Solved Paper 2015*

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

		No. of Questions
Part I	Physics	1–40
Part II	Chemistry	41–80
Part III	a. English Proficiency	81–95
	b. Logical Reasoning	96–105
Part IV	Mathematics	106–150

- All questions are Multiple Choice Questions having four options out of which **only one** is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.
- Time allotted to complete this paper is 3 hrs.

PART I

Physics

1. A body cools from 60°C to 50°C in 10 min. What will be its temperature at the end of the next 10 min, if the room temperature is 25°C?

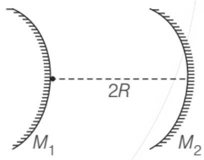
(Assume Newton's law of cooling holds.)

a. 49°C ***b.*** 42.8°C
c. 44°C ***d.*** 45.8°C

2. Two tuning forks with natural frequencies 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards the observer at the same speed. The observer hears beats of frequency of 3 Hz. Find the speed of the tuning forks.

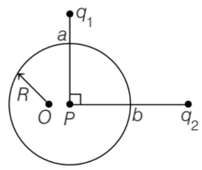
a. 1.5 m/s
b. 2 m/s
c. 1 m/s
d. 2.5 m/s

3. Two spherical mirrors, one convex and other concave are placed coaxially at a distance $2R$ from each other. Both the mirrors have same radius of curvature R. What is the radius of 3rd image from first three images of the circle, if the small circle of radius a is drawn on the convex mirror in the figure?



a. $a/41$ ***b.*** $a/43$ ***c.*** $a/56$ ***d.*** $a/53$

4. In the given figure, two point charges q_1 and q_2 are placed at distance a and b from the centre of a metallic sphere having charge Q. Find the electric field due to the metallic sphere at the point P.



a. $\frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{q_1}{a^2}\right)^2+\left(\frac{q_2}{b^2}\right)^2}$

b. $\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}$

c. $\frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{Q}{R^2}\right)^2+\left(\frac{q_1}{a^2}+\frac{q_2}{b^2}\right)^2}$

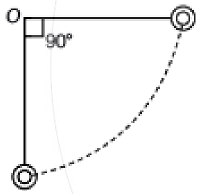
d. None of the above

5. A body is thrown vertically upwards from A, the top of the tower, it reaches the ground in time t_1. If it is thrown vertically downwards from A with the same speed, it reaches the ground in time t_2. If it is allowed to fall freely from A, then the time it takes to reach the ground is given by

a. $t=\frac{t_1+t_2}{2}$ b. $t=\frac{t_1-t_2}{2}$

c. $t=\sqrt{t_1t_2}$ d. $t=\sqrt{\frac{t_1}{t_2}}$

6. A simple pendulum is vibrating with an angular amplitude of 90° as shown in the figure. For what value of α, the acceleration is directed horizontally?



a. Zero b. 90°

c. $\cos^{-1}(1/\sqrt{3})$ d. $\sin^{-1}(1/\sqrt{3})$

7. A charged particle moving in a uniform magnetic field and losses 4% of its kinetic energy. The radius of curvature of its path changes by

a. 2% b. 4%

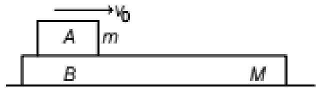
c. 10% d. None of these

8. Calculate the wavelength of light used in an interference experiment from the following data : fringe width = 0.03 cm. Distance between the slits and eye-piece through which the interference pattern in observed is 1m. Distance betweeen the images of the virtual source when a convex lens of focal length 16 cm is used at a distance of 80 cm from the eye-piece is 0.8 cm.

a. 0.0006 Å b. 0.0006 m

c. 600 cm d. 6000 Å

9. The masses of blocks A and B are m and M, respectively. Between A and B, there is a constant frictional force F and B can slide on a smooth horizontal surface. A is set in motion with velocity, while B is at rest. What is the distance moved by A relative to B before they move with the same velocity?



a. $\frac{mMv_0^2}{F(m-M)}$ b. $\frac{mMv_0^2}{2F(m-M)}$

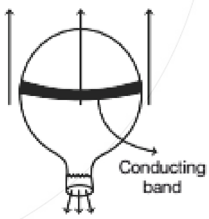
c. $\frac{mMv_0^2}{F(m+M)}$ d. $\frac{mMv_0^2}{2F(M+m)}$

10. A particle of mass m is projected from the surface of the earth with a speed (v_0 > escape velocity). Find the speed of particle at height $h=R$ (radius of the earth).

a. $\sqrt{gR}$ b. $\sqrt{v_0^2-2gR}$

c. $\sqrt{v_0^2-gR}$ d. None of these

11. An elasticised conducting band is around a spherical balloon. Its plane passes through the centre of balloon. A uniform magnetic field of magnitude 0.04 T is directed perpendicular to the plane of band. Air is let out of the balloon at 100 cm^3/s at an instant, when the radius of the balloon is 10 cm. The induced emf in the band is



a. 20 μV b. 25 μV

c. 10μV d. 15 μV

12. An elastic string of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is

a. $1/2\,ky^2$ b. $1/2\,ky(2x+y)$

c. $1/2\,k(x^2+y^2)$ d. $1/2\,k(x+y)^2$

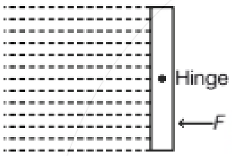
13. The resistance of a 50 cm long wire is 10 Ω. The wire is stretched to uniform wire of length 100 cm. The new resistance will be

a. 15 Ω b. 30 Ω c. 20 Ω d. 40 Ω

14. A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed amongst the three wavelengths. The beam falls normally on the area 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in 2s.

a. 2×10^9 *b.* 1.075×10^{12}
c. 9×10^8 *d.* 3.75×10^6

15. A square gate of size $1\text{ m}\times 1\text{ m}$ is hinged at its mid-point. A fluid of density ρ fills the space to the left of the gate. The force F required to hold the gate stationary is



a. $\frac{\rho g}{3}$ *b.* $\frac{\rho g}{2}$
c. $\frac{\rho g}{6}$ *d.* $\frac{\rho g}{8}$

16. When 0.50 Å X-rays strike a material, the photoelectrons from the k-shell are observed to move in a circle of radius 23 mm in a magnetic field of 2×10^{-2} T acting perpendicularly to the direction of emission of photoelectrons. What is the binding energy of k-shell electrons?

a. 3.5 keV *b.* 6.2 keV
c. 2.9 keV *d.* 5.5 keV

17. In *CE*-transistor amplifier, the audio signal voltage across the collector resistance of $2\text{ k}\Omega$ is 2 V. If the base resistance is 1 kΩ and the current amplification of the transistor is 100, the input signal voltage is

a. 2 mV *b.* 3 mV
c. 10 mV *d.* 0.1 mV

18. The equation of motion of projectile is

$$y = 12x - \frac{3}{4}x^2$$

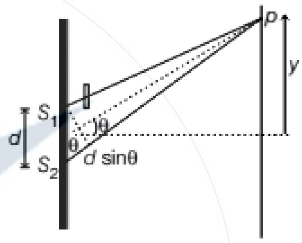
The horizontal component of velocity is 3 ms^{-1}. What is the range of the projectile?

a. 18 m *b.* 21.6 m
c. 12 m *d.* 16 m

19. A vessel of volume 20 L contains a mixture of hydrogen and helium at temperature of 27°C and pressure 2 atm. The mass of mixture is 5 g. Assuming the gases to be ideal, the ratio of mass of hydrogen to that of helium in the given mixture will be

a. 1 : 2 *b.* 2 : 3 *c.* 2 : 1 *d.* 2 : 5

20. In a YDSE, light of wavelength $\lambda = 5000$ Å is used, which emerges in phase from two slits a distance $d = 3\times 10^{-7}$ m apart. A transparent sheet of thickness $t = 1.5\times 10^{-7}$ m, refractive index $\mu = 1.17$ is placed over one of the slits. What is the new angular position of the central maxima of the interference pattern, from the centre of the screen? find the value of y.

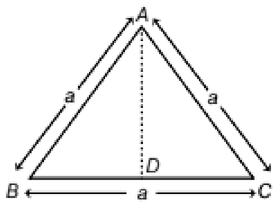


a. 4.9° and $\frac{D(\mu-1)t}{2d}$ *b.* 4.9° and $\frac{D(\mu-1)t}{d}$
c. 3.9° and $\frac{D(\mu+1)t}{d}$ *d.* 2.9° and $\frac{2D(\mu-1)t}{d}$

21. A particle moves with a simple harmonic motion in a straight line. In the first second starting from rest, it travels a distance a and in the next second, it travels a distance b in the same direction. The amplitude of motion is

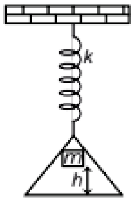
a. $\frac{2a^2}{3a-b}$ *b.* $\frac{3a^2}{3b-a}$ *c.* $\frac{2a^3}{3b-a}$ *d.* $\frac{3a^3}{3a-b}$

22. At the corners of an equilateral triangle of side a (1 m), three point charges are placed (each of 0.1 C). If this system is supplied energy at the rate of 1 kW, then calculate the time required to move one of the charges to the mid-point of the line joining the other two.



a. 50 h *b.* 60 h
c. 48 h *d.* 54 h

23. A load of mass m falls from a height h onto the scale pan hung from the spring as shown in the Figure. If the spring constant is k and mass of the scale pan is zero and the mass m does not bounce relative to the pan, then the amplitude of vibration is



a. mg/k
b. $\frac{mg}{k}\sqrt{\left(1+\frac{2hk}{mg}\right)}$
c. $\frac{mg}{k}+\frac{mg}{k}\sqrt{\left(\frac{1+2hk}{mg}\right)}$
d. $\frac{mg}{k}\sqrt{\left(\frac{1+2hk}{mg}-\frac{mg}{k}\right)}$

24. In an ore containing uranium, the ratio of U^{238} to Pb^{206} is 3. Calculate the age of the ore, assuming that all the lead present in the ore is the final stable product of U^{238}. Take, the half-life of U^{238} to be 4.5×10^{9} yrs.

a. 1.6×19^{3} yrs
b. 1.5×10^{4} yrs
c. 1.867×10^{9} yrs
d. 2×10^{5} yrs

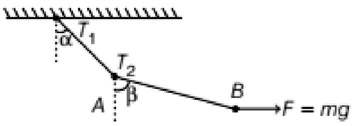
25. A concave mirror is palced on a horizontal table with its axis directed vertically upward. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has real image, also located at C. If the mirror is now filled with water, the image will be

a. real and will remain at C
b. real and located at a point between C and ∞
c. real and located at a point between C and O
d. None of the above

26. A direct current of 5A is superposed on an alternating current $I=10\sin\omega t$ flowing through the wire. The effective value of the resulting current will be

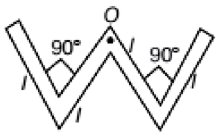
a. (15/2)A
b. $5\sqrt{3}$ A
c. $5\sqrt{5}$ A
d. 15 A

27. Two particles A and B, each of mass m, are kept stationary by applying a horizontal force $F=mg$ on the particle B as shown in the figure. Then,



a. $2\tan\beta=\tan\alpha$
b. $2T_1=5T_2$
c. $T_1\sqrt{2}=T_2\sqrt{5}$
d. None of these

28. A thin rod of length $4l$ and mass $4M$ is bent at the points as shown in figure. What is the moment of inertia of the rod about the axis passes through point O and perpendicular to the plane of paper?



a. $\frac{Ml^2}{3}$
b. $\frac{10Ml^2}{3}$
c. $\frac{Ml^2}{12}$
d. $\frac{Ml^2}{24}$

29. The wavelength of the second line of Balmer series in the hydrogen spectrum is 4861 Å. The wavelength of the first line is

a. $\frac{27}{20}\times4861$ Å
b. $\frac{20}{27}\times4861$ Å
c. 20×4861 Å
d. 4861 Å

30. Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2, respectively. The ratio of masses of X and Y is

a. $(R_1/R_2)^{1/2}$
b. (R_2/R_1)
c. $(R_1/R_2)^2$
d. (R_1/R_2)

31. A glass capillary tube of internal radius $r=0.25$ mm is immersed in water. The top end of the tube projects by 2cm above the surface of the water. At what angle does the liquid meet the tube?

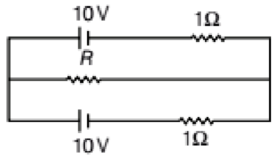
Surface tension of water = 0.07 N/m.

a. $\theta=90°$
b. $\theta=70°$
c. $\theta=45°$
d. $\theta=35°$

32. A particle of mass 2 m is projected at an angle of 45° with the horizontal with a velocity of $20\sqrt{2}$ m/s. After 1 s, explosion takes place and the particle is broken into two equal pieces. As a result of explosion, one part comes to rest. The maximum height from the ground attained by the other part is

a. 50 m
b. 25 m
c. 40 m
d. 35 m

33. Maximum power developed across variable resistance R in the circuit as shown in the figure.



a. 50 W *b.* 75 W *c.* 25 W *d.* 60 W

34. A neutron moving with speed v makes a head on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of the neutron for which inelastic collision takes place is

a. 10.2 eV *b.* 20.4 eV
c. 12.1 eV *d.* 16.8 eV

35. Vertical displacement of a plank with a body of mass m on it is varying according to law $y = \sin \omega t + \sqrt{3} \cos \omega t$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after $t = 0$, are given by

a. $\sqrt{g/2}, \frac{\sqrt{2}}{6}\frac{\pi}{\sqrt{g}}$ *b.* $\frac{g}{\sqrt{2}}, \frac{2}{3}\sqrt{\pi/g}$
c. $\sqrt{g/2}, \frac{\pi}{3}\sqrt{2/g}$ *d.* $\sqrt{2g}, \sqrt{2\pi/3g}$

36. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is similarly charged to a potential difference $2V$. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

a. Zero *b.* $\frac{3}{2}CV^2$
c. $\frac{25}{6}CV^2$ *d.* $\frac{9}{2}CV^2$

37. A man drives a car y towards x at speed 60 km/h. A car leaves station x for station y every 10 min. The distance between x and y is 60 km. The car travels at the speed of 60 km/h. A man drives a car from y towards x at speed of 60 km/h. If he starts at the moment when first car leaves the station y, then how many cars would be meet on the route?

a. 20 *b.* 7
c. 10 *d.* 2

38. The ratio of the energy required to raise a satellite upto a height h above the earth to the kinetic energy of the satellite in the orbit is

a. $h : R$ *b.* $R : 2h$
c. $2h : R$ *d.* $R : h$

39. The frequency of a sonometer wire is 100 Hz. When the weights producing the tensions are completely immersed in water, the frequency becomes 80 Hz and on immersing the weights in a certain liquid, the frequency becomes 60 Hz. The specific gravity of the liquid is

a. 1.42 *b.* 1.77
c. 1.82 *d.* 1.21

40. A wire of length l is bent in the form of a circular coil of some turns. A current i flows through the coil. The coil is placed in a uniform magnetic field B. The maximum torque on the coil can be

a. $\frac{iBl^2}{4\pi}$ *b.* $\frac{iBl^2}{\pi}$
c. $\frac{iBl^2}{2\pi}$ *d.* $\frac{2iBl^2}{\pi}$

PART II

Chemistry

41. Which of the following pollutants is main product of automobiles exhaust?

a. CO *b.* CO_2
c. NO *d.* Hydrocarbons

42. The harmful chemical present in tobacco is

a. morphine *b.* zinc
c. arsenic *d.* phosphate

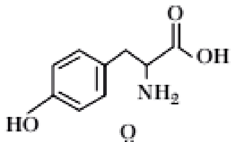
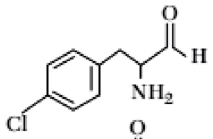
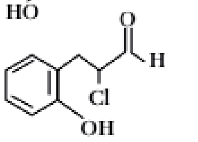
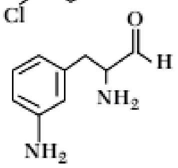
43. The disease causes by the high concentration of hydrocarbon pollutants in atmosphere is

a. silicosis *b.* TB *c.* cancer *d.* asthma

44. In disulphide bond formation of protein, the functional group that participates is

a. amino group *b.* thio ether
c. thiol *d.* thio ester

45. The correct structure of tyrosine is

a. 
b. 
c. 
d. 

46. Which law of the thermodynamics helps in calculating the absolute entropies of various substances at different temperatures?

a. First law *b.* Second law
c. Third law *d.* Zeroth law

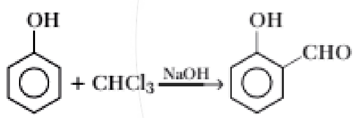
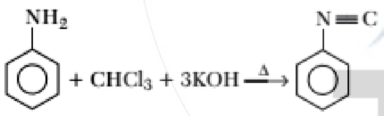
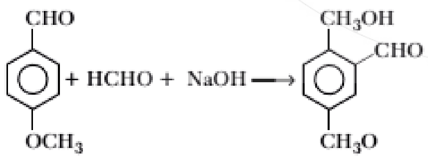
47. The boiling point of alkyl halides are higher than those of corresponding alkanes because of

a. dipole-dipole interaction
b. dipole-induced dipole interaction
c. H-bonding
d. None of the above

48. Lucas test is given by

a. *n*-alcohol *b.* *sec*-alcohol
c. *tert*-alcohol *d.* Both (b) and (c)

49. The carbylamine reaction is

a. 
b. 
c. 
d. 

50. The colour of $CoCl_3 \cdot 5NH_3 \cdot H_2O$ is

a. red *b.* orange
c. yellow *d.* pink

51. Some salts containing two different metallic elements give test for only one of them in solution, such salts are

a. double salts *b.* normal salts
c. complex salts *d.* None of these

52. The metal present in the chlorophyll is

a. magnesium *b.* zinc
c. cobalt *d.* copper

53. The common name of $K[PtCl_3(\eta^2\text{-}C_2H_4)]$ is

a. potassium salt *b.* Zeise's salt
c. complex salt *d.* None of these

54. The metal present in vitamin B_{12} is

a. magnesium *b.* cobalt *c.* copper *d.* zinc

55. The anthracene is purified by

a. crystallisation *b.* filtration
c. distillation *d.* sublimation

56. Which is an intensive property?

a. Mass *b.* Volume
c. Energy *d.* Temperature

57. Laughing gas is

a. nitrogen pentoxide *b.* nitrous oxide
c. nitrogen trioxide *d.* nitric oxide

58. Brown ring test is used to detect

a. nitrate *b.* bromide *c.* iron *d.* iodide

59. Semiconductor materials, like Si and Ge are usually purified by

a. distillation *b.* zone refining
c. liquation *d.* electrolytic refining

60. Which of the following is not a method of concentration of metal?

a. Smelting *b.* Froth floatation
c. Distillation *d.* Electromagnetic method

61. Which of the following according to Le-Chatelier's principle is correct?

a. Increase in temperature favours the endothermic reaction.
b. Increase in temperature favours the exothermic reaction.
c. Increase in pressure shifts the equilibrium in that side in which number of gaseous moles increases
d. All of the above

62. The van't Hoff equation is

a. $\log \frac{K_1}{K_2} = \frac{\Delta H}{2.303R}\left[\frac{T_2 - T_1}{T_1T_2}\right]$

b. $\log \frac{K_2}{K_1} = \frac{\Delta H}{2.303R}\left[\frac{T_2 - T_1}{T_1T_2}\right]$

c. $\log \frac{K_2}{K_1} = \frac{\Delta H}{2.303R}\left[\frac{T_1 - T_2}{T_1T_2}\right]$

d. None of the above

63. Which of the following is a strong base?
a. PH_3 *b.* AsH_3
c. NH_3 *d.* SbH_3

64. What is the correct increasing order of Bronsted bases ?
a. $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$
b. $ClO_4^- > ClO_3^- > ClO_2^- > ClO^-$
c. $ClO_3^- < ClO_4^- < ClO_2^- < ClO^-$
d. $ClO^- > ClO_3^- > ClO_2^- > ClO_4^-$

65. The mass of the substance deposited when one Faraday of charge is passed through its solution is equal to
a. relative equivalent weight
b. gram equivalent weight
c. specific equivalent weight
d. None of the above

66. The efficiency of fuel cell is given by the expression, η is
a. $\eta = -\frac{nFE_{cell}}{\Delta H} \times 100$
b. $\eta = -\frac{nFE_{cell}}{\Delta S} \times 100$
c. $\eta = -\frac{nFE_{cell}}{\Delta A} \times 100$
d. None of the above

67. For a gaseous reaction, the units of rate of reaction is/ are
a. atm time^{-1} *b.* bar time^{-1}
c. atm^{-1} time *d.* Both (a) and (b)

68. The unit of rate constant for reactions of second order is
a. L mol^{-1} s^{-1} *b.* L^{-1} mol s^{-1}
c. L mol s^{-1} *d.* s^{-1}

69. In a first order reaction with time the concentration of the reactant decreases
a. linearly *b.* exponentially
c. no change *d.* None of these

70. *Aqua-regia* is a mixture of
a. conc. HNO_3(3 parts) + conc. HCl (1 part)
b. conc. HNO_3 (1 part) + conc. HCl (3 parts)
c. conc. HNO_3 (1 part) + conc. H_2SO_4 (3 parts)
d. conc. HNO_3 (3 parts) + conc. H_2SO_4 (1 part)

71. The P—P—P angle in P_4 molecule and S — S — S angle in S_8 molecule is (in degree) respectively
a. 60°, 107°
b. 107°, 60°
c. 40°, 60°
d. 60°, 40°

72. What is the formula of borax?
a. $NaAlSi_3O_8$
b. $Na_2B_4O_4 \cdot 10H_2O$
c. $Na_2SO_4 \cdot 10H_2O$
d. $NaNO_3$

73. The byproduct of Solvay-ammonia process is
a. CO_2 *b.* NH_3
c. $CaCl_2$ *d.* $CaCO_3$

74. The number of elements present in the *d*-block of the periodic table is
a. 40 *b.* 41
c. 45 *d.* 46

75. Chromite ore has the formula
a. $Cr_2O_3 \cdot FeO$ *b.* $FeOCr_2O_3$
c. Cr_2O_3 *d.* None of these

76. Which of the following represents hexadentate ligand?
a. EDTA *b.* DMG
c. Ethylenediamine *d.* None of these

77. Prussian blue is
a. $K_4[Fe(CN)_6]$
b. $KFe[Fe(CN)_6]$
c. $K_3[Fe(CN)_6]$
d. $Fe_4[Fe(CN)_6]_3$

78. Which one of given elements shows maximum number of different oxidation states in its compounds?
a. Am *b.* Fu
c. La *d.* Gd

79. $K_4[Fe(CN)_6]$ is used in detecting
a. Fe^{3+} ion *b.* Cu^+ ion
c. Cu^{3+} ion *d.* Fe^{2+} ion

80. For a process to be spontaneous
a. ΔG = + ve *b.* ΔH = − ve
c. ΔS = − ve *d.* ΔG = − ve

PART III

a. English Proficiency

Directions (Q. Nos. 81-83) *Choose the word which best expresses the meaning of the underlined word in the sentence.*

81. He is a good-looking but <u>insipid</u> young man.
a. Arrogant
b. Unscrupulous
c. Sick
d. Lacking in spirit

82. It was an <u>astute</u> move to sell the property at that stage.
a. Shrewd *b.* Unwise
c. Dishonest *d.* Inexplicable

83. Even the most careful researcher cannot predict the possible future <u>ramifications</u> of his findings.
a. Uses *b.* Developments
c. Consequences *d.* Conclusions

Directions (Q. Nos. 84-86) *Fill in the blanks.*

84. Do you prefer .... or traditional art forms?
a. archaic *b.* contemporary
c. foreign *d.* simultaneous

85. She remained .... all her life.
a. spinster *b.* bachelor *c.* unmarried *d.* single

86. He was accused of bringing money into the country in .... of Foreign Exchange Rules.
a. anticipation *b.* compensation
c. perpetration *d.* violation

Directions (Q. Nos. 87-89) *Choose the word which is closest to the opposite in meaning of the following sentence.*

87. Do not stay in the grasslands after dark, as some animals become .... when they see humans.
a. provoked *b.* alerted
c. aggressive *d.* threatened

88. In all places and at all times, there is a <u>profusion</u> of talents.
a. Plenty *b.* Scarcity
c. Aversion *d.* Generosity

89. Unlike the other candidates, his manner was entirely <u>languid</u>.
a. energetic *b.* lazy
c. liquid *d.* slow

Directions (Q. Nos. 90-92) *In each of the following questions, out of the four alternatives, choose the one which can be substituted for the given words/sentences.*

90. A person who loves everybody.
a. Egoist *b.* Fatalist
c. Humanist *d.* Altruist

91. A small village or a group of houses
a. Community *b.* Settlement
c. Hamlet *d.* Colony

92. Wild Imagination
a. Whim *b.* Fantasy
c. Fancy *d.* Memory

Directions (Q. Nos. 93-95) *Choose the order of the sentences marked A, B, C, D and E to form a logical paragraph.*

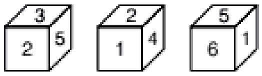
93. Technology is
A : investors, workers, companies and policymakers
B : are clueless about how to safeguard jobs
C : difficult to comprehend that
D : changing work at a pace so
a. DCBA *b.* CDAB
c. CDBA *d.* DCAB

94. Since Independence,
A : on the necessity of
B : application of Uniform Civil Code
C : We have been debating
D : for all the citizens
a. CADB *b.* ABCD
c. CABD *d.* ACBD

95. The ruling party,
A : for power than by the sincere desire
B : in a democracy is obviously
C : to serve the people in a real sense
D : guided more by the lust
a. DBCA *b.* BDCA
c. DBAC *d.* BDAC

b. Logical Reasoning

96. Three positions of a dice are given below. Identify the number on the face opposite to 1.



a. 3 *b.* 4 *c.* 5 *d.* 2

97. In the following question, choose the missing terms out of the given alternatives.

2	9	11	7
8	5	13	–3
7	?	10	– 4
6	4	10	?

a. 3 and –2 *b.* –3 and –2
c. 3 and 2 *d.* –3 and 2

98. On evening before sunset, 2 friends Raman and Arjun were talking to each other face to face. If Raman's shadow was exactly to his left side, which direction was Arjun facing?

a. West *b.* East *c.* North *d.* South

99. Sunil is the son of Keshav. Simran, Keshav's sister has a son Maruti and daughter Sita. Prem is the maternal uncle of Maruti. How is Sunil related to Maruti?

a. Cousin *b.* Uncle *c.* Brother *d.* Nephew

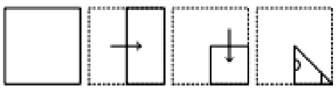
100. If the following equation has to be balance, then the signs of which of the following options will be used?

24_6_12_16=0

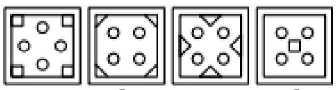
a. ÷, + and – *b.* –, ÷ and +
c. –, – and – *d.* ÷, + and ÷

101. A piece of paper is folded and punched as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.

Question figures

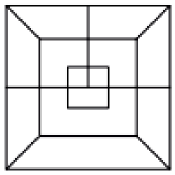


Answer figures



a. *b.* *c.* *d.*

102. How many squares are there in the following figure?



a. 5 *b.* 9
c. 7 *d.* 8

103. Arrange the following words in a logical and meaningful order.

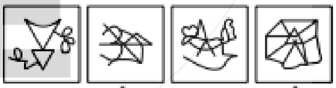
1. Country
2. Furniture
3. Forest
4. Wood
5. Trees

a. 1, 3, 5, 4, 2
b. 1, 4, 3, 2, 5
c. 2, 4, 3, 1, 5
d. 5, 2, 3, 1, 4

104. From the given answer figures, select the one in which the question figure is hidden/embedded.

Question figure

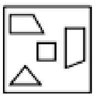
Answer figures



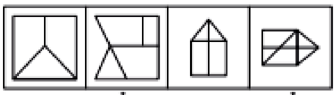
a. *b.* *c.* *d.*

105. In the question given below, find out which of the figures can be formed from the pieces gives in the problem figure.

Problem figure



Answer figures



a. *b.* *c.* *d.*

Mathematics

106. $\lim\limits_{x \to \infty} \dfrac{\int_0^{2x} xe^{x^2}\,dx}{e^{4x^2}}$ equals

a. 0 *b.* ∞ *c.* 2 *d.* $\frac{1}{2}$

107. If ω is the complex cube root of unity, then the value of $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\ldots\right)}$ is

a. -1 *b.* 1 *c.* $-i$ *d.* i

108. Let $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$, $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$. Then, the equation whose roots are α and β, is

a. $x^2 - x + 2 = 0$ *b.* $x^2 + x - 2 = 0$
c. $x^2 - x - 2 = 0$ *d.* $x^2 + x + 2 = 0$

109. The value of $\frac{3}{4} + \frac{15}{16} + \frac{63}{64} + \ldots$ upto n terms is

a. $n - \frac{4^n}{3} - \frac{1}{3}$ *b.* $n + \frac{4^{-n}}{3} - \frac{1}{3}$
c. $n + \frac{4^n}{3} - \frac{1}{3}$ *d.* $n - \frac{4^{-n}}{3} + \frac{1}{3}$

110. The period of the function $f(x) = \cos x + \{x\}$ is

a. 2π *b.* 1
c. π *d.* non-existent

111. If a function $f(x)$ is given by
$$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots \infty,$$ then at $x = 0$, $f(x)$

a. has no limit
b. is not continuous
c. is continuous but not differentiable
d. is differentiable

112. If g is the inverse of function f and $f'(x) = \sin x$, then $g'(x)$ is equal to

a. $\operatorname{cosec}\{g(x)\}$ *b.* $\sin\{g(x)\}$
c. $\frac{1}{\sin\{g(x)\}}$ *d.* None of these

113. A bag contains $(2n+1)$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $(n+1)$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then n is equal to

a. 10 *b.* 11 *c.* 12 *d.* 13

114. If $\phi(x)$ is a differentiable function, then the solution of the differential equation $dy + \{y\,\phi'(x) - \phi(x)\,\phi'(x)\}\,dx = 0$, is

a. $y = \{\phi(x) - 1\} + Ce^{-\phi(x)}$
b. $y\phi(x) = \{\phi(x)\}^2 + C$
c. $ye^{\phi(x)} = \phi(x)\,e^{\phi(x)} + C$
d. $y - \phi(x) = \phi(x)\,e^{-\phi(x)}$

115. The area of the region $R = \{(x, y): |x| \le |y|$ and $x^2 + y^2 \le 1\}$ is

a. $\frac{3\pi}{8}$ sq units *b.* $\frac{5\pi}{8}$ sq units
c. $\frac{\pi}{2}$ sq units *d.* $\frac{\pi}{8}$ sq unit

116. A and B are any two non-empty sets and A is proper subset of B. If $n(A) = 5$, then find the minimum possible value of $n(A \Delta B)$.

a. 1 *b.* 5
c. Cannot be determined *d.* None of these

117. If $\alpha = 2\tan^{-1}(\sqrt{2} - 1)$, $\beta = 3\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\left(-\frac{1}{2}\right)$ and $\gamma = \cos^{-1}\frac{1}{3}$, then

a. $\alpha < \beta < \gamma$ *b.* $\alpha < \gamma < \beta$
c. $\beta < \gamma < \alpha$ *d.* $\gamma < \beta < \alpha$

118. If $\frac{e^x + e^{5x}}{e^{3x}} = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$, then the value of $2a_1 + 2^3a_3 + 2^5a_5 + \ldots$ is

a. $e^2 + e^{-2}$ *b.* $e^4 - e^{-4}$
c. $e^4 + e^{-4}$ *d.* 0

119. Let **a**, **b** and **c** be three vectors satisfying $\mathbf{a} \times \mathbf{b} = 2(\mathbf{a} \times \mathbf{c})$, $|\mathbf{a}| = |\mathbf{c}| = 1$, $|\mathbf{b}| = 4$ and $|\mathbf{b} \times \mathbf{c}| = \sqrt{15}$. If $\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$, then λ equals

a. 1 *b.* -1 *c.* 2 *d.* -4

120. The total number of 4-digit numbers in which the digits are in descending order, is

a. ${}^{10}C_4 \times 4!$ *b.* ${}^{10}C_4$
c. $\frac{10!}{4!}$ *d.* None of these

121. The line which is parallel to X-axis and crosses the curve $y=\sqrt{x}$ at an angle of 45°, is

a. $x=\frac{1}{4}$ b. $y=\frac{1}{4}$
c. $y=\frac{1}{2}$ d. $y=1$

122. In a ΔABC, the lengths of the two larger sides are 10 and 9 units, respectively. If the angles are in AP, then the length of the third side can be

a. $5\pm\sqrt{6}$ b. $3\sqrt{3}$
c. 5 d. None of these

123. The arithmetic mean of the data 0, 1, 2, …, n with frequencies 1, ${}^nC_1$, ${}^nC_2$, …, ${}^nC_n$ is

a. n b. $\frac{2^n}{n}$
c. $n+1$ d. $\frac{n}{2}$

124. The mean square deviation of a set of n observations $x_1, x_2, \ldots, x_n$ about a point c is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i-c)^2$.

The mean square deviations about -2 and 2 are 18 and 10 respectively, the standard deviation of this set of observations is

a. 3 b. 2
c. 1 d. None of these

125. Let S be the focus of the parabola $y^2=8x$ and PQ be the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. The area of ΔPQS is

a. 4 sq units b. 3 sq units
c. 2 sq units d. 8 sq units

126. The number of real roots of the equation $e^{x-1}+x-2=0$ is

a. 1 b. 2
c. 3 d. 4

127. Minimise $Z=\sum_{j=1}^{n}\sum_{i=1}^{m}c_{ij}\,x_{ij}$

Subject to $\sum_{i=1}^{m}x_{ij}=b_j,\ j=1,\ 2,\ \ldots,n$

$\sum_{j=1}^{n}x_{ij}=b_j,\ i=1,2,\ldots,m$ is a LPP with number of constraints

a. $m-n$ b. mn
c. $m+n$ d. m/n

128. India plays two matches each with West Indies and Australia. In any match, the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent, the probability of India getting atleast 7 points is

a. 0.8750 b. 0.0875 c. 0.0625 d. 0.0250

129. Let M be a 3×3 non-singular matrix with $\det(M)=\alpha$. If M^{-1} adj (adj A) $=KI$, then the value of K is

a. α^3 b. α
c. α^2 d. None of these

130. Tangents are drawn from the origin to the curve $y=\cos x$. Their points of contact lie on

a. $x^2y^2=y^2-x^2$ b. $x^2y^2=x^2+y^2$
c. $x^2y^2=x^2-y^2$ d. None of these

131. The slope of the tangent to the curve $y=e^x\cos x$ is minimum at $x=a$, $0\le a\le 2\pi$, then the value of a is

a. 0 b. π
c. 2π d. $3\pi/2$

132. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$,

$L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$

are coplanar. Then, α can take value (s)

a. 1, 4, 5 b. 1, 2, 5 c. 3, 4, 5 d. 2, 4, 5

133. Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e. If A, A' are the vertices and S, S' are the foci of the ellipse, then area of $\Delta PSS'$: area of $\Delta APA'$ is equal to

a. $e^3:1$ b. $e^2:1$
c. $e:1$ d. $1/e:1$

134. The largest interval for which $x^{12}-x^9+x^4-x+1>0$ is

a. $(-4, 0]$ b. $(-100, 100)$
c. $(0, 1)$ d. $(-\infty, \infty)$

135. If $f(\theta)=\sin\left[\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right]$, where $-\frac{\pi}{4}<\theta<\frac{\pi}{4}$. Then, the value of $\frac{d}{d(\tan\theta)}f(\theta)$ is

a. 1 b. 2
c. 3 d. 4

136. If $(1+3+5+\ldots+p)+(1+3+5+\ldots+q)$ $=(1+3+5+\ldots+r)$, then the least possible value of $p+q+r$, given $p>6$, is

a. 12 b. 21
c. 45 d. 54

137. For $x>0$, $\lim_{x\to 0}\left\{(\sin x)^{1/x}+\left(\frac{1}{x}\right)^{\sin x}\right\}$ is equal to

a. 0 b. -1
c. 1 d. 2

138. If $0<x<\frac{\pi}{2}$, then

a. $\tan x<x<\sin x$ b. $x<\sin x<\tan x$
c. $\sin x<\tan x<x$ d. None of these

139. The degree of the differential equation satisfying the relation $\sqrt{1+x^2}+\sqrt{1+y^2}$ $=\lambda(x\sqrt{1+y^2}-y\sqrt{1+x^2})$ is

a. 1 b. 2
c. 3 d. 4

140. Let $f(x)$ be a polynomial of degree three satisfying $f(0)=-1$ and $f(1)=0$. Also, 0 is a stationary point of $f(x)$. If $f(x)$ does not have an extremum at $x=0$, then the value of $\int\frac{f(x)}{x^3-1}dx$ is

a. $\frac{x^2}{2}+C$ b. $x+C$
c. $\frac{x^3}{6}+C$ d. None of these

141. If $e^x+e^{f(x)}=e$, then the domain of $f(x)$ is

a. $(-\infty, 1)$ b. $(-\infty, 0)$
c. $(1, \infty)$ d. None

142. The equation of the line with gradient $-3/2$, which is concurrent with the lines $4x+3y-7=0$ and $8x+5y-1=0$, is

a. $3x+2y-2=0$
b. $3x+2y-63=0$
c. $2y-3x-2=0$
d. $2y-3x-63=0$

143. Area of the circle in which a chord of length $\sqrt{2}$ makes an angle $\pi/2$ at the centre, is

a. $\pi/2$ sq units b. 2π sq units
c. π sq units d. $\pi/4$ sq units

144. The ratio of the greatest value of $2-\cos x+\sin^2 x$ is to its least value, is

a. $7/4$ b. $11/4$
c. $13/4$ d. None of these

145. If z_1, z_2, z_3 and z_4 represent the vertices of a rhombus taken in the anti-clockwise order, then

a. $z_1+z_2+z_3+z_4=0$
b. $z_1+z_2=z_3+z_4$
c. $\arg\left(\frac{z_2-z_4}{z_1-z_3}\right)=\frac{\pi}{2}$
d. $\arg\left(\frac{z_1-z_2}{z_3-z_4}\right)=\frac{\pi}{4}$

146. If $\rho=\{(x, y)\mid x^2+y^2=1; x, y\in R\}$. Then, ρ is

a. reflexive
b. symmetric
c. transitive
d. anti-symmetric

147. A line makes the same angle θ with each of the X and Z-axes. If the angle β, which it makes with Y-axis, is such that $\sin^2\beta=3\sin^2\theta$, then $\cos^2\theta$ equals

a. $2/5$ b. $1/5$
c. $3/5$ d. $2/3$

148. If in a binomial distribution $n=4$, $P(X=0)=\frac{16}{81}$, then $P(X=4)$ equals

a. $\frac{1}{16}$ b. $\frac{1}{81}$
c. $\frac{1}{27}$ d. $\frac{1}{8}$

149. Let $f: R\to R$ be a function such that $f(x+y)=f(x)+f(y), \forall x, y\in R$

If $f(x)$ is differentiable at $x=0$, then which one of the following is incorrect?

a. $f(x)$ is continuous, $\forall x\in R$
b. $f'(x)$ is constant, $\forall x\in R$
c. $f(x)$ is differentiable, $\forall x\in R$
d. $f(x)$ is differentiable only in a finite interval containing zero

150. If binomial coefficients of three consecutive terms of $(1+x)^n$ are in HP, then the maximum value of n is

a. 1 b. 2
c. 0 d. None of these

Answers

Physics

1. (b)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (c)
11. (a)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (b)
21. (a)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (c)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (c)
31. (b)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (a)

Chemistry

41. (c)	**42.** (c)	**43.** (c)	**44.** (c)	**45.** (a)	**46.** (c)	**47.** (a)	**48.** (d)	**49.** (b)	**50.** (d)
51. (c)	**52.** (a)	**53.** (b)	**54.** (b)	**55.** (d)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (b)	**60.** (a)
61. (a)	**62.** (b)	**63.** (c)	**64.** (a)	**65.** (b)	**66.** (a)	**67.** (d)	**68.** (a)	**69.** (b)	**70.** (b)
71. (a)	**72.** (b)	**73.** (c)	**74.** (a)	**75.** (b)	**76.** (a)	**77.** (d)	**78.** (a)	**79.** (a)	**80.** (d)

English Proficiency

81. (d)	**82.** (a)	**83.** (c)	**84.** (b)	**85.** (a)	**86.** (d)	**87.** (c)	**88.** (b)	**89.** (a)	**90.** (d)
91. (c)	**92.** (b)	**93.** (d)	**94.** (c)	**95.** (d)					

Logical Reasoning

96. (a)	**97.** (a)	**98.** (c)	**99.** (a)	**100.** (a)	**101.** (a)	**102.** (b)	**103.** (a)	**104.** (d)	**105.** (b)

Mathematics

106. (d)	**107.** (a)	**108.** (d)	**109.** (b)	**110.** (d)	**111.** (b)	**112.** (c)	**113.** (a)	**114.** (a)	**115.** (c)
116. (a)	**117.** (b)	**118.** (d)	**119.** (d)	**120.** (b)	**121.** (c)	**122.** (a)	**123.** (d)	**124.** (a)	**125.** (a)
126. (a)	**127.** (c)	**128.** (b)	**129.** (b)	**130.** (c)	**131.** (b)	**132.** (a)	**133.** (c)	**134.** (d)	**135.** (a)
136. (b)	**137.** (c)	**138.** (d)	**139.** (a)	**140.** (b)	**141.** (a)	**142.** (a)	**143.** (c)	**144.** (c)	**145.** (c)
146. (b)	**147.** (c)	**148.** (b)	**149.** (d)	**150.** (d)					

Hints & Solutions

Physics

1. ***(b)*** According to Newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\Rightarrow \quad \log(\theta - \theta_0) = -kt + C \quad \ldots(i)$$

Given, at $t = 0$, $\theta = 60°C$

$$\log(60 - 25) = C$$

$$\Rightarrow \quad C = \log 35 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad kt = \log 35 - \log(\theta - 25)$$

$$\log(\theta - 25) = -kt + \log 35$$

$$kt = \log 35 - \log(\theta - 25) \quad \ldots(iii)$$

At, $t = 10$ min, $\theta = 50°C$

$$\therefore \quad k \times 10 = \log 35 - \log 25$$

$$\Rightarrow \quad k = \frac{1}{10}\log\left(\frac{7}{5}\right)$$

$$\left(\frac{1}{10}\log\frac{7}{5}\right)t = \log 35 - \log(\theta - 25)$$

When $t = 20$ min, $\theta = ?$

$$\therefore \quad \left(\frac{1}{10}\log\frac{7}{5}\right)20 = \log\left(\frac{35}{\theta - 25}\right)$$

$$\Rightarrow \quad 2\log\left(\frac{7}{5}\right) = \log\left(\frac{35}{\theta - 25}\right)$$

$$\Rightarrow \quad \log\left(\frac{7}{5}\right)^2 = \log\left(\frac{35}{\theta - 25}\right)$$

$$\Rightarrow \quad \left(\frac{7}{5}\right)^2 = \frac{35}{\theta - 25} \Rightarrow \theta = 42.8\ °C$$

2. ***(a)*** Let v be the speed of sound and v_s be the speed of forks. The apparent frequency of fork which moves towards the observer is

$$n_1 = \left(\frac{v}{v - v_s}\right) n$$

The apparent frequency of the fork which moves away from observer is

$$n_2 = \left(\frac{v}{v + v_s}\right) n$$

If x is the number of beats heard per second,

then $x = n_1 - n_2$

$$\Rightarrow \quad x = \left(\frac{v}{v - v_s}\right) n - \left(\frac{v}{v + v_s}\right) n$$

$$\Rightarrow \quad x = \frac{v(v + v_s) - v(v - v_s)}{v^2 - v_s^2}(n)$$

$$\Rightarrow \quad \frac{2vv_s n}{v^2 - v_s^2} = x \Rightarrow 2\left(\frac{v_s}{v}\right) n = x \qquad \{\because \text{ if } v_s << v\}$$

$$\Rightarrow \quad v_s = \frac{xv}{2n}$$

Given, $v = 340$ m / s, $x = 3$, $n = 340$ Hz.

$$\therefore \quad v_s = \frac{340 \times 3}{2 \times 340} = 1.5 \text{ m/s}$$

3. ***(a)*** First image is formed by reflection from the concave mirror (M_2).

$$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v_1} + \frac{1}{-2R} = \frac{-2}{R} \quad \left[\because f_1 = \frac{-R}{2}, u_1 = -2R\right]$$

$$\Rightarrow \quad v_1 = -\frac{2R}{3}$$

$$m = -\frac{v_1}{u_1}$$

$$\Rightarrow \quad \frac{-I}{O} = -\left(\frac{-2R}{3}\right) \cdot \frac{1}{(-2R)}$$

$$\Rightarrow \quad \frac{-I}{a} = \frac{-1}{3}$$

$$\Rightarrow \quad I = \frac{a}{3}$$

Radius of image circle = $a/3$ and image is formed by the convex mirror (M_1).

Object distance $= 2R - \frac{2R}{3} = \frac{4R}{3}$

$$f = \frac{-R}{2}, \quad v = ?$$

$$\frac{3}{4R} + \frac{1}{v_2} = -\frac{2}{R} \Rightarrow v_2 = -\frac{4R}{11}$$

$$\therefore \quad m_2 = -\frac{v_2}{u_2} = -\left(\frac{-4R/11}{4R/3}\right)$$

$$\Rightarrow \quad m_2 = \frac{3}{11}$$

New, object distance $= 2R - \left(-\frac{4R}{11}\right) = \frac{26R}{11}$

$$\frac{1}{v_3} - \frac{1}{26R} = -\frac{2}{R} \Rightarrow v_3 = -\frac{26R}{41}$$

$$m_3 = -\frac{v_3}{u_3} = -\frac{\left(-\frac{26R}{41}\right)}{\left(\frac{26R}{41}\right)} = \frac{11}{41}$$

$\because$ Radius of 3rd image $= \frac{a}{11} \times \frac{11}{41} = \frac{a}{41}$

4. ***(a)*** The net electric field at P is zero.

$$\therefore \quad E_P = E_1 + E_2 + E_{\text{sphere}} = 0$$

$$0 = E_1 + E_2 + E_{\text{sphere}}$$

$$\therefore \quad E_{\text{sphere}} = -E_1 - E_2$$

$$\therefore \quad |E_{\text{sphere}}| = \sqrt{E_1^2 + E_2^2}$$

$$= \sqrt{\left(\frac{q_1}{4\pi\varepsilon_0 a^2}\right)^2 + \left(\frac{q_2}{4\pi\varepsilon_0 b^2}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{q_1}{a^2}\right)^2 + \left(\frac{q_2}{b^2}\right)^2}$$

$$\therefore \quad E_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{q_1}{a^2}\right)^2 + \left(\frac{q_2}{b^2}\right)^2}$$

5. ***(c)*** Condsider that the body is projected vertically upwards from A with a speed u_0.

Using equation, $s = ut + \frac{1}{2}at^2$, we get

For first case, $-h = u_0 t_1 - \frac{1}{2}gt_1^2$ ...(i)

For second case, $-h = -u_0 t_2 - \frac{1}{2}gt_2^2$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$0 = u_0(t_2 + t_1) + \frac{1}{2}g(t_2^2 - t_1^2)$$

$$\Rightarrow \quad u_0 = \frac{1}{2}g(t_1 - t_2) \qquad \text{...(iii)}$$

Putting the value of u_0 in Eq. (ii), we get

$$-h = -\left(\frac{1}{2}\right)g(t_1 - t_2)t_2 - \left(\frac{1}{2}\right)gt_2^2$$

$$\Rightarrow \quad h = \frac{1}{2}g(t_1 t_2) \qquad \text{...(iv)}$$

For third case, $u = 0$, $t = ?$

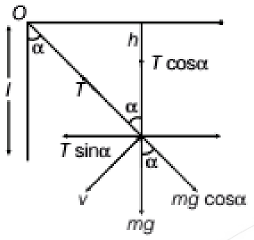
$$-h = 0 \times t - \left(\frac{1}{2}\right)gt^2$$

$$\Rightarrow \quad h = \left(\frac{1}{2}\right)gt^2 \quad \ldots(v)$$

On comparing Eqs. (iv) and (v), we get

$$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \Rightarrow t = \sqrt{t_1 t_2}$$

6. ***(c)*** For horizontal acceleration, net force in vertical direction should be zero.



$$T \cos\alpha = mg$$

Loss in potential energy = Gain in kinetic energy.

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad mgl\cos\alpha = \frac{1}{2}mv^2$$

$$\therefore \quad v = \sqrt{2gl\cos\alpha}$$

The net force towards O, is

$$T - mg\cos\alpha = mv^2/l$$

$$T - mg\cos\alpha = \frac{m\,2gl\cos\alpha}{l} = 2mg\cos\alpha$$

$$\Rightarrow \quad T = 3\,mg\cos\alpha$$

$$\therefore \quad T\cos\alpha = mg$$

$$3\,mg\cos^2\alpha = mg \Rightarrow \alpha = \cos^{-1}(1/\sqrt{3})$$

7. ***(a)*** Since, we know that, $r = \frac{mv}{Bq}$ ...(i)

$$\Rightarrow \quad KE = K = \frac{1}{2}mv^2$$

$$\therefore \quad mv = \sqrt{2Km}$$

$\therefore$ From Eq. (i), $r = \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB}$

$\Rightarrow \quad r \propto \sqrt{K}$ or $r = cK^{1/2}$ [where, c is a constant.]

$$\frac{dr}{dr} = c\frac{dK^{1/2}}{dr}$$

$$\text{or} \quad \frac{c\Delta k}{\Delta r} = 2\sqrt{K}$$

$$\text{or} \quad \frac{\Delta r}{r} = \frac{c\Delta K}{2\sqrt{K}\,c\sqrt{K}} = \frac{\Delta K}{2K}$$

$$\text{or} \quad \frac{\Delta r}{r} \times 100 = \frac{\Delta K}{2K} \times 100 = \frac{1}{2} \times 4\% = 2\%$$

8. ***(d)*** $\beta = 0.03$ cm, $D = 1$ m = 100 cm

Distance between images of the source = 0.8 cm

Distance of image from lens, $v = 80$ cm

Convex lens of focal length, $f = 16$ cm

Distance of slit from lens = u

By lens formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} + \frac{1}{u} = \frac{1}{16}$$

$$\Rightarrow \quad u = 20 \text{ cm}$$

$$\because \quad \text{Magnification, } m = \frac{v}{u} = \frac{80}{20} = 4$$

$$\text{Magnification} = \frac{\text{Distances between images of slits}}{\text{Distance between slits}}$$

$$= \frac{0.8}{d} = 4$$

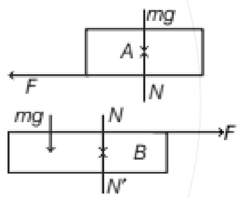
$$\Rightarrow \quad d = 0.2 \text{ cm}$$

$$\because \quad \beta = \frac{D\lambda}{d}$$

$$\Rightarrow \quad \beta = \frac{100\lambda}{0.2} \times 0.03 \times 10^{-2}$$

$$\text{or} \quad \lambda = 6000 \text{ Å}$$

9. ***(d)*** Free body diagram for the blocks A and B are as shown below



Equations of motion, $a_B = \frac{F}{M}$ (in $+x$-direction)

$$a_A = \frac{F}{m} (\text{in} - x\text{-direction}).$$

Relative velocity of A w.r.t. B,

$$a_{A,B} = a_A - a_B = -\frac{F}{m} - \frac{F}{M}$$

$$= -F\left(\frac{M+m}{Mm}\right) (\text{along} - x\text{-direction})$$

Initial relative velocity of A w.r.t. B,

$$u_{AB} = v_0$$

Final relative velocity of A w.r.t. B,

$$v^2 = u^2 + 2as$$

$$0 = v_0^2 - \frac{2F(m+M)s}{Mm}$$

$$\Rightarrow \quad s = \frac{Mmv_0^2}{2F(m+M)} \quad [\because GM = gR^2]$$

10. ***(c)*** During motion of the particle, total mechanical energy remains constant.

At the surface of the earth, total mechanical energy is

$$E_i = -\frac{GmM}{R} + \frac{1}{2}mv_0^2 = -\frac{GMmR}{R^2} + \frac{1}{2}mv_0^2$$

$$= -gmR + \frac{1}{2}mv_0^2 \qquad [\because GM = gR^2]$$

Total mechanical energy at height $h = R$ is

$$E_f = -\frac{GmM}{2R} + \frac{1}{2}mv^2 = -\frac{gmR}{2} + \frac{1}{2}mv^2.$$

According to the principle of conservation of energy,

$$E_i = F_F$$

$$\therefore \quad -gmR + \frac{1}{2}mv_0^2 = -\frac{gmR}{2} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad -2gR + v_0^2 = -gR + v^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 - gR}$$

11. ***(a)*** Volume of the balloon at any instant, when radius is r

$$V = \frac{4}{3}\pi r^3$$

Time rate of change of volume, $\frac{dV}{dt} = 4\pi r^2 . \frac{dr}{dt}$

Time rate of change of radius of balloon,

$$\frac{dr}{dt} = \frac{1}{4\pi r^2} . \frac{dV}{dt}$$

Flux through rubber band at given instant,

$$\phi = B(\pi r^2)$$

Induced emf,

$$E_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\pi r^2) = -2\pi r B\left(\frac{dr}{dt}\right)$$

$$= -2\pi r B\left(\frac{1}{4\pi r^2} . \frac{dV}{dt}\right) = -\frac{B}{2r} . \frac{dV}{dt}$$

As, volume of the balloon is decreasing, $\frac{dV}{dt}$ is negative.

$$\Rightarrow \quad E_{\text{induced}} = -\frac{(0.04)}{2 \times 10 \times 10^{-2}} \times (-100 \times 10^{-6})$$

$$= 20\,\mu V$$

12. ***(b)*** Elastic force in the string is conservative in nature.

$$\therefore \quad W = -\Delta U$$

where, W = work done by elastic force of string.

$$\therefore \quad W = -(U_F - U_i) = U_i - U_F$$

$$= \frac{1}{2}kx^2 - \frac{k}{2}(x+y)^2$$

$$= \frac{1}{2}kx^2 - \frac{1}{2}k(x^2 + y^2 + 2xy)$$

$$= \frac{1}{2}kx^2 - \frac{1}{2}ky^2 - \frac{1}{2}kx^2 - \frac{1}{2}k(2xy)$$

$$= -kxy - \frac{1}{2}ky^2$$

$\therefore$ The work done against elastic force is

$$W_{\text{external}} = -W = \frac{ky}{2}(2x + y)$$

13. ***(d)*** When the wire is stretched, its area and length changes in such a way that volume always remains same. Wire A has resistance R, length = l, radius = r.

When wire A is stretched, Resistance = R', length = l', radius = r_1

$\therefore$ Volume of original wire = Volume of stretched wire

$$V = V'$$

$$\Rightarrow \quad Al = A'l'$$

$$\text{or} \quad \pi r^2 l = \pi r_1^2 l' \quad \text{or} \quad \frac{l}{l'} = \frac{r_1^2}{r^2} = \frac{50}{100} = \frac{1}{2}$$

$$\therefore \quad R = \rho\frac{l}{A},\ R' = \rho\frac{l'}{A'}$$

$$\text{or} \quad \frac{R}{R'} = \frac{l/A}{l'/A'} = \frac{l}{A} \times \frac{A'}{l'}$$

$$= \frac{l}{l'} \times \frac{r_1^2}{r^2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$R' = 4R = 4 \times 10$$

$$= 40\,\Omega$$

14. ***(b)*** We know that, threshold wavelength, $\lambda_0 = \frac{hc}{\phi}$

$$\Rightarrow \quad \lambda_0 = \frac{(6.63 \times 10^{-34}) \times 3 \times 10^8}{2.3 \times (1.6 \times 10^{-19})}$$

$$= 5.404 \times 10^{-7}\,\text{m} = 5404\,\text{Å}$$

Thus, wavelength 4144 Å and 4972 Å will emit electron from the metal surface.

Energy incident on the surface per unit time for each wavelength

= Intensity of each × Area of the surface wavelength

$$= \frac{3.6 \times 10^{-3}}{3} \times 1 \times 10^{-4} = 1.2 \times 10^{-7}$$

Energy incident on the surface for each wavelength in 2 s

$$E = (1.2 \times 10^{-7}) \times 2 = 2.4 \times 10^{-7}\,\text{J}$$

Number of photons n_1 due to the wavelength 4144 Å

$$n_1 = \frac{E}{hc/\lambda_1} = \frac{E\lambda_1}{hc} = \frac{(2.4 \times 10^{-7})(4144 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)}$$

$$= 0.5 \times 10^{12}$$

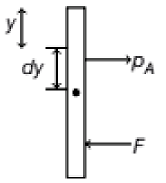
Number of photons n_2 due to wavelength 4972 Å

$$n_2 = \frac{E}{hc/\lambda_2} = \frac{E\lambda_2}{hc} = \frac{(2.4 \times 10^{-7})(4972 \times 10^{-10})}{(6.63 \times 10^{-34})(3 \times 10^8)}$$

$$N = n_1 + n_2 = 0.5 \times 10^{12} + 0.575 \times 10^{12}$$

$$= 1.075 \times 10^{12}$$

15. ***(c)*** At a depth y from the surface of the fluid, the net force acting on the gate element of width dy is



$$dF = (p_0 + \rho g y - p_o) \times 1\, dy = \rho g y\, dy$$

Torque of this force about the hinge is

$$d\tau = dF \times \left(\frac{1}{2} - y\right) = \rho g y\, dy \times \left(\frac{1}{2} - y\right)$$

Net torque experienced by the gate is

$$\tau_{\text{net}} = \int d\tau + F \times \frac{1}{2}$$

$$= \int_0^1 \rho g\, y\, dy \left(\frac{1}{2} - y\right) + F \times \frac{1}{2} = 0$$

$$\Rightarrow \quad F = \frac{\rho g}{6}$$

16. ***(b)*** The velocity of the photoelectrons is found by the relation,

$$evB = m\frac{v^2}{R} \Rightarrow v = \frac{e}{m}BR$$

The kinetic enregy of the photoelectrons is

$$k = \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

$$= \frac{1}{2}\frac{(1.6 \times 10^{-19})^2(2 \times 10^{-2})^2(23 \times 10^{-3})^2}{(9.1 \times 10^{-31})}$$

$$= 2.97 \times 10^{-15}\ \text{J}$$

or $$k = \frac{2.97 \times 10^{-15}}{1.6 \times 10^{-19}} = 18.36\ \text{keV}$$

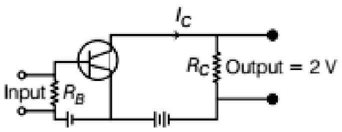
The energy of the incident photon is $E_v = \dfrac{hc}{\lambda}$.

$$\Rightarrow \quad E_v = \frac{12.4}{0.50} = 24.8\ \text{keV}$$

Since, the binding energy is the difference between these two values.

$$\text{BE} = E_v - k = 24.8 - 18.6 = 6.2\ \text{keV}$$

17. ***(c)***



$$V_0 = I_C R_C = 2$$

$$\Rightarrow \quad I_C = \frac{2}{2 \times 10^3} = 10^{-3}\ \text{A}$$

$$\text{Current gain} = \frac{I_C}{I_B} = 100$$

$$\Rightarrow \quad I_B = \frac{I_C}{100} = \frac{10^{-3}}{100} = 10^{-5}\ \text{A}$$

$$V_i = R_B I_B$$

$$= 1 \times 10^3 \times 10^{-5} = 10^{-2}\ \text{V}$$

$$\Rightarrow \quad V_i = 10\ \text{mV}$$

18. ***(d)*** Given, $y = 12x - \dfrac{3}{4}x^2$, $u_x = 3\ \text{ms}^{-1}$.

$$v_y = \frac{dy}{dt} = 12\frac{dx}{dt} - \frac{3}{4} \times (2x)\left(\frac{dx}{dt}\right)$$

$$= 12\frac{dx}{dt} - \frac{3}{2}x\frac{dx}{dt}$$

At $x = 0$, $v_y = u_y = 12\dfrac{dx}{dt}$

$$= 12u_x = 12 \times 3 = 36\ \text{ms}^{-1}$$

and $a_y = \dfrac{d}{dt}v_y = \dfrac{d}{dt}\left(\dfrac{dy}{dt}\right)$

$$= 12\frac{d^2x}{dt^2} - \frac{3}{2}\left(\left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2}\right)$$

But, $\dfrac{d^2x}{dt^2} = a_x = 0$, hence

$$a_y = -\frac{3}{2}\left(\frac{dx}{dt}\right)^2 = -\frac{3}{2}u_x^2$$

$$\Rightarrow \quad a_y = -\frac{3}{2} \times (3)^2 = -\frac{27}{2}\text{ms}^{-1}$$

$$\Rightarrow \quad |a_y| = \frac{27}{2}\text{ms}^{-1}$$

Range, $$R = \frac{2u_x u_y}{|a_y|}$$

$$= \frac{2 \times 3 \times 36}{27/2} = 16\ \text{m} = 16\ \text{m}$$

19. ***(d)*** Let us consider that there are n_1 moles of hydrogen and n_2 moles of helium in the given mixture. Then, the pressure of the mixture will be

$$p = \frac{n_1RT}{V} + \frac{n_2RT}{V} = (n_1 + n_2)\frac{RT}{V}$$

$$\Rightarrow 2 \times 101.3 \times 10^3 = (n_1 + n_2) \times \frac{(8.3 \times 300)}{20 \times 10^{-3}}$$

or $$(n_1 + n_2) = \frac{2 \times 101.3 \times 10^3 \times 20 \times 10^{-3}}{(8.3)\,(300)}$$

or $$n_1 + n_2 = 1.62 \qquad \ldots\text{(i)}$$

The mass of the mixture is (in grams),

$$n_1 \times m_1 + n_2 \times m_2 = 5$$

$$\Rightarrow \quad n_1 \times 2 + n_2 \times 4 = 5$$

$\Rightarrow \quad (n_1 + 2n_2) = 2.5 \quad \ldots(ii)$

Solving the Eqs. (i) and (ii), we get

$$n_1 = 0.74, n_2 = 0.88$$

Hence, $\frac{m_H}{m_{He}} = \frac{0.74 \times 2}{0.88 \times 4} = \frac{1.48}{3.52} = \frac{2}{5}$

20. ***(b)*** The path difference introduced due to the introduction of transparent sheet is given by $\Delta x = (\mu - 1)t$

If the central maxima occupies position of nth fringe, then $(\mu - 1)\, t = n\lambda = d \sin\theta$

$$\Rightarrow \sin\theta = \frac{(\mu - 1)\, t}{d} = \frac{(1.17 - 1) \times 1.5 \times 10^{-7}}{3 \times 10^{-7}} = 0.085$$

Hence, the angular position of central maxima is

$$\theta = \sin^{-1}(0.085) = 4.88^\circ = 4.9$$

For small angles, $\sin\theta = \theta = \tan\theta$

$$\Rightarrow \quad \tan\theta = \frac{y}{D}$$

$$\therefore \quad \frac{y}{D} = \frac{(\mu - 1)\, t}{d}$$

Shift of central maxima is $y = \frac{D(\mu - 1)\, t}{d}$

21. ***(a)*** Let the acceleration be a, hence $a = -\omega^2 x$.

Therefore, distance of the particle from the centre at any time t is given by

$x = r\cos(\omega t)$, where r is the amplitude

when $t = 1$ s, $x = r - a$

$\therefore (r - a) = r\cos\omega$

$$\cos\omega = \frac{r - a}{r} \quad \ldots(i)$$

when $t = 2$s, $x = r - a - b$

Therefore, $r - a - b = \cos 2\omega$

$$\therefore \quad r - a - b = r(2\cos^2\omega - 1) \quad \ldots(ii)$$

On substituting the value of $\cos\omega$ from Eq. (i) to Eq. (ii), we get

$$r - a - b = r\left[\frac{2(r - a)^2}{r^2} - 1\right]$$

$$\Rightarrow \quad r - a - b = \frac{2(r - a)^2}{r} - r$$

$$\therefore \quad r(3a - b) = 2a^2 \Rightarrow r = \frac{2a^2}{3a - b}$$

22. ***(a)*** U_i = intial potential energy of the system

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{a} + \frac{q^2}{a} + \frac{q^2}{a}\right] = \frac{1}{4\pi\varepsilon_0}\left(\frac{3q^2}{a}\right)$$

$$= 9 \times 10^9 \left(3 \times \frac{(0.1)^2}{1}\right) = 27 \times 10^7 \text{ J}$$

Let charge at A is moved to mid-point D.

$$\text{Then,} U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{(a/2)} + \frac{q^2}{a}\right]$$

$$= 5 \times \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{a^2}\right) = 5 \times 9 \times 10^9 \times \frac{(0.1)^2}{1^2}$$

$$= 45 \times 10^7 \text{ J}$$

Work done

$= U_f - U_i = 95 \times 10^7 - 27 \times 10^7 = 18 \times 10^7$ J

Also, energy supplied per second = 1000 J

$$\text{Time taken} = \frac{18 \times 10^7}{1000} = 18 \times 10^4 \text{ s}$$

$$= \frac{18 \times 10^4}{60 \times 60} \text{h} = 50 \text{ h}$$

23. ***(b)*** According to the conservation principle,

$$mgh = \frac{1}{2}kx^2 - mgx$$

where, x is maximum elongation in the spring (when the particle is in its lowest extreme position).

$$\Rightarrow \quad \frac{1}{2}kx^2 - mgx - mgh = 0$$

$$\text{or} \quad x^2 - \frac{2mg}{k}x - \frac{2mg}{k}\cdot h = 0$$

$$\therefore \quad x = \frac{\frac{2mg}{k} \pm \sqrt{\left[\left(\frac{2mg}{k}\right)^2 + 4 \times \frac{2mg}{k}h\right]}}{2}$$

$\because$ Amplitude = elongation in the spring for lowest extreme position – elongation in spring for equilibrium position.

$$= x - x_0 = \frac{mg}{k}\sqrt{\left(1 + \frac{2hk}{mg}\right)} \quad \left[\because x_0 = \frac{mg}{k}\right]$$

24. ***(c)*** Given, $\frac{M_U}{M_{Pb}} = 3$

Let the initial mass of uranium be M_0.

Final mass of uranium after time t, $M = \frac{3}{4}M_0$.

According to the law of disintegration,

$$\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{M_0}{M} = (2)^{t/T}$$

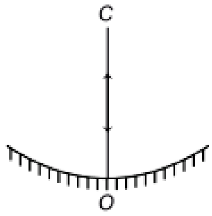
$$\therefore \log_{10}\left(\frac{M_0}{M}\right) = \frac{t}{T}\log_{10}(2)$$

$$t = T\frac{\log_{10}\left(\frac{M_0}{M}\right)}{\log_{10}(2)} = \frac{T\log_{10}\left(\frac{4}{3}\right)}{\log_{10}(2)}$$

$$= \frac{T\log_{10}(1.333)}{\log_{10}(2)} = 4.5 \times 10^9\left(\frac{0.1249}{0.3010}\right)$$

$$\Rightarrow \quad t = 1.867 \times 10^9 \text{ yrs}$$

25. ***(c)*** When there is no water in the mirror, the rays of light are incident normally on the mirror and retrace their path. So, we get an image coincident with the object as shown in the figure.



When the mirror is filled with the water, then equivalent focal length f is given by

$$\frac{1}{f} = \frac{1}{f_{\text{water lens}}} + \frac{1}{f_{\text{concave mirror}}} + \frac{1}{f_{\text{water lens}}}$$

$$\Rightarrow \quad \frac{1}{f} = 2 \times \left(\frac{1}{f_{\text{water lens}}}\right) + \left(\frac{1}{f_{\text{concave mirror}}}\right)$$

$$\Rightarrow \quad \frac{1}{f} = 2(\mu - 1)\left(\frac{1}{R}\right) + \frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{2(\mu - 1)}{R} + \frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{2\mu}{R} \Rightarrow f = \frac{R}{2\mu}$$

Clearly, focal length of the new optical system is less than the original. So, the object is effectively at a distance greater than twice the focal length. So, the real image will be formed between F and $2F$ or we can say between C and O.

26. ***(b)*** Total current when direct current of 5A is superimposed on alternating current,

$$I = (5 + 10 \sin \omega t)$$

$$\Rightarrow \quad I_{\text{eff}} = \left[\frac{\int_0^T I^2 dt}{\int_o^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (5 + 10 \sin \omega t)^2 dt\right]^{1/2}$$

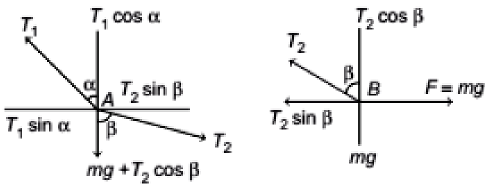
$$= \left[\frac{1}{T}\int_0^T (25 + 100 \sin \omega t + 100 \sin^2 \omega t)\right]^{1/2}$$

But, $\frac{1}{T}\int_0^T \sin \omega t \cdot dt = 0$ and $\frac{1}{T}\int_0^T \sin^2 \omega t \cdot dt = \frac{1}{2}$

So, $$I_{\text{eff}} = \left[25 + \frac{1}{2} \times 100\right]^{1/2}$$

$$= 5\sqrt{3} \text{ A}$$

27. ***(c)*** Consider FBD of A and B as shown below.



Balancing forces, we have

From FBD of A,

$$T_1 \sin\alpha = T_2 \sin\beta \quad \ldots\text{(i)}$$

$$T_1 \cos\alpha = T_2 \cos\beta + mg \quad \ldots\text{(ii)}$$

From FBD of B,

$$T_2 \cos\beta = mg \quad \ldots\text{(iii)}$$

$$T_2 \sin\beta = mg \quad \ldots\text{(iv)}$$

$$\Rightarrow \quad \tan\beta = 1 \quad \text{[From Eqs. (iii) and (iv)]}$$

$$\therefore \quad \sin\beta = \frac{1}{\sqrt{2}}$$

Hence, $T_2 = \frac{mg}{\sin\beta} = \sqrt{2}$

Using Eqs. (iv) in (i) and (iii) in (ii), we get

$$T_1 \sin\alpha = mg \text{ and } T_1 \cos\alpha = mg + mg = 2mg$$

$$\tan\alpha = \frac{1}{2} \Rightarrow \cos\alpha = \frac{2}{\sqrt{5}}$$

$$\therefore \quad T_1 = \frac{2mg}{\cos\alpha} = \frac{2mg}{\frac{2}{\sqrt{5}}}$$

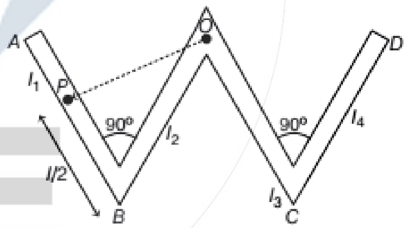
$$\Rightarrow \quad T_1 = mg\sqrt{5}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{\sqrt{5}}{\sqrt{2}} \Rightarrow \sqrt{2}T_1 = \sqrt{5}T_2$$

28. ***(b)*** Total moment of inertia

$$= I_1 + I_2 + I_3 + I_4 = 2I_1 + 2I_2$$

$$= 2(I_1 + I_2)[I_3 = I_2, I_1 = I_4]$$



Now, $$I_2 = I_3 = \frac{Ml^2}{3}$$

Using parallel axis theorem, we have

$$I = I_{CM} + Mx^2$$

and $$x = \sqrt{l^2 + \frac{l^2}{4}}$$

$$I_1 = I_4 = \frac{Ml^2}{12} + M\left[\sqrt{l^2 + \left(\frac{l}{2}\right)^2}\right]^2$$

Putting all the values, we get

$$I = 10\left(\frac{Ml^2}{3}\right)$$

29. ***(a)*** Wavelength of second line of Balmer series is

$$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] \Rightarrow \frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\frac{1}{\lambda_2} = R\left[\frac{3}{16}\right] \quad \Rightarrow \quad \lambda_2 = \frac{16}{3R}$$

For first line of Balmer series,

$$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda_1} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda_1} = \frac{5R}{36} \quad \Rightarrow \quad \lambda_1 = \frac{36}{5R}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{36}{5R} \times \frac{3R}{16} = \frac{27}{20}$$

$$\Rightarrow \quad \lambda_1 = \frac{27}{20} \times 4861 \text{ Å}$$

30. ***(c)*** If v be the velocity of charge particle of charge q accelerated through potential difference V, then

$$\frac{1}{2}mv^2 = qV$$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

As we know that, when a charge particle is allowed to move in a uniform magnetic field, then it describes spiral (or circular) path.

Centripetal force, $\frac{mv^2}{R} = qvB$

$$\therefore \quad v = \left(\frac{qB}{m}\right)R$$

Hence, $$\sqrt{\frac{2qV}{m}} = \left(\frac{qB}{m}\right)R$$

$$\Rightarrow \quad R = \left(\frac{2mV}{q}\right)^{1/2} \times \frac{1}{B}$$

Here, V, q and B are constant.

Hence, $\quad m \propto R^2$

So, $$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

31. ***(b)*** Water wets glass and so the angle of contact is zero. Neglecting the small mass in the meniscus, for full rise,

$$2\pi rT = \pi r^2 h\rho g$$

$$\Rightarrow \quad h = \frac{2T}{r\rho g}$$

$$= \frac{2 \times 0.07}{0.25 \times 10^{-3} \times 1000 \times 9.8}$$

$$= 0.057 \text{ m} = 5.7 \text{ cm}$$

But here the tube is only 2 cm above the water and so, water will rise by 2 cm and meet the tube at an angle θ such that

$$2\pi rT \cos\theta = \pi r^2 h' \rho g$$

$$\Rightarrow \quad 2T \cos\theta = h' r\rho g$$

$$\Rightarrow \quad \cos\theta = \frac{h' r\rho g}{2T}$$

$$\Rightarrow \quad \cos\theta = \frac{2 \times 10^{-2} \times 0.25 \times 10^{-3} \times 1000 \times 9.8}{2 \times 0.07}$$

$$= 0.35$$

$$\Rightarrow \quad \theta = \cos^{-1}(0.35)$$

$$= 69.5 \simeq 70$$

32. ***(d)*** Horizontal and vertical components of initial velocity are

$$u_x = 20\sqrt{2}\cos 45° = 20 \text{ m/s}$$

and $$u_y = 20\sqrt{2}\sin 45° = 20 \text{ m/s}$$

After 1s, horizontal component remains unchanged, while the vertical component becomes

$$v_y = u_y - gt$$

Due to explosion, one part comes to rest. Hence, from the conservation of linear momentum, vertical component of second part will become $v'_y = 20$ m/s. Therefore, maximum height attained by the second part will be

$$H = h_1 + h_2$$

Here, h_1 = height attained in 1 s.

Using, $h = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad h_1 = (20 \times 1) - \frac{1}{2} \times 10 \times (1)^2$$

$$h_1 = 15 \text{ m}$$

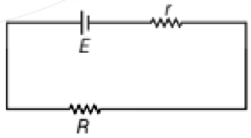
Now, $\quad a = g = 10 \text{ m/s}^2$

$$h_2 = \frac{v_y'^2}{2g}$$

$$= \frac{(20)^2}{2 \times 10} = 20 \text{ m}$$

$$\Rightarrow \quad H = 20 + 15 = 35 \text{ m}$$

33. ***(a)***



Here, $$r = \frac{r_1 r_2}{r_1 + r_2} = \frac{(1)(1)}{1+1} = \frac{1}{2}\,\Omega$$

and $$E = \frac{\Sigma E/r}{\Sigma 1/r} = \frac{\left(\frac{10}{1}\right) + \left(\frac{10}{1}\right)}{1+1} = 10 \text{ V}$$

Maximum power will be developed across R, when

$$R = r = \frac{1}{2}\,\Omega$$

$$i_{max} = \frac{E}{R+r} = \frac{10}{1/2 + 1/2} = 10 \text{ A}$$

$$P_{max} = i_{max}^2\, R = (10)^2 \left(\frac{1}{2}\right) = 50 \text{ W}$$

34. ***(b)*** Let v = speed of neutron before collision.

v_1 = speed of neutron after collision.

v_2 = speed of proton or hydrogen atom after collision.

and ΔE = energy of excitation.

From the conservation of linear momentum,

$$mv = mv_1 + mv_2$$

$$\Rightarrow \quad v = v_1 + v_2 \qquad \ldots\text{(i)}$$

From conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$v^2 = v_1^2 + v_2^2 \qquad \ldots\text{(ii)}$$

From squaring Eq. (i), we get

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2$$

From Eq. (ii), we get

$$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$$

$$\therefore \quad 2v_1v_2 = \frac{2\Delta E}{m}$$

$$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$$

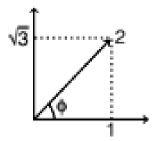
As, $v_1 - v_2$ must be real, $v^2 - \frac{4\Delta E}{m} \geq 0$

$$\Rightarrow \quad \frac{1}{2}mv^2 \geq 2\Delta E$$

The minimum energy that can be absorbed by the hydrogen atom in the ground state to go into the excited state is 2eV. Therefore, the maximum kinetic energy needed is

$$\frac{1}{2}mv_{min}^2 = 2 \times 10.2 = 20.4 \text{ eV}$$

35. ***(a)*** $A_R = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$



$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$$\therefore \quad y = 2\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\frac{d^2y}{dt^2} = a = -2\omega^2 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$a_{max} = -2\omega^2 = g$$

The minimum value of ω for which mass just breaks off the plank.

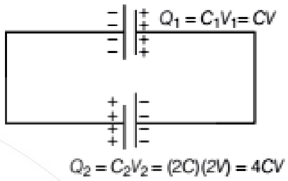
$$\omega = \sqrt{\frac{g}{2}}$$

This will be happen for the first time when

$$\omega t + \frac{\pi}{3} = \frac{\pi}{2} \text{ or } \omega t = \frac{\pi}{6}$$

$$\therefore \quad t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{2}{g}}$$

36. ***(b)*** The diagrammatic representation of the given problem is shown in the figure.



The net charge shared between the two capacitors

$$Q' = Q_2 - Q_1 = 4\,CV - CV = 3\,CV$$

The two capacitors will have same potential, say V'.

The net capacitance of the parallel combination of the two capacitors will be

$$C' = C_1 + C_2 = C + 2C = 3C$$

The potential of the capacitors will be

$$V' = \frac{Q'}{C'} = \frac{3CV}{3C} = V$$

The electrostatic energy of the capacitors will be

$$E' = \frac{1}{2}C'V'^2 = \frac{1}{2}(3C)V^2 = \frac{3}{2}CV^2$$

37. ***(b)*** Distance between two cars leaving from the station x is

$$d = \frac{1}{6} \times 60 = 10 \text{ km}$$

Man meets the first car after time,

$$t_1 = \frac{60}{60 + 60} = \frac{1}{2} \text{ h}$$

He will meet the next car after time,

$$t_2 = \frac{10}{60 + 60} = \frac{1}{12} \text{ h}$$

In the remaining half an hour, the number of cars he will meet again is, $n = \frac{1/2}{1/12} = 6$

$\therefore$ Total number of cars would be meet on route will be 7.

38. ***(c)*** Energy required to raise a satellite upto height h,

$$E_1 = \Delta U = \frac{mgh}{1 + \frac{h}{R}} \qquad \ldots\text{(i)}$$

E_2 = energy required to put in orbit $= \frac{1}{2} m v_0^2$

$$= \frac{1}{2} m \left(\frac{GM}{r}\right) \left[\text{as, } v_0 \text{ orbital speed} = \sqrt{\frac{GM}{r}}\right]$$

$$= \frac{1}{2} m \left(\frac{GM}{R+h}\right) = \frac{1}{2} m \left(\frac{GM}{R^2}\right) \times \frac{R}{1+\frac{h}{R}}$$

$$= \frac{1}{2} m . \frac{gR^2}{R^2} \times \frac{R}{1+h/R} \qquad [\because GM = gR^2]$$

$$E_2 = \frac{mgR}{2\left(1+\frac{h}{R}\right)} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{E_1}{E_2} = \frac{2h}{R}$$

39. ***(b)*** As, frequency,

$$f \propto \sqrt{mg} \text{ or } f \propto \sqrt{g}$$

In water, $f_w = 0.8 f_{\text{air}}$

$$\frac{g'}{g} = (0.8)^2 = 0.64 \Rightarrow 1 - \frac{\rho_w}{\rho_m} = 0.64$$

$$\Rightarrow \frac{\rho_w}{\rho_m} = 0.36 \qquad \ldots\text{(i)}$$

In liquid, $\frac{g'}{g} = (0.6)^2 = 0.36$

$$1 - \frac{\rho_l}{\rho_m} = 0.36 \Rightarrow \frac{\rho_l}{\rho_m} = 0.64 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\rho_l}{\rho_w} = \frac{0.64}{0.36} \Rightarrow \rho_l = 1.77$$

Here, ρ_w = relative density of water = 1

ρ_m = relative density of mass

ρ_l = relative density of liquid.

40. ***(a)*** Let N be the number of turns and R be the radius of the coil.

Then, $l = 2\pi RN$

$$\Rightarrow R = \frac{l}{2\pi N} \qquad \ldots\text{(i)}$$

Now, magnetic moment of the coil is

$$M = NiA = Ni(\pi R^2)$$

$$\Rightarrow M = (Ni\pi)\left(\frac{l^2}{4\pi^2 N^2}\right) = \frac{il^2}{4\pi N}$$

Maximum value of M can be

$$M_{\max} = \frac{il^2}{4\pi} \quad \text{at} \quad N = 1$$

$$\therefore \tau_{\max} = M_{\max} B \sin 90° = \frac{iBl^2}{4\pi}$$

$$\Rightarrow \tau_{\max} = \frac{iBl^2}{4\pi}$$

Chemistry

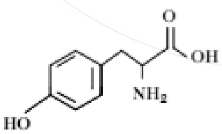
41. ***(c)*** NO pollutant is the main product of automobiles exhaust.

42. ***(c)*** The harmful chemical present in tobacco is arsenic.

43. ***(c)*** The disease causes by the high concentration of hydrocarbon pollutants in atmosphere is cancer. Hydrocarbon causes irritation and hence, abnormal growth of tissues.

44. ***(c)*** Thiol, i.e. R—SH participates in disulphide bond formation of protein.

45. ***(a)*** Structure of tyrosine is



46. ***(c)*** The third law of thermodynamics helps to calculate absolute entropies of pure substance at different temperature.

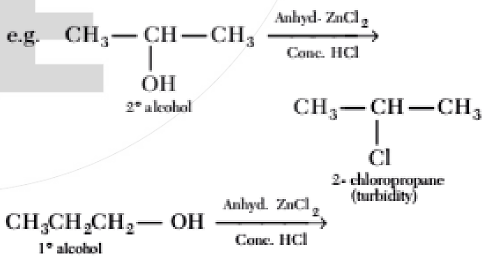
$$\Delta S = S_T - S_0 = \int_0^T \frac{C_p \cdot dt}{T} \Rightarrow \Delta S = C_p \ln T$$

$$\Delta S = 2.303\, C_p \log T$$

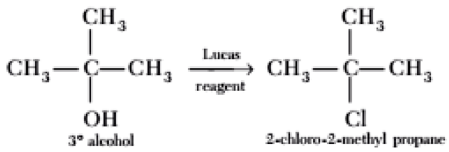
47. ***(a)*** Dipole-dipole interaction occur in the alkyl halides. Due to this interaction, the boiling point of alkyl halide increases as compared to other alkanes.

48. ***(d)*** Lucas test is given by both (b) and (c) but not by *n*-alcohol.

e.g.

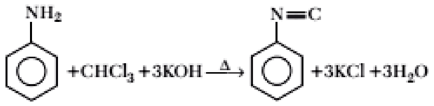


No turbidity at room temperature



The precipitate formed is based on the difference in reactivity of the three classes of alcohol. The difference in reactivity reflects the different case of formation of the corresponding carbocations. Tertiary carbocation is far more stable than 2° and 1°, while primary carbocation is least stable.

49. ***(b)*** When a primary amine (aromatic or aliphatic) is warmed with chloroform and alc. KOH, it forms an isocyanide or carbylamine having offensive smell. This reaction is called carbylamine reaction.



50. ***(d)*** The colour of $CoCl_3 \cdot 5NH_3 \cdot H_2O$ is pink.

51. ***(c)*** Complex salts has two different metallic elements but give test for only one element.

e.g. $K_4[Fe(CN)_6]$ gives test for K^+ only not for Fe^{2+} ions. $K_4[Fe(CN)_6] \rightleftharpoons 4K^+ + [Fe(CN)_6]^{4-}$

52. ***(a)*** Mg metal is present in chlorophyll.

53. ***(b)*** Zeise's salt is the common name of $K[PtCl_3(\eta^2 - C_2H_4)]$.

54. ***(b)*** Cobalt metal is present in vitamin B_{12}.

55. ***(d)*** Anthracene is purified by sublimation because in this method, a substance directly changes from (or transient from) the solid to the gas phase without passing through the intermediate liquid phase.

56. ***(d)*** Temperature is an intensive property. Intensive property is that which does not depend upon the quantity of matter present in the system. e.g. temperature, density, refractive index, boiling point, etc.

57. ***(b)*** Nitrous oxide is the laughing gas (i.e. N_2O).

58. ***(a)*** Brown ring test is used to detect nitrate.

$$NO_3^- + 3Fe^{2+} + 4H^+ \longrightarrow 3Fe^{3+} + NO + 2H_2O$$

$$[Fe(H_2O)_6]^{2+} + NO \longrightarrow \underset{\text{(Brown ring)}}{[Fe(H_2O)_5.(NO)]^{2+}} + H_2O$$

59. ***(b)*** Semiconductor materials, like Si and Ge are usually purified by zone refining. Zone refining method of purifying crystal or to prepare high pure materials, mainly semiconductor, in which a nano region of a crystal is melted and this molten zone is moved along the crystal.

The molten region will melt impure solid at its forward edge and leaves a pure materials behind. The impurity will be removed at one end.

60. ***(a)*** Smelting method is not used for concentration of metal. In smelting, oxides of less electropositive metals are reduced by strong heating with coal/coke. Flux is added to reduce melting point of impurities and to change them into slag.

61. ***(a)*** The Le-Chatelier's principle states that, if a system at equilibrium is subjected to a change in temperature, pressure or concentration, the equilibrium shifts in the direction that tends to under the effect of change.

According to this, only statement (a) is correct, i.e. increase in temperature favours the endothermic reaction while decrease in temperature favours the exothermic reaction.

Increase of pressure shifts, the equilibrium in that side in which number of gaseous moles decreases.

62. ***(b)*** van't Hoff equation shows variation of chemical equilibrium with temperature.

$$\log \frac{K_2}{K_1} = \frac{\Delta H}{2.303R}\left[\frac{T_2 - T_1}{T_1 \cdot T_2}\right]$$

where, ΔH = enthalpy change, K_1 and K_2 = equilibrium constants at temperature T_1 and T_2 respectively and

R = universal gas constant.

63. ***(c)*** NH_3 is strong base as it has one lone pair. Basic character decreases from N to Bi due to increase in atomic size.

64. ***(a)*** $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$ is the correct order of Bronsted base because $HClO_4 > HClO_3 > HClO_2 > HClO$ is the correct order of conjugated acid.

(Since, $HClO_4$ can easily give H^+ ion.) In ClO_4^-, the negative charge is not localised because of the resonance.

But in ClO^-, there is no resonance. Thus, the basicity will increase.

65. ***(b)*** The mass of the substance deposited when one Faraday of charge is passed through its solution is equal to gram equivalent weight.

66. ***(a)*** $\eta = -\dfrac{nFE_{cell}}{\Delta H} \times 100$

67. ***(d)*** For a gaseous reaction, the units of rate of reaction are atm $time^{-1}$ or bar $time^{-1}$.

68. ***(a)*** Unit of rate constant for reactions of second order is $L\ mol^{-1}\ s^{-1}$.

General formula = $(\text{concentration})^{1-n}\ time^{-1}$

$= [mol\ L^{-1}]^{1-n}\ time^{-1}$

where, n = order of reaction

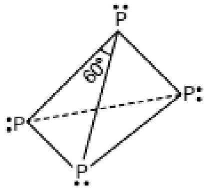
69. ***(b)*** The formula of first order reaction is $A = [A_0]e^{-kt}$.

∴ The concentration of reactants will exponentially decreases with time.

$$A \propto e^{-t}$$

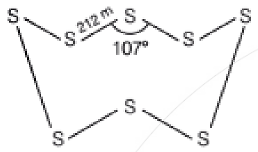
70. ***(b)*** *Aqua-regia* is a mixture of conc. HNO_3 (1 part) + conc. HCl (3 parts). Its IUPAC name is nitric acid hydrochloride.

71. ***(a)*** P_4 **molecule** The four sp^3-hybridised phosphorus atoms lie at the corners of a regular tetrahedron with $\angle PPP = 60°$. Each phosphorus atom is linked to three other P-atoms by covalent bonds.



So, that each P-atom complete its octet.

S_8 molecule Sulphur exists as octa-atomic solid because sulphur does not form $p\pi$-$p\pi$ multiple bonds due to its larger size and hence, do not exist as diatomic molecule instead S form complex structure by forming single bond resulting a puckered 8-membered crown shaped rings. Thus $\angle SSS = 107°$.



72. ***(b)*** The formula of borax is $Na_2B_4O_4 \cdot 10H_2O$.

$$NaAlSi_3O_8 \longrightarrow \text{Albite}$$
$$Na_2SO_4 \cdot 10H_2O \longrightarrow \text{Glauber's salt}$$
$$NaNO_3 \longrightarrow \text{Chile saltpetre}$$

73. ***(c)*** The Solvay-ammonia process is used for the preparation of sodium carbonate. $CaCl_2$ is produced as a byproduct which has no use.

The main steps of Solvay-ammonia process are

(i) $NaCl + CO_2 + NH_3 + H_2O \longrightarrow NaHCO_3 + NH_4Cl$

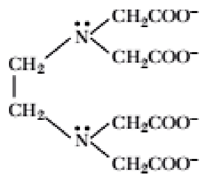
(ii) $CaCO_3 \longrightarrow CO_2 + CaO$

(iii) $2NH_4Cl + CaO \longrightarrow 2NH_3 + \underset{\text{(By product)}}{CaCl_2} + H_2O$

74. ***(a)*** 40 elements are present in *d*-block.

75. ***(b)*** $FeOCr_2O_3 \rightarrow$ Chromite

76. ***(a)*** Ethylenediammine tetraacetate (EDTA) represent the hexadentar ligand.



77. ***(d)*** Prussian blue is $Fe_4[Fe(CN)_6]_3$.

78. ***(a)*** Am has maximum number of oxidation states, i.e. + 3, + 4, + 5, + 6.

79. ***(a)*** $K_4[Fe(CN)_6]$ is used in detecting Fe^{3+} ion only not other.

$$4Fe^{3+} + 3K_4[Fe(CN)_6] \longrightarrow Fe_4[Fe(CN)_6]_3 + 12K^+$$

80. ***(d)*** For a process to be spontaneous, $\Delta G = -$ ve.

ΔH	ΔS	ΔG = T · ΔS	Nature of reaction
(−)	(+)	(−)	Spontaneous at all temperatures
(−)	(−)	(−) at low temperature (+) at high temperature	Spontaneous at low temperatures non-spontaneous at high temperature
(+)	(+)	(+) at low temperature (−) at high temperature	Non-spontaneous at low temperatures, spontaneous at high temperatures
(+)	(−)	(+)	Non-spontaneous at all temperatures

a. English Proficiency

81. ***(d)*** 'Insipid' means lacking in spirit.

82. ***(a)*** 'Astute' means having or showing shrewdness and an ability to notice and understand things clearly. So, 'shrewd' would be its correct answer.

83. ***(c)*** 'Ramification' means a complex or unwelcome consequence of an action or event. So, 'consequence' would be its correct synonym.

84. ***(b)*** The word 'contemporary' means belonging to the same time. So, option (b) is the suitable word for the given blank.

85. ***(a)*** For women, 'spinster' is used. 'Spinster' means a woman who is not married.

86. ***(d)*** The word 'violate' means to go against or refuse to obey a law. Hence, 'violation' should be used in the given blank.

87. ***(c)*** The word 'aggressive' means angry and behaving in a threatening way. So, 'aggressive' should be used in the given blank.

88. ***(b)*** 'Profusion' means in plenty or in ample. Hence, 'scarcity' is its correct antonym that means less in quantity.

89. ***(a)*** 'Languid' means lazy and thus 'energetic' would be its correct antonym.

90. ***(d)*** A person who loves everybody is called 'Altruist'.

91. ***(c)*** A small village or a group of houses is called 'Hamlet'.

92. ***(b)*** The one which can be substituted for the given words/sentence is given in option (b) i.e. fantasy.

93. ***(d)*** The correct sequence is DCAB.

94. ***(c)*** The correct sequence is CABD.

95. ***(d)*** The correct sequence is BDAC.

b. Logical Reasoning

96. ***(a)*** 1 is adjacent to 2, 4, 5 and 6. So, 1 is opposite to 3.

97. ***(a)*** The pattern is as follows,

$$2+9=11$$
$$9-2=7$$

Also, $8+5=13$

$$5-8=-3$$

Now, $7+?=10$

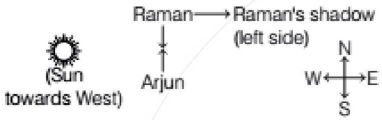
$$\boxed{?=3}$$
$$3-7=-4$$

Also, $6+4=10$

$$\boxed{4-6=-2}$$

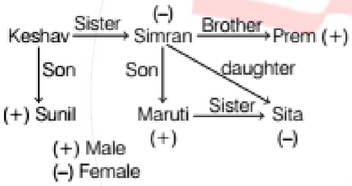
So, the correct options are $(3, -2)$.

98. ***(c)*** According to the question,



$\because$ Raman's shadow towards East, so Arjun was facing North.

99. ***(a)***



From the above family diagram, it is clear that Sunil and Maruti are cousins.

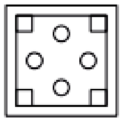
100. ***(a)*** From option (a),

$$24 \div 6+12-16=0$$

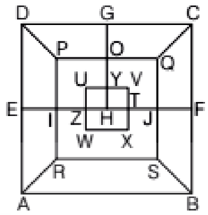
$\Rightarrow$ $4+12-16=0 \Rightarrow 16-16=0$

$\therefore$ LHS = RHS

101. ***(a)*** The unfolded figure of question figure is depicted in the answer figure (a).



102. ***(b)***



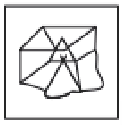
Number of squares

DGHE, GCFH, PQSR, UVXW, UYHZ, VTHY, POHI, QOHJ, ABCD = 9

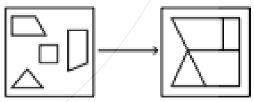
103. ***(a)*** From the given words, it is deduced that a country contains forests, a forest has trees, trees give wood that is used to make furniture.

So, the correct logical order is 1, 3, 5, 4, 2.

104. ***(d)*** Clearly, question figure is embedded in the answer figure (d).



105. ***(b)*** Figure given in option (b) can be formed by joining pieces given in the question figure as shown below.



Mathematics

106. ***(d)*** $\lim_{x \to \infty} \dfrac{\int_0^{2x} x e^{x^2}\, dx}{e^{4x^2}} = \lim_{x \to \infty} \dfrac{\int_0^{2x} e^{x^2}\, d(x^2)}{2e^{4x^2}}$

$$= \lim_{x \to \infty} \frac{[e^{x^2}]_0^{2x}}{2e^{4x^2}} = \lim_{x \to \infty} \frac{e^{4x^2}-1}{2e^{4x^2}}$$

$$= \lim_{x \to \infty} \left(\frac{1}{2} - \frac{1}{2e^{4x^2}}\right) = \frac{1}{2}$$

Alternate Method

Using $\dfrac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x)dx\right) = f(b)b'(x) - f(a)a'(x)$,

we get, $\lim_{x \to \infty} \dfrac{\int_0^{2x} x e^{x^2}\, dx}{e^{4x^2}} = \lim_{x \to \infty} \dfrac{2x e^{(2x)^2}(2) - 0}{e^{4x^2}(8x)}$

$$= \lim_{x \to \infty} \frac{1}{2} \frac{e^{4x^2}}{e^{4x^2}} = \lim_{x \to \infty} \frac{1}{2} = \frac{1}{2}$$

107. ***(a)*** We have, $\dfrac{1}{2} + \dfrac{3}{8} + \dfrac{9}{32} + \dfrac{27}{128} + \dots$

$$= \frac{3^0}{2^1} + \frac{3^1}{2^3} + \frac{3^2}{2^5} + \frac{3^3}{2^7} + \dots$$

$$= \frac{1}{2}\left[1 + \frac{3}{2^2} + \frac{3^2}{2^4} + \frac{3^3}{2^6} + \dots\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-\frac{3}{2^2}}\right] = \frac{1}{2}\left[\frac{1}{1-\frac{3}{4}}\right] = 2$$

Now, $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\ldots\right)} = \omega + \omega^2$

$= -1 \qquad [\because 1 + \omega + \omega^2 = 0]$

108. ***(d)*** We have, $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$

$\Rightarrow \quad a^7 = \left(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}\right)^7$

$= \cos 2\pi + i\sin 2\pi$ [by De-Moivre's theorem]

$= 1 + i\cdot 0 = 1$

Now, $\alpha + \beta = a + a^2 + a^4 + a^3 + a^5 + a^6$

$= a + a^2 + a^3 + a^4 + a^5 + a^6$

$= a\left(\frac{1-a^6}{1-a}\right) = \frac{a-a^7}{1-a} = \frac{a-1}{1-a} \; [\because a^7 = 1]$

$= \frac{a-1}{-(a-1)} = -1$

and $\alpha\cdot\beta = (a + a^2 + a^4)\cdot(a^3 + a^5 + a^6)$

$= a^4(1 + a + a^3)(1 + a^2 + a^3)$

$= a^4(1 + a^2 + a^3 + a + a^3 + a^4 + a^3 + a^5 + a^6)$

$= a^4(1 + a + a^2 + 3a^3 + a^4 + a^5 + a^6)$

$= a^4 + a^5 + a^6 + 3a^7 + a^8 + a^9 + a^{10}$

$= 3 + a + a^2 + a^3 + a^4 + a^5 + a^6$

$[\because a^7 = 1$, so $a^8 = a^7\cdot a = a$, $a^9 = a^7\cdot a^2 = a^2$, $a^{10} = a^7\cdot a^3 = a^3]$

$= 3 + a\left(\frac{1-a^6}{1-a}\right) = 3 + \frac{a-a^7}{1-a}$

$= 3 + \frac{a-1}{1-a} = 3 - 1 = 2$

So, the required equation is

$x^2 - (\alpha+\beta)x + \alpha\beta = 0 \Rightarrow x^2 + x + 2 = 0$

109. ***(b)*** We have, $\frac{3}{4} + \frac{15}{16} + \frac{63}{64} + \ldots$ upto n terms

$= \frac{2^2-1}{2^2} + \frac{2^4-1}{2^4} + \frac{2^6-1}{2^6} + \ldots$ upto n terms

$= \left(1-\frac{1}{2^2}\right) + \left(1-\frac{1}{2^4}\right) + \left(1-\frac{1}{2^6}\right) + \ldots$ upto n terms

$= (1 + 1 + 1 + \ldots$ upto n terms$)$

$- \left(\frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} - + \ldots n \text{ terms}\right)$

$$= n - \frac{1}{2^2}\left[\frac{1-\left(\frac{1}{2^2}\right)^n}{1-\frac{1}{2^2}}\right] = n - \frac{1}{3}(1 - 4^{-n})$$

$= n + \frac{4^{-n}}{3} - \frac{1}{3}$

110. ***(d)*** We have, $f(x) = \cos x + \{x\}$

$\Rightarrow \quad f(x) = \cos x + x - [x]$

Here, $\cos x$ is a periodic function with period 2π and $x - [x]$ is periodic function with period 1. But LCM of 2π and 1 does not exist.

Hence, $f(x)$ is not a periodic function and its period does not exist.

111. ***(b)*** We have,

$$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots\infty$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{x}{[(r-1)x+1](rx+1)}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left[\frac{1}{[(r-1)x+1]} - \frac{1}{rx+1}\right]$$

$$= \lim_{n\to\infty}\left[1 - \frac{1}{nx+1}\right] = 1$$

For $x = 0$, we have $f(x) = 0$

Thus, we have $f(x) = \begin{cases}1, & x \neq 0\\ 0, & x = 0\end{cases}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \neq f(0)$

So, $f(x)$ is not continuous at $x = 0$.

112. ***(c)*** Since, g is the inverse of function f. Therefore, $fog(x) = x$, for all x

$\Rightarrow \quad \frac{d}{dx}\{fog(x)\} = \frac{d}{dx}(x)$, for all x

$\Rightarrow \quad \frac{d}{dx} f'\{g(x)\}\cdot g'(x) = 1$, for all x

$\Rightarrow \quad \sin\{g(x)\}\, g'(x) = 1$, for all x

$\Rightarrow \quad g'(x) = \frac{1}{\sin\{g(x)\}}$

113. ***(a)*** Consider the following events:

E_1 = Getting a coin having head on both sides from the bag

E_2 = Getting a fair coin from the bag

A = Toss results in a head

Then, $P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$

$\Rightarrow \quad \frac{31}{42} = \frac{n}{2n+1}\times 1 + \frac{n+1}{2n+1}\times\frac{1}{2}$

$\Rightarrow \quad \frac{31}{21} = \frac{2n}{2n+1} + \frac{n+1}{2n+1}$

$\Rightarrow \quad \frac{31}{21} = \frac{3n+1}{2n+1}$

$\therefore \quad n = 10$

114. *(a)* We have, $dy + \{y\phi'(x) - \phi(x)\,\phi'(x)\}\,dx = 0$

$\Rightarrow \quad \frac{dy}{dx} + \phi'(x)\,y = \phi(x)\,\phi'(x)$

which is a linear differential equation with

$\text{IF} = e^{\int \phi'(x)\,dx} = e^{\phi(x)}$

$\therefore$ Solution is $y \cdot e^{\phi(x)} = \int \phi(x) \cdot \phi'(x)\, e^{\phi(x)} dx + C$

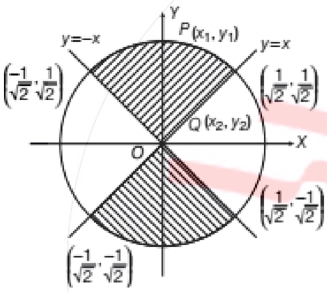
$\Rightarrow \quad y \cdot e^{\phi(x)} = \int \underbrace{\phi(x) \cdot e^{\phi(x)}}_{\text{I}}\ \underbrace{\phi'(x)}_{\text{II}}\, dx + C$

$\Rightarrow \quad y \cdot e^{\phi(x)} = \phi(x)\, e^{\phi(x)} - \int \phi'(x)\, e^{\phi(x)}\, dx + C$

$\Rightarrow \quad y \cdot e^{\phi(x)} = \phi(x)\, e^{\phi(x)} - e^{\phi(x)} + C$

$\Rightarrow \quad y = \{\phi(x) - 1\} + Ce^{-\phi(x)}$

115. *(c)*



Required area = 4 (Area of the shaded region in first quadrant)

$= 4\int_0^{1/\sqrt{2}} (y_1 - y_2)\, dx = 4\int_0^{1/\sqrt{2}} (\sqrt{1-x^2} - x)\, dx$

$= 4\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x - \frac{x^2}{2}\right]_0^{1/\sqrt{2}}$

$= 4\left[\frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{\pi}{4} - \frac{1}{4}\right]$

$= \frac{\pi}{2}$ sq units

116. *(a)* It is given that A is a proper subset of B.

$\therefore \quad A - B = \phi$

$\Rightarrow \quad n(A - B) = 0$

We have, $n(A) = 5$, so the minimum number of elements in B is 6.

Minimum possible value of

$n\,(A \Delta B) = n(B) - n(A)$

$= 6 - 5 = 1$

117. *(b)* Given, $\alpha = 2\tan^{-1}(\sqrt{2} - 1)$

$= 2\tan^{-1}\left(\tan\frac{\pi}{8}\right) = \frac{\pi}{4} = \cos^{-1}\frac{1}{\sqrt{2}}$

$\beta = 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= 3 \times \frac{\pi}{4} - \frac{\pi}{6} = \frac{7\pi}{12}$

$\gamma = \cos^{-1}\left(\frac{1}{3}\right)$

Clearly, $\quad \alpha < \beta$

Also, $\cos^{-1} x$ is a decreasing function on $[-1, 1]$

and $\frac{1}{3} < \frac{1}{\sqrt{2}}$.

$\therefore \quad \cos^{-1}\frac{1}{3} > \cos^{-1}\frac{1}{\sqrt{2}} \Rightarrow \gamma > \alpha$

We observe that, $0 < \gamma < \frac{\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$

$\Rightarrow \quad \gamma < \beta$

Thus, we have $\alpha < \gamma < \beta$.

118. *(d)* We have, $\frac{e^x + e^{5x}}{e^{3x}} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

$\Rightarrow \quad e^{-2x} + e^{2x} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

$\Rightarrow \quad 2\left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \ldots\right]$

$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$

$\Rightarrow \quad a_1 = a_3 = a_5 = \ldots = 0$

Hence, $2a_1 + 2^3 a_3 + 2^5 a_5 + \ldots = 0$

119. *(d)* Let θ be the angle between **b** and **c**.

Then, $\quad |\mathbf{b} \times \mathbf{c}| = \sqrt{15}$

$\Rightarrow \quad |\mathbf{b}||\mathbf{c}| \sin\theta = \sqrt{15}$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{15}}{4 \times 1} = \frac{\sqrt{15}}{4}$

$\therefore \quad \cos\theta = \frac{1}{4}$

Now, $\mathbf{b} - 2\mathbf{c} = \lambda\,\mathbf{a} \Rightarrow |\mathbf{b} - 2\mathbf{c}|^2 = |\lambda\,\mathbf{a}|^2$

$\Rightarrow \quad |\mathbf{b}|^2 + 4|\mathbf{c}|^2 - 4\,(\mathbf{b} \cdot \mathbf{c}) = \lambda^2\ |\mathbf{a}|^2$

$\Rightarrow \quad 16 + 4 - 4|\mathbf{b}||\mathbf{c}| \cos\theta = \lambda^2$

$\Rightarrow \quad 20 - 16\cos\theta = \lambda^2$

$\Rightarrow \quad 20 - 4 = \lambda^2$

$\Rightarrow \quad \lambda^2 = 16$

$\Rightarrow \quad \lambda = \pm 4$

120. *(b)* Total number of arrangements of 10 digits 0, 1, 2, ..., 9 by taking 4 at a time $= {}^{10}C_4 \times 4!$

We observe that in every arrangement of 4 selected digits there is just one arrangement in which the digits are in descending order.

∴ Required number of 4-digit numbers

$$= \frac{{}^{10}C_4 \times 4!}{4!} = {}^{10}C_4$$

121. *(c)* Given equation of a line parallel to X-axis is $y = k$. On solving this equation with $y = \sqrt{x}$, we get $x = k^2$. Thus, the line $y = k$ intersects with $y = \sqrt{x}$ at (k^2, k).

It is given that the line $y = k$ intersects the curve $y = \sqrt{x}$ at an angle of $\pi/4$. This means that the slope of the tangent to $y = \sqrt{x}$ at (k^2, k) is $\tan\left(+\frac{\pi}{4}\right) = +1$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(k^2,\, k)} = +1 \Rightarrow \left(\frac{1}{2\sqrt{x}}\right)_{(k^2,\, k)} = +1$$

$$\Rightarrow k = +\frac{1}{2}$$

Hence, the required line is $y = +\frac{1}{2}$.

122. *(a)* Let $a = 10$ and $b = 9$

It is given that the angles are in AP.

$$\therefore \quad 2B = A + C$$
$$\Rightarrow \quad 3B = A + B + C$$
$$\Rightarrow \quad 3B = 180°$$
$$\Rightarrow \quad B = 60°$$

Now, $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$$\Rightarrow \cos 60° = \frac{10^2 + c^2 - 9^2}{2 \times 10 \times c}$$

$$\Rightarrow \frac{1}{2} = \frac{100 + c^2 - 81}{20c}$$

$$\Rightarrow c^2 - 10c + 19 = 0 \Rightarrow c = 5 \pm \sqrt{6}$$

123. *(d)* The required mean is given by

$$\overline{X} = \frac{0 \cdot 1 + 1 \cdot {}^nC_1 + 2 \cdot {}^nC_2 + \ldots + n \cdot {}^nC_n}{1 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n}$$

$$= \frac{\sum_{r=0}^{n} r \cdot {}^nC_r}{\sum_{r=0}^{n} {}^nC_r} = \frac{\sum_{r=1}^{n} r \cdot \frac{n}{r} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r}$$

$$= \frac{n \sum_{r=1}^{n} {}^{n-1}C_{r-1}}{\sum_{r=0}^{n} {}^nC_r} = \frac{n \cdot 2^{n-1}}{2^n} = \frac{n}{2}$$

124. *(a)* We have, $\frac{1}{n}\sum_{i=1}^{n}(x_i + 2)^2 = 18$

and $\quad \frac{1}{n}\sum_{i=1}^{n}(x_i - 2)^2 = 10$

$$\Rightarrow \sum_{i=1}^{n}(x_i + 2)^2 = 18n$$

and $\quad \sum_{i=1}^{n}(x_i - 2)^2 = 10n$

$$\Rightarrow \sum_{i=1}^{n}(x_i + 2)^2 + \sum_{i=1}^{n}(x_i - 2)^2 = 28n$$

and $\quad \sum_{i=1}^{n}(x_i + 2)^2 - \sum_{i=1}^{n}(x_i - 2)^2 = 8n$

$$\Rightarrow 2\sum_{i=1}^{n}(x_i^2 + 4) = 28n \quad \text{and} \quad 2\sum_{i=1}^{n} 4x_i = 8n$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 + 4n = 14n \quad \text{and} \quad \sum_{i=1}^{n} x_i = n$$

$$\Rightarrow \sum_{i=1}^{n} x_i^2 = 10n \quad \text{and} \quad \sum_{i=1}^{n} x_i = n$$

$$\therefore \quad \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2}$$

$$= \sqrt{\frac{10n}{n} - \left(\frac{n}{n}\right)^2} = 3$$

125. *(a)* The parametric equations of the parabola $y^2 = 8x$ are $x = 2t^2$ and $y = 4t$.

On putting $x = 2t^2$ and $y = 4t$ in $x^2 + y^2 - 2x - 4y = 0$, we get

$$4t^4 + 16t^2 - 4t^2 - 16t = 0$$
$$4t^4 + 12t^2 - 16t = 0$$
$$\Rightarrow 4t(t^3 + 3t - 4) = 0$$
$$\Rightarrow t(t - 1)(t^2 + t + 4) = 0$$
$$\Rightarrow t = 0, t = 1 \quad [\because t^2 + t + 4 \neq 0]$$

Thus, the coordinates of points of intersection of the circle and the parabola are $O(0, 0)$ and $P(2, 4)$.

Clearly, these are diametrically opposite points on the circle.

The coordinates of the focus S of the parabola are $(2, 0)$ which lies on the circle. So, OSP is a right triangle.

∴ Area of $\Delta OSP = \frac{1}{2} \times OS \times SP$

$$= \frac{1}{2} \times 2 \times 4 = 4 \text{ sq units}$$

126. ***(a)*** Let $f(x) = e^{x-1} + x - 2$

Then, $f(1) = e^0 + 1 - 2 = 0$

So, $x = 1$ is a real roots of the equation $f(x) = 0$.
Let $x = \alpha$ be the other root of $f(x) = 0$ such that $\alpha > 1$ or $\alpha < 1$.

Consider, the interval $[1, \alpha]$ or $[\alpha, 1]$. Clearly, $f(1) = f(\alpha) = 0$

By Rolle's theorem, $f'(x) = 0$ has a root in $(1, \alpha)$ or in $(\alpha, 1)$.

But $f'(x) = e^{x-1} + 1 > 0$, for all x. Thus, $f'(x) \neq 0$, for any $x \in (1, \alpha)$ or $x \in (\alpha, 1)$, which is a contradiction.
Hence, $f(x) = 0$ has no real root other than 1.

127. ***(c)*** Constraints will be

$$\begin{aligned} x_{11} + x_{21} + \dots + x_{m1} &= b_1 \\ x_{12} + x_{22} + \dots + x_{m2} &= b_2 \\ &\vdots \\ x_{1n} + x_{2n} + \dots + x_{mn} &= b_n \\ x_{11} + x_{12} + \dots + x_{1n} &= b_1 \\ x_{21} + x_{22} + \dots + x_{2n} &= b_2 \\ &\vdots \\ x_{m1} + x_{m2} + \dots + x_{mn} &= b_n \end{aligned}$$

So, total number of constraints $= m + n$

128. ***(b)*** We have, probability of getting atleast seven points

= Probability of getting 7 points or 8 points

Seven points in four matches can be obtained in the following four different ways

2, 2, 2, 1 ; 2, 2, 1, 2; 2, 1, 2, 2; 1, 2, 2, 2

The probability of each of these ways

$$= (0.50)^3 \times (0.05) = 0.00625$$

$\therefore$ Probability of getting 7 points

$$= 4 \times 0.00625 = 0.0250$$

Eight points in four matches can be obtained only in one way i.e. 2, 2, 2, 2.

$\therefore$ Probability of getting 8 points

$$= (0.50)^4 = 0.0625$$

Thus, the required probability

$$= 0.0250 + 0.0625 = 0.0875$$

129. ***(b)*** We know that, $M\,(\text{adj}\,M) = |M|\,I$

Replacing M by adj M, we get

$$\text{adj}\,M\,[\text{adj}(\text{adj}\,M)] = \det(\text{adj}\,M)\,I$$

$$\Rightarrow \det(M)\,M^{-1}[\text{adj}(\text{adj}\,M)] = \alpha^2 I$$

$$\left[\because M^{-1} = \frac{1}{|M|}\text{adj}(M)\right]$$

$$\Rightarrow \alpha M^{-1}\cdot[\text{adj}(\text{adj}\,M)] = \alpha^2 I$$

$$\Rightarrow M^{-1}[\text{adj}(\text{adj}\,M)] = \alpha I$$

But $M^{-1}[\text{adj}(\text{adj}\,M)] = KI$

Hence, $K = \alpha$

130. ***(c)*** Let (x_1, y_1) be one of the points of contact. Then, the equation of the tangent at (x_1, y_1) is

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)}(x - x_1)$$

$\Rightarrow$ $y - y_1 = -\sin x_1\,(x - x_1)$

which passes through the origin.

$\therefore$ $0 - y_1 = -\sin x_1\,(0 - x_1)$

$\Rightarrow$ $y_1 = -x_1 \sin x_1$ ...(i)

Also, point (x_1, y_1) lies on $y = \cos x$.

$\therefore$ $y_1 = \cos x_1$ ...(ii)

From Eqs. (i) and (ii), we get

$$\sin^2 x_1 + \cos^2 x_1 = \frac{y_1^2}{x_1^2} + y_1^2 = 1$$

$$\Rightarrow x_1^2 = y_1^2 + y_1^2 x_1^2$$

Hence, the locus of (x_1, y_1) is

$$x^2 = y^2 + y^2x^2 \Rightarrow x^2y^2 = x^2 - y^2$$

131. ***(b)*** Let m be the slope of the tangent to the curve $y = e^x \cos x$.

Then, $m = dy/dx = e^x(\cos x - \sin x)$

$$\Rightarrow \frac{dm}{dx} = e^x(\cos x - \sin x) + e^x(-\sin x - \cos x)$$

$$= -2e^x \sin x$$

and $\dfrac{d^2m}{dx^2} = -2e^x(\sin x + \cos x)$

$\therefore$ $\dfrac{dm}{dx} = 0 \Rightarrow \sin x = 0$

$\Rightarrow$ $x = 0, \pi, 2\pi$

Clearly, $\dfrac{d^2m}{dx^2} > 0$ for $x = \pi$

Thus, slope of the tangent m is minimum at $x = \pi$.

Hence, $a = \pi$.

132. ***(a)*** The equations of given lines are

$$L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2},$$

$$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

Since, these lines are coplanar.

Therefore, $\begin{vmatrix} 5-\alpha & 0-0 & 0-0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$

$\Rightarrow$ $(5-\alpha)[(3-\alpha)(2-\alpha) - 2] = 0$

$\Rightarrow$ $(5-\alpha)(6 - 3\alpha - 2\alpha + \alpha^2 - 2) = 0$

$\Rightarrow$ $(5-\alpha)(\alpha^2 - 5\alpha + 4) = 0$

$\Rightarrow$ $(5-\alpha)(\alpha - 1)(\alpha - 4) = 0$

$\Rightarrow$ $\alpha = 1, 4, 5$

133. ***(c)*** Let $P(a\cos\theta, b\sin\theta)$ be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e. Then, the coordinates of A, A′, S and S′ are $(a, 0), (-a, 0), (ae, 0)$ and $(-ae, 0)$, respectively.

Now, area of $\Delta PSS'$

$$=\frac{1}{2}\left\|\begin{matrix} a\cos\theta & b\sin\theta & 1\\ ae & 0 & 1\\ -ae & 0 & 1\end{matrix}\right\|$$

$$= abe\sin\theta$$

and area of $\Delta APA' = \frac{1}{2}\left\|\begin{matrix} a\cos\theta & b\sin\theta & 1\\ a & 0 & 1\\ -a & 0 & 1\end{matrix}\right\|$

$$= ab\sin\theta$$

$\therefore$ Area of $\Delta PSS'$: Area of $\Delta APA' = e:1$

134. ***(d)*** $x^{12}-x^9+x^4-x+1$

$= x^9(x^3-1)+x(x^3-1)+1 = (x^3-1)(x^9+x)+1$

$= x(x^3-1)(x^8+1)+1 \geq 1 > 0$, for all $x \geq 1$

Also, $x^{12}-x^9+x^4-x+1$

$= x^{12}+x^4(1-x^5)+(1-x) > 0$, for all $0 \leq x < 1$

Clearly, $x^{12}-x^9+x^4-x+1>0, \forall x<0$

$\therefore \quad x^{12}-x^9+x^4-x+1>0, \forall x \in (-\infty, \infty)$

135. ***(a)*** We have, $f(\theta)=\sin\left[\tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-2\sin^2\theta}}\right)\right]$

$$=\sin\left[\sin^{-1}\left(\frac{\sin\theta}{\sqrt{\sin^2\theta+1-2\sin^2\theta}}\right)\right]$$

$$=\sin[\sin^{-1}(\tan\theta)]=\tan\theta$$

$\because \quad \tan^{-1}x=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$\therefore \frac{d}{d(\tan\theta)}f(\theta)=1$

136. ***(b)*** Since, p, q and r are odd natural numbers.

Let $p=2x-1$, $q=2y-1$ and $r=2z-1$.

Then, $(1+3+5+\ldots+p)+(1+3+5+\ldots+q)$

$=(1+3+5+\ldots+r) \Rightarrow x^2+y^2=z^2$

$\Rightarrow x, y$ and z are pythagorean triplets.

It is given that, $p>6 \Rightarrow x \geq 4$.

First pythagorean triplet containing a number greater than or equal to 4 is (4, 3, 5).

$\therefore \quad x=4, y=3$ and $z=5$

$\Rightarrow \quad p=7, q=5$ and $r=9$

$\Rightarrow \quad p+q+r=21$

137. ***(c)*** Let $a=\lim_{x\to 0}\left\{(\sin x)^{\frac{1}{x}}+\left(\frac{1}{x}\right)^{\sin x}\right\}$

$\Rightarrow a=\lim_{x\to 0}(\sin x)^{1/x}+\lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x} \Rightarrow a=l+m$

Now, $l=\lim_{x\to 0}(\sin x)^{1/x}$ and $m=\lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x}$

$l=0$

$\Rightarrow \log m=\lim_{x\to 0}(-\sin x\log x)=\lim_{x\to 0}\frac{-\log x}{\operatorname{cosec} x}$

$$=\lim_{x\to 0}\frac{-1/x}{-\operatorname{cosec} x\cot x}=\lim_{x\to 0}\frac{\sin^2 x}{x\cos x}$$

$$=\lim_{x\to 0}\frac{\tan x}{x}\times\sin x=1\times 0=0$$

$\Rightarrow \quad m=e^0=1 \quad \Rightarrow \quad a=0+1=1$

138. ***(d)*** Consider the functions $f(x)$ and $g(x)$ given by $f(x)=\tan x-x$

and $\quad g(x)=x-\sin x$, for $0<x<\frac{\pi}{2}$

We have, $f'(x)=\sec^2 x-1$ and

$g'(x)=1-\cos x$

$\Rightarrow \quad f'(x)>0$ and $g'(x)>0, \forall x\in\left(0, \frac{\pi}{2}\right)$

$\Rightarrow \quad f(x)>f(0)$ and $g(x)>g(0), \forall x\in\left(0, \frac{\pi}{2}\right)$

$\Rightarrow \quad \tan x-x>0$ and $x-\sin x>0, \forall x\in\left(0, \frac{\pi}{2}\right)$

$\Rightarrow \quad \tan x>x$ and $x>\sin x, \forall x\in\left(0, \frac{\pi}{2}\right)$

$\Rightarrow \quad \sin x<x<\tan x, \forall x\in\left(0, \frac{\pi}{2}\right)$

139. ***(a)*** On putting $x=\tan A$ and $y=\tan B$ in the given relation, we get $\sqrt{1+\tan^2 A}+\sqrt{1+\tan^2 B}$

$=\lambda[\tan A\sqrt{1+\tan^2 B}\; - \tan B\sqrt{1+\tan^2 A}]$

$\Rightarrow \quad \sec A+\sec B=\lambda[\tan A\sec B-\tan B\sec A]$

$[\because \sec^2\theta-\tan^2\theta=1]$

$\Rightarrow \cos A+\cos B=\lambda[\sin A-\sin B]$

$\Rightarrow 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$$=\lambda\left(2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right)$$

$$\left[\begin{matrix}\because \cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\\ \text{and } \sin x-\sin y=2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\end{matrix}\right]$$

$\Rightarrow \quad \tan \frac{A-B}{2} = \frac{1}{\lambda} \Rightarrow A - B = 2\tan^{-1}(1/\lambda)$

$\Rightarrow \quad \tan^{-1} x - \tan^{-1} y = 2\tan^{-1}(1/\lambda)$

On differentiating w.r.t. x, we get

$$\frac{1}{1+x^2} - \frac{1}{1+y^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

Hence, the degree is 1.

140. ***(b)*** Let $f(x) = ax^3 + bx^2 + cx + d$

Then, $f(0) = -1$ and $f(1) = 0$

$\Rightarrow \quad d = -1$ and $a + b + c + d = 0$

So, $\quad a + b + c = 1 \quad \ldots$(i)

It is given that $x = 0$ is a stationary point of $f(x)$ but it is not a point of extremum.

Therefore, $f'(0) = 0 = f''(0)$ and $f'''(0) \neq 0$

Now, $\quad f(x) = ax^3 + bx^2 + cx + d$

$\Rightarrow \quad f'(x) = 3ax^2 + 2bx + c,$

$\Rightarrow \quad f''(x) = 6ax + 2b$ and $f'''(x) = 6a$

$\because \quad f'(0) = 0, f''(0) = 0$ and $f'''(0) \neq 0$

$\Rightarrow \quad c = 0, b = 0$ and $a \neq 0 \quad \ldots$(ii)

From Eqs. (i) and (ii), we get

$$a = 1, b = c = 0 \text{ and } d = -1$$

$\therefore \quad f(x) = x^3 - 1$

Hence, $\int \frac{f(x)}{x^3-1}dx = \int \frac{x^3-1}{x^3-1}dx = \int 1\,dx = x + C$

141. ***(a)*** We have, $e^x + e^{f(x)} = e$

$\Rightarrow \quad e^{f(x)} = e - e^x$

$\Rightarrow \quad f(x) = \log_e(e - e^x) = \log[e(1 - e^{x-1})]$

$= \log_e e + \log_e(1 - e^{x-1})$

$= 1 + \log_e(1 - e^{x-1})$

Clearly, for $f(x)$ to be real, we must have

$$1 - e^{x-1} > 0$$

$\Rightarrow \quad e^x < e \Rightarrow x < 1 \Rightarrow x \in (-\infty, 1)$

142. ***(a)*** The equation of a line concurrent with the lines $4x + 3y - 7 = 0$ and $8x + 5y - 1 = 0$ is

$(4x + 3y - 7) + \lambda(8x + 5y - 1) = 0$

$\Rightarrow (4 + 8\lambda)x + (3 + 5\lambda)y - 7 - \lambda = 0$

The gradient of this line is $-\frac{3}{2}$, therefore

$$-\frac{8\lambda + 4}{5\lambda + 3} = -\frac{3}{2} \Rightarrow \lambda = 1$$

So, the required line is

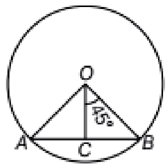
$$12x + 8y - 8 = 0 \Rightarrow 3x + 2y - 2 = 0$$

which has gradient $\frac{-3}{2}$.

143. ***(c)*** Let AB be the chord of length $\sqrt{2}$, O be the centre of the circle and let OC be the perpendicular from O on AB.

Then, $AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\because \quad \angle AOB = \frac{\pi}{2} = \angle BOC = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$



In ΔOBC, we have

$$OB = BC \text{ cosec } 45° = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$$

$\therefore$ Area of the circle $= \pi(OB)^2 = \pi$ sq units

144. ***(c)*** Let $y = 2 - \cos x + \sin^2 x$

Then, $\quad y = 2 - \cos x + 1 - \cos^2 x$

$= 3 - (\cos^2 x + \cos x)$

$= \frac{13}{4} - \left(\cos x + \frac{1}{2}\right)^2$

Now, $-1 \leq \cos x \leq 1$, for all x.

$\Rightarrow \quad -\frac{1}{2} \leq \cos x + \frac{1}{2} \leq \frac{3}{2}$

$\Rightarrow \quad 0 \leq \left(\cos x + \frac{1}{2}\right)^2 \leq \frac{9}{4}$

$\Rightarrow \quad -\frac{9}{4} \leq -\left(\cos x + \frac{1}{2}\right)^2 \leq 0$

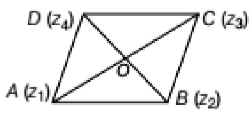
$\Rightarrow \quad \frac{13}{4} - \frac{9}{4} \leq \frac{13}{4} - \left(\cos x + \frac{1}{2}\right)^2 \leq \frac{13}{4}$

$\Rightarrow \quad 1 \leq y \leq \frac{13}{4}$

$\therefore \quad y_{max} = \frac{13}{4}$ and $y_{min} = 1$

Hence, the required ratio is 13/4.

145. ***(c)*** Since, the diagonals of a rhombus bisect each other at right angle.



$\therefore \quad \frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2} \Rightarrow z_1 + z_3 = z_2 + z_4$

Also, $\quad \angle AOB = \frac{\pi}{2} \Rightarrow \arg\left(\frac{z_2 - z_4}{z_1 - z_3}\right) = \frac{\pi}{2}$

146. ***(b)*** Let $x = 2$, $y = 2$ but $2^2 + 2^2 \neq 1 \Rightarrow p$ is not reflexive

Let $(0, 1)$ and $(1,0) \in p$

$\Rightarrow$ but $(0, 0) \notin p$

$\Rightarrow p$ is not transitive the relation is not reflexive and transitive, but it is symmetric, because

$$x^2 + y^2 = 1$$

$$\Rightarrow \quad y^2 + x^2 = 1$$

147. ***(c)*** Let l, m and n be the direction cosines.

Then, $\quad l = \cos\theta,\ m = \cos\beta,\ n = \cos\theta$

$$\therefore \quad l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \quad \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$$

$$\Rightarrow \quad 2\cos^2\theta + 1 - \sin^2\beta = 1$$

$$\Rightarrow \quad 2\cos^2\theta - \sin^2\beta = 0$$

$$[\because \cos^2\theta + \sin^2\theta = 1]$$

$$\Rightarrow \quad 2\cos^2\theta - 3\sin^2\theta = 0$$

$$[\because \sin^2\beta = 3\sin^2\theta \text{ (given)}]$$

$$\Rightarrow \quad \tan^2\theta = 2/3$$

$$\therefore \cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{1}{1+2/3} = \frac{3}{5}$$

148. ***(b)*** We have, $n = 4$ and $P(X = 0) = \frac{16}{81}$

Let p be the probability of success and q that of failure in a trial.

Then, $\quad P(X = 0) = \frac{16}{81}$

$$\Rightarrow \quad {}^4C_0\, p^0 q^4 = \frac{16}{81}$$

$$\Rightarrow \quad q^4 = \left(\frac{2}{3}\right)^4$$

$$\Rightarrow \quad q = \frac{2}{3} \Rightarrow p = \frac{1}{3} \quad \{\because p + q = 1\}$$

$$\therefore \quad P(X = 4) = {}^4C_4\, p^4 q^0$$

$$= p^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$$

149. ***(d)*** We have,

$$f(x+y) = f(x) + f(y),\ \forall\ x, y \in R$$

$$\Rightarrow \quad f(0) = f(0) + f(0)$$

[replacing x and y both by zero]

$$\Rightarrow \quad f(0) = 0$$

Now, $\quad f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$$f'(0) = \lim_{h \to 0} \frac{f(h)}{h}$$

Now, $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{f(x) + f(h) - f(x)}{h}$$

$$\Rightarrow \quad f'(x) = \lim_{h \to 0} \frac{f(h)}{h} = f'(0)$$

$$\Rightarrow \quad f(x) = x f'(0) + C$$

But $\quad f(0) = 0$

$\therefore \quad C = 0$

Hence, $f(x) = x f'(0),\ \forall\ x \in R$.

Clearly, $f(x)$ is everywhere continuous and differentiable and $f'(x)$ is constant, $\forall\ x \in R$.

150. ***(d)*** Let the coefficients of rth, $(r+1)$th and $(r+2)$th terms be in HP.

$\Rightarrow$ Coefficients of $\frac{1}{r\text{th}}, \frac{1}{(r+1)\text{th}}, \frac{1}{(r+2)\text{th}}$ term will be in AP.

Then, $\quad \frac{2}{{}^nC_r} = \frac{1}{{}^nC_{r-1}} + \frac{1}{{}^nC_{r+1}}$

$$\Rightarrow \quad 2 = \frac{{}^nC_r}{{}^nC_{r-1}} + \frac{{}^nC_r}{{}^nC_{r+1}}$$

$$\Rightarrow \quad 2 = \frac{n-r+1}{r} + \frac{r+1}{n-r}$$

$$\Rightarrow \quad n^2 - 4nr + 4r^2 + n = 0$$

$$\Rightarrow \quad (n-2r)^2 + n = 0$$

which is not possible for any value for n.