Application of Derivatives

Excerise-1: Single Choice Problems

1. The difference between the maximum and minimum value of the function

   $f(x) = 3\sin^4 x = \cos^6 x$ is:

   (a) $\frac{3}{2}$ (b) $\frac{5}{2}$

   (c) 3 (d) 4

2. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$. If its graph passes through the point $(2, 1)$ and at that point the tangent to the graph is $y = 3x - 5$, then the function is :

   (a) $(x-1)^2$ (b) $(x-1)^3$

   (c) $(x+1)^3$ (d) $(x+1)^2$

3. If the subnormal at any point on the curve $y = 3^{1-k}.x^k$ is of constant length then k equals to :

   (a) $\frac{1}{2}$ (b) 1

   (c) 2 (d) 0

4. If $x^5 - 5qx + 4r$ is divisible by $(x-c)^2$ then which of the following must hold true $\forall\ q, r, c \in R$?

   (a) $q = r$ (b) $q + r = 0$

   (c) $q^5 = r^4$ (d) $q^4 = r^5$

5. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\text{cm}^3/\text{min}$. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is :

   (a) $\frac{1}{36\pi}\text{cm/min}$ (b) $\frac{1}{18\pi}\text{cm/min}$

   (c) $\frac{1}{54\pi}\text{cm/min}$ (d) $\frac{5}{6\pi}\text{cm/min}$

6. If $f(x)=\frac{(x-1)(x-2)}{(x-3)(x-4)}$, then number of local extremes for $g(x)$, where

$g(x)=f(|x|)$:

(a) 3 (b) 4

(c) 5 (d) None of these

7. Two straight roads OA and OB intersect at an angle 60°. A car approaches O from A, where $OA = 700$m at a uniform speed of 20 m/s, Simultaneously, a runner starts running from O towards B at a uniform speed of 5 m/s. The time after start when the car and the runner are closest is :

(a) 10 sec (b) 15 sec

(c) 20 sec (d) 30 sec

8. Let $f(x)=\begin{cases} a-3x & ; \quad -2 \le c < 0 \\ 4x+3 & ; \quad 0 \le x < 1 \end{cases}$; if $f(x)$ has smallest value at $x=0$, then range of a, is:

(a) $(-\infty, 3)$ (b) $(-\infty, 3]$

(c) $(3, \infty)$ (d) $[3, \infty)$

9. $f(x)=\begin{cases} 3+|x-k| & , \quad x \le k \\ a^2-2+\frac{\sin(x-k)}{(x-k)} & , \quad x > k \end{cases}$ has maximum at $x=k$, then :

(a) $a \in R$ (b) $|a| < 2$

(c) $|a| > 2$ (d) $1 < |a| < 2$

10. For a certain curve $\frac{d^2y}{dx^2}=6x-4$ and curve has local minimum value 5 at $x=1$. Let the global maximum and global minimum values, where $0 \le x \le 2$; are M an m. Then the value of $(M-m)$ equals to :

(a) -2 (b) 2

(c) 12 (d) -12

11. The tangent to $y = ax^2 + bx + \frac{7}{2}$ at $(1, 2)$ is parallel to the normal at the point $(-2, 2)$ on the curve $y = x^2 + 6x + 10$. Then the value of $\frac{a}{2} - b$ is:

(a) 2
(b) 0
(c) 3
(d) 1

12. If (a, b) be the point on the curve $9y^2 = x^3$ where normal to the curve make equal intercepts with the axis, then the value of $(a + b)$ is :

(a) 0
(b) $\frac{10}{3}$
(c) $\frac{20}{3}$
(d) None of these

13. The curve $y = f(x)$ satisfies $\frac{d^2y}{dx^2} = 6x - 4$ and $f(x)$ has local minimum value 5 when $x = 1$. Then $f(0)$ is equal to:

(a) 1
(b) 0
(c)
(d) None of these

14. Let A be the point where the curve $5a^2x^3 + 10\alpha x^2 + x + 2y - 4 = 0$ $(\alpha \epsilon R, \alpha \neq 0)$ meets the $y -$ axis, then the equation of tangent to the curve at the point where normal at A meets the curve again, is :

(a) $x - ay + 2a = 0$
(b) $ax + y - 2 = 0$
(c) $2x - y + 2 = 0$
(d) $x + 2y - 4 = 0$

15. The difference between the greatest and the least value of the function $f(x) = \cos x + \frac{1}{2} \cos 2x - \frac{1}{3} \cos 3x$

(a) $\frac{11}{5}$
(b) $\frac{13}{6}$
(c) $\frac{9}{4}$
(d) $\frac{7}{3}$

16. The x co-ordinate of the point on the curve $y = \sqrt{x}$ which is closest to the point (2, 1) is :

(a) $\frac{2+\sqrt{3}}{2}$
(b) $\frac{1+\sqrt{3}}{2}$
(c) $\frac{-1+\sqrt{3}}{2}$
(d) 1

17. The tangent at a point P on the curve $y = \text{In}\left(\frac{2+\sqrt{4-x^2}}{2-\sqrt{4-x^2}}\right) - \sqrt{4-x^2}$ meets the $y-$ axis at T; then PT^2 equals to :

(a) 2 (b) 4

(c) 8 (d) 16

18. Let $f(x) = \int_{x^2}^{x^3} \frac{dt}{\text{In } t}$ for $x > 1$ and $g(x) = \int_1^x (2t^2 - \text{In}t)f(t)dt$ $(x > 1)$, then :

(a) g is increasing on $(1, \infty)$

(b) g is decreasing on $(1, \infty)$

(c) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$

(d) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

19. Let $f(x) = x^3 + 6x^2 + ax + 2,$ if $(-3, -1)$ is the largest possible interval for which f(x) is decreasing function, then $a =$

(a) 3 (b)9

(c) -2 (d) 1

20. Let $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right)$. Then difference of the greatest and least value of f(x) on $[0, 1]$ is :

(a) $\pi/2$ (b) $\pi/4$

(c) π (d) $\pi/3$

21.The number of integral values of a for which $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is monotonic in $\forall\, x \in R$.

(a) 2 (b) 4

(c) 6 (d) 7

22.The number of critical point of $f(x) = \left(\int_0^x \left(\cos^2 t - \sqrt[3]{t}\right) dt\right) + \frac{3}{4}x^{4/3} - \frac{x+1}{2}$ in $[0, 6\pi]$ is:

(a) 10 (b) 8

(c) 6 (d) 12

23. Let $f(x) = \min\left(\frac{1}{2} - \frac{3x^2}{4}, \frac{5x^2}{4}\right)$ for $0 \le x \le 1$, then maximum value of f(x) is

(a) 0
(b) $\frac{5}{64}$
(c) $\frac{5}{4}$
(d) $\frac{5}{16}$

24. Let $f(x) = \begin{cases} 2 - |x^2 + 5x + 6| & x \neq -2 \\ b^2 + 1 & x \neq -2 \end{cases}$

Has relative maximum at $x = -2$, then complete set of values b can take is:

(a) $|b| \ge 1$
(b) $|b| < 1$
(c) $b > 1$
(d) $b < 1$

25. Let for the function $f(x) = \begin{bmatrix} \cos^{-1}x & ; & -1 \le x \le 0 \\ mx + c & ; & 0 < x \le 1 \end{bmatrix}$;

Lagrange's mean value theorem is applicable in $[-1, 1]$ then ordered pair (m, c) is:

(a) $\left(1, -\frac{\pi}{2}\right)$
(b) $\left(1, \frac{\pi}{2}\right)$
(c) $\left(-1, -\frac{\pi}{2}\right)$
(d) $\left(-1, \frac{\pi}{2}\right)$

26. Tangents are drawn to $y = \cos x$ from origin then points of contact of these tangents will always lie on :

(a) $\frac{1}{x^2} = \frac{1}{y^2} + 1$
(b) $\frac{1}{x^2} = \frac{1}{y^2} - 2$
(c) $\frac{1}{y^2} = \frac{1}{x^2} + 1$
(d) $\frac{1}{y^2} = \frac{1}{x^2} - 2$

27. Least natural number a for which $x + ax^{-2} > 2 \forall x \epsilon (0, \infty)$ is :

(a) 1
(b) 2
(c) 5
(d) None of these

28. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at points $(2, 0)$ and $(3, 0)$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

29. Difference between the greatest and least values of the function $f(x) = \int_0^x (\cos^2 t + \cos t + 2)dt$ in the interval $[0, 2\pi]$ is $K\pi$, then K is equal to :

(a) 1 (b) 3

(c) 5 (d) None of these

30. The range of the function $f(\theta) = \frac{\sin\theta}{\theta} + \frac{\theta}{\tan\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is equal to :

(a) $(0, \infty)$ (b) $\left(\frac{1}{\pi}, 2\right)$

(c) $(2, \infty)$ (d) $\left(\frac{2}{\pi}, 2\right)$

31. Number of integers in the range of c so that the equation $x^3 - 3x + c = 0$ has all its roots real and distinct is:

(a) 2 (b) 3

(c) 4 (d) 5

32. Let $f(x) = \int e^x(x-1)(x-2)dx$. Then $f(x)$ decreases in the interval:

(a) $(2, \infty)$ (b) $(-2, -1)$

(c) $(1, 2)$ (d) $(-\infty, 1) \cup (2, \infty)$

33. If the cubic polynomial $y = ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R$ has only one critical point in entire domain and $ac = 2$, then the value of $|b|$ is:

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) $\sqrt{5}$ (d) $\sqrt{6}$

34. On the curve $y = \frac{1}{1+x^2}$, the point at which $\left|\frac{dy}{dx}\right|$ is greatest in the first quadrant is :

(a) $\left(\frac{1}{2}, \frac{4}{5}\right)$ (b) $\left(1, \frac{1}{2}\right)$

(c) $\left(\frac{1}{\sqrt{2}}, \frac{4}{5}\right)$ (d) $\left(\frac{1}{\sqrt{3}}, \frac{3}{4}\right)$

35. If $f(x) = 2x, g(x) = 3\sin x - x\cos x$, then for $x \in \left(0, \frac{\pi}{2}\right)$:

(a) $f(x) > g(x)$ (b) $f(x) < g(x)$

(c) $f(x) = g(x)$ (d) $f(x) = g(x)$ has exactly two real roots

36. Let $f(x) = \sin^{-1}\left(\frac{2g(x)}{1+g(x)^2}\right)$ then which are correct ?

(a) $f(x)$ is decreasing if $g(x)$ is increasing and $|g(x)| > 1$

(b) $f(x)$ is increasing function if $g(x)$ is increasing and $|g(x)| \leq 1$

(c) $f(x)$ is decreasing function if $g(x)$ is decreasing and $|g(x)| > 1$

(a) (i) and (iii) (b) (i) and (ii) (c) (i), (ii) and (iii) (d) (iii)

37. The graph of the function $y = f(x)$ has a unique tangent at $(e^a, 0)$ through which the graph passes then $\lim_{x \to e^a} \frac{\text{In}\left(1+7f(x)\right)-\sin(f(x))}{3f(x)}$ is equal to :

(a) 1 (b) 3

(c) 2 (d) 7

38. Let $f(x)$ be a function such that $f'(x) = \log_{/3}(\log_3(\sin x + a))$. The complete set of values of 'a' for which $f(x)$ is strictly decreasing for all real values of x is:

(a) $[4, \infty)$ (b) $[3, 4]$

(c) $(\infty, 4)$ (d) $[2, \infty)$

39. If $f(x) = a\,\text{In}|x| + bx^2 + x$ has extremas at $x = 1$ and $x = 3$, then:

(a) $a = \frac{3}{4}, b = -\frac{1}{8}$ (b) $a = \frac{3}{4}, b = \frac{1}{8}$

(c) $a = -\frac{3}{4}, b = -\frac{1}{8}$ (d) $a = -\frac{3}{4}, b = \frac{1}{8}$

40. Let $f(x) = \begin{cases} 1 + \sin x, & x < 0 \\ x^2 - x + 1, & x \geq 0 \end{cases}$, then :

(a) f has a local maximum at $x = 0$ (b) f has a local minimum at $x = 0$

(c) f is increasing everywhere (b) f is increasing everywhere

41. If m and n are positive integers and $f(x) = \int_1^x (t-a)^{2n}\,(t-b)^{2m+1}dt, a \neq b$, then :

(a) $x = b$ is a point of local minimum

(b) $x = b$ is a point of local maximum

(c) $x = a$ is a point of local minimum

(d) $x = a$ is a point of local maximum

42. For any real θ, the maximum value of $\cos^2(\cos\theta) + \sin^2(\sin\theta)$ is:

(a) 1 (b) $1 + \sin^2 1$

(c) $1 + \cos^2 1$ (d) Does not exist

43. If the tangent at P of the curve $y^2 = x^3$ intersects the curve again at Q and the straight line OP, OQ have inclinations a, b where O is origin, then $\left(\frac{\tan\alpha}{\tan\beta}\right)$ has the value, equals to :

(a) -1 (b) -2

(c) 2 (d) $\sqrt{2}$

44. If $x + 4y = 14$ is a normal to the curve $y^2 = ax^3 - \beta$ at $(2, 3)$, then value of $\alpha + \beta$ is:

(a) 9 (b) -5

(c) 7 (d) -7

45. The tangent to the curve $y = e^{kx}$ at a point $(0, 1)$ meets the x-axis at $(a, 0)$ where $a \in [-2, -1]$ then $k \in$:

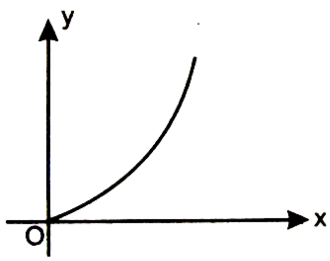
(a) $\left[-\frac{1}{2}, 0\right]$ (b) $\left[-1, -\frac{1}{2}\right]$

(c) $[\,0, 1]$ (d) $\left[\frac{1}{2}, 1\right]$

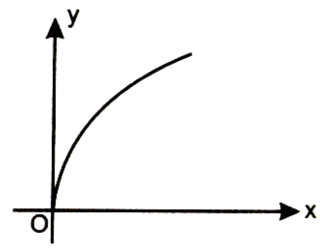
46. Which of the following graph represent the function

$f(x) = \int_0^{\sqrt{x}} e^{-\frac{u^2}{x}}\, du$, for $x > 0$ and $f(0) = 0$?

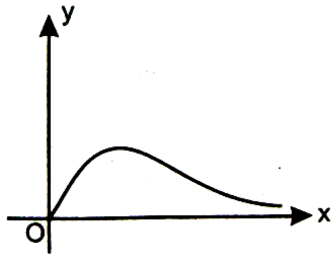
(a)



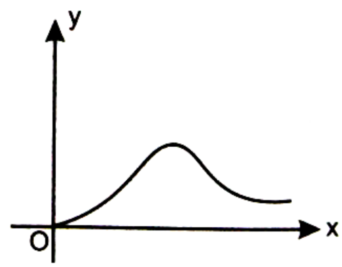
(b)



(c)



(d)



47. Let $f(x) = (x-a)(x-b)(x-c)$ be a real valued function where $a < b < c (a, b, c \in R)$ such that $f''(a) = 0$. Then if $a \in (c_1, c_2)$, which one of the following is correct?

(a) $a < c_1 < b$ and $b < c_2 < c$ (b) $a < c_1, c_2 < b$

(c) $b < c_1, c_2 < c$ (d) None of these

48. $f(x) = x^6 - x - 1, x \in [1, 2]$. Consider the following statements :

(a) f is increasing on $[1, 2]$ (b) f has a root in $[1, 2]$

(c) f is decreasing on $[1, 2]$ (b) f has no root in $[1, 2]$

49. Which one of the following curves is the orthogonal trajectory of straight lines passing through a fixed point (a, b)?

(a) $x - a = k(y - b)$ (b) $(x-a)(y-b) = k$

(c) $(x-a)^2 = k(y-b)$ (d) $(x-a)^2 + (y-b)^2 = k$

50. The function $f(x) = \sin^3 x - m\sin x$ is defined on open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and if assumes only 1 maximum value and only 1 minimum value on this interval. Then, which one of the following must be correct?

(a) $0 < m < 3$ (b) $-3 < m < 0$

(c) $m > 3$ (d) $m < -3$

51. The greatest of the numbers $1, 2^{1/2}, 3^{1/4}, 4^{1/4}, 5^{1/5}, 6^{1/6}$ and $7^{1/7}$ is :

(a) $2^{1/2}$ (b) $3^{1/3}$

(c) $7^{1/7}$ (d) $6^{1/6}$

52. Let l be the line through (0, 0) and tangent to the curve $y = x^3 + x + 16$. Then the slope of l equal to :

(a) 10 (b) 11

(c) 17 (d) 13

53. The slope of the tangent at the point of inflection of $y = x^3 - 3x^2 + 6x + 2009$ is equal to :

(a) 2 (b) 3

(c) 1 (d) 4

54. Let f be a real valued function with $(n + 1)$ derivatives at each point of R. For each pair of real numbers $a, b, a < b$, such that

$$\text{In}\left[\frac{f(b)+f'(b)+\cdots+f^{(n)}(b)}{f(a)+f'(a)+\cdots+f^{(n)}(a)}\right] = b - a$$

Statement-1 : There is a number $c \in (a, b)$ for which $f^{(n)}(c) = f(c)$

because

Statement-2 : If $h(x)$ be a derivable function such that $h(p) = h(q)$ then by Rolle's theorem $h'(d) = 0; d \in (p, q)$

(a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(b) Statement-1 is true, statement-2 is true and statement-2 is not correct explanation for statement-1

(c) Statement-1 is true, statement-2 is false

(d) Statement-1 is false, statement-2 is true

55. If $g(x)$ is twice differentiable real valued function satisfying $g''(x) - 3g'(x) > 3\ \forall\ x \geq 0$ and $g'(0) = -1$, then $h(x) = g(x) + x\ \forall x > 0$ is:

(a) Strictly increasing
(b) Strictly decreasing
(c) Non monotonic
(d) Data insufficient

56. If the straight line joining the points (0,3) and (5,-2) is tangent to the curve

$y = \frac{c}{x+1}$; then the value of c is:

(a) 2
(b) 3
(c) 4
(d) 5

57. Number of solutions(s) of $\text{In}\,|\sin x| = -x^2$ if $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is/are :

(a) 2
(b) 4
(c) 6
(d) 8

58. The equation $\sin^{-1}x = |x - a|$ will have atleast one solution then complete set of values of a be:

(a) $[-1, 1]$
(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(c) $\left[1 - \frac{\pi}{2}, 1 + \frac{\pi}{2}\right]$
(d) $\left[\frac{\pi}{2} - 1, \frac{\pi}{2} + 1\right]$

59. For any real number b, let f(b) denotes the maximum of $\left|\sin x + \frac{2}{3+\sin x} + b\right|\forall\ x \in R$. Then the minimum value of $f(b) \forall b \in R$ is :

(a) $\frac{1}{2}$
(b) $\frac{3}{4}$
(c) $\frac{1}{4}$
(d) 1

60. Which of the following are correct

(a) $x^4 + 2x^2 - 6x + 2 = 0$ has exactly four real solution

(b) $x^5 + 5x + 1 = 0$ has exactly three real solution

(c) $x^n + ax + b = 0$ where n is an even natural number has atmost two real solution $a, b, \in R$.

(d) $x^3 - 3x + c = 0, c > 0$ has two real solution for $x \in (0,1)$

61. For any real number b, let f(b) denotes the maximum of $\left|\sin x + \frac{2}{3+\sin x} + b\right| \forall x \in R$. Then the minimum value of $f(b) \forall b \in$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{1}{4}$ (d) 1

62. If p be a point on the graph of $y = \frac{x}{1+x^2}$, then coordinates of 'p' such that tangent drawn to curve at p has the greatest slope in magnitude is :

(a) $(0, 0)$ (b) $\left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

(c) $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$ (d) $\left(1, \frac{1}{2}\right)$

63. Let $f: [0, 2\pi] \to [-3, 3]$ be a given function defined as $f(x) = 3\cos\frac{x}{2}$. The slope of the tangent to the curve $y = f^{-1}(x)$ at the point where the curve crosses the y- axis is :

(a) -1 (b) $-\frac{2}{3}$

(c) $-\frac{1}{6}$ (d) $-\frac{1}{3}$

64. Number of stationary points in $[0, \pi]$ for the function $f(x) = \sin x + + \tan x - 2x$ is :

(a) 0 (b) 1

(c) 2 (d) 3

65. If $a, b, c, d \in R$ such that $\frac{a+2c}{b+3d}+\frac{4}{3}=0$, then the equation $ax^3+bx^2+cx+d=0$ has

(a) atleast one root in $(-1,0)$

(b) atleast one root in $(0,1)$

(c) no root in $(-1,1)$

(d) no root in $(0,2)$

66. If $f'(x)=\emptyset(x)(x-2)^2$. Where $\emptyset(2)\neq 0$ and $\emptyset(x)$ is continuous at $x=2$, then in the neighborhood of $x=2$

(a) f is increasing if $\emptyset(2)<0$

(b)f is decreasing if $\emptyset(2)>0$

(c) f is neither increasing nor decreasing

(d) f is increasing if $\emptyset(2)>0$

67. If $f(x)=x^3-6x^2+ax+b$ is defined on $[1,3]$ satisfies Rolle's theorem for $c=\frac{2\sqrt{3}+1}{\sqrt{3}}$ then

(a) $a=-11, b=6$

(b) $a=-11, b=-6$

(c) $a=11, b \in R$

(d) $a=22, b=-6$

68. For which of the following function(s) Lagrange's mean value theorem is not applicable in $[1,2]$?

(a) $f(x)=\begin{cases} \frac{3}{2}-x & , \quad x<\frac{3}{2} \\ \left(\frac{3}{2}-x\right)^2 & , \quad x\geq\frac{3}{2} \end{cases}$

(b) $f(x)=\begin{cases} \frac{\sin(x-1)}{x-1} & , \quad x\neq 1 \\ 1 & , \quad x=1 \end{cases}$

(c) $f(x)=(x-1)|x-1|$

(d) $f(x)=|x-1|$

69. If the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^2=16x$ intersect at right angles, then :

(a) $a=\pm 1$

(b) $a=\pm\sqrt{3}$

(c)$a=\pm\frac{1}{\sqrt{3}}$

(d) $a=\pm\sqrt{2}$

70. If the line $x\cos\alpha + y\sin\alpha = P$ touches the curve $4x^3 = 27ay^2$, then $\frac{P}{a} =$:

(a) $\cot^2\alpha\cos\alpha$
(b) $\cot^2\alpha\sin\alpha$
(c) $\tan^2\alpha\cos\alpha$
(d) $\tan^2\alpha\sin\alpha$

Answer

1.	(d)	2.	(b)	3.	(a)	4.	(c)	5.	(b)	6.	(c)	7.	(d)	8.	(d)	9.	(c)	10.	(b)
11.	(c)	12.	(c)	13.	(c)	14.	(c)	15.	(c)	16.	(a)	17.	(b)	18.	(a)	19.	(b)	20.	(b)
21.	(b)	22.	(d)	23.	(d)	24.	(a)	25.	(d)	26.	(c)	27.	(b)	28.	(d)	29.	(c)	30.	(d)
31.	(b)	32.	(c)	33.	(d)	34.	(d)	35.	(a)	36.	(b)	37.	(c)	38.	(a)	39.	(c)	40.	(a)
41.	(a)	42.	(b)	43.	(b)	44.	(a)	45.	(d)	46.	(b)	47.	(a)	48.	(a)	49.	(d)	50.	(a)
51.	(b)	52.	(d)	53.	(b)	54.	(a)	55.	(a)	56.	(c)	57.	(b)	58.	(c)	59.	(b)	60.	(c)
61.	(b)	62.	(a)	63.	(b)	64.	(c)	65.	(b)	66.	(d)	67.	(c)	68.	(a)	69.	(d)	70.	(a)

Excerise-2: One or More than One Answer is/are Correct

1. Common tangent (s) to $y = x^3$ and $x = y^3$ is/are :

(a) $x - y = \frac{1}{\sqrt{3}}$ (b) $x - y = -\frac{1}{\sqrt{3}}$

(c) $x - y = \frac{2}{3\sqrt{3}}$ (d) $x - y = \frac{-2}{3\sqrt{3}}$

2. Let $f: [0, 8] \to R$ be differentiable function such that $f(0) = 0, f(4) = 1, f(8) = 1$, then which of the following hold(s) good ?

(a) There exist some $c_1, c_2 \in (0,8)$ where $f'(c_1) = \frac{1}{4}$

(b) There exist some $c \in (0, 8)$ where $f'(c) = \frac{1}{12}$

(c) There exist $c_1, c_2 \in [0, 8]$ where $8f'(c_1)f(c_2) = 1$

(d) There exist some $\alpha, \beta \in (0, 2)$ such that $\int_0^8 f(t)dt = 3\left(\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3)\right)$

3. If $f(x) = \begin{cases} \sin^{-1}(\sin x) & x > 0 \\ \frac{\pi}{2} & x = 0 \\ \cos^{-1}(\cos x) & x < 0 \end{cases}$, then

(a) $x = 0$ is a point of maxima

(b) $f(x)$ is continuous $\forall\ x \in R$

(c) global maximum value of $f(x) \forall\ x \in R$ is π

(d) global minimum value of $f(x) \forall\ x \in R$ is 0

4. A function $f: R \to R$ is given by $f(x) = \begin{cases} x^4\left(2 + \sin\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$, then

(a) f has a continuous derivative $\forall\ x \in R$

(b) f is a bounded function

(c) f has an global minimum at $x = 0$

(d) f'' is continuous $\forall\ x \in R$

5. If $|f''(x)| \leq 1\ \forall\ x \in R$, and $f(0) = 0 = f'(0)$, then which of the following can not be true ?

(a) $f\left(-\frac{1}{2}\right) = \frac{1}{6}$ (b) $f(2) = -4$

(c) $f(-2) = 3$ (d) $f\left(\frac{1}{2}\right) = \frac{1}{5}$

6. Let f: $[-3, 4] \rightarrow R$ such that $f''(x) > 0$ for all $x \epsilon[-3, 4]$, then which of the following are always true ?
(a) $f(x)$ has a relative minimum on $(-3, 4)$
(b) $f(x)$ has a minimum on $[-3, 4]$
(c) $f(x)$ has a maximum on $[-3, 4]$
(d) if $f(3) = f(4)$, then $f(x)$ has a critical point on $[-3, 4]$

7. Let $f(x)$ be twice differentiable function such that $f''(x) > 0$ in $[0, 2]$. Then :
(a) $f(0) + f(2) = 2f(c)$, for atleast one $c, c \epsilon (0, 2)$
(b) $f(0) + f(2) < 2f(1)$
(c) $f(0) + f(2) > 2f(1)$
(d) $2f(0) + f(2) > 3f\left(\frac{2}{3}\right)$

8. Let $g(x)$ be cubic polynomial having local maximum at $x = -1$ and $g'(x)$ has a local minimum at $x = 1$. If $g(-1) = 10, g(3) = -32$, then :
(a) perpendicular distance between it two horizontal tangents is 12
(b) perpendicular distance between its two horizontal tangents is 32
(c) $g(x) = 0$ has atleast one real root lying in interval $(-1, 0)$
(d) $g(x) = 0$ has 3 distinct real roots

9. The function $f(x) = 2x^3 - 3(\lambda + 2)x^2 + 2\lambda x + 5$ has a maximum and a minimum for :
(a) $\lambda \epsilon (-4, \infty)$
(b) $\lambda \epsilon (-\infty, 0)$
(c) $\lambda \epsilon (-3, 3)$
(d) $\lambda \epsilon (1, \infty)$

10. The function $f(x) = 1 + x\, In\left(x + \sqrt{1 + x^2}\right) - \sqrt{1 - x^2}$ is :
(a) strictly increasing $\forall\, x \epsilon (0, 1)$
(b) strictly decreasing $\forall\, x \epsilon (-1, 0)$
(c) strictly decreasing for $x \epsilon (-1, 0)$
(d) strictly decreasing for $x \epsilon (0, 1)$

11. Let m and n be positive integers and $x, y > 0$ and $x + y = k$, where k is constant. Let $f(x, y) = x^m y^n$, then :

(a) $f(x, y)$ is maximum when $x = \frac{mk}{m+n}$

(b) $f(x, y)$ is maximum where $x = y$

(c) maximum value of $f(x, y)$ is $\frac{m^n n^m k^{m+n}}{(m+n)^{m+n}}$

(d) maximum value of $f(x, y)$ is $\frac{k^{m+n} m^m n^n}{(m+n)^{m+n}}$

12. The straight line which is both tangent and normal to the curve $x = 3t^2, y = 2t^3$ is :

(a) $y + \sqrt{3}(x - 1) = 0$ (b) $y - \sqrt{3}(x - 1) = 0$

(c) $y + \sqrt{2}(x - 2) = 0$ (d) $y - \sqrt{3}(x - 2) = 0$

13. A curve is such that the ratio of the subnormal at any point to the sum of its co-ordinates is equal to the ratio of the ordinate of this point to its abscissa. If the curve passes through (1, 0), then possible equation of the curve(s) is :

(a) $y = x\, In\, x$ (b) $y = \frac{In\, x}{x}$

(c) $y = \frac{2(x-1)}{x^2}$ (b) $y = \frac{1-x^2}{2x}$

14. A parabola of the form $y = ax^2 + bx + c(a > 0)$ intersects the graph of $(x) = \frac{1}{x^2-4}$. The number of possible distinct intersection(s) of these graph can be :

(a) 0 (b) 2

(c) 3 (d) 4

15. Gradient of the line passing through the point (2, 8) and touching the curve $y = x^3$, can be "

(a) 3 (b) 6

(c) 9 (d) 12

16. The equation $x + cos\, x = a$ has exactly one positive root, then :

(a) $a \in (0, 1)$
(b) $a \in (2, 3)$
(c) $a \in (1, \infty)$
(d) $a \in (-\infty, 1)$

17. Given that $f(x)$ is a non-constant linear function. Then the curves :

(a) $y = f(x)$ and $y = f^{-1}(x)$ are orthogonal
(b) $y = f(x)$ and $y = f^{-1}(-x)$ are orthogonal
(c) $y = f(-x)$ and $y = f^{-1}(x)$ are orthogonal
(d) $y = f(-x)$ and $y = f^{-1}(-x)$ are orthogonal

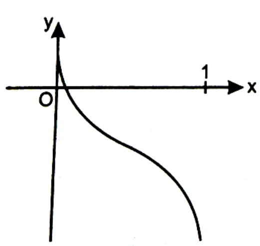
18. Let $f(x) = \int_0^x e^{t^3}(t^2 - 1)\, t^2(t+1)^{2011}(t-2)^{2012}$ at $(x > 0)$ then :

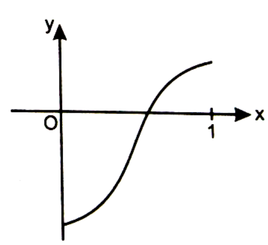
(a) The number of point of inflections is atleast 1
(b) The number of point of inflections is 0
(c) The number of point of local maxima is 1
(d) The number of point of local minima is 1

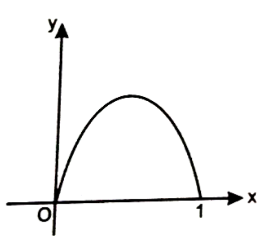
19. Let $f(x) = sin\, x + ax + b$. Then $f(x) = 0$ has :

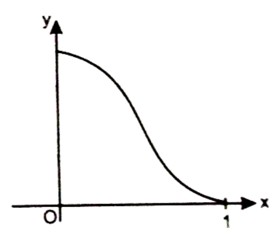
(a) only one real root which is positive if $a > 1, b < 0$
(b) only one real root which is negative if $a > 1, b > 0$
(c) only one real root which is negative if $a < -1, b < 0$
(d) only one real root which is positive if $a < -1, b < 0$

20. Which of the following graphs represents function whose derivatives have a maximum in the interval $(0, 1)$?

(a)


(b)


(c)


(d)


21. Consider $f(x) = sin^5 x + cos^5 x - 1, x \epsilon \left[0, \frac{\pi}{2}\right]$, which of the following is/are correct ?

(a) f is strictly decreasing in $\left[0, \frac{\pi}{4}\right]$

(b) f is strictly increasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(c) There exist a number 'c' in $\left[0, \frac{\pi}{2}\right]$ such that $f'(c) = 0$

(d) The equation $f(x) = 0$ has only two roots in $\left[0, \frac{\pi}{2}\right]$

22. Let $f(x) = \begin{bmatrix} x^{2\alpha+1} In\, x & ; & x > 0 \\ 0 & ; & x = 0 \end{bmatrix}$

If $f(x)$ satisfies rolle's theorem in interval $[0, 1]$, then α can be :

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$

(c) $-\frac{1}{4}$ (d) -1

23. Which of the following is/are true for the function $(x) = \int_0^x \frac{\cos t}{t} dt \ (x > 0)$?

(a) $f(x)$ is monotonically increasing in $\left((4n-1)\frac{\pi}{2}\right) \forall\, n \epsilon N$

(b) $f(x)$ has a local minima at $x = (4n-1)\frac{\pi}{2} \forall\, n \epsilon N$

(c) The points of inflection of the curve $y = f(x)$ in $(0, 10\pi)$ are 19

(d) Number of critical points of $y = f(x)$ in $(0, 10\pi)$ are 19

24. Let $F(x) = \left(f'(x)\right)^2, F(0) = 6$, where $f(x)$ is a thrice differentiable function such that $|f(x)| \leq 1 \ \forall\, x \epsilon [-1, 1]$, then choose the correct statement(s)

(a) there is atleast one point in each of the intervals $(-1, 0)$ and $(0, 1)$ where $|f'(x)| \leq 2$

(b) there is atleast one point in each of the intervals $(-1, 0)$ and $(0, 1)$ where $F(x) \leq 5$

(c) there is no point of local maxima of $F(x)$ in $(-1, 1)$

(d) for some $c \epsilon (-1, 1), F(c) \geq 6, F'(c) = 0$ and F"(c)≤ 0

25. Let $f(x) = \begin{cases} x^3 + x^2 - 10x; & 0 \le \mathrm{x} < 0 \\ \sin \mathrm{x}; & 0 \le \mathrm{x} \le \frac{\pi}{2} \\ 1 + \cos \mathrm{x}; & \frac{\pi}{2} \le \mathrm{x} \le \pi \end{cases}$

then f(x) has :

(a) local maximum at $\mathrm{x} = \frac{\pi}{2}$

(b) local minimum at $\mathrm{x} = \frac{\pi}{2}$

(c) absolute maximum at $\mathrm{x} = 0$

(d) absolute maximum at $\mathrm{x} = -1$

26. Minimum distance between the curves $\mathrm{y}^2 = \mathrm{x} - 1$ and $\mathrm{x}^2 = \mathrm{y} - 1$ is equal to :

(a) $\frac{\sqrt{2}}{4}$

(b) $\frac{3\sqrt{2}}{4}$

(c) $\frac{5\sqrt{2}}{4}$

(d) $\frac{7\sqrt{2}}{4}$

27. For the equation $\frac{\mathrm{e}^{-\mathrm{x}}}{1+\mathrm{x}} = \lambda$ which of the following statement(s) is/are correct ?

(a) When $\lambda \in (0, \infty)$ equation has 2 real and distinct roots

(b) When $\lambda \in (-\infty, -\mathrm{e}^2)$ equation has 2 real and distinct roots

(c) When $\lambda \in (0, \infty)$ equation has 1 real root

(d) When $\lambda \in (-\mathrm{e}, 0)$ equation has no real root

28. If $\mathrm{y} = \mathrm{mx} + 5$ is a tangent to the curve $\mathrm{x}^3\mathrm{y}^3 = \mathrm{ax}^3 + \mathrm{by}^3$ at $\mathrm{P}(1, 2)$, then

(a) $\mathrm{a} + \mathrm{b} = \frac{18}{5}$

(b) $\mathrm{a} > \mathrm{b}$

(c) $\mathrm{a} < \mathrm{b}$

(d) $\mathrm{a} + \mathrm{b} = \frac{19}{5}$

29. If $(\mathrm{f(x)} - 1(\mathrm{x}^2 + \mathrm{x} + 1)^2 - (\mathrm{f(x)} + 1)(\mathrm{x}^4 + \mathrm{x}^2 + 1) = 0$
$\forall\ \mathrm{x} \in \mathrm{R} - \{0\}$ and $\mathrm{f(x)} \neq \pm 1$, then which of the following statement(s) is/are correct ?

(a) $|\mathrm{f(x)}| \ge 2 \forall\ \mathrm{x} \in \mathrm{R} - \{0\}$

(b) f(x) has a local maximum at $\mathrm{x} = -1$

(c) f(x) has a local minimum at $\mathrm{x} = 1$

(d) $\int_{-\pi}^{\pi} (\cos \mathrm{x})\mathrm{f(x)}\mathrm{dx} = 0$

Answers

1.	(c, d)	2.	(a, c, d)	3.	(a, c)	4.	(a, c)	5.	(a, b, c, d)	6.	(b, c, d)
7.	(c, d)	8.	(b, d)	9.	(a, b, c, d)	10.	(a, c)	11.	(a, d)	12.	(c, d)
13.	(a, d)	14.	(b, c, d)	15.	(a, d)	16.	(b, c)	17.	(b, c)	18.	(a, d)
19.	(a, b, c)	20.	(a, b)	21.	(a, b, c, d)	22.	(b, c)	23.	(a, b, c)	24.	(a, b, d)
25.	(a, d)	26.	(b)	27.	(b, c, d)	28.	(a, d)	29.	(a, b, c, d)		