c)*** We have,

$$a + b + c = 0 \text{ and } 2ab + 2ac - bc = 0$$

$$\Rightarrow \quad a = -(b + c) \text{ and } 2a(b + c) - bc = 0$$

$$\Rightarrow \quad -2(b + c)^2 - bc = 0$$

$\Rightarrow 2b^2 + 5bc + 2c^2 = 0 \Rightarrow (2b + c)(b + 2c) = 0$

$\Rightarrow \quad 2b + c = 0$ or $b + 2c = 0$

If $2b + c = 0$, then $a = -(b + c) \Rightarrow a = b$

$\therefore \quad a = b$ and $c = -2b \Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{-2}$

If $b + 2c = 0$, then $a = -(b + c) \Rightarrow a = c$

$\therefore \quad a = c$ and $b = -2c \Rightarrow \frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$

Thus, the direction ratio of two lines are proportional to 1, 1, -2 and 1, -2, 1 respectively. So, the angle θ between them is given by

$$\cos\theta = \frac{1 - 2 - 2}{\sqrt{1+1+4}\sqrt{1+4+1}} = -\frac{1}{2} = \cos\frac{2\pi}{3}$$

$$\Rightarrow \quad \theta = \frac{2\pi}{3}$$

137. ***(a)*** Since $f(x)$ satisfies the conditions of Rolle's theorem.

$\therefore \quad f(1) = f(3)$,

where $f(x) = x^3 - 6x^2 + ax + b$

$\Rightarrow \quad 1 - 6 + a + b = 27 - 54 + 3a + b$

$\Rightarrow \quad a = 11$

Now, $f'(x) = 3x^2 - 12x + a = 3x^2 - 12x + 11$

Clearly, $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0.$

Hence, $a = 11$

138. ***(d)*** We have, $f^{-1}(f(x)) = x$ for all x

On differentiating w.r.t. 'x'

$$(f^{-1})'(f(x))\, f'(x) = 1 \text{ for all } x$$

$$(f^{-1})'(f(x)) = \frac{1}{f(x)} \text{ for all } x \quad [\because f'(x) = f(x)]$$

Differentiating both sides w.r.t. x

$$(f^{-1})''(f(x))\, f'(x) = -\frac{1}{\{f(x)\}^2} f'(x)$$

$$\Rightarrow \quad (f^{-1})''(f(x)) = -\frac{1}{\{f(x)\}^2}$$

$$\Rightarrow \quad (f^{-1})''(x) = -\frac{1}{x^2}$$

139. ***(c)*** Let $f(x) = 2x^3 + 5$ and $g(x) = 9x^2 - 12x$.

Then, $f(x)$ increases less rapidly than $g(x)$ means that

$$\frac{d}{dx}(f(x)) < \frac{d}{dx}(g(x))$$

$$\Rightarrow \quad \frac{d}{dx}(f(x) - g(x)) < 0$$

$$\Rightarrow \quad f'(x) - g'(x) < 0 \Rightarrow 6x^2 - (18x - 12) < 0$$

$$\Rightarrow \quad x^2 - 3x + 2 < 0$$

$$\Rightarrow \quad x \in (1, 2)$$

140. ***(a)*** We have,

$$f(x) = \sin^{-1} x^2 \Rightarrow f'(x) = \frac{2x}{\sqrt{1 - x^4}}$$

$\therefore \quad f'(x) = 0 \Rightarrow x = 0$

Now, $\quad f\left(-\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6},$

$$f\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

and $\quad f(0) = \sin^{-1} 0 = 0$

Hence, the greatest value is $\pi/6$.

141. ***(a)*** We have,

$$f(x) = [x]^2 + [x + 1] - 3$$

$$\Rightarrow \quad f(x) = [x]^2 + [x] + 1 - 3$$

$[\because [x + n] = [x] + n$, where $n \in Z]$

$$\Rightarrow \quad f(x) = [x]^2 + [x] - 2$$

$$\Rightarrow \quad f(x) = ([x] + 2)([x] - 1)$$

Clearly, $f(x) = 0$ for all $x \in [1, 2) \cup [-2, -1)$

So, f is a many-one function.

Also, $f(x)$ assumes only integral values

$\therefore \quad$ Range of $f \neq R$.

Hence, $f(x)$ is a many-one into function.

142. ***(a)*** The equation of a line through $P(2, 4)$ and parallel to $y - x = 1$ is

$$\frac{x - 2}{\cos\frac{\pi}{4}} = \frac{y - 4}{\sin\frac{\pi}{4}}$$

The coordinates of Q are given by

$$\frac{x - 2}{\cos\frac{\pi}{4}} = \frac{y - 4}{\sin\frac{\pi}{4}} = -3\sqrt{2} \Rightarrow x = -1, y = 1$$

Thus, the coordinates of Q are $(-1, 1)$.

R is the image of Q with respect to the line $y - x = 1$

Therefore, coordinates of R are given by

$$\frac{x + 1}{-1} = \frac{y - 1}{1} = \frac{-2(1 + 1 - 1)}{(-1)^2 + (1)^2}$$

$$\Rightarrow \quad \frac{x + 1}{-1} = \frac{y - 1}{1} = -1$$

$$\Rightarrow \quad x = 0 \text{ and } y = 0$$

Hence, the coordinates of R are $(0, 0)$.

143. ***(c)*** The equation of lines making equal intercepts of length l on the coordinate axes is

$$x \pm y = \pm l$$

$$\Rightarrow \quad r = \sqrt{32} = 4\sqrt{2}$$

This will be a chord of the circle $x^2 + y^2 = 32$, if length of the perpendicular from the centre is less than radius.

i.e. $\left|\frac{\pm l}{\sqrt{1+l}}\right| < 4\sqrt{2}$

$\Rightarrow \quad |l| < 8 \Rightarrow l \in (-8, 8)$

144. ***(a)*** $x^2 - (a-2)x - a + 1 = 0$

Let α and β be the roots of the given equation, so that

$$\alpha + \beta = a - 2 \text{ and } \alpha\beta = -(a-1)$$

Let $S = (\alpha^2 + \beta^2)$. Then,

$$S = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= (a-2)^2 + 2(a-1) = a^2 - 2a + 2$$

$\therefore \quad dS/da = 2a - 2$

Now, $dS/da = 0 \Rightarrow a = 1$

Also, $d^2S/da^2 = 2 > 0$ for all a

Hence, S is minimum when $a = 1$.

145. ***(c)*** We have,

$$y = \int_x^{x^2} \cos^{-1} t^2 \, dt$$

$\Rightarrow \quad dy/dx = 2x \cos^{-1}(x^4) - \cos^{-1}(x^2)$

[by Leibnitz's rule]

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x = 2^{-\frac{1}{4}}} = 2 \times 2^{-\frac{1}{4}} \cos^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x = 2^{-\frac{1}{4}}} = 2^{\frac{3}{4}} \times \frac{\pi}{3} - \frac{\pi}{4} = \left(\frac{\sqrt[4]{8}}{3} - \frac{1}{4}\right)\pi$$

146. ***(c)*** $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\{(\cos x + \sin x)^2\}^{5/2} - (2)^{5/2}}{(1 + \sin 2x) - 2}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(1 + \sin 2x)^{5/2} - 2^{5/2}}{(1 + \sin 2x) - 2}$$

$$= \lim_{y \to 2} \frac{y^{5/2} - 2^{5/2}}{y - 2}, \text{ where } y = 1 + \sin 2x$$

$$= \frac{5}{2} \times 2^{5/2 - 1} = 5\sqrt{2}$$

147. ***(c)*** We have,

$f(x) + f(y) + f(z) + f(x) f(y) f(z) = 14$

for all $x, y, z \in R$ ...(i)

Putting $x = y = z = 0$ in Eq. (i), we get

$$3f(0) + \{f(0)\}^3 = 14$$

$\Rightarrow \quad \{f(0)\}^3 + 3f(0) - 14 = 0$

$\Rightarrow \quad f(0) = 2$

Now, putting $y = z = x$ in Eq. (i), we get

$3f(x) + \{f(x)\}^3 = 14$ for all $x \in R$

On differentiating w.r.t. x, we get

$$3f'(x) + 3\{f(x)\}^2 f'(x) = 0 \text{ for all } x \in R$$

$\Rightarrow \quad [\{f(x)\}^2 + 1] f'(x) = 0$ for all $x \in R$

$\Rightarrow \quad f'(x) = 0$ for all $x \in R$

148. ***(d)*** We have, $6\int_1^x f(t)\,dt = 3x f(x) - x^3$

On differentiating both sides w.r.t. x, we get

$$6f(x) = 3f(x) + 3xf'(x) - 3x^2$$

$\Rightarrow \quad 3f(x) = 3x f'(x) - 3x^2$

$\Rightarrow \quad f'(x) - \frac{1}{x} f(x) = x$

$\Rightarrow \quad \frac{d}{dx} f(x) + \left(-\frac{1}{x}\right) f(x) = x$

This is a linear differential equation with integrating factor

$$e^{\int -\frac{1}{x} dx} = e^{-\log x} = 1/x$$

Solution is given by,

$$\frac{1}{x} f(x) = \int x \times \frac{1}{x} dx + C$$

$\Rightarrow \quad f(x) = x^2 + Cx$

$\therefore \quad f(1) = 1 + C \Rightarrow 2 = 1 + C$

$\Rightarrow \quad C = 1$

Thus, $f(x) = x^2 + x$

$\therefore \quad f(2) = 4 + 2 = 6$

149. ***(b)*** Since g is the inverse of f.

$\therefore \quad fog(x) = x$ for all x

$\Rightarrow \quad \frac{d}{dx}\{fog(x)\} = 1$ for all x

$\Rightarrow \quad f'(g(x)) \cdot g'(x) = 1 \Rightarrow f'\{g(x)\} = \frac{1}{g'(x)}$

$\Rightarrow \quad \frac{1}{1 + [g(x)]^n} = \frac{1}{g'(x)} \quad t\left[\because f'(x) = \frac{1}{1 + x^n}\right]$

$\Rightarrow \quad g'(x) = 1 + [g(x)]^n$

150. ***(d)*** For $f(x)$ to be continuous at $x = 0$, we must have $\lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow \lim_{x \to 0} \left(1 + xe^{-1/x^2} \sin \frac{1}{x^4}\right)^{e^{1/x^2}} = k$$

$$\Rightarrow e^{\lim_{x \to 0}\left(xe^{1/x^2} e^{-1/x^2} \sin \frac{1}{x^4}\right)} = k$$

$$\Rightarrow e^{\lim_{x \to 0} x \sin \frac{1}{x^4}} = k \Rightarrow e^{\lim_{x \to 0} x \sin \frac{1}{x^4}} = k$$

$\Rightarrow \quad e^0 = k \Rightarrow k = 1$