(Paper - 02)

SOLUTIONS TO JEE(ADVANCED) – 2021

PHYSICS

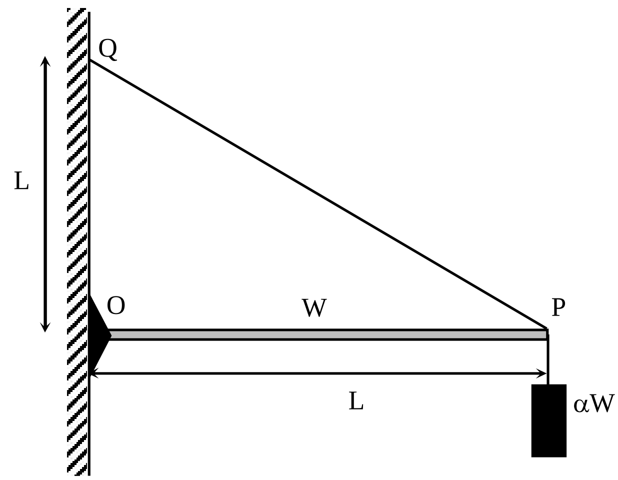
SECTION 1

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

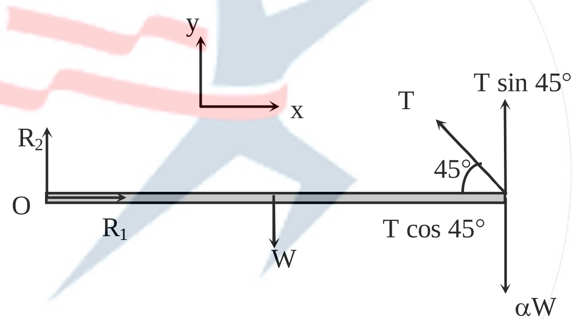
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.

*Q.1 One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $\left(2\sqrt{2}\right)W$. Which of the following statement(s) is(are) correct?



(A) The vertical component of reaction force at O does **not** depend on α
(B) The horizontal component of reaction force at O is equal to W for α = 0.5
(C) The tension in the rope is 2W for α = 0.5
(D) The rope breaks if α > 1.5

Sol. **A, B, D**
Free body diagram of rod



$\Sigma F_x = 0$
$R_1 = T \cos 45°$
$$R_1 = \frac{T}{\sqrt{2}} \qquad \ldots(i)$$
$\Sigma F_y = 0$
$R_2 + T \sin 45° = W + \alpha W$
$$R_2 + \frac{T}{\sqrt{2}} = W(1+\alpha) \qquad \ldots(ii)$$
$\Sigma \tau_0 = 0$
$$W\frac{L}{2} + \alpha WL = \frac{T}{\sqrt{2}}L$$
$$T = \sqrt{2}W\left[\alpha + \frac{1}{2}\right] \qquad \ldots(iii)$$
From (ii) and (iii)
$$R_2 + W\left[\alpha + \frac{1}{2}\right] = W(1+\alpha)$$
$$R_2 = \frac{W}{2}$$
Hence, option (A) is correct.

From (i) and (iii)

$$R_1 = W\left[\alpha + \frac{1}{2}\right]$$

$\alpha = 0.5$, $R_1 = W$

Hence, option (B) is correct.

From equation (iii) if $\alpha = 0.5$

$$T = \sqrt{2}W$$

$$T_{max} = 2\sqrt{2}\ W$$

For rope to break

$$T > 2\sqrt{2}W$$

$$\sqrt{2}W\left[\alpha + \frac{1}{2}\right] > 2\sqrt{2}W$$

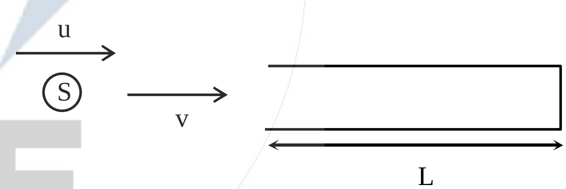
$$\alpha > \frac{3}{2}$$

Hence, option (D) is correct.

*Q.2 A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_S, will the sound reaching the pipe lead to a resonance?

(A) $u = 0.8v$ and $f_s = f_0$
(B) $u = 0.8v$ and $f_s = 2f_0$
(C) $u = 0.8v$ and $f_s = 0.5f_0$
(D) $u = 0.5v$ and $f_s = 1.5f_0$

Sol. **A, D**



Natural frequency of closed pipe

$f = (2n + 1)f_0$

f_0 is fundamental frequency

$n = 0, 1, 2, \ldots\ldots\ldots\ldots$

frequency of source received by pipe

$$f' = f_s\left[\frac{v - 0}{v - u}\right]$$

For resonance

$f' = f$

$$f_s\left[\frac{v}{v - u}\right] = (2n + 1)f_0$$

If $u = 0.8$ v $\qquad f_s = f_0$

$$f' = \frac{v}{0.2\,v}f_0 = 5f_0$$

for n = 2 pipe can be in resonance

Hence, option (A) is correct.

If $u = 0.8v$ $\qquad f_s = 2f_0$

$$f' = \frac{v}{0.2v} \times 2f_0 = 10f_0$$

If $u = 0.8$ v, $f_S = 0.5\ f_0$

$$f' = \frac{v}{0.2v} \times 0.5f_0 = 2.5f_0$$

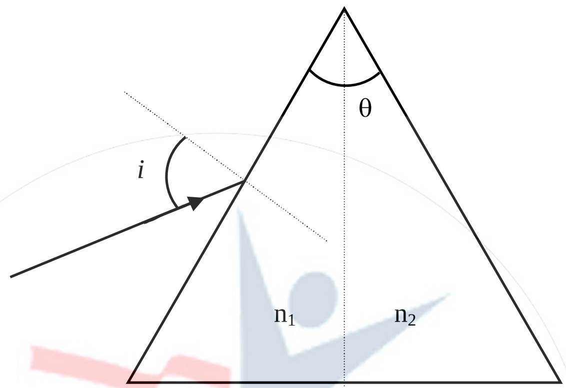
Not possible

If $u = 0.5$ v, $f_S = 1.5\ f_0$

$$f' = \frac{v}{0.5v} \times 1.5f_0 = 3f_0$$

for n = 1 f = $3f_0$
pipe can be in resonance
Hence, option (D) is correct.

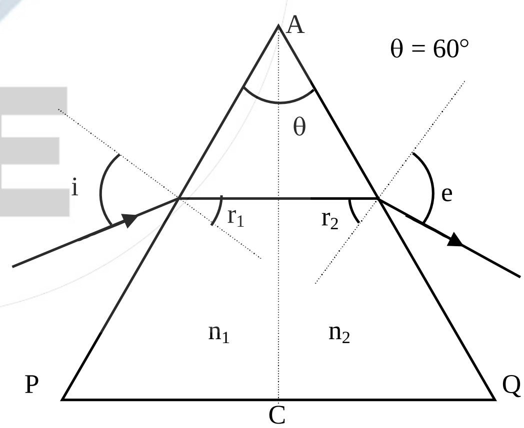
Q.3 For a prism of prism angle θ = 60°, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \ge n_1$) as shown in the figure. The angle of incidence *i* is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n << n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is(are) correct?



(A) The value of Δe (in radians) is greater than that of Δn
(B) Δe is proportional to Δn
(C) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$
(C) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

Sol. **B, C**

Diagram at minimum deviation for $n_1 = n_2 = n$



n = 1.5
$r_1 = r_2 = \theta/2 = 30°$
for face AQ
$n\sin r_2 = \sin e$

$$1.5 \sin 30° = \frac{3}{2} \times \frac{1}{2} = \sin e$$

$$\sin e = \frac{3}{4}, \qquad \cos e = \frac{\sqrt{7}}{4}$$

When n_2 is given small variation there will be no change in path of light ray inside prism. As deviation on face AC is zero.
So, $r_2 = 30°$
Now for face AQ
$n_2 \sin 30° = \sin e$
for small change in n_2 change in e is given by
$dn_2 \sin 30° = \cos e \, de$
or $dn_2 = \Delta n$ $de = \Delta e$
$\Delta n \sin 30° = \cos e \, \Delta e$

$$\Delta n \frac{1}{2} = \frac{\sqrt{7}}{4} \Delta e$$

$$\Delta n = \frac{\sqrt{7}}{2} \Delta e \qquad \ldots(i) \; \Delta n > \Delta e$$

$\Delta n \propto \Delta e$

Hence, option (B) is correct.

$\Delta e = \dfrac{2.8\times10^{-3}\times 2}{\sqrt{7}}$

Hence, option (C) is correct.

Q.4 A physical quantity $\vec{S}$ is defined as $\vec{S} = \left(\vec{E}\times\vec{B}\right)/\mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity(ies)?

(A) $\dfrac{\text{Energy}}{\text{Charge}\times\text{Current}}$

(B) $\dfrac{\text{Force}}{\text{Lenght}\times\text{Time}}$

(C) $\dfrac{\text{Energy}}{\text{Volume}}$

(D) $\dfrac{\text{Power}}{\text{Area}}$

Sol. **B, D**

$\vec{S} = \vec{E}\times\vec{B}$

Method – 1:

$\vec{S}$ is pointing vector which is defined as energy flowing per unit area in unit time.

S.I. unit of S is Watt/m^2.

Method 2:

$\because \quad B = \mu_0 ni \Rightarrow \dfrac{1}{\mu_0} = \dfrac{ni}{B}$

$S = \dfrac{1}{\mu_0}EB\cos\theta \quad = niE\cos\theta$

S.I. unit of S $= \dfrac{1}{\text{meter}}\times\dfrac{q}{\text{time}}\times\dfrac{\text{Force}}{q} = \dfrac{\text{Force}}{\text{meter}-\text{time}}$

Q.5 A heavy nucleus N, at rest, undergoes fission $N \rightarrow P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P and E_Q are the kinetic energies of P and Q, respectively. The speeds of P and Q are v_P and v_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct ?

(A) $E_P + E_Q = c^2\delta$

(B) $E_P = \left(\dfrac{M_P}{M_P + M_Q}\right)c^2\delta$

(C) $\dfrac{v_P}{v_Q} = \dfrac{M_Q}{M_P}$

(D) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu = \dfrac{M_P M_Q}{(M_P + M_Q)}$

Sol. **A, C, D**

Energy released during process

$Q = \delta c^2 \qquad \ldots(1)$

$\because$ momentum is conserved in process.

$\Rightarrow \; 0 = m_p v_p - m_Q . v_Q$

$$\Rightarrow \quad \frac{v_P}{v_Q} = \frac{m_Q}{m_P} \qquad \text{… (2)}$$

$$E_P = \frac{1}{2} m_P v_P^2$$

$$E_Q = \frac{1}{2} m_Q v_Q^2$$

$$\Rightarrow \quad \frac{E_P}{E_Q} = \frac{m_Q}{m_P} \qquad \text{…(3)}$$

Solving (1) and (3),

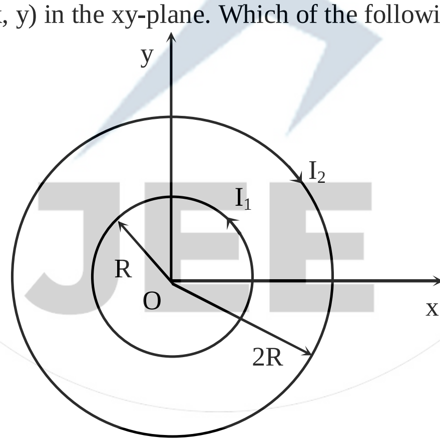
$$E_P = \frac{m_Q}{m_P + m_Q} \delta c^2$$

$$E_Q = \frac{m_P}{m_P + m_Q} \delta c^2$$

$$\text{Momentum } P = \sqrt{2m_P E_P} = \sqrt{2m_Q E_Q}$$

$$= \sqrt{2m_P \frac{m_Q}{m_P + m_Q} .\delta c^2} = c\sqrt{\frac{2m_P m_Q}{m_P + m_Q} .\delta}$$

Q.6 Two concentric circular loops, one of radius R and the other of radius 2R, lie in the xy-plane with the origin as their common centre, as shown in the figure. The smaller loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 in the clock wise direction, with $I_2 > 2I_1$. $\vec{B}(x, y)$ denotes the magnetic field at a point (x, y) in the xy-plane. Which of the following statement(s) is(are) correct?



(A) $\vec{B}(x, y)$ is perpendicular to the xy-plane at any point in the plane

(B) $|\vec{B}(x, y)|$ depends on x and y only through the radial distance $r = \sqrt{x^2 + y^2}$

(C) $|\vec{B}(x, y)|$ is non-zero at all points for $r < R$

(D) $\vec{B}(x, y)$ points normally outward from the xy-plane for all the points between the two loops

Sol. **A, B**

Consider a circular loop of radius r in x-y plane and having centre at origin

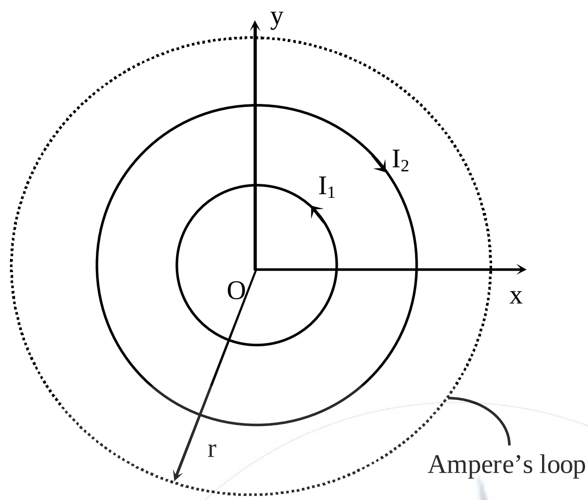
$$\oint \vec{B}.\vec{d\ell} = 0$$

$$B\oint d\ell \cos\theta = 0$$

$\because \quad B \neq 0$ for given r

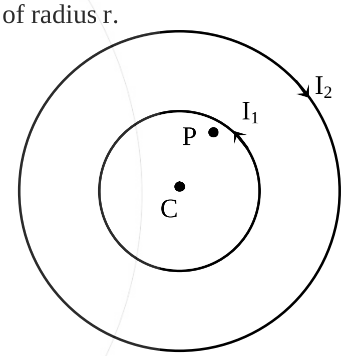
$\Rightarrow \cos\theta = 0$

$\theta = 90^0$



Here $d\ell$ is in xy plane $\Rightarrow$ B is normal to plane (B can't be in xy plane as its magnetic lines would have been in radial direction)

Also, for given r, B must be same in magnitude for all points on loop of radius r.



At centre $B = \left(\frac{\mu_0 i_1}{2R} - \frac{\mu_0 i_2}{4R}\right)$ (inwards)

For point P, Let field of inner loop increases x_1 times and that of outer loop increases x_2 times

$\Rightarrow$ magnetic field at P

$B_P = \left(x_1 \frac{\mu_0 i_1}{2R} - x_2 \frac{\mu_0 i_2}{4R}\right)$

For $B_P = 0$, $i_2 = \left(\frac{x_1}{x_2}\right).(2i_1)$

$\because$ B changes more rapidly as point P come closer to circumference.

$\Rightarrow x_1 > x_2$

Or $i_2 > 2i_1$ (which is given condition)

So, there are points inside inner loop where magnetic field will be zero.

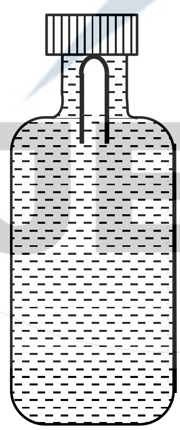
SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Mark : +2 If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

A Soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $v_0 = 3.3$ cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $p_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$.
Let Δv = X cc and $\Delta p = Y \times 10^3$ Pa.



*Q.7 The value of X is _______.

Sol. **0.30**

For equilibrium of the test tube

$$mg = (v_{tube} + v_{air})\rho_w g$$

$$\text{So,} \quad 5 = \left(\frac{5}{2.5} + v_{air}\right)(1)$$

So, $v_{air} = 3$cc

So, $\Delta v = 0.3$ cc

*Q.8 The value of Y is _______.

Sol. **10.00**

For isothermal process

$(10^5)(3.3) = (P)(3)$

So, $P = 1.1 \times 10^5$ Pa

So, $\Delta P = (1.1 - 1) \times 10^5 = 10 \times 10^3$ Pa

Question Stem for Question Nos. 9 and 10

Question Stem

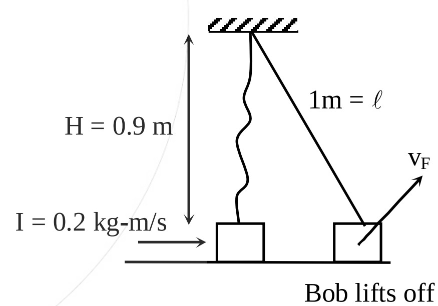
A pendulum consists of a bob of mass m = 0.1 kg and a massless inextensible string of length L = 1.0 m. It is suspended from a fixed point at height H = 0.9 m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse P = 0.2 kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg-m^2/s. The kinetic energy of the pendulum just after the lift-off is K Joules.

*Q.9 The value of J is ______.

Sol. **0.18**

$J_i = MV_iH\sin 90°$

$= 0.2 \times 0.9 = 0.18$ kg-m^2/s



*Q.10 The value of K is ______.

Sol. **0.16**

As bob lifts off due to impulse of string angular momentum will be conserved about suspension point

$\Rightarrow J_f = J_i$

$\Rightarrow Mv_f\ell \sin 90° = 0.18$

$\Rightarrow v_f = 1.8$ m/s

Kinetic energy after lifting off

$$\Rightarrow K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\times 0.1 \times (1.8)^2 = 0.162\text{ J}$$

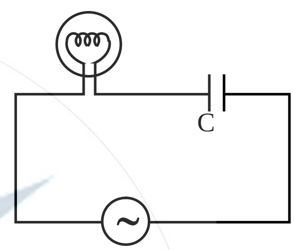
Question Stem for Question Nos. 11 and 12

Question Stem

In the circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take *rms* values of the voltages. The magnitude of the phase-angle (in degrees) between the current and supply voltage is ϕ. Assume, $\pi\sqrt{3} \approx 5$.

Q.11 The value of C is _____.

Sol. **100.00**



$P_{lamp} = 500$ W

$\Rightarrow V_{lamp} i_{lamp} = 500$

$\Rightarrow (100) i_{lamp} = 500$

$i_{lamp} = 5A$

Impedance of circuit $Z = \frac{V_{source}}{i_{source}} = \frac{200}{5} = 40\ \Omega$

Resistance of lamp $R = \frac{V_{lamp}}{i_{source}} = 20\Omega$

Reactance of capacitance

$X_C = \sqrt{Z^2 - R^2} = 20\sqrt{3}\Omega$

$\frac{1}{\omega C} = 20\sqrt{3}$

$C = \frac{1}{20\sqrt{3} \times 2 \times \pi \times 50} = 100\mu F$

Q.12 The value of ϕ is _____.

Sol. **60.00**

$\cos\phi = \frac{R}{Z} = \frac{20}{40} = \frac{1}{2}$

$\Rightarrow \phi = 60°$

SECTION 3

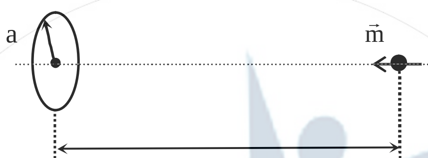
- This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+3	If ONLY the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	−1	In all other cases.

Paragraph

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its centre at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r(>> a) from the centre of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of magnetic field of a dipole m, at a point on its axis at distance r, is $\frac{\mu_0}{2\pi}\frac{m}{r^3}$, where μ_0 is the permeability of free space. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the common axis, with their north poles facing each other, is $\frac{km_1m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



Q.13 When the dipole m is placed at a distance r from the center of the loop (as shown in the figure), the current induced in the loop will be proportional to

(A) m/r^3 (B) m^2/r^2 (C) m/r^2 (D) m^2/r

Sol. **A**

$$\frac{\mu_0}{2\pi}\frac{m}{r^3} = \frac{\mu_0 i_1}{2a}$$

$$\Rightarrow i_1 \propto \frac{m}{r^3}$$

Q.14 The work done in bringing the dipole from infinity to a distance r from the center of the loop by the given process is proportional to

(A) m/r^5 (B) m^2/r^5 (C) m^2/r^6 (D) m^2/r^7

Sol. **C**

$$dW = F.dx = \frac{-Km(i_1\pi a^2)}{r^4}(dr)$$

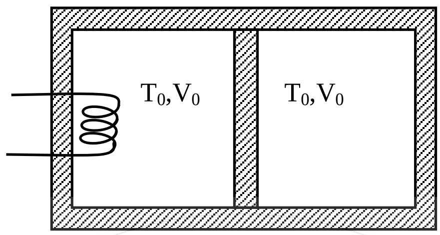
$$i_1 = \frac{ma}{\pi r^3}$$

$$W = \int_{\infty}^{r}\frac{km\left(\frac{ma}{\pi r^3}\times\pi a^2\right)dr}{r^3} = km^2a^3\int\frac{dr}{r^7}$$

$$W\alpha\frac{m^2}{r^6}$$

Paragraph

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_V = 2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition.



*Q.15 The value of $\frac{T_R}{T_0}$ is

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 3

Sol. **A**

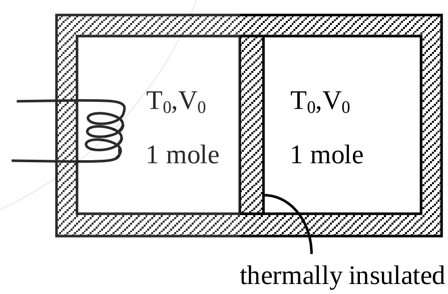
*Q.16 The value of $\frac{Q}{RT_0}$ is

(A) $4\left(2\sqrt{2}+1\right)$ (B) $4\left(2\sqrt{2}-1\right)$ (C) $\left(5\sqrt{2}+1\right)$ (D) $\left(5\sqrt{2}-1\right)$

Sol. **B**

15-16. Pressure on either side is equal

$C_V = 2R$; $C_P = 3R \Rightarrow \gamma = 3/2$



Left chamber

$Q = \Delta U_1 + \Delta W_1$

Right chamber

$0 = \Delta U_2 + \Delta W_2$

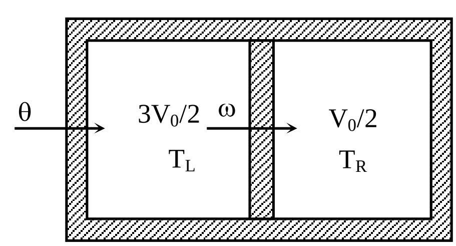
$\Delta W_1 + \Delta W_2 = 0$

$\Rightarrow Q = \Delta U_1 + \Delta U_2 = 2R(T_L - T_0) + 2R(T_R - T_0)$ …(i)

Also pressure each side of piston is equal

$\Rightarrow \frac{RT_L}{3V_0/2} = \frac{RT_R}{V_0/2}$ …(ii)

$\Rightarrow (T_L/3) = T_R$



15. For right chamber $\Rightarrow$ adiabatic compression $\Rightarrow TV^{\gamma-1}$ = constant

$$T_0 V_0^{0.5} = T_R\left(\frac{V_0}{2}\right)^{0.5} \Rightarrow T_R = \sqrt{2}T_0$$

$$\Rightarrow T_L = 3\sqrt{2}T_0$$

$$\frac{T_R}{T_0} = \sqrt{2}$$

16. $Q = 2R\left(3\sqrt{2}T_0 - T_0\right) = 2R\left(\sqrt{2}T_0 - T_0\right) = 8\sqrt{2}RT_0 - 4RT_0$

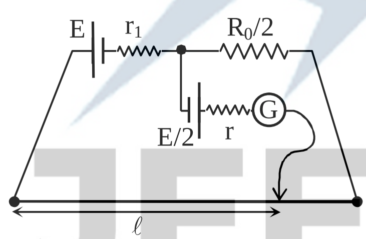
$$\Rightarrow \frac{Q}{RT_0} = 8\sqrt{2} - 4$$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designed to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	In all other cases.

Q.17 In order to measure the internal resistance r_1 of a cell of emf E, a meter bridge of wire resistance $R_0 = 50\ \Omega$, a resistance $R_0/2$, another cell of emf E/2 (internal resistance r) and a galvanometer G are used in a circuit, as shown in the figure. If the null point is found at $\ell = 72$ cm, then the value of r_1 = ___Ω.



Sol. **3**

$R_{AB} = 50\ \Omega$

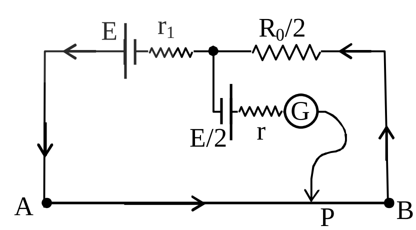
So, $R_{AP} = \frac{50}{100}\times 72 = 36\Omega$

$I = \frac{\varepsilon}{r_1 + 50 + 25}$ …(i)

$-36I - \frac{\varepsilon}{2} - Ir_1 + \varepsilon = 0$ …(ii)

Solving equation (i) and (ii)

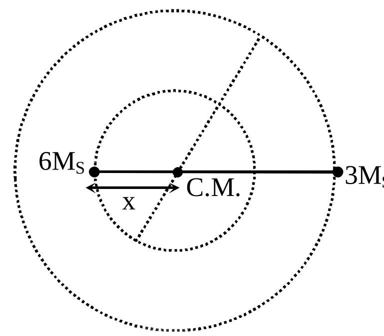
$r_1 = 3\Omega$



*Q.18 The distance between two stars of masses $3M_S$ and $6M_S$ is 9R. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common centre of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun. The value of n is ______.

Sol. **9**

Both will revolve about common center of mass



$$x = \frac{3M_s}{6M_s + 3M_S} \times 9R = 3R \qquad \ldots(i)$$

$$\frac{G6M_s \times 3M_S}{(9R)^2} = 6M_S(\omega^2 x) \qquad \ldots(ii)$$

Solving equation (i) and (ii) we get

$$\omega^2 = \frac{GM_S}{81\,R^3}$$

$$T'^2 = \frac{4\pi^2 R^3}{GM_S} \times 81 \qquad \ldots(iii)$$

For the motion of earth around Sun

$$T^2 = \frac{4\pi^2 R^3}{GM_S} \qquad \ldots(iv)$$

From (iii) and (iv)

$$T'^2 = 81T^2$$

$$T' = 9T$$

Q.19 In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P, Q and R are E_P, E_Q and E_R, respectively, and they are related by $E_P = 2E_Q = 2E_R$. In this experiment, the same source of monochromatic light is used for metal P and Q while a different source of monochromatic light is used for the metal R. The work functions for metals P, Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R, in eV, is ______.

Sol. **6**

Let $E_R = E$

Then $E_Q = E$

$E_P = 2E$

From Einstein's equation for P, Q and R

$2E = h\nu - 4.0 \qquad \ldots(i)$

$E = h\nu - 4.5 \qquad \ldots(ii)$

$E = h\nu' - 5.5 \qquad \ldots(iii)$

Solving equation (i) and (ii) we get E = 0.5

So, $h\nu' = 6.0$ eV

PART II: CHEMISTRY

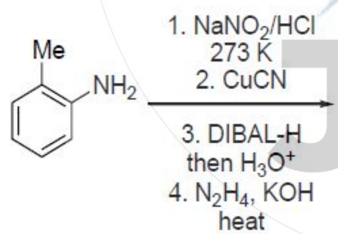
SECTION 1

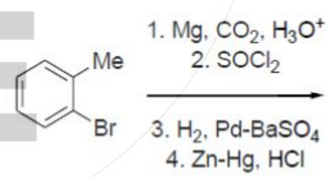
- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

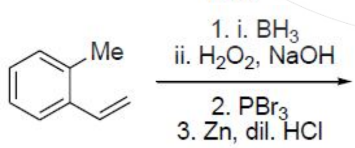
Full Mark	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.

1. The reaction sequence(s) that would lead to *o*-xylene as the major product is(are

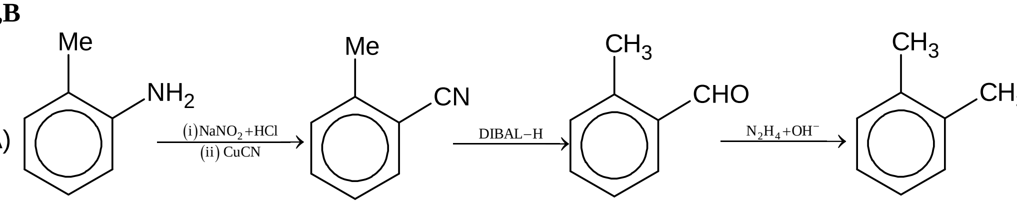
(A) 

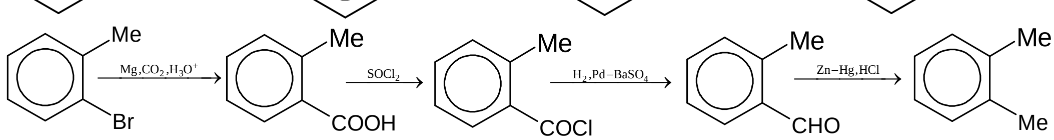
(B) 

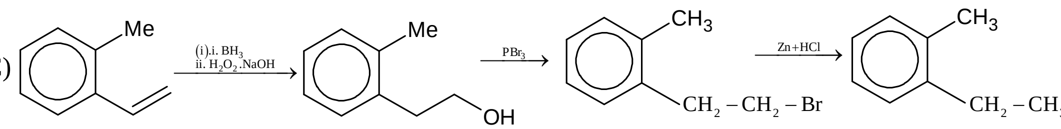
(C) 

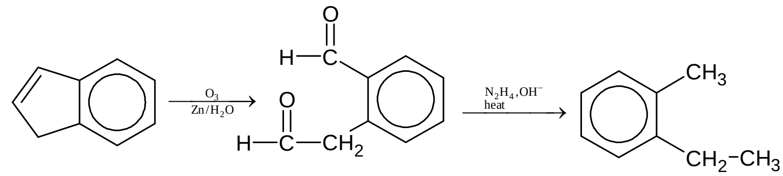
(D) 

Sol.: **A,B**

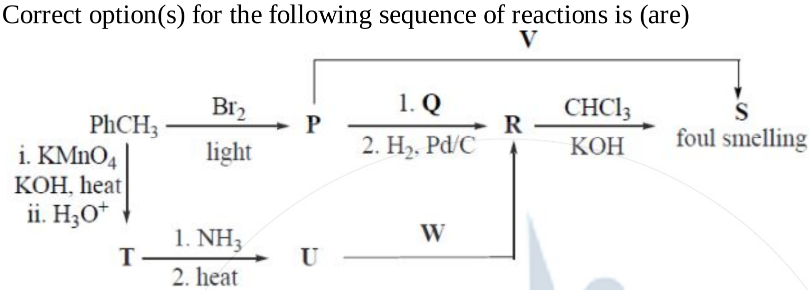
(A) 

(B) 

(C) 

(D) 

2. Correct option(s) for the following sequence of reactions is (are)



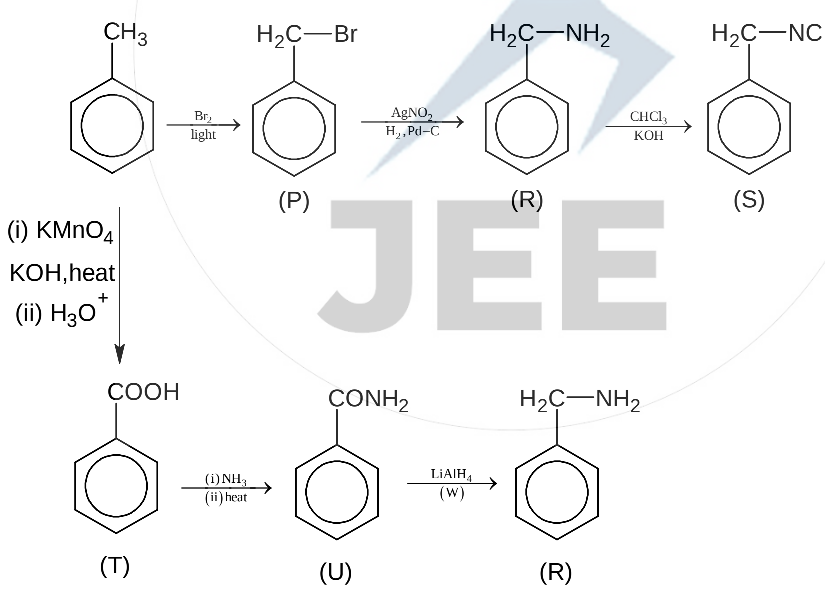
(A) $Q = KNO_2, W = LiA1H_4$

(B) R=benzenamine, V = KCN

(C) $Q = AgNO_2$, R = phenylmethanamine

(D) $W = LiAlH_4$, V = AgCN

Sol.: **C, D**



*3. For the following reaction $2X + Y \xrightarrow{k} P$ the rate of reaction is $\frac{d[P]}{dt} = k[X]$. Two moles of **X** are mixed with one mole of **Y** to make 1.0 L of solution. At 50 s, 0.5 mole of **Y** is left in the reaction mixture. The correct statement(s) about the reaction is(are)

(Use: ln 2 = 0.693)

(A) The rate constant, k, of the reaction is $13.86 \times 10^{-4} s^{-1}$

(B) Half-life of X is 50s

(C) At 50 s, $-\frac{d[X]}{dt} = 13.86 \times 10^{-3} \text{mol L}^{-1} \text{ s}^{-1}$.

(D) At 100 s, $-\frac{d[Y]}{dt} = 3.46 \times 10^{-3} \text{mol L}^{-1} \text{ s}^{-1}$.

Sol.: **B,C,D**

	2X	+	y	$\longrightarrow$	P
$t = 0$	2		1		0
$t = 50$	$2 - 2 \times 0.5$		$1 - 0.5$		
	1mole		0.5		

$$\text{rate} = -\frac{1}{2}\frac{dx}{dt} = -\frac{dy}{dt} = \frac{dP}{dt} = K[X]$$

$$-\frac{1}{2}\frac{dx}{dt} = K[X]$$

$$-\frac{dx}{dt} = 2K[X] = K^{1}[X]$$

Half life is t = 50 sec

$$2K = \frac{0.653L}{50}$$

$$K = \frac{0.6932}{100} = 6.332 \times 10^{-3}$$

t =50 sec

$$-\frac{dx}{dt} = 2K[X]$$

$$-\frac{dx}{dt} = 2 \times 6.332 \times 10^{-3} \times 1 = 13.864 \times 10^{-3} \text{ mole / L / Sec}$$

$$-\frac{dy}{dt} = K[X] = 6.332 \times 10^{-3}\left(\frac{1}{2}\right) = 3.46 \times 10^{-3} \text{ mole / L / Sec}^{-1}$$

4. Some standard electrode potentials at 298 K are given below:

Pb^{2+}/Pb – 0.13 V

Ni^{2+}/Ni – 0.24 V

Cd^{2+}/Cd – 0.40 V

Fe^{2+}/Fe – 0.44 V

To a solution containing 0.001 M of $\mathbf{X}^{2+}$ and 0.1 M of $\mathbf{Y}^{2+}$, the metal rods **X** and **Y** are inserted (at 298 K) and connected by a conducting wire. This resulted in dissolution of **X**. The correct combination(s) of **X** and **Y**, respectively, is(are)

(Given: Gas constant, R = 8.314 J K^{-1} mol^{-1},

Faraday constant, F = 96500 C mol^{-1})

(A) Cd and Ni

(B) Cd and Fe

(C) Ni and Pb

(D) Ni and Fe

Sol.: **A,B,C**

(A) $Cd + Ni^{+2} \longrightarrow Cd^{+2} + Ni$

$$E_{cell} = 0.40 + (-24) - \frac{0.0591}{2}\log\frac{0.001}{0.1}$$

$$= 0.16 + \frac{0.0591}{2} \times 2 = 6.64 + 0.551 = 0.71(+ve)$$

(B) $$E_{cell} = 0.40 + (-0.44) - \frac{0.591}{2}\log\frac{0.01}{0.1}$$

$$= -0.04 + \frac{0.591}{2} \times 2 = -0.04 + 0.06 = 0.02(+ve)$$

(C) $$E_{cell} = 0.24 + (-0.13) + \frac{.0591}{2} \times 2$$

$$= 0.11 + 0.06 = 0.33(+ve)$$

(D) $$E_{cell} = 0.24 + (-0.44) + \frac{0.0591}{2} \times 2$$

$$= -0.20 + 0.06 = -0.14(-ve)$$

5. The pair(s) of complexes wherein both exhibit tetrahedral geometry is(are)
(Note: py = pyridine
Given: Atomic numbers of Fe, Co, Ni and Cu are 26, 27, 28 and 29, respectively)

(A) $[FeCl_4]^-$ and $[Fe(CO)_4]^{2-}$

(B) $[Co(CO)_4]^-$ and $[CoCl_4]^{2-}$

(C) $[Ni(CO)_4]$ and $[Ni(CN)_4]^{2-}$

(D) $[Cu(py)_4]^+$ and $[Cu(CN)_4]^{3-}$

Sol.: **A, B, D**

(A) $[FeCl_4]^-$

$Fe^{+3} = 3d^5, Cl^-$ weak field liganol sp^3

$[Fe(CO)_4]^{-2}$

$Fe^{-2} = 3d^8 4s^2$

CO strong field liganol pairing occurs

$Fe^{-2} = 3d^{10}$ hence sp^3

(B) $[Co(CO)_4]^-$

$Co = 3d^8 4s^2$ due to CO pairing occurs

Hence = $3d^{10}$

$[CoCl_4]^{-2}$

$Co^{+2} = 3d^7$, Cl^- weak field liganol sp^3

(D) $[Cu(Py)_4]^{+1}$

$Cu^{+1} = 3d^{10}$, sp^3

$[Cu(CN)_4]^{-3}$

$Cu^{+1} = 3d^{10}$, sp^3

6. The correct statement(s) related to oxoacids of phosphorous is(are)

(A) Upon heating, H_3PO_3 undergoes disproportionation reaction to produce H_3PO_4 and PH_3.

(B) While H_3PO_3 can act as reducing agent, H_3PO_4 cannot.

(C) H_3PO_3 is a monobasic acid.

(D) The H atom of P–H bond in H_3PO_3 is not ionizable in water.

Sol.: **A,B,D**

$$H_3PO_3 \xrightarrow{\Delta} H_3PO_4 + PH_3$$

P–H bond is responsible for it is reducing character, H_3PO_4 doesnot have.

H_3PO_3 is a dibasic acid in oxyacids of phosphors O–H bond is ionisalbe where as P–H bond is non ionizable.

SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+2	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

At 298 K, the limiting molar conductivity of a weak monobasic acid is 4×10^2 S cm^2 mol^{-1}. At 298 K, for an aqueous solution of the acid the degree of dissociation is α and the molar conductivity is $y \times 10^2 S\ cm^2 mol^{-1}$. At 298 K, upon 20 times dilution with water, the molar conductivity of the solution becomes $3y \times 10^2 S\ cm^2 mol^{-1}$

7. The value of α is _______.

Sol.: **0.22**

$$\alpha_1 = \frac{\Lambda_m^c}{\Lambda_m^0} = \frac{y \times 10^2}{4 \times 10^2} = \frac{y}{4} = \alpha$$

On dilution conductivity increases three times

$$\alpha_2 = \frac{3y \times 10^2}{4 \times 10^2} = 3\alpha_1 = 3\alpha$$

HA	$\rightleftharpoons$	H^+	+	A^-
C		O		O
$C(1-\alpha)$		$C\alpha$		$C\alpha$

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

Since temperature is constant K_a will be constant

$$\frac{C_1\alpha_1^2}{1-\alpha_1} = \frac{C_2\alpha_2^2}{1-\alpha_2}$$

$$\frac{C \times \alpha^2}{1-\alpha} = \frac{\left(\frac{C}{20}\right)(3\alpha)^2}{1-3\alpha}$$

$$\frac{1}{1-\alpha} = \frac{9}{20} - \frac{1}{(1-3\alpha)}$$

$$20 - 60\alpha = 9 - 9\alpha;$$

$$\alpha = \frac{11}{51} = 0.2156$$

$$\alpha = 0.22$$

8. The value of **y** is ________.

Sol.: **0.88**

$$\alpha = \frac{y}{4}; \quad y = 4\alpha = 4 \times 0.22 = 0.88$$

Question Stem for Question Nos. 9 and 10

Question Stem

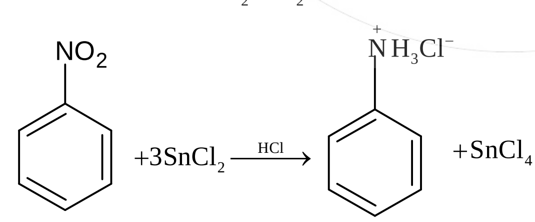
Reaction of **x** g of Sn with HCl quantitatively produced a salt. Entire amount of the salt reacted with **y** g of nitrobenzene in the presence of required amount of HCl to produce 1.29 g of an organic salt (quantitatively).

(Use Molar masses (in g mol^{-1}) of H, C, N, O, Cl and Sn as 1, 12, 14, 16, 35 and 119, respectively).

9. The value of **x** is ________.

Sol.: **3.57**

$$Sn + 2HCl \longrightarrow SnCl_2 + H_2$$



$$\text{Moles of Sn} = \frac{x}{119}$$

$$\text{Moles of nitrobenzene} = \frac{x}{119} \times \frac{1}{3}$$

$$\text{Moles of anilium chloride} = \frac{x}{119} \times \frac{1}{3}$$

$$\text{Moles of nitrobenzene} = \frac{x}{357}$$

$$\frac{y}{123} = \frac{x}{357} \quad \ldots (1)$$

Moles of anilium chloride $= \frac{x}{357}$ … (2)

$$\frac{1.29}{129} = \frac{x}{357}$$

x = 3.57

10. The value of **y** is _______.

Sol.: **1.23**

$y = 1.225 \cong 1.23$

Question Stem for Question Nos. 11 and 12

Question Stem

A sample (5.6 g) containing iron is completely dissolved in cold dilute HCl to prepare a 250 mL of solution. Titration of 25.0 mL of this solution requires 12.5 mL of 0.03 M $KMnO_4$ solution to reach the end point. Number of moles of Fe^{2+} present in 250 mL solution is **x** $\times 10^{-2}$ (consider complete dissolution of $FeCl_2$). The amount of iron present in the sample is **y**% by weight.

(Assume: $KMnO_4$ reacts only with Fe^{2+} in the solution

Use: Molar mass of iron as 56 g mol^{-1})

*11. The value of **x** is _______.

Sol.: **1.875**

meq of Fe^{+2} = meq of $KMnO_4$

$x \times 10^{-2} \times 1000 \times 1 = 12.5 \times 0.03 \times 5 \times 10$

x = 1.875 mole

*12. The value of **y** is _______.

Sol.: **18.75**

Moles of $Fe^{+2} = x \times 10^{-2} = 1.875 \times 10^{-2}$

wt.of $Fe^{+2} = 1.875 \times 10^{-2} \times 56$

Hence percentage of Fe^{+2}

$$= \frac{1.875 \times 10^{-2} \times 56}{5.6} \times 100\% = 18.75\%$$

SECTION 3

- This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+3	If ONLY the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Paragraph

The amount of energy required to break a bond is same as the amount of energy released when the same bond is formed. In gaseous state, the energy required for *homolytic cleavage* of a bond is called Bond Dissociation Energy (BDE) or Bond Strength. BDE is affected by *s*-character of the bond and the stability of the radicals formed. Shorter bonds are typically stronger bonds. BDEs for some bonds are given below:

$$H_3C-H(g) \longrightarrow H_3C^{\bullet}(g) + H^{\bullet}(g) \quad \Delta H^\circ = 105 \text{ kcal mol}^{-1}$$

$$Cl-Cl(g) \longrightarrow Cl^{\bullet}(g) + Cl^{\bullet}(g) \quad \Delta H^\circ = 58 \text{ kcal mol}^{-1}$$

$$H_3C-Cl(g) \longrightarrow H_3C^{\bullet}(g) + Cl^{\bullet}(g) \quad \Delta H^\circ = 85 \text{ kcal mol}^{-1}$$

$$H-Cl(g) \longrightarrow H^{\bullet}(g) + Cl^{\bullet}(g) \quad \Delta H^\circ = 103 \text{ kcal mol}^{-1}$$

*13. Correct match of the **C–H** bonds (shown in bold) in Column **J** with their BDE in Column K is

Column **J** Molecule	Column **K** BDE (kcl mol^{-1})
(P) $H-CH(CH_3)_2$	(i) 132
(Q) $H-CH_2Ph$	(ii) 110
(R) $H-CH=CH_2$	(iii) 95
(S) $H-C\equiv CH$	(iv) 88

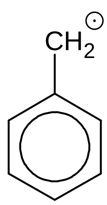
(A) P – iii, Q – iv, R – ii, S – I
(B) P – i, Q – ii, R – iii, S – iv
(C) P – iii, Q – ii, R –i, S – iv
(D) P – ii, Q – i, R – iv, S – iii

Sol.: **A**

As S character increases bond dissociation energy increases

$H-C\equiv CH$ (sp) $\quad H-CH=CH_2$ (sp^2)

$H-CH(CH_3)_2$ (sp^3) $\quad H-CH_2-Ph$ (sp^3)

And  is more stable then $CH_3-\overset{\displaystyle CH_3}{\underset{\displaystyle H}{C^{\odot}}}$

*14. For the following reaction

$$CH_4(g) + Cl_2(g) \xrightarrow{\text{light}} CH_3Cl(g) + HCl(g)$$

The correct statement is

(A) Initiation step is exothermic with $\Delta H° = -58$ kcal mol^{-1}.

(B) Propagation step involving $^{\bullet}CH_3$ formation is exothermic with $\Delta H° = -2$ kcal mol^{-1}.

(C) Propagation step involving CH_3Cl formation is endothermic with $\Delta H° = +27$ kcal mol^{-1}.

(D) The reaction is exothermic with $\Delta H° = -25$ kcal mol^{-1}.

Sol.: **D**

$CH_4 + Cl_2 \xrightarrow{\text{light}} CH_3Cl + HCl$ this reaction is obtained from given reaction.

$CH_3 - H \longrightarrow CH_3^{\odot} + H^{\odot}$ ...(1)

$Cl - Cl \longrightarrow Cl^{\odot} + Cl^{\odot}$ ...(2)

$CH_3 - Cl \longrightarrow CH_3^{\odot} + Cl^{\odot}$ ...(3)

$H - Cl \longrightarrow H^{\odot} + Cl^{\odot}$ ...(4)

$(1) + (2) - (3) - (4)$

Hence $\Delta H = 105 + 58 - 85 - 103 = -25$ K Cal / mole

Paragraph

The reaction of $K_3[Fe(CN)_6]$ with freshly prepared $FeSO_4$ solution produces a dark blue precipitate called Turnbull's blue. Reaction of $K_4[Fe(CN)_6]$ with the $FeSO_4$ solution in complete absence of air produces a white precipitate **X**, which turns blue in air. Mixing the $FeSO_4$ solution with $NaNO_3$, followed by a slow addition of concentrated H_2SO_4 through the side of the test tube produces a brown ring.

15. Precipitate **X** is

(A) $Fe_4[Fe(CN)_6]_3$

(B) $Fe[Fe(CN)_6]$

(C) $K_2Fe[Fe(CN)_6]$

(D) $KFe[Fe(CN)_6]$

Sol.: **C**

$FeSO_4 + K_4[Fe(CN)_6] \longrightarrow K_2Fe[Fe(CN)_6]$

While ppt.

16. Among the following, the brown ring is due to the formation of

(A) $[Fe(NO)_2(SO_4)_2]^{2-}$

(B) $[Fe(NO)_2(H_2O)_4]^{3+}$

(C) $[Fe(NO)_4(SO_4)_2]$

(D) $[Fe(NO)(H_2O)_5]^{2+}$

Sol.: **D**

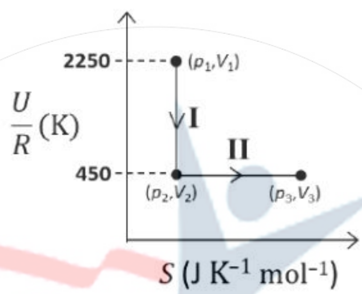
$FeSO_4 + NaNO_3 + H_2SO_4 \longrightarrow [Fe(H_2O)_5 NO]^{+2} + SO_4^{-2}$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designed to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	In all other cases.

*17. One mole of an ideal gas at 900 K, undergoes two reversible processes, **I** followed by **II**, as shown below. If the work done by the gas in the two process are same, the value of ln $\frac{V_3}{V_2}$ ___



(U: internal energy, S: entropy, p: pressure, V: volume, R: gas constant)

(Given: molar heat capacity at constant volume, $C_{v,m}$ of the gas is $\frac{5}{2}R$)

Sol.: **10**

1 st process is adiabatic since entropy is constant.

$W_1 = \Delta U$

$\Delta U = 450R - 2250R = -1800R$

$W_1 = -1800R$ … (1)

In 2nd process internal energy is constant it means it is a isothermal process.

$W_2 = -2.303nRT\log\frac{V_3}{V_2}$ ….(2)

$= -nRT\ln\frac{V_3}{V_2}$ ….(3)

Given, n = 1 mole, here temperature is unknown

$U = nC_V T$ for process II

$450R = 1\times\frac{5}{2}RT$

$T = \frac{450\times 2}{5} = 180K$

Equation (1) = equation (2)

$W_1 = W_2$

$-1800R = -1\times R\times 180\ln\frac{V_3}{V_2}$

$\ln\frac{V_3}{V_2} = \frac{1800}{180} = 10$

*18. Consider a helium (He) atom that absorbs a photon of wavelength 330 nm. The change in the velocity (in cm s^{-1}) of He atom after the photon absorption is ___.
(Assume: Momentum is conserved when photon is absorbed.
Use: Planck constant = 6.6×10^{-34} J s, Avogadro number = 6×10^{23} mol^{-1},
Molar mass of He = 4 g mol^{-1})

Sol.: **30**

$$\lambda = \frac{h}{m(\Delta V)}$$

$$330 \times 10^{-9} = \frac{6.6 \times 10^{-34}}{\left(\frac{4 \times 10^{-3}}{6 \times 10^{23}}\right) \times \Delta V}$$

$$\Delta V = \frac{6.6 \times 6 \times 10^{23} \times 10^{-34}}{4 \times 10^{-3} \times 330 \times 10^{-9}}$$

$$= 0.30 \text{ m/s}$$

$$= 0.30 \times 100 \text{ cm/s}$$

$$= 30 \text{ cm/s}$$

19. Ozonolysis of ClO_2 produces an oxide of chlorine. The average oxidation state of chlorine in this oxide is ___.

Sol.: **6**

$$2ClO_2 + 2O_3 \longrightarrow Cl_2O_6 + 2O_2$$

Average oxidization state of Cl atom is +6

PART III: MATHEMATICS

SECTION 1

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 mark;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.

*1. Let $S_1 = \{(i, j, k) : i, j, k \in \{1, 2, ..., 10\}\}$,

$S_2 = \{(i, j) : 1 \le i < j + 2 \le 10,\ i, j \in \{1, 2, ..., 10\}\}$,

$S_3 = \{(i, j, k, l) : 1 \le i < j < k < l,\ i, j, k, l \in \{1, 2, ..., 10\}\}$ and

$S_4 = \{(i, j, k, l) : i, j, k \text{ and } l \text{ are distinct elements in } \{1, 2, ..., 10\}\}$

If the total number of elements in the set S_r is n_r, r = 1, 2, 3, 4, then which of the following statements is(are) **TRUE**?

(A) $n_1 = 1000$ (B) $n_2 = 44$ (C) $n_3 = 220$ (D) $\frac{n_4}{12} = 420$

Sol. **A, B, D**

$n_1 = 10 \times 10 \times 10 = 10^3$

$n_2 = {}^8C_2 + 2\ {}^8C_1 = 44$

$n_3 = {}^{10}C_4 = 210$

$n_4 = {}^{10}P_4 = 5040$

*2. Consider a triangle PQR having sides of lengths p, q and r opposite to the angles P, Q and R, respectively. Then which of the following statements is(are) **TRUE**?

(A) $\cos P \geq 1 - \frac{p^2}{2qr}$

(B) $\cos R \geq \left(\frac{q-r}{p+q}\right)\cos P + \left(\frac{p-r}{p+q}\right)\cos Q$

(C) $\frac{q+r}{p} < 2\frac{\sqrt{\sin Q \sin R}}{\sin P}$

(D) If p < q and p < r, then $\cos Q > \frac{p}{r}$ and $\cos R > \frac{p}{q}$

Sol. **A, B**

(A) $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$, $q^2 + r^2 \geq 2qr$, by A.M. ≥ G.M

$\Rightarrow \cos P \geq 1 - \frac{p^2}{2qr}$

(B) By triangle inequality q + p > r

by projection formula

r cosP + p cosR + q cosR + rcosQ > p cosQ + q cosP

$\Rightarrow \cos R > \frac{(q-r)}{(p+q)}\cos P + \frac{(p-r)}{(p+q)}\cos Q$

So inequality is true (equality does not hold)

(C) By AM. ≥ GM.

$q + r \geq 2\sqrt{qr}$

$\frac{q+r}{p} \geq \frac{2\sqrt{qr}}{p}$

$\frac{\sin P}{p} = \frac{\sin Q}{q} = \frac{\sin R}{r}$ by sine rule

$\frac{q+r}{p} \geq \frac{2\sqrt{\sin Q \sin R}}{\sin P}$

(D) If $\cos Q > \frac{p}{r} \Rightarrow r\cos Q > p$ … (i)

$\cos R > \frac{p}{q} \Rightarrow q\cos R > p$ … (ii)

Add (i) and (ii)

r cosQ + q cosR > 2P ⇒ p > 2p

⇒ p < 0, false

3. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to R$ be a continuous function such that $f(0)=1$ and $\int_0^{\frac{\pi}{3}} f(t)dt=0$. Then which of the following statements is(are) **TRUE**?

(A) The equation f(x) – 3 cos 3x = 0 has at least one solution in $\left(0,\frac{\pi}{3}\right)$

(B) The equation f(x) – 3 sin 3x = $-\frac{6}{\pi}$ has at least one solution in $\left(0,\frac{\pi}{3}\right)$

(C) $\lim\limits_{x\to 0}\frac{x\int_0^x f(t)dt}{1-e^{x^2}}=-1$

(D) $\lim\limits_{x\to 0}\frac{\sin x\int_0^x f(t)dt}{x^2}=-1$

Sol. **A, B, C**

(A) Let g(x) = f(x) – 3 cos3x

$$\int_0^{\pi/3} g(x)dx=\int_0^{\pi/3}(f(x)-3\cos 3x)dx=\int_0^{\pi/3} f(x)dx-\int_0^{\pi/3} 3\cos 3x\,dx=0$$

$\Rightarrow$ g(x) = 0 has at least one solution in $[0,\pi/3]$

(B) Let h(x) = f(x) – 3 sin3x + 6/π

$$\int_0^{\pi/3} h(x)=\int_0^{\pi/3}\left(f(x)-3\sin 3x+\frac{6}{\pi}\right)dx=0$$

$\Rightarrow$ h(x) = 0 has atleast one solution in [0, π/3]

(C) $$\lim_{x\to 0}\frac{\int_0^x f(t)\,dt}{x}=\lim_{x\to 0}\frac{f(x)}{1}=f(0)=1$$

$$\Rightarrow \lim_{x\to 0}\frac{x\int_0^x f(t)\,dt}{x^2\left(\frac{1-e^{x^2}}{x^2}\right)}=\lim_{x\to 0}\frac{-\int_0^x f(t)}{x}\cdot\frac{x^2}{\left(e^{x^2}-1\right)}=-1$$

(D) $$\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\int_0^x f(t)}{x}=1$$

4. For any real numbers α and β, let $y_{\alpha,\beta}(x)$, x ∈ R, be the solution of the differential equation

$$\frac{dy}{dx}+\alpha y=xe^{\beta x},\ \ y(1)=1.$$

Let $S=\{y_{\alpha,\beta}(x):\alpha,\beta\in R\}$. Then which of the following functions belong(s) the set S?

(A) $f(x)=\frac{x^2}{2}e^{-x}+\left(e-\frac{1}{2}\right)e^{-x}$

(B) $f(x)=-\frac{x^2}{2}e^{-x}+\left(e+\frac{1}{2}\right)e^{-x}$

(C) $f(x)=\frac{e^x}{2}\left(x-\frac{1}{2}\right)+\left(e-\frac{e^2}{4}\right)e^{-x}$

(D) $f(x)=\frac{e^x}{2}\left(\frac{1}{2}-x\right)+\left(e+\frac{e^2}{4}\right)e^{-x}$

Sol. **A, C**

$$\frac{dy}{dx}+\alpha y = xe^{\beta x}$$

$d\left(e^{\alpha x}\cdot y\right)xe^{\alpha x}\cdot e^{\beta x}$

$d\left(e^{\alpha x}\cdot y\right) = xe^{(\alpha+\beta)x}$ … (i)

Case–I : $\alpha + \beta \neq 0$

$d\left(e^{\alpha x}\cdot y\right) = xe^{(\alpha+\beta)x}$

$$e^{\alpha x}\cdot y = \frac{xe^{(\alpha+\beta)x}}{(\alpha+\beta)}-\frac{e^{(\alpha+\beta)x}}{(\alpha+\beta)^2}+C$$

$$y = \frac{xe^{\beta x}}{(\alpha+\beta)}-\frac{e^{\beta x}}{(\alpha+\beta)^2}+Ce^{-\alpha x}$$

$\alpha = 1, \beta = 1 \Rightarrow y = \frac{xe^{x}}{2}-\frac{e^{x}}{4}+Ce^{-x}$

as $y(1) = 1 \Rightarrow C = e\left(1-\frac{e}{4}\right)$

$y(x) = \frac{xe^{x}}{2}-\frac{e^{x}}{4}+\left(e-\frac{e^2}{4}\right)e^{-x}$

Case–II : $\alpha + \beta = 0$

$\Rightarrow \quad \frac{dy}{dx}-\beta y = xe^{\beta x}$

$d\left(e^{-\beta x}y\right) = x$

$e^{-\beta x}\cdot y = \frac{x^2}{2}+C$

$y = \frac{e^{\beta x}x^2}{2}+Ce^{\beta x}$

$y(1) = 1$

$\Rightarrow \quad C = \left(1-\frac{e}{2}\right)\frac{1}{e} \Rightarrow y = e^{\beta x}\cdot\frac{x^2}{2}+\left(1-\frac{e}{2}\right)\frac{1}{e}\cdot e^{\beta x}$

Take $\beta = -1 \Rightarrow y = \frac{x^2}{2}e^{-x}+\left(1-\frac{e}{2}\right)e^{-x}$

5. Let O be the origin and $\overrightarrow{OA} = 2\hat{i}+2\hat{j}+\hat{k}$, $\overrightarrow{OB} = \hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{OC} = \frac{1}{2}\left(\overrightarrow{OB}-\lambda\overrightarrow{OA}\right)$ for some $\lambda > 0$. If $\left|\overrightarrow{OB}\times\overrightarrow{OC}\right| = \frac{9}{2}$, then which of the following statements is(are) **TRUE**?

(A) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

(B) Area of the triangle OAB is $\frac{9}{2}$

(C) Area of the triangle ABC is $\frac{9}{2}$

(D) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

Sol. **A, B, C**

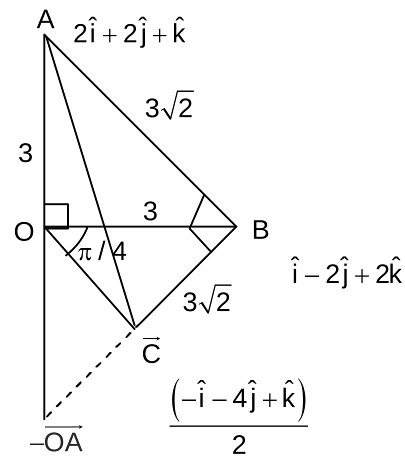
$\overrightarrow{OA}\cdot\overrightarrow{OB}=0 \Rightarrow \overrightarrow{OA}\perp\overrightarrow{OB}$

$\overrightarrow{OC}=\frac{1}{2}\left((1-2\lambda)\hat{i}+(-2-2\lambda)\hat{j}+(2-\lambda)\hat{k}\right)$

$\left|\overrightarrow{OB}\times\overrightarrow{OC}\right|=\frac{9|\lambda|}{2}=\frac{9}{2}$

$\Rightarrow$ $\lambda = \pm 1$, as $\lambda > 0$, $\lambda = 1$

$\overrightarrow{OC}=\frac{\overrightarrow{OB}-\overrightarrow{OA}}{2}$



(A) Projection $\overrightarrow{OC}$ on $\overrightarrow{OA}$

$\overrightarrow{OC}\cdot\widehat{OA}=-\frac{3}{2}$

(B) Area of $\Delta OAB = 9/2$

(C) Area of $\Delta ABC = 9/2$

(D) $\overrightarrow{OA}+\overrightarrow{OC}=\frac{3\hat{i}+3\hat{x}}{2}$, $\overrightarrow{OA}-\overrightarrow{OC}=\frac{5\hat{i}+8\hat{j}+\hat{k}}{2}$

$\left(\overrightarrow{OA}+\overrightarrow{OC}\right)\left(\overrightarrow{OA}-\overrightarrow{OC}\right)=\left|\overrightarrow{OA}+\overrightarrow{OC}\right|\left|\overrightarrow{OA}-\overrightarrow{OC}\right|\cos\theta$

$\cos\theta = \frac{1}{\sqrt{5}}$

*6. Let E denote the parabola $y^2 = 8x$. Let P = (–2, 4) and let Q and Q′ be two distinct points on E such that the lines PQ and PQ′ are tangents to E. Let F be the focus of E. Then which of the following statements is(are) **TRUE**?

(A) The triangle PFQ is a right-angled triangle
(B) The triangle QPQ′ is a right-angle triangle
(C) The distance between P and F is $5\sqrt{2}$
(D) F lies on the line joining Q and Q′

Sol. **A, B, D**

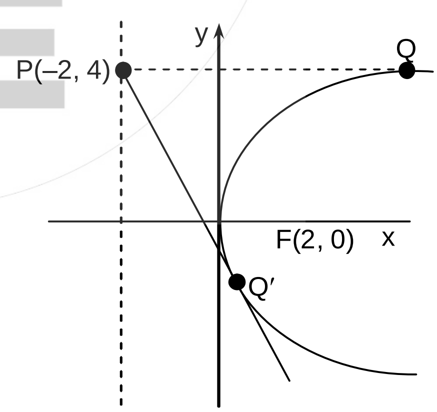
$E = y^2 - 8x = 0$

$a = 2$

Let $Q\left(2t_1^2, 4t_1\right)$, $Q'\left(2t_2^2, 4t_2\right)$

$t_1t_2 = -1$, $t_1 + t_2 = 2$

$t_1 = 1+\sqrt{2}$, $t_2 = 1-\sqrt{2}$



(A) (slope of PF) (slope of FQ) = –1

$\Rightarrow \angle PFQ = \frac{\pi}{2}$

(B) (slope of PQ′)(slope of PQ) = –1

$\angle QPQ' = \frac{\pi}{2}$

(C) $PF = 4\sqrt{2}$

(D) slope of Q′F = slope of FQ

$\Rightarrow$ Q, F, Q′ are collinear

SECTION 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Mark : +2 If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

Consider the region $R = \{(x, y) \in R \times R : x \geq 0 \text{ and } y^2 \leq 4 - x\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F. Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

*7. The radius of the circle C is _____

Sol. **1.5**

*8. The value of α is _____

Sol. **2**

Solution (7 and 8)

Let the equation of circle $(x - r)^2 + y^2 = r^2$

Equation of parabola $y^2 = 4 - x$

Solving them

$(x - r)^2 + 4 - x = r^2$

$\Rightarrow$ $x^2 - x(2r + 1) + 4 = 0$

Since circle and parabola meet tangentially hence

$(2r + 1)^2 - 16 = 0$

$\Rightarrow$ $2r + 1 = 4$

$\Rightarrow$ $r = 3/2$ and $x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$\Rightarrow$ $\alpha = 2, r = 3/2 = 1.5$

Question Stem for Question Nos. 9 and 10

Question Stem

Let $f_1:(0,\infty)\to R$ and $f_2:(0,\infty)\to R$ be defined by $f_1(x)=\int_0^x \prod_{j=1}^{21}(t-j)^j\,dt,\ x>0$ and $f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450,\ x > 0$, where, for any positive integer n and real numbers $a_1, a_2, \ldots, a_n$, $\prod_{i=1}^{n} a_i$ denotes the product of $a_1, a_2, \ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function f_i, i = 1, 2, in the interval $(0, \infty)$.

9. The value of $2m_1 + 3n_1 + m_1n_1$ is _____

Sol. **57**

$f_1(x) = \int_0^x \prod_{j=1}^{21}(t-j)^j\,dt$

$\Rightarrow\quad f_1'(x)=\prod_{j=1}^{21}(x-j)^j$

Therefore $m_1 = 6$, $n_1 = 5$

$2m_1 + 3n_1 + m_1n_1$

$= 2(6) + 3(5) + 30$

$= 12 + 15 + 30 = 57$

10. The value of $6m_2 + 4n_2 + 8m_2n_2$ is _____

Sol. **6**

$f_2'(x) = (98)(50)(x-1)^{49} - (600)(49)(x-1)^{48}$

$= (49)(100)(x-1)^{48}(x-1-6)$

$m_2 = 1,\ n_2 = 0$

$6m_2 + 4n_2 + 8m_2n_2$

$6(1) + 4(0) + 8(0) = 6$

Question Stem for Question Nos. 11 and 12

Question Stem

Let $g_i:\left[\frac{\pi}{8},\frac{3\pi}{8}\right]\to R$, i = 1, 2 and $f:\left[\frac{\pi}{8},\frac{3\pi}{8}\right]\to R$ be functions such that

$g_1(x) = 1$, $g_2(x) = |4x-\pi|$ and $f(x) = \sin^2 x$, for all $x\in\left[\frac{\pi}{8},\frac{3\pi}{8}\right]$. Define $S_i=\int_{\pi/8}^{3\pi/8} f(x)\cdot g_i(x)\,dx,\ i=1,2$

11. The value of $\frac{16S_1}{\pi}$ is _____

Sol. **2**

$g_1 : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R, i = 1, 2, \ f : \left[\frac{\pi}{r}, \frac{3\pi}{r}\right] \to R$

$g_1 = 1, g_2 = |4x - \pi|, f(x) = \sin^2 x$

$$S_i = \int_{\pi/8}^{3\pi/8} f(x) \cdot g_i(x)\, dx$$

$$S_1 = \int_{\pi/8}^{3\pi/8} \sin^2 x\, dx = \int_{\pi/8}^{3\pi/8} \sin^2\left(\frac{\pi}{2} - x\right) dx \Rightarrow 2S_1 = \int_{\pi/8}^{3\pi/8} 1\, dx$$

$$\Rightarrow S_1 = \frac{1}{2}\left(\frac{3\pi}{8} - \frac{\pi}{8}\right) = \frac{\pi}{8} \Rightarrow \frac{16S_1}{\pi} = 2$$

12. The value of $\frac{48 S_2}{\pi^2}$ is ______

Sol. **1.5**

$$S_2 = \int_{\pi/8}^{3\pi/8} \sin^2 x |4x - \pi| dx = \int_{\pi/8}^{3\pi/8} \sin^2\left(\frac{\pi}{2} - x\right)\left|4\left(\frac{\pi}{2} - x\right) - \pi\right| dx$$

$$= \int_{\pi/8}^{3\pi/8} \cos^2 x |4x - \pi|\, dx$$

$$\Rightarrow 2S_2 = \int_{\pi/8}^{3\pi/8} |4x - \pi| dx = 2\int_{\pi/8}^{\pi/4} (\pi - 4x)\, dx$$

$$S_2 = \left(\pi x - 2x^2\right)\Big|_{\pi/8}^{\pi/4} = \pi\left(\frac{\pi}{4} - \frac{\pi}{8}\right) - 2\left(\frac{\pi^2}{16} - \frac{\pi^2}{64}\right)$$

$$S_2 = \frac{4\pi^2}{8} - \frac{3\pi^2}{32} = \frac{\pi^2}{32} = \frac{48S_2}{\pi^2} = \frac{48}{\pi^2} \times \frac{\pi^2}{32} = \frac{3}{2}$$

$$\frac{48S_2}{\pi^2} = 1.5$$

SECTION 3

- This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+3	If ONLY the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Paragraph

Let $M = \{(x, y) \in R \times R : x^2 + y^2 \le r^2\}$, where $r > 0$. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}$, $n = 1, 2, 3, ...$ Let $S_0 = 0$ and for $n \ge 1$, let S_n denote the sum of the first n terms of this progression. For $n \ge 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

*13. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then

(A) $k + 2l = 22$ (B) $2k + l = 26$ (C) $2k + 3l = 34$ (D) $3k + 2l = 40$

Sol. **D**

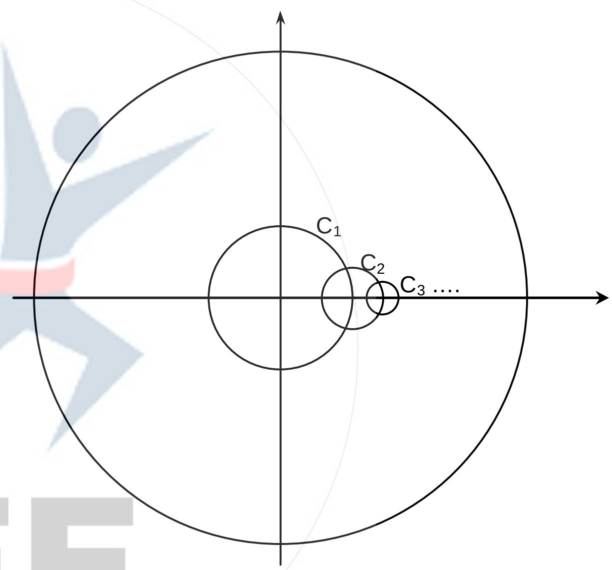
$C_1 \to (0, 0), r = 1$

$C_2 \to (1, 0), r = 1/2$

$C_3 \to (3/2, 0), r = 1/4$

⋮

$C_n\left(2\left(1-\frac{1}{2^{n-1}}\right), 0\right), r = \frac{1}{2^{n-1}}$



$\Rightarrow \quad 2\left(1-\frac{1}{2^{n-1}}\right) + \frac{1}{2^{n-1}} < r$

$2 - \frac{1}{2^{n-1}} < r$

$\Rightarrow \quad 2 - \frac{1}{2^{n-1}} < \frac{1025}{513}$

$\Rightarrow \quad \frac{1}{2^{n-1}} > \frac{1}{513}$

$\Rightarrow \quad 2^{n-1} < 513 \Rightarrow n - 1 \le 9$

$\Rightarrow \quad n \le 10 \Rightarrow k = 10$

Also no two by C_1, C_3, C_5, C_7, C_9 intersect each other. And no two of C_2, C_4, C_6, C_8, C_{10} intersect each other

For both, we get $l = 5$

$\Rightarrow 3k + 2l = 40$

*14. Consider M with $r = \frac{(2^{199}-1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

(A) 198 (B) 199 (C) 200 (D) 201

Sol. **B**

$\sqrt{2}S_{n-1} + a_n < r$

$\Rightarrow \sqrt{2}\left(2\left(1-\frac{1}{2^{n-1}}\right)\right)+\frac{1}{2^{n-1}} < \left(\frac{2^{199}-1}{2^{198}}\right)\sqrt{2}$

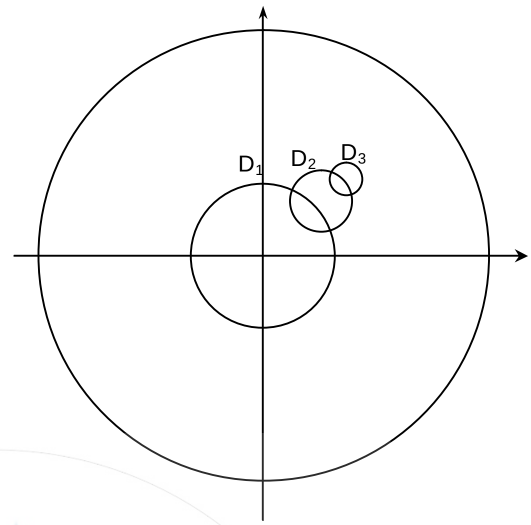
$\Rightarrow \sqrt{2}\left(1-\frac{1}{2^{n-1}}\right)+\frac{1}{2^{n}} < \left(1-\frac{1}{2^{199}}\right)\sqrt{2}$

$\Rightarrow \frac{1}{\left(\sqrt{2}\right)^{2n}}-\frac{1}{\left(\sqrt{2}\right)^{2n-3}} < \frac{-1}{\left(\sqrt{2}\right)^{397}}$

$\Rightarrow \frac{2\sqrt{2}-1}{\left(\sqrt{2}\right)^{2n}} > \frac{1}{\left(\sqrt{2}\right)^{397}}$

$\Rightarrow \left(\sqrt{2}\right)^{2n-397} < 2\sqrt{2}-1$

$\Rightarrow 2n - 397 \le 1 \Rightarrow n \le 199.$



Paragraph

Let $\Psi_1 : [0, \infty) \to R$, $\Psi_2 : [0, \infty) \to R$, $f : [0, \infty) \to R$ and $g : [0, \infty) \to R$ be functions such that $f(0) = g(0) = 0$,

$$\psi_1(x) = e^{-x} + x,\ \ x \ge 0,$$
$$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2,\ \ x \ge 0,$$
$$f(x) = \int_{-x}^{x} \left(|t| - t^2\right)e^{-t^2}\,dt,\ \ x > 0$$

and $g(x) = \int_0^{x^2} \sqrt{t}\, e^{-t}dt,\ \ x > 0.$

15. Which of the following statements is **TRUE**?

(A) $f\left(\sqrt{\ln 3}\right) + g\left(\sqrt{\ln 3}\right) = \frac{1}{3}$

(B) For every $x > 1$, there exists and $\alpha \in (1, x)$ such that $\psi_1(x) = 1 + \alpha x$

(C) For every $x > 0$, there exists a $\beta \in (0, x)$ such that $\psi_2(x) = 2x(\psi_1(\beta) - 1)$

(D) f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

Sol. **C**

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2,\ x \ge 0$

$\psi'_2(x) = 2x - 2 + 2e^{-x} = 2(x + e^{-x} - 1)$

$\psi'_2(\beta) = 2(\psi_1(\beta) - 1)$

Since $\psi_2(x)$ is a continuous and differentiable function $\forall\ x \in [0, x]$

$\psi_2(0) = 0,\ \psi_2(x) = x^2 - 2x - 2e^{-x} + 2$

Hence according to LMVT there exist atleast one $\beta \in (0, x)$ such that

$$\left(\frac{\psi_2(x) - \psi_2(0)}{x}\right) = \psi'_2(\beta)$$

$$= \frac{\psi_2(x)}{x} = 2(\psi_1(\beta) - 1)$$

$= \psi_2(x) = 2x(\psi_1(\beta) - 1)$.

16. Which of the following statements is **TRUE**?

(A) $\psi_1(x) \le 1$, for all $x > 0$

(B) $\psi_2(x) \le 0$, for all $x > 0$

(C) $f(x) \ge 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

(D) $g(x) \le \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

Sol. **D**

$\psi'_1(x) = 1 - e^{-x} > 0$

$\psi_2(x) = (x - 1)^2 + 1 - 2e^{-x} > 0$ for $x = 1$

$\because \quad e^{-t} = 1 - t + \frac{t^2}{2} - \frac{t^3}{3} + \ldots$

$$\sqrt{t}e^{-t} = \sqrt{t} - t^{3/2} + \frac{1}{2}t^{5/2} \ldots.$$

So, $\sqrt{t}e^{-t} < \sqrt{t} - t^{3/2} + \frac{1}{2}t^{5/2}$ for $t \in (0,1)$

$$\int_0^{x^2} \sqrt{t}e^{-t}dt < \int_0^{x^2} \left(\sqrt{t} - t^{3/2} + \frac{1}{2}t^{5/2}\right) dt$$

$$= \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designed to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	In all other cases.

17. A number is chosen at random from the set {1, 2, 3, ….., 2000}. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of 500p is _____

Sol. **214**

Let A = {1, 2, 3, …., 2000}

Let $E_1 = 3m$, $1 \le m \le 666$, $m \in N$

$E_2 = 7k$, $1 \le k \le 285$, $k \in N$

$= n(E_1 \cup E_2) = 666 + 285 - 95 = 856$

$= P(E_1 \cup E_2) = 856 / 2000 = 500\beta = 856 / 4 = 214$

18. Let E be the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. For any three distinct points P, Q and Q′ on E, let M(P, Q) be the mid-point of the line segment joining P and Q, and M(P, Q′) be the mid-point of the line segment joining P and Q′. Then the maximum possible value of the distance between M(P, Q) and M(P, Q′), as P, Q and Q′ vary on E, is _____

Sol. **4**

M(P, Q) is mid-point of PQ
M(P, Q′) is mid-point of PQ′
In a Δ PQQ′ since M(P, Q) & M(P, Q′) are mid-point of PQ & PQ′ hence line joining M(P,Q), M(P,Q′) is parallel to QQ′ and half of it.

$\Rightarrow$ max distance $=\frac{1}{2}QQ'=\frac{1}{2}(8)=4$

19. For any real number x, let [x] denote the largest integer less than or equal to x. If

$$I=\int_0^{10}\left[\sqrt{\frac{10x}{x+1}}\right]dx,$$

then the value of 9I is _____

19. **182**

$$I=\int_0^{10}\left[\sqrt{\frac{10x}{x+1}}\right]dx$$

$\left[\sqrt{\frac{10x}{x+1}}\right]=n \Rightarrow \frac{n^2}{10-n^2}\le x<\frac{(n+1)^2}{10-(n+1)^2}$ where n ∈ I

For n = 0, 0 ≤ x < 1/9
n = 1; 1/9 ≤ x < 2/3
n = 2; 2/3 ≤ x < 9, n = 3, x ≥ 9

$\Rightarrow \quad I=\int_0^{1/9}0\cdot dx+\int_{1/9}^{2/3}1\cdot dx+\int_{2/3}^{9}2\cdot dx+\int_9^{10}3\cdot dx$

$=\left(\frac{2}{3}-\frac{1}{9}\right)+2\left(9-\frac{2}{3}\right)+3(10-9)=\frac{182}{9}$ = 9I = 182

END OF THE QUESTION PAPER