(Paper - 01)

SOLUTIONS TO JEE(ADVANCED) – 2021

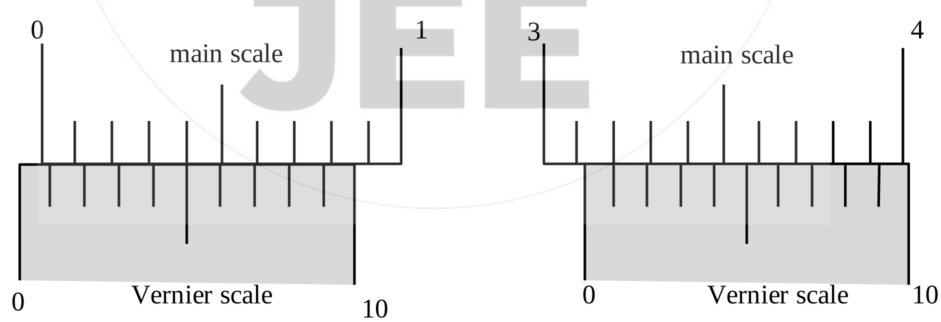
PHYSICS

SECTION 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- Four each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+3	If only the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered)
Negative Marks	:	–1	In all other cases.

Q. 1 The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calipers with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is



(A) 3.07 cm (B) 3.11 cm (C) 3.15 cm (D) 3.17 cm

Sol. **C**

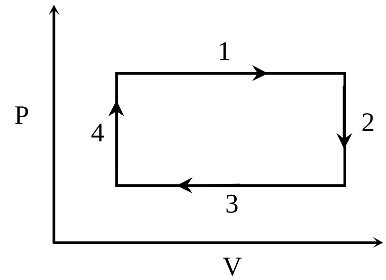
Least count = $\left(1-\frac{9}{10}\right)(0.1) = 0.01$ cm

Zero error = –0.1 + 0.06 = –0.04 cm

Final reading = 3.1 + 0.01 × 1 = 3.11 cm

So correct measurement = 3.11 + 0.04 = 3.15 cm

*Q.2 An ideal gas undergoes a four step cycle as shown in the P-V diagram below. During this cycle, heat is absorbed by the gas in

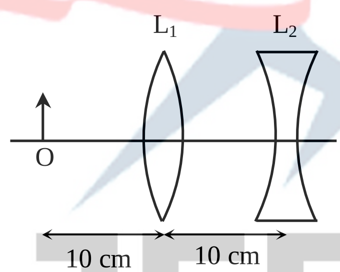


(A) steps 1 and 2
(B) steps 1 and 3
(C) steps 1 and 4
(D) steps 2 and 4

Sol. **C**

$\Delta Q_1 = nC_P\Delta T > 0$
$\Delta Q_2 = nC_V\Delta T < 0$
$\Delta Q_3 = nC_P\Delta T < 0$
$\Delta Q_4 = nC_V\Delta T > 0$

Q.3 An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is



(A) 0.4
(B) 0.8
(C) 1.3
(D) 1.6

Sol. **B**

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{1}{20}$$

$$\frac{1}{f_2} = (1.5-1)\left(-\frac{1}{20}-\frac{1}{20}\right) = -\frac{1}{20}$$

So, $\frac{1}{v}-\frac{1}{-10}=\frac{1}{20}$

So, v = – 20 cm

and $\frac{1}{v'}-\frac{1}{-30}=\frac{1}{-20}$

So, v′ = –12 cm

So total magnification $= \left(\frac{-20}{-10}\right)\left(\frac{-12}{-30}\right) = 0.8$

Q.4 A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decay of Q in the first one hour is

(A) 50 (B) 75 (C) 350 (D) 525

Sol. **D**

Total no. of decays in 60 minutes = $1000 - 1000\left(\frac{1}{2}\right)^3$ = 875

So, no. of α-decay = 875 × 0.6 = 525

SECTION 2

- This section contains **THREE (03)** question stems
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark : +2 If ONLY the correct numerical value is entered at the designated place;
Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}$ m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity g = 10 m/s^2.

*Q.5 The value of t is ____.

Sol. **0.50**

After splitting 1st mass takes 0.5 sec to reach ground.

Initial velocity is same for both mass at the highest point in vertical direction. Displacement and acceleration in vertical direction is also same

So, 2nd mass will also take 0.5 sec to reach ground.

*Q. 6 The value of x is ____.

Sol. **7.50**

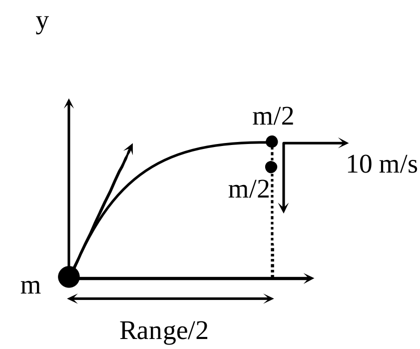
6. Velocity of projectile at highest point $5\text{ m/s}\,\hat{i}$

Since, there is no external force in horizontal direction so by conservation of momentum

$$m(5) = \frac{m}{2}(0) + \frac{m}{2}(v)$$

$$\vec{v} = 10\text{ m/s }\hat{i}$$

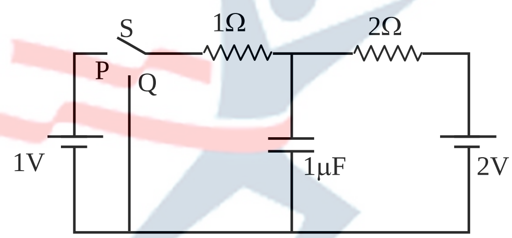
Distance covered by second mass before landing = $\frac{\text{Range}}{2} + 10(t) = 7.5\text{ m}$



Question Stem for Question Nos. 7 and 8

Question Stem

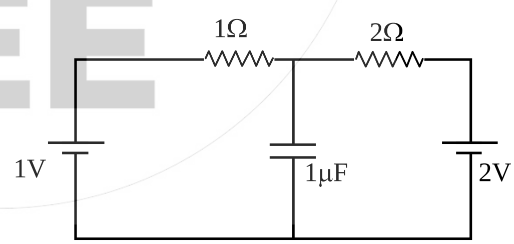
In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC.



Q.7 The magnitude of q_1 is ____.

Sol. **1.33**

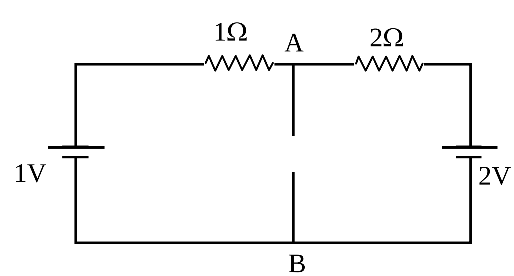
After long time we can replace the capacitor by open circuit.



i current in circuit = 1/3 ampere

$$v_{AB} = 2 - 2\left(\frac{1}{3}\right) = \frac{4}{3}\text{ volt}$$

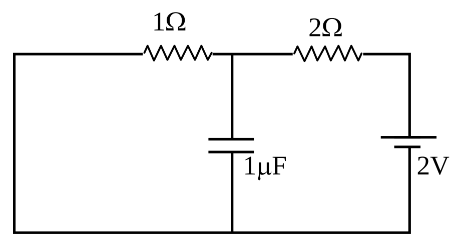
So, $q_1 = CV = \frac{4}{3}\mu C$



Q. 8 The magnitude of q_2 is ____.

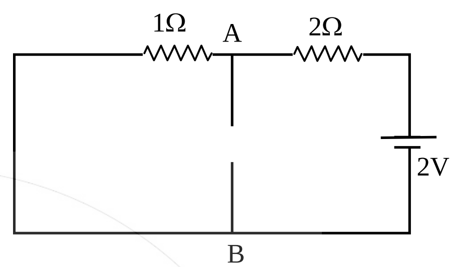
Sol. **0.67**

After long time i circuit = $\frac{2}{3}$ ampere



$v_{AB} = 2 - 2\left(\frac{2}{3}\right) = \frac{2}{3}$ volt

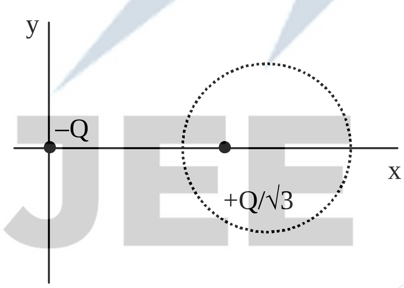
So, $q_2 = CV = (1)\left(\frac{2}{3}\right) = \frac{2}{3}\mu C$



Question Stem for Question Nos. 9 and 10

Question Stem

Two point charges –Q and $+Q/\sqrt{3}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V = 0 in the xy-plane with its center at (b, 0). All lengths are measured in meters.



Q.9 The value of R is _______ meter.

Sol. **1.73**

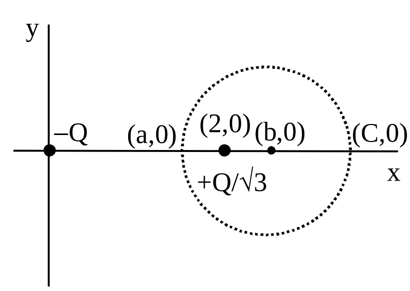
Lets take two points (a, 0) and (C, 0) on equipotential circle.

Net potential at (C, 0) = 0

$$\frac{K(-q)}{C} + \frac{Kq}{\frac{\sqrt{3}}{(C-2)}} = 0$$

$$\frac{1}{C} = \frac{1}{\sqrt{3}(C-2)}$$

$$\Rightarrow \sqrt{3}C - 2\sqrt{3} = C$$



$\Rightarrow (\sqrt{3}-1)C = 2\sqrt{3}$

$\Rightarrow C = \dfrac{2\sqrt{3}}{\sqrt{3}-1}$

Potential net at (a, 0) = 0

$$\frac{K(-q)}{a} + \frac{K\frac{q}{\sqrt{3}}}{(2-a)} = 0$$

$\Rightarrow \dfrac{1}{a} = \dfrac{1}{\sqrt{3}(2-a)}$

$\Rightarrow 2\sqrt{3} - \sqrt{3}a = a$

$\Rightarrow a = \dfrac{2\sqrt{3}}{1+\sqrt{3}}$

So, Radius = $\dfrac{C-a}{2} = \dfrac{\dfrac{2\sqrt{3}}{\sqrt{3}-1} - \dfrac{2\sqrt{3}}{\sqrt{3}+1}}{2}$

$= \sqrt{3}\left(\dfrac{1}{\sqrt{3}-1} - \dfrac{1}{\sqrt{3}+1}\right) = \sqrt{3}\left(\dfrac{\sqrt{3}+1-\sqrt{3}+1}{3-1}\right)$

Radius = $\sqrt{3}$

Q. 10 The value of b is _______ meter.

Sol. **3.00**

b = a + radius

$= \dfrac{2\sqrt{3}}{\sqrt{3}+1} + \sqrt{3} = \dfrac{2\sqrt{3}+3+\sqrt{3}}{1+\sqrt{3}}$

$= \dfrac{3\sqrt{3}+3}{1+\sqrt{3}} = 3$

Center = (3, 0)

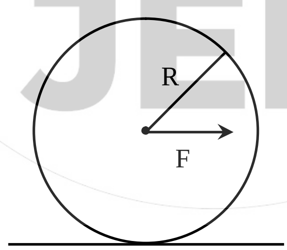
SECTION 3

- This section contains **SIX (06)** question.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.

*Q.11 A horizontal force F is applied at the centre of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The center of mass of the object has an acceleration *a*. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct?



(A) For the same F, the value of *a* does not depend on whether the cylinder is solid or hollow

(B) For a solid cylinder, the maximum possible value of *a* is 2μg

(C) The magnitude of the frictional force on the object due to the ground is always μmg

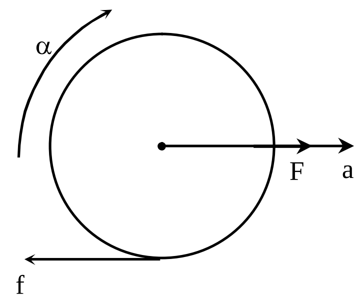
(D) For a thin-walled hollow cylinder, $a = \dfrac{F}{2m}$

Sol. **B, D**

$F - f = ma$

$fR = I\alpha$ (about center of mass)

$a = R\alpha$ (pure rolling)



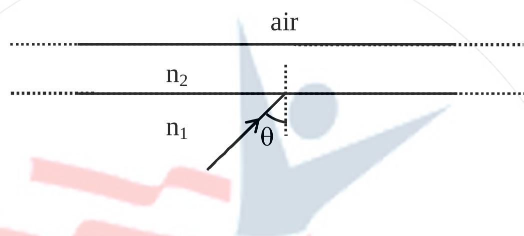
For hollow cylinder $a = \frac{F}{2m},\ f = \frac{F}{2}$

For solid cylinder, $a = \frac{2F}{3m},\ f = \frac{F}{3}$

Also for solid cylinder $\frac{F}{2} \le \mu mg$

Therefore $a \le 2\mu g$

Q.12 A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where $\sin\theta$ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is(are) correct?



(A) The light ray enters air if $n_2 = n_1$

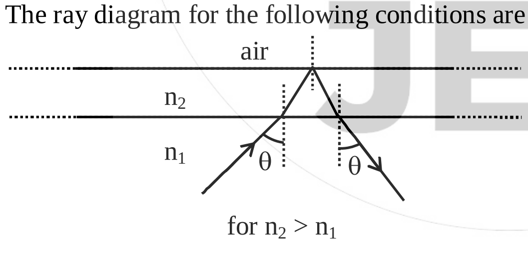
(B) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$

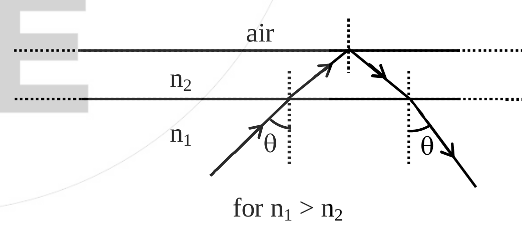
(C) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$

(D) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

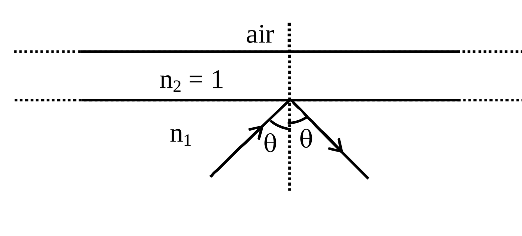
Sol. **B, C, D**

The ray diagram for the following conditions are





The light finally must reflected back in medium of refractive index n_1 for all values of n_2.



*Q.13 A particle of mass M = 0.2 kg is initially at rest in the xy-plane at a point ($x = -\ell$, $y = -h$), where $\ell = 10$ m and h = 1 m . The particle is accelerated at time t = 0 with a constant acceleration $a = 10\ m/s^2$ along the positive x-direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$ respectively. $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z-directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct?

(A) The particle arrives at the point ($x = \ell$, $y = -h$) at time t = 2 s.

(B) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point ($x = \ell$, $y = -h$)

(C) $\vec{L} = 4\hat{k}$ when the particle passes through the point ($x = \ell$, $y = -h$)

(D) $\vec{\tau} = \hat{k}$ when the particle passes through the point ($x = 0$, $y = -h$)

Sol. **A, B, C**

Time taken to reach from ($-\ell$, $-h$) to (ℓ, $-h$) is given by

$$2\ell = \frac{1}{2}at^2$$

$$20 = \frac{1}{2}(10)t^2 \qquad\qquad t = 2 \text{ sec}$$

$$\vec{L} = mvh\hat{k} = math\hat{k}$$

$$\vec{L} = (0.2)(10)2(1)\hat{k} = 4\hat{k}$$ [when particle passes through (ℓ, $-h$)]

$$\vec{\tau} = \frac{d\vec{L}}{dt} = mah\hat{k} = (0.2)(10)(1)\hat{k} = 2\hat{k} \text{ (always)}$$

14. Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom?

(A) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9/5

(B) There is an overlap between the wavelength ranges of Balmer and Paschen series

(C) The wavelength of Lyman series are given by $\left(1+\frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer

(D) The wavelength ranges of Lyman and Balmer series do not overlap

Sol. **A, D**

For hydrogen atom, z = 1

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$, where $R = 1.0973 \times 10^7\ m^{-1} = 1.1 \times 10^7\ m^{-1}$ = Rydberg constant.

For the Lyman series, $n_1 = 1$ and $n_2 = 2, 3, 4, \ldots\ldots\infty$

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{n_2^2}\right)$$

$\lambda = \lambda_{max}$, when $n_2 = 2$

$$\frac{1}{\lambda_{max}} = \frac{3R}{4} \Rightarrow \lambda_{max} = \frac{4}{3R} = 121.5 \text{ nm}$$

$\lambda = \lambda_{min}$, when $n_2 = \infty$

$$\frac{1}{\lambda_{min}} = R \Rightarrow \lambda_{min} = \frac{1}{R} = 91.1 \text{ nm}$$

Also, $\lambda = \left(\frac{n_2^2}{n_2^2 - 1}\right)\frac{1}{R} = \left[1 + \frac{1}{(n_2^2 - 1)}\right]\lambda_0 = \left(1 + \frac{1}{m^2}\right)\lambda_0$

$\lambda = \left(1 + \frac{1}{m^2}\right)\lambda_0$

where,

$m^2 = \left(n_2^2 - 1\right)$ = an integer

$m = \sqrt{n_2^2 - 1}$ = not an integer

For the Balmer series, $n_1 = 2$ and $n_2 = 3, 4, 5, 6, \ldots\ldots\ldots, \infty$

$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n_2^2}\right)$

$\lambda = \lambda_{max}$, when $n_2 = 3$

$\frac{1}{\lambda_{max}} = \frac{5R}{36}$

$\lambda_{max} = \frac{36}{5R} = 656.2$ nm

$\lambda = \lambda_{min,}$ when $n_2 = \infty$

$\Rightarrow \lambda_{min} = \frac{4}{R} = 364.5$ nm

Hence, for the Balmer series,

$\frac{\lambda_{max}}{\lambda_{min}} = \frac{36/5R}{4/R} = \frac{9}{5}$

For the Paschen series, $n_1 = 3$ and $n_2 = 4, 5, 6, \ldots\ldots\ldots, \infty$

$\frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{n_2^2}\right)$

$\lambda = \lambda_{max}$, when $n_2 = 4$

$\frac{1}{\lambda_{max}} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{144}$

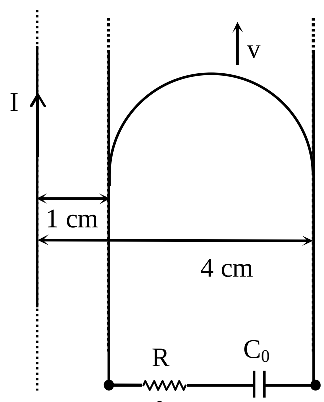
$\Rightarrow \lambda_{max} = \frac{144}{7R} = 1874.7$ nm

$\lambda = \lambda_{min,}$ when $n_2 = \infty$

$\frac{1}{\lambda_{min}} = \frac{R}{9} \Rightarrow \lambda_{min} = \frac{9}{R} = 820.2$ nm

15. A long straight wire carries a current, I = 2 ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1 cm and 4 cm from the wire. At time t = 0, the rod starts moving on the rails with a speed v = 3.0 m/s (see the figure)

A resistor R = 1.4 Ω and a capacitor C_0 = 5.0 μF are connected in series between the rails. At time t = 0, C_0 is uncharged. Which of the following statement(s) is(are) correct? [$\mu_0 = 4\pi \times 10^{-7}$ SI units. Take $\ell n2 = 0.7$]



(A) Maximum current through R is 1.2×10^{-6} ampere
(B) Maximum current through R is 3.8×10^{-6}ampere
(C) Maximum charge on capacitor C_0 is 8.4×10^{-12} coulomb
(D) Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb

Sol. **A, C**

Emf induced across the semi-circular conducting rod.

$$\varepsilon = \int_1^4 \frac{\mu_0 I v dx}{2\pi x} = \frac{\mu_0 I v}{2\pi}\ln(4) = \frac{\mu_0 I v}{\pi}\ln(2)$$

Since the semi-circular conducting rod is moving with a constant speed v = 3 m/s, then

$$\varepsilon = \frac{\mu_0 I v}{\pi}\ln(2) = \text{constant}$$

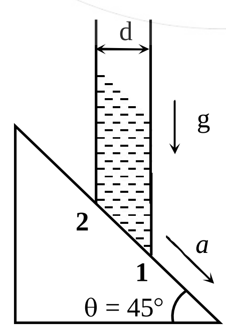
Maximum current through the resistor R is

$$i_{max} = \frac{\varepsilon}{R} = \frac{\mu_0 I v}{\pi R}\ln(2) = \frac{4\times10^{-7}\times2\times3\times0.7}{1.4} = 1.2\times10^{-6} \text{ ampere.}$$

Maximum charge on the capacitor C_0 is

$$q_{max} = C_0\varepsilon = C_0\left(\frac{\mu_0 I v}{\pi}\ln(2)\right) = 5\times10^{-6}\times4\times10^{-7}\times2\times3\times0.7 = 8.4\times10^{-12} \text{ coulomb.}$$

*16. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration *a* along a fixed inclined plane with angle $\theta = 45°$. P_1 and P_2 are pressures at points **1** and **2**, respectively located at the base of the tube. Let $\beta = (P_1 - P_2) / (\rho g d)$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct?



(A) $\beta = 0$ when $a = g/\sqrt{2}$
(B) $\beta > 0$ when $a = g/\sqrt{2}$
(C) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = g/2$
(D) $\beta = \frac{1}{\sqrt{2}}$ when $a = g/2$

Sol. **A, C**

$(P_1 - P_2)ds = \rho dsd\sqrt{2}(g\sin 45 - a)$

$(P_1 - P_2) = \rho d(g - a\sqrt{2})$

$\beta = \frac{(P_1 - P_2)}{\rho gd} = \left(1 - \frac{a\sqrt{2}}{g}\right)$

When $a = g/\sqrt{2}$, $\beta = 0$

When $a = g/2$, $\beta = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designed to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	In all other cases.

17. An α-particle (mass 4 amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of uniform magnetic field which is normal to the velocities of the particles. Within this region, the α-particle and the sulfur ion move in circular orbits of radii r_α and r_S, respectively. The ratio (r_S/r_α) is _____.

Sol. **4**

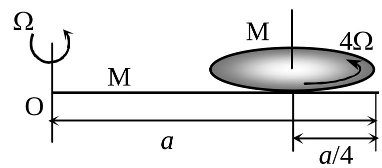
$r_\alpha = \frac{\sqrt{2m_\alpha q_\alpha V}}{q_\alpha B}$

$r_S = \frac{\sqrt{2m_S q_S V}}{q_S B}$

$\frac{r_S}{r_\alpha} = \sqrt{\frac{m_S}{q_S}\frac{q_\alpha}{m_\alpha}} = \sqrt{\left(\frac{32}{1}\right)\left(\frac{2}{4}\right)}$

$\frac{r_S}{r_\alpha} = 4$

*18. A thin rod of mass M and length *a* is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius *a*/4 is pivoted on this rod with its center at a distance *a*/4 from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)n$. The value of n is______.



Sol. **49**

Angular momentum of disc about O is

$$L_{DO} = M\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\Omega + \frac{M}{2}\left(\frac{a}{4}\right)^2(4\Omega)$$

Angular momentum of rod about O is

$$L_{RO} = \frac{Ma^2}{3}\Omega$$

So, $L_0 = L_{DO} + L_{RO} = \frac{49}{48}(Ma^2\Omega)$

So, n = 49

*Q.19 A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time t = 0, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2/t_1) is ____,

Sol. **9**

$$-C\frac{dT}{dt} = (T^4 - T_S^4)$$

$$\int_{200}^{100}\frac{dT}{T^4 - T_S^4} = \int_0^{t_1} -\frac{1}{c}\,dt$$

$$-\frac{1}{3}\left[\frac{1}{T^3}\right]_{200}^{100} = -\frac{1}{c}(t_1)$$

$$\Rightarrow \left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right] = \frac{3}{c}t_1$$

Similarly, $\left[\frac{1}{(50)^3} - \frac{1}{(200)^3}\right] = \frac{3}{c}t_2$

$$\frac{t_2}{t_1} = \frac{\left[\frac{1}{(50)^3} - \frac{1}{(200)^3}\right]}{\left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]} = 9$$

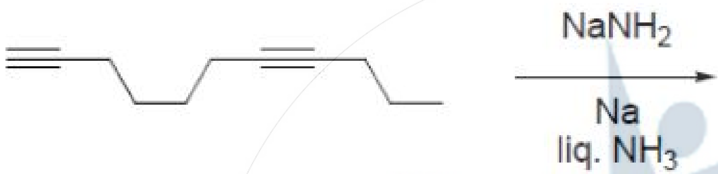
PART II: CHEMISTRY

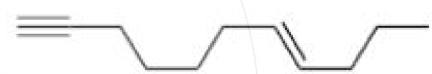
SECTION 1

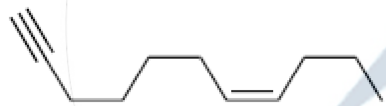
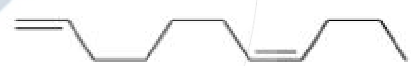
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- Four each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+3	If only the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered)
Negative Marks	:	−1	In all other cases.

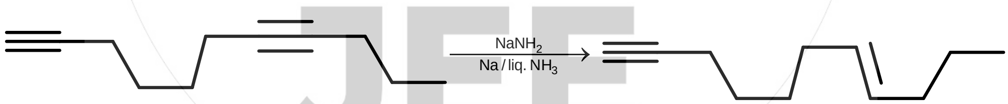
*Q.1 The major product formed in the following reaction is



(A)  (B) 

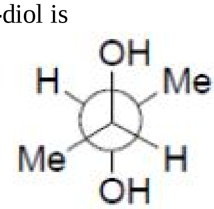
(C)  (D) 

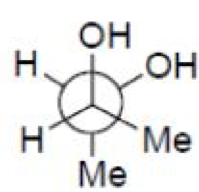
Sol. **B**

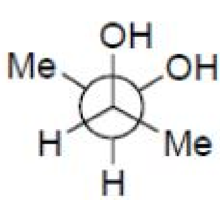


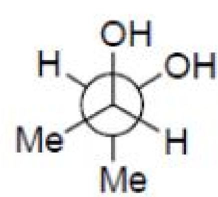
Na in liquid NH_3 reduces non terminal alkyne into trans alkene.

*Q.2 Among the following, the conformation that corresponds to the most stable conformation of *meso*-butane-2,3-diol is

(A) 

(B) 

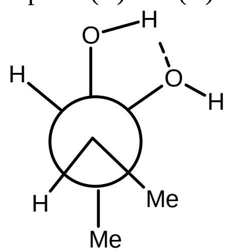
(C) 

(D) 

Sol. **B**

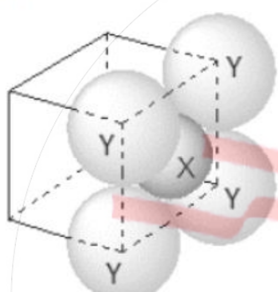
In option (B), given configuration represents meso – butane – 2, 3-diol and due to intramolecular hydrogen bonding, the gauche form is more stable.

Option (C) and (D) does not represent meso – isomer.



Q.3 For the given close packed structure of a salt made of cation **X** and anion **Y** shown below (ions of only one face are shown for clarity), the packing fraction is approximately

$$\left(\text{packing fraction} = \frac{\text{packing efficiency}}{100}\right)$$



(A) 0.74
(B) 0.63
(C) 0.52
(D) 0.48

Sol. **B**

$$\text{Packing fraction (f)} = \frac{3\times\frac{4}{3}\pi r_+^3 + 1\times\frac{4}{3}\pi r_-^3}{a^3}$$

$$= \frac{1\times\frac{4}{3}\pi\left[3\left(\frac{r_+}{r_-}\right)^3 + 1\right]}{\left(\frac{a}{r_-}\right)^3}$$

Now $2r_- = a$

$\therefore \frac{a}{r_-} = 2$

Also, $\frac{r_+}{r_-} = 0.414$

So, $f = \frac{1\times\frac{4}{3}\times 3.14\left[3\times(0.414)^3 + 1\right]}{(2)^3}$

$= 0.634 \approx 0.63$

Q.4 The calculated spin only magnetic moments of $\left[Cr\left(NH_3\right)_6\right]^{3+}$ and $\left[CuF_6\right]^{3-}$ in BM, respectively, are
(Atomic number of Cr and Cu are 24 and 29, respectively)
(A) 3.87 and 2.84
(B) 4.90 and 1.73
(C) 3.87 and 1.73
(D) 4.90 and 2.84

Sol. **A**
$\left[Cr\left(NH_3\right)_6\right]^{3+}$ has d^3 configuration, so as per CFT,
N = 3 and $\mu = \sqrt{3(3+2)} = 3.87$ BM
$\left[CuF_6\right]^{3-}$, has d^8 configuration and weak field ligand.
So N = 2 and $\mu = \sqrt{2(2+2)} = 2.84$ BM

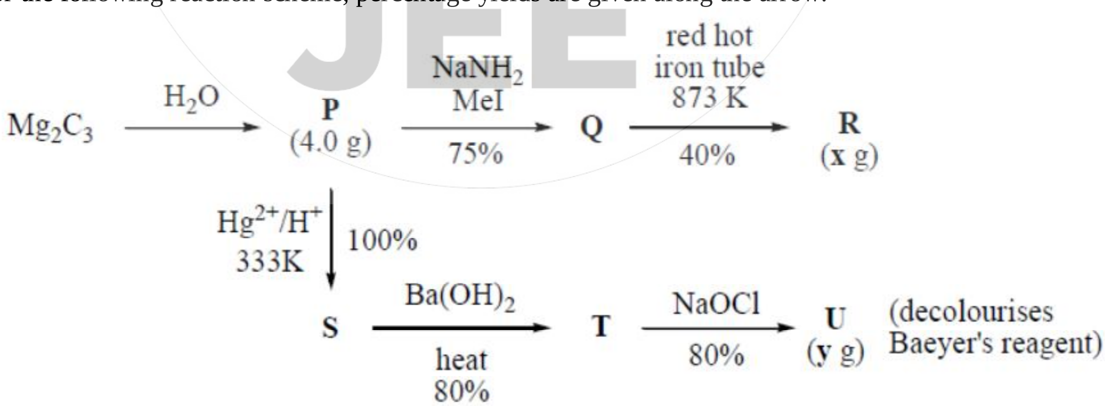
SECTION 2

- This section contains **THREE (03)** question stems
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Mark : +2 If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

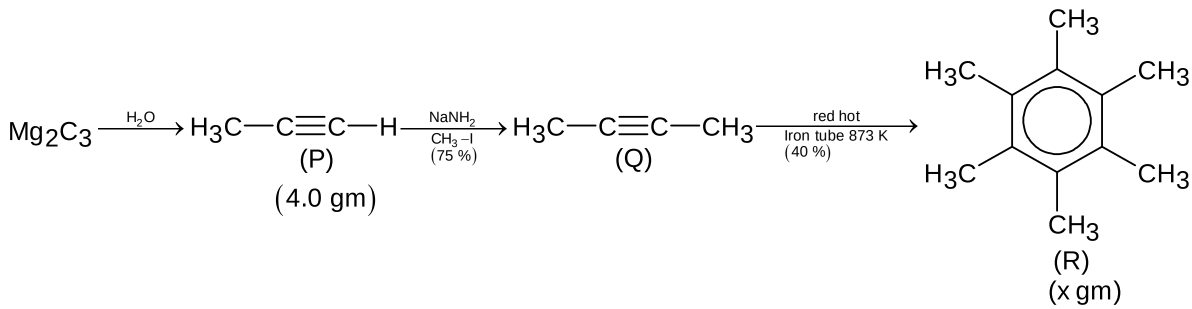
For the following reaction scheme, percentage yields are given along the arrow:

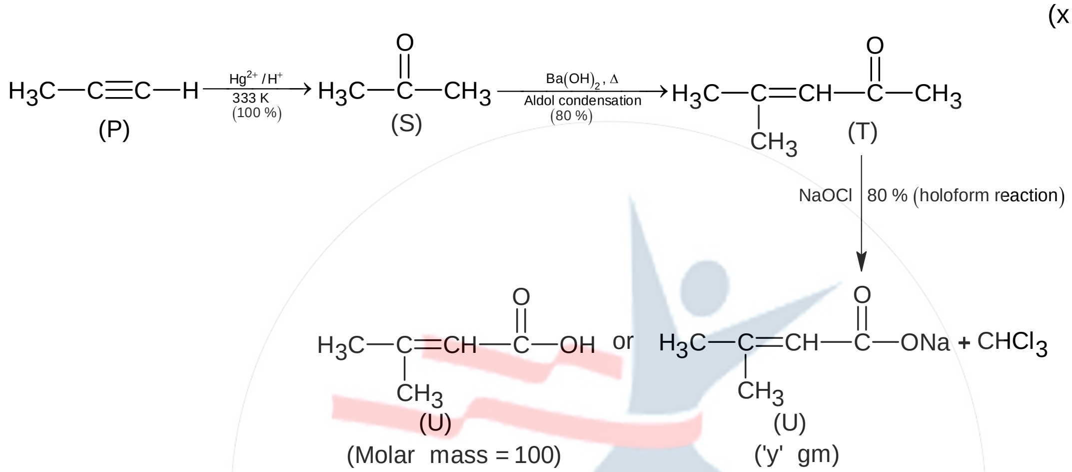


X g and **y** g are mass of **R** and **U,** respectively.
(Use : Molar mass (in g mol^{-1}) of H, C and O as 1, 12 and 16, respectively)

Q.5 The value of **x** is ___.

Sol. **1.62**





Molar mass of P = 40

$$\underset{(0.1\,\text{mole})}{P} \xrightarrow{75\,\%} \underset{\left(0.1\times\frac{3}{4}\right)}{Q}$$

$$\underset{\left(0.1\times\frac{3}{4}\right)}{3Q} \xrightarrow{40\,\%} \underset{\left(\frac{1}{3}\times 0.1\times\frac{3}{4}\times 0.4\right)}{R}$$

So, moles of R = 0.01 mole

Molar mass of (R) = 162

So, x = 0.01 × 162 = 1.62 g

Q.6 The value of **y** is ___.

Sol. **3.20 - 3.90**

Molar mass of 'U' = 122 g or 100 g

$$\underset{(0.1\,\text{mole})}{P} \xrightarrow{100\,\%} \underset{(0.1\,\text{mole})}{S}$$

$$\underset{(0.1\,\text{mole})}{2S} \xrightarrow[(80\,\%)]{\text{Aldol condensation}} \underset{\left(\frac{0.1}{2}\times 0.8\right)}{T} \xrightarrow{80\,\%} \underset{\left(\frac{0.1}{2}\times 0.8\times 0.8\right)}{U}$$

So, mass of 'U' = $\frac{0.1}{2}\times 0.8\times 0.8\times 100 = 3.20$ gm

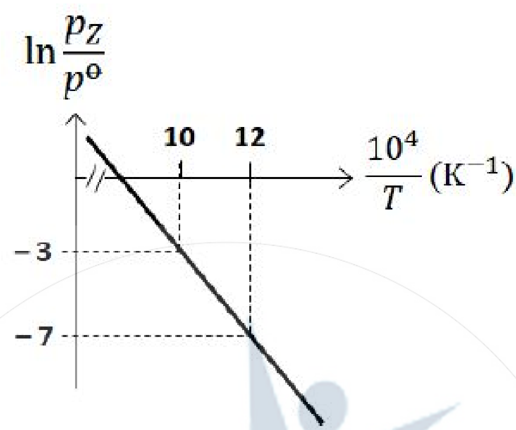
Or

Mass of 'U' = $\frac{0.1}{2}\times 0.8\times 0.8\times 122 = 3.90$ gm

Question Stem for Question Nos. 7 and 8

Question Stem

For the reaction, $\mathbf{X}(s) \rightleftharpoons \mathbf{Y}(s) + \mathbf{Z}(g)$, the plot of $\ln \frac{p_{\mathbf{Z}}}{p^{\ominus}}$ versus $\frac{10^4}{T}$ is given below (in solid line), where $p_{\mathbf{Z}}$ is the pressure (in bar) of the gas $\mathbf{Z}$ at temperature T and $p^{\ominus} = 1$ bar.



(Given, $\frac{d(\ln K)}{d\left(\frac{1}{T}\right)} = -\frac{\Delta H^{\ominus}}{R}$, where the equilibrium constant, $K = \frac{p_Z}{p^{\ominus}}$ and the gas constant, $R = 8.314\ \mathrm{J\ K^{-1}\ mol^{-1}}$)

* Q.7 The value of standard enthalpy, $\Delta H^{\ominus}$ (in $\mathrm{kJ\ mol^{-1}}$) for the given reaction is ___.

Sol. **166.28**

$$X(s) \rightleftharpoons Y(s) + Z(g)$$

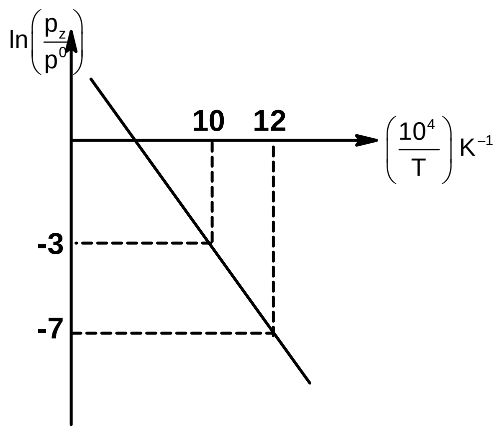
$$K_p = \frac{p_z}{p^0}, \text{ also } \Delta G^0 = -RT \ln k_p$$

$$= -RT \ln\left(\frac{p_z}{p^0}\right)$$

Now, $\Delta G^0 = \Delta H^0 - T\Delta S^0$

$$-RT\ kn\left(\frac{p_z}{p^0}\right) = \Delta H^0 - T\Delta S^0$$

$$\ln\left(\frac{p_z}{p^0}\right) = -\left(\frac{\Delta H^0}{R}\right)\frac{1}{T} + \frac{\Delta S^0}{R} \qquad ...(1)$$



$(1) \Rightarrow \ln\left(\frac{p_z}{p^0}\right) = -\left(\frac{\Delta H^0}{10^4 R}\right) \times \frac{10^4}{T} + \frac{\Delta S^0}{T}$ ....(2)

Slope of the line $= -\frac{\Delta H^0}{10^4 R} = \frac{[-7-(-3)]}{12-10} = -2$

$\therefore \Delta H^0 = 2\ R \times 10^4$

$= 2 \times 8.314 \times 10^{-3} \times 10^4 = 1.66.28$ kJ mol^{-1} K^{-1}

* Q.8 The value of $\Delta S^\ominus$ (in J K^{-1} mol^{-1}) for the given reaction, at 1000 K is ___.

Sol. **141.34**

Putting the value of ΔH^0 in equation (2), we get

$-3 = -\left(\frac{2R \times 10^4}{10^4 R}\right) \times \frac{10^4}{7} + \frac{\Delta S^0}{R}$

$-3 = -2R \times \frac{10^4}{T} + \frac{\Delta S^0}{R}$

$-3 = -2 \times \frac{10^4}{1000} + \frac{\Delta S^0}{R}$

$-3 = -20 + \frac{\Delta S^0}{R}$

$\therefore \frac{\Delta S^0}{R} = 17$

$\therefore \Delta S^0 = 17 \times 8.314 = 141.34$ JK^{-1} mol^{-1}

Question Stem for Question Nos. 9 and 10

Question Stem

The boiling point of water in a 0.1 molal silver nitrate solution (solution **A)** is $\mathbf{x}^{o}C$. To this solution **A,** an equal volume of 0.1 molal aqueous barium chloride solution is added to make a new solution **B**. The difference in the boiling points of water in the two solutions **A** and **B** is $\mathbf{y}\times10^{-2}\,^{o}C$.

(Assume: Densities of the solutions **A** and **B** are the same as that of water and the soluble salts dissociate completely.

Use: Molal elevation constant (Ebullioscopic Constant), $K_b = 0.5\ K\ kg\ mol^{-1}$;

Boiling point of pure water as $100^{o}C$.)

Q.9 The value of **x** is ___.

Sol. **100.10 °C**

0.1 molal $AgNO_3$ (aq) solution

$AgNO_3 \longrightarrow Ag^{+}(aq) + NO_3^{-}(aq)$

$i = 1 + (2-1)\times 1 = 2(\alpha = 1, \text{given})$

$\Delta T_b = i \times k_b \times m$

$\Delta T_b = 2 \times 0.5 \times 0.1 = 0.1$

So, boiling point of solution 'A' is = 100.10°C = x

Q.10 The value of |y| is ___.

Sol. **2.5**

Let solution 'B' is prepared by mixing 1 L (=1000 g) of solution 'A' with 1 L (= 1000 g) of solution of $BaCl_2$.

	$BaCl_2$	$+\ 2AgNO_3$	$\longrightarrow 2AgCl(s)$	$+\ Ba(NO_3)_2$
Initial moles	0.1 moles	0.1 moles	–	0
Moles after reaction	$0.1 - 0.05 = 0.05$ moles	0	–	0.05 moles

So, molality of new solution = $\left(\dfrac{i_1 \times m_1 + i_2 \times m_2}{2}\right)$

$= \left(\dfrac{3\times 0.05 + 3\times 0.05}{2}\right)$

= 0.15

Now, Elevation of boiling point of solution 'B' be (ΔT_b^{1})

$\Delta T_b^{1} = 0.15 \times k_b$

$= 0.15 \times \dfrac{1}{2}$

= 0.075

Now, T_b^{1} = 100.075 °C

So, difference of boiling point of 'A' and 'B' = 100.10 – 100.075 = 0.025 = $y \times 10^{-2}$ (given)

So, y = 2.5

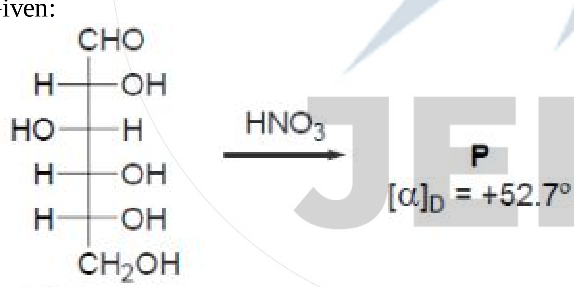
SECTION 3

- This section contains **SIX (06)** question.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.

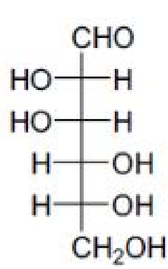
Q.11 Given:



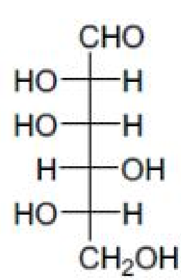
D-Glucose

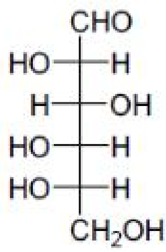
The compound(s), which on reaction with HNO_3 will give the product having degree of rotation, $[\alpha]_D = -52.7^{o}$ is(are)

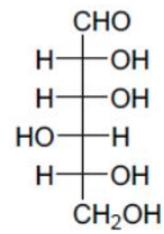
(A)



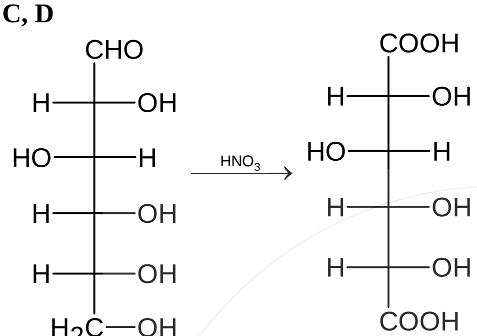
(B)



(C)


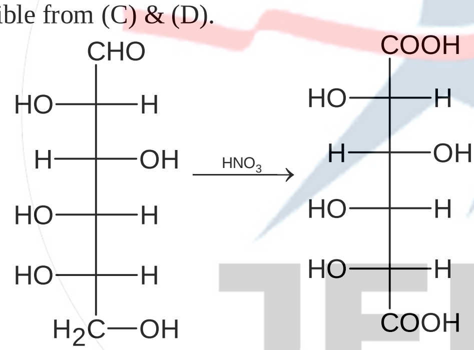
(D)


Sol. **C, D**

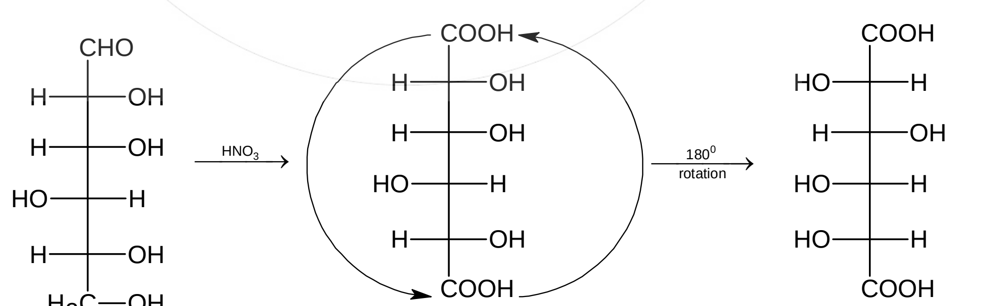


$(d-\text{glucose})$ $\quad$ $(\alpha)_D = 52.7^0$

Since, we have to get the product (x) of $(\alpha)_D = -52.7^0$, i.e. the enantiomer of above product. Which is only possible from (C) & (D).

(C)


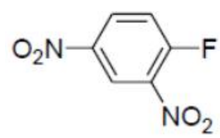
enantiomer of (x)

(D)


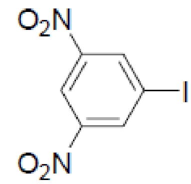
enantiomer of (x)

Q.12 The reaction of **Q** with PhSNa yields an organic compound (major product) that gives positive Carius test on treatment with Na_2O_2 followed by addition of $BaCl_2$. The correct option(s) for **Q** is(are)

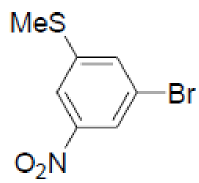
(A)



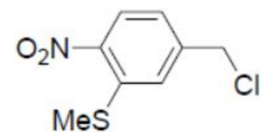
(B)



(C)



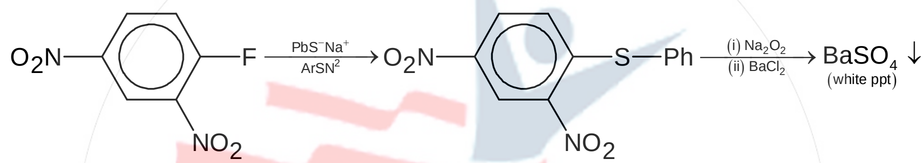
(D)



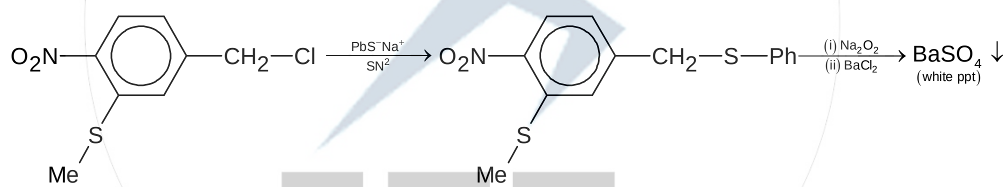
Sol. **A, D**

In option (B) and (C), NO_2 group (an EWG) is not present ortho or para position wrt the leaving group, so $ArSN^2$ reaction will not be possible.

(A)



(D)



Q.13 The correct statement(s) related to colloids is(are)

(A) The process of precipitating colloidal sol by an electrolyte is called peptization.

(B) Colloidal solution freezes at higher temperature than the true solution at the same concentration.

(C) Surfactants form micelle above critical micelle concentration (CMC). CMC depends on temperature.

(D) Micelles are macromolecular colloids.

Sol. **B, C**

(A) Process of precipitating colloidal solution by using an electrolyte is called "COAGULATION" and not peptisation.

(B) Since, molar mass of sol is much higher than true solutions, so magnitude of any colligative properties is smaller than true solutions.

$$(\Delta T_f)_{sol} < (\Delta T_f)_{true\ solution}$$

So, freezing point of sols > freezing point of true solution.

So, option (B) is correct.

(C) Micells are formed greater than or equal to CMC and above KRAFT temperature.

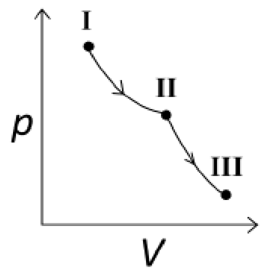
So option (C) is also correct.

(D) Micelles are ASSOCIATED colloids and not Macromolecular colloids.

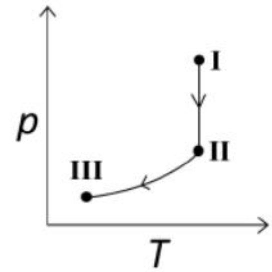
*Q.14 An ideal gas undergoes a reversible isothermal expansion from state **I** to state **II** followed by a reversible adiabatic expansion from state **II** to state **III**. The correct plot(s) representing the changes from state **I** to state **III** is(are)

(p: pressure, V: volume, T: temperature, H: enthalpy, S: entropy)

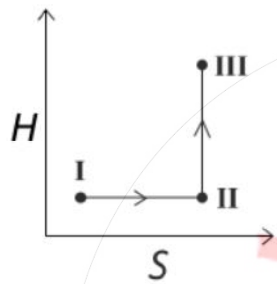
(A)



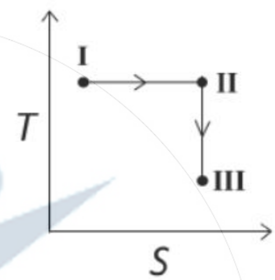
(B)



(C)



(D)

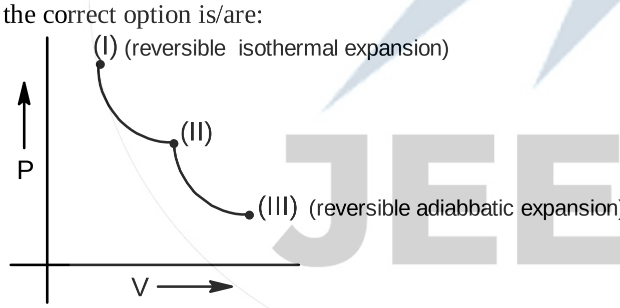


Sol. **A, B, D**

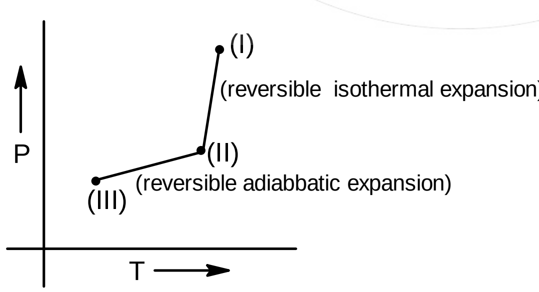
$(\text{I}) \xrightarrow[\text{expansion}]{\text{Reversible isothermal}} (\text{II}) \xrightarrow[\text{expansion}]{\text{Reversible adiabatic}} (\text{III})$

So, the correct option is/are:

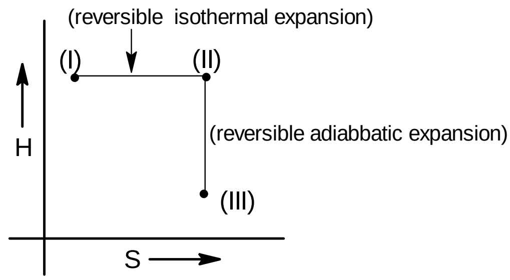
(A)



(B)



(D)



Reversible isothermal process is isoenthalpic while reversible adiabatic process is isoentropic.

Q.15 The correct statement(s) related to the metal extraction processes is(are)

(A) A mixture of PbS and PbO undergoes self-reduction to produce Pb and SO2.

(B) In the extraction process of copper from copper pyrites, silica is added to produce copper silicate.

(C) Partial oxidation of sulphide ore of copper by roasting, followed by self-reduction produces blister copper.

(D) In cyanide process, zinc powder is utilized to precipitate gold from $Na\left[Au(CN)_2\right]$.

Sol. **A, C, D**

(A) $PbS + 2PbO \xrightarrow{\text{self reduction}} 3Pb + SO_2 \uparrow$

So option (A) is correct.

(B) In the extraction Cu from copper pyrite $CuFeS_2$

SiO_2 is added to remove FeO as slag $FeSiO_3$.

So option (B) is wrong.

(C) $\underset{\substack{\text{concentrated}\\ \text{ore}}}{CuFeS_2} \xrightarrow{\text{roasting}} \underbrace{Cu_2S + FeS}_{\text{(matte)}} + SO_2$

$FeS + O_2 \longrightarrow FeO + SO_2$

$Cu_2S + O_2 \longrightarrow Cu_2O + SO_2$

$Cu_2S + 2Cu_2O \longrightarrow \underset{\text{(Blister copper)}}{6Cu} + SO_2$

So, option (C) is correct.

(D) $Zn + 2Na\left[Au(CN)_2\right] \longrightarrow Na_2\left[Zn(CN)_4\right] + 2Au \downarrow$

So, option (D) is correct.

Q.16 A mixture of two salts is used to prepare a solution **S**, which gives the following results:

$$\begin{matrix}\text{White}\\ \text{precipitate(s)}\\ \text{only}\end{matrix} \xleftarrow[\text{Room temperature}]{\text{Dilute NaOH(aq)}} \begin{matrix}\textbf{S}\\ \text{(aq solution}\\ \text{of the salts)}\end{matrix} \xrightarrow[\text{Room temperature}]{\text{Dilute HCl(aq)}} \begin{matrix}\text{White}\\ \text{precipitate(s)}\\ \text{only}\end{matrix}$$

The correct option(s) for the salt mixture is(are)

(A) $Pb(NO_3)_2$ and $Zn(NO_3)_2$

(B) $Pb(NO_3)_2$ and $Bi(NO_3)_3$

(C) $AgNO_3$ and $Bi(NO_3)_3$

(D) $Pb(NO_3)_2$ and $Hg(NO_3)_2$

Sol. **A, B, C**

$Pb(OH)_2$, $Zn(OH)_2$ and $Bi(OH)_3$ are white precipitates but $Hg(OH)_2$ (unstable) is not.

$PbCl_2$ is white ppt.

So, option (A), (B) and (C) are correct.

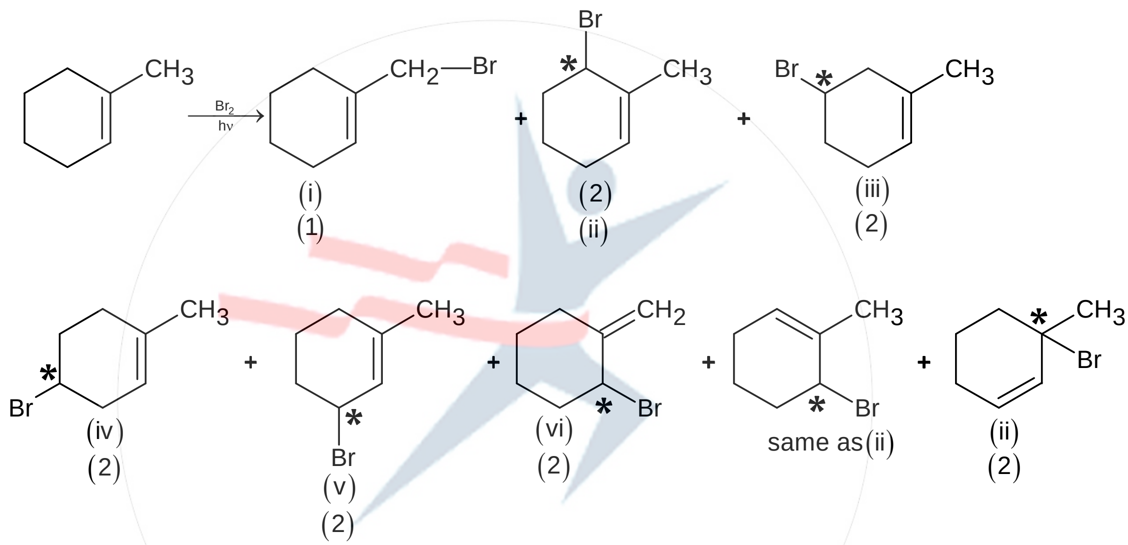
SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designed to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

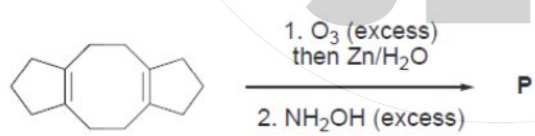
Full Mark	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	In all other cases.

*Q.17 The maximum number of possible isomers (including stereoisomers) which may be formed on *mono*-bromination of 1-methylcyclohex-1-ene using Br_2 and UV light is ____.

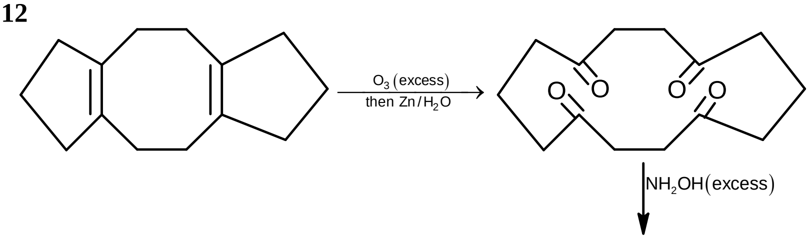
Sol. **13**

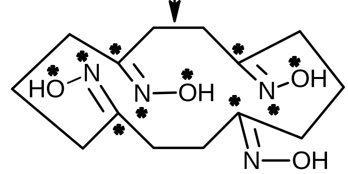


Q.18 In the reaction given below, the total number of atoms having sp^2 hybridization in the major product **P** is ____.



Sol. **12**





✱ represents sp^2 – hybrid atom

Q.19 The total number of possible isomers for $[Pt(NH_3)_4Cl_2]Br_2$ is ____.

Sol. **6**

Possible structural isomers are:

$[Pt(NH_3)_4Cl_2]Br_2$	$[Pt(NH_3)_4(Cl)(Br)]Cl.Br$	$[Pt(NH_3)_4Br_2]Cl_2$
Cis (achiral), trans (achiral)	Cis (achiral), trans (achiral)	Cis (achiral), trans (achiral)

PART III: MATHEMATICS

SECTION 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- Four each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

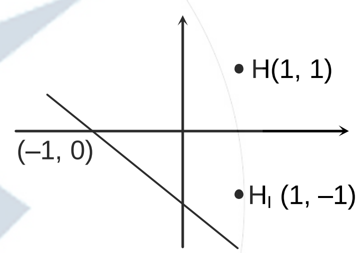
Full Mark	:	+3	If only the correct option is chosen;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered)
Negative Marks	:	−1	In all other cases.

*1. Consider a triangle Δ whose two sides lie on the x-axis and the line $x + y + 1 = 0$. If the orthocentre of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is

(A) $x^2 + y^2 - 3x + y = 0$
(B) $x^2 + y^2 + x + 3y = 0$
(C) $x^2 + y^2 + 2y - 1 = 0$
(D) $x^2 + y^2 + x + y = 0$

Sol. **B**

Image of orthocentre about any side of the triangle lies on its circumcircle.
We can observe that
$x^2 + y^2 + x + 3y = 0$ is satisfied by (–1, 0) and (1, –1) both
So, option (B) is the correct choice



2. The area of the region $\left\{(x, y)\,;\quad 0 \le x \le \frac{9}{4},\quad 0 \le y \le 1,\quad x \ge 3y,\quad x + y \ge 2\right\}$ is

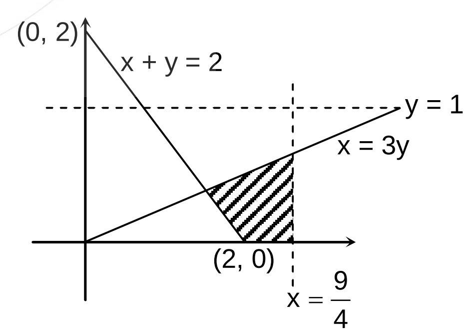
(A) $\frac{11}{32}$ (B) $\frac{35}{96}$ (C) $\frac{37}{96}$ (D) $\frac{13}{32}$

Sol. **A**

Area of shaded region

$= \frac{1}{2}\left(\frac{2}{3} + \frac{3}{4}\right)\frac{1}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2}$

$= \frac{11}{32}$



3. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1 and let S_1 denote the set of these chose elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.
Let $G_2 = F_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.
Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Sol. **A**

	S_1	$F_2 = F_1 \cup S_1$	S_2		$G_1 \cup S_2 = G_2$	S_3
(i)	{1, 2}	{1, 2, 3, 4)	{1, x}		{1, 2, 3, 4, 5}	{1, 2}
(ii)	{2, 3}	{1, 2, 3, 4}	{1, x}		{1, 2, 3, 4, 5}	{2, 3}
			{x, y}	(where x and y	or {2, 3, 4,5}	
(iii)	{1, 3}	{1, 3, 4}	{1, x}	are other than 1)	{1, 2, 3, 4, 5}	{1, 3}

(i) $P_1 = \frac{{}^2C_2}{{}^3C_2} \cdot \frac{{}^3C_1}{{}^4C_2} \cdot \frac{{}^2C_1}{{}^5C_2} = \frac{1}{60}$

(ii) $P_2 = \frac{1}{3}\left(\frac{{}^3C_1}{{}^4C_2} \times \frac{{}^2C_2}{{}^5C_2} + \frac{{}^3C_2}{{}^4C_2} \times \frac{{}^2C_2}{{}^4C_2}\right) = \frac{2}{45}$

(iii) $P_3 = \frac{1}{{}^3C_1} \times \frac{{}^2C_1}{{}^3C_2} \times \frac{{}^2C_2}{{}^5C_2} = \frac{1}{45}$

Conditional probability = $\frac{P_1}{P_1 + P_2 + P_3} = \frac{1}{5}$

*4. Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \ldots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1}e^{i\theta_k}$ for k = 2, 3, ..., 10, where $i = \sqrt{-1}$. Consider the statements P and Q given below:

$$P : |z_2 - z_1| + |z_3 - z_2| + \ldots + |z_{10} - z_9| + |z_1 - z_{10}| \le 2\pi$$
$$Q : |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \le 4\pi$$

Then

(A) P is **TRUE** and Q is **FALSE** (B) Q is **TRUE** and P is **FALSE**
(C) both P and Q are **TRUE** (D) both P and Q are **FALSE**

Sol. **C**

$\because\ z_1 = e^{i\theta_1}$

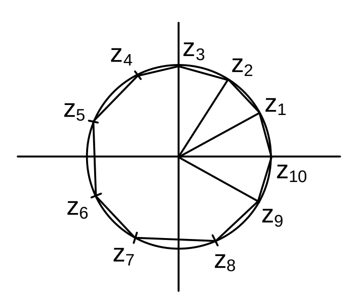
So, $z_2 = e^{i(\theta_1+\theta_2)}$

$z_3 = e^{i(\theta_1+\theta_2+\theta_3)}$

$\vdots$

$z_{10} = e^{i(\theta_1+\theta_2+\ldots+\theta_{10})} = e^{i(2\pi)}$

Sum of all the chord length < Circumference



So, $\sum|z_2 - z_1| \le 2\pi$

Also, $2|z_2 - z_1| \ge |z_2^2 - z_1^2|$

Hence, $2\left(|z_2 - z_1| + \ldots.. + |z_{10} - z_1|\right) \le 2(2\pi) = 4\pi$

So for, we have $P \le 2\pi$ and $Q \le 4\pi$

SECTION 2

- This section contains **THREE (03)** question stems
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+2	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots.., 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

5. The value of $\frac{625}{4}p_1$ is ______

Sol. **76.25**

$P_1 = 1 -$ (Probability that 3 chosen numbers are less than 81)

$= 1 - \left(\frac{80}{100}\right)^3 = 1 - \frac{64}{125} = \frac{61}{125}$

So, $\frac{625}{4}$

$P_1 = \frac{625}{4} \times \frac{61}{125} = 76.25$

6. The value of $\frac{125}{4}p_2$ is ______

Sol. **24.5**

$P_2 = 1 -$ (Probability that 3 chosen numbers are greater than 40)

$= 1 - \left(\frac{60}{100}\right)^3 = 1 - \left(\frac{3}{5}\right)^3 = \frac{98}{125}$

So, $\frac{125}{4}$

$$P_2 = \frac{125}{4} \times \frac{98}{125} = 24.5$$

Question Stem for Question Nos. 7 and 8

Question Stem

Let α, β and γ be real numbers such that the system of linear equation

$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let |M| represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the **square** of the distance of the point (0, 1, 0) from the plane P.

7. The value of |M| is _____

8. The value of D is _____

Sol. (7. to 8.)

$x + 2y + 3z = \alpha$ ..... (1)

$4x + 5y + 6z = \beta$ ..... (2)

$7x + 8y + 9z = \gamma - 1$ ..... (3)

Equation (1) + (3) – (2) = 0. Equation (2) provides

$\alpha + \gamma - 1 - 2\beta = 0$

7. **1**

$$|M| = \begin{vmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} (C_1 \to C_1 + C_3) = \begin{vmatrix} \alpha+\gamma & 2 & \gamma \\ \beta & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} (R_1 \to R_1 - 2R_2)$$

$$\begin{vmatrix} \alpha+\gamma-2\beta & 0 & \gamma \\ \beta & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & \gamma \\ \beta & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

8. **1.5**

Plane P is x – 2y + z – 1 = 0

$$D = \left(\frac{|-2-1|}{\sqrt{1+4+1}}\right)^2 = 1.5$$

Question Stem for Question Nos. 9 and 10

Question Stem

Consider the lines L_1 and L_2 defined by

$$L_1 : x\sqrt{2} + y - 1 = 0 \text{ and } L_2 : x\sqrt{2} - y + 1 = 0$$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line y = 2x + 1 meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R′ and S′. Let D be the **square** of the distance between R′ and S′

*9. The value of λ^2 is _____

Sol. **9**

Locus $C = \left|\frac{(x\sqrt{2}+y-1)(x\sqrt{2}-y+1)}{\sqrt{3}}\right| = \lambda^2$

$2x^2 - (y-1)^2 = \pm 3\lambda^2$ for intersection with y = 2x + 1

$2x^2 - (2x)^2 = \pm 3\lambda^2$

$-2x^2 = -3\lambda^2$ (taking –ve sign)

$x = \pm\sqrt{\frac{3}{2}}\lambda$

Distance between R and S = $2\left|\sqrt{\frac{3}{2}}\lambda\right|\sec\theta$ (tan θ is slope of line)

$= \sqrt{6}|\lambda|\sqrt{5}$

So, $\sqrt{30}|\lambda| = \sqrt{270}$ ($\lambda = \pm 3$)

$\lambda^2 = 9$

*10. The value of D is _____

Sol. **77.14**

Equation of perpendicular bisector y = $-\frac{1}{2}x + 1$

For point of intersection $2x^2 - \frac{1}{4}x^2 = \pm 3\lambda^2$

$x = \pm\sqrt{\frac{12}{7}}\lambda$ (taking +ve sign)

Distance = $\left|2\cdot\sqrt{\frac{12}{7}}\cdot 3\cdot\sec\theta\right| = 2\cdot\sqrt{\frac{12}{7}}\cdot 3\cdot\sqrt{\frac{5}{2}} = 3\cdot\sqrt{\frac{60}{7}}$

$D = \frac{9\times 60}{7} = 77.14$

SECTION 3

- This section contains **SIX (06)** question.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Mark	:	+4	If only (all) the correct option(s) is(are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial Marks	:	+2	If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If unanswered;
Negative Marks	:	–2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get –2 marks.

11. For any 3×3 matrix M, let |M| denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a non-singular matrix of order 3×3, then which of the following statements is(are) **TRUE**?

(A) F = PEP and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\left|EQ + PFQ^{-1}\right| = |EQ| + \left|PFQ^{-1}\right|$

(C) $\left|(EF)^3\right| > |EF|^2$

(D) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

Sol. **A, B, D**

Let A = [C_1 C_2 C_3], $\quad B = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$

AP = [C_1 C_3 C_2] and $\quad PB = \begin{bmatrix} R_1 \\ R_3 \\ R_2 \end{bmatrix}$

and $P^2 = I$

$$P(EP) = P\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 3 \\ 8 & 18 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix} = F$$

$$|E| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{vmatrix} (R_3 \to R_3 - 3R_2 - 2R_1) = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow |F| = 0$

$|EQ + PFQ^{-1}| = |EQ + PPEPQ^{-1}| = |EQ + EPQ^{-1}| = |E||Q + PQ^{-1}| = 0$

$|EQ| = |E|\,|Q| = 0$, $|PFQ^{-1}| = |P||F||Q^{-1}| = 0$

(D) $P^{-1}EP + F = PEP + F = 2F$ (as $P^{-1} = P$)

$E + P^{-1}FP = E + P^{-1}PEPP = 2E$ (trace (E) = trace (F))

12. Let $f : R \to R$ be defined by $f(x) = \dfrac{x^2 - 3x - 6}{x^2 + 2x + 4}$. Then which of the following statements is(are) **TRUE**?

(A) f is decreasing in the interval (–2, –1)

(B) f is increasing in the interval (1, 2)

(C) f is onto

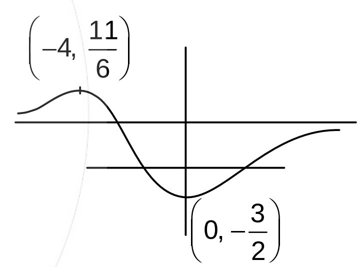
(D) Range of f is $\left[-\dfrac{3}{2}, 2\right]$

Sol. **A, B**

$f'(x) = \dfrac{5x(x+4)}{(x^2 + 2x + 4)^2}$

$f(0) = -\dfrac{3}{2}$ (point of local minima)

$f(-4) = \dfrac{11}{6}$ (point of local maxima)



13. Let E, F and G be three events having probabilities

$P(E) = \dfrac{1}{8}$, $P(F) = \dfrac{1}{6}$ and $P(G) = \dfrac{1}{4}$, and let $P(E \cap F \cap G) = \dfrac{1}{10}$.

For any event H, if H^C denotes its complement, then which of the following statements is(are) **TRUE**?

(A) $P(E \cap F \cap G^C) \le \dfrac{1}{40}$

(B) $P(E^C \cap F \cap G) \le \dfrac{1}{15}$

(C) $P(E \cup F \cup G) \le \dfrac{13}{24}$

(D) $P(E^C \cap F^C \cap G^C) \le \dfrac{5}{12}$

Sol. **A, B, C**

$P(E) = \dfrac{1}{8}$, $P(F) = \dfrac{1}{6}$, $P(G) = \dfrac{1}{4}$, $P(E \cap F \cap G) = \dfrac{1}{10}$

(A) $P(E \cap F \cap G^C) = P(E \cap F) - P(E \cap F \cap G)$

$\le P(E) - P(E \cap F \cap G)$

$\le \dfrac{1}{8} - \dfrac{1}{10}$

$\leq \frac{5-4}{40}$

$\leq \frac{1}{40}$

(B) $P\left(E^C \cap F \cap G\right) = P(F \cap G) - P(E \cap F \cap G)$

$\leq P(F) - P(E \cap F \cap G)$

$\leq \frac{1}{6} - \frac{1}{10}$

$\leq \frac{10-6}{60}$

$\leq \frac{4}{60}$

$\leq \frac{1}{15}$

(C) $P(E \cup F \cup G) \leq P(E) + P(E) + P(G)$

$\leq \frac{1}{8} + \frac{1}{6} + \frac{1}{4}$

$\leq \frac{13}{24}$

(D) $P\left(E^C \cap F^C \cap G^C\right) = 1 - P(E \cup F \cup G)$

$\geq 1 - \frac{13}{24}$

$\geq \frac{11}{24} > \frac{10}{24} > \frac{5}{12}$

14. For any 3×3 matrix M, let |M| denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that (I – EF) is invertible. If G = $(I - EF)^{-1}$, then which of the following statements is(are) **TRUE**?

(A) |FE| = |I – FE| |FGE|
(B) (I – FE)(I + FGE) = I
(C) EFG = GEF
(D) (I – FE)(I – FGE) = I

Sol. **A, B, C**

G(I – EF) = (I – EF)G = I

⇒ G – GEF = G – EFG = I ..... (1)

(A) |FE| = |I – FE| |FGE| = |FGE – FE FGE|

= |FGE – F(G – I)E| = |FGE – FGE + FE| = |FE|

(B) (I – FE)(I + FGE) = I + FGE – FE – FEFGH

= I + FGE – FE – F(G – I)E = I + FGE – FE – FGE + FE = I

(C) From (I) it is true

(D) (I – FE)(I – FGE) = I – FGE – FE + FEFGE

= I – FGE – FE + F(G – I)E = I – FGE – FE + FGE – FE

= I – 2FE

15. For any positive integer n, let $S_n : (0, \infty) \to R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right),$$

where for any $x \in R$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statements is(are) **TRUE**?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim_{n\to\infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

(D) $\tan(S_n(x)) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

Sol. **A, B**

$$S_n = \sum_{k=1}^{n} \cot^{-1}\left\{\frac{1+k(k+1)x^2}{x}\right\} ; \sum_{k=1}^{n} \cot^{-1}\left\{\frac{k(k+1)x^2+1}{(k+1)x-kx}\right\}$$

$$S_n = \sum_{k=1}^{n} \cot^{-1}(kx) - \cot^{-1}(k+1)x$$

$$t_1 = \cot^{-1}(x) - \cot^{-1}(2x)$$

$$t_2 = \cot^{-1}(2x) - \cot^{-1}(3x)$$

$$t_3 = \cot^{-1}(3x) - \cot^{-1}(4x)$$

$\vdots$

$$t_n = \cot^{-1}(nx) - \cot^{-1}((n+1)x)$$

$$S_n = \cot^{-1}(x) - \cot^{-1}((n+1)x)$$

$$\Rightarrow S_n = \cot^{-1}\left(\frac{(n+1)x^2+1}{nx}\right)$$

$$S_{10} = \cot^{-1}\left(\frac{11x^2+1}{10x}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{11x^2+1}{10x}\right)$$

(B) $\lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty} \cot\left(\cot^{-1}\left(\frac{(n+1)x^2+1}{nx}\right)\right) = \lim_{n\to\infty} \frac{nx^2+x^2+1}{nx} = x$

(C) $S_3(x) = \cot^{-1}\left(\frac{4x^2+1}{3x}\right) = \frac{\pi}{4} \Rightarrow \frac{1+4x^2}{3x} = 1$

$\Rightarrow 4x^2 - 3x + 1 = 0$ have imaginary roots

(D) $\tan(S_n(x)) = \tan\left(\cot^{-1}\left(\frac{1+(n+1)x^2}{nx}\right)\right) = \frac{nx}{1+(n+1)x^2} = \dfrac{1}{\frac{1}{nx} + \frac{(n+1)x}{n}}$

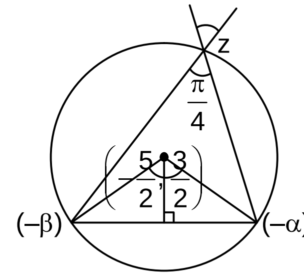
*16. For any complex number $w = c + id$, let $\arg(w) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, then ordered pair (x, y) lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$. Then which of the following statements is(are) TRUE?

(A) $\alpha = -1$ (B) $\alpha\beta = 4$ (C) $\alpha\beta = -4$ (D) $\beta = 4$

Sol. **B, D**

w = c + id, arg(w) ∈ (–π, π]

$\arg\left(\frac{z+\alpha}{z+\beta}\right)=\frac{\pi}{4}$; $z_0=-\frac{5}{2}+\frac{3}{2}i$



$z_0+\alpha=(z_0+\beta)i$; $z_0+\alpha=z_0 i+\beta i$; $z_0(1-i)=\beta i-\alpha$ (by Rotation)

$-\frac{5}{2}+\frac{3}{2}i+\alpha=\left(-\frac{5}{2}+\frac{3}{2}i\right)i+\beta i$

$-\frac{5}{2}+\frac{3}{2}i+\alpha=-\frac{5}{2}i-\frac{3}{2}+\beta i$

$-\frac{5}{2}+\frac{3}{2}+\frac{3}{2}i+\frac{5}{2}i+\alpha=\beta i$; $-1+4i=\beta i-\alpha$ (As α, β are real number)

⇒ –α = –1, α = 1 ; β = 4

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designed to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

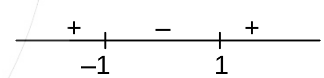
  Full Mark : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

*17. For x ∈ R, the number of real roots of the equation $3x^2-4|x^2-1|+x-1=0$ is ______

Sol. **4**

$3x^2-4|x^2-1|+x-1=0$



Case-I: |x| ≥ 1

$3x^2-4x^2+4+x-1=0$

$-x^2+x+3=0$

$x^2-x-3=0$

$x=\frac{1\pm\sqrt{1+12}}{2}=\frac{1\pm\sqrt{13}}{2}=\frac{1+\sqrt{13}}{2},\frac{1-\sqrt{13}}{2}$

|x| < 1

$3x^2+4x^2-4+x-1=0$

$7x^2+x-5=0$

$x=\frac{-1\pm\sqrt{1+20\times 7}}{14}=\frac{-1\pm\sqrt{141}}{14}$

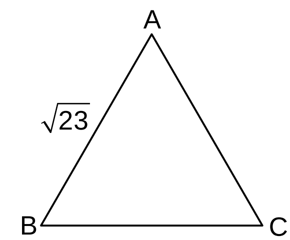
So, number of real roots = 4

*18. In a triangle ABC, let $AB=\sqrt{23}$, BC = 3 and CA = 4. Then the value of $\frac{\cot A+\cot C}{\cot B}$ is ______

Sol. **2**

$AB = \sqrt{23} = C$
BC = 3 = a
CA = 4 = b

$$\frac{\cot A + \cot C}{\cot B}; \frac{\frac{\cos A}{\sin A} + \frac{\cos C}{\sin C}}{\frac{\cos B}{\sin B}}$$



$$= \frac{\cos A \sin C + \cos C \sin A}{\sin A \cdot \sin C \cdot \frac{\cos B}{\sin B}} = \frac{\sin(A+C)\cdot \sin B}{\sin A \cdot \sin C \cdot \cos B} = \frac{\sin B \cdot \sin B}{\sin A \cdot \sin C \cdot \cos B}$$

$$= \frac{b^2}{ac \cdot \frac{(a^2+c^2-b^2)}{2ac}} = \frac{2b^2}{a^2+c^2-b^2} = \frac{2\times 16}{9+23-16} = \frac{2\times 16}{32-16} = \frac{2\times 16}{16} = 2$$

19. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u}\cdot\vec{w} = 1$, $\vec{v}\cdot\vec{w} = 1$, $\vec{w}\cdot\vec{w} = 4$.
If the volume of the parallelepiped, whose adjacent sides are represented by the vectors $\vec{u}, \vec{v}$ and $\vec{w}$ is $\sqrt{2}$, then the value of $|3\vec{u}+5\vec{v}|$ is _____

Sol. **7**

$$[\vec{u}\ \vec{v}\ \vec{w}]^2 = \begin{vmatrix} \vec{u}\cdot\vec{u} & \vec{u}\cdot\vec{v} & \vec{u}\cdot\vec{w} \\ \vec{v}\cdot\vec{u} & \vec{v}\cdot\vec{v} & \vec{v}\cdot\vec{w} \\ \vec{w}\cdot\vec{u} & \vec{w}\cdot\vec{v} & \vec{w}\cdot\vec{w} \end{vmatrix} = \begin{vmatrix} 1 & \vec{u}\cdot\vec{v} & 1 \\ \vec{v}\cdot\vec{u} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$$

$$= 1(3) - \vec{u}\cdot\vec{v}(4\vec{u}\cdot\vec{v}-1) + 1(\vec{u}\cdot\vec{v}-1) = 2$$

$$= 3 - 4(\vec{u}\cdot\vec{v})^2 + \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{v} - 1 = 2 = -4(\vec{u}\cdot\vec{v})^2 + 2\vec{u}\cdot\vec{v} = 0$$

$$(\vec{u}\cdot\vec{v})(2-4(\vec{u}\cdot\vec{v})) = 0;\ \vec{u}\cdot\vec{v} = 0,\ \vec{u}\cdot\vec{v} = \frac{1}{2}$$

$$|3\vec{u}+5\vec{v}|^2 = 9|\vec{u}|^2 + 30\vec{u}\cdot\vec{v} + 25|\vec{v}|^2 = 9 + 15 + 25 = 49$$

$$\therefore |3\vec{u}+5\vec{v}| = 7$$

END OF THE QUESTION PAPER