Solved Paper 2014*

Instructions

- There are 150 questions in all. The number of questions in each part is as given below.

		No. of Questions
Part I	Physics	1-40
Part II	Chemistry	41-80
Part III	a. English Proficiency	81-95
	b. Logical Reasoning	96-105
Part IV	Mathematics	106-150

- All questions are Multiple Choice Questions having four options out of which **only one** is correct.
- Each correct answer fetches 3 marks while incorrect answer has a penalty of 1 mark.
- Time allotted to complete this paper is 3 hrs.

PART I

Physics

1. In process of amplitude modulation of signal to be transmitted. Signal to be modulated is given by $m(t) = A_m \sin \omega_m t$, carrier wave is given by $C(t) = A_c \sin \omega_c t$, modulated signal $C_m(t)$ is given by

a. $C_m(t) = A_c \sin \omega_c t + A_m \sin \omega_m t$

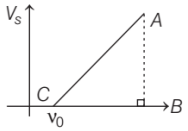
b. $C_m(t) = (A_c + A_m) \sin \omega_c t$

c. $C_m(t) = [A_c + m(t)] \sin \omega_c t$

d. None of the above

2. Graph of stopping potential for most energetic emitted photoelectron (V_S) with frequency of incident radiation on metal is given below

Value of $\frac{AB}{BC}$, in graph is



a. h ***b.*** e ***c.*** $\frac{h}{e}$ ***d.*** $\frac{e}{h}$

(h = Planck's constant, e = electronic charge)

3. A block of mass 0.18 kg is attached to a spring of force constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block. The block slides a distance of 0.06 m and comes to rest for the first time the initial velocity of the block in m/s is $v = N/10$. Then, N is

a. 2 ***b.*** 3 ***c.*** 4 ***d.*** 6

4. Ohm's law says

a. $V = IR$

b. V/I = constant

c. Both (a) and (b) are correct

d. Both (a) and (b) are incorrect

5. A train accelerating uniformly from rest attains a maximum speed of 40 ms^{-1} in 20 s. It travels at this speed for 20 s and is brought to rest with uniform retardation in further 40 s. What is the average velocity during this period?

a. 80 m/s ***b.*** 25 m/s ***c.*** 40 m/s ***d.*** 30 m/s

6. The strain-stress curves of three wires of different materials are shown in the figure. P, Q, R are the elastic limits of the wires, the figure shows that

a. elasticity of wire P is maximum
b. elasticity of wire Q is maximum
c. tensile strength of R is maximum
d. None of the above

7. For the equation $F \propto A^a v^b d^c$, where F is the force, A is the area, v is the velocity and d is the density, the values of a, b and c are respectively

a. 1, 2, 1 ***b.*** 2, 1, 1 ***c.*** 1, 1, 2 ***d.*** 0, 1,1

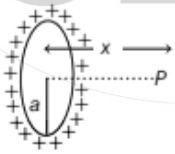
8. In hydrogen atom, an electron jumps from bigger orbit to smaller orbit so that radius of smaller orbit is one-fourth of radius of bigger orbit. If speed of electron in bigger orbit was v then speed in smaller orbit is

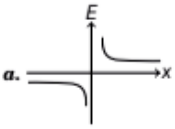
a. $v/4$ ***b.*** $v/2$ ***c.*** v ***d.*** $2v$

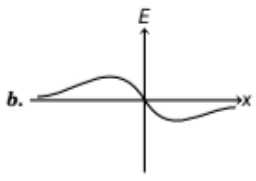
9. A steel wire of length 4.7 m and cross-section 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-section 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel, so that of copper ?

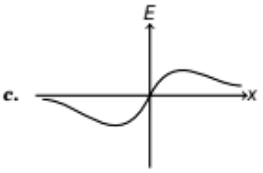
a. 1.5 : 2 ***b.*** 1.8 : 2 ***c.*** 1.5 : 1 ***d.*** 1.8 : 1

10. A ring shaped conductor with radius a carries a net positive charge q uniformly distributed on it as shown in figure. A point P is situated at a distance x from its centre. Which of the following graph shows the correct variation of electric field (E) with distance (x)?



a.


b.


c.


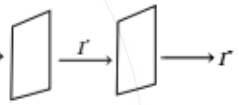
d. None of the above

11. .....A..... is the essential condition for coherent sources. Here, A refers to

a. constant phase difference
b. equal amplitude
c. Both (a) and (b) are correct
d. Both (a) and (b) are incorrect

12. The angular size of the central maxima due to a single slit diffraction is (a = slit width)

a. $\frac{\lambda}{a}$ ***b.*** $\frac{2\lambda}{a}$
c. $\frac{3\lambda}{2a}$ ***d.*** $\frac{\lambda}{2a}$

13. Light $\xrightarrow[I_0]{\text{unpolarised}}$  $\longrightarrow I''$

Find the final intensity of light I'', if the angle between the axes of two polaroids is 60°.

a. $\frac{3I_0}{2}$ ***b.*** $\frac{I_0}{2}$
c. $\frac{I_0}{4}$ ***d.*** $\frac{I_0}{8}$

14. Resolving power of a telescope will be more, if the diameter a of the objective is

a. larger
b. smaller
c. resolving power does not depend on a
d. None of the above

15. Let binding energy per nucleon of nucleus is denoted as E_{bn} and radius of nucleus is denoted as r. If mass number of nuclei A, B are 64 and 125 respectively, then

a. $r_A < r_B$, $E_{bnA} < E_{bnB}$
b. $r_A > r_B$, $E_{bnA} > E_{bnB}$
c. $r_A = \frac{4}{5} r_B$, $E_{bnA} < E_{bnB}$
d. $r_A < r_B$, $E_{bnA} > E_{bnB}$

16. The heat energy

a. is a state variable
b. does not depend on the state of the system
c. is equal to internal energy of the system
d. None of the above

17. A current 4.0 A exists in a wire of cross-sectional area 2.0 mm^2. If each cubic metre of the wire contains 12.0×10^{28} free electrons, then the drift speed is

a. 2×10^{-8} m/s

b. 0.5×10^{-3} m/s

c. 1.04×10^{-4} m/s

d. None of the above

18. In an experiment on the specific heat of a metal a 0.2 kg block of the metal at 150°C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cc of water at 27°C. The final temperature is 40°C. Calculate the specific heat of the metal. If heat losses to the surroundings are not negligible, is our answer greater or smaller than the actual value of specific heat of the metal?

a. 0.02 *b.* 0.2

c. 0.01 *d.* 0.1

19. Electric field in a region is given by $\mathbf{E} = \left(\frac{M}{x^3}\right)\hat{\mathbf{i}}$, then the correct expression for the potential in the region is
(assume potential at infinity is zero).

a. $\frac{M}{2x^2}$ *b.* Mx^2

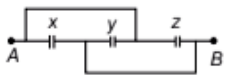
c. $\frac{M}{3x^4}$ *d.* None of these

20. A ball is projected upwards from the top of tower with a velocity 50 m/s making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds from the instant of throwing will the ball reach the ground?

a. 2 s *b.* 5 s

c. 7 s *d.* 9 s

21. Three capacitors $X = 1\,\mu F$, $Y = 2\,\mu F$ and $Z = 3\,\mu F$ are connected as shown in figure, then the equivalent capacitance between points A and B is



a. $6\,\mu F$ *b.* $12\,\mu F$

c. $3\,\mu F$ *d.* None of these

22. The work done in blowing a soap bubble of surface tension 0.06 Nm^{-1} from 2 cm radius to 5 cm radius is

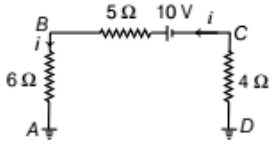
a. 0.004168 J *b.* 0.003168 J

c. 0.003158 J *d.* 0.004568 J

23. The sum of the magnitudes of two forces acting at a point is 16 N. The resultant of these forces is perpendicular to the smaller force which has a magnitude of 8 N. If the smaller force is magnitude x, then the value of x is

a. 2 N *b.* 4 N *c.* 6 N *d.* 7 N

24. What will be the value of current i in the circuit shown?



a. 0.67 A *b.* 1 A

c. 0.32 A *d.* None of these

25. The average depth of Indian Ocean is about 3000 m. Bulk modulus of water is 2.2×10^4 N/m^2, $g = 10$ m/s^2, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of the Indian Ocean will be

a. 1.36% *b.* 20.6% *c.* 13.9% *d.* 0.52%

26. If **A** and **B** denote the sides of a parallelogram and its area is $\frac{1}{2}AB$ (A and B are magnitude of **A** and **B** respectively), the angle between **A** and **B** is

a. 30° *b.* 45° *c.* 60° *d.* 90°

27. If edge lengths of a cuboid are measured to be 1.2 cm, 1.5 cm and 1.8 cm, then volume of the cuboid is

a. 3.240 cm^3 *b.* 3.24 cm^3

c. 3.2 cm^3 *d.* 3.0 cm^3

28. A ball thrown upwards from the top of a tower with speed v reaches the ground in t_1 second. If this ball is thrown downwards from the top of the same tower with speed v, it reaches the ground in t_2 sec. In what time the ball shall reach the ground, if it is allowed to fall freely under gravity from the top of the tower?

a. $\frac{t_1 + t_2}{2}$ *b.* $\frac{t_1 - t_2}{2}$ *c.* $\sqrt{t_1 t_2}$ *d.* $t_1 + t_2$

29. One end of steel wire is fixed to ceiling of an elevator moving up with an acceleration 2 m/s^2 and a load of 10 kg hangs from other end. Area of cross-section of the wire is 2 cm^2. The longitudinal strain in the wire is
($g = 10$ m/s^2 and $Y = 2 \times 10^{11}$ Nm^{-2})

a. 4×10^{11} *b.* 3×10^{-6}

c. 8×10^{-6} *d.* 2×10^{-6}

30. The electrostatic force of repulsion between two positively charged ions carrying equal charge is 3.7×10^{-9} N, when they are separated by a distance of 5 Å. How much electrons are missing from each ion?

a. 10 *b.* 8 *c.* 2 *d.* 1

31. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. The ratio of the radii of the circular paths described by them is

a. 1 : 2 *b.* 1 : 1 *c.* 2 : 1 *d.* 1 : 3

32. A solenoid of length 1.0 m has a radius of 1 cm and has a total of 1000 turn wound on it. It carries a current of 5 A. If an electron were to move with a speed of 10^4 ms^{-1} along the axis of this current carrying solenoid, the force experienced by this electron is

a. 2 N *b.* 1.2 N
c. zero *d.* 2.5 N

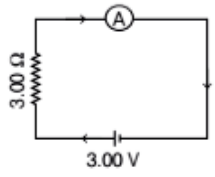
33. Magnetic field

a. can increase the speed of charge particle
b. can accelerate a charge particle
c. Both (a) and (b) are correct
d. Both (a) and (b) are incorrect

34. A square coil of side 10 cm has 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil, makes an angle θ with the direction of a uniform horizontal magnetic field of 0.80 T. If the torque, experienced by the coil, equals 0.96 N-m, the value of θ is

a. 0° *b.* $\frac{\pi}{2}$ rad *c.* $\frac{\pi}{3}$ rad *d.* $\frac{\pi}{6}$ rad

35. In the circuit (figure) the current is to be measured. The ammeter shown is a galvanometer with a resistance $R_G = 60.00\,\Omega$ converted to an ammeter by a shunt resistance $r_s = 0.02\,\Omega$. The value of the current is

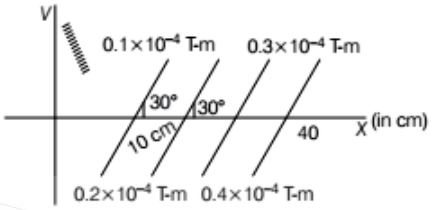


a. 0.79 A *b.* 0.29 A
c. 0.99 A *d.* 0.8 A

36. The susceptibility of a magnetism at 300 K is 1.2×10^{-5}. The temperature at which the susceptibility increases to 1.8×10^{-5} is

a. 150 K *b.* 200 K
c. 250 K *d.* 20 K

37. Figure shows some of the equipotential surface of the magnetic scalar potential. The magnetic field **B** at a point in the region is



a. 1×10^{-4} T
b. 2×10^{-4} T
c. 3×10^{-4} T
d. 4×10^{-4} T

38. In a thermodynamic process, the pressure of a fixed mass of a gas is changed in such a manner that the gas release 20 J of heat and 8 J of work is done on the gas. If the initial internal energy of the gas was 30 J, then the final internal energy will be

a. 2 J *b.* 42 J
c. 18 J *d.* 58 J

39. Two balloons are filled one with pure He gas and other with air respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is

a. more in He filled balloon
b. same in both balloons
c. more in air filled balloon
d. in the ratio 1 : 4

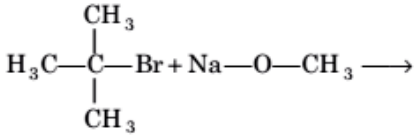
40. The equation of a simple harmonic motion is given by $y = 3\sin\frac{\pi}{2}(50t - x)$, where x and y are in metres and t is in seconds, the ratio of maximum particle velocity to the wave velocity is

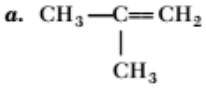
a. 2π *b.* $\frac{3}{2}\pi$
c. 3π *d.* $\frac{2}{3}\pi$

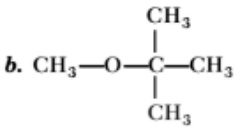
PART II

Chemistry

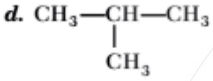
41. What will be the product of the reaction?



a. 

b. 

c. $CH_3—CH_2—CH_2—CH_3$

d. 

42. Which of the following relation is incorrect regarding Bohr's theory?

a. Velocity of electron $\propto \frac{1}{n}$

b. Frequency of revolution $\propto \frac{1}{n^2}$

c. Radius of orbit $\propto n^2Z$

d. Force on electron $\propto \frac{1}{n^4}$

43. Which of the following pair have identical shape?

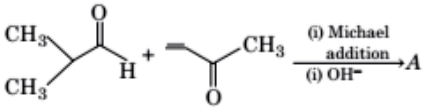
a. CH_4, SF_4 *b.* BCl_3, ClF_3

c. XeF_2, $ZnCl_2$ *d.* SO_2, CO_2

44. 10 g of sample of mixture of $CaCl_2$ and NaCl is treated to precipitate all the calcium as $CaCO_3$. This $CaCO_3$ is heated to convert all the Ca to CaO and the final mass of CaO is 1.62 g. The percent by mass of $CaCl_2$ in the original mixture is

a. 32.1% *b.* 16.2% *c.* 21.8% *d.* 12.0%

45. How many chiral centre are possible for the product of following reaction?



a. 1 *b.* 0 *c.* 3 *d.* 2

46. Elements/ions having same number of electrons are known as isoelectronic species. Arrange the following elements in correct order of atomic/ionic radii and choose the correct choice from the four choices given below

O^{2-}, Na^+, Mg^{2+}, F^-, Al^{3+}

a. $Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-}$

b. $Al^{3+} < Na^+ < Mg^{2+} < F^- < O^{2-}$

c. $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

d. None of the above

47. The ratio of oxidation states of Cl in potassium chloride to that in potassium chlorate is

a. $+\frac{1}{5}$ *b.* $-\frac{1}{5}$ *c.* $-\frac{2}{5}$ *d.* $+\frac{3}{5}$

48. A reaction, $Cu^{2+} + 2e^- \longrightarrow Cu$ is given. For this reaction, graph between E_{red} *versus* $\ln[Cu^{2+}]$ is a straight line of intercept 0.34 V, then the electrode oxidation potential of the half cell Cu/Cu^{2+} (0.1 M) will be

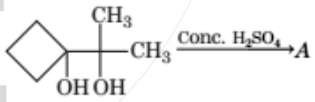
a. 0.34 *b.* $0.34+\frac{0.0591}{2}$

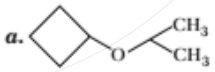
c. $-0.34-\frac{0.0591}{2}$ *d.* $-0.34+\frac{0.0591}{2}$

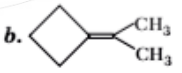
49. Which one of the following silanes on hydrolysis produces cross linked polymers ?

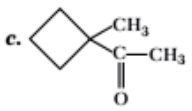
a. $RSiCl_3$ *b.* R_2SiCl_2 *c.* R_3SiCl *d.* R_4Si

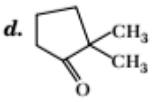
50. Identify the correct product formed during the following reaction.



a. 

b. 

c. 

d. 

51. Usually, $CaCl_2$ is preferred over NaCl for cleaning snow on roads particulary in very cold countries. This is because

a. NaCl makes the road slippery but $CaCl_2$ does not

b. $CaCl_2$ is hygroscopic but NaCl is not

c. $CaCl_2$ is less soluble in H_2O than NaCl

d. eutectic mixture of $CaCl_2/H_2O$ freezes at $-55°C$ while that of $NaCl/H_2O$ freezes at $-18°C$.

52. The gold numbers of a few protective colloids are given

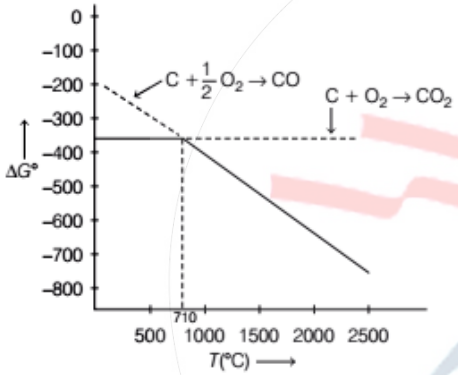
x	0.005
y	3.5
z	40

The protective nature of these colloidal solutions follow the order

a. $z > x > y$ *b.* $x < y > z$

c. $z > y > x$ *d.* $x > y > z$

53. Consider the following Ellingham diagram for carbon.



Which of the statement is incorrect for the above Ellingham diagram?

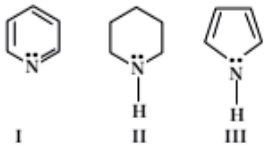
a. Upto 710°C, the reaction of formation of CO_2 is energetically more favourable but above 710°C, the formation of CO is preferred.

b. Carbon can be used to reduce any metal oxide at a sufficiently high temperature.

c. Carbon reduces many oxides at elevated temperature because $\Delta G°$ *vs* temperature line has a negative slope.

d. $\Delta S°[C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g)] < \Delta S°[C(s) + O_2(g) \longrightarrow CO_2(g)]$

54. Arrange the following in correct order of basicity.



a. I > II > III

b. III > II > I

c. II > I > III

d. I > III > II

55. What is the density of Na_2O having antifluorite type crystal structure, if the edge length of the cube is 100 pm and what is the effect on density by 0.05% Frenkel defect ?

a. 823.5 g cm^{-3}, density increases

b. 414.16 g cm^{-3}, density decreases

c. 823.5 g cm^{-3}, density remains same

d. 414.16 g cm^{-3}, density remains same

56. A swimmer coming out from a pool is covered with a film of water weiging about 18 g. Calculate the internal energy of vaporisation at 100°C.

[$\Delta_{vap}H^{\ominus}$ for water at 373 K = 40.66 kJ mol^{-1}]

The correct option is

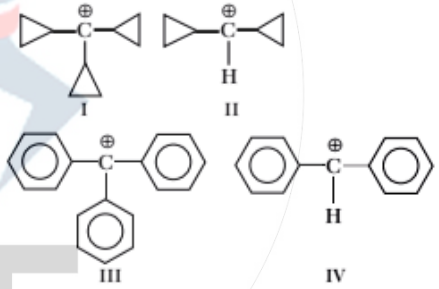
a. 35.67 kJ mol^{-1}

b. 37.56 kJ mol^{-1}

c. 36.57 kJ mol^{-1}

d. 38.75 kJ mol^{-1}

57. Which of the following is correct order of stability of carbocation?



a. IV > III > II > I *b.* I > II > III > IV

c. III > II > I > IV *d.* I > III > II > IV

58. Mercury is a liquid metal because

a. it has a completely filled *d*-orbital that causes *d-d* overlapping

b. it has completely filled *d*-orbital that prevents *d-d* overlapping

c. it has a completely filled *s*-orbital

d. it has a small atomic size

59. The volume of 10 N and 4 N HCl required to make 1 L of 7 N HCl are

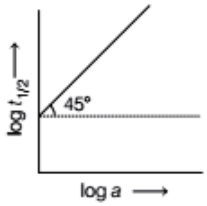
a. 0.75 L of 10 N HCl and 0.25 L of 4 N HCl

b. 0.50 L of 10 N HCl and 0.50 L of 4 N HCl

c. 0.65 L of 10 N HCl and 0.35 L of 4 N HCl

d. 0.85 L of 10 N HCl and 0.15 L of 4 N HCl

60. Following is the graph between $\log T_{50}$ and $\log a$ (a = initial concentration) for a given reaction at 27° C. Hence, order is



a. 1 *b.* 2 *c.* 3 *d.* 0

61. The catalyst used for olefin polymerisation is
a. Ziegler-Natta catalyst
b. Raney nickel catalyst
c. Wilkinson catalyst
d. Merrified resin

62. Which one of the following is a covalent hydride?
a. CaH_2 *b.* NaH
c. BH_3 *d.* BeH_2

63. Which one of the following is used for the separation of noble gas mixture from air ?
a. Charcoal
b. 90% CaC_2 + 10% $CaCl_2$
c. Soda lime + potash solution
d. 90% $CaCO_3$ + 10% urea

64. Consider the following statements.
I. NCl_5 does not exist while PCl_5 does.
II. Both O_2^+ and NO are paramagnetic.
III. The three C—O bonds are not equal in carbonate ion.
IV. Lead prefers to form tetravalent compound.
Which of the above statements are incorrect?
a. I and III *b.* I, III and IV
c. II and III *d.* III and IV

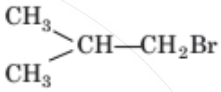
65. The liquefied metal that expand on solidification is
a. Al *b.* Zn
c. Ga *d.* Cu

66. Point out the correct statement for the set of characteristics of ZnS crystal.
a. Coordination number (4 : 4) : ccp ; Zn^{2+} ion in the alternate tetrahedral voids.
b. Coordination number (6 : 6); hcp; Zn^{2+} ion in all tetrahedral voids.
c. Coordination number (6 : 4); hcp; Zn^{2+} ion in all octahedral voids.
d. Coordination number (4 : 4); ccp; Zn^{2+} ion in all tetrahedral voids.

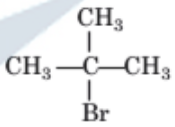
67. Arrange the following compounds in the increasing order of nucleophilic addition reaction.
I. HCHO
II. CH_3COCH_3
III. $C_6H_5COCH_3$
IV. $C_2H_5COC_6H_5$
a. I< II< III< IV *b.* IV< III< II< I
c. IV< II< III< I *d.* III< IV< II< I

68. The heat of combustion of sucrose, $C_{12}H_{22}O_{11}(s)$ at constant volume is 1348.9 kcal mol^{-1} at 25°C, then the heat of reaction at constant pressure when steam is produced
a. −1348.9 kcal *b.* −1342.34 kcal
c. + 1250 kcal *d.* None of these

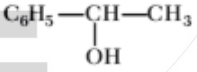
69. Arrange the following compounds in increasing order of their boiling points.

I. 

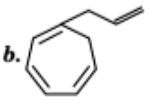
II. $CH_3CH_2—CH_2—CH_2—Br$

III. 

a. II < I < III *b.* I < II < III
c. III < I < II *d.* III < II < I

70. Which of the following compounds will give positive iodoform test with I_2 and NaOH?
a. $C_6H_5COC_6H_5$
b. CH_3CH_2CHO
c. $C_6H_5COCH_2CH_3$
d. 

71. What will be the product when most acidic species among following will react with 3-chloroprop-1-ene?

a. 
b. 
c. 
d. 

72. $CaCO_3(s) \xrightarrow{\text{Heat}} X(s) + Z(g)$

$X(s) \xrightarrow{\text{Carbon, heat}} C(s) + D(g)$

$C(s) + H_2O \longrightarrow E(g)$.

Then, compound $E(g)$ and $C(s)$ will be

a. C_2H_2, CaO ***b.*** C_2H_2, CaC_2

c. CH_4, CaC_2 ***d.*** CH_4, CaO

73. Which of the following will not form optical isomers ?

a. $[Co(en)_3]^{3+}$ ***b.*** $[Co(NH_3)_3(NO_2)_3]$

c. $[Pt(en)_2Cl_2]^{2+}$ ***d.*** $[CrCl_2(ox)_2]^{3-}$

74. $C_6H_5NH_2 \xrightarrow[180°C]{H_2SO_4} \underset{\textit{Para}\text{-form}}{NH_2C_6H_4(SO_3H)}$

The true statement about the product is

a. It does not exist as Zwitter ion

b. It does not act as inner salt

c. $-SO_3$ diminishes the basic character of $-NH_2$

d. $-NH_2$ displays a powerful basic character

75. A copolymer of ethene and vinyl chloride contains alternate monomers of each type. What is the mass percentage of vinyl chloride in this copolymer ?

a. 38% ***b.*** 69% ***c.*** 72% ***d.*** 82%

76. The number of disulphide linkages present in insulin are

a. 1 ***b.*** 2 ***c.*** 3 ***d.*** 4

77. Which of the following statement is not true about the drug barbital?

a. It is used in sleeping pills.

b. It is a non-hypnotic drug.

c. It is transquillizer.

d. It causes addiction.

78. Calculate the pH at the equivalence point during the titration of 0.1 M, 25 mL CH_3COOH with 0.05 M NaOH solution.
$[K_a(CH_3COOH) = 1.8 \times 10^{-5}]$

a. 9.63 ***b.*** 8.63 ***c.*** 10.63 ***d.*** 11.63

79. The temperature 30.98°C is called critical temperature (T_c) of carbon dioxide. The critical temperature is the

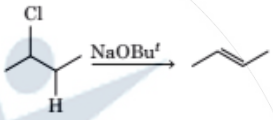
a. lowest temperature at which liquid carbon dioxide is observed

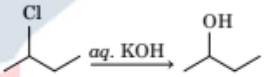
b. highest temperature at which gas carbon dioxide is observed

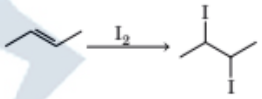
c. highest temperature at which solid carbon dioxide is observed

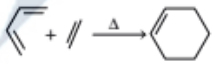
d. highest temperature at which liquid carbon dioxide is observed

80. The types of reactions for these are

I. 

II. 

III. 

IV. 

a. elimination, substitution, addition, addition

b. addition, elimination, addition, substitution

c. elimination, addition, substitution, addition

d. substitution, elimination, addition, addition

PART III

a. English Proficiency

Directions (Q. Nos. 81-83) *Out of the four alternatives, choose the one which expresses the right meaning of the word.*

81. Augment

a. Increase ***b.*** Decrease ***c.*** Save ***d.*** Mention

82. Consolation

a. Comfort ***b.*** Problem ***c.*** Sadness ***d.*** Solution

83. Auxiliary

a. Chief ***b.*** Supplemental

c. Negligible ***d.*** Separate

Directions (Q. Nos. 84-86) *Choose the word apposite is meaning to the given word.*

84. Auspicious

a. Prosperous ***b.*** Unfavourable

c. Improper ***d.*** New

85. Recompense

a. Emolument ***b.*** Reward

c. Payment ***d.*** Penalty

86. Impede

a. Block ***b.*** Delay ***c.*** Push ***d.*** Freeze

Directions (Q. Nos. 87-89) *A part of sentence is underlined. Balance are given alternatives to the underlined part a, b, c and d which may improve the sentence. Choose the correct alternative.*

87. They requested me to follow them.
- **a.** ordered
- **b.** urged
- **c.** asked
- **d.** No improvement

88. She did not believed me.
- **a.** believing
- **b.** believe to
- **c.** believe
- **d.** No improvement

89. I am fine, what about you?
- **a.** your
- **b.** your's
- **c.** yours
- **d.** No improvement

Directions (Q. Nos. 90-92) *Sentence Completion*

90. They were afraid ..... the lion, so they dropped the idea of hunting in jungle.
- **a.** in
- **b.** to
- **c.** from
- **d.** of

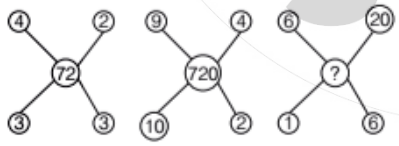
91. Our company signed a profitable ..... last month.
- **a.** issue
- **b.** agenda
- **c.** deal
- **d.** paper

92. What is your ..... for tonight?
- **a.** principle
- **b.** motto
- **c.** plan
- **d.** objective

b. Logical Reasoning

96. In a certain code language, 'SAFER' is written as '5@3#2' and 'RIDE' is written as '2©%#', how would 'FEDS' be written in that code?
a. 3#©5 **b.** 3@%5 **c.** 3#%5 **d.** 3#%2

97. Find the missing number from the given response.



- **a.** 72
- **b.** 720
- **c.** 7200
- **d.** 38

98. If the first and second letters in the word DEPRESSION were interchanged, also the third and fourth letters, the fifth and the sixth letters and so on, then which of the following would be seventh letter from the right?
a. O **b.** P **c.** R **d.** S

Directions (Q. Nos. 93-95) *Arrange the following sentences in correct pattern and mark at the correct combination.*

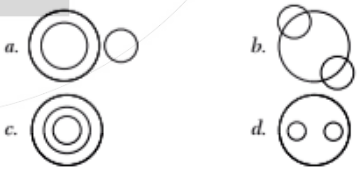
93. 1. Today we live in modern technology era.
P. We have a lot of problems now.
Q. We want to get everything in one day.
R. Ancient time was quite pleasant.
S. We had no problems then.
C. Perhaps greed is the main cause for this.
- **a.** PQRS
- **b.** PRSQ
- **c.** SRQP
- **d.** RPQS

94. 1. He is a common man.
P. Yesterday our city saw a brutal crime.
Q. Police is trying to arrest innocent persons.
R. The criminals are well known.
S. Police as well as whole system in corrupt.
C. Police will arrest him as he is an easy target because of being a common man.
a. PRSQ **b.** PQSR **c.** PQRS **d.** PSQR

95. 1. I want to change the room.
P. Last month I got a job.
Q. I had been living there for six months.
R. The office is far from the room.
S. I want to cut expenses of travelling.
C. Hopefully I will do this next week.
- **a.** PQRS
- **b.** PRSQ
- **c.** QPRS
- **d.** PQSR

99. Today is Thrusday. The day after 59 days will be
- **a.** Sunday
- **b.** Monday
- **c.** Tuesday
- **d.** Wednesday

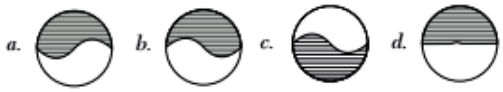
100. Which of the following represents coal mines, factories and fields?



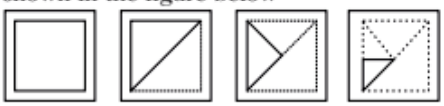
101. Find out the missing term in the series
1, 8, 27, $\frac{?}{}$, 125, 216
a. 52 **b.** 58 **c.** 64 **d.** 65

102. If '+' means '×', '−' means '+', '×' means '÷' and '÷' means '−', then $6-9+8\times3\div20=?$
a. −2 **b.** 6 **c.** 10 **d.** 12

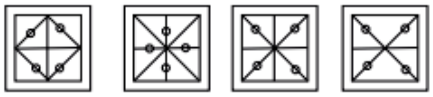
103. What is the water image of



104. A piece of paper is folded and penched as shown in the figure below



How will it appear when unfolded?



105. Here are some words translated from an artificial language.
mallon piml means blue light
mallon tifl means blue berry
arpan tifl means rasp berry
Which word could means 'light house'?

a. tifl mallon
b. piml arpan
c. mallon arpan
d. piml doken

PART IV

Mathematics

106. If p, q, r and s are positive real numbers such that $p+q+r+s=2$, then $M=(p+q)(r+s)$ satisfies the relation

a. $0<M\le 1$ ***b.*** $1\le M\le 2$
c. $2\le M\le 3$ ***d.*** $3\le M\le 4$

107. The complex number $z=x+iy$ which satisfies the equation $\left|\frac{z-3i}{z+3i}\right|=1$, lie on

a. the X-axis
b. the straight line $y=3$
c. a circle passing through origin
d. None of the above

108. If $f(x)$ is an odd periodic function with period 2, then $f(4)$ equals to

a. -4 ***b.*** 4
c. 2 ***d.*** 0

109. The solution of the differential equation

$$\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}+\ldots}{1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots}=\frac{dx-dy}{dx+dy} \text{ is}$$

a. $2ye^{2x}=Ce^{2x}+1$ ***b.*** $2ye^{2x}=Ce^{2x}-1$
c. $ye^{2x}=Ce^{2x}+2$ ***d.*** None of these

110. The value of k such that the lines $2x-3y+k=0$, $3x-4y-13=0$ and $8x-11y-33=0$ are concurrent, is

a. 20 ***b.*** -7 ***c.*** 7 ***d.*** -20

111. Two lines, whose equations are $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{\lambda}$ and $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$ lie in the same plane. Then, the value of $\sin^{-1}\sin\lambda$ is equal to

a. 3 ***b.*** $\pi-3$
c. 4 ***d.*** $\pi-4$

112. If $\frac{e^x}{1-x}=B_0+B_1x+B_2x^2+\ldots+B_nx^n+\ldots$, then the value of B_n-B_{n-1} is

a. 1 ***b.*** $\frac{1}{n}$
c. $\frac{1}{n!}$ ***d.*** None of these

113. $2^{3n}-7n-1$ is divisible by

a. 64 ***b.*** 36
c. 49 ***d.*** 25

114. If $\int_0^{25}e^{x-[x]}\,dx=k(e-1)$, then the value of k is equal to

a. 12 ***b.*** 25
c. 23 ***d.*** 24

115. A variable chord PQ of the parabola $y^2=4ax$ subtends a right angle at the vertex, then the locus of the points of intersection of the normal at P and Q is

a. a parabola ***b.*** a hyperbola
c. a circle ***d.*** None of these

116. If $\omega \neq 1$ is a cube root of unity, then

$$A=\begin{bmatrix} 1+2\omega^{100}+\omega^{200} & \omega^2 & 1 \\ 1 & 1+2\omega^{100}+\omega^{200} & \omega \\ \omega & \omega^2 & 2+\omega^{100}+2\omega^{200} \end{bmatrix}$$

a. A is singular
b. $|A| \neq 0$
c. A is symmetric
d. None of the above

117. $\lim\limits_{x \to \tan^{-1}3} \dfrac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$ equals to

a. 1 *b.* 2
c. 0 *d.* 3

118. The locus of the points of intersection of the tangents at the extremities of the chords of the ellipse $x^2 + 2y^2 = 6$ which touches the ellipse $x^2 + 4y^2 = 4$, is

a. $x^2 + y^2 = 4$ *b.* $x^2 + y^2 = 6$
c. $x^2 + y^2 = 9$ *d.* None of these

119. Number of roots of the equation $|\sin x \cdot \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$, where $x \in [0, 4\pi]$, are

a. 1 *b.* 2
c. 3 *d.* None of these

120. If $f(x) = \begin{cases} \left[\tan\left(\dfrac{\pi}{4}+x\right)\right]^{1/x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

For what value of k, $f(x)$ is continuous at $x = 0$?

a. 1 *b.* 0
c. e *d.* e^2

121. The period of $\sin^2\theta$ is

a. π^2 *b.* π
c. 2π *d.* $\dfrac{\pi}{2}$

122. Five persons A, B, C, D and E are in queue of a shop. The probability that A and E are always together, is

a. $\dfrac{1}{4}$ *b.* $\dfrac{2}{3}$
c. $\dfrac{2}{5}$ *d.* $\dfrac{3}{5}$

123. $\lim\limits_{x \to \infty} \dfrac{x^4 \cdot \sin\left(\dfrac{1}{x}\right) + x^2}{1 + |x|^3}$ equals to

a. 0 *b.* -1 *c.* 2 *d.* 1

124. The sum of the series $\log_4 2 - \log_8 2 + \log_{16} 2 \ldots$ is

a. e^2 *b.* $\log_e 2 + 1$
c. $\log_e 3 - 2$ *d.* $1 - \log_e 2$

125. The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is

a. $\bar{x} + n$ *b.* $\bar{x} + \dfrac{n}{2}$
c. $\bar{x} + \dfrac{n+1}{2}$ *d.* None of these

126. The greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ are respectively

a. $\dfrac{\pi^2}{4}$ and 0 *b.* $\dfrac{\pi}{2}$ and $\dfrac{-\pi}{2}$
c. $\dfrac{5\pi^2}{4}$ and $\dfrac{\pi^2}{8}$ *d.* $\dfrac{\pi^2}{4}$ and $\dfrac{-\pi^2}{4}$

127. The probability of simultaneous occurrence of atleast one of two events A and B is p. If the probability that exactly one of A and B occurs is q, then $P(A') + P(B')$ is equal to

a. $2 - 2p + q$ *b.* $2 + 2p - q$
c. $3 - 3p + q$ *d.* $2 - 4p + q$

128. The solution of the differential equation

$$x = 1 + xy\frac{dy}{dx} + \frac{x^2y^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{x^3y^3}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots$$

is

a. $y = \log x + C$ *b.* $y^2 = (\log x)^2 + C$
c. $y = \log x + xy$ *d.* $xy = x^y + C$

129. If n is a positive integer, then $n^3 + 2n$ is divisible by

a. 2 *b.* 6
c. 15 *d.* 3

130. The sum of the coefficients in the expansion of $(5x - 4y)^n$, where n is a positive integer, is

a. 0 *b.* n
c. 1 *d.* -1

131. The distance of the point $(1, -5, 9)$ from the plane $x + y + z = 5$ measured along a straight line $x = y = z$ is $2\sqrt{3k}$, then the value of k is

a. 5 *b.* 6
c. $\sqrt{3}$ *d.* 4

132. $\lim_{x\to 1}\frac{x^m-1}{x^n-1}$ is equal to

a. $\frac{n}{m}$ *b.* $\frac{m}{n}$

c. $\frac{2m}{n}$ *d.* $\frac{2n}{m}$

133. The value of $x>1$ satisfying the equation $\int_1^x t\log t\,dt=\frac{1}{4}$, is

a. $\sqrt{e}$ *b.* $e^{3/2}$

c. e^2 *d.* $2e-1$

134. $a=\sum_{n=0}^{\infty}\frac{x^{3n}}{3n!}$, $b=\sum_{n=1}^{\infty}\frac{x^{3n-2}}{(3n-2)!}$ and $c=\sum_{n=1}^{\infty}\frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3+b^3+c^3-3abc$ is

a. 1 *b.* 0

c. -1 *d.* -2

135. The unit vector perpendicular to the vectors $\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}$ forming a right handed system is

a. $\hat{\mathbf{k}}$ *b.* $-\hat{\mathbf{k}}$

c. $\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{\sqrt{2}}$ *d.* $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$

136. The number of solutions of the equation $x^3+2x^2+5x+2\cos x=0$ in $[0,2\pi]$ are

a. 0 *b.* 1

c. 2 *d.* 3

137. $\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}$ is equal to

a. 8 *b.* 6

c. 4 *d.* 2

138. The area bounded by the curves $y=-\sqrt{-x}$ and $x=-\sqrt{-y}$, where $x, y\le 0$, is

a. 1/3 *b.* 1/4

c. 1/5 *d.* 1/2

139. If α and β are the roots of the equation $x^2-px+q=0$, then the value of

$$(\alpha+\beta)x-\left(\frac{\alpha^2+\beta^2}{2}\right)x^2+\left(\frac{\alpha^3+\beta^3}{3}\right)x^3+\ldots, \text{ is}$$

a. $\log(1-px+qx^2)$

b. $\log(1+px-qx^2)$

c. $\log(1+px+qx^2)$

d. None of the above

140. If $\cos^{-1}x>\sin^{-1}x$, then

a. $\frac{1}{\sqrt{2}}<x\le 1$ *b.* $0\le x<\frac{1}{\sqrt{2}}$

c. $-1\le x<\frac{1}{\sqrt{2}}$ *d.* $x>0$

141. A student is allowed to select atmost n books from a collection of $(2n+1)$ books. If the number of ways in which he can do this, is 64, then the value of n is

a. 6 *b.* n

c. 3 *d.* None of these

142. Let $R=\{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A=\{3,6,9,12\}$. The relation is

a. an equivalence relation

b. reflexive and symmetric

c. reflexive and transitive

d. only reflexive

143. The value of a, so that the sum of squares of the roots of the equation $x^2-(a-2)x-a+1=0$ assume the least value, is

a. 2 *b.* 0 *c.* 3 *d.* 1

144. The form of the differential equation of the central conics $ax^2+by^2=1$ is

a. $x=y\frac{dy}{dx}$

b. $x\left(\frac{dy}{dx}\right)^2+xy\frac{d^2y}{dx^2}-y\frac{dy}{dx}=0$

c. $x+y\frac{d^2y}{dx^2}=0$

d. None of the above

145. A furniture dealer deals in only two items namely tables and chairs. He has ₹ 5000 to invest and space to store at the most 60 pieces. A table cost him ₹ 250 and a chair ₹ 60. He can sell a table at a profit of ₹ 15. Assume that he can sell all the items that he produced. The number of constraints in the problem are

a. 2 *b.* 3 *c.* 4 *d.* 5

146. $10^n+3(4^{n+2})+5$ is divisible by $(n\in N)$

a. 7 *b.* 5 *c.* 9 *d.* 17

147. The total number of subsets of a finite set A has 56 more elements than the total number of subsets of another finite set B. What is the number of elements in the set A ?

a. 5 *b.* 6

c. 7 *d.* 8

148. A sample of 35 observations has the mean 80 and standard deviation as 4. A second sample of 65 observations from the same population has mean 70 and standard deviation 3. Then, the standard deviation of the combined sample is

a. 5.85 *b.* 5.58

c. 34.2 *d.* None of these

149. Three straight lines $2x + 11y - 5 = 0$, $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$

a. from a triangle

b. are only concurrent

c. are concurrent with one line bisecting the angle between the other two

d. None of the above

150. If the plane $x + y + z = 1$ is rotated through an angle 90° about its line of intersection with the plane $x - 2y + 3z = 0$, then the new position of the plane is

a. $x - 5y + 4z = 1$

b. $x - 5y + 4z = -1$

c. $x - 8y + 7z = 2$

d. $x - 8y + 7z = -2$

Answers

Physics

1. (c)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (c)
11. (a)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (*)	**19.** (a)	**20.** (c)
21. (a)	**22.** (b)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (a)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (c)
31. (b)	**32.** (c)	**33.** (b)	**34.** (d)	**35.** (c)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (b)

Chemistry

41. (a)	**42.** (c)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (b)	**48.** (d)	**49.** (a)	**50.** (d)
51. (d)	**52.** (d)	**53.** (d)	**54.** (c)	**55.** (d)	**56.** (b)	**57.** (d)	**58.** (b)	**59.** (b)	**60.** (d)
61. (a)	**62.** (c)	**63.** (b)	**64.** (d)	**65.** (c)	**66.** (a)	**67.** (c)	**68.** (b)	**69.** (c)	**70.** (d)
71. (a)	**72.** (b)	**73.** (b)	**74.** (c)	**75.** (b)	**76.** (b)	**77.** (b)	**78.** (b)	**79.** (d)	**80.** (a)

English Proficiency

81. (a)	**82.** (a)	**83.** (b)	**84.** (b)	**85.** (d)	**86.** (c)	**87.** (a)	**88.** (c)	**89.** (d)	**90.** (d)
91. (c)	**92.** (c)	**93.** (b)	**94.** (a)	**95.** (c)					

Logical Reasoning

96. (c)	**97.** (b)	**98.** (b)	**99.** (a)	**100.** (d)	**101.** (c)	**102.** (c)	**103.** (b)	**104.** (b)	**105.** (d)

Mathematics

106. (a)	**107.** (a)	**108.** (d)	**109.** (b)	**110.** (b)	**111.** (d)	**112.** (c)	**113.** (c)	**114.** (b)	**115.** (a)
116. (a)	**117.** (b)	**118.** (c)	**119.** (d)	**120.** (d)	**121.** (b)	**122.** (c)	**123.** (b)	**124.** (d)	**125.** (c)
126. (c)	**127.** (a)	**128.** (b)	**129.** (d)	**130.** (c)	**131.** (a)	**132.** (b)	**133.** (a)	**134.** (a)	**135.** (a)
136. (a)	**137.** (c)	**138.** (a)	**139.** (a)	**140.** (c)	**141.** (c)	**142.** (c)	**143.** (d)	**144.** (b)	**145.** (c)
146. (c)	**147.** (b)	**148.** (a)	**149.** (c)	**150.** (d)					

Note *(*) None of the option is correct.*

Hints & Solutions

Physics

1. ***(c)*** In amplitude modulation, amplitude of carrier wave varies with signal to be modulated, so modulated signal $C_m(t)$ is given by

$$C_m(t) = [A_c + m(t)] \sin \omega_c t$$

$$\Rightarrow \quad C_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$$

$$[\because \text{given } m(t) = A_m \sin \omega_m t]$$

2. ***(c)*** By Einstein's photoelectric effect equation,

$$KE_{max} = eV_S = h\nu - h\nu_0$$

$$\Rightarrow \quad V_S = \left(\frac{h}{e}\right)\nu - \frac{h\nu_0}{e}$$

Graph of V_s with ν_0 is straight line whose slope $= \frac{h}{e}$

From given graph slope $= \frac{AB}{BC} \Rightarrow \frac{AB}{BC} = \frac{h}{e}$

3. ***(c)*** Here, $m = 0.18$ kg, $K = 2$ N / m, $\mu = 0.1$, $x = 0.06$ m.

According to conservation of mechanical energy principle, we know

Decrease in mechanical energy = Work done against friction

$$\frac{1}{2}mv^2 - \frac{1}{2}kx^2 = \mu mgx$$

$$\Rightarrow \quad v = \sqrt{\frac{2\mu mgx + Kx^2}{m}}$$

Substituting the values of m, μ, g, x and K, we get

$$v = \sqrt{\frac{2 \times 0.1 \times 0.18 \times 9.8 \times 0.06 + 2 \times 0.06}{0.18}}$$

$$v = (4/10) \text{ m/s} = \frac{N}{10} \text{ (given)}$$

So, $N = 4$

4. ***(b)*** Ohm's law says

$$\frac{V}{I} = R = \text{constant}$$

5. ***(b)*** From equation of motion,

(i) $v = u + at_1$

$$40 = 0 + a \times 20$$

$$\therefore \quad a = 2 \text{ m/s}^2$$

and $\quad v^2 - u^2 = 2as$

$$40^2 - 0 = 2 \times 2s_1$$

$$\therefore \quad s_1 = 400 \text{ m}$$

(ii) $s_2 = v \times t_2 = 40 \times 20 = 800$ m

(iii) $v = u + at$

$$0 = 40 + a \times 40$$

$$\therefore \quad a = -1 \text{ m/s}^2$$

$$0^2 - 40^2 = 2(-1)s_3$$

$$\therefore \quad s_3 = 800 \text{ m}$$

Total distance travelled

$$= s_1 + s_2 + s_3$$

$$= 400 + 800 + 800 = 2000 \text{ m}$$

Total time taken $= 20 + 20 + 40 = 80$ s

$\therefore$ Average velocity $= \frac{2000}{80} = 25$ m/s

6. ***(c)*** As stress is shown on X-axis and strain on Y-axis.

So, we can say that Young's modulus of elasticity,

$$Y = \frac{\text{stress}}{\text{strain}} = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$$

So, elasticity of wire P is minimum and R is maximum. Hence, we can say tensile strength of R is maximum.

7. ***(a)*** Given, $F \propto A^a v^b d^c$

$$F = kA^a v^b d^c$$

$$\Rightarrow \quad [F] = [A]^a [v]^b [d]^c$$

$$[MLT^{-2}] = [L^{2a}] \times [L^b T^{-b}] [M^c L^{-3c}]$$

$$= [M^c L^{2a+b-3c} T^{-b}]$$

Comparing powers of M, L and T, we get

$$c = 1, \; 2a + b - 3c = 1$$

$$-b = -2 \text{ or } b = 2$$

Also, $\quad 2a + 2 - 3(1) = 1$

$$2a = 2 \text{ or } a = 1$$

$\therefore$ This is 1, 2, 1

8. ***(d)*** Radius of nth orbit, $r_n \propto n^2$

$$\frac{r_{n\,big}}{r_{n\,small}} = \frac{n_{big}^2}{n_{small}^2} = \frac{4}{1} \quad \text{(given)}$$

$$\Rightarrow \quad \frac{n_{big}}{n_{small}} = 2$$

$$\Rightarrow \quad \frac{n_{small}}{n_{big}} = \frac{1}{2}$$

Velocity of electron in nth orbit,

$$v_n \propto \frac{1}{n}$$

$$\frac{v_{n\,big}}{v_{n\,small}} = \frac{n_{small}}{n_{big}} = \frac{1}{2}$$

$$v_{n\,small} = 2(v_{n\,big}) = 2v$$

9. ***(d)*** As given for steel wire,

$A_1 = 3\times 10^{-5}\ \text{m}^2,\ l_1 = 4.7\ \text{m},\ \Delta l_1 = \Delta l;\ F_1 = F$

For copper wire,

$A_2 = 4\times 10^{-5}\ \text{m}^2,\ l_2 = 3.5\ \text{m},\ \Delta l_2 = \Delta l,\ F_2 = F$

Let Y_1 and Y_2 be the Young's modulus of steel wire and copper wire, respectively.

So, $$Y_1 = \frac{F_1}{A_1}\times\frac{l_1}{\Delta l_1} = \frac{F}{3\times 10^{-5}}\times\frac{4.7}{\Delta l}$$

and $$Y_2 = \frac{F_2\times l_2}{A_2\times\Delta l_2} = \frac{F\times 3.5}{4\times 10^{-5}\times\Delta l}$$

$$\therefore\quad \frac{Y_1}{Y_2} = \frac{4.7\times 4\times 10^{-5}}{3.5\times 3.0\times 10^{-5}} = 1.8$$

So, $Y_1 : Y_2 = 1.8 : 1$

10. ***(c)*** The net electric field at point P is given by

$$\mathbf{E} = \frac{qx}{4\pi\varepsilon_0(x^2+a^2)^{3/2}}\hat{\mathbf{i}}$$

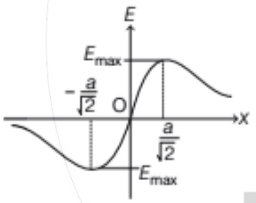
$\therefore$ At centre of ring, $x = 0 \Rightarrow \mathbf{E} = 0$

It $x >> a$, $$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

$$\Rightarrow\quad E \propto \frac{L}{x^2}$$

At, $x << a$, $E = \dfrac{qx}{4\pi\varepsilon_0 a^3}$

$$\Rightarrow\quad E \propto x$$



and E will be maximum, where

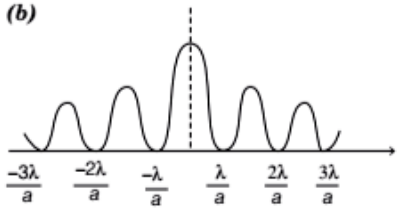
$$\frac{dE}{dx} = 0$$

or $$x = \pm\frac{a}{\sqrt{2}}$$

and $$E_{\max} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}a^2}$$

11. ***(a)*** Constant phase difference is the essential condition for coherent sources.

12. ***(b)***



So, angular size of central maxima is

$$= \frac{\lambda}{a} - \left(-\frac{\lambda}{a}\right) = 2\left(\frac{\lambda}{a}\right) = \frac{2\lambda}{a}$$

13. ***(d)*** From first polaroid, the unpolarised light will become polarised with half the intensity.

So, $$I' = \frac{I_0}{2}$$

From second polaroid,

$$I'' = I'\cos^2\theta = \frac{I_0}{2}\cos^2(60) = \frac{I_0}{2}\frac{1}{4} = \frac{I_0}{8}$$

14. ***(a)*** Resolving power, $\text{RP} = \dfrac{a}{1.22\,\lambda}$

$$\text{RP} \propto a$$

15. ***(d)*** $r = r_0\,(A)^{1/3}$

r increases with increasing A (= mass number).

So, $r_A < r_B$ as mass number of A is smaller.

E_{bn} decreases with increasing A for $A > 56$.

So, ^{56}Fe has highest E_{bn} value.

So, E_{bn} for $A = 64$ is larger as compared to E_{bn} for nucleus with $A = 125$

$$E_{bnA} > E_{bnB}$$

16. ***(b)*** Heat energy is not a state variable, i.e., unlike temperature, pressure and volume, it is not a intrinsic property of a system. It has meaning only as it describes the transfer of energy into or out of a system.

17. ***(c)*** The current density in the wire is

$$J = \frac{I}{A} = \frac{4}{2\times 10^{-6}} = 2\times 10^6\ \text{Am}^{-2}$$

We know that, $I = ne\,A\,v_d$

$$\Rightarrow\quad v_d = \frac{I}{neA} = \frac{J}{ne}$$

$$= \frac{2\times 10^6}{12\times 10^{28}\times 1.6\times 10^{-19}}$$

$$\Rightarrow\quad v_d = \frac{10^6}{6\times 1.6\times 10^9} = 1.04\times 10^{-4}\ \text{m/s}$$

18. (*) Mass of metal, $m = 0.2\ \text{kg} = 200\ \text{g}$

Fall in temperature of metal

$$\Delta T = 150 - 40 = 110^\circ\text{C}$$

If s is specific heat of metal, the heat lost by the metal

$$\Delta Q = ms\,\Delta T = 200\times 110\times s \qquad \ldots(\text{i})$$

Volume of water = 150 cc

Mass of water $m' = 150\ \text{g}$

Water equivalent of calorimeter

$$w = 0.025\ \text{kg} = 25\ \text{g}$$

Rise in temperature of water in calorimeter

$$\Delta T' = 40 - 27 = 13^\circ\text{C}$$

Heat gained by water and calorimeter

$$\Delta Q' = (m' + w)\,\Delta T' \times s_w$$
$$= (150 + 25) \times 13 \times 4.18$$
$$\Delta Q' = 175 \times 13 \times 4.18 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\Delta Q' = \Delta Q$$

So, $200 \times s \times 100 = 175 \times 13 \times 4.18$

$$\Rightarrow \quad s = \frac{175 \times 13 \times 4.18}{200 \times 100} = 0.42$$

Hence, no option is correct.

19. ***(a)*** $V(x, y, z) = -\int_{\infty}^{(x, y, z)} \mathrm{E} \cdot d\mathbf{r}$

$$= -\int_{\infty}^{(x, y, z)} \frac{M dx}{x^3} = \frac{M}{2x^2}$$

20. ***(c)*** Taking vertical downward motion of projectile from point of projection to ground, we have

$$u = -50 \sin 30° = -25 \text{ m/s}$$
$$a = g = +10 \text{ m/s}^2, s = 70 \text{ m}, t = ?$$

From equation of motion,

$$s = ut + \frac{1}{2}at^2$$

So, $$70 = -25 \times t + \frac{1}{2} \times 10 \times t^2$$

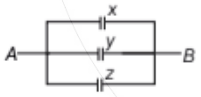
or $$5t^2 - 25t - 70 = 0$$

or $$t^2 - 5t - 14 = 0$$

On solving, we get

$$t = 7\text{s}$$

21. ***(a)*** The equivalent circuit of the given figure is as follow.



$$\therefore \quad C_{eq} = X + Y + Z = 1 + 2 + 3 = 6\,\mu\text{F}$$

22. ***(b)*** As given surface tension, $S = 0.06\ \text{Nm}^{-1}$,

$r_1 = 2 \text{ cm} = 0.02 \text{ m}$, $r_2 = 5 \text{ cm} = 0.05 \text{ m}$

Since, bubble has two surface

Initial surface area of the bubble $= 2 \times 4\pi r_1^2$

$$= 2 \times 4\pi \times (0.02)^2 = 32\pi \times 10^{-4}\ \text{m}^2$$

Final surface area of the bubble $= 2 \times 4\pi r_2^2$

$$= 2 \times 4 \times \pi \times (0.05)^2$$
$$= 200 \times \pi \times 10^{-4}\ \text{m}^2$$

So, work done = $S \times$ increase in surface

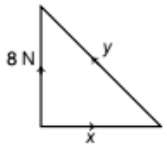
$$= 0.06 \times (200\,\pi \times 10^{-4} - 32\,\pi \times 10^{-4})$$
$$= 0.06 \times 168\,\pi \times 10^{-4}$$
$$= 0.003168\ \text{J}$$

23. ***(c)*** The given situation is shown below

$$x + y = 16$$

Also, $$y^2 = 8^2 + x^2$$

or $$y^2 = 64 + (16 - y)^2 \qquad (\because x = 16 - y)$$



or $$y^2 = 64 + 256 + y^2 - 32y$$

or $$32y = 320$$

$$Y = 10\ \text{N}$$

$$\therefore \quad x + 10 = 16 \Rightarrow x = 6\ \text{N}$$

24. ***(a)*** By Kirchhoff's laws,

$$V_A - V_D = -6i - 5i + 10 - 4i$$

Here, $$V_A = V_D$$

Since, points A and D are centred.

$$-6i - 5i + 10 - 4i = 0 \Rightarrow 15i = 10$$

$$\Rightarrow \quad i = \frac{10}{15} = 0.67\ \text{A}$$

25. ***(a)*** The pressure exerted by a 3000 m column of water on the bottom layer.

$$p = h\rho g = 3000 \times 1000 \times 10$$
$$= 3 \times 10^7 = 3 \times 10^7\ \text{N/m}^2$$

Fractional compression $\left(\frac{\Delta V}{V}\right)$

$$= \frac{\text{Stress}}{B} = \frac{3 \times 10^7}{2.2 \times 10^9}$$
$$= 1.36 \times 10^{-2}$$

$$\frac{\Delta V}{V} \times 100 = 1.36\%$$

26. ***(a)*** Area of parallelogram $= |\mathbf{A} \times \mathbf{B}|$

$$\Rightarrow \quad AB \sin\theta = \frac{1}{2} AB$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2},\ \theta = 30°$$

27. ***(c)*** Volume of cuboid $= l \times b \times h$

$$= 1.8 \times 1.5 \times 1.2\ \text{cm}^3$$
$$= 2.70 \times 1.2 = 3.240\ \text{cm}^3$$

Using concept of significant figures, product is reported in number of significant figures present in measurement which has least number of significant figures, here all measurement have 2 significant figures.

So, volume = $3.2\ \text{cm}^3$

(keeping 2 significant figures only.)

28. ***(c)*** According to first condition,

$$h = -vt_1 + \frac{1}{2}gt_1^2 \text{ or } \frac{h}{t_1} = -v + \frac{1}{2}gt_1 \quad \ldots(i)$$

Similarly, according to second condition,

$$h = vt_2 + \frac{1}{2}gt_2^2 \text{ or } \frac{-h}{t_2} = +v + \frac{1}{2}gt_2 \quad \ldots(ii)$$

$$\therefore \quad \frac{h}{t_1} + \frac{h}{t_2} = \frac{1}{2}g(t_1 + t_2) \quad \text{[From Eqs. (i) and (ii)]}$$

or $$h = \frac{1}{2}gt_1t_2$$

For falls under gravity from the top of the tower,

$$h = \frac{1}{2}gt^2$$

$$\therefore \quad \frac{1}{2}gt_1t_2 = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{t_1t_2}$$

29. ***(b)*** When elevator is moving up with acceleration a_0, then tension in the wire.

$$T = m(g + a_0) = 10(10 + 2) = 120 \text{ N}$$

$$\text{Stress} = \frac{T}{A} = \frac{120}{2 \times 10^{-4}} = 60 \times 10^4 \text{ Nm}^{-2}$$

$$\text{and } Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow \text{Strain} = \frac{\text{Stress}}{Y}$$

$$= \frac{60 \times 10^4}{2 \times 10^{11}}$$

$$= 30 \times 10^{-7} = 3 \times 10^{-6}$$

30. ***(c)*** Here, $F = 3.7 \times 10^{-9}$ N

Let, $q_1 = q_2 = q$

$$r = 5 \text{ Å} = 5 \times 10^{-10} \text{ m}$$

$$\because \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\Rightarrow \quad 3.7 \times 10^{-9} = 9 \times 10^9 \frac{q \times q}{(5 \times 10^{-10})^2}$$

$$q^2 = \frac{3.7 \times 10^{-9} \times 25 \times 10^{-20}}{9 \times 10^9}$$

$$= 10.28 \times 10^{-38}$$

or $$q = 3.2 \times 10^{-19} \text{ C}$$

Now, $$q = ne$$

$$\therefore \quad n = \frac{q}{e} = \frac{3.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 2$$

31. ***(b)*** Since, the radius of circular path of charge particle in magnetic field, $r = \frac{mv}{qB} = \frac{p}{qB}$

Now, the radius of circular path of charge particle of given momentum p and magnetic field B is given by

$$r \propto \frac{1}{q}$$

But charge on both charge particles protons and deuterons, is same. Therefore,

$$\frac{r_p}{r_D} = \frac{q_D}{q_p} = \frac{1}{1}$$

32. ***(c)*** Here, $L = 1$ m, $N = 1000$

The number of turn per unit length

$$n = N/L = 1000 \text{ turn/m}$$

Magnetic field inside the solenoid

$$B = \mu_0 nI = \mu_0 \times 1000 \times 5$$

$$= 2\pi \times 10^{-3} \text{ T}$$

The direction of magnetic field is along the axis of solenoid.

For electron $q = -e$, $v = 10^4$ ms^{-1}

Magnetic Lorentz force, $F = -evB \sin 0° = 0$ as the angle between B and v is 0°.

33. ***(b)*** Magnetic field can accelerate a charge particle by changing the direction of its velocity but it cannot change the speed of charged particle as magnetic force always acts perpendicular to the velocity of charged particle.

34. ***(d)*** Area of coil, $A = \text{side}^2 = (0.1)^2 = 0.01 \text{ m}^2$

Number of turns, $N = 20$

Current, $I = 12$ A

Normal to the coil make an angle θ with the direction of B, magnetic field $B = 0.80$ T

Torque, experienced by the coil, $\tau = 0.96$ N-m

Since, total torque on the coil $\tau = (NIA)B \sin\theta$

Substituting the values in above formula,

$$0.96 \text{ N-m} = 20 \times 12 \text{ A} \times 0.01 \text{ m}^2 \times 0.80 \text{ T} \times \sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{0.96}{1.92} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6} \text{ rad}$$

35. ***(c)*** $R_G = 60.00\ \Omega$; shunt resistance, $r_s = 0.02\ \Omega$

Total resistance in the circuit is $R_G + 3 = 63\ \Omega$

Hence, $I = \frac{3}{63} = 0.048$ A

Resistance of the galvanometer converted to an ammeter is

$$\frac{R_G r_s}{R_G + r_s} = \frac{60\ \Omega \times 0.02\ \Omega}{(60 + 0.02)\ \Omega} = 0.02\ \Omega$$

Total resistance in the circuit is

$$= 0.02 + 3 = 3.02\ \Omega$$

Hence, current, $I = \frac{3}{3.02} = 0.99$ A

36. ***(b)*** $\chi = \frac{C}{T} \Rightarrow \frac{\chi_1}{\chi_2} = \frac{T_2}{T_1}$

$\Rightarrow \quad \frac{1.2\times 10^{-5}}{1.8\times 10^{-5}} = \frac{T_2}{300}$

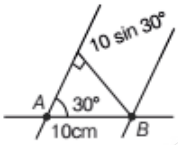
$\Rightarrow \quad T_2 = \frac{12}{18}\times 300 = 200\ \text{K}$

37. ***(b)*** Since perpendicular distance between two equipotentials is

$$dx = 10 \sin 30° \text{ cm} = 5 \text{ cm}$$

Since the potential gradient gives magnetic field (**B**) as

$$\frac{dV}{dx} = B$$



Substituting the values,

$$\frac{dV}{dx} = B = \frac{0.1\times 10^{-4}\ \text{T-m}}{5\times 10^{-2}\ \text{m}}$$

$$B = 2\times 10^{-4}\ \text{T}$$

Since B is perpendicular to equipotential surface. Here, it is at angle 120° with (+ ve)X-axis.

38. ***(c)*** Given, $Q = -20$ J, $W = -8$ J

Using first law of thermodynamics,

$$Q = \Delta U + W$$

$$\Rightarrow \quad -20 = \Delta U + (-8)$$

$$\Rightarrow \quad \Delta U = -12 \text{ J}$$

$$\Rightarrow \quad U_f - U_i = -12 \text{ J}$$

$$\Rightarrow \quad U_f - 30 = -12 \text{ J}$$

$$\Rightarrow \quad U_f = 18 \text{ J}$$

39. ***(b)*** Assuming the balloons have the same volume, as $pV = nRT$. If p, V and T are the same, n the number of moles present will be the same, whether it is He or air. Hence, number of molecules per unit volume will be same in both the balloons.

40. ***(b)*** The given wave equation is

$$y = 3\sin\frac{\pi}{2}(50t - x)$$

$$\Rightarrow \quad y = 3\sin\left(25\pi\, t - \frac{\pi}{2}x\right) \quad \ldots(\text{i})$$

Comparing with standard equation,

$$y = a\sin(\omega t - kx) \quad \ldots(\text{ii})$$

We get, $\omega = 25\pi$, $k = \pi/2$

Wave velocity, $v = \frac{\omega}{K} = \frac{25\pi}{\pi/2} = 50$ m/s

Particle velocity,

$$v_P = \frac{dy}{dt} = \frac{d}{dt}\left[3\sin\left(25\pi t - \frac{\pi}{2}x\right)\right]$$

$$v_P = 75\ \pi\cos\left(25\pi t - \frac{\pi}{2}x\right)$$

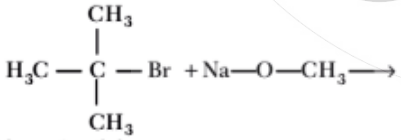
Maximum particle velocity, $(v_P)_{\text{max}} = 75\pi$ m/s

$$\Rightarrow \quad \frac{(v_P)_{\text{max}}}{v} = \frac{75\pi}{50}$$

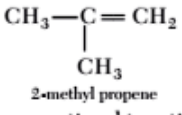
$$\Rightarrow \quad \frac{(v_p)_{\text{max}}}{v} = \frac{3}{2}\pi$$

Chemistry

41. ***(a)*** When tertiary alkyl halide is treated with sodium alkoxide than elimination reaction competes over substitution reaction because alkoxides are not only nucleophiles but strong base as well. Therefore, alkenes are formed instead of ethers as follows.



2-bromo-2-methyl propane



2-methyl propene

42. ***(c)*** Radius of orbit is directly proportional to ratio of square of principal quantum number (n) and atomic number (z).

i.e., Radius of orbit $\propto \frac{n^2}{Z}$

∴ Option (c) is incorrect regarding Bohr's theory.

43. ***(c)*** Molecules/compounds and their shape can be arranged as

Compound	Shape
XeF_2	Linear
$ZnCl_2$	Linear
BCl_3	Triangular planar
ClF_3	T-shaped
CH_4	Tetrahedral
SF_4	*See-saw*
SO_2	Bent
CO_2	Linear

∴ XeF_2 and $ZnCl_2$ both have same shape (linear).

44. ***(a)*** $CaCl_2 + NaCl = 10$ g

Let weight of $CaCl_2 = x$ g

$$CaCl_2 \longrightarrow CaCO_3 \longrightarrow CaO$$

1 mol 1 mol 1 mol

$\frac{x}{111}$ mol $\frac{x}{111}$ mol $\frac{x}{111}$ mol

$$\text{Mole of } CaO = \frac{1.62}{56}$$

$$\therefore \quad \frac{x}{111} = \frac{1.62}{56}$$

$$x = 3.21 \text{ g}$$

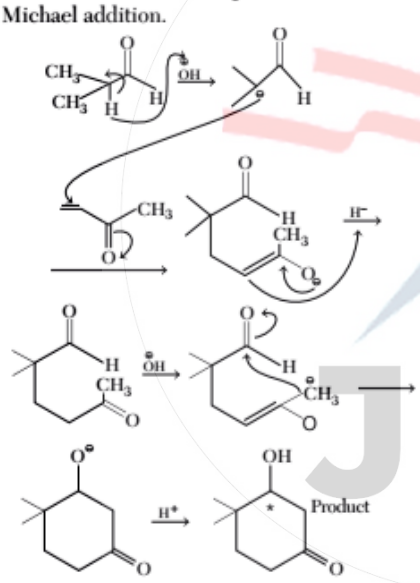
$$\% \text{ of } CaCl_2 = \frac{3.21}{10} \times 100$$

$$= 32.1\%$$

45. ***(a)*** This problems includes conceptual mixing of Michael addition and number of chiral centre.

- Complete the reaction using the concept of Michael addition.
- Then, complete intramolecular aldol condensation.
- Now, count number of chiral centre.

Michael addition Addition of nucleophile to enone system is done in such a way that the addition looks like addition at 1st and 4th position of enone is known as Michael addition.



The number of chiral centre in product is 1 represented by asterisk (*).

46. ***(a)*** Among isoelectronic species, cations having highest charge are smallest while anion having highest charge are largest.

Cation < Neutral atom < Anion

Hence, correct choice is

$$Al^{3+} < Mg^{2+} < Na^+ < F^- < O^{2-}$$

47. ***(b)*** Oxidation state of Cl in KCl = − 1

Oxidation state of Cl in $KClO_3$ = + 5

$$\therefore \text{ Ratio of oxidation states of Cl} = \frac{-1}{5}$$

48. ***(d)*** $Cu^{2+} + 2e^- \longrightarrow Cu$

$$E_{Cu^{2+}/Cu} = E_{Cu^{2+}/Cu} - \frac{0.059}{2} \log \frac{1}{[Cu^{2+}]}$$

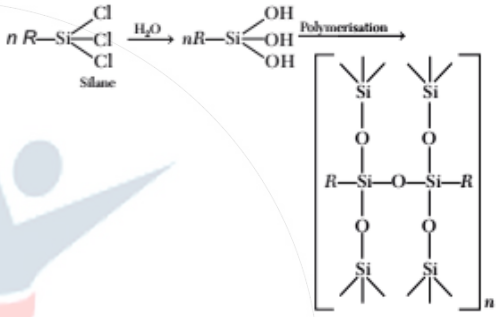
$$= E^\circ_{Cu^{2+}/Cu} - \frac{RT}{2F} \ln [Cu^{2+}]$$

Intercept = 0.34 $\Rightarrow E^\circ_{Cu^{2+}/Cu} = 0.34$

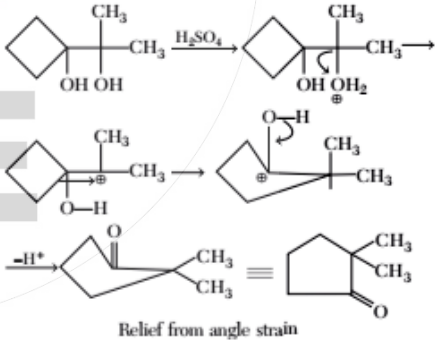
$$E_{Cu^{2+}/Cu} = 0.34 + \frac{0.059}{2} \log 0.1 = 0.31 \text{ V}$$

$$E_{Cu/Cu^{2+}} = -E^\circ_{Cu^{2+}/Cu} = -0.34 + \frac{0.059}{2} \text{V}$$

49. ***(a)*** $RSiCl_3$ on hydrolysis produces cross linked polymer.



50. ***(d)*** **Pinacol-pinacolone rearrangement** The diol is converted into α-hydroxy ketone when reacted in presence of acid is believed to proceeds through rearrangement of carbocation as shown.



Relief from angle strain

Basic of carbocationic rearrangement is due to relief from angle strain.

51. ***(d)*** A mixture of chemical compounds having a single chemical composition, solidifies at a lower temperature than any other composition made up of the same ingredients. This mixture is called **eutectic mixture**.

Freezing point of eutectic mixture of $NaCl/H_2O$ is only − 18°C but the ambient temperature of very cold countries is much lower than − 18°C. In such situations, NaCl will be ineffective. Thus, for such

situations eutectic mixture $CaCl_2$ / H_2O is used because it has freezing point of $-55\,°C$ which is much lower than NaCl.

This mixture lowers the freezing point of ice that allows street snow or ice to melt at lower temperature.

52. ***(d)*** **Caution Point** Gold number is used for calculating the protective powers of lyophillic colloids.

Smaller the gold number, greater is its protective power. Hence, the order would be

$$x > y > z$$

53. ***(d)*** Since, $\Delta G° = \Delta H° - T\Delta S°$

where, $\Delta G°$ = standard Gibbs' free energy of the reaction

$\Delta S°$ = standard entropy of the reaction

$\Delta H°$ = standard enthalpy of the reaction

T = temperature

From, the above equation, it is clear that, $\Delta G°$ will be more negative when $\Delta S°$ is less negative (or $\Delta S°$ is high].

In the diagram, $\Delta G°$ value for $C + \frac{1}{2}O_2 \longrightarrow CO$ is less negative, (lower) than that for

$$C + O_2 \longrightarrow CO_2$$

Therefore, $\Delta S°$ would be higher for

$$C + \frac{1}{2}O_2 \longrightarrow CO$$

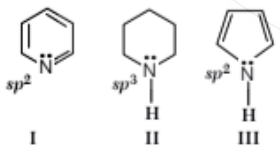
than that for $C + O_2 \longrightarrow CO_2$.

Hence, $\Delta S°\,[C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g)]$

$$> \Delta S°\,[C(s) + O_2(g) \longrightarrow CO_2(g)]$$

54. ***(c)*** This problem includes conceptual mixing of basic strength, hybridisation of nitrogen atom and extent of conjugation. While solving such problem, students are advised to draw the structure and mark the type of hybridisation on N-atom, then answer the question by using combined concept of hybridisation and conjugation.

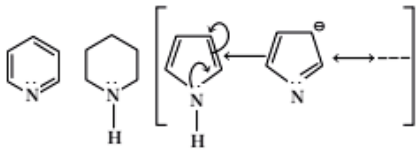
Hybridisation of N-atom in below compounds are sp^2, sp^3 and sp^2 respectively.



I II III

Greater the s-character more will be electronegativity of N-atom and lesser will be its basicity. On this basis, I and III are less basic than II.

Conjugation If lone pairs of electron of N is involved in conjugation causes decrease in basicity of compound due to lesser availability of lone pair for donation to show basic nature.



Lone pair involved in formation of aromatic sextet of 6π-electron (least basic).

∴ Correct order of basicity is II > I > III.

55. ***(d)*** As, density $(\rho) = \dfrac{Z_{eff} \times \text{molecular weight}}{N_A \times a^3}$

(For antifluorite, $Z_{eff} = \dfrac{4}{\text{unit cell}}$

$$\rho = \frac{4 \times (23 \times 2 + 16)}{6 \times 10^{23} \times (10^{-10}\,\text{cm})^3}$$

$$= 414.16\ \text{g cm}^{-3}$$

$$[1\ \text{picometre (pm)} = 10^{-12}\ \text{m} = 10^{-10}\ \text{cm}]$$

Frenkel defect is the type of stoichiometric defect in which density of the crystal does not change.

56. ***(b)*** We can represent the process of evaporation as

$$18\ \text{g}\ H_2O(l) \xrightarrow{\text{vaporisation}} 18\ \text{g}\ H_2O(g)$$

Number of moles in 18 g $H_2O(l)$ is

$$= \frac{18\ \text{g}}{18\ \text{g mol}^{-1}} = 1\ \text{mol}$$

$$\Delta_{vap}U = \Delta_{vap}H^{\ominus} - p\Delta V$$

$$= \Delta_{vap}H^{\ominus} - \Delta n_g RT$$

Assume steam behave as an ideal gas.

$$\Delta_{vap}U = (40.66) - (1)(8.314 \times 10^{-3})(373)$$

$$= 40.66 - 3.10$$

$$= 37.56\ \text{kJ mol}^{-1}$$

57. ***(d)*** In case of cyclopropyl carbocation, stability of carbocation depends upon conjugation between bent orbitals of cyclopropyl ring and vacant p-orbital of cationic carbon. This type of bonding is known as banana bonding.

In case of carbocation III and IV, stability is because of the resonace effect. Three R-effect dominates over two banana bondings so, III is more stable than II.

∴ Correct order of stability of carbocation is

$$\text{I} > \text{III} > \text{II} > \text{IV}$$

58. ***(b)*** The electronic configuration of mercury is $[Xe]4f^{10}, 5d^{10}, 6s^2$. Its d-subshell is completely filled thus, it prevents the overlapping of d-orbitals (d-d overlapping). Hence, it is liquid metal at room temperature.

59. ***(b)*** Let V litre of 10 N HCl be mixed with $(1 - V)$ litre of 4 N HCl to give $(V + 1 - V) = 1$ L of 7 N HCl

As we know that,

$$N_1V_1 + N_2V_2 = NV$$
$$10V + 4(1 - V) = 7 \times 1$$
$$10V + 4 - 4V = 7$$
$$6V = 7 - 4$$
$$V = \frac{3}{6} = 0.50 \text{ L}$$

Volume of 10 N HCl = 0.50 L

Volume of 4 N HCl = 1 − 0.50 = 0.50 L

60. ***(d)*** $t_{1/2} \propto \left(\frac{1}{a}\right)^{n-1}$ or $t_{1/2} = k(a)^{1-n}$

$$\log t_{1/2} = \log k + (1 - n) \log a$$

(It represents straight line equation; $y = c + mx$)

slope, $m = (1 - n) = \tan 45° = 1$

$\therefore \quad (1 - n) = 1$

$\Rightarrow \quad n = 0$

61. ***(a)*** Ziegler-Natta catalyst $[TiCl_4 + Al(C_2H_5)_3]$ is used as a catalyst in the polymerisation of olefins.

62. ***(c)*** Hydrides are binary compounds of hydrogen. These can be classified into four groups

(i) Ionic hydrides : NaH, CaH_2, LiH

(ii) Covalent hydrides : B_2H_6 , NH_3, SbH_3

(iii) Polynuclear hydrides : $LiAlH_4$, $NaBH_4$

(iv) Interstitial hydrides : those in which hydrogen is trapped in the interstitial spaces of transition metals.

Here, B_2H_6 is a dimeric form of BH_3. BH_3 covalently combined with another BH_3 molecule to form B_2H_6. B_2H_6 contain 3 centre 2 electrons bond.

63. ***(b)*** The method used to separate noble gas mixture from air is called Fischer-Ringe's method. When air free from moisture and CO_2 is passed over a heated mixture (800°C) of 90% CaC_2 + 10% $CaCl_2$ in an iron sealed tube, the following reactions take place

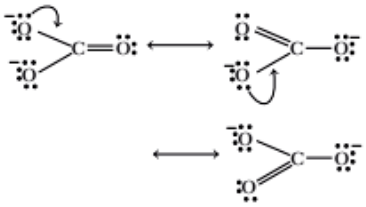
$$CaC_2 + N_2 \xrightarrow{800°C} CaCN_2 + C$$
$$2C + O \longrightarrow 2CO$$
$$C + O_2 \longrightarrow CO_2$$
$$2CaC_2 + 3CO_2 \longrightarrow 2CaCO_3 + 5C$$
$$CuO + CO \longrightarrow Cu + CO_2$$

CO_2 gas is absorbed by KOH solution. Thus, a mixture of inert gases are obtained.

64. ***(d)*** I. In nitrogen, d-orbitals are absent, hence, it does not form NCl_5. Thus, NCl_5 does not exist but PCl_5 does.

II. O_2^+ and NO are isoelectronic and contains one unpaired electron each, thus both are paramagnetic.

III. In carbonate ion, CO_3^{2-} all three C—O bonds are identical due to resonance.



IV. Pb^{2+} is more stable than Pb^{4+} due to inert pair effect, hence prefers to form divalent compounds.

Thus, the incorrect statements are III and IV.

65. ***(c)*** Gallium, Ga is a soft silvery white metal and is liquid at room temperature. When it solidifies, expands by 3.1%. Thus, it should not be stored in glass or metal containers.

66. ***(a)*** ZnS has zinc blende type structure (*i.e.*, ccp structure). The S^{2-} ions are present at the corners of the cube and at the centre of each face. Zinc ions occupy half of the tetrahedral sites. Each zinc ion is surrounded by four sulphide ions which are disposed towards the corner of regular tetrahedron. Similarly, S^{2-} ion is surrounded by four Zn^{2+} ions.

$\therefore$ Option (a) is correct statement.

67. ***(c)*** Reactivity of nucleophilic addition reaction depends upon the electron deficiency of carbonyl group and steric hinderance. Steric hinderance decreases the rate of reaction. This steric hinderance is minimum in methanal and maximum in benzophenone.

$\therefore$ Increasing order of nucleophilic addition reaction of the given compound is

$$\text{IV} < \text{II} < \text{III} < \text{I}$$

68. ***(b)*** The combustion equation of sucrose is

$$C_{12}H_{22}O_{11}(s) + 12O_2(g) \longrightarrow 12CO_2(g) + 11H_2O(g)$$

Here, $\Delta n = 12 + 11 - 12 = 11$

As we know,

$$\Delta H = \Delta E + \Delta nRT$$
$$\Delta H = (-1348.9 \times 10^3) + 11 \times 2 \times 298$$
$$= -1348900 + 6556$$
$$= -1342344 \text{ cal}$$
$$= -1342.344 \text{ kcal}$$

69. ***(c)*** Boiling point decreases with increase in branching. Compound (III) has two branches, compound (I) has one branch and compound (II) is a normal alkyl halide with no branch. So, the boiling point is minimum for compound (III) and maximum for compound (II).

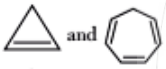
70. ***(d)*** Aldehydes and ketones possess $\left(-\overset{\overset{O}{\|}}{C}-CH_3\right)$ group will give positive iodoform test. Apart from methylated carbonyl compounds, alcohols with $\left(CH_3-\underset{OH}{\underset{|}{CH}}-\right)$ group also give positive iodoform test.

$$C_6H_5-\underset{OH}{\underset{|}{CH}}-CH_3 \xrightarrow[NaOH]{I_2 \text{ and}} \underset{\text{Iodoform}}{CHI_3} + C_6H_5COONa$$

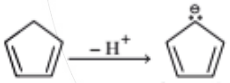
∴ Option (d) will give iodoform test.

71. ***(a)*** This problem includes conceptual mixing of acidic character, aromaticity and nucleophilic substitution reaction.
Students are advised to identify the most stable intermediate obtained among all (after the removal of H^+) keeping in mind the concept of conjugation and aromaticity. Then complete the reaction further using concept of nucleophilic substitution reaction.

Acidic character The species which easily donate its hydrogen and produces stable conjugate base is acid. The species which produces more stable conjugate base is more stronger acid.

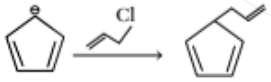
 and do not looses H^+ hence are not acidic.

 looses the H^+ easily and produces more stable aromatic cyclopentadienyl anion.



Non-aromatic (cyclopentadienyl anion) Aromatic

Now, cyclopentadienyl anion on reaction with 3-chloro prop-1-ene produces the product *via* nucleophilic substitution reaction.



Product

72. ***(b)*** The given reaction take place as follows :

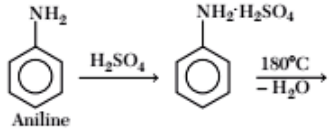
$$\underset{}{CaCO_3} \xrightarrow{\text{Heat}} \underset{(X)}{CaO} + \underset{(Z)}{CO_2}$$

$$\underset{}{CaO} + 3C \xrightarrow{\text{Heat}} \underset{(C)}{CaC_2} + \underset{(D)}{CO}$$

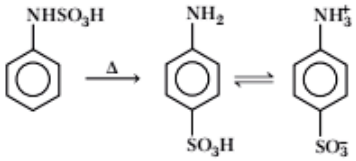
$$\underset{(C)}{CaC_2} + 2H_2O \longrightarrow Ca(OH)_2 + \underset{(E)}{C_2H_2}$$

73. ***(b)*** Optical isomerism is shown by only those complexes which lack symmetry. Complex $[Co(NH_3)_3(NO_2)_3]$ shows facial and meridional isomerism. Both isomers of this complex contain plane of symmetry. So, it will not form optical isomers.

74. ***(c)***



Aniline



Sulphanilic acid

Sulphanilic acid exists as a dipolar ion which has acidic as well as basic groups in the same molecule. Such ions are called Zwitter ions or inner salts.

Thus, $-SO_3$ diminishes the basic character of $-NH_2$.

75. ***(b)*** The structure of copolymer of ethene and vinyl chloride is shown below

$$\underset{\text{Ethene}}{n(CH_2=CH_2)} + \underset{\text{Vinyl chloride}}{n(CH_2=CHCl)} \longrightarrow \underset{\text{Copolymer}}{\left(-CH_2-CH_2-CH_2-\underset{Cl}{\underset{|}{CH}}-\right)_n}$$

Molecular weight of ethene $(CH_2CH_2) = 28$

Molecular weight of vinyl chloride (CH_2CHCl)

$$= 62.5$$

Empirical formula weight of copolymer

$$= 28 + 62.5 = 90.5$$

Mass % of vinyl chloride in the copolymer

$$= \frac{62.5 \times 100}{90.5} = 69.06 = 69\%$$

76. ***(b)*** Insulin is composed of two peptide chains referred to chain *A* and *B*. Chain *A* of 21 residues and chain *B* of 30 residues are cross linked by two disulphide bridges.

77. ***(b)*** Barbital is a sleep-producing drug, *i.e.*, hypnotic tranquillizer. It causes addiction.

78. ***(b)*** Since, at equivalence point

(for acid) $N_1V_1 = N_2V_2$ (for base)

∴ Volume of NaOH required to reach

equivalence point $= \frac{0.1 \times 25}{0.05} = 50$ mL

∴ Concentration of salt formed $= \frac{\text{millimoles of acid}}{\text{total volume in mL}}$

$$= \frac{25 \times 0.1}{75} = \frac{0.1}{3}$$

$$\text{Since, } [H^+] = \sqrt{\frac{K_w \times K_a}{C}} = \sqrt{\frac{10^{-14} \times 1.8 \times 10^{-5} \times 3}{0.1}}$$

$$\therefore \quad = 2.32 \times 10^{-9} \text{ M}$$

$$pH = \log [H^+]$$

$$= -\log (2.32 \times 10^{-9})$$

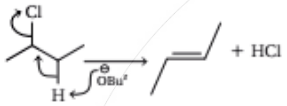
$$= 9 - \log 2.32$$

$$= 9 - 0.37$$

$$pH = 8.63$$

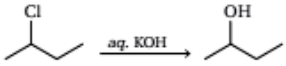
79. ***(d)*** Critical temperature of a gas is highest temperature at which liquification of the gas first occurs. The temperature 30.98°C is called critical temperature of carbon dioxide because this is the highest temperature at which liquid carbon dioxide is observed. Above this temperature it is gas.

80. ***(a)*** I. **Elimination reaction**



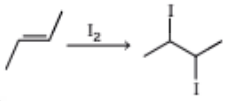
The reaction in which smaller neutral molecule are removed during the reaction is known as elimination reaction.

II. **Substitution reaction** The reaction in which one nucleophilic group is replaced by another nucleophile is known as nucleophilic substitution reaction.

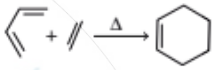


Here, OH replaces Cl.

III. **Addition reaction** The reaction in which reactant undergo addition with reagent to give a single product.



IV. **Addition reaction**



Hence, correct option is (a).

a. English Proficiency

81. ***(a)*** Augment means make bigger, so 'increase' is its correct synonoym.

82. ***(a)*** Consolation means 'comfort received by a person after a loss', so 'comfort' is the correct option.

83. ***(b)*** Auxiliary means 'providing additional help', so 'supplemental' is the correct option.

84. ***(b)*** Auspicious means 'favourable', so 'unfavourable' is best opposite word for it.

85. ***(d)*** Recompense means 'payment', so 'penalty' is the correct opposite word for it.

86. ***(c)*** Impede means delay or prevent (someone or something) by obstructing them, so 'push' is correct opposite word for it. Push means a vigorous effort to do or obtain something.

87. ***(a)*** Here a sense of command is depicted in sentence, so we should use 'ordered' for proper meaning of sentence.

88. ***(c)*** Sentence is in past tense and first form is used in those negative sentence which contain 'did', so option (c) is correct.

89. ***(d)*** No improvement is needed in the given sentence, as it is right.

90. ***(d)*** Afraid agrees with preposition 'of ', so option (d) is correct.

91. ***(c)*** Normally, company signs a contract or deal, so use of 'deal' is proper here.

92. ***(c)*** The question gives a sense of query about normal routine of some special/specific day, so use of 'plan' is more proper here.

93. ***(b)*** According to the events of sentence, PRSQ is best arrangement.

94. ***(a)*** According to events of sentence, PRSQ is best arrangement.

95. ***(c)*** According to sequence of events in the sentence, QPRS is best arrangement.

96. ***(c)*** Given,

SAFER = 5@3#2 and RIDE = 2©%#

$\boxed{S \longrightarrow 5}$ $R \longrightarrow 2$

$A \longrightarrow @$ $I \longrightarrow ©$

$\boxed{F \longrightarrow 3}$ $\boxed{D \longrightarrow \%}$

$\boxed{E \longrightarrow \#}$ $E \longrightarrow \#$

$R \longrightarrow 2$

$\therefore$ Code for FEDS = 3#%5

97. ***(b)*** From the given responses,

$4 \times 2 \times 3 \times 3 = 72$

$9 \times 4 \times 2 \times 10 = 720$

Similarly, $6 \times 20 \times 1 \times 6 = \boxed{720}$

98. ***(b)*** Since, consecutive two letters are interchanged. Therefore,

DE PR ES SI ON

↓ ↓ ↓ ↓ ↓

ED R[P]SE IS NO ← On counting

Now, on counting from right hand side P is the 7th letter from right.

99. ***(a)*** Every day of week repeats after seven days.

Hence, $59 = 7 \times 8 + 3$ = 56 days + 3 odd days

$\therefore$ It will be Thrusday after 56 days.

$\therefore$ 57th day = Thrusday

$\Rightarrow$ 58th day = Friday

59th day = Saturday

$\Rightarrow$ 60th day = Sunday

$\therefore$ It will be Sunday after 59 days.

100. ***(d)*** Both coal mines and factories are located in the fields.

101. ***(c)*** From the given series,

$1^3 \longrightarrow 1$

$2^3 \longrightarrow 8$

$3^3 \longrightarrow 27$

$\boxed{4^3 \longrightarrow 64}$

$5^3 \longrightarrow 125$

$6^3 \longrightarrow 216$

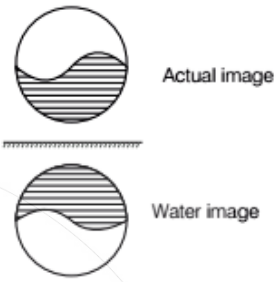
Therefore, 64 will come in place of questions mark.

102. ***(c)*** Interchanging the symbols as given in the above question, the above equation becomes

$$6 + 9 \times 8 \div 3 - 20 = 6 + 9 \times \frac{8}{3} - 20$$

$$= 6 + 24 - 20 = 10$$

103. ***(b)*** Water image is the reflection of image in water.



104. ***(b)*** On unfolding layer 1,

On unfolding layer 2,

On unfolding layer 3,

105. ***(d)***

mallon piml = blue light

mallon tifl = blue berry

arpan tifl = rasp berry

$\therefore$ From the above analysis, we see codes as

blue = Mallon

Light = piml

berry = tifl

arpan = rasp

$\therefore$ Light *i.e.*, piml is given in two options (b) and (d) but in option (b) other code given is of rasp, hence it cannot be the code of house. So, option (d) is correct.

106. ***(a)*** Since, AM ≥ GM, then

$$\frac{(p+q)+(r+s)}{2} \geq \sqrt{(p+q)(r+s)}$$

$$\Rightarrow \quad 2/2 \geq \sqrt{M} \Rightarrow \sqrt{M} \leq 1 \Rightarrow M \leq 1$$

Also, $(p+q)(r+s) > 0 \quad (\because p, q, r, s > 0)$

$\therefore \quad M > 0$

Hence, $0 < M \leq 1$

107. ***(a)*** Given, $\left|\frac{z-3i}{z+3i}\right| = 1 \Rightarrow |z-3i| = |z+3i|$

(if $|z-z_1| = |z-z_2|$, then it is a perpendicular bisector of z_1 and z_2)

Hence, perpendicular bisector of $(0, 3)$ and $(0, -3)$ is X-axis.

108. ***(d)*** Since, $f(x)$ is an odd periodic function with period 2.

$\therefore \quad f(-x) = -f(x)$ and $f(x+2) = f(x)$

$\therefore \quad f(2) = f(0+2) = f(0)$

and $\quad f(-2) = f(-2+2) = f(0)$

Now, $\quad f(0) = f(-2) = -f(2) = -f(0)$

$\Rightarrow \quad 2f(0) = 0$, *i.e.*, $f(0) = 0$

$\therefore \quad f(4) = f(2+2) = f(2) = f(0) = 0$

Thus, $\quad f(4) = 0$

109. ***(b)*** We have, $\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots} = \frac{dx - dy}{dx + dy}$

On applying componendo and dividendo, we get

$$\frac{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right) + \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right)}{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right) - \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right)}$$

$$= \frac{(dx - dy) + (dx + dy)}{(dx - dy) - (dx + dy)}$$

$$\Rightarrow \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\right)}{-\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\right)} = \frac{2dx}{-2dy}$$

$$\Rightarrow \frac{e^x}{-e^{-x}} = -\frac{dx}{dy}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-x}}{e^x} \Rightarrow \frac{dy}{dx} = e^{-2x}$$

$$\Rightarrow \quad dy = e^{-2x}dx$$

On integrating both sides, we get

$$y = \frac{e^{-2x}}{(-2)} + C_1 \Rightarrow 2y = -e^{-2x} + 2C_1$$

$$\Rightarrow \quad 2y = -e^{-2x} + C \text{ (where, } C = 2C_1)$$

$$\Rightarrow \quad 2ye^{2x} = -1 + e^{2x}C \Rightarrow 2ye^{2x} = Ce^{2x} - 1$$

110. ***(b)*** Given lines are concurrent,

So, $$\begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix} = 0$$

$$\Rightarrow 2(132 - 143) + 3(-99 + 104) + k(-33 + 32) = 0$$

$$\Rightarrow -22 + 15 - k = 0 \Rightarrow \quad k = -7$$

111. ***(d)*** Given lines are

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda} \qquad \ldots(i)$$

and $$\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3} \qquad \ldots(ii)$$

These lines lie in the same plane. So, both are coplanar.

$\therefore$ $$\begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2(-2+3) - 3(-3-3) + \lambda(-3-2) = 0$$

$$\Rightarrow \quad 2 + 18 - 5\lambda = 0 \Rightarrow 5\lambda = 20 \Rightarrow \lambda = 4$$

$$\therefore \sin^{-1}\sin\lambda = \sin^{-1}\sin 4$$

$$= \sin^{-1}\sin(\pi - 4)$$

$$= \pi - 4$$

112. ***(c)*** We have,

$$e^x = (1-x)(B_0 + B_1x + B_2x^2 + \ldots + B_{n-1}x^{n-1} + B_nx^n + \ldots)$$

By the expansion of e^x, we get

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$= (1-x)(B_0 + B_1x + B_2x^2 + \ldots + B_{n-1}x^{n-1} + B_nx^n + \ldots)$$

Equating the coefficient of x^n on both sides, we get

$$B_n - B_{n-1} = \frac{1}{n!}$$

113. ***(c)*** Let $P(n) = 2^{3n} - 7n - 1 \Rightarrow P(1) = 0, P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) = 2^{3k} - 7k - 1 = 49I$

$$P(k+1) = 2^{3k+3} - 7k - 8$$

$$= 8(49I + 7k + 1) - 7k - 8$$

$$= 49(8I) + 49k = 49\lambda$$

(where, $\lambda = 8I + k$, which is an integer.)

114. ***(b)*** We know that, $x-[x]$ is periodic function with period one.

$\therefore\ e^{x-[x]}$ has period one

[since, $f(x)$ is periodic with period T, then

$$\int_0^{nT} f(x)dx = n\int_0^T f(x)dx]$$

$$\therefore\ \int_0^{25\times 1} e^{x-[x]}dx = 25\int_0^1 e^{x-[x]}dx$$

$$= 25\int_0^1 e^{x-0}dx$$

$$= 25[e^x]_0^1 = 25[e-1]$$

$\therefore\quad 25(e-1) = k(e-1)$

Hence, $\quad k = 25$

115. ***(a)*** Let P be $(at_1^2,\ 2at_1)$ and Q be $(at_2^2,\ 2at_2)$. Since, PQ subtends a right angle at the vertex $(0,\ 0)$.

Hence, $\quad t_1t_2 = -4 \qquad \ldots(i)$

If $(h,\ k)$ is the point of intersection of normals at P and Q, then $\quad h = 2a + a(t_1^2 + t_2^2 + t_1t_2) \qquad \ldots(ii)$

and $\quad k = -at_1t_2(t_1 + t_2) \qquad \ldots(iii)$

In order to find the locus of $(h,\ k)$, we have to eliminate t_1 and t_2 between Eqs. (i), (ii) and (iii),

$$k = 4a(t_1 + t_2) \qquad \ldots(iv)$$

[from Eqs. (i) and (iii)]

and $\quad h - 2a = a[(t_1 + t_2)^2 - t_1t_2]$

$$\Rightarrow\quad h - 2a = a\left[\frac{k^2}{16a^2} + 4\right] \qquad \text{[from Eq. (iv)]}$$

$$\Rightarrow\quad h - 6a = \frac{k^2}{16a}$$

Hence, the required locus is $y^2 = 16a(x - 6a)$.

Which represents a parabola.

116. ***(a)*** Given matrix can be rewritten as

$$A = \begin{bmatrix} 1+2\omega+\omega^2 & \omega^2 & 1 \\ 1 & 1+\omega^2+2\omega & \omega \\ \omega & \omega^2 & 2+\omega+2\omega^2 \end{bmatrix}$$

$$\therefore\ |A| = \begin{vmatrix} \omega & \omega^2 & 1 \\ 1 & \omega & \omega \\ \omega & \omega^2 & 1+\omega^2 \end{vmatrix} \qquad (\because 1+\omega+\omega^2 = 0)$$

$$= \omega\begin{vmatrix} \omega & \omega & 1 \\ 1 & 1 & \omega \\ \omega & \omega & -\omega \end{vmatrix} \qquad (\because 1+\omega+\omega^2 = 0)$$

$\Rightarrow \omega(0) = 0$

117. ***(b)*** $\lim\limits_{x\to \tan^{-1}3} \dfrac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$

$$= \lim_{\tan x\to 3} \frac{(\tan x - 3)(\tan x + 1)}{(\tan x - 3)(\tan x - 1)}$$

$$= \lim_{\tan x\to 3} \frac{\tan x + 1}{\tan x - 1}$$

$$= \frac{3+1}{3-1} = \frac{4}{2} = 2$$

118. ***(c)*** The given equation of second ellipse can be rewritten as

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Equation of tangent to this ellipse is

$$\frac{x}{2}\cos\theta + y\sin\theta = 1 \qquad \ldots(i)$$

Equation of the first ellipse can be rewritten as

$$\frac{x^2}{6} + \frac{y^2}{3} = 1 \qquad \ldots(ii)$$

Let ellipse (i) meets the first ellipse at P and Q and the tangents at P and Q to the second ellipse intersected at $(h,\ k)$, then Eq. (i) is the chord of contact of $(h,\ k)$ with respect to the ellipse (ii) and thus, its equation is

$$\frac{hx}{6} + \frac{ky}{3} = 1 \qquad \ldots(iii)$$

Since, Eqs. (i) and (iii) represent the same line

$$\frac{h/6}{\frac{1}{2}\cos\theta} = \frac{k/3}{\sin\theta} = 1$$

$$h = 3\cos\theta$$

and $\quad k = 3\sin\theta$

Hence, locus is $x^2 + y^2 = 9$.

119. ***(d)*** We have, $|\sin x\cos x| + |\tan x + \cot x| = \sqrt{3}$

$$\Rightarrow\quad |\sin x\cos x| + \frac{1}{|\sin x\cdot\cos x|} = \sqrt{3}$$

But $\quad |\sin x\cdot\cos x| + \dfrac{1}{|\sin x\cdot\cos x|} \geq 2$

Hence, there is no solution.

120. ***(d)*** $\lim\limits_{x\to 0}\left[\tan\left(\dfrac{\pi}{4} + x\right)\right]^{1/x} = \lim\limits_{x\to 0}\left[\dfrac{1+\tan x}{1-\tan x}\right]^{1/x}$

$$= \lim_{x\to 0}\left[(1+\tan x)^{\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}} \times \lim_{x\to 0}\left[(1-\tan x)^{-\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}}$$

$$= e\cdot e = e^2$$

121. ***(b)*** Since, $\sin^2\theta = \dfrac{1-\cos 2\theta}{2} = \dfrac{1}{2} - \dfrac{1}{2}\cos 2\theta$

$\therefore$ Period of $\sin^2\theta = \dfrac{2\pi}{2} = \pi$

122. ***(c)*** $\because$ Total number of ways $= 5!$

and favourable number of ways $= 2\cdot 4!$

$\therefore$ Required probability $= \dfrac{2\cdot 4!}{5!} = \dfrac{2}{5}$

123. *(b)* $\lim_{x\to\infty}\frac{x^4\cdot\sin\left(\frac{1}{x}\right)+x^2}{1+|x|^3}=\lim_{x\to\infty}\left[\frac{x\sin\left(\frac{1}{x}\right)+\frac{1}{x}}{\frac{1}{x^3}+\frac{|x|^3}{x^3}}\right]$

[dividing numerator and denominator by x^3]

$$=\frac{\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}+\lim_{x\to\infty}\frac{1}{x}}{\lim_{x\to\infty}\frac{1}{x^3}+\lim_{x\to\infty}\frac{|x|^3}{x^3}}=\frac{1-0}{0-1}=-1$$

124. *(d)* Given that, $\log_4 2-\log_8 2+\log_{16}2-\ldots$

$$=\frac{1}{\log_2 4}-\frac{1}{\log_2 8}+\frac{1}{\log_2 16}-\ldots$$

$$=\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\ldots$$

$$=1-\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots\right]$$

$$=1-\log_e 2$$

125. *(c)* Let the observation be $x_1, x_2, x_3,\ldots,x_n$.

Now, mean $(\bar{x})=\frac{x_1+x_2+\ldots+x_n}{n}$

When first term increased by 1, second by 2 and so on, then observations will be

$(x_1+1),(x_1+2),(x_1+3),\ldots,(x_n+n)$

Then, new mean

$$(\bar{x}_1)=\frac{(x_1+1)+(x_2+2)+\ldots+(x_n+n)}{n}$$

$$=\frac{(x_1+x_2+\ldots+x_n)+(1+2+3+\ldots+n)}{n}$$

$$\Rightarrow \bar{x}_1=\frac{x_1+x_2+\ldots+x_n}{n}+\frac{1+2+3+\ldots+n}{n}$$

$$\Rightarrow \bar{x}_1=\bar{x}+\frac{n(n+1)}{2n}\quad\left[\because 1+2+3+\ldots+n=\frac{n(n+1)}{2}\right]$$

$$\Rightarrow \bar{x}_1=\bar{x}+\frac{n+1}{2}$$

126. *(c)* We have, $(\sin^{-1}x)^2+(\cos^{-1}x)^2$

$$=(\sin^{-1}x+\cos^{-1}x)^2-2\sin^{-1}x\cdot\cos^{-1}x$$

$$=\frac{\pi^2}{4}-2\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)$$

$$=\frac{\pi^2}{4}-\pi\sin^{-1}x+2(\sin^{-1}x)^2$$

$$=2\left[(\sin^{-1}x)^2-\frac{\pi}{2}\sin^{-1}x+\frac{\pi^2}{8}\right]$$

$$=2\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right]$$

Thus, the least value is $2\left(\frac{\pi^2}{16}\right)$ *i.e.*, $\frac{\pi^2}{8}$ and the greatest value is $2\left[\left(\frac{-\pi}{2}-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right]$ *i.e.*, $\frac{5\pi^2}{4}$.

127. *(a)* Since, P (exactly one of A, B occurs) $=q$ (given), we get

$$P(A\cup B)-P(A\cap B)=q$$

$$\Rightarrow\quad p-P(A\cap B)=q$$

$$\Rightarrow\quad P(A\cap B)=p-q$$

$$\Rightarrow\quad 1-P(A'\cup B')=p-q$$

$$\Rightarrow\quad P(A'\cup B')=1-p+q$$

$$\Rightarrow P(A')+P(B')-P(A'\cap B')=1+q-p$$

$$\Rightarrow P(A')+P(B')=(1-p+q)+[1-P(A\cup B)]$$

$$=(1-p+q)+(1-p)$$

$$=2-2p+q$$

128. *(b)* The given equation is reduced to $x=e^{xy\left(\frac{dy}{dx}\right)}$

$$\Rightarrow\quad \log x=xy\frac{dy}{dx}\Rightarrow y\,dy=\frac{\log x}{x}dx$$

$$\Rightarrow\quad \int y\,dy=\int\frac{1}{x}\log x\,dx\Rightarrow\frac{y^2}{2}=\frac{(\log x)^2}{2}+C'$$

$$\Rightarrow\quad y^2=(\log x)^2+2C'$$

$$\Rightarrow\quad y^2=(\log x)^2+C\qquad(\text{where, } C=2C')$$

129. *(d)* Let $P(n)=n^3+2n$

$$P(1)=1+2=3$$

$$P(2)=8+4=12$$

$$P(3)=27+6=33$$

Clearly, we see that all these numbers are divisible by 3.

130. *(c)* ∵ Using Binomial theorem,

$$(5x-4y)^n={}^nC_0(5x)^n+{}^nC_1(5x)^{n-1}(-4y)+{}^nC_2(5x)^{n-2}(-4y)^2+\ldots+{}^nC_n(-4y)^n$$

Sum of coefficients

$$={}^nC_0 5^n+{}^nC_1 5^{n-1}(-4)+{}^nC_2 5^{n-2}\cdot(-4)^2+\ldots+{}^nC_n(-4)^n$$

$$=(5-4)^n=1^n=1$$

131. *(a)* Given equation of plane is $x+y+z=5$.

The distance measured along the line $x=y=z$.

Direction ratio's of the given line is (1, 1, 1).

So, the equation of line PQ is

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}$$

Now, let $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

$$x=\lambda+1,\ y=\lambda-5,\ z=\lambda+9$$

lies on the plane $x+y+z=5$

$\lambda+1-\lambda+5+\lambda+9=5 \Rightarrow \lambda=-10$

The coordinate of Q is $(-9,-15,-1)$ and the coordinate of P is $(1,-5,9)$.

$$PQ=\sqrt{(10)^2+(10)^2+(10)^2}=10\sqrt{3}$$

$\therefore \quad 2\sqrt{3}k=10\sqrt{3} \Rightarrow k=5$

132. ***(b)*** $\lim\limits_{x\to 1}\dfrac{x^m-1}{x^n-1}=\lim\limits_{x\to 1}\dfrac{mx^{m-1}}{nx^{n-1}}$ [by L' Hospital rule]

$=m/n$

133. ***(a)*** Consider that, $I=\int_1^x t\log t\,dt$

$$=\left[\log t\cdot\frac{t^2}{2}\right]_1^x-\int_1^x\frac{1}{t}\cdot\frac{t^2}{2}dt=\frac{x^2}{2}\log x-\frac{1}{2}\left[\frac{t^2}{2}\right]_1^x$$

$$=\frac{x^2}{2}\log x-\frac{1}{2}\left[\frac{x^2}{2}-\frac{1}{2}\right]$$

$$\Rightarrow \quad \frac{1}{4}=\frac{x^2}{2}\log x-\frac{1}{4}(x^2-1)$$

$$\Rightarrow \quad \frac{1}{2}x^2\log x-\frac{1}{4}x^2=0 \qquad (\text{as } x>1)$$

$$\Rightarrow \quad x^2(2\log x-1)=0 \Rightarrow 2\log x-1=0$$

$$\Rightarrow \quad \log x=\frac{1}{2} \Rightarrow x=e^{1/2} \Rightarrow x=\sqrt{e}$$

134. ***(a)*** We have,

$$a=\sum_{n=0}^{\infty}\frac{x^{3n}}{(3n)!},\ b=\sum_{n=1}^{\infty}\frac{x^{3n-2}}{(3n-2)!} \text{ and } c=\sum_{n=1}^{\infty}\frac{x^{3n-1}}{(3n-1)!}$$

$$\text{Now, } a+b+c=\sum_{n=0}^{\infty}\frac{x^{3n}}{3n!}+\sum_{n=1}^{\infty}\frac{x^{3n-2}}{(3n-2)!}+\sum_{n=1}^{\infty}\frac{x^{3n-1}}{(3n-1)!}$$

$$=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots=e^x$$

$$a+b\omega+c\omega^2=1+\omega x+\frac{\omega^2x^2}{2!}+\frac{\omega^3x^3}{3!}+\ldots=e^{\omega x}$$

and $a+b\omega^2+c\omega=e^{\omega^2 x}$, ω is imaginary cube root of unity.

Now, $a^3+b^3+c^3-3abc$

$=(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$

$=e^x\cdot e^{\omega x}\cdot e^{\omega^2 x}=e^{x(1+\omega+\omega^2)}=e^{0\cdot x}=1$

135. ***(a)*** Required unit vector is $\dfrac{(\hat{i}-\hat{j})\times(\hat{i}+\hat{j})}{|(\hat{i}-\hat{j})\times(\hat{i}+\hat{j})|}$

$$=\frac{\hat{k}+\hat{k}}{2}=\frac{2\hat{k}}{2}=\hat{k}$$

136. ***(a)*** $f(x)=x^3+2x^2+5x+2\cos x$

$f'(x)=3x^2+4x+5-2\cdot\sin x$

$$=3\left(x+\frac{2}{3}\right)^2+\frac{11}{3}-2\cdot\sin x$$

$\Rightarrow \quad f'(x)>0, \forall\, x$

$f(x)$ is increasing for all $x\in R$.

Also, $f(0)=2 \Rightarrow f(x)=0$

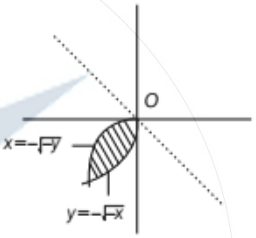
So, $f(x)$ has no solution.

137. ***(c)*** $\lim\limits_{x\to 0}\dfrac{(1+x)^8-1}{(1+x)^2-1}$

$$=\lim_{x\to 0}\frac{[(1+x)^4+1][(1+x)^2+1][(1+x)^2-1]}{(1+x)^2-1}$$

$=2\times 2=4$

138. ***(a)*** Given that, $y=-\sqrt{-x}$



$\Rightarrow \quad y^2=-x$, where x and y both negative.

Now, $x=-\sqrt{-y}$

$\Rightarrow \quad x^2=-y$, where x and y both negative.

$\therefore$ Area $=\left|\int_{-1}^{0}-\sqrt{-x}\,dx-\int_{-1}^{0}-x^2\,dx\right|=\frac{1}{3}$

139. ***(a)*** Given series

$$\left[\alpha x-\frac{1}{2}(\alpha x)^2+\frac{1}{3}(\alpha x)^3-\ldots\right]+\left[\beta x-\frac{1}{2}(\beta x)^2+\frac{1}{3}(\beta x)^3-\ldots\right]$$

$=\log(1-\alpha x)+\log(1-\beta x)$

$=\log[1-(\alpha+\beta)x+\alpha\beta x^2]$

Now, $\alpha+\beta=p$ and $\alpha\beta=q$

Given series $=\log(1-px+qx^2)$

140. ***(c)*** We know that,

$\sin^{-1}x+\cos^{-1}x=\pi/2, \forall\, x\in[-1,1]$

$\therefore \quad \cos^{-1}x>\sin^{-1}x$

$$\Rightarrow \quad \frac{\pi}{2}-\sin^{-1}x>\sin^{-1}x \Rightarrow \frac{\pi}{2}>2\sin^{-1}x$$

$$\Rightarrow \quad \sin^{-1}x<\frac{\pi}{4} \Rightarrow x<\frac{1}{\sqrt{2}}$$

$$\therefore \quad -1\le x<\frac{1}{\sqrt{2}}$$

141. ***(c)*** Number of ways

$$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n$$

$$= \frac{1}{2}(2^{2n+1}) = 2^{2n}$$

Thus, $2^{2n} = 64$ i.e., $2^{2n} = 2^6$

On comparing, $2n = 6 \Rightarrow n = 3$

142. ***(c)*** R is reflexive as $(3, 3), (6, 6), (9, 9), (12, 12) \in R$.

R is not symmetric as $(6, 12) \in R$ but $(12, 6) \notin R$.

R is transitive as the only pair which needs verification is $(3, 6)$ and $(6, 12) \in R \Rightarrow (3, 12) \in R$.

143. ***(d)*** Let α and β be the roots of the equation

$$x^2 - (a-2)x - a + 1 = 0$$

$$\therefore \quad \alpha + \beta = a - 2 \text{ and } \alpha\beta = -(a-1)$$

$$s = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (a-2)^2 + 2(a-1)$$

$$= a^2 - 4a + 4 + 2a - 2 = a^2 - 2a + 2$$

Now, $ds/da = 2a - 2$

For maximum and minimum, $ds/da = 0$

$$\Rightarrow \quad 2a - 2 = 0 \Rightarrow a = 1$$

Also, $d^2s/da^2 = 2 > 0$

Hence, at $a = 1$, s will have minimum value.

144. ***(b)*** We have, $ax^2 + by^2 = 1$

On differentiating both sides w.r.t. x, we get

$$2ax + 2by\frac{dy}{dx} = 0 \Rightarrow ax + by\frac{dy}{dx} = 0 \quad \ldots(i)$$

$$\Rightarrow \quad \frac{-a}{b} = \frac{y}{x}\frac{dy}{dx} \quad \ldots(ii)$$

Again, differentiating Eq. (i) w.r.t. x, we get

$$a + b\left(\frac{dy}{dx}\right)^2 + by\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \quad -\frac{a}{b} = \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} \quad \ldots(iii)$$

From Eqs. (ii) and (iii), we get

$$\frac{y}{x}\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2}$$

or $$x\left(\frac{dy}{dx}\right)^2 + xy\left(\frac{d^2y}{dx^2}\right) - \frac{dy}{dx} = 0$$

145. ***(c)*** If x tables and y chairs are purchased for maximum profit.

Then, $x + y \le 60$

$$5x + \frac{6y}{5} \le 100 \Rightarrow x \ge 0, y \ge 0$$

So, number of constraints are four.

146. ***(c)*** For $n = 1$, $10^n + 3 \cdot 4^{n+2} + 5 = 10 + 3 \cdot 4^3 + 5$

$= 207$, which is divisible by 9

So, by induction, the result is divisible by 9.

147. ***(b)*** Let sets A and B have m and n elements, respectively.

$$2^m - 2^n = 56$$

$$2^n(2^{m-n} - 1) = 56 = 8 \times 7 = 2^3 \times 7$$

Comparing both sides, we get

$$2^n = 2^3 \text{ and } 2^{m-n} - 1 = 7$$

$$\Rightarrow \quad n = 3 \text{ and } 2^{m-n} = 8$$

$$\Rightarrow \quad 2^{m-n} = 2^3 \Rightarrow m - n = 3$$

$$\Rightarrow \quad m - 3 = 3 \Rightarrow m = 6$$

Number of the elements in A is 6.

148. ***(a)*** Mean of the combined sample

$$\overline{X} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2} = \frac{35 \times 80 + 65 \times 70}{35 + 65} = 73.5$$

Standard deviation of the combined sample is given by

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

where, $d_1^2 = (\overline{x}_1 - \overline{x})^2 = (80 - 73.5)^2 = 42.25$

$$d_2^2 = (\overline{x}_2 - \overline{x})^2 = (70 - 73.5)^2 = 12.25$$

$$\therefore \quad \sigma^2 = \frac{35(4^2 + 42.25) + 65(3^2 + 12.25)}{35 + 65}$$

$$\Rightarrow \quad \sigma^2 = 34.2 \Rightarrow \sigma = 5.85$$

149. ***(c)*** For the two lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$, the angle bisectors are given by

$$\frac{24x + 7y - 20}{25} = \pm\frac{4x - 3y - 2}{5}$$

Taking positive sign, we get $2x + 11y - 5 = 0$

So, the given three lines are concurrent with one line bisecting the angle between the other two.

150. ***(d)*** The new position of plane is

$$(x - 2y + 33) + \lambda(x + y + z - 1) = 0$$

$$\Rightarrow \quad (1+\lambda)x + (\lambda - 2)y + (3+\lambda)z - \lambda = 0$$

Given that this is perpendicular to

$$x + y + z = 1$$

$$\therefore \quad (1+\lambda)\cdot 1 + (\lambda - 2)\cdot 1 + (3+\lambda)\cdot 1 = 0$$

$$\Rightarrow \quad 1 + \lambda + \lambda - 2 + 3 + \lambda = 0$$

$$\Rightarrow \quad 3\lambda + 2 = 0$$

$$\Rightarrow \quad \lambda = \frac{-2}{3}$$

Hence, the new position of the plane is

$$(x - 2y + 3z) - \frac{2}{3}(x + y + z - 1) = 0$$

$$\Rightarrow \quad 3x - 2x - 6y - 2y + 9z - 2z + 2 = 0$$

$$\therefore \quad x - 8y + 7z = -2$$