CONTENTS

BASIC MATHS

JEE (Main/Advanced) Syllabus :

Numbers (Rational, Irrational, Even ,Odd ,Prime, Composite,Co-prime / Relatively Prime, Twin prime), Divisibility Rules, LCM and HCF, Surds, Indices, Zeros / Roots of a polynomial / equation, Remainder theorem, Factor theorem, Factorization , Proportions, Intervals, theorems on circles and triangles.

BASIC MATHS

1. NUMBER SYSTEM :

Natural Numbers : (N) = {1, 2, 3....}

Whole Numbers : (W) = {0, 1, 2, 3.....}

Integers : (I) = {........–3, –2, –1, 0, 1, 2, 3.....}

Positive Integers : (I^+) = {1, 2, 3 ... }

Negative Integers : (I^-) = {.... –3, –2, –1}

Non-negative Integers : {0, 1, 2, 3........}

Non-positive Integers : {.... –3, –2, –1, 0}

Even Integers = {.... –6, –4, –2, 0, 2, 4, 6 ...}

Odd Integers = {........ –5, –3, –1, 1, 3, 5........}

Note :

(i) Zero is neither positive nor negative.

(ii) Zero is even number.

(ii) Positive means > 0.

(iv) Non-negative means ≥ 0.

2. FRACTION $\left(\frac{p}{q}\right)$:

(a) Proper Fraction = $\frac{3}{5}$: $N^r < D^r$

(b) Improper Fraction = $\frac{5}{3}$: $N^r > D^r$

(c) Mixed Fraction : $2+\frac{3}{5}$

(d) Compound Fraction : $\frac{\frac{2}{3}}{\frac{5}{6}}$

(e) Complex Fraction : $2\frac{1}{3}$

(f) Continued Fraction : $2+\cfrac{2}{2+\cfrac{2}{+.....}}$

3. RATIONAL NUMBERS (Q) :

All the numbers that can be represented in the form p/q, where p and q are integers and q ≠ 0, are called rational numbers. Integers, Fractions, Terminating decimal numbers, Non-terminating but repeating decimal numbers are all rational numbers. $Q = \left\{\frac{p}{q} : p,\ q \in I \text{ and } q \neq 0\right\}$

Note :

(i) Integers are rational numbers, but converse need not be true.

(ii) A rational number always exists between two distinct rational numbers, hence infinite rational numbers exist between two rational numbers.

4. IRRATIONAL NUMBERS (Q^C) :

There are real numbers which can not be expressed in p/q form. Non-Terminating non repeating decimal numbers are irrational number e.g. $\sqrt{2}, \sqrt{5}, \sqrt{3}, \sqrt[3]{10}$; e, π.

$e \approx 2.71$ is called Napier's constant and $\pi \approx 3.14$

Note :

(i) Sum of a rational number and an irrational number is an irrational number e.g. $2 + \sqrt{3}$

(ii) If $a \in Q$ and $b \notin Q$, then ab = rational number, only if a = 0.

(iii) Sum, difference, product and quotient of two irrational numbers need not be an irrational number or we can say, result may be a rational number also.

5. REAL NUMBERS (R) :

The complete set of rational and irrational number is the set of real numbers, $R = Q \cup Q^C$. The real numbers can be represented as a position of a point on the real number line.

6. COMPLEX NUMBERS. (C) :

A number of the form a + ib, where a, b $\in$ R and $i = \sqrt{-1}$ is called a complex number. Complex number is usually denoted by z and the set of all complex numbers is represented by $C = \{(x + iy) : x, y \in R, i = \sqrt{-1}\}$

$$\boxed{N \subset W \subset I \subset Q \subset R \subset C}$$

7. EVEN NUMBERS :

Numbers divisible by 2, unit's digit 0, 2, 4, 6, 8 & represented by 2n.

8. ODD NUMBERS :

Not divisible by 2, last digit 1, 3, 5, 7, 9 represented by $(2n \pm 1)$

(a) even $\pm$ even = even

(b) even $\pm$ odd = odd

(c) odd $\pm$ odd = even

(d) even $\times$ odd = even

(e) even $\times$ even = even

(f) odd $\times$ odd = odd

9. PRIME NUMBERS :

Let 'p' be a natural number, 'p' is said to be prime if it has exactly two distinct positive integral factors, namely 1 and itself. e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 .....

10. COMPOSITE NUMBERS :

A number that has more than two divisors

Note :

(i) '1' is neither prime nor composite.

(ii) '2' is the only even prime number.

(iii) '4' is the smallest composite number.

(iv) Natural numbers which are not prime are composite numbers (except 1)

11. CO-PRIME NUMBERS/ RELATIVELY PRIME NUMBERS :

Two natural numbers (not necessarily prime) are coprime, if their H.C.F. is one
e.g. (1, 2), (1, 3), (3, 4) (5, 6) etc.

Note :

(i) Two distinct prime number(s) are always co-prime but converse need not be true.

(ii) Consecutive natural numbers are always co-prime numbers.

12. TWIN PRIME NUMBERS :

If the difference between two prime numbers is two, then the numbers are twin prime numbers.

e.g. {3, 5}, {5, 7}, {11, 13} etc.

13. NUMBERS TO REMEMBER :

Number	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Square	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400
Cube	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375	4096	4913	5832	6859	8000
Sq. Root	1.41	1.73	2	2.24	2.45	2.65	2.83	3	3.16										

Note :

(i) Square of a real number is always non negative (i.e. $x^2 \geq 0$)

(ii) Square root of a positive number is always positive e.g. $\sqrt{4} = 2$

(iii) $\sqrt{x^2} = |x|,\ x \in R$

14. DIVISIBILITY RULES :

Divisible by	**Remark.**
2	Last digit of number is 0, 2, 4, 6 or 8
3	Sum of digits of number divisible by 3 (Remainder will be same when number is divided by 3 or sum of digits is divided by 3.)
4	Number formed by last two digits divisible by 4 (Remainder will be same whether we divide the number or its last two digits)
5	Last digit 0 or 5
6	Divisible by 2 and 3 simultaneously.
8	Number formed by last three digits is divisible by 8 (Remainder will be same whether we divide the number or its last three digits)
9	Sum of digits divisible by 9. (Remainder will be same when number is divided by 9 or sum of digit is divided by 9)
10	Last digit 0
11	(Sum of digits at even places) – (sum of digits at odd places) = 0 or divisible by 11

15. LCM AND HCF :

(a) HCF is the highestw common factor between any two or more numbers or algebraic expressions. When dealing only with numbers, it is also called "Greatest common divisor" (GCD).

(b) LCM is the lowest common multiple of two or more numbers or algebraic expressions.

(c) The product of HCF and LCM of two numbers (or expressions) is equal to the product of the numbers.

16. SURDS AND INDICES

Radicals

A radical expression (or simply a radical) is an expression of the type $\sqrt[n]{x}$. The sign '$\sqrt[n]{\ }$' is called the radical sign. The number under this sign, i.e., 'x' is called the radical and the number in the angular part of the sign, i.e., 'n' is the order of the radical.

Note that in the expression $\sqrt[n]{x}$, $n \in N$, $n \geq 2$.

17. DESCRIPTION OF SURDS

If 'a' is rational number, which is not the nth power (n is any natural number) of any rational number, then the irrational number $\pm\sqrt[n]{a}$ are called simple surds or monomial surds. Every surd is an irrational number (but every irrational number is not a surd). So, the representation of monomial surd on a number line is same that of irrational numbers.

Eg.

(a) $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.

(b) $\sqrt[3]{5}$ is surd and $\sqrt[3]{5}$ is an irrational number.

(c) π is an irrational number, but it is not a surd.

(d) $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

18. ORDER OF SURD

In the surd $\sqrt[n]{a}$, n is called the order of the surd. Thus the order of $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$ are 2, 3 and 4 respectively.

19. COMPARISON OF MONOMIAL SURDS

If two simple surds are of the same order, then they can easily be compared. If $a < b$, $\sqrt[n]{a} < \sqrt[n]{b}$ for all positive integral values of n. E.g. $\sqrt[4]{2} < \sqrt[4]{7}$, $\sqrt[3]{3} < \sqrt[3]{5}$, $\sqrt[5]{10} < \sqrt[5]{13}$ etc.

If two simple surds of different orders have to be compared, they have to be expressed as radicals of the same order.

Thus to compare $\sqrt[4]{6}$ and $\sqrt[3]{5}$, we express both as the radicals of 12th (LCM of 3, 4) order.

$\sqrt[4]{6} = \sqrt[12]{6^3}$ and $\sqrt[3]{5} = \sqrt[12]{5^4}$

As $6^3 < 5^4$, $\sqrt[4]{6} < \sqrt[3]{5}$

Illustration-1 :

If $x = \dfrac{1}{2+\sqrt{3}}$, find the value of $x^3 - x^2 - 11x + 4$

Solution :

as $x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$x = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$x - 2 = -\sqrt{3}$ squaring both sides ; we get

$(x-2)^2 = \left(-\sqrt{3}\right)^2 \Rightarrow x^2 + 4 - 4x = 3$

$\Rightarrow x^2 - 4x + 1 = 0$

Now $x^3 - x^2 - 11x + 4$

$= x^3 - 4x^2 + x + 3x^2 - 12x + 4$

$= x(x^2 - 4x + 1) + 3(x^2 - 4x + 1) + 1 = x \times 0 + 3(0) + 1 = 0 + 0 = 0 = 1$

Illustration-2 :

If $x = 3 - 2\sqrt{2}$, find $x^2 + \frac{1}{x^2}$

Solution :

We have, $x = 3 - 2\sqrt{2}$.

$\therefore \frac{1}{x} = \frac{1}{3-2\sqrt{2}} = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2} = \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$

Thus, $x^2 + \frac{1}{x^2} = (3-2\sqrt{2})^2 + (3+2\sqrt{2})^2$

$= (3)^2 + (2\sqrt{2})^2 - 2 \times 3 \times 2\sqrt{2} + (3)^2 + (2\sqrt{2})^2$
$+ 2 \times 3 \times 2\sqrt{2}$

$= 9 + 8 - 12\sqrt{2} + 9 + 8 + 12\sqrt{2} = 34$

Illustration-3 :

Rationalise the denominator of $\frac{1}{\sqrt{3}-\sqrt{2}-1}$

Solution :

$\frac{1}{\sqrt{3}-\sqrt{2}-1} = \frac{1}{\sqrt{3}-\sqrt{2}-1} \times \frac{\sqrt{3}-\sqrt{2}+1}{\sqrt{3}-\sqrt{2}+1} = \frac{\sqrt{3}-\sqrt{2}+1}{(\sqrt{3}-\sqrt{2}-1)(\sqrt{3}-\sqrt{2}+1)}$

$= \frac{\sqrt{3}-\sqrt{2}+1}{(\sqrt{3}-\sqrt{2})^2-1^2}$

$= \frac{\sqrt{3}-\sqrt{2}+1}{3+2-2\sqrt{3}\times\sqrt{2}-1} = \frac{\sqrt{3}-\sqrt{2}+1}{4-2\sqrt{6}}$

$= \frac{\sqrt{3}-\sqrt{2}+1}{2(2-\sqrt{6})} = \frac{\sqrt{3}-\sqrt{2}+1}{2(2-\sqrt{6})} \times \frac{2+\sqrt{6}}{2+\sqrt{6}}$

$$= \frac{(\sqrt{3}-\sqrt{2}+1)(2+\sqrt{6})}{2(2^2-(\sqrt{6})^2)}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}+2+\sqrt{3}\times\sqrt{6}-\sqrt{2}\times\sqrt{6}+\sqrt{6}}{2(4-6)}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}+2+\sqrt{3\times 6}-\sqrt{2\times 6}+\sqrt{6}}{-4}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}+2+\sqrt{3^2\times 2}-\sqrt{2^2\times 3}+\sqrt{6}}{-4}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}+2+3\sqrt{2}-2\sqrt{3}+\sqrt{6}}{-4}$$

$$= \frac{\sqrt{2}+\sqrt{6}+2}{-4} = -\left(\frac{\sqrt{2}+\sqrt{6}+2}{4}\right)$$

20. INDICES

Some useful Formulae

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = (a^n)^m = a^{mn}$

(iv) $\left(\frac{a}{b}\right)^{-\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$

(v) $a^m \div b^{-n} = a^m \times b^n$

(vi) $a^0 = 1,\ a \neq 0$

Illustration-4 :

Evaluate the following

(i) $(\sqrt[3]{64})^{\frac{-1}{2}}$

(ii) $\left(\frac{121}{169}\right)^{-3/2}$

Solution :

(i) $(\sqrt[3]{64})^{\frac{-1}{2}} = \left[(64)^{\frac{1}{3}}\right]^{\frac{-1}{2}} = (64)^{\frac{1}{3}\times\frac{-1}{2}} = (64)^{\frac{-1}{6}}$

$= (2^6)^{\frac{-1}{6}} = 2^{6\times\left(\frac{-1}{6}\right)} = 2^{-1} = \frac{1}{2}$

(ii) $\left(\frac{11\times 11}{13\times 13}\right)^{-3/2} = \left(\frac{11^2}{13^2}\right)^{-3/2} = \left(\frac{11}{13}\right)^{2\times\frac{-3}{2}} = \left(\frac{11}{13}\right)^{-3} = \left(\frac{13}{11}\right)^{3} = \frac{2197}{1331}$

Illustration-5 :

If $a^x = b$, $b^y = c$, $c^z = a$, prove that $xyz = 1$ where a,b,c are distinct numbers

Solution :

We have, $a^{xyz} = (a^x)^{yz}$

$\Rightarrow a^{xyz} = (b)^{yz}$ $[\because a^x = b]$

$\Rightarrow a^{xyz} = (b^y)^z$

$\Rightarrow a^{xyz} = c^z \quad [\because b^y = c]$

$\Rightarrow a^{xyz} = a \quad [\because c^z = a]$

$\therefore a^{xyz} = a^1$

$\Rightarrow xyz = 1$

21. POLYNOMIAL IN ONE VARIABLE

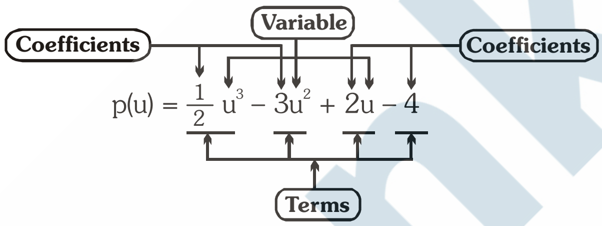
An algebraic expression of the form

$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1x^1 + a_0x^0$, where

(i) $a_n \neq 0$

(ii) power of x is whole number, is called a polynomial in one variable.

Hence, $a_n, a_{n-1}, a_{n-2}, ..., a_0$ are coefficients of $x^n, x^{n-1}, ..., x^0$ respectively and $a_nx^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ...$ are terms of the polynomial. Here the term a_nx^n is called the ***leading term*** and its coefficient a_n, the ***leading coefficient.***



22. ZERO POLYNOMIAL

Constants $2, -2, \sqrt{2}, \frac{3}{2}$ and a can be written as $2x°, -2x°, \sqrt{2}\,x°, \frac{3}{2}x°$ and $ax°$ respectively. Therefore, these constants are expressed as polynomials which contain single term in variable x and the exponent of the variable is 0. Thus, we can define a constant as a constant polynomial.

In particular, the constant number 0 as the zero polynomial.

23. DEGREE OF POLYNOMIALS

Degree of the polynomial in one variable is the largest exponent of the variable. For example, the degree of the polynomial $3x^7 - 4x^6 + x + 9$ is 7 and the degree of the polynomial $5x^6 - 4x^2 - 6$ is 6.

Polynomials classified by degree

Degree	Name	General form	Example
(undefined)	Zero polynomial	0	0
0	(Non-zero) constant polynomial	$a; (a \neq 0)$	1
1	Linear polynomial	$ax + b; (a \neq 0)$	$x + 1$
2	Quadratic polynomial	$ax^2 + bx + c; (a \neq 0)$	$x^2 + 1$
3	Cubic polynomial	$ax^3 + bx^2 + cx + d; (a \neq 0)$	$x^3 + 1$

Usually, a polynomial of degree n, for n greater than 3, is called a polynomial of degree n, although the phrases quartic polynomial and quintic polynomial are sometimes used.

24. SOME SPECIAL TYPES OF POLYNOMIALS

Monomials : Polynomials having only one term are called monomials. E.g. 2, 2x, $7y^5$, $12t^7$ etc.

Binomials : Polynomials having exactly two dissimilar terms are called binomials.

E.g. $p(x) = 2x + 1$, $r(y) = 2y^7 + 5y^6$. etc.

Trinomials : Polynomials having exactly three distinct terms are called trinomials.

E.g. $p(x) = 2x^2 + x + 6$, $q(y) = 9y^6 + 4y^2 + 1$ etc.

25. ZEROS / ROOTS OF A POLYNOMIAL / EQUATION

Consider a polynomial $f(x) = 3x^2 - 4x + 2$. If we replace x by 3 everywhere in the above expression, we get

$f(3) = 3 \times (3)^2 - 4 \times 3 + 2 = 27 - 12 + 2 = 17$

We can say that the value of the polynomial f(x) at x = 3 is 17.

Similarly the value of polynomial $f(x) = 3x^2 - 4x + 2$

at $x = -2$ is $f(-2) = 3(-2)^2 - 4 \times (-2) + 2 = 12 + 8 + 2 = 22$

at $x = 0$ is $f(0) = 3(0)^2 - 4(0) + 2 = 0 - 0 + 2 = 2$

at $x = \frac{1}{2}$ is $f\left(\frac{1}{2}\right) = 3 \times \left(\frac{1}{2}\right)^2 - 4 \times \left(\frac{1}{2}\right) + 2 = \frac{3}{4} - 2 + 2 = \frac{3}{4}$

In general, we can say $f(\alpha)$ is the value of the polynomial f(x) at $x = \alpha$, where α is a real number.

A real number α is zero of a polynomial f(x) if the value of the polynomial f(x) is zero at $x = \alpha$ i.e. $f(\alpha) = 0$.

OR

The value of the variable x, for which the polynomial f(x) becomes zero is called zero of the polynomial.

E.g. : consider, a polynomial $p(x) = x^2 - 5x + 6$; replace x by 2 and 3.

$p(2) = (2)^2 - 5 \times 2 + 6 = 4 - 10 + 6 = 0$,

$p(3) = (3)^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$

$\therefore$ 2 and 3 are the zeros of the polynomial p(x).

26. ROOTS OF A POLYNOMIAL EQUATION

An expression $f(x) = 0$ is called a polynomial equation if f(x)is a polynomial of degree $n \geq 1$.

A real number α is a root of a polynomial $f(x) = 0$ if $f(\alpha) = 0$ i.e. α is a zero of the polynomial f(x).

E.g. consider the polynomial $f(x) = 3x - 2$, then $3x - 2 = 0$ is the corresponding polynomial equation.

Here, $f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) - 2 = 0$

i.e. $\frac{2}{3}$ is a zero of the polynomial $f(x) = 3x - 2$

or $\frac{2}{3}$ is a root of the polynomial equation $3x - 2 = 0$

27. REMAINDER THEOREM

Statement : Let p(x) be a polynomial of degree ≥ 1 and 'a' is any real number. If p(x) is divided by (x – a), then the remainder is p(a).

E.g. Let p(x) be $x^3 - 7x^2 + 6x + 4$

Divide p(x) with (x – 6) and to find the remainder, put x = 6 in p(x) i.e. p(6) will be the remainder.

∴ required remainder be

$p(6) = (6)^3 - 7.6^2 + 6.6 + 4 = 216 - 252 + 36 + 4 = 256 - 252 = 4$

$$
\begin{array}{r|l|l}
x-6 & x^3 - 7x^2 + 6x + 4 & x^2 - x \\
 & \underline{\overset{-}{x^3} \overset{+}{-\,6x^2}} & \\
 & \quad -x^2 + 6x + 4 & \\
 & \quad \underline{\overset{+}{-x^2} \overset{-}{+\,6x}} & \\
 & \text{Remainder} = 4 &
\end{array}
$$

Thus, p(a) is remainder on dividing p(x) by (x – a).

Remark (i) p(–a) is remainder on dividing p(x) by (x + a)

$[\because x + a = 0 \Rightarrow x = -a]$

(ii) $p\left(\frac{b}{a}\right)$ is remainder on dividing p(x) by (ax – b)

$[\because ax - b = 0 \Rightarrow x = b/a]$

(iii) $p\left(\frac{-b}{a}\right)$ is remainder on dividing p(x) by (ax + b)

$[\because ax + b = 0 \Rightarrow x = -b/a]$

(iv) $p\left(\frac{b}{a}\right)$ is remainder on dividing p(x) by (b – ax)

$[\because b - ax = 0 \Rightarrow x = b/a]$

Illustration-6 :

Find the remainder when

$x^3 - ax^2 + 6x - a$ is divided by x – a

Solution :

Let $p(x) = x^3 - ax^2 + 6x - a$

$p(a) = a^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a = 5a$

So, by the Remainder theorem, remainder = 5a

28. FACTOR THEOREM

Statement : Let f(x) be a polynomial of degree ≥ 1 and a be any real constant such that f(a) = 0, then (x – a) is a factor of f(x). Conversely, if (x – a) is a factor of f(x), then f(a) = 0.

Proof : By Remainder theorem, if f(x) is divided by (x – a), the remainder will be f(a). Let q(x) be the quotient. Then, we can write,

f(x) = (x – a) × q(x) + f(a)   ($\because$ Dividend = Divisor × Quotient + Remainder)

If f(a) = 0, then f(x) = (x – a) × q(x)

Thus, (x – a) is a factor of q(x).

Converse Let (x – a) is a factor of f(x).

Then we have a polynomial q(x) such that f(x) = (x – a) × q(x)

Replacing x by a, we get f(a) = 0.

Hence, proved.

Illustration-7 :

Use the factor theorem to determine whether (x – 1) is a factor of f(x) = $2\sqrt{2}x^3 + 5\sqrt{2}x^2 - 7\sqrt{2}$

Solution :

By using factor theorem, (x – 1) is a factor of f(x), only when f(1) = 0

$f(1) = 2\sqrt{2}(1)^3 + 5\sqrt{2}(1)^2 - 7\sqrt{2} = 2\sqrt{2} + 5\sqrt{2} - 7\sqrt{2} = 0$

Hence, (x – 1) is a factor of f(x).

29. FACTORIZATION :

Formulae :

(a) $(a \pm b)^2 = a^2 \pm 2ab + b^2 = (a \mp b)^2 \pm 4ab$

(b) $a^2 - b^2 = (a+b)(a-b)$

- If $a^2 - b^2 = 1$ then $a + b = \dfrac{1}{a-b}$

For example : $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$ or $\sqrt{3} + \sqrt{2} = \dfrac{1}{\sqrt{3} - \sqrt{2}}$

(c) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(d) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(e) $a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3ab(a+b)$

(f) $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3ab(a-b)$

(g) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(h) $\mathbf{a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)}$

$$\mathbf{= \frac{1}{2}(a + b + c)\{(a - b)^2 + (b - c)^2 + (c - a)^2\}}$$

(i) $\mathbf{(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)}$

(j) $\mathbf{a^4 + a^2 + 1 = (a^2 + 1)^2 - a^2 = (1 + a + a^2)(1 - a + a^2)}$

30. CYCLIC FACTORS :

If an expression remain same after replacing a by b, b by c & c by a, then it is called cyclic expression and its factors are called cyclic factors. e.g. a(b – c) + b(c – a) + c(a – b)

Do yourself-1 :

(i) If $x = 3-\sqrt{8}$, then find the value of $\frac{1}{\sqrt{x}} - \sqrt{x}$.

(ii) The value of $\frac{a^3+b^3+c^3-3abc}{ab+bc+ca-a^2-b^2-c^2}$ is (where a = –5, b = –6, c = 10)

(iii) Find solution(s) of equation $64\,(9^x) - 84(12^x) + 27(16^x) = 0$

(iv) Find solution(s) of equation $3.2^{x/2} - 7.2^{x/4} = 20$, $x \in R$

(v) If $\sqrt{9+\sqrt{48}-\sqrt{32}-\sqrt{24}} = \sqrt{a}-\sqrt{b}+2$, where a,b ∈ N, then find the value of a + b.

Factorization :

Type-1 $a^2 - b^2 = (a-b)(a+b)$

Illustration-8 :

$(3x-y)^2 - (2x-3y)^2$

Solution :

Use $a^2 - b^2 = (a-b)(a+b)$

$(3x-y)^2 - (2x-3y)^2 = (3x-y+2x-3y)(3x-y-2x+3y)$

$= (5x-4y)(x+2y)$

Type -2 : $a^3 \pm b^3 \equiv (a \pm b)(a^2 \mp ab + b^2)$

Illustration-9 :

$a^6 - b^6$

Solution :

$a^6 - b^6 = (a^2)^3 - (b^2)^3$

$= (a^2 - b^2)(a^4 + a^2b^2 + b^4)$

$= (a-b)(a+b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

Type -3 : Factorising the quadratic

Illustration-10 :

$x^2 + 6x - 187$

Solution :

$x^2 + 6x - 187 = x^2 + 17x - 11x - 187$

$= x(x+17) - 11(x+17)$

$= (x+17)(x-11)$

Type -4 : Factorisationaly by converting the given expression into a perfect square.

Illustration-11 :

$9x^4 - 10x^2 + 1$

Solution :

$9x^4 - 10x^2 + 1 = (9x^2-1)(x^2-1)$

$= (3x-1)(3x+1)(x-1)(x+1)$

Type -5 : Using factor Theorem

Illustration-12 :

$x^3 - 13x - 12$

Solution :

$x^3 - 13x - 12 \because x = -1$ satisfies given expression

$\Rightarrow x + 1$ is a factor

$$
\begin{array}{r|l|l}
x+1 & x^3 - 13x - 12 & x^2 - x - 12 \\
 & \underline{x^3 + x^2} & \\
 & -x^2 - 13x - 12 & \\
 & \underline{-x^2 - x} & \\
 & -12x - 12 & \\
 & \underline{-12x - 12} & \\
 & \quad 0 &
\end{array}
$$

$\therefore x^3 - 13x - 12 = (x + 1)(x^2 - x - 12)$

$= (x + 1)(x - 4)(x + 3)$

Type -6 : $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Illustration-13 :

$8x^3 + y^3 + 27z^3 - 18xyz$

Solution :

$8x^3 + y^3 + 27z^3 - 18xyz = (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$

$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 6xz - 3yz)$

Type-7 :

Illustration-14 :

$x(x + 1)(x + 2)(x + 3) - 8$

Solution :

$$
\begin{aligned}
x(x + 1)(x + 2)(x + 3) - 8 &= x(x + 1)(x + 2)(x + 3) - 8 \\
&= x(x + 3)(x + 1)(x + 2) - 8 \\
&= (x^2 + 3x)(x^2 + 3x + 2) - 8 \\
&= (x^2 + 3x)^2 + 2(x^2 + 3x) - 8 \\
&= (x^2 + 3x)^2 + 4(x^2 + 3x) - 2(x^2 + 3x) - 8 \\
&= (x^2 + 3x)(x^2 + 3x + 4) - 2(x^2 + 3x + 4) \\
&= (x^2 + 3x - 2)(x^2 + 3x + 4)
\end{aligned}
$$

31. RATIO :

A ratio is a comparision of two quantities by division. It is a relation that one quantity bears to another with respect to magnitude. In other words, ratio means what part one quantity is of another. The quantities may be of same kind or different kinds. For example, when we consider the ratio of the weight 45 kg of a bag of rice to the weight 29 kg of a bag of sugar we are considering the quantities of same kind but when we talk of allotting 2 cricket bats to 5 sportsmen, we are considering quantities of different kinds. Normally, we consider the ratio between quantities of the same kind.

If a and b are two numbers, the ratio of a to b is $\frac{\mathbf{a}}{\mathbf{b}}$ of a ÷ b and is denoted by a : b. The two quantities that are being compared are called terms. The first is called antecedent and the second term is called consequent. For example, the ratio 3 : 5 represents $\frac{\mathbf{3}}{\mathbf{5}}$ with antecedent 3 and consequent 5.

Notes:

1. A ratio is a number, so to find the ratio of two quantities, they must be expressed in the same units.
2. A ratio does not change if both of its terms are multiplied or divided by the same number. Thus,

   $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$ etc.

32. PROPORTION

The equality of two ratios is called proportion.

If $\frac{a}{b} = \frac{c}{d}$, then a, b, c and d are said to be in proportion and we write a : b :: c : d. This is read as "a is to b as c is to d".

For example, since $\frac{3}{4} = \frac{6}{8}$, we write 3 : 4 :: 6 : 8 and say 3, 4, 6 and 8 are in proportion.

Each term of the ratio $\frac{a}{b}$ and $\frac{c}{d}$ is called a proportional. a, b, c and d are respectively, the first, second, third and fourth proportionals.

Here a, d are known as extremes and b, c are known as means.

(a) If four quantities are in proportion, then

Product of means = Product of extremes

For example, in the proportion a : b :: c : d, we have bc = ad.

Product of means

a : b : : c : d

Product of extremes

From this relation, we see that if any three of the four quantities are given, the fourth can be determined.

(b) Fourth proportional

If a : b :: c : x, x is called the fourth proportional of a, b, c.

We have $\frac{a}{b} = \frac{c}{x}$ or, $x = \frac{b \times c}{a}$.

Thus, fourth proportional of a, b, c is $\frac{b \times c}{a}$

Illustration-15 :

Find a fourth proportional to the numbers 2, 5, 4.

Solution :

Let x be the fourth proportional, then

$2 : 5 :: 4 : x$ or $\frac{2}{5} = \frac{4}{x}$.

$x = \frac{5 \times 4}{2} = 10.$

(c) Third proportional

If a : b:: b : x, x is called the third proportional of a, b.

We have $\frac{a}{b} = \frac{b}{x}$ or $x = \frac{b^2}{a}$.

Thus, third proportional of a, b is $\frac{b}{a}$.

Illustration-16 :

Find a third proportional to the numbers 2.5, 1.5.

Solution :

Let x be the third proportional, then

$2.5 : 1.5 :: 1.5 : x$ or $\frac{2.5}{1.5} = \frac{1.5}{x}$.

$x = \frac{1.5 \times 1.5}{2.5} = 0.9$

(d) Mean proportional

If a : x :: x : b, x is called the mean or second proportional of a, b.

We have $\frac{a}{x} = \frac{x}{b}$ **or** $x^2 = ab$ **or** $x = \sqrt{ab}$.

$\therefore$ **Mean proportional of a and b is** $\sqrt{ab}$.

We also say that a, x, b are in continued proportion.

Illustration-17 :

Find the mean proportional between 48 and 12.

Solution :

Let x be the mean proportional. Then,

$48 : x : : x : 12$ or $\frac{48}{x} = \frac{x}{12}$

or, $x^2 = 576$ or, $x = 24$.

(e) If $\frac{a}{b} = \frac{c}{d}$**, then**

(i) $\frac{a+b}{b} = \frac{c+d}{d}$ **(Componendo)**

(ii) $\frac{a-b}{b} = \frac{c-d}{d}$ **(Dividendo)**

(iii) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ **(Componendo and dividendo)**

(iv) $\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$.

(v) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ... = \frac{\lambda_1 a + \lambda_2 c + \lambda_3 e...}{\lambda_1 b + \lambda_2 d + \lambda_3 f...}$, where $\lambda_1, \lambda_2, \lambda_3$......... are real numbers

(vi) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=......,$ then each ratio $=\left(\frac{a^n+c^n+e^n}{b^n+d^n+f^n}\right)^{\frac{1}{n}}$

Example : $\frac{a}{b}=\frac{c}{d}=\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}=\frac{a+c}{b+d}=\frac{a-c}{b-d}$

Illustration-18 :

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then show that $\frac{a^3b+2c^2e-3ae^2f}{b^4+2d^2f-3bf^3}=\frac{ace}{bdf}$ (wherever defined)

Solution :

$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k \quad \Rightarrow \quad a = bk, c = dk, e = fk$

$\therefore \quad \frac{a^3b+2c^2e-3ae^2f}{b^4+2d^2f-3bf^3}=\frac{k^3(b^4+2d^2f-3bf^3)}{b^4+2d^2f-3bf^3}=k^3=\frac{ace}{bdf}$

Illustration-19 :

The sum of two numbers is c and their quotient is $\frac{p}{q}$. Find the numbers.

Solution :

Let the numbers be x, y.

Given $\quad x+y=c \qquad$ ...(1)

and, $\quad \frac{x}{y}=\frac{p}{q} \qquad$ ...(2)

$\therefore \quad \frac{x}{x+y}=\frac{p}{p+q}$

$\Rightarrow \quad \frac{x}{c}=\frac{p}{p+q}$ [Using (1)]

$\Rightarrow \quad x=\frac{pc}{p+q}$ and $y=\frac{qc}{p+q}$

Illustration-20 :

Two positive numbers are in the ratio of 4 : 5. If the difference between these numbers is 24, then find the numbers.

Solution :

Here a = 4, b = 5 and x = 24.

$\therefore$ The first number $=\frac{ax}{b-a}=\frac{4\times 24}{5-4}=96.$

and the second number $=\frac{bx}{b-a}=\frac{5\times 24}{5-4}=120.$

32. INTERVALS

Intervals are basically subsets of R. If there are two numbers a, b $\in$ R such that a < b, we can define four types of intervals as follows :

(a) Open interval : (a, b) = $\{x : a < x < b\}$ i.e. end points are not included.

(b) Closed interval : [a, b] = $\{x : a \le x \le b\}$ i.e. end points are also included.

This is possible only when both a and b are finite.

(c) Semi open or semi closed interval : $(a, b] = \{x : a < x \le b\}$; $[a, b) = \{x : a \le x < b\}$

(d) The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x : x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(-\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = R$

Note :

(i) For some particular values of x, we use symbol { } e.g. If $x = 1, 2$ we can write it as $x \in \{1, 2\}$

(ii) If there is no values of x, then we say $x \in \phi$ (null set)

Illustration-21 :

Let $A \equiv \{1,2,4,6\}$, $B \equiv \{2,4,5,7,9\}$, then find (i) $A \cup B$ (ii) $A \cap B$

Ans. (i) $\{1,2,4,5,6,7,9\}$ (ii) $\{2,4\}$

Do yourself : 2

(i) Factorize the given expression :

(a) $x^3 - 5x^2 + 6x$

(b) $25x^4 + 5x^2 + 1$

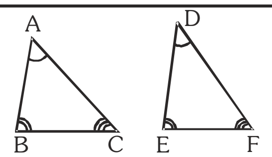
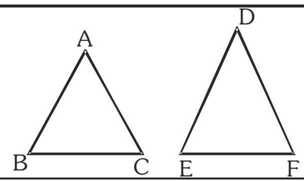
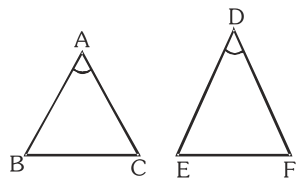
(ii) Find a third proportional to the numbers 3,8

(iii) Find the mean proportional between 9 and 16.

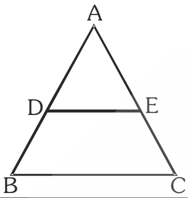
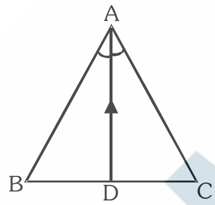
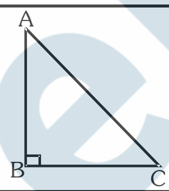
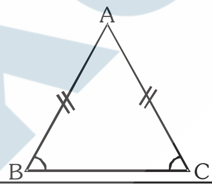
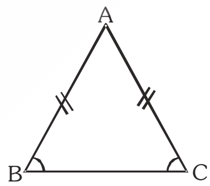
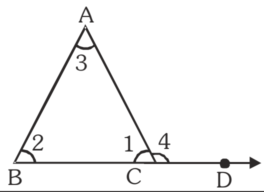
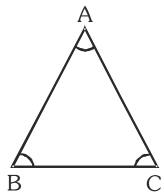
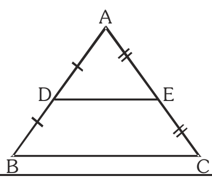
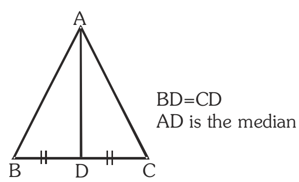
(iv) The sum of two numbers is 10 and their quotient is $\frac{2}{3}$. Find the numbers.

(v) Let $A \equiv (2,5]$, $B \equiv (-\infty,4)$, then find (a) $A \cup B$ (b) $A \cap B$

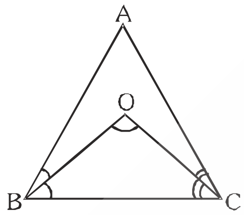
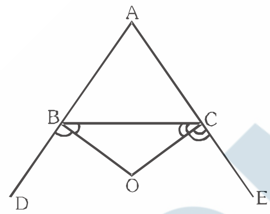
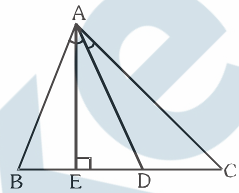
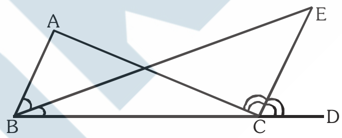
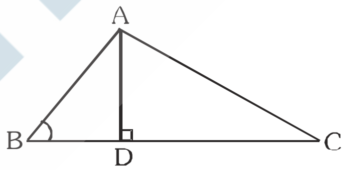
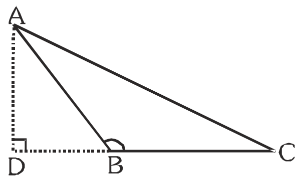
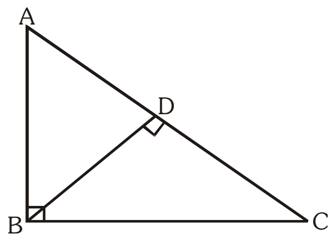
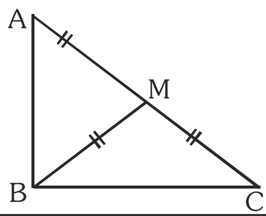
■ **Theorems related to similar triangles**

Test	Property	Diagram
A – A – A (similarity)	If in two triangles corresponding angles are equal i.e., the two triangles are equiangular, then the triangles are similar. $\angle A = \angle D$, $\angle B = \angle E$ & $\angle C = \angle F$ $\Delta ABC \sim \Delta DEF$	
S – S – S (Similarity)	If the corresponding sides of two triangles are proportional, then they are similar.. $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ $\Delta ABC \sim \Delta DEF$	
S – A – S (Similarity)	If in two triangles, one pair of corresponding sides are proportional and the included angles are equal then the two triangles are similar.. $\frac{AB}{DE} = \frac{AC}{DF}$ & $\angle BAC = \angle EDF$ $\Delta ABC \sim \Delta DEF$	

■ Theorems related to triangles

Theorem	Statement/Expanation	Diagram
Basic proportionality theorem	In a triangle, a line drawn parallel to one side, will divide the other two sides in same ratio. If DE||BC, then $\frac{AD}{DB} = \frac{AE}{EC}$	
Vertical angle bisector	The bisector of the vertical angle of a triangle divides the base in the ratio of other two sides. $\frac{BD}{DC} = \frac{AB}{AC}$	
Pythagoras theorem	In a right angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides. $AC^2 = AB^2 + BC^2$	
Theorem	Angles opposite to equal sides of a triangle are equal. If AB = AC then $\angle B = \angle C$	
Theorem	If two angles of a triangle are equal, then the sides opposite to them are also equal.If $\angle B = \angle C$ then AB = AC	
Exterior angle	If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles. $\angle 4 = \angle 2 + \angle 3$	
Theorem	The sum of three angles in a triangle is 180°. $\angle A + \angle B + \angle C = 180°$	
Mid-point theorem	If the mid-points of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side. i.e., if AD=BD and AE=CE then DE||BC and BC = 2DE	
Apollonius theorem	In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side. i.e. $AB^2+AC^2=2(AD^2+BD^2)$	

■ Some useful results

S.No.	Statement	Diagram
(1)	In a ΔABC, if the internal angle bisectors of $\angle B$ and $\angle C$ meet at O then $\angle BOC = 90° + (\angle A)/2$	
(2)	In a ΔABC, if sides AB and AC are produced to D and E respectively and the bisectors of $\angle DBC$ and $\angle ECB$ intersect at O, then $\angle BOC = 90° - (\angle A)/2$	
(3)	In a ΔABC, if AD is the angle bisector of $\angle BAC$ and $AE \perp BC$, $\angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$	
(4)	In a ΔABC, if side BC is produced to D and bisectors of $\angle ABC$ and $\angle ACD$ meet at E, then $\angle BEC = \frac{1}{2}\angle BAC$	
(5)	In an acute angle ΔABC, AD is a perpendicular dropped on the opposite side of $\angle A$ then $AC^2 = AB^2 + BC^2 - 2BD.\ BC \quad (\angle B < 90°)$	
(6)	In a obtuse angle ΔABC, AD is perpendicular dropped on BC. BC is produce to D to meet AD, then $AC^2 = AB^2 + BC^2 + 2\ BD\ BC\ (\angle B > 90°)$	
(7)	In a right angle ΔABC, $\angle B = 90°$ and AC is hypotenuse the perpendicular BD is dropped on hypotenuse AC from right angle vertex B, then (i) $BD = \frac{AB \times BC}{AC}$ (ii) $AD = \frac{AB^2}{AC}$ (iii) $CD = \frac{BC^2}{AC}$ (iv) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$	
(8)	In a right angled triangle, the median to the hypotenuse $= \frac{1}{2} \times$ hypotenuse i.e, $BM = \frac{AC}{2}$	

ANSWERS FOR DO YOURSELF

1 : **(i)** 2 **(ii)** 1 **(iii)** 1,2 **(iv)** 8 **(v)** 5

2 : **(i)** (a) $x(x-2)(x-3)$ (b) $\left(5x^2+\sqrt{5}x+1\right)\left(5x^2-\sqrt{5}x+1\right)$

(ii) $\frac{64}{3}$ **(iii)** 12 **(iv)** 4,6 **(v)** **(a)** $(-\infty,5]$ **(b)** $(2,4)$

EXERCISE (O-1)

1. $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+..........\infty \text{ times}}}}} =$

(1) 3 (2) 2 (3) 1 (4) ± 3

2. If $x = 8-\sqrt{60}$, then $\frac{1}{2}\left[\sqrt{x}+\frac{2}{\sqrt{x}}\right] =$

(1) $\sqrt{5}$ (2) $\sqrt{3}$ (3) $2\sqrt{5}$ (4) $2\sqrt{3}$

3. If $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a+b\sqrt{5}$, a, b are rational numbers, then (a, b) =

(1) $\left(\frac{61}{29},\frac{-24}{29}\right)$ (2) $\left(\frac{-61}{29},\frac{24}{29}\right)$ (3) $\left(\frac{61}{29},\frac{24}{29}\right)$ (4) $\left(\frac{-61}{29},\frac{-24}{29}\right)$

4. The square root of $11+\sqrt{112}$ is -

(1) $\sqrt{7}+2$ (2) $\sqrt{7}+\sqrt{2}$ (3) $2-\sqrt{7}$ (4) None

5. The square root $5+2\sqrt{6}$ is :

(1) $\sqrt{3}+2$ (2) $\sqrt{3}-\sqrt{2}$ (3) $\sqrt{2}-\sqrt{3}$ (4) $\sqrt{3}+\sqrt{2}$

6. $\sqrt{21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}} =$

(1) $\sqrt{5}-2+2\sqrt{3}$ (2) $-\sqrt{5}-\sqrt{4}-\sqrt{12}$ (3) $-\sqrt{5}+\sqrt{4}+\sqrt{12}$ (4) $-\sqrt{5}-\sqrt{4}+\sqrt{12}$

7. If $\frac{4}{2+\sqrt{3}+\sqrt{7}} = \sqrt{a}+\sqrt{b}-\sqrt{c}$, then which of the following can be true -

(1) a = 1, b = 4/3, c = 7/3 (2) a = 1, b = 2/3, c = 7/9
(3) a = 2/3, b = 1, c = 7/3 (4) a = 7/9, b = 4/3, c = 1

8. If $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3} = 10$, then x =

(1) 2, 2 (2) $\sqrt{2}$, $-\sqrt{2}$ (3) 2, $+\sqrt{2}$ (4) 2, –2, $\sqrt{2}$, $-\sqrt{2}$

9. If $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0$ then $(a+b+c)^3 =$

(1) abc (2) 3abc (3) 9ac (4) 27abc

10. If $3^{2x^2}-2.3^{x^2+x+6}+3^{2(x+6)}=0$ then the value of x is

(1) – 2 (2) 3 (3) Both (1) and (2) (4) None of these

11. The numerical value of $\left(x^{1/(a-b)}\right)^{1/(a-c)} \times \left(x^{1/(b-c)}\right)^{1/(b-a)} \times \left(x^{1/(c-a)}\right)^{1/(c-b)}$ is (a, b, c are distinct real numbers)

(1) 1 (2) 8 (3) 0 (4) None

12. $\left(\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right)^{2} =$

(1) 4 (2) 5 (3) 2 (4) 3

13. $\left(5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^{3}\right)^{\frac{1}{4}} =$

(1) 3 (2) 6 (3) 5 (4) 4

14. $(1^3 + 2^3 + 3^3 + 4^3)^{-3/2} =$

(1) 10^{-3} (2) 10^{-2} (3) 10^{-4} (4) 10^{-1}

15. $(0.000729)^{-3/4} \times (0.09)^{-3/4} =$

(1) $\frac{10^3}{3^3}$ (2) $\frac{10^5}{3^5}$ (3) $\frac{10^2}{3^2}$ (4) $\frac{10^6}{3^6}$

16. $\left\{\sqrt[4]{\left(\frac{1}{x}\right)^{-12}}\right\}^{-2/3} =$

(1) $\frac{1}{x^2}$ (2) $\frac{1}{x^4}$ (3) $\frac{1}{x^3}$ (4) $\frac{1}{x}$

17. $\frac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}} =$

(1) $x^{76/15}$ (2) $x^{78/15}$ (3) $x^{79/15}$ (4) $x^{77/15}$

18. $\left(\frac{5}{6}\right)^{3/4}$ when divided by $\left(\frac{5}{6}\right)^{7/6}$ becomes $\left(\frac{5}{6}\right)^{7-x}$, the value of x is

(1) $\frac{7}{12}$ (2) $\frac{89}{12}$ (3) $\frac{8}{12}$ (4) $\frac{10}{12}$

19. If $\sqrt[4]{\sqrt[3]{x^2}} = x^k$, then k =

(1) $\frac{2}{6}$ (2) 6 (3) $\frac{1}{6}$ (4) 7

20. $\left(7^{\left(-\frac{1}{2}\right)} \times 5^2\right)^2 \div \sqrt{25^3} =$

(1) $\frac{5}{7}$ (2) $\frac{7}{5}$ (3) 35 (4) $-\frac{5}{7}$

21. $(2d^2e^{-1})^3 \times \left(\frac{d^3}{e}\right)^{-2} =$

(1) $8e^{-2}$ (2) $8e^{-3}$ (3) $8e^{-1}$ (4) $8e^{-4}$

22. If $\sqrt{9^x} = \sqrt[3]{9^2}$, then x =

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$

23. If $a = x + \frac{1}{x}$, then $x^3 + x^{-3} =$

(1) $a^3 + 3a$ (2) $a^3 - 3a$ (3) $a^3 + 3$ (4) $a^3 - 3$

24. If $x^y = y^x$ and x = 2y, then the values of x and y are (x, y > 0)

(1) x = 4, y = 2 (2) x = 3, y = 2 (3) x = 1, y = 1 (4) None of these

25. If $(a^m)^n = a^{m^n}$, then express 'm' in the terms of n is $(a > 0, a \neq 0, m > 1, n > 1)$

(1) $n^{\left(\frac{1}{n-1}\right)}$ (2) $n^{\left(\frac{1}{n+1}\right)}$ (3) $n^{\left(\frac{1}{n}\right)}$ (4) None

26. If $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$, then x =

(1) –2 (2) 4 (3) –6 (4) –4

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	4	1	4	3	1	4	4	3	1	2	3	1	4
Que.	16	17	18	19	20	21	22	23	24	25	26				
Ans.	1	4	2	3	1	3	2	2	1	1	4				

EXERCISE (S-1)

1. Match the values of x given in **Column-II** satisfying the exponential equation in Column-I (Do not verify). Remember that for a > 0, then term a^x is always greater than zero $\forall\ x \in R$.

Column-I	Column-II
(A) $5^x - 24 = \frac{25}{5^x}$	(P) –3
(B) $(2^{x+1})(5^x) = 200$	(Q) –2
(C) $4^{2/x} - 5(4^{1/x}) + 4 = 0$	(R) –1
	(S) 0
(D) $\frac{2^{x-1}.4^{x+1}}{8^{x-1}} = 16$	(T) 1
	(U) 2
(E) $4^{x^2+2} - 9\left(2^{x^2+2}\right) + 8 = 0$	(V) 3
(F) $5^{2x} - 7^x - 5^{2x}(35) + 7^x(35) = 0$	(X) None

2. Which of the following equation(s) has (have) only unity as the solution.

(A) $2(3^{x+1}) - 6(3^{x-1}) - 3^x = 9$

(B) $7(3^{x+1}) - 5^{x+2} = 3^{x+4} - 5^{x+3}$

3. Which of the following equation (s) has (have) only natural solution(s)

(A) $6.9^{1/x} - 13.6^{1/x} + 6.\ 4^{1/x} = 0$

(B) $4^x.\sqrt{8^{x-1}} = 4$

4. If $(5+2\sqrt{6})^{x^2-8} + (5-2\sqrt{6})^{x^2-8} = 10, x \in R$

On the basis of above information, answer the following questions :

(a) Number of solution(s) of the given equation is/are-

(A) 1 (B) 2 (C) 4 (D) infinite

(b) Sum of positive solutions is

(A) 3 (B) $3+\sqrt{7}$ (C) $2+\sqrt{5}$ (D) 2

(c) If $x \in (-3,5]$, then number of possible values of x, is-

(A) 1 (B) 2 (C) 3 (D) 4

5. **Factorize following expressions**

(i) $x^4 - y^4$ (ii) $9a^2 - (2x-y)^2$ (iii) $4x^2 - 9y^2 - 6x - 9y$

6. **Factorize following expressions**

(i) $8x^3 - 27y^3$ (ii) $8x^3 - 125y^3 + 2x - 5y$

7. **Factorize following expressions**

(i) $x^2 + 3x - 40$ (ii) $x^2 - 3x - 40$ (iii) $x^2 + 5x - 14$

(iv) $x^2 - 3x - 4$ (v) $x^2 - 2x - 3$ (vi) $3x^2 - 10x + 8$

(vii) $12x^2 + x - 35$ (viii) $3x^2 - 5x + 2$ (ix) $3x^2 - 7x + 4$

(x) $7x^2 - 8x + 1$ (xi) $2x^2 - 17x + 26$ (xii) $3a^2 - 7a - 6$

(xiii) $14a^2 + a - 3$

8. **Factorize following expressions**

(i) $a^2 - 4a + 3 + 2b - b^2$ (ii) $x^4 + 324$ (iii) $x^4 - y^2 + 2x^2 + 1$

(iv) $4a^4 - 5a^2 + 1$ (v) $4x^4 + 81$

9. **Factorize following expressions**

(i) $x^3 - 6x^2 + 11x - 6$ (ii) $2x^3 + 9x^2 + 10x + 3$

(iii) $2x^3 - 9x^2 + 13x - 6$ (iv) $x^6 - 7x^2 - 6$

10. **(i) Factorize the expressions** $8a^6 + 5a^3 + 1$

(ii) Show that $(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$.

11. **Factorize following expressions**

(i) $(x + 1)(x + 2)(x + 3)(x + 4) - 15$ (ii) $4x(2x + 3)(2x - 1)(x + 1) - 54$

(iii) $(x - 3)(x + 2)(x + 3)(x + 8) + 56$

ANSWER KEY

1. **Ans. (A)→(U); (B)→(U); (C)→(T); (D)→(P,Q,R,S,T,U,V);(E)→(R,T); (F)→(S)**
2. A
3. B
4. **(a)** C **(b)** B **(c)** C
5. (i) $(x^2 + y^2)(x + y)(x - y)$ (ii) $(3a + 2x - y)(3a - 2x + y)$ (iii) $(2x + 3y)(2x - 3y - 3)$
6. (i) $(2x - 3y)(4x^2 + 6xy + 9x^2)$ (ii) $(2x - 5y)(4x^2 + 10xy + 25y^2 + 1)$
7. (i) $(x + 8)(x - 5)$ (ii) $(x - 8)(x + 5)$ (iii) $(x + 7)(x - 2)$ (iv) $(x - 4)(x + 1)$
(v) $(x - 3)(x + 1)$ (vi) $(x - 2)(3x - 4)$ (vii) $(4x + 7)(3x - 5)$ (viii) $(3x-2)(x-1)$
(ix) $(x - 1)(3x - 4)$ (x) $(x - 1)(7x - 1)$ (xi) $(2x - 13)(x - 2)$ (xii) $(a-3)(3a+2)$
(xiii) $(2a + 1)(7a - 3)$
8. (i) $(a - b - 1)(a + b - 3)$ (ii) $(x^2 + 6x + 18)(x^2 - 6x + 18)$
(iii) $(x^2 + 1 + y)(x^2 + 1 - y)$ (iv) $(2a + 1)(2a - 1)(a + 1)(a - 1)$
(v) $(2x^2 + 6x + 9)(2x^2 - 6x + 9)$
9. (i) $(x - 1)(x - 2)(x - 3)$ (ii) $(x + 1)(x + 3)(2x + 1)$
(iii) $(x - 1)(x - 2)(2x - 3)$ (iv) $(x^2 + 2)(x - \sqrt{3})(x^2 + 1)(x + \sqrt{3})$
10. (i) $(2a^2 - a + 1)(4a^4 + 2a^3 - a^2 + a + 1)$
11. (i) $(x^2 + 5x + 1)(x^2 + 5x + 9)$ (ii) $2(2x^2 + 2x + 3)(4x^2 + 4x - 9)$
(iii) $(x^2 + 5x - 22)(x + 1)(x + 4)$